when $\omega > f$

$$\xi = \frac{(\omega^2-f^2)^{1/2}}{N}\,\frac{x}{h}$$ all others ths

$$\cancel{N^2}\frac{(\omega^2-f^2)}{\cancel{N^2h^2}}K^2\cancel{\omega}\frac{\partial^2\tilde{\psi}}{\partial\xi} - (\omega^2-f^2)\frac{1}{K^2}K^2\cancel{\omega} = -\frac{(\omega^2-f^2)^{1/2}}{NK}K\omega^2\frac{\partial\tilde{Q}}{\partial\xi}$$

$$\frac{\partial^2\tilde{\psi}}{\partial\xi^2} \quad \frac{\partial^2\tilde{\psi}}{\partial\xi^2} = -\frac{1}{\frac{N}{\omega}\left(1-\frac{f^2}{\omega^2}\right)^{1/2}}\frac{\partial\tilde{Q}}{\partial\xi} = -\beta\frac{\partial\tilde{Q}}{\partial\xi}$$

Fourier Transform

$$\beta = \frac{1}{\frac{N}{\omega}\left(1-\frac{f^2}{\omega^2}\right)^{1/2}}$$

$$\hat{F}(K) = \int_{-\infty}^{\infty} F(\xi)\,e^{-iK\xi}\,d\xi$$

inverse $\Rightarrow$ $$F(\xi) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{F}(K)\,e^{iK\xi}\,dK$$

$$\frac{\partial^2\hat{\tilde{\psi}}}{\partial\xi^2} + K^2\hat{\tilde{\psi}} = \beta\frac{\partial\hat{\tilde{Q}}}{\partial\xi}$$

$\hat{\tilde{\psi}}(K,0) = 0$

Sol'n by Green's Function Method.

$$G_{\xi\xi} + K^2 G = \delta(\xi-\xi')$$

for $\xi < \xi'$ $\quad G = A\sin K\xi$

$\xi > \xi'$ $\quad G = B_+e^{iK\xi} + B_-e^{-iK\xi}$

if $K>0$ $B_+=0$, $K<0$ $B_-=0$ — Apply Radiation condition here.

$$G = B\,e^{-i\alpha\xi}$$

$$\alpha = K\,\mathrm{sgn}(K)$$

Green's Functions

$$G(\xi,\xi') = -\frac{e^{-i\alpha\xi'}\sin K\xi}{K} \quad \xi<\xi'$$

$$= -\frac{e^{-i\alpha\xi}\sin K\xi'}{\alpha} \quad \xi>\xi'$$

$$\hat{\tilde{\psi}}(k,\zeta) = -\frac{\beta}{k}\left\{ e^{-ik\zeta}\int_0^{\zeta} \frac{\partial\hat{\tilde{Q}}}{\partial\xi}\sin k\zeta' d\zeta' + \sin k\zeta \int_{\zeta}^{\infty} \frac{\partial\hat{\tilde{Q}}}{\partial\xi} e^{-ik\zeta'} d\zeta' \right\}$$

$$\tilde{\psi}(\xi,\zeta) = \frac{C\beta}{2\pi}\left[\int_{-\infty}^{0}\left\{ \frac{e^{+ik\zeta}}{k}\int_0^{\zeta} \frac{\partial\hat{\tilde{Q}}}{\partial\xi'}\sin k\zeta' d\zeta' + \frac{\sin k\zeta}{k}\int_{\zeta}^{\infty}\frac{\partial\hat{\tilde{Q}}}{\partial\xi} e^{ik\zeta'} d\zeta' \right\} e^{ik\xi}\, dk \right.$$

$$\left. + \int_0^{\infty}\left\{ \frac{e^{-ik\zeta}}{k}\int_0^{\zeta} \frac{\partial\hat{\tilde{Q}}}{\partial\xi}\sin k\zeta' d\zeta' + \frac{\sin k\zeta}{k}\int_{\zeta}^{\infty}\frac{\partial\hat{\tilde{Q}}}{\partial\xi} e^{-ik\zeta'} d\zeta' \right\} e^{ik\xi}\, dk \right]$$

$$\boxed{\tilde{\psi}(\xi,\zeta,\tau) = \mathrm{Re}\left[-\frac{\beta}{\pi} e^{-i(\tau-\frac{\pi}{2})}\int_0^{\infty} \cos k\xi \left\{ e^{-ik\zeta}\int_0^{\zeta} \frac{\partial\hat{\tilde{Q}}}{\partial\xi}\sin k\zeta' d\zeta' + \sin k\zeta \int_{\zeta}^{\infty} \frac{\partial\hat{\tilde{Q}}}{\partial\xi} e^{-ik\zeta'} d\zeta' \right\} \frac{dk}{k} \right]}$$

Rotunno (1983) Eq. (37)

$$\frac{\partial\tilde{Q}}{\partial\xi} = \frac{A}{h\omega^3}\,\xi_0\,\frac{1}{\xi^2+\xi_0^2}\,e^{-\zeta}$$

$$\frac{\partial\hat{\tilde{Q}}}{\partial\xi} = \frac{A}{h\omega^3}\,\xi_0\,\frac{\pi}{\xi_0}\,e^{-\xi_0|k|}\,e^{-\zeta}$$

$$\tilde{\psi}(\xi,\zeta,\tau) = \mathrm{Re}\left[-\beta\tilde{A}\, e^{-i(\tau-\frac{\pi}{2})}\int_0^{\infty} \cos k\xi \left\{ e^{-ik\zeta} \int_0^{\zeta} e^{-\zeta'}\sin k\zeta' d\zeta' \right.\right.$$

$$\left.\left. + \sin k\zeta \int_{\zeta}^{\infty} e^{-\zeta'} e^{-ik\zeta'} d\zeta' \right\} e^{-\xi_0 k}\frac{dk}{k} \right]$$

$$\tilde{\psi}(\xi,\zeta,\tau) = \mathrm{Re}\left[-\beta\tilde{A}\, e^{-i(\tau-\frac{\pi}{2})}\int_0^{\infty} \cos k\xi\, e^{-\xi_0 k}\frac{dk}{1+k^2}\left\{ e^{-ik\zeta} - e^{-\zeta} \right\} \right]$$

$$\boxed{\tilde{\psi}(\xi,\zeta,\tau) = -\beta\tilde{A}\int_0^{\infty} \frac{dk \cos k\xi\, e^{-\xi_0 k}}{1+k^2}\left\{ \sin(k\zeta+\tau) - e^{-\zeta}\sin(\tau) \right\}}$$

10/8/82

Eq. (38)