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A Helicon Solid-State Plasma Experiment for the Advanced Laboratory

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This paper reports an advanced undergraduate laboratory experiment concerning the helicon solid-state plasma wave. Student results are reported. The experiment is performed as an inexpensive dip in a liquid helium storage vessel. The experiment is made possible by the design of a small, superconducting magnet. The magnet is capable of attaining 18 kG and costs about \$25. The whole experiment can be built by students in a week's time. When the experiment is completed, the student not only understands helicon propagation but also understands the use of the helicon to measure magnetoresistance and Hall coefficients in pure metals.

INTRODUCTION

There has been considerable research interest in solid-state plasmas in recent years. Little of this work has been used in advanced undergraduate laboratories. This paper reports the development and use of an advanced laboratory experiment concerning the solid-state plasma wave called the helicon propagating in pure metals. The helicon wave is of interest not only because of its inherent wave properties, but also because the wave can be used to measure parameters of the host solid. This paper reports the results of a student-built and student-performed experiment using dip techniques in a liquid helium storage Dewar. The experiment is made possible by the production of a miniature superconducting magnet that surrounds the experiment in the storage vessel.[1]

The theory of the helicon has been amply covered in review article by Maxfield.[2] That article also serves as a résumé of research work in the field.

The student experiment reported here uses flat plate samples of pure indium metal immersed in a large dc magnetic field at liquid helium temperatures. Inside pure metals in large dc magnetic fields, an electromagnetic wave motion called the helicon is possible. This is the same wave motion commonly known as the whistler in ionospheric physics. For propagation in an infinite sample with no resistivity, in the presence of a static magnetic field in the z direction, the dispersion relation for the wave is

$$\omega = (RB_0/\mu_0)kk_z, \qquad (1)$$

where ω is the angular frequency, k is the magnitude of the wave vector with z component k_z, R is the Hall coefficient for the metal, B_0 is the magnitude of the (uniform) external dc magnetic field, and the equation is in mks units. The wave is circularly polarized with the direction of rotation being that of the cyclotron resonance rotation of the majority charge carriers; the wave is transverse and propagates (essentially) down the magnetic field.

Standing helicon waves can be set up inside a metal sample. For a flat plate of indium whose thickness is much less than its lateral dimensions, the following numbers apply: for a 1-mm-thick sample, perpendicular to a 10-kG magnetic field, one-half wavelength fits in the sample at a frequency of 600 Hz. There is a series of such standing wave resonances—each with an integral number of half-wavelengths fitting into the sample. The frequency and Q of the resonances depend not only on sample dimensions and the restoring force RB_0, but also on the damping, which in this case is due to the sample resistivity ρ. For standing waves in a slab, placed perpendicular to $\mathbf{B}_0$, one finds

$$f_{\text{res}} = (N^2/8\times10^{-7}d^2)(R^2B_0^2+\rho^2)^{1/2} \qquad (2a)$$

$$Q = f_{\text{res}}/\Delta f = \tfrac{1}{2}[1+(R^2B_0^2/\rho^2)]^{1/2}. \qquad (2b)$$

One can obtain the resistivity ρ from the full width of the resonance at half-power, Δf:

$$\rho = 8\times10^{-7}d^2\Delta f. \qquad (2c)$$

In these equations, d is the thickness of the slab, f_{res} is the resonant frequency for the mode, N is an integer labelling the number of half-waves fitting in the sample, and ρ is the resistivity in the magnetic field B_0. Thus the properties of the standing-wave resonances measure two parameters of great

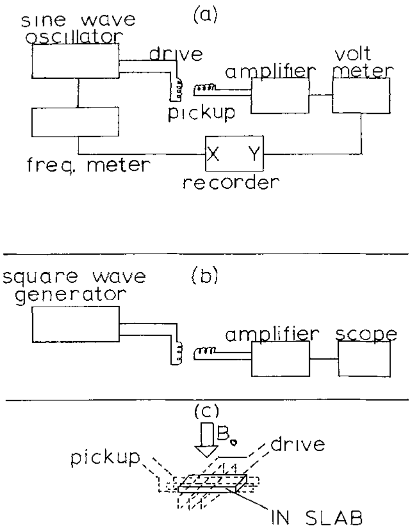

FIG. 1. Block diagram of instrumentation. (a) Instrumentation for continuous-wave swept-frequency helicon experiments is shown. The equipment plots resonant helicon response as a function of frequency. The drive and pickup coils surround the metal slab sample and are dipped into a liquid helium storage vessel along with the small superconducting magnet. The amplifier is differential; an oscilloscope plug-in serves well. (b) Instrumentation for pulsed-decay helicon experiments. The square wave is of very low frequency. The oscilloscope plots response as a function of time and observes the ringing of the helicon standing waves in the sample. (c) Geometry of the metal slab and enclosing coils. The coils are perpendicular to each other and to the large dc magnetic field from the superconducting magnet.

interest in solid state—the Hall coefficient R (related to the densities of the various charge carriers in the metal) and the magnetoresistance $\rho(B_0)$ (related to the shape of the Fermi surface). The experiment demonstrates the properties of the helicon wave and then measures the Hall coefficient and resistivity as functions of magnetic field and temperature.

EXPERIMENT

Figure 1 is a block diagram showing the instrumentation used to perform helicon experiments in the two most frequently used methods. The two methods of observing standing helicon waves are the continuous-wave, swept-frequency method, and the pulsed-decay method. Figure 1(a) shows the instrumentation used for the continuous-wave swept-frequency method. As shown, the instrumentation plots the response of the helicon as a function of frequency for fixed magnetic field.[3] The experiment is performed by slowly ramping the frequency of a small ac magnetic drive field at the sample. This drive field is in addition to the large external dc magnetic field. It is possible to do without the frequency meter (frequency to dc converter), the $X-Y$ recorder, and the ac voltmeter.[4] Figure 1(b) shows the instrumentation used to perform so-called decay helicon experiments. In this method, the helicon is excited by a small step-function magnetic drive field and the sample responds by ringing at all its resonant frequencies. The square-wave generator can be replaced by a battery and a switch; the ringing is observed by plotting response vs time on an oscilloscope.

Figure 1(c) shows the configuration of the sample, drive coil, and pickup coil. The large dc magnetic field B_0 is provided by a superconducting dip magnet capable of providing 18 kG. The design of this magnet is reported elsewhere.[1] Briefly it has an o.d. of $\frac{3}{4}$ in., an i.d. of $\frac{5}{8}$ in., a length of 2 in., and is wound out of $Nb_{52}Ti_{48}$ superconducting wire. The magnet is corrected to give 0.1% uniformity over all the full bore radially and 0.6 in. axially. The total cost of the magnet was about $25, and all the work to build the magnet was performed by students in 2 days. No professional machining was required; the magnet reaches 18 kG before going normal (at liquid helium temperature).

The standing helicon is a transverse wave propagating parallel to $\mathbf{B}_0$. Thus the drive and pickup coils are arranged perpendicular to $\mathbf{B}_0$. The drive and pickup can be perpendicular to each

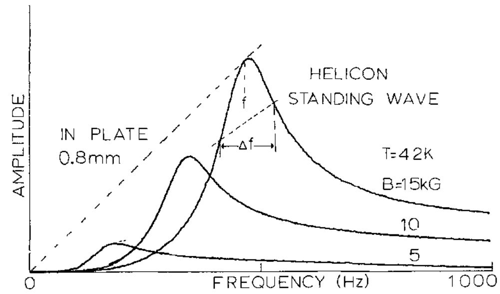

FIG. 2. The resonant response of the helicon is plotted against frequency for three magnetic fields. The dashed lines demonstrate the method of data reduction. All three resonances shown have one-half wavelength of the helicon fitting inside the In slab. The resonances can be used to measure the sample resistivity and Hall coefficient as functions of magnetic field.

other (minimizing direct pickup) since the helicon is circularly polarized. The drive field is produced by a current of a few tens of millamperes through the drive coil.

In the continuous-wave, swept-frequency method, the output voltage from the pickup coil is proportional not only to the helicon amplitude in the sample, but also to the drive frequency (due to the time derivative in Faraday's law). The extra frequency factor complicates the analysis, but to remove this extra factor experimentally is more trouble than it is worth. No such problem exists in the decay method.

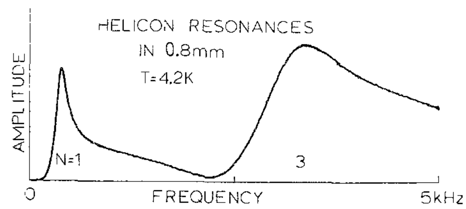

FIG. 3. Two of the series of helicon resonances are shown as a function of frequency for a magnetic field of 15 kG. The lower resonance has one-half wavelength fitting in the sample. Even harmonics do not appear due to flux cancellation within the pickup coil.

RESULTS

The results reported here were obtained by an undergraduate student. The results are typical of what students in the laboratory obtain. The sample was a 0.8 mm ($\frac{1}{32}$ in.) thick slab of Cominco 99.9999% pure In. This purity is necessary to perform the temperature-variation experiment; otherwise, 99.999% is adequate. The sample was 10×10 mm and was made by squeezing the In between $\frac{3}{8}$-in. brass plates separated by $\frac{1}{32}$-in. spacers. The coils for the experiment were wound by students and were formed of #40 Formvar covered Cu wire. The coils were cast in Epoxy during winding and, when hard, were free-standing. Heavier leads connect the coils to the leads leading from the experiment to room temperature. The pickup coil was wound on a $\frac{1}{32}$-in. by 10-mm former covered by Teflon sheet; it had about 800 turns and was about 7 mm long. The drive coil was wound on a former large enough so that the pickup coil would slip inside. The drive coil had about 400 turns and was about 7 mm long. The coils as well as the superconducting magnet were wound on a simple lathe.

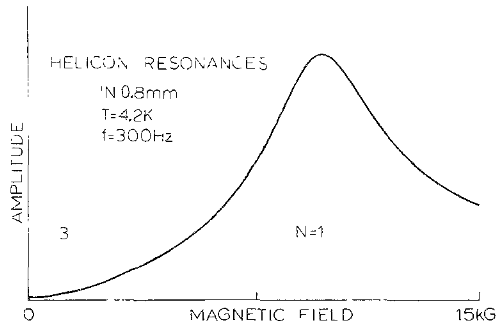

FIG. 4. Helicon resonances as a function of ramped magnetic field. The drive frequency was 300 Hz. The higher order resonances ($N=3, 5, \cdots$) are present but very small.

It is useful to glue the drive coil into a plastic or wood boat which will freeze tightly into the magnet. The pickup coil with the sample inside is inserted into the drive coil, perpendicularized by observing ac coupling of the coils, and frozen in with Vaseline and liquid nitrogen. Such a system minimizes the problem of mechanical resonances.

Figures 2–6 give typical student results. Figure 2 shows the resonance with one-half wave fitting in the sample for several magnetic fields. As the field increases, the resonances move to higher frequency ($f_{\text{res}}\approx B_0$) and sharpen up ($Q\approx RB_0/\rho$). The amplitude is proportional to the ratio of restoring force to resistive loss (RB_0/ρ) as well as to frequency. Because of the extra frequency factor, the resonant frequency f_{res} is determined by finding the tangency point between the resonance curve and a straight line through the origin. The full width at half-power, Δf, is given by the intersections of the resonance curve and a straight line from the origin through a point $1/\sqrt{2}$ down at f_{res}. The dashed lines in Fig. 2 demonstrate this analysis.

Figure 3 shows two of the series of resonances for a given field. The two resonances shown have one and three half-wavelengths inside the sample ($N=1$ and 3). All even-numbered modes are not observed since the flux due to such modes cancels inside the pickup coil.

Figure 4 shows the helicon resonance for a fixed frequency and ramped magnetic field. This is an easy method of observation, but the analysis necessary to get information on the Hall coefficient and magnetoresistivity is very complicated.

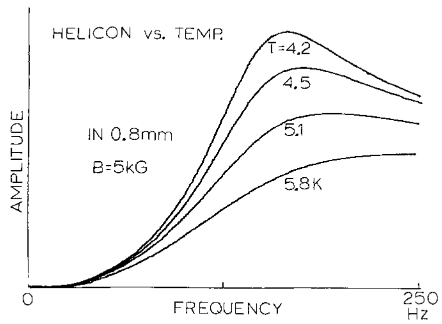

FIG. 5. Helicon resonances at 5 kG for several temperatures. Due to the change in sample resistivity with temperature, the resonances change shape. The resonances can be used to measure the resistivity and the Hall coefficient as a function of temperature.

Figure 5 shows the effect on the $N=1$ helicon resonance at 5 kG as the temperature of the sample is raised above 4.2 K. The temperature is varied by lifting the magnet and experiment out of the liquid helium bath. The In is so pure that the resistivity of the sample changes rapidly with temperature; the Hall coefficient changes slightly. The result is that the position of the resonance curve shifts slightly in position and its Q decreases dramatically.

Figure 6 shows the results of pulsed-decay method helicon observations. The three magnetic fields are those used in Fig. 2 also. One can see the effect of the $N=1$ resonance, and also, at the very start of the trace, the effects of $N=3$ and 5 resonances. Because the higher N resonances have

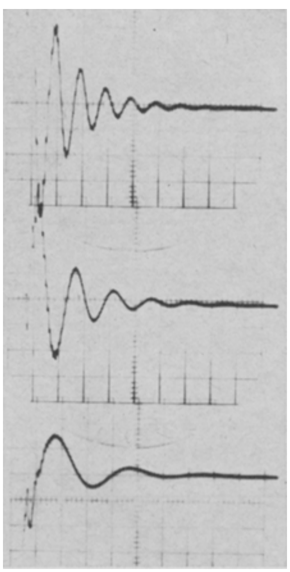

FIG. 6. Helicon pulse-decay experimental results. The response of the helicon standing waves to a small step-function drive field is displayed against time. The three magnetic fields are those of Fig. 1 (5, 10, and 15 kG). The sample rings at several frequencies. The most prominent oscillation is the $N=1$ mode (one-half wavelength fitting the sample); at the start of the trace, oscillations due to $N=3$ and 5 can be seen. The time scale is 2 msec/div.

higher frequency, the oscillations from the higher N resonances die out sooner in time. The amplitudes go essentially as $\exp(-\omega_{res}t/2Q)$.

Curves such as those in Fig. 2 can be reduced to determine values of the Hall coefficient and magnetoresistivity of In as functions of magnetic field. The results show that, as a function of field from 1 to 18 kG, the Hall coefficient is essentially constant at $+16\times10^{-11}$ mks units. This corresponds to about 0.95 holes/ atom in the metal.[5] The resistivity varies by a factor of almost 2 over that range of fields at 4.2 K; the resistivity curve has a knee at about 4–7 kG with this pure In.[6] Over the range of temperatures 4.2–6 K, the Hall coefficient changes by a few percent; the resistivity by about 50%.

After obtaining these experimental results, the student understands the principles involved in the propagation of plasma waves in solids. He must also explain: (a) the meaning of holes, (b) the number of holes per atom in In, (c) the importance of Fermi-surface shape in conduction properties, and (d) the variation of solid-state parameters with temperature (thermal scattering effects).

CONCLUSION

This paper has presented an advanced laboratory experiment concerning plasma wave propagation in solids. The wave studied was the helicon in pure In metal. The student not only learns how such waves are produced and how and why they propagate, but also uses the wave to study electrical conduction parameters of the solid (the Hall coefficient and magnetoresistance).

The experiment is made feasible by the production of a small, corrected, superconducting magnet. The magnet with the experiment inside is dipped into a storage Dewar while the experiment is performed. Such a method costs less than one-half liter of liquid helium per student experiment.

The results reported here were from an experiment performed by an undergraduate student. Students constructed the magnet as well as the whole experimental system. The total time necessary to produce the experiment from scratch is less than one week.

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No. GY 5819 at Dartmouth College. He is an undergraduate physics major at the University of Vermont, Burlington, Vt.

[1] J. R. Merrill, Rev. Sci. Instr. **41,** 25 (1970).

[2] B. W. Maxfield, Amer. J. Phys. **37,** 241 (1969).

[3] Typical instrumentation might include a Hewlett–Packard 200 CD oscillator, a 1A7 Tektronix plug-in amplifier, a Hewlett–Packard frequency meter, a Hewlett–Packard 400E ac voltmeter, and a Moseley X–Y recorder.

[4] R. Bowers, C. Legendy, and F. Rose, Phys. Rev. Letters **7,** 339 (1961).

[5] R. G. Chambers and B. K. Jones, Proc. Roy. Soc. (London) **A270,** 417 (1962).

[6] For an introductory explanation of the Hall coefficient and magnetoresistivity, see J. Olsen, *Electron Transport in Metals* (Wiley–Interscience, Inc., New York, 1962).

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Relations among Systems of Electromagnetic Equations

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The equations of electromagnetism, whether written in the electrostatic, electromagnetic, or symmetric system, whether in rationalized or nonrationalized form, express an invariant set of physical relationships. The same set of letter symbols is employed in each system of equations; these symbols represent related, but different, quantities in the various systems. The relationships among corresponding symbols are given and applied to precise statements about the relation between the oersted and the ampere per meter, the abampere and the ampere, etc.

The increasing use of the International System of Units (SI units), sometimes from preference, sometimes for reasons of institutional policy, has raised questions among some physicists and electrical engineers about the nature of the conversions from old units to new units. Since the SI is intimately related to the use of equations having a standardized rationalization, there is a *conceptual* change associated with the unit change. The relations between the "new" quantities and the "old" quantities need to be set forth explicitly in teaching electromagnetism.

There are four systems of writing the equations of electromagnetism that have had considerable use: the so-called electrostatic, electromagnetic, symmetric or Gaussian, and the (Giorgi) rationalized system. The first three are called, by the International Union of Pure and Applied Physics (IUPAP), "systems of equations with three basic quantities". IUPAP applies the term "system of equations with four basic quantities" to the modern Giorgi system, and also to a fifth variant which is the nonrationalized form most closely related to the Giorgi system.

These various designations for the systems of equations can be misleading; they refer to the philosophies behind the formulation of the equations, rather than describing the equations or systems themselves. The properties of a system of equations are independent of the units and the dimensional-analysis procedures preferred by different scientists. *Quantities* are invariant to changes of units; their *measures* depend on the choice of units. Similarly, the *kind*, or *dimension*, of a quantity is independent of the means of expressing it. For example, the *dimension* of energy can be expressed in terms of the dimensions length, mass, time, and current, as

$$\mathrm{D}(W)=\mathrm{L}^2\mathrm{M}^1\mathrm{T}^{-2}\mathrm{I}^0$$

or in terms of force, length, current, and time:

$$\mathrm{D}(W)=\mathrm{F}^1\mathrm{L}^1\mathrm{I}^0\mathrm{T}^0.$$

The dimension $\mathrm{D}(W)$ can be thought of as an element $[W]$ in a group; the element is invariant to the choice of group generators. These concepts of the invariance of a quantity under changes in our mode of description or measurement form the logical foundation for expressing physical relations as *quantity equations*.[1] The content of a quantity equation is independent of any consideration of unit systems and of choice of base dimensions