Initial Conditions of Long Term simulations of the Solar System

Eric Yeung*
Department of Physical Sciences, University of Toronto, Toronto M1C 1A4, Canada
(Dated: June 3, 2015)

The main research in PHYD01 explores the work done predominantly by J. Laskar and H. Rein. The project will consist of Examples will be shown using Rebound.

I use the IAS15 integrator [2], which is the default integrator in Rebound, to demonstrate the changes in orbits with differing initial conditions. The code will follow closely Rebound's python tutorials.

I. PLAN

- Introduce integrators (SABA4, IAS15, WHFast)?
-
-
-

* eric.yeung@mail.utoronto.ca

II. SOME NOTES...

Due to the precession of mercury's perihelion caused by the $g_1 - g_5$ resonance with Jupiter, Mercury's eccentricity can reach large values of e_{max}. In Laskar's simulation, there are incremental eccentricities from $e_{m0} = 0.35$ to $e_{m0} = 0.90$ with timescales from 500 Myr to 5000 Myr. The supplementary information implies that the initial condition difference is around 3.8 cm for the case without general relativity (201 orbital solutions). For the case with general relativity, which has 2501 orbital solutions (over 10 times the no general relativity case), the initial condition difference is around 0.038 cm (now 10 times smaller than the noGR case). Two initially close orbital solutions have x10 difference in the respective distances. On the extrema case of $e_{m0} = 0.90$, which occurs 1% of the orbital solutions [1], the Solar System becomes destabilised.

To demonstrate the impact of initial conditions, I create two massive particles– akin to the Sun and Jupiter. The simulation runs in $\mathcal{O}(n)$ complexity time.

```
#!/usr/bin/env
from __future__ import division
from math import *
import rebound
rebound.reset()
rebound.integrator = "ias15"
rebound.add("Sun") # arguments are mass, semi major axis , and orbital eccentricity
rebound.add("Jupiter")
rebound.move_to_com()
rebound.dt = 0.1 # positive, non-zero time step
```

Now adding an arbitrary amount of test particles, let's say 500.

```
import numpy as np
N_testparticles = 500
a_initial = np.linspace(0.5, 4, N_testparticles) # initial and final semi major axes that will change later
for a in a_initial:
    rebound.add(a=a,anom=np.random.rand()*2.*pi) # no arg for m because massless
rebound.N_active = 2
```

Simulating and plotting for a relatively long timescale (3.4 thousand years).

```
t_max = 3.4e3
N_out = 10 # number of times to store test particle positions
xy = np.zeros((N_out, N_testparticles, 2))
times = np.linspace(0, t_max, N_out)
for i, time in enumerate(times):
    rebound.integrate(time)
    for j, p in enumerate(rebound.particles[2:500]):
        xy[i][j] = [p.x, p.y]

import matplotlib.pyplot as plt
fig = plt.figure(figsize=(5,5))
ax = plt.subplot(111)
ax.set_xlim([-3,3])
ax.set_ylim([-3,3])
plt.scatter(xy[:,:,0],xy[:,:,1],marker=".")
```

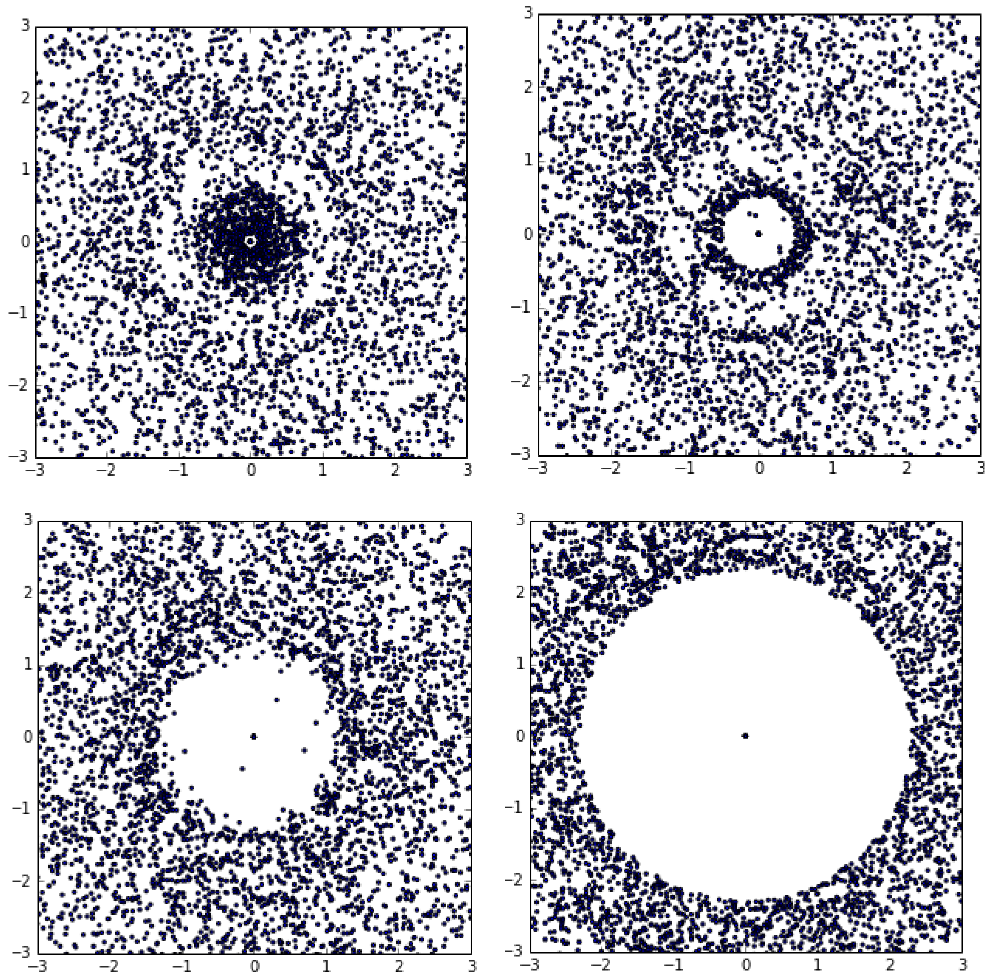

FIG. 1: Plots of the positions with intiail semi major axes 0.1, 0.5, 1.1, and 2.3 respectively.

Laskar uses the SABA4 integrator which is comparable to the– also symplectic integrator– IAS15.
In Arminjon's paper, he discusses the

III. REFERENCES

[1] J. Laskar et al., "Existence of collisional trajectories of Mercury, Mars and Venus with the Earth," Nature, 2009. **459**: 817-9.

[2] Rein, H., Spiegel, "IAS15: a fast, adaptive, high-order integrator for gravitational dynamics, accurate to machine precision over a billion orbits," MNRAS, 2015. **446**: 14241437.1409.4779

[3] Arminjon, M., "Proper initial conditions for long-term integrations of the solar system," Astron.Astrophys, 2002. **383**: 729-737