A Design of Model Driven Cascade PID Controllers Using a Neural Network

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Abstract—Since most process systems have nonlinearities, it is necessary to consider the design of schemes to deal with such systems. In this paper, a new design scheme of PID controllers is proposed. This scheme is designed based on the IMC which is a kind of the model driven controllers. The internal model consists of the design-oriented model and the full model. The full model is deigned by using the neural network. The inner PID control system is first constructed for the augmented system which is composed of the controlled object and the internal model, and this control system is designed by the pole-assignment method. Furthermore, the outer PID controller is designed in order to remove the steady state error. Finally, the effectives of the newly proposed control scheme is numerically evaluated on a simulation example.

I. INTRODUCTION

In recent years, development of computers enables us to easily mount complicated control algorithms based on adaptive control theory or robust control theory. However, in the industrial processes, PID controllers[1] are still used widely and are employed for about 80% or more of control loops. The reasons are as follows. (1) the control structure is quit easy, (2) the physical meaning of the control parameters is clear, and (3) The know-how which an operators have can be harnessed in parameter adjustment. Therefore, it is still attractive to design PID controllers.

By the way, man's cerebral structure and cerebral mechanism have been partially clarified with development of physiology. These results are realized in engineering as a neural network(NN), and are applied to various problems, such as pattern matching, pattern recognition and learning control. Especially, since the NN is composed of some neurons which are nonlinear elements, it has an advantage that the nonlinear system can be dealt with. Therefore, various neural-net based controllers have been proposed until now. They are mainly classified into two types. One is that control inputs are directly generated by the NN[2]. Another is that control parameters are tuned by the NN[3]. However, according to these methods, the information about the sign of the system Jacobian is needed in advance, and it is a serious obstacle in implementing them.

On the other hand, the so-called model driven controller has been proposed[4], where the full model which describes the controlled object in detail is inserted in the control system. The model driven controller is attractive[5], because the control structure is simple and it has the high robustness for system uncertainties. As one of the model driven controllers, the internal model control(IMC) has been proposed[6]. How-

ever, to the best of our knowledge, there are few studies of IMC schemes for nonlinear systems. According to every IMC scheme considered in reference [7] and [8], the linear internal models are switched corresponding to the equilibrium point in order to deal with nonlinear systems. Although the relatively good control performance can be obtained, there is troublesome to design some linear models, and the stability of the control system in switching the models is not strictly guaranteed.

The main motivation in this study is to consider a new design scheme of nonlinear controllers based on the idea of the IMC which is a kind of the model driven controller. Concretely, a model driven cascade PID controller is proposed for nonlinear systems. The internal model is designed by using the NN. According to the newly proposed control scheme, it is not necessary to switch some models. Furthermore, the information about the sign of the system Jacobian is not also required.

This paper is organized as follows. The outline of the control system is firstly explained, and followed by the design scheme of the internal model. The inner PID controller is designed based on the pole-assignment scheme for the augmented system constructed by the controlled object and the internal model. Furthermore, the outer PID controller is succeedingly designed in order to remove the steady state error. Finally, the behavior of the newly proposed control scheme is examined on a simulation example.

II. CONTROLLER DESIGN

A. Outline

The block diagram of the proposed control system is shown in Fig.1. The internal model which is composed with the full model and the design-oriented model is firstly designed.

The full model describes the controlled object as exactly as possible. The transfer function $(y_a(t))$ thru u(t) of the augmented system surrounded by the dotted line becomes equivalent to one of the design-oriented model in the case where the full model is strictly designed. Here, the design-oriented model is freely designed.

Next, the inner PID control system is constructed for the augmented system, whose PID parameters are adjusted based on the pole-assignment. Then, the control system surrounded by the dashed and dotted line realizes $y_a(t) \to w(t)$. Here, the control objective is to make y(t) follow the reference signal. The steady state error occurs because $y(t) \neq y_a(t)$. In

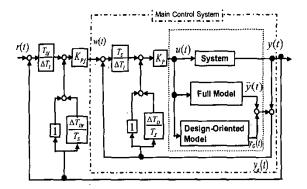


Fig. 1. Block diagram of the proposed control system

order to remove the steady state error, PID controller is further designed for the control system surrounded by the dashed and dotted line. That is, the cascade PID control system is constructed. Such a cascade PID controller is usually employed for controlling the reactor and/or the jacket temperature in chemical processes[9], and the control system shown in Fig.1 is also constructed based on this idea. In Fig.1, $\Delta (:= 1 - z^{-1})$ denotes the differential operator. PID parameters included in the outer PID controller are adjusted by the similar way to the inner PID controller.

B. Internal model design

The full model which describes the controlled object as exactly as possible is designed by using the NN. In order to reduce the burden in training the NN, the nonlinear system is firstly approximated by the linear model. And also, remaining nonlinear components are compensated by the NN. Then, the output of the full model, $\hat{y}(t)$, is given by the following equation:

$$\hat{y}(t) = y_L(t) + \hat{y}_n(t), \tag{1}$$

where $\hat{y}_n(t)$ and $y_L(t)$ denote the output of the NN and the output of the linear model, respectively.

The linear model is designed by linearizing around an equilibrium point, and given by

$$y_L(t) = G_n(z^{-1})u(t) \tag{2}$$

$$y_L(t) = G_n(z^{-1})u(t)$$
(2)
$$G_n(z^{-1}) = \frac{z^{-(k+1)}\widetilde{B}(z^{-1})}{\widetilde{A}(z^{-1})},$$
(3)

where u(t) and k denote the control input and the time-delay, respectively. And also, $\widetilde{A}(z^{-1})$ and $\widetilde{B}(z^{-1})$ are given by

$$\widetilde{A}(z^{-1}) = 1 + \widetilde{a_1}z^{-1} + \dots + \widetilde{a_n}z^{-n}$$
(4)

$$\widetilde{B}(z^{-1}) = \widetilde{b_0} + \widetilde{b_1}z^{-1} + \dots + \widetilde{b_m}z^{-m}. \tag{5}$$

The linear model can be more exactly designed if the parameters included in (4) and (5), n and m, are set as large values. However, the number of estimates becomes large corresponding to the values of n and m. Therefore, these parameters, nand m are determined by using the AIC criterion[10] in which the model structure and the accuracy of the model can be

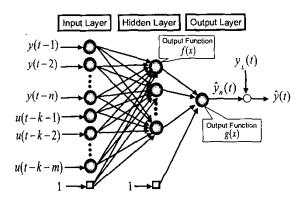


Fig. 2. Structure of the neural network.

simultaneously evaluated. The parameters \tilde{a}_i and \tilde{b}_i included in the linear model are estimated using the least squares method.

On the other hand, the NN has a multilayered structure as shown in Fig.2. The weighting coefficients, V_{ij} and W_j , included in the NN are trained by using the error back propagation(BP) method, so that $\hat{y}(t)$ approaches to the output of the system, y(t). The error criterion is as follows:

$$J = \frac{1}{2}\tilde{\varepsilon}^2(t) \tag{6}$$

$$\widetilde{\varepsilon}(t) := y(t) - \hat{y}(t).$$
 (7)

Furthermore, the following sigmoidal functions are employed:

$$f(x) = 2\left\{\frac{1}{1 + e^{-ax}} - \frac{1}{2}\right\} \tag{8}$$

$$g(x) = c \left\{ \frac{1}{1 + e^{-bx}} - \frac{1}{2} \right\},\tag{9}$$

where a, b and c are the user-specified parameters. Then, the update rules of the weighting coefficients V_{ij} and W_i are given

$$\Delta W_j = \eta_1 \tilde{\varepsilon} \left(\frac{c}{2} - \hat{y}_n\right) \left(\frac{c}{2} + \hat{y}_n\right) \frac{b}{c} H_j \tag{10}$$

$$\Delta V_{ij} = \eta_2 \tilde{\varepsilon} \frac{b}{c} \left(\frac{c}{2} - \hat{y}_n \right) \left(\frac{c}{2} + \hat{y}_n \right) \cdot \frac{a}{2} \left(1 - H_j \right) \left(1 + H_j \right) I_i W_j,$$
(11)

where I_i and H_i denote units in the input and hidden layers, respectively. And also, $\eta_1(>0)$ and $\eta_2(>0)$ are learning coefficients which are user-specified parameters. The full model is designed by the linear model and the NN mentioned above.

According to the conventional neural-net based controllers, the information with respect to the sign of the system Jacobian is required in designing the controller. Then, the system Jacobian is estimated by using an extra emulator or the least squares method. Thus, in order to obtain the information about the sign of the system Jacobian, an extra device is required. However, according to the newly proposed scheme, the information with the respect to the sign of the system Jacobian is not necessary in the update rules (10) and (11), and the above troublesomeness can be avoided. Especially, the effectiveness of the proposed scheme is remarkable in employing in real systems, e.g., process systems, because the time-delay is relatively large and/or can not be estimated exactly in advance in such systems.

Next, the following design-oriented model is designed:

$$y_G(t) = G_L(z^{-1})u(t)$$
 (12)

$$G_L(z^{-1}) = \frac{z^{-1}b_0}{A(z^{-1})},$$
 (13)

where $y_G(t)$ denotes the output of the design-oriented model, and $A(z^{-1})$ is given by

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2}. (14)$$

Note that the time-delay is set as unity in the design-oriented model (13). Therefore, the proposed controller can compensate the time-delay. Here, although the design-oriented model can be designed as $G_L(z^{-1})=1$, the control system is designed with high gains and the robustness for disturbances can not be necessarily guaranteed. Especially, in process systems, the rapid change in a control loop gives the influence to other control loops, and there is a case where the whole control system falls into unstable state. Therefore, the process control systems are usually constructed so that the controllers does not have high gains. In this paper, by using the least squares method, $A(z^{-1})$ is designed based on the historical data which can be obtained around an equilibrium point.

C. Inner PID controller design

The inner PID controller is designed for the augmented system surrounded by the dotted line. The control law is as follows:

$$u(t) = K_P \left\{ \frac{T_s}{\Lambda T_r} (w(t) - y_a(t)) - (1 + \frac{\Delta T_D}{T_s}) y_a(t) \right\}, (15)$$

where K_P , T_I and T_D are the proportional gain, the reset time and the derivative time, respectively. T_s denotes the sampling interval. Furthermore, the output of the augmented system, $y_a(t)$, is given by

$$y_a(t) = y(t) - \hat{y}(t) + y_G(t).$$
 (16)

Substituting (13) and (16) into (15) yields

$$u(t) = \frac{K_I A(z^{-1})}{T(z^{-1})} w(t) - \frac{(K_I + \Delta K_P + \Delta^2 K_D) A(z^{-1})}{T(z^{-1})} \varepsilon(t),$$

$$K_I = K_I T / T_I \text{ and } K_I = K_I T_I / T_I \text{ Furthermore}$$

where $K_I = K_P T_s / T_I$ and $K_D = K_P T_D / T_s$. Furthermore, $\varepsilon(t)$ and $T(z^{-1})$ are respectively defined by

$$\varepsilon(t) := y(t) - \hat{y}(t), \tag{18} [A.2]$$

$$T(z^{-1}) := \Delta A(z^{-1}) + z^{-1}b_0(K_I + \Delta K_P + \Delta^2 K_D).$$
 (19)

 $T(z^{-1})$ is the characteristic polynomial of 'Main Control System' shown in Fig.1. Therefore, by designing $T(z^{-1})$ desirably and solving (19), K_P , K_I and K_D are obtained.

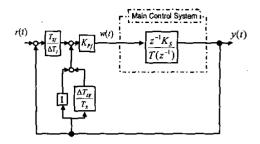


Fig. 3. Block diagram of outer PID control system.

Then, the property of the control system depends on zeros of $T(z^{-1})$. $T(z^{-1})$ is designed based on the following two features.

- (i) rise-time
- (ii) damping property

Thus, let $T(z^{-1})$ be the denominator part of the discrete-time version of the following desirable second-order continuous-time transfer function M(s)[11]:

$$M(s) = \frac{1}{1 + \sigma s + \mu(\sigma s)^2}.$$
 (20)

That is, $T(z^{-1})$ is defined as follows[12]:

$$T(z^{-1}) = 1 + t_1 z^{-1} + t_2 z^{-2}$$
 (21)

$$t_1 = -2\exp(-\frac{\rho}{2\mu})\cos(\frac{\sqrt{4\mu - 1}}{2\mu}\rho) \tag{22}$$

$$t_2 = \exp \frac{-\rho}{\mu} \tag{23}$$

$$\rho := T_s / \sigma \tag{24}$$

$$\mu := 0.25(1 - \delta) + 0.51\delta \tag{25}$$

where σ denotes the rise-time. μ denotes the damping coefficient which is adjustable by changing δ . In the case where $\delta=0$ and $\delta=1$, the step response of M(s) shows the Binomial model response and the Butterworth model response, respectively.

D. Outer PID controller design

In order to remove the steady state error, the outer PID controller is designed. Fig.3 show the block diagram of the outer PID control system. In Fig.3, K_S denotes the gain of the main closed-loop, that is, 'Main Control System'. The following assumptions are introduced in order to derive K_S .

[A.1] The reference signal r(t) is piecewise constant.

$$\overline{y} = a(\overline{u})\widehat{u} \tag{26}$$

 \bar{u} and \bar{y} denote respectively u(t) and y(t) in the steady state. And also, $a(\bar{u})$ is the static gain between \bar{u} and \bar{y} .

$$\lim_{t \to \infty} \varepsilon(t) = \overline{\varepsilon} < \infty \tag{27}$$

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(27) means that $\varepsilon(t)$ converges to the constant $\bar{\varepsilon}$ in the steady state.

Here, these assumptions are concluded in the only case where the controlled object is asymptotically stable around every equilibrium point. Then, from (17) and these assumptions, the control input in the steady state, \bar{u} , is expressed as:

$$\overline{u} = \frac{K_I A(1)}{T(1)} \overline{w} = \frac{1}{G_L(1)} \overline{w}, \qquad (28)$$

where T(1) and $G_L(1)$ are the static gains of $T(z^{-1})$ and $G_L(z^{-1})$,respectively. The second equation in (28) can be immediately derived by calculating the static gain of (19).

Other side, Using (26) and (28) yields the following relation:

$$\overline{y} = a(\overline{u})\overline{u} = \frac{a(\overline{u})}{G_{L}(1)}\overline{w}.$$
(29)

Then, the main closed-loop gain, K_S , is calculated by

$$K_S = \frac{a(\overline{u})T(1)}{G_L(1)}. (30)$$

Since the controlled object is nonlinear, $a(\bar{u})$ changes corresponding to the equilibrium point. Furthermore, $a(\bar{u})$ also changes by disturbances. For such reasons, $a(\bar{u})$ is replaced by the maximum value of $a(\bar{u})$, or the somewhat larger value than it, $a(\bar{u})_{\max}$. Mathematically,

$$K_S' = \frac{a(\overline{u})_{\text{max}}T(1)}{G_L(1)}. (31)$$

The characteristic polynomial $P(z^{-1})$ of the closed-loop system shown in Fig.3 is given by

$$P(z^{-1}) = \Delta T(z^{-1}) + z^{-1} K_S' K_{Pf} \left(\frac{T_s}{T_{If}} + \Delta + \Delta^2 \frac{T_D f}{T_s}\right). \tag{32}$$

Here, by designing desirable $P(z^{-1})$ and solving (32), K_{Pf} , T_{If} and T_{Df} can be obtained with the similar way to the case where the inner PID controller is designed. Note that the strictly pole-assignment can not be realized because K_S is replaced by K_S' , although the polynomial $P(z^{-1})$ can be designed by following σ and μ as a measure. However, the stability of the control system is guaranteed by setting larger K_S' than a real system gain.

III. SIMULATION EXAMPLE

In order to evaluate the effectiveness of the newly proposed scheme, a simulation example for the nonlinear system is considered.

The following Hammerstein method is discussed:

$$y(t) = 0.6y(t-1) - 0.1y(t-2) +1.2x(t-3) - 0.1x(t-4) + \xi(t) x(t) = 1.5u(t) - 1.5u^{2}(t) + 0.5u^{3}(t) ,$$
 (33)

where $\xi(t)$ denotes the white Gaussian noise with zero mean and variance 0.001. Note that this system includes the time-delay. The static property of the Hammerstein model is shown in Fig.4. ¿From Fig.4, it is clear that the model (33) includes nonlinearities.

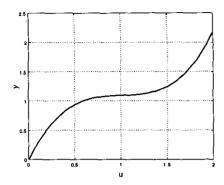


Fig. 4. Static property of Hammerstein model.

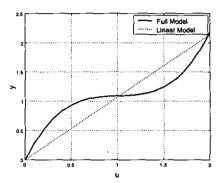


Fig. 5. Static property of the full model.

A. Controller design

First, the linear model was designed as follows:

$$\widetilde{A}(z^{-1}) = 1 - 0.608z^{-1} - 0.0327^{-2}$$

$$\widetilde{b_0} = 0.384.$$
 (34)

Next, the neural network was designed for the purpose of compensating remaining nonlinear properties. The user-specified parameters included in the neural network to be designed in this paper were determined as shown in Table 1

Table 1 User-specified parameters on the neural network
Note | Variables | Value |

Note	variables	value
Number of units in input layer	р	6
Number of units in hidden layer	q	20
User-specifi ed parameters	a	1
included in sigmoidal functions	(b (1
	c	4
Learnig rate	η_1	0.01
	η_2	0.01
Threshold in input layer	I(p)	1
Threshold in hidden layer	H(q)	1

By training the NN, the full model was obtained. The behavior of the full model is evaluated on Fig.5 and Fig.6, where Fig.5 and Fig.6 show the static and the dynamical properties, respectively.

In Fig.5, the dashed line and the solid lines denotes the static property using only the linear model (34) and one by

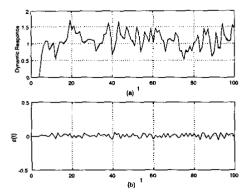


Fig. 6. The dynamical property of the full model.

the full model. On the other hand, the first figure in Fig.6 shows the dynamic behavior of the full model, and the second figure shows the error between the real system output and the full model output. From these figures, it is clear that the full model is designed adequately by using the linear model and the neural network.

Next, the design-oriented model is designed as follows:

$$A(z^{-1}) = 1 - 0.470z^{-1} - 0.103^{-2}$$

$$b_0 = 1.7507.$$
(35)

Furthermore, the desired characteristic polynomial $T(z^{-1})$ was designed as follows:

$$T(z^{-1}) = 1 - 0.271z^{-1} + 0.0183z^{-2},$$
 (36)

where $T(z^{-1})$ was designed by setting $\sigma=1.0$ and $\delta=0.0$ in (24) and (25). Then, PID parameters included in the inner PID controller were calculated as

$$K_P = 0.199, \quad T_I = 0.467, \quad T_D = 0.294.$$
 (37)

 $P(z^{-1})$ in (32) was also designed as the same polynomial as (37), that is, $P(z^{-1}) = T(z^{-1})$, add then PID parameters included in the outer PID controller are given by

$$K_{Pf} = 0.108, \quad T_{If} = 0.313, \quad T_{Df} = 0.0783.$$
 (38)

Furthermore, $a(\overline{u})_{max}=8$ and the reference signal r(t) is given as follows:

$$r(t) = \begin{cases} 0.5(0 \le t < 200) \\ 1.0(200 \le t < 400) \\ 1.5(400 \le t < 600) \\ 1.2(600 \le t \le 800). \end{cases}$$
 (39)

In order to examine the behavior for disturbances the following disturbances d(t) were put on the system:

$$d(t) = \begin{cases} 0 & t \le 300\\ 0.05 & t > 300. \end{cases}$$
 (40)

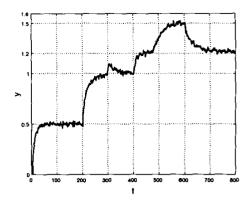


Fig. 7. Control result using the fixed PID control whose PID parameters are tuned by CHR method.

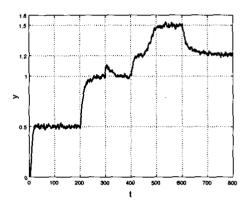


Fig. 8. Control result using the fixed PID control whose PID parameters are tuned by the IMC turning method.

B. Control results

For the purpose of comparison with the conventional schemes, the fixed PID control scheme widely used in industrial processes was employed, whose PID parameters were tuned by using Chien, Hrones & Reswick(CHR) method[13]. Then, PID parameters are $K_P=0.116,\ T_I=1.728$ and $T_D=1.250$.

The control result is shown in Fig.7. Owing to the nonlinearities, the control result is not good around 1.2.

Moreover, the fixed PID control whose PID parameters were adjusted using the IMC tuning scheme, was also employed, where PID parameters were $K_P=0.260,\ T_I=2.978,\ T_D=0.725$ and $T_f=0.273.$ Here, T_f denotes the pre-filter parameter, and refer the reference[14] in detail. The control result is shown in Fig.8. Although the relatively good control result in comparison with Fig.7, it is clear that Fig.8 also includes the influence by the nonlinear properties.

Furthermore, the conventional neural-net based controller which generated the control input using the NN[2], was employed. The control result is shown in Fig.9. In this case, since the time-delay is contained in the system, 3 sets of weighting coefficients corresponding to the time-delay are required, these sets are employed in turn. Therefore, the control result is

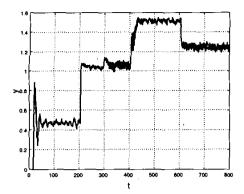


Fig. 9. Control result using the conventional neural-net based controller.

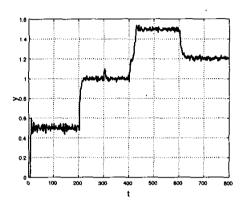


Fig. 10. Control result using the newly proposed control scheme.

oscillatory and cannot necessarily say it as a good control result.

On the other hand, the newly proposed control scheme was employed for this system, and then the control result as shown in Fig.10 and Fig.11 was obtained. Fig.10 and Fig.11 show the output signal and the corresponding trajectories of w(t), respectively. Even if the nonlinear properties and the time-delay are included in the controlled object, the good control performance can be obtained using the newly proposed scheme. Especially, it is clear that w(t) shown in Fig.11 is adequately adjusted corresponding to the nonlinearities.

The generalized capability of the newly proposed control scheme for non-trained reference signals have been already examined. The control results are omitted due to the page limit.

IV. CONCLUSIONS

In this paper, a new design scheme of model driven cascade PID controllers using the NN for nonlinear systems has been proposed. The proposed scheme is based on the idea of the internal model control, and the internal model is composed of the full model and the design-oriented model. The full model is designed using linear model and the NN. The NN plays a role of compensating nonlinearities which cannot be expressed by using the linear model. According to the newly

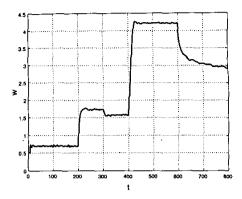


Fig. 11. Trajectory of w(t).

proposed control scheme, there is a strong advantage such that a priori information with respect to the system Jacobian is not required. Especially, the advantage gives us a suggestion that the newly proposed scheme enables us to deal with uncertain time-delay systems, and it is useful in implementing the proposed scheme to real systems. The investigation on this point is currently in our work.

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