Draft version January 7, 2022
Typeset using LaTeX **twocolumn** style in AASTeX631

Schwarzschild or Ledoux: composition gradients are fragile

Evan H. Anders,[1,2] Adam S. Jermyn,[3,2] Daniel Lecoanet,[1,4,2] Adrian E. Fraser,[5,2] Imogen G. Cresswell,[6,2] and J. R. Fuentes[7]

[1] *CIERA, Northwestern University, Evanston IL 60201, USA*
[2] *Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA*
[3] *Center for Computational Astrophysics, Flatiron Institute, New York, NY 10010, USA*
[4] *Department of Engineering Sciences and Applied Mathematics, Northwestern University, Evanston IL 60208, USA*
[5] *University of California, Santa Cruz, Santa Cruz, California 95064, U.S.A*
[6] *Department Astrophysical and Planetary Sciences & LASP, University of Colorado, Boulder, CO 80309, USA*
[7] *Department of Physics and McGill Space Institute, McGill University, 3600 rue University, Montreal, QC H3A 2T8, Canada*

(Received July 28, 2021; Revised October 19, 2021; Accepted; Published)

Submitted to ApJ

ABSTRACT

This will be an abstract.

Keywords: UAT keywords

1. INTRODUCTION

2. THEORY

3. CONCLUSIONS

4. RESULTS

We thank Meridith Joyce, Anne Thoul, Dominic Bowman, ETC ETC ETC. EHA is funded as a CIERA Postdoctoral fellow and would like to thank CIERA and Northwestern University. This research was supported in part by the National Science Foundation under Grant No. PHY-1748958, and we acknowledge the hospitality of KITP during the Probes of Transport in Stars Program. Computations were conducted with support from the NASA High End Computing (HEC) Program through the NASA Advanced Supercomputing (NAS) Division at Ames Research Center on Pleiades with allocation GID s2276. The Flatiron Institute is supported by the Simons Foundation.

APPENDIX

A. MODEL & INITIAL CONDITIONS

In this work we study the simplest possible system: incompressible, Boussinesq convection with a composition field and a height-varying background radiative conductivity, similar to that used in Fuentes & Cumming (2020); Anders

Corresponding author: Evan H. Anders
evan.anders@northwestern.edu

et al. (2021). These equations are

$$\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0, \tag{A1}$$

$$\partial_t \boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u} = -\frac{1}{\rho_0}\boldsymbol{\nabla} p + \frac{\rho_1}{\rho_0}\boldsymbol{g} + \nu\boldsymbol{\nabla}^2\boldsymbol{u}, \tag{A2}$$

$$\partial_t T + \boldsymbol{u}\cdot\boldsymbol{\nabla} T + w\nabla_{\rm ad} + \boldsymbol{\nabla}\cdot[-\kappa_{T,0}\boldsymbol{\nabla}\overline{T}] = \kappa_T\boldsymbol{\nabla}^2 T', \tag{A3}$$

$$\partial_t C + \boldsymbol{u}\cdot\boldsymbol{\nabla} C = \kappa_{C,0}\boldsymbol{\nabla}^2\overline{C} + \kappa_C\boldsymbol{\nabla}^2 C', \tag{A4}$$

$$\frac{\rho_1}{\rho_0} = -|\alpha|T + |\beta|C. \tag{A5}$$

Here, $\boldsymbol{u}$ is the vector velocity, T is the temperature, C is the composition, ρ_0 is the constant background density, p is the kinematic pressure which enforces Eqn. A1, ρ_1 are density fluctuations which act only on the buoyant term, and α and β are the thermal and compositional expansion coefficients, and $\nabla_{\rm ad}$ is the adiabatic gradient. Diffusive terms are controlled by the kinematic viscosity ν, as well as the thermal diffusivity κ_T and compositional diffusivity κ_C. On the horizontally-invariant ($n_x = 0$ and $n_y = 0$) mode, we use a height-depended thermal diffusion coefficient $\kappa_{T,0}$ (which allows $\nabla_{\rm rad}$ to vary with height) and a lower compositional diffusivity $\kappa_{C,0} < \kappa_C$ to ensure that the evolution of the mean composition profile is due to advection rather than diffusion.

We nondimensionalize Eqns. A1-A5 according to

$$\begin{aligned}
&T^* = (\Delta T)T, \qquad C^* = (\Delta C)C, \qquad \partial_{t^*} = \tau_{\rm ff}^{-1}\partial_t, \qquad \boldsymbol{\nabla}^* = L_s^{-1}\boldsymbol{\nabla}, \qquad p^* = \rho_0 u_{\rm ff}^2\varpi, \\
&\boldsymbol{u}^* = u_{\rm ff}\boldsymbol{u} = \frac{L_s}{\tau_{\rm ff}}\boldsymbol{u}, \qquad \tau_{\rm ff} = \left(\frac{L_s}{|\alpha|g\Delta T}\right)^{1/2}, \qquad \kappa_{T,0}^* = (L_s^2\tau_{\rm ff}^{-1})\kappa_{T,0}.
\end{aligned} \tag{A6}$$

For convenience, here we define quantities with $*$ (e.g., T^*) as being the "dimensionful" quantities of Eqns. A1-A5. Henceforth, quantities without $*$ (e.g., T) are dimensionless. Here, L_s is the length scale of the initial Schwarzschild-unstable convection zone and $\tau_f f$ is the buoyant freefall timescale. The temperature and composition are set by the destabilizing radiative temperature gradient $\Delta T = L_s(\partial_z T + \boldsymbol{\nabla}_{\rm ad})$ and the stabilizing composition gradient ($\Delta C = L_s\partial_z C$). Within this nondimensionalization, the dynamical control parameters are

$$\mathcal{P} = \frac{u_{\rm ff}L_s}{\kappa_T}, \qquad \mathrm{R}_0 = \frac{|\alpha|\Delta T}{|\beta|\Delta C}, \qquad \mathrm{Pr} = \frac{\nu}{\kappa_T}, \qquad \tau = \frac{\kappa_C}{\kappa_T}, \qquad \tau_0 = \frac{\kappa_{C,0}}{\kappa_T} \tag{A7}$$

The dimensionless equations of motion are

$$\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0 \tag{A8}$$

$$\partial_t\boldsymbol{u} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u} = -\boldsymbol{\nabla}\varpi + (T - \mathrm{R}_0^{-1}C)\hat{z} + \frac{\mathrm{Pr}}{\mathcal{P}}\boldsymbol{\nabla}^2\boldsymbol{u} \tag{A9}$$

$$\partial_t T + \boldsymbol{u}\cdot\boldsymbol{\nabla} T + w\boldsymbol{\nabla}_{\rm ad} + \boldsymbol{\nabla}\cdot[-\kappa_{T,0}\boldsymbol{\nabla}\overline{T}] = \frac{1}{\mathcal{P}}\boldsymbol{\nabla}^2 T'., \tag{A10}$$

$$\partial_t C + \boldsymbol{u}\cdot\boldsymbol{\nabla} C = -\frac{\tau_0}{\mathcal{P}}\boldsymbol{\nabla}^2\overline{C} + \frac{\tau}{\mathcal{P}}\boldsymbol{\nabla}^2 C'. \tag{A11}$$

We define the thermal and compositional gradients

$$\nabla_T \equiv -\frac{\partial T}{\partial z}, \qquad \nabla_C \equiv -\mathrm{R}_0^{-1}\frac{\partial C}{\partial z}, \tag{A12}$$

and stability is determined by the sign of the Brunt-Väisälä frequency,

$$N^2 = N^2_{\rm structure} + N^2_{\rm composition}, \text{ with } N^2_{\rm structure} = -(\nabla_T - \nabla_{\rm ad}),\ N^2_{\rm composition} = \nabla_C, \tag{A13}$$

where $N^2 > 0$ is buoyantly stable, so the stability criterion is $\nabla_C - (\nabla_T - \nabla_{\rm ad}) > 0$, as in stellar models (Salaris & Cassisi 2017).

In this work, we study a three-layer model in $z = [0, 3]$. We want to construct a simulation with

$$N^2 = \begin{cases} -1 & z \le 1 \\ R_0^{-1} - 1 & 1 < z \le 2 \\ R_0^{-1} & 2 < z \end{cases}, \qquad N^2_{\rm composition} = \begin{cases} 0 & z \le 1 \\ R_0^{-1} & 1 < z \le 2 \\ 0 & 2 < z \end{cases}, \qquad N^2_{\rm structure} = \begin{cases} -1 & z \le 1 \\ -1 & 1 < z \le 2 \\ R_0^{-1} & 2 < z \end{cases} \tag{A14}$$

To achieve this, we set $\partial_z C = -\mathrm{R}_0 N^2_{\text{composition}}$ and $\partial_z T = N^2_{\text{structure}} - \nabla_{\text{ad}}$, where we set $\nabla_{\text{ad}} = 5[\mathrm{R}_0^{-1} - 2]$ as a constant so that $\nabla_{\text{ad}} > 0$ for all values of R_0 studied. We furthermore enforce that $\nabla_T = \nabla_{\text{rad}}$ in the initial state, where

$$\nabla_{\text{rad}} = -\frac{F_{\text{tot}}}{\kappa_{T,0}}, \tag{A15}$$

is the radiative gradient and F_{tot} is the total vertical energy flux through the system. We set the total flux $F_{\text{tot}} = -(1+\nabla_{\text{ad}})/\mathcal{P}$ and the convective flux $F_{\text{conv}} = 1/\mathcal{P}$, so $\kappa_{T,0}(z) = -((1+\nabla_{\text{ad}})/\partial_z T)\mathcal{P}^{-1}$.

B. SIMULATION DETAILS & DATA AVAILABILITY

We time-evolve equations ?? using the Dedalus pseudospectral solver (Burns et al. 2020) using timestepper SBDF2 (**?**) and safety factor 0.3. All fields are represented as spectral expansions of n_z Chebyshev coefficients in the vertical (z) direction and as (n_x, n_y) Fourier coefficients in the horizontal (x, y) directions; our domain is therefore horizontally periodic. We use a domain with an aspect ratio of two so that $x \in [0, L_x]$, $y \in [0, L_y]$, and $z \in [0, L_z]$ with $L_x = L_y = 2L_z$. The initial convection zone spans initially spans 1/3 of the domain depth and in the evolved state spans 2/3 of the domain depth, so it has an initial aspect ratio of 6 and a final aspect ratio of 3. To avoid aliasing errors, we use the 3/2-dealiasing rule in all directions. To start our simulations, we add random noise temperature perturbations with a magnitude of 10^{-6} to the initial temperature profile (discussed in ??).

Spectral methods with finite coefficient expansions cannot capture true discontinuities. In order to approximate discontinuous functions such as Eqns. A14, we must use smooth transitions. We therefore define a smooth Heaviside step function,

$$H(z; z_0, d_w) = \frac{1}{2}\left(1 + \text{erf}\left[\frac{z - z_0}{d_w}\right]\right). \tag{B16}$$

where erf is the error function. In the limit that $d_w \to 0$, this function behaves identically to the classical Heaviside function centered at z_0. Throughout this work, we set $d_w = 0.05$.

A table describing all of the simulations presented in this work can be found in Appendix C. We produce figures ?? and ?? using matplotlib (Hunter 2007; Caswell et al. 2021). We produce figure ?? using TODO. All of the Python scripts used to run the simulations in this paper and to create the figures in this paper are publicly available in a git repository[1], and in a Zenodo repository (**?**).

C. TABLE OF SIMULATION PARAMETERS

Input parameters and summary statistics of the simulations presented in this work are shown in Table **??**.

REFERENCES

Anders, E. H., Jermyn, A. S., Lecoanet, D., & Brown, B. P. 2021, arXiv e-prints, arXiv:2110.11356. https://arxiv.org/abs/2110.11356

Burns, K. J., Vasil, G. M., Oishi, J. S., Lecoanet, D., & Brown, B. P. 2020, Physical Review Research, 2, 023068, doi: 10.1103/PhysRevResearch.2.023068

Caswell, T. A., Droettboom, M., Lee, A., et al. 2021, matplotlib/matplotlib: REL: v3.3.4, v3.3.4, Zenodo, doi: 10.5281/zenodo.4475376

Fuentes, J. R., & Cumming, A. 2020, Physical Review Fluids, 5, 124501, doi: 10.1103/PhysRevFluids.5.124501

Hunter, J. D. 2007, Computing in Science and Engineering, 9, 90, doi: 10.1109/MCSE.2007.55

Salaris, M., & Cassisi, S. 2017, Royal Society Open Science, 4, 170192, doi: 10.1098/rsos.170192

[1] https://github.com/evanhanders/convective_penetration_paper