CS513:	**Homework 2**	Due in class, 9/26/13

Reading Please read Sections 2.1 through 2.3 by the time this assignment is due. I suggest that you only read one section at a sitting and that you think about all of the exercises – assigned or not – to help make sure that you are understanding all the concepts as you read.

Problems

1. Do the following two resolution exercises:
   (a) exercise 33, p. 36.
   (b) exercise 34, p. 36.

2. This problem is meant to reinforce the idea of the inductive step in the proof of the completeness of resolution in the text. Consider the following CNF formula represented as a set of clauses:

$$\varphi = \{\{\neg x_3, x_1\}, \{\neg x_1, x_2\}, \{\neg x_2, \neg x_3\}, \{x_3, \neg x_1\}, \{x_1, \neg x_2\}, \{x_2, x_3\}\}$$

   (a) Compute $\varphi_0 = \varphi[x_3/\bot]$ and $\varphi_1 = \varphi[x_3/\top]$
   (b) Give a resolution derivation of $\varphi_0 \vdash \Box$.
   (c) Give a resolution derivation of $\varphi_1 \vdash \Box$.
   (d) Now show using the above two that $\varphi \vdash \{x_3\}$ and $\varphi \vdash \{\neg x_3\}$.

   Thus, you get a proof of $\varphi \vdash \Box$.

3. Exercise 42, p. 43: Let free(F) be the set of all variables that occur freely in F. Define free(F) formally by induction on terms and then on formulas.

4. Exercise 45, p. 49: Consider the following sentences R, S, T which express that the predicate P is reflexive, symmetric, and transitive:

$$\begin{aligned} R &= \forall x P(x,x) \\ S &= \forall xy(P(x,y) \rightarrow P(y,x)) \\ T &= \forall xyz(P(x,y) \wedge P(y,z) \rightarrow P(x,z)) \end{aligned}$$

   Show that these sentences are independent by constructing three structures that satisfy each possible pair of the sentences but not the third.

   [Note that $\wedge$ and $\vee$ bind more tightly than $\rightarrow$, so I didn't use extra parentheses in the definition of T.]

5. Exercise 55, p. 52: show that the following pairs of formulas are not equivalent by constructing structures that satisfy one of them but not the other:

$$\begin{aligned} (\forall x P(x) \vee \forall x Q(x)) &\not\equiv \forall x(P(x) \vee Q(x)) \\ (\exists x P(x) \wedge \exists x Q(x)) &\not\equiv \exists x(P(x) \wedge Q(x)) \end{aligned}$$

$6^\star$. Two sets of formulas M_1 and M_2 are **equivalent** iff they have the same set of models:

$$\{\mathcal{A} \mid \mathcal{A} \models M_1;\ \mathcal{A} \text{ suitable for } M_1 \cup M_2\} \quad = \quad \{\mathcal{A} \mid \mathcal{A} \models M_2;\ \mathcal{A} \text{ suitable for } M_1 \cup M_2\}$$

We also say that M_2 is an axiom system for M_1 if they are equivalent. We say that M is **finitely axiomatizable** iff it has a finite axiom system. Suppose that $\{\alpha_1, \alpha_2, \ldots\}$ is an axiom system for M, and for all i, $M \models \alpha_{i+1} \to \alpha_i$ but $M \not\models \alpha_i \to \alpha_{i+1}$. Show that M is not finitely axiomatizable.