Lab #1: Characterising Helicons in Indium

Anonymous
Department of Physics, University of Toronto, Toronto M5S 1A7, Canada
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This experiment studied the conduction properties, the Hall coefficient and resistivity, of a metal with high purity. This was done by letting Helicons propagate through Indium at very low temperatures, so that phonon and electron scattering disappears, and by applying a large magnetic field to minimise the attenuation of the Helicon waves. In order to do this, the resonance frequencies for the Helicons were found for fixed values of magnetic field. I compared my best value for the Hall coefficient at B = 3245 G which is $R_H = (+5.64 \times 10^{-10} \pm 1.128 \times 10^{-11})\mathrm{m}^3/\mathrm{C}$, with Kittel's, which is $R_H = +1.602 \times 10^{-10}\mathrm{m}^3/\mathrm{C}$ [3]. This value is within 1 magnitude of Kittel's $\mathcal{O}(10^{-10})$. The carrier density was also calculated to be n = $1.11 \times 10^{28} \pm 3\%$ compared to the value for metals n $\sim 10^{22}$ quoted in the lab manual [1]. I discovered that the indium foil was a p-type semi-conductor because of the positive sign of R_H and also found the average resistivity of the indium foil $(1.48 \times 10^{-11} \pm 4.41 \times 10^{-13})\Omega{\cdot}\mathrm{m}$. Then, I compared it with Swenson's and White & Woods' published values [2, 5].

I. INTRODUCTION

Metals are known for being good conductors because the electrons flow freely in the metal's electron sea. From earlier physics courses, the electric field inside a conductor is known to be equal to 0. The electromagnetic wave penetration in a metal is given by the skin depth [Eq. 1]

$$\delta = \sqrt{\frac{2\rho}{\omega\mu}} \tag{1}$$

where $\omega = 2\pi f_{res}$ and $\mu = \mu_0$ since Indium is not magnetic. This experiment, introduced by Merrill et al., works because helicons propagate with higher penetration depth with the condition that the cyclotron frequency is much greater than the mean time between electron collisions [1], which is characterised by the temperature. So a much lower temperature would imply a lower kinetic energy– yielding less collisions for phonons and electrons. It was for this reason that liquid nitrogen, and even liquid helium, was used. At liquid helium temperature, the electrons only scatter off the impurities in the metal.

The experiment applied a large magnetic field B via a superconducting magnet to produce a transverse electric field E_y, also known as the Hall field. The Hall coefficient is a quantity that can be studied with Helicons. The Hall coefficient R_H is the ratio of the Hall field E_y and both the current density j_x and the magnetic field B. Using the equations obtained from [4],

$$R_H = \frac{E_y}{j_x B} = \frac{2\mu_0}{N^2\pi B_0} d^2 f_{res} \tag{2}$$

where N is the resonance, which can only be odd integers in our experiment, and B_0 is the static magnetic field. Note that the "current" is actually positive holes moving instead of negative electrons in the case of Indium. As $R_H > 0$ for Indium, it is a p-type semi-conductor. It is an intrinsic semi-conductor doped with acceptors, as opposed to being doped with donors for the case of the n-type semi-conductor.

Since Indium's donor is an electron acceptor, the electrons are attracted towards it– yielding a positive region free of electrons. This positive region is known as a hole. In a p-type semi-conductor, it is the majority carrier; with electrons being the minority. Therefore, the carrier density n is just hole density,

$$n = \frac{1}{R_H e} \tag{3}$$

The magnetoresistance, and more importantly the resistivity ρ, of Indium is another quantity that can be found from the experiment,

$$\rho = \frac{\mu_0}{N^2\pi} d^2 \Delta f \tag{4}$$

where Δf is the resonance width and all the other parameters are the same as in [Eq. 2]. Helicons can be used to study these two quantities when electrical contacts to the sample cannot be made [1].

Following the derivation of Maxfield [7], the angle between the Hall field and the conduction electric field is given by u,

$$u = \frac{R_H B}{\rho} \tag{5}$$

The Helicon wave is made up of complex and real parts. The real part, caused by $u \gg 1$, represents propagation of the wave. Conversely, the imaginary part– in which u is not much greater than unity– represents attenuation of the wave. The propagating Helicon wave is characterised by q_+ for holes,

$$q_+ = \sqrt{\frac{\omega\mu_0}{\rho\mu}} \tag{6}$$

The purpose of this experiment was to find the Hall coefficients and magnetoresistances of an Indium slab by determining the resonance frequencies and resonance widths of the propagating Helicons.

II. APPARATUS

A Helicon probe made of a drive coil, pickup coil, and a third unused probe surrounded an indium foil. The drive coil was labelled by the white and red banana cables, while the pickup coil was labelled by the black and green banana cables. The circuit was arranged such that the lock-in amplifier received the signal from the pickup coil [Fig. 1].

The drive coil was connected to the reference generator HP 33120A (±0.05%), and then connected to the lock-in as well. The voltmeter Wavetech BDM40 (±0.03%) reads the voltage drop from the lock-in amplifier. The lab manual suggested to use an oscilloscope; however, I neglected to use it as the voltmeter sufficed and I wanted to avoid any ground loops.

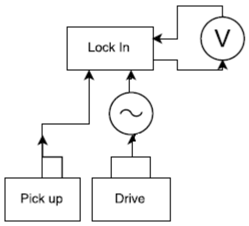

FIG. 1: Drawn block diagram of the circuit

I noted that the amplitude of the reference signal had to be $\geq$ 105mV at 100Hz in order for the signal to remain stable. I noticed that as the frequency of the reference signal increases, so must the amplitude in order to keep the signal stable. I found that 300mV was sufficient for the range of frequencies used [50, 5000]Hz inclusive.

The 99.9999% pure Indium slab and its dimensions are illustrated in [Fig. 2]. Note that the experiment used Indium with such high purity because there is scattering only off impurities at T $<$ 15K.

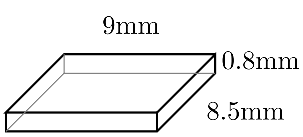

FIG. 2: The diagram of the slab was found in MP 226.

With the resistance of superconducting magnet known $R_{mag} = (9.985 \times 10^{-4} \pm 1.997 \times 10^{-7})\Omega$, the voltage was read across the magnet using a digital multimeter Meterman X38R (±0.1%). Ohm's law was then used to get the current. This gives a more precise reading of the current instead of reading the current off the power supply. Because the magneto resistor in the Helicon probe was broken, the graph and fit from [1] were used to infer the magnetic field [7].

$$B = \left(324 \pm 2 \;\; G \cdot A^{-1}\right) I \tag{7}$$

The resistances of the drive and pickup coils were recorded at room temperature, after they were dipped in liquid nitrogen, and after they were dipped in liquid helium [Table. I].

Temperature (K)	$R_{drive}(\Omega)$	$R_{pickup}(\Omega)$
293	99.54	173.31
77	13.87	23.20
4.2	1.69	2.43

TABLE I: Resistances of the Drive and Pickup Coils at different Temperatures

These values confirmed that the drive and pickup coils were indeed connected and continuous. Then, the two sets of measurements were made. The frequency from the reference signal was first fixed, and the magnetic field was varied. Then, the magnetic field was fixed, and the frequency was allowed to vary.

III. RESULTS

The plots of fixed frequencies and varying magnetic fields are shown below [Fig. 3].

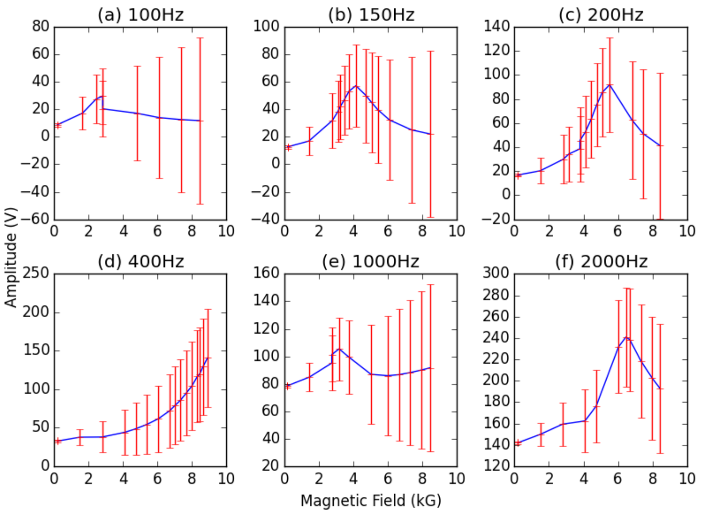

FIG. 3: Varying the magnetic field while fixing frequency. (d) behaves strangely as there is no peak, only exponential growth. This can be explained by the peak being at a higher value of magnetic field, or the fact that 400Hz is not a resonance frequency. The error bars convey the instability of the measurements at high frequencies and high magnetic fields. This was expected as I used only a signal with 300mV amplitude.

Now for the plots for fixed magnetic fields and varying frequency. The two resonances plotted are shown to have one and three half wavelengths. For B = 3244.87 G, the N = 1 and N = 3 resonances were found to be at f_{res} = 136 Hz and f_{res} = 1100 Hz respectively. The resonance widths were also found to be Δf = 60Hz for N = 1, but the N = 3 resonance was not well defined so the resonance width was not clear. See [Fig. 4].

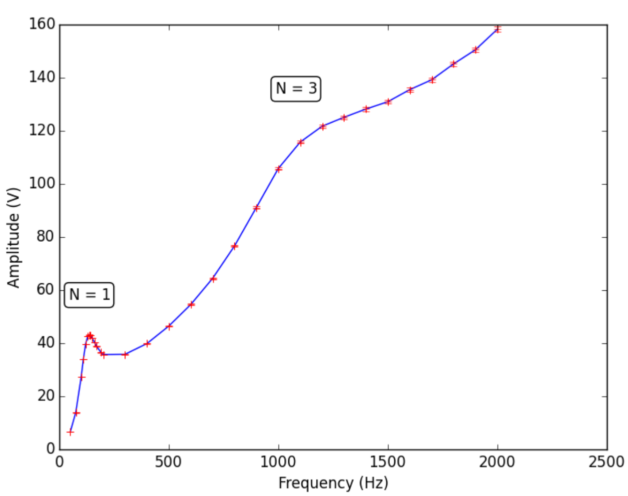

FIG. 4: Varying frequency for B = 3244.87 G

Similarly for B = 5191.79 G shown in [Fig. 5], the N = 1 and N = 3 resonances were found to be at f_{res} = 203 Hz and f_{res} = 1800 Hz respectively. The resonance widths were Δf = 80Hz and Δf = 200Hz respectively.

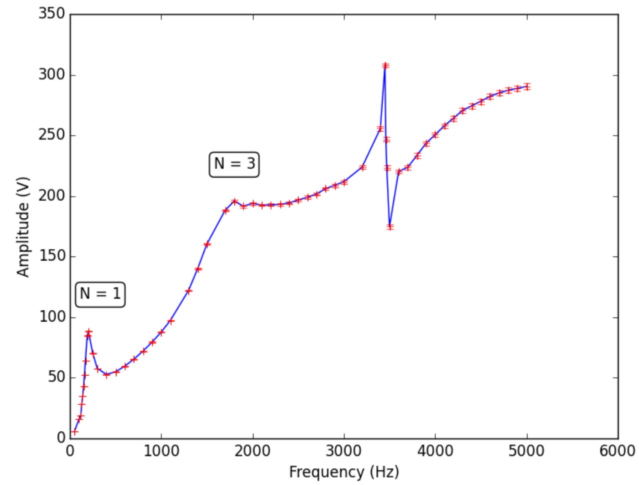

FIG. 5: Varying frequency for B = 5191.79 G. Note that there is a spike around 3450Hz. This might be a higher order frequency or just an outlier.

Lastly for the B = 8436.65 G scenario shown in [Fig. 6], the N = 1 and N = 3 resonances were found to be at f_{res} = 321 Hz and f_{res} = 2730 Hz respectively. The resonance widths were also found to be Δf = 30Hz and Δf = 600Hz respectively for the N = 1 and N = 3 resonances. As one can see, the resonance frequencies and widths are proportional to the magnetic field.

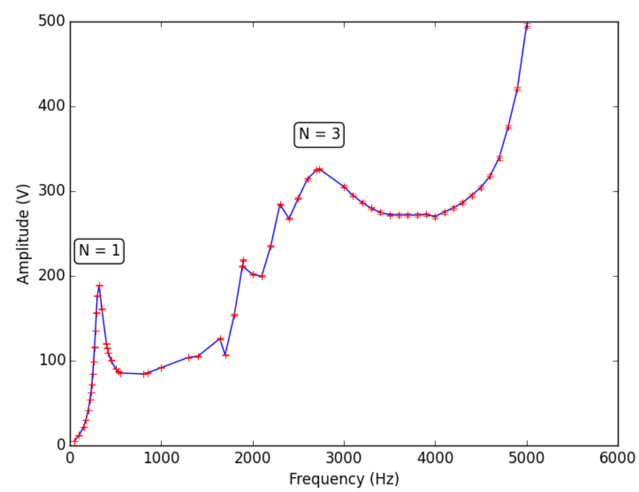

FIG. 6: Varying frequency for B = 8436.65 G

Note that there are some spikes leading to the N = 3 resonance in the B = 8436.65 G case, but this cannot be another resonance (i.e., N = 2) since there is flux cancellation for even harmonics [1]. There are higher order resonances, but the N = 1 resonances are the most well-defined. So I truncated the plots [Fig. 7] at 1000Hz to show only the first resonances for each magnetic field.

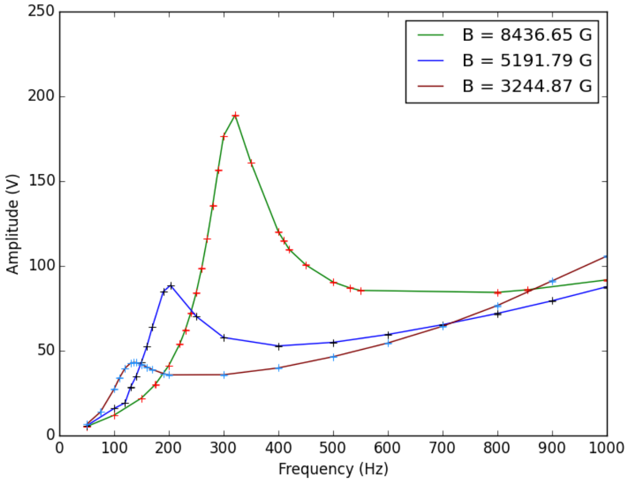

FIG. 7: Truncated Plot for the N = 1 resonances

The error bars were plotted on all of the graphs above; error propagation calculations and justifications can be found in Appendix A. The Hall coefficients R_H were calculated from [Eq. 2], and the resistivitys ρ were calculated from [Eq. 4]. The magnetoresistances were calculated for only the N = 1 resonances, since all of its' resonance widths Δf were well defined. And for consistency, the Hall coefficients were calculated for only the N = 1 resonances as well. The hole densities were computed using [Eq. 3]. And the results, for the N = 1 resonance, were recorded in [Table. II].

B (G)	R_H ($\pm3\%$ $\frac{\text{m}^3}{\text{C}}$)	ρ ($\pm3\%$ Ω·m)	Hole Density ($\pm3\%$)
3245	5.64×10^{-10}	7.78×10^{-12}	1.11×10^{28}
5192	8.42×10^{-10}	2.07×10^{-11}	7.41×10^{27}
8436	1.33×10^{-9}	1.56×10^{-11}	4.70×10^{27}
Mean	9.12×10^{-10}	1.48×10^{-11}	7.74×10^{27}

TABLE II: Results for Hall coefficient, resistivity and carrier density for different magnetic fields.

A quantity that Merrill et at., suggested to calculate was the quality constant Q = $f_{res}/\Delta f$. This value is dimensionless and it resembles a slope. It can also be thought of the ratio between the resonance frequency and width. Therefore, the greater Q is, the more sharp the resonance is. For 3244 G, 5192 G, and 8436 G, the quality constant was 4.53, 2.54, and 5.35 respectively.

IV. DISCUSSION

With the resistivity of indium at 293K known to be $8.37 \times 10^{-8}\Omega$·m, it is reasonable to say that the resistivity at 273K is of order $\mathcal{O}(10^{-8})$. From Table 2 in Swenson [5], R/$R_{273.2}$ = 0.001 at 4.2K. So the resistivity at 4.2K is $\mathcal{O}(10^{-3})\cdot\mathcal{O}(10^{-8}) = \mathcal{O}(10^{-11})$ according to Swenson.

Similarly in White and Woods' experiment [2], they found that $R_{4.2}/R_{273} \sim 10^{-4}$. So the resistivity at 4.2K is $\mathcal{O}(10^{-4})\cdot\mathcal{O}(10^{-8}) = \mathcal{O}(10^{-12})$ according to White and Woods. In contrast, White and Woods' value is 1 order of magnitude smaller than Swenson's. Looking at my results, I have 3 resistivities; the higher magnetic fields B = 8436 G and B = 5192 G are of the order $\mathcal{O}(10^{-11})$ which is within 1 magnitude of Swenson's values. The lower magnetic field B = 3244 G is of the order $\mathcal{O}(10^{-12})$ which is within 1 magnitude of White and Woods' values.

Compared to copper which has a resistivity $1.68 \times 10^{-8}\Omega$·m at 293K, Indium has a resistivity $8.37 \times 10^{-8}\Omega$·m at 293K which is typical of metals.

As all my values of R_H were positive, I can classify Indium as a p-type semi-conductor with electron holes as carriers. This is because $R_H > 0$ implies a positive charge on the carriers, which in this case are just holes. Kittel's published value for the Hall coefficient of Indium is $R_H = +1.602\times10^{-10}\text{m}^3/\text{C}$ [3], my closest empirical value was at B = 3245 G which is $R_H = (+5.64\times10^{-10} \pm 1.128\times10^{-11})\text{m}^3/\text{C}$. This is unexpected as I would expect higher magnetic fields giving better results, since a larger magnetic field would imply a larger Hall angle tangent [Eq. 5] such that $u \gg 1$. This results in less attenuation and more propagation of the Helicon wave; thus, leading to better results.

The main experimental error was the rare fluctuating pickup amplitude at higher frequencies as shown by the spike in [Fig. 5]. This could be solved by increasing the reference signal amplitude. I also took less data points around some of the peaks due to time constraints, resulting in a smaller quality constant Q and less sharp, well defined resonances.

This experiment followed closely Merrill et al.'s; although unlike Merrill et al., I did not take data for the Helicon resonances as a function of temperature. This is because there was no reliable way to measure the temperature near 4.2K if I pulled the Helicon probe out of the helium dewar.

V. CONCLUSION

After letting Helicons propagate through Indium, the average Hall coefficient was found to be $(+9.12 \times 10^{-10} \pm 2.74 \times 10^{-11})\text{m}^3/\text{C}$, which is within 1 magnitude of Kittel's. The average resistivity was found to be $(1.48 \times 10^{-11} \pm 4.41 \times 10^{-13})\Omega$·m, which agrees more with Swenson's values $\mathcal{O}(10^{-11})$ than White and Woods' $\mathcal{O}(10^{-12})$. The average hole density was found to be $7.74\times10^{27} \pm 2.32\times10^{26}$ compared to the general carrier density n $\sim 10^{22}$ in metals [1].

The metal, Indium, was identified as a p-type semi-conductor because of the positive sigh of R_H. It follows that the charge carriers in Indium are positive, also known as holes.

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[3] C. Kittel,“Introduction to Solid State Physic”, 6th edition (1986), 150-151
[4] J. R. Merrill, D. Pierce, and D. Giovanielli, “A Helicon Solid-State Plasma Experiment for the Advanced Laboratory”, Amer. J. Phys. **38** (1970), 417-421
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VI. APPENDIX A: ERROR PROPAGATION

Uncertainty of voltage across magnet: $\delta V_{mag} = \pm 0.1\%$
Uncertainty of resistance of magnet: $\delta R_{mag} = \pm 0.02\%$
Uncertainty of current:

$$I = \frac{V_{mag}(\pm 0.1\%)}{R_{mag}(\pm 0.02\%)} = I \pm 0.1\%$$

Uncertainty of magnetic field:

$$B = (324 \pm 0.617\%)(I \pm 0.1\%) = (324 \cdot I) \pm 0.7\%$$

Uncertainty of pickup amplitude: $\delta V_{Pickup} = \pm 0.03\%$
Uncertainty of frequency: $\delta f = \pm 0.05\%$