CS513:	**Homework 1**	Due in class, Thurs, Sept. 12, 2013

Cooperation: Students should talk to each other about the subject matter of this class and help each other. It is fine to discuss the problems and ask questions about them. I encourage such questions in class and office hours. However, there is a line past which you must not go, e.g., sharing or copying a solution is not okay and could result in failure. If a significant part of one of your solutions is due to someone else, or something you've read then **you must acknowledge your source!** Failure to do so is a serious academic violation, likely to result in failure of the course or worse. Furthermore, **all solutions must be written by yourself, in your own words**. You may get an idea from somewhere or someone and acknowledge that, but you must still understand it and explain it yourself. A copied solution, even with the source acknowledged will be considered plagiarism. The exception is if it is in quotation marks and cited specifically. But in this case, don't bother because you won't get credit for quoting someone else's solution.

General Strategy for doing the homeworks: My desire for the homeworks is that they help you **understand** the concepts from lecture and readings, i.e., that these concepts not only seem believable, but you can employ them. Please read the homework early and make sure that you understand all the questions.

For almost all the problems I give, you should be able to look at a few tiny examples, and try to solve the problem on those examples. If you can do it for the tiny examples, then you are part way to giving a rule that will solve the problem in general. If you cannot solve or are confused by what the problem means on one of your small examples, then that would be a great time to ask me a question about it.

Reading Please read the whole first chapter of Schöning by the time this assignment is due. I suggest that you only read one section at a sitting and that you think about all of the exercises – assigned or not – to help make sure that you are understanding all the concepts as you read.

Class notes or text? When I don't exactly follow the text I would prefer if you use the definitions I make in class rather than those in the text, but please point out to me if there is an interesting issue, or if something would be easier if you follow the way the book did it. For example, the definition of propositional formulas that I want you to use is the one I gave in class:

0. $\top, \bot$ are formulas.
1. All atomic formulas are formulas.
2. If F is a formula, then so is $\neg F$.
3. If F, G are formulas, then so is $(F \vee G)$.

Problems [The starred problems at the end are just for 690LG students. Other students may do them for extra credit, but don't feel obliged.]

0. **I'd like each student to do this today, so that I've heard from everyone enrolled in the class.** Please take a look at the course webpage and then send me a brief email. I would like to know anything you are concerned about, or any topic that you would particularly like to learn about. This will help me as I decide which topics to emphasize, etc. Thanks.

1. Show from our definition of semantics and our abbreviation for "$\wedge$", that for any truth assignment $\mathcal{A}$ that is suitable for $F \wedge G$, $\mathcal{A}(F \wedge G) = \min(\mathcal{A}(F), \mathcal{A}(G))$.

2. Prove or give a counter example:

   (a) if $(F \to G)$ is valid and F is valid, then G is valid.

   (b) if $(F \to G)$ is satisfiable and F is satisfiable, then G is satisfiable.

   (c) if $(F \to G)$ is valid and F is satisfiable, then G is satisfiable.

3. Prove the following Theorem: If $\mathcal{A}$ and $\mathcal{B}$ are suitable truth assignments for propositional formula F and $\mathcal{A}$ and $\mathcal{B}$ agree on all the atomic formulas that occur in F, then $\mathcal{A}(F) = \mathcal{B}(F)$.

   [Hint: you should prove this by induction on the structure of the formula F.]

4. Horn formulas:

   (a) Apply the linear-time Horn-SAT algorithm from the text to the following Horn formula:

   $$F \;=\; (\neg A \vee \neg B \vee \neg D) \wedge \neg E \wedge (\neg C \vee A) \wedge C \wedge B \wedge (\neg G \vee D) \wedge G.$$

   Please show your work and either produce a satisfying assignment or state that is is not satisfiable.

   (b) Give an example of a formula that does not have an equivalent Horn formula. Give a convincing argument that it is not equivalent to a Horn formula.

5*. How many inequivalent propositional formulas are there on n atomic formulas: $\{A_1, \ldots, A_n\}$? Prove your answer.

6*. (cf. Exercise 13, p. 14) Prove Craig's Interpolation Theorem for Propositional Logic: Suppose that $\models (F \to G)$ for propositional formulas F and G. Prove that there is a formula I – called the interpolant – such that $\models (F \to I)$, $\models (I \to G)$, and $\text{atom}(I) = \text{atom}(F) \cap \text{atom}(G)$. [Hint: prove this by induction on $n = |\text{atom}(F) - \text{atom}(G)|$, i.e, the number of atomic formulas that occur in F but not in G. Note that your base case should be $n = 0$. In Exercise 13, Schöning just states this theorem for $n \geq 1$ because he doesn't have $\top$ and $\bot$ as propositional formulas, but we do.]