Studies of Quantum Dots using Machine Learning

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Outline

- Motivation
- Quantum Theory
- Machine Learning
- Methods
- Results
- Conclusion
- (Code)

Motivation

Why quantum dots?

- Technology[1]
- Theoretically
- Experimentally (2D)

Why is it challenging?

- Quantum many-body problem
- Fermi-Dirac statistics

How to overcome the challenges?

- Efforts..
- gh
- Our approach: Machine Learning[2,3]

Quantum Theory

The Time-independent Schrödinger Equation

A stationary quantum mechanical system is described by

$$E_n = \frac{\int d\boldsymbol{R}\Psi_n^*(\boldsymbol{R})\hat{\mathcal{H}}\Psi_n(\boldsymbol{R})}{\int d\boldsymbol{R}\Psi_n^*(\boldsymbol{R})\Psi_n(\boldsymbol{R})}$$

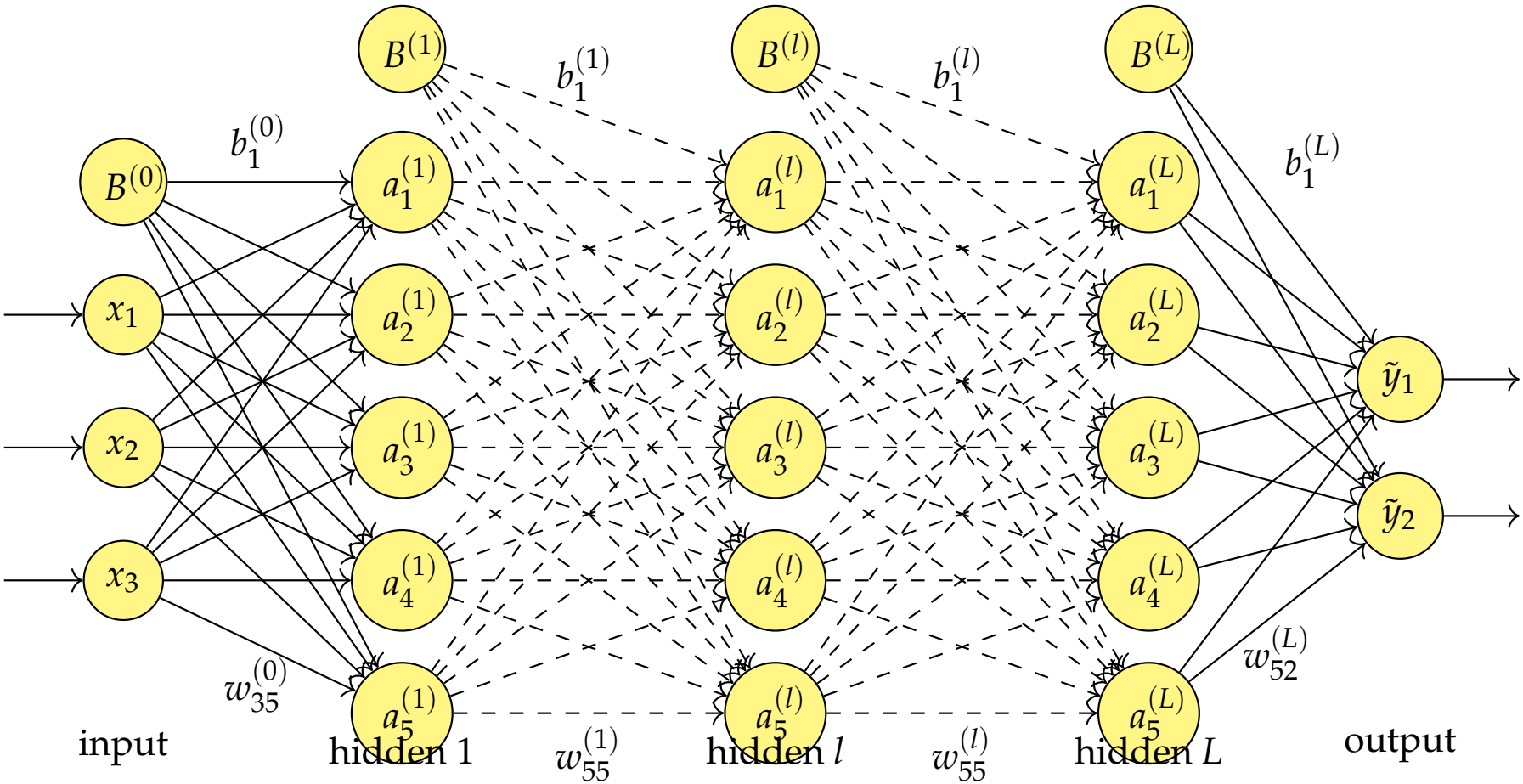
which gives the energy of state n.

Machine Learning Theory

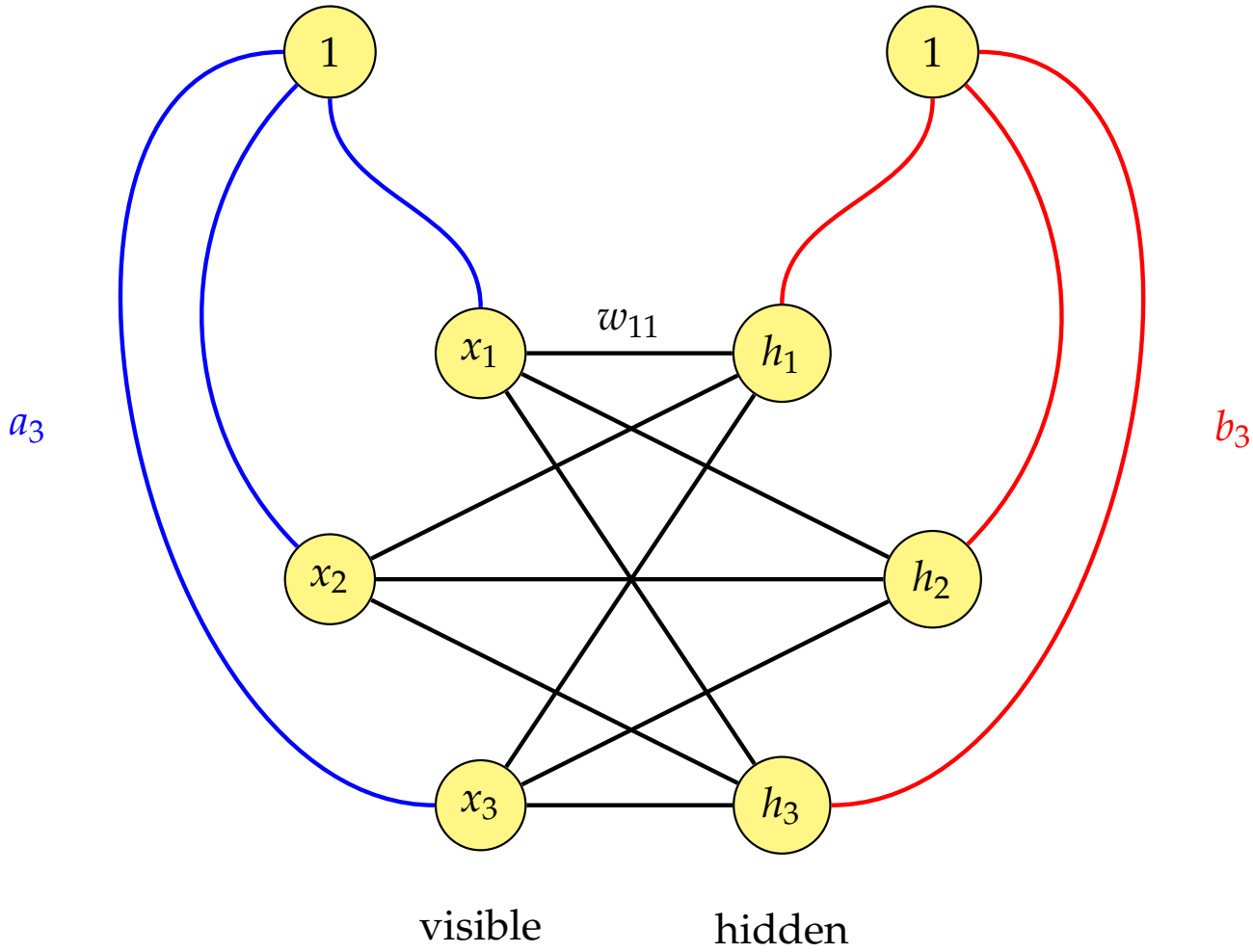
Machine Learning

> *Machine learning is the science of getting computers to act without being explicitly programmed.*

Feed-forward Neural Network



Restricted Boltzmann machines



Methods

Variational Monte Carlo (VMC)

Exploit the variational principle in order to obtain the ground state energy

$$
\begin{aligned}
E_0 < E_{\mathrm{VMC}} &= \frac{\int d\boldsymbol{R}\Psi_T(\boldsymbol{R})^*\hat{\mathcal{H}}\Psi_T(\boldsymbol{R})}{\int d\boldsymbol{R}\Psi_T(\boldsymbol{R})^*\Psi_T(\boldsymbol{R})} \\
&= \int d\boldsymbol{R}\underbrace{\frac{\Psi_T(\boldsymbol{R})^*\Psi_T(\boldsymbol{R})}{\int d\boldsymbol{R}\Psi_T(\boldsymbol{R})^*\Psi_T(\boldsymbol{R})}}_{P(\boldsymbol{R})}\cdot\underbrace{\frac{1}{\Psi_T(\boldsymbol{R})}\hat{\mathcal{H}}\Psi_T(\boldsymbol{R})}_{E_L(\boldsymbol{R})}
\end{aligned}
$$

Monte Carlo Integration

We attempt to solve the integral by sampling from the probability density function $P(\boldsymbol{r})$

$$E_{\mathrm{VMC}} = \int d\boldsymbol{R} E_L(\boldsymbol{R}) P(\boldsymbol{R})$$
$$\approx \frac{1}{M} \sum_{i=1}^{M} E_L(\boldsymbol{R}_i)$$

Trial Wave Function

$$P(\boldsymbol{R}) \propto \Psi_T(\boldsymbol{R})^* \Psi_T(\boldsymbol{R})$$

Use the Slater-Jastrow function as our trial wave function

$$\Psi_T(\boldsymbol{R}) = |\hat{D}(\boldsymbol{R})| J(\boldsymbol{R})$$

where the Slater matrix, $\hat{D}(\boldsymbol{R})$, contains all the single-particle functions

$$\hat{D}(\boldsymbol{R}) = \begin{pmatrix} \phi_1(\boldsymbol{r}_1) & \phi_2(\boldsymbol{r}_1) & \dots & \phi_N(\boldsymbol{r}_1) \\ \phi_1(\boldsymbol{r}_2) & \phi_2(\boldsymbol{r}_2) & \dots & \phi_N(\boldsymbol{r}_2) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_1(\boldsymbol{r}_N) & \phi_2(\boldsymbol{r}_N) & \dots & \phi_N(\boldsymbol{r}_N) \end{pmatrix}$$

Single-particle Functions

The Hermite functions,

$$\phi_n(\boldsymbol{r}) \propto H_n(\boldsymbol{r}) \exp\left(-\frac{1}{2}\alpha\omega|\boldsymbol{r}|^2\right),$$

are used as the single-particle functions for quantum dots in standard VMC. The Gaussian can be factorized out from the Slater determinant.

$$|\hat{D}(\boldsymbol{R};\alpha)| \propto \exp\left(-\frac{1}{2}\alpha\omega|\boldsymbol{R}|^2\right) \begin{vmatrix} H_1(\boldsymbol{r}_1) & H_2(\boldsymbol{r}_1) & \dots & H_N(\boldsymbol{r}_1) \\ H_1(\boldsymbol{r}_2) & H_2(\boldsymbol{r}_2) & \dots & H_N(\boldsymbol{r}_2) \\ \vdots & \vdots & \ddots & \vdots \\ H_1(\boldsymbol{r}_N) & H_2(\boldsymbol{r}_N) & \dots & H_N(\boldsymbol{r}_N) \end{vmatrix}$$

Restricted Boltzmann Machine

We use the marginal distribution of the visible units as the single-particle functions in the Slater determinant, and see if them can model the correlations

$$\phi_n(\boldsymbol{r}) \propto H_n(\boldsymbol{r})P(\boldsymbol{r};\boldsymbol{a},\boldsymbol{b},\boldsymbol{W})$$

where $P(\boldsymbol{r})$ is the marginal distribution of the visible units.

$$|\hat{D}(\boldsymbol{r};\boldsymbol{a},\boldsymbol{b},\boldsymbol{W})| \propto P(\boldsymbol{r};\boldsymbol{a},\boldsymbol{b},\boldsymbol{W}) \begin{vmatrix} H_1(\boldsymbol{r}_1) & H_2(\boldsymbol{r}_1) & \dots & H_N(\boldsymbol{r}_1) \\ H_1(\boldsymbol{r}_2) & H_2(\boldsymbol{r}_2) & \dots & H_N(\boldsymbol{r}_2) \\ \vdots & \vdots & \ddots & \vdots \\ H_1(\boldsymbol{r}_N) & H_2(\boldsymbol{r}_N) & \dots & H_N(\boldsymbol{r}_N) \end{vmatrix}$$

Jastrow Factor

The Jastrow factor is added to account for the correlations
Simple Jastrow factor

$$J(\boldsymbol{r};\boldsymbol{\beta}) = \exp\left(\sum_{i=1}^{N}\sum_{j>i}^{N}\beta_{ij}r_{ij}\right).$$

Padé-Jastrow factor

$$J(\boldsymbol{r};\beta) = \exp\left(\sum_{i=1}^{N}\sum_{j>i}^{N}\frac{a_{ij}r_{ij}}{1+\beta r_{ij}}\right).$$

Results

Ground State Energy

Table: $N = 2$

ω	RBM	RBM+SJ	RBM+PJ	HF *	Exact †
1/6	0.7036(1)	0.67684(7)	0.66715(6)	0.768675	2/3
1	3.0803(2)	3.02108(5)	2.999587(5)	3.1690	3

*Mariadason[4]

†Taut[5]

Ground State Energy

Table: $N = 20$

ω	RBM	RBM+SJ	RBM+PJ	HF ‡	VMC	
0.1	30.824(2)	30.567(3)	30.1553(9)	31.1902	30.0403(2)	29.
1.0	159.428(3)	156.816(4)	156.104(1)	158.004	155.8900(4)	155.

‡Mariadason[4]

§Høgberget[6]

Energy distribution

Distribution between various energy sources

One-body density

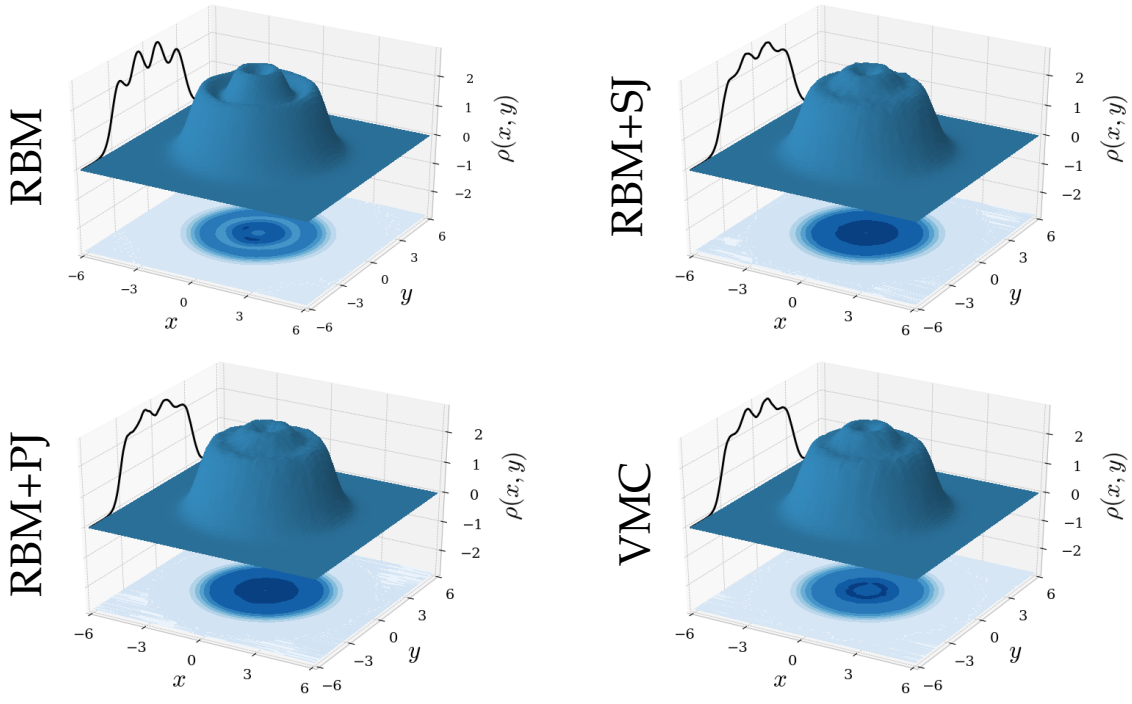


Figure: $N = 12$, $\omega = 1.0$

Low-frequency dots

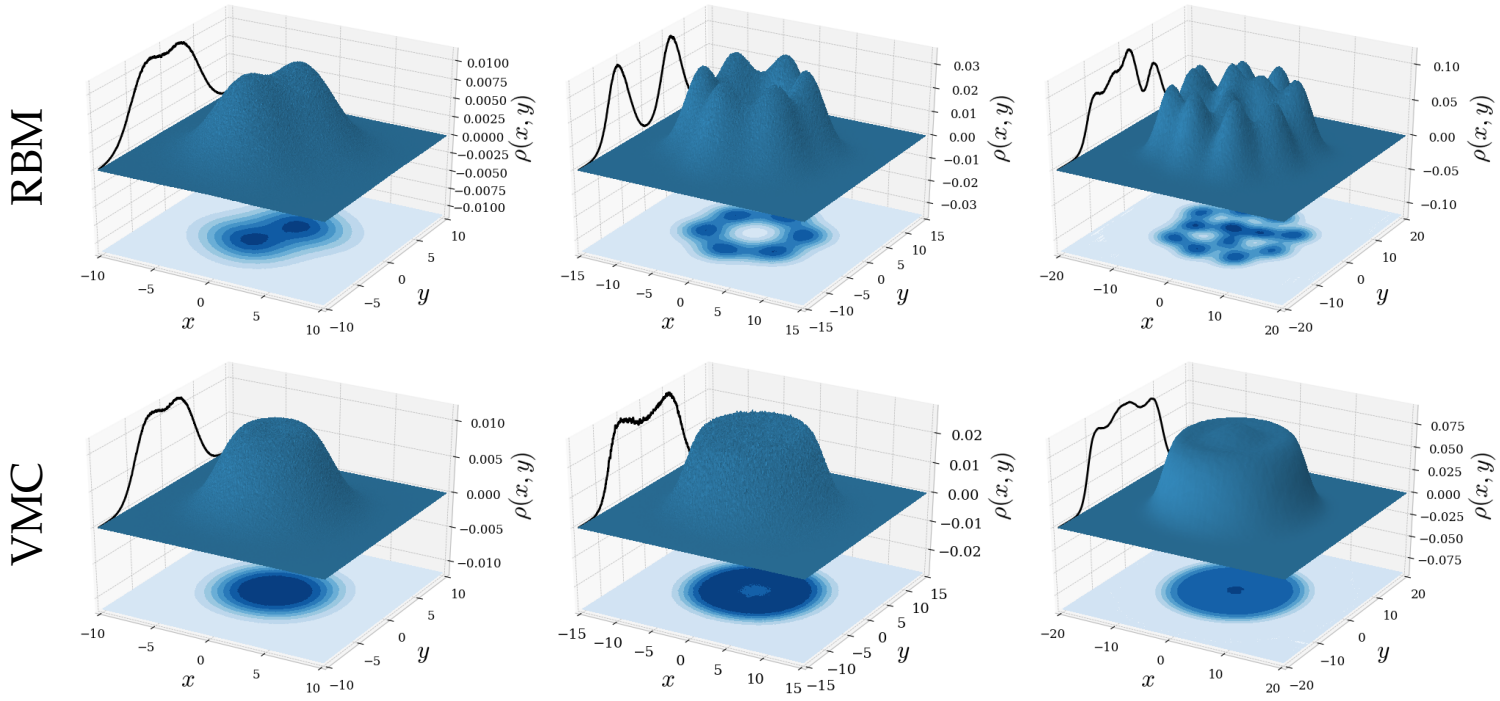


Figure: $\omega = 0.1$

Two-body density

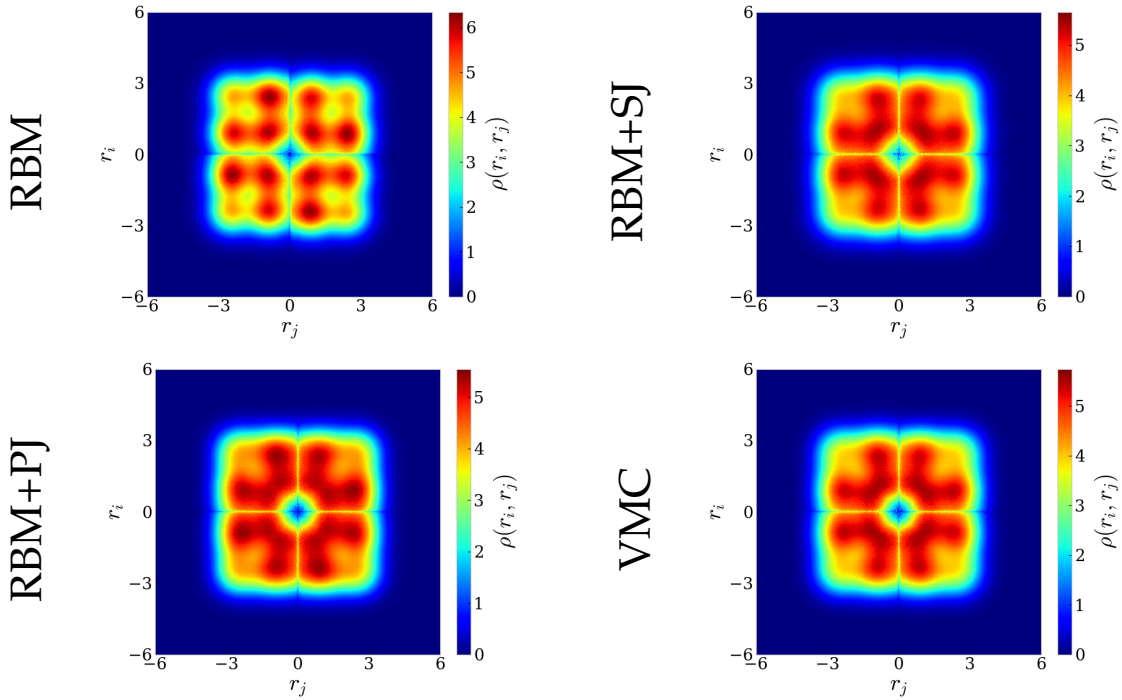
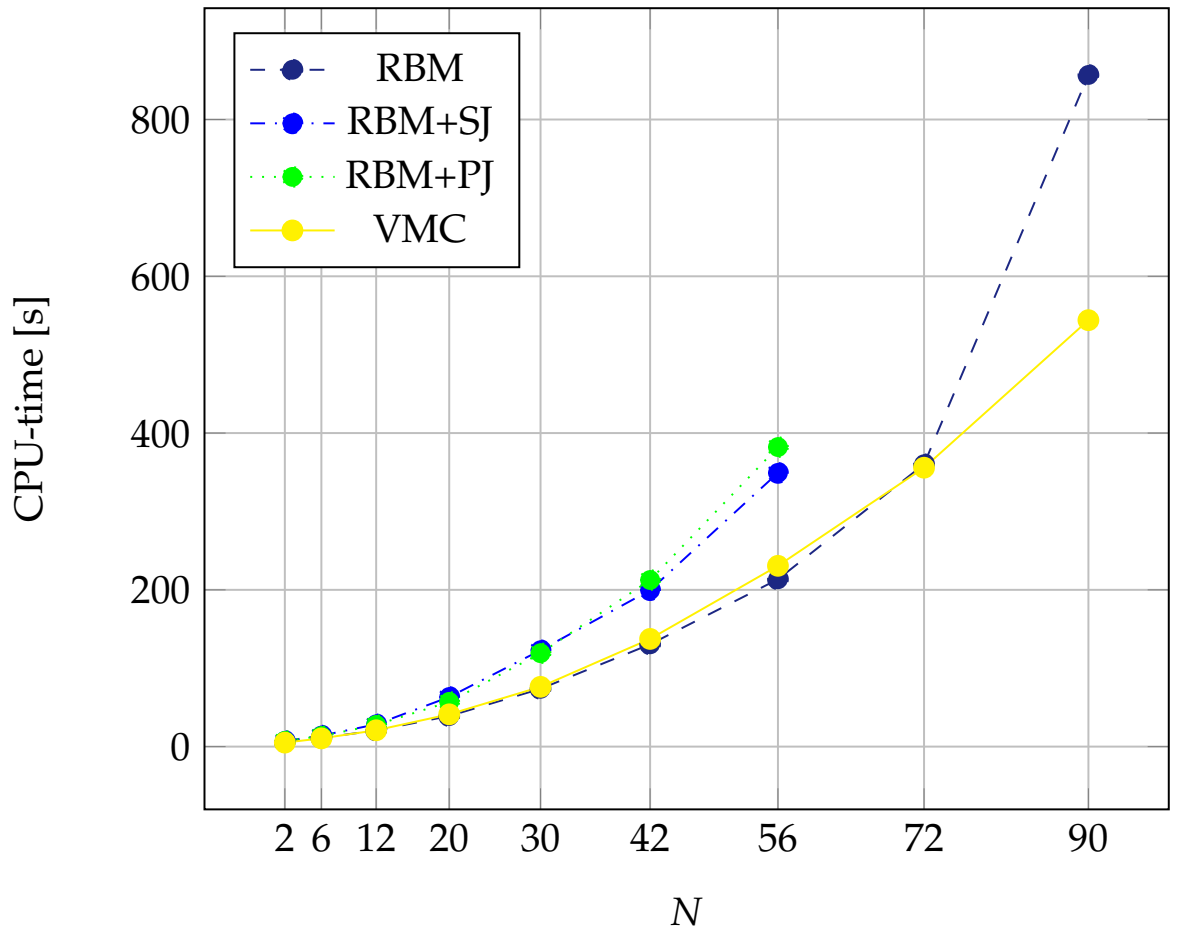


Figure: $N = 20$, $\omega = 1.0$

Computational Cost



Conclusion

Conclusions

- RBM is able to account for most of the correlations
- RBM+PJ implies to give a lower ground state energy and model the correlations better than a traditional VMC
- RBM+SJ is both more expensive and less accurate than its fellow methods, and we see no reason to choose it

Future Work

- Repeat the exercise using spherical coordinates - interactions are easier to model in spherical coordinates
- Check the ability of modeling the three-body correlations, considering nuclear systems
- Reduce the computational cost

Thank you!

References

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