5/22 Initial notes + first steps

1 General dynamical systems background

(a) Read Chapter 1 (pg 5-16) of "Introduction to Applied Nonlinear Dynamical Systems and Chaos" by Wiggins. Pay especial attention to Eq (1.1.1), section 1.2A Linearization, and Theorem 1.2.5

(b) Evaluate the steady states (aka fixed points) of

$$\dot{x} = \mu x - M x^2 = x(\mu - M x), \qquad (1)$$

and evaluate their stability. This is the logistic equation.

2 2-dimensional generalized Lotka-Volterra equations

The 2D gLV equations are:

$$\begin{aligned} \dot{x} &= x(\mu_x + M_{xx} x + M_{xy} y) \\ \dot{y} &= y(\mu_y + M_{yx} x + M_{yy} y) \end{aligned} \qquad (2)$$

(a) $(0,0)$ is a trivial steady state. Identify the singly-existing SSs $(\bar{x}, 0)$ and $(0, \bar{y})$ by identifying the value of $\bar{x}$ and $\bar{y}$. Find the matrix equation that's solution gives the coexistent steady state $(\bar{\bar{x}}, \bar{\bar{y}})$, but you don't need to solve the matrix equation by hand.

(b) Evaluate the stability of $(0,0)$, $(\bar{x}, 0)$, and $(0, \bar{y})$ a in terms of the parameters $\mu_x, \mu_y, M_{xx}, M_{xy}, M_{yx}, M_{yy}$

(c) What happens for $M_{xx} > 0$ versus $M_{xx} < 0$? Think about how the equations react to positive or negative parameter values

3 N-dimensional gLV equations

The ND gLV equations are, for $i = 1, \ldots, N$,

$$\dot{x}_i = x_i\left(\mu_i + \sum_{j=1}^{N} M_{ij} x_j\right) \qquad (3)$$

(a) Ensure these reduce to Equation (2) for $N=2$

(b) Show that there are 2^N steady states. (How many SSs are there for Eq (2)? What are they?)

4 Simulation

(a) Simulate Eq 1 in Python. Use 'numpy', 'np.linspace', 'scipy.integrate', 'scipy.integrate.odeint'. Google some examples with "numerical simulation in python." Use an initial condition (IC) of $x_0 = 1$, $\mu = 1$, $M = .5$.

Plot output as a pdf with 'pyplot.matplotlib' and 'plt.plot' and 'plt.savefig'

(b) Simulate Eq (2). Use $\mu_x = \mu_y = M_{xx} = M_{yy} = 1$. Try ICs of $(x_0, y_0) = (.1, .9)$ and $(.9, .1)$.

negative

Use $(M_{xy}, M_{yx}) = (.5, .5)$ and $(1.5, 1.5)$.

5 Github

(a) Create a ssh key and link it with your github account

(b) Become comfortable using git from the command line (git add, git commit, git push, git pull). You should be able to do this without entering your username/password due to

(c) 'git clone git@github.com:erijones/ssr.git'

(d) Share your code with me on our shared github (github.com/erijones/ssr)