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Schwarzschild or Ledoux: composition gradients are fragile

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ABSTRACT

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2. THEORY

3. CONCLUSIONS

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APPENDIX

A. MODEL & INITIAL CONDITIONS

In this work we study the simplest possible system: incompressible, Boussinesq convection with a composition field and a height-varying background radiative conductivity, similar to that used in Fuentes & Cumming (2020); Anders

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et al. (2021). These equations are

$$\nabla \cdot \boldsymbol{u} = 0, \tag{A1}$$

$$\partial_t \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho_0}\nabla p + \frac{\rho_1}{\rho_0}\boldsymbol{g} + \nu \nabla^2 \boldsymbol{u}, \tag{A2}$$

$$\partial_t T + \boldsymbol{u} \cdot \nabla T + w\nabla_{\rm ad} + \nabla \cdot [-\kappa_{T,0}\nabla \overline{T}] = \kappa_T \nabla^2 T', \tag{A3}$$

$$\partial_t C + \boldsymbol{u} \cdot \nabla C = \kappa_{C,0}\nabla^2 \overline{C} + \kappa_C \nabla^2 C', \tag{A4}$$

$$\frac{\rho_1}{\rho_0} = -|\alpha| T + |\beta| C. \tag{A5}$$

Here, $\boldsymbol{u}$ is the vector velocity, T is the temperature, C is the composition, ρ_0 is the constant background density, is the kinematic pressure which enforces Eqn. A1, ρ_1 are density fluctuations which act only on the buoyant term, and α and β are the thermal and compositional expansion coefficients, and $\nabla_{\rm ad}$ is the adiabatic gradient. Diffusive terms are controlled by the kinematic viscosity ν, as well as the thermal diffusivity κ_T and compositional diffusivity κ_C. On the horizontally-invariant ($n_x = 0$ and $n_y = 0$) mode, we use a height-depended thermal diffusion coefficient $\kappa_{T,0}$ (which allows $\nabla_{\rm rad}$ to vary with height) and a lower compositional diffusivity $\kappa_{C,0} < \kappa_C$ to ensure that the evolution of the mean composition profile is due to advection rather than diffusion.

We nondimensionalize Eqns. A1-A5 on the length scale of the initial Schwarzschild-unstable convection zone L_s, the timescale of freefall across that convection zone

$$\tau_{\rm ff} = \left(\frac{L_s}{|\alpha| g \Delta T}\right)^{1/2}, \tag{A6}$$

and the temperature scale set by the temperature gradient at the bottom boundary $\Delta T = L_s(\partial_z T)_{\rm bot}$; mass is nondimensionalized so that the freefall ram pressure $\rho_0 (L_s/\tau_{\rm ff})^2 = 1$, and composition is nondimensionalized so that its value is initially 1 in the convection zone and zero in the Schwarzschild-stable radiative zone. This nondimensionalization is

$$\begin{aligned}
&T^* = (\Delta T)T = (L_s[\partial_z T]_{\rm bot})tQ_0\tau_{\rm ff}T, \qquad C^* = (\Delta C)C, \qquad \partial_{t^*} = \tau_{\rm ff}^{-1}\partial_t, \qquad \nabla^* = L_s^{-1}\nabla, \\
&\boldsymbol{u}^* = u_{\rm ff}\boldsymbol{u} = \frac{L_s}{\tau_{\rm ff}}\boldsymbol{u}, \qquad p^* = \rho_0 u_{\rm ff}^2 \varpi, \qquad \kappa_T^* = (L_s^2 \tau_{\rm ff}^{-1})\kappa_T, \qquad \kappa_C^* = (L_s^2\tau_{\rm ff}^{-1})\kappa_C.
\end{aligned} \tag{A7}$$

For convenience, here we define quantities with $*$ (e.g., T^*) as being the "dimensionful" quantities of Eqns. A9-A5. Henceforth, quantities without $*$ (e.g., T) are dimensionless. Within this nondimensionalization, we define the following control parameters for our simulations

$$\mathcal{P} = \frac{u_{\rm ff}L_s}{\kappa_T}, \qquad \mathrm{R}_0 = \frac{|\alpha|\Delta T}{|\beta|\Delta C}, \qquad \mathrm{Pr} = \frac{\nu}{\kappa_T}, \qquad \tau = \frac{\kappa_C}{\kappa_T}, \qquad \tau_0 = \frac{\kappa_{C,0}}{\kappa_T} \tag{A8}$$

The dimensionless equations of motion are

$$\nabla \cdot \boldsymbol{u} = 0 \tag{A9}$$

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla \varpi + (T - \mathrm{R}_0^{-1} C)\hat{z} + \frac{\mathrm{Pr}}{\mathcal{P}}\nabla^2 \boldsymbol{u} \tag{A10}$$

$$\partial_t T + \boldsymbol{u} \cdot \nabla T + w\nabla_{\rm ad} + \nabla \cdot [-\kappa_{T,0}\nabla \overline{T}] = \frac{1}{\mathcal{P}}\nabla^2 T'., \tag{A11}$$

$$\partial_t C + \boldsymbol{u} \cdot \nabla C = -\frac{\tau_0}{\mathcal{P}}\nabla^2 \overline{C} + \frac{\tau}{\mathcal{P}}\nabla^2 C', \tag{A12}$$

In this work, we study a three-layer model in $z = [0, 3]$. We set

$$\nabla_\mu = \begin{cases} 0 & z \le 1 \\ \mathrm{R}_0^{-1} & 1 < z \le 2 \\ 0 & 2 < z \end{cases}, \qquad \nabla_{\rm rad} = \begin{cases} \nabla_{\rm ad} + 1 & z \le 2 \\ \nabla_{\rm ad} - \mathrm{R}_0^{-1} & z > 2 \end{cases}, \tag{A13}$$

where in our simple boussinesq system, we define

$$\nabla_\mu \equiv -\mathrm{R}_0^{-1}\frac{\partial \mu}{\partial z}, \qquad \nabla_T \equiv -\frac{\partial T}{\partial z}, \qquad \nabla_{\rm ad} = 5[\mathrm{R}_0^{-1} - 2] \text{ (a constant).} \tag{A14}$$

and $\nabla_T = \nabla_{\rm rad}$ if conduction carries all of the energy flux. We fix the flux carried by convection to be

$$F_{\rm conv} = \kappa_{T,0}, \tag{A15}$$

so the total flux through the system is $F_{\rm tot} = F_{\rm conv}(\nabla_{\rm ad} + 1)$. We set $\nabla_T = \nabla_{\rm rad}$ in the initial state.

B. SIMULATION DETAILS & DATA AVAILABILITY

We time-evolve equations ?? using the Dedalus pseudospectral solver (Burns et al. 2020) using timestepper SBDF2 (**?**) and safety factor 0.3. All fields are represented as spectral expansions of n_z Chebyshev coefficients in the vertical (z) direction and as (n_x, n_y) Fourier coefficients in the horizontal (x, y) directions; our domain is therefore horizontally periodic. We use a domain with an aspect ratio of two so that $x \in [0, L_x]$, $y \in [0, L_y]$, and $z \in [0, L_z]$ with $L_x = L_y = 2L_z$. The initial convection zone spans initially spans 1/3 of the domain depth and in the evolved state spans 2/3 of the domain depth, so it has an initial aspect ratio of 6 and a final aspect ratio of 3. To avoid aliasing errors, we use the 3/2-dealiasing rule in all directions. To start our simulations, we add random noise temperature perturbations with a magnitude of 10^{-6} to the initial temperature profile (discussed in ??).

Spectral methods with finite coefficient expansions cannot capture true discontinuities. In order to approximate discontinuous functions such as Eqns. **??**, **??**, and **??**, we must use smooth transitions. We therefore define a smooth Heaviside step function,

$$H(z; z_0, d_w) = \frac{1}{2}\left(1 + \mathrm{erf}\left[\frac{z - z_0}{d_w}\right]\right). \tag{B16}$$

where erf is the error function. In the limit that $d_w \to 0$, this function behaves identically to the classical Heaviside function centered at z_0. For Eqn. **??** and Eqn. **??**, we use $d_w = 0.02$; while for Eqn. **??** we use $d_w = 0.075$. In all other cases, we use $d_w = 0.05$.

A table describing all of the simulations presented in this work can be found in Appendix C. We produce figures ?? and ?? using matplotlib (Hunter 2007; Caswell et al. 2021). We produce figure ?? using TODO. All of the Python scripts used to run the simulations in this paper and to create the figures in this paper are publicly available in a git repository[1], and in a Zenodo repository (**?**).

C. TABLE OF SIMULATION PARAMETERS

Input parameters and summary statistics of the simulations presented in this work are shown in Table **??**.

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