CS513:	**Homework 4**	Due in class, Oct. 31, 2013

Problems

1. Do Exercise 85, p. 96. Assume that the whole universe consists of dragons. Use the predicates: $H(x)$, $G(y)$, $F(z)$, $C(v, w)$ to mean x is happy, y is green, z can fly, and v is a child of w, respectively.

2. Do Exercise 86, p. 96.

3. Write a Prolog program to compute path length, i.e., p(x,y,n) should be true whenever $n \geq 1$ and there is a path from x to y of length exactly n. (It does not have to be a simple path, i.e., reusing vertices is okay.) Assume that a graph is given with the edge predicate, e. [Hint: you may use the "is" predicate as in the example on the top of page 143 of the text.]

4. Prove: $\mathcal{A} \cong \mathcal{B} \;\Rightarrow\; \mathcal{A} \equiv \mathcal{B}$

   Recall some definitions: let $\mathcal{A}, \mathcal{B}$ be structures of the same vocabulary, τ. We say that $\mathcal{A}$ is **isomorphic** to $\mathcal{B}$, in symbols $\mathcal{A} \cong \mathcal{B}$, iff there is a 1:1 and onto function $g : |\mathcal{A}| \to |\mathcal{B}|$ with the following properties:

   for every function symbol $f^r \in \tau$, and all elements $e_1, \ldots e_r \in |\mathcal{A}|$,

$$g(f^{\mathcal{A}}(e_1, \ldots, e_r)) \quad = \quad f^{\mathcal{B}}(g(e_1), \ldots, g(e_r)),$$

   and for every relation symbol $R^a \in \tau$, and all elements $e_1, \ldots e_a \in |\mathcal{A}|$,

$$\langle e_1, \ldots, e_a \rangle \in R^{\mathcal{A}} \quad \Leftrightarrow \quad \langle g(e_1), \ldots, g(e_a) \rangle \in R^{\mathcal{B}} \ .$$

   The map g is called an **isomorphism** from $\mathcal{A}$ to $\mathcal{B}$.

   We say that $\mathcal{A}$ is **elementarily equivalent** to $\mathcal{B}$, in symbols $\mathcal{A} \equiv \mathcal{B}$, iff for all first-order sentences φ of vocabulary τ, $\mathcal{A} \models \varphi \Leftrightarrow \mathcal{B} \models \varphi$.

   Note that the definition of isomorphism says that g honors all symbols of τ. Thus $\mathcal{A}$ and $\mathcal{B}$ are identical except for the names of the elements of the universe. Thus it should be no surprise that two isomorphic structures satisfy exactly the same sentences.

   To do this I suggest that you show by induction on φ that $\mathcal{A} \models \varphi \Leftrightarrow \mathcal{B} \models \varphi$. To do this by induction you will have to prove it not just for sentences but for all formulas. For this, you should make the added assumption that g preserves the default value of all variables, i.e., for all $x_i \in$ VAR,

$$g(x_i^{\mathcal{A}}) \quad = \quad x_i^{\mathcal{B}}$$

   Note that since the default values of the variables do not affect the truth of sentences, you can always adjust the default values to meet the above condition.

5*. Show with an example that the converse of 4 is not true, i.e., give a pair of structures for which $\mathcal{A} \equiv \mathcal{B}$ but $\mathcal{A} \not\cong \mathcal{B}$.

   Show however that the converse of 4 is true for finite structures.

6*. Do Exercise 89, p. 106. I suggest you use the first hint, i.e., imagine running the Horn-SAT marking algorithm on an unsatisfiable formula, φ, and then build an SLD refutation of φ by simulating that run backwards.