

# Applying genetic algorithm to on-line updated PID neural network controllers for ball and plate system

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## Abstract

The PID (proportional-integral-derivative) neural network (PIDNN) controller based on genetic algorithm (GA) for ball and plate system is proposed in this paper. Genetic algorithm is applied in training weighting factor of multilayered forward neural network, thus the disadvantage of BP algorithm that easily fall into partial extreme value can be overcome and at the same time, the advantage of PID neural network controller that has simple structure and good dynamic and static performance is provided with. The simulation results show that the proposed controller has the adaptability, strong robustness and satisfactory control performance in the ball and plate system.

## 1. Introduction

Ball and plate system is a typical multi-variable plant, which is the extension of the traditional ball and beam system [1]. Stabilization and desired trajectory tracking control of the ball and plate system are challenging problems because the system possesses more degrees of freedom than the number of actuators. The system is also influenced by frictions, backlash effect in transmission, measurement time delay, parameters uncertainty and so on. The platform can be used as a benchmark to test various control schemes.

Stabilization control of the ball and plate system is to hold the ball in a specific position on the plate. Trajectory tracking control demands the ball follow the given position reference. Since the system is inherent nonlinear, the mathematical model is difficult to be derived. A practical result on static position control as well as smooth paths tracking was given in [2]. Ball position was measured by a touch pad rather than a camera. A linear quadratic state feedback regulator was designed after the system was linearized around a few operating points. Accuracy of stabilization control was  $5(mm)$ . Steady error in average was  $18(mm)$  in cir-

cle following while given tracking velocity was less than  $4.2(mm/s)$ . A hierarchical fuzzy control scheme with a T-S type fuzzy tracking controller was proposed in [3]. The fuzzy controller was built based on local linearized models in different state space regions. A fuzzy controller with online learning capability was reported on CE 151 [4]. Instead of fuzzy logic, a sliding mode controller was utilized to cope with system uncertainty [5]. A trajectory tracking experiment system was constructed based on single input rule modules (SIRMs) fuzzy logic. Maximum tracking error of circles was less than  $50(mm)$ .

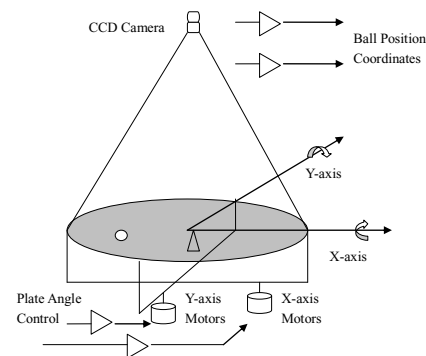


Figure 1. ball and plate system

As is well known conventional PID controllers have many advantages, which are simplicity and robustness, so that they are most widely used in various fields the industrial process control. Although PID controllers have strong abilities they are not suitable for the control of some multi-variable nonlinear systems, in which the P, I, and D parameters are difficult to choose and can hardly adapt to time varying of characteristics in wide range. Artificial neural networks can perform adaptive control through learning processes. According to advantages of Neural Network (NN) and PID controller, the PID Neural Network (PIDNN) control is proposed in this paper. It consists of proportional (P),

integral (I) and derivative (D) neurons and its weights are adjusted by the GA algorithms. Genetic algorithm is applied in training weighting factor of multilayered forward neural network, thus the disadvantage of BP algorithm that easily fall into partial extreme value can be overcome and at the same time, the advantage of PID neural network controller that has simple structure and good dynamic and static performance is provided with.

## 2. The Mathematics Model Of Ball And Plate System

Dynamical equations of system are obtained using Lagrange equation:

$$(m + \frac{I_b}{r^2})\ddot{x} - m\dot{x}\dot{\alpha}^2 - m\dot{y}\dot{\alpha}\dot{\beta} + mg\sin\alpha = 0 \quad (1)$$

$$(m + \frac{I_b}{r^2})\ddot{y} - m\dot{x}\dot{\beta}^2 - m\dot{y}\dot{\alpha}\dot{\beta} + mg\sin\beta = 0 \quad (2)$$

$$\tau_x = (I_p + I_b + mx^2)\ddot{\alpha} + 2mx\dot{x}\dot{\alpha} + mxy\ddot{\beta} + m\dot{x}y\dot{\beta} + m\dot{x}\dot{y}\dot{\beta} + mgx\cos\alpha \quad (3)$$

$$\tau_y = (I_p + I_b + mx^2)\ddot{\beta} + 2my\dot{y}\dot{\beta} + mxy\ddot{\alpha} + m\dot{x}y\dot{\alpha} + m\dot{x}\dot{y}\dot{\alpha} + mgy\cos\beta \quad (4)$$

System variables are selected as following:  $x$  is the displacement of the ball along the  $x$ -axis,  $y$  is the displacement of the ball along the  $y$ -axis,  $\alpha$  is the angle between  $x$ -axis of the plate and horizontal plane,  $\beta$  is the angle between  $y$ -axis of the plate and horizontal plane,  $\tau_x$  is the torque exerted on the plate in  $x$ -axis,  $\tau_y$  is the torque exerted on the plate in the  $y$ -axis.  $m$  is the mass of the ball,  $r$  is the radius of the ball,  $I_b$  is the ball inertia and  $I_p$  is the plate inertia. Usually,  $|\alpha| < 5^\circ, |\beta| < 5^\circ$ .

Define  $B = m/(m + I_b/r^2)$ , Then the system can be modeled in the state-space form as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{bmatrix} = \begin{bmatrix} x_2 \\ B(x_1x_4^2 + x_4x_5x_8 - g\sin x_3) \\ x_4 \\ 0 \\ x_6 \\ B(x_5x_8^2 + x_1x_4x_8 - g\sin x_7) \\ x_8 \\ 0 \end{bmatrix} \quad (5)$$

$$Y = C[x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8]^T \quad (6)$$

Where

$$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)^T = (x, \dot{x}, \alpha, \dot{\alpha}, y, \dot{y}, \beta, \dot{\beta})^T$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

## 3. PID Neural Network Control System Structure And Algorithm

The PID neural network (Fig.2.) is a 3-layer forward network accept modulated value  $y$  and given value  $r$ , while the hidden layer has 3 neurons with input and output functions being proportion ( $P$ ) integral ( $I$ ) differential ( $D$ ) function. There is one neuron in the output layer to give control value the output object needed. In PID neural network, the number of neurons within each layer, the connection method and the connection weight is determined by basic principal of PID control rule and previous experience, making sure the stability and fast convergence of system. PID neural network does not take advantage of traditional PID controller, but also has the parallel structure, the ability of learning and remembering and the ability of approaching any function with multiple network of neural network.

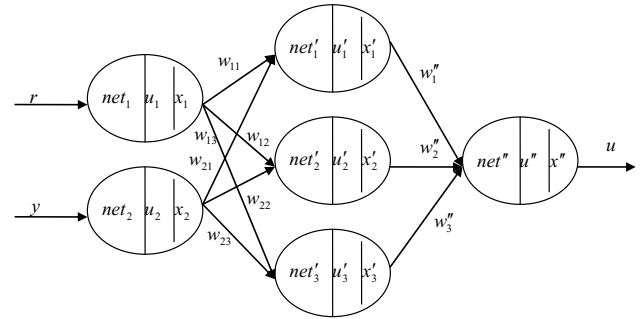


Figure 2. PID neural network structure

### 3.1. PID neural network structure

If controlled object has 2 input and 1 output, the forward function of this PID neural network at sampling time  $k$  is as follows:

1) Input layer

The input layer has 2 same neurons, its input relationship is:

$$\begin{cases} net_1(k) = r(k) \\ net_2(k) = y(k) \end{cases} \quad (7)$$

The input layer neurons state is:

$$u_i(k) = net_i(k) \quad (8)$$

The output of the input layer:

$$x_i(k) = \begin{cases} 1 & u_i(k) > 1 \\ u_i(k) & -1 \leq u_i(k) \leq 1 \\ -1 & u_i(k) < -1 \end{cases} \quad (9)$$

Where  $i = 1, 2$ .

## 2) Hidden layer

The hidden layer of PID neural network is consisted with  $3 \times 1$  neurons, which are 1 proportion neurons, 1 integral neurons and 1 differential neurons respectively. The formula of hidden layer input relationship is:

$$net'_j(k) = \sum_{i=1}^2 \omega_{ij}x_i(k) \quad (10)$$

Where  $j = 1, 2, 3$ . The  $\omega_{ij}$  is the weight between inputs layer and hidden layer.

There are three kinds of input output function in hidden layer.

Proportional neurons state is:

$$u'_1 = net'_1(k) \quad (11)$$

Integral neurons state is:

$$u'_2 = net'_2(k-1) + net'_2(k) \quad (12)$$

Derivative neurons state is:

$$u'_3 = net'_3(k) - net'_3(k-1) \quad (13)$$

Hidden-layer neurons output is:

$$x'_j(k) = \begin{cases} 1 & u'_j(k) > 1 \\ u'_j(k) & -1 \leq u'_j(k) \leq 1 \\ -1 & u'_j(k) < -1 \end{cases} \quad (14)$$

Where  $j = 1, 2, 3$ .

## 3) Output layer

The output layer only has one neuron. The output of the neuron state is:

$$net''(k) = \sum_{j=1}^3 \omega'_j x'_j(k) \quad (15)$$

Where  $x'_j(k)$  is the output of the hidden layer, The  $\omega'_j$  is the weight between hidden layer and output layer.

Hidden-layer neurons output is:

$$x''(k) = \begin{cases} 1 & u''(k) > 1 \\ u''(k) & -1 \leq u''(k) \leq 1 \\ -1 & u''(k) < -1 \end{cases} \quad (16)$$

## 3.2. The GA Online Optimization

Genetic algorithms, introduced by Holland, are based on the idea of engendering new solutions from parent solutions, employing mechanisms inspired by genetics. The best offsprings of the parent solutions are selected for a next iteration of mating, thereby proceeding in an evolutionary fashion that encourages the survival of the fittest, according to Darwin's theory.

In the following text the controller will be optimized by genetic algorithm. The optimization parameters are  $\omega_{ij}$ , the number of variables is  $2 \times 3 + 3 = 9$ . And  $K_r, K_y, K_u$ . So the total number of variables is 12.

Eight bits binary code is adopted in encoding. Here, 12 parameters need training. So the length of each sample is 96 bit. The fitness function is shown as formula. Evaluate the fitness value of each output.

$$J = \frac{1}{2} \sum_{i=1}^N (u_i^* - \hat{u}_i^*)^2 \quad (17)$$

Where  $\hat{u}_i^*$  is expectation output,  $u_i^*$  is reality output.

Select those offspring individuals with a less fitness-value for the next generation. In this paper, Roulette wheel is adopted.

In order to improve training speed, Crossover and mutation probability  $p_c, p_m$  are adjusted according follow formula.

$$p_c = \exp(-N/M) \quad (18)$$

$$p_m = \exp(-0.05 * N/M) - 1 \quad (19)$$

where  $N$  is the currently number of generation, and  $M$  is the total number of generation.

## 4. Control System Structure And Design

In the ball and plate system, it is supposed that the ball remains in contact with the plate and the rolling occurs without slipping, which imposes a constraint on the rotation acceleration of the plate. Because of the low velocity and acceleration of the plate rotation, the mutual interactions of both coordinates can be negligible. So motion of ball on the plate can be divided into motion along the  $x$ -axis and motion along the  $y$ -axis. The  $x$ -axis motion is regulated :

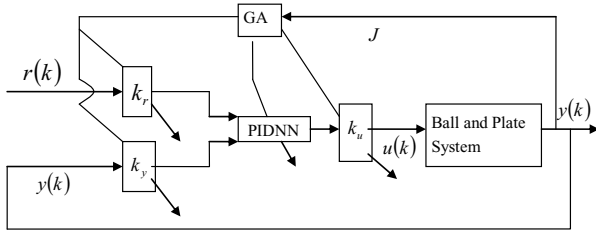
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ B(x_1 x_4^2 - g \sin x_3) \\ x_4 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_x \quad (20)$$

Namely, the ball-plate system can be regarded as two individual sub-systems and we can control both coordinates independently. Due to the symmetry of  $x$  direction and  $y$  direction, hereafter we will discuss  $x$  direction only.

The entire control system is shown in Fig.3.

## 5. Simulation Results

The initialization module is used for determining the number of input variable and output variable of neural network. Meanwhile, the initial value selection of network connection weight is according to PID control rule.



**Figure 3. PID neural network controller based on GA**

For the PID network controller, the scaling factors were also obtained as  $K_r = 1$ ,  $K_y = 1$ ,  $K_u = 1$ . The initial value of weight between input layer and hidden layer follows the  $(r, y) \rightarrow e$  mapping requirement:  $\omega_{1j} = 1$ ,  $\omega_{2j} = -1$ .

In PID neural network, the initial weight between hidden layer and output layer is determined by basic principal of PID control rule and previous experience:  $\omega'_1 = K_p$ ,  $\omega'_2 = K_i = K_p/T_i$ ,  $\omega'_3 = K_d = K_p \times T_d$ .

So the initial output of the PID neural network is:

$$x''(k) = K_p e(k) + \frac{K_p}{T_i} \sum_{i=0}^K e(i) + K_p T_d [e(k) - e(k-1)] \quad (21)$$

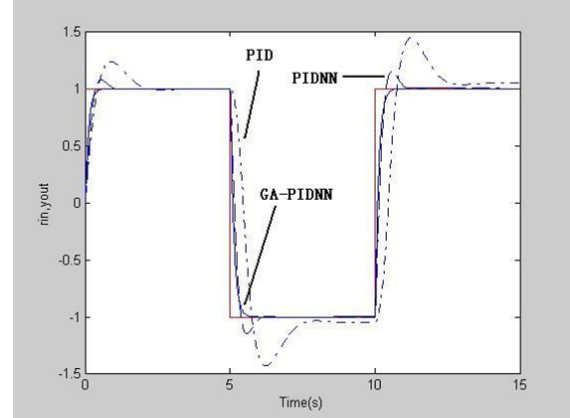
We train the PID neural network on GA, Let chromosome number in each generation Npop is 30, the number of new generation is 50, the input variable interval is  $\Delta\omega_{ij} \in [-1, 1]$ ,  $\Delta\omega'_j \in [-10, 10]$ ,  $\Delta K_r \in [0, 1]$ ,  $\Delta K_y \in [0, 1]$ ,  $\Delta K_u \in [0, 10]$  and  $f = 1/(1 + J)^2$ .

The structure of PID neural network and the initial value of input parameters were given. In order to improve the predicting accuracy, we use the GA algorithm mentioned to update the parameters in PID neural network.

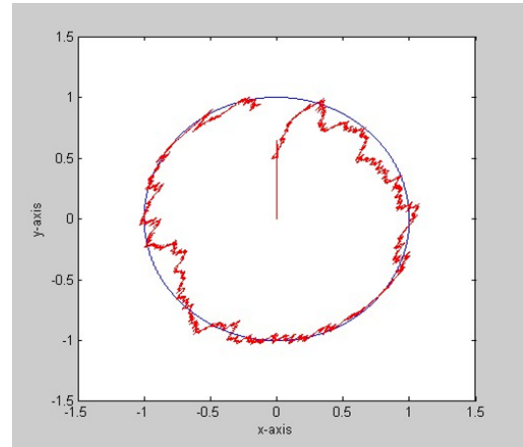
When the given inputs of control system are square wave, the simulation results are shown in Fig.4. The simulation results of the proposed control method in this paper is compared with PID control method and conventional PID neural network control method, the contrast simulation results show that the proposed method has achieves efficaciously control in ball and plate system, and has satisfactory control performance.

For the tracking problem, it is especially challenging. Because motion of the steel ball on rigid plate is subject to environment change, accordingly anterior standpoint, it belongs a nonholonomic under-actuated dynamics system. When the ball is set a circle trajectory, its velocity need to change at all time. Therefore, this trajectory is the most difficult than others.

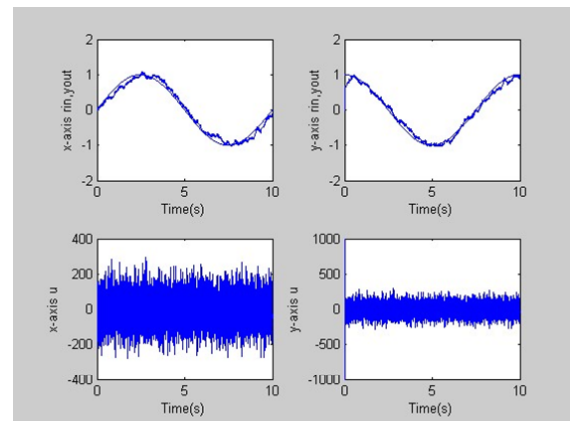
As mentioned above, we had fulfilled two texts. Its



**Figure 4. response on a square reference signal**

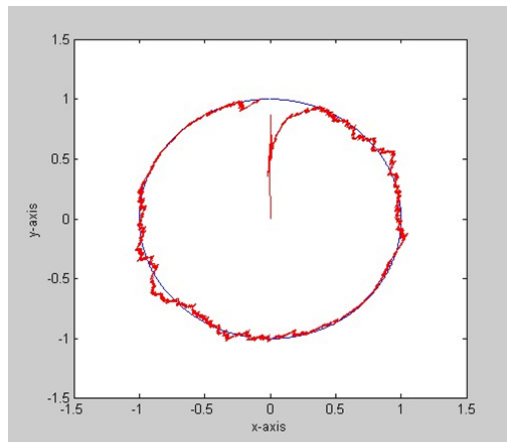


**Figure 5. Tracking results using conventional PIDNN controller**

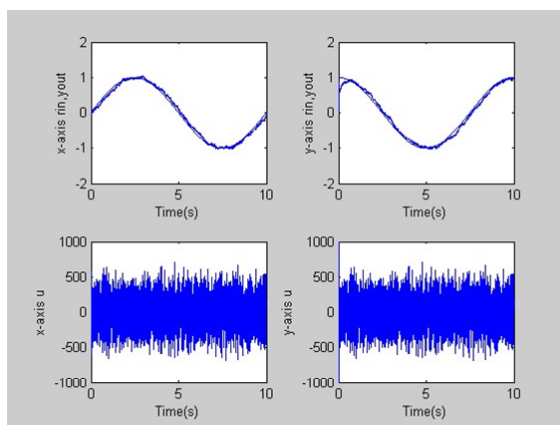


**Figure 6. the x-axis and y-axis results using conventional PIDNN controller**

two dimension data curves are shown in Fig.5-8. The contrast simulation results show the GA-PIDNN controller presented excellent static and dynamic performance.



**Figure 7. Tracking results using GA-PIDNN controller**



**Figure 8. the x-axis and y-axis results using GA-PIDNN controller**

## 6. Conclusion

In this paper a GA-PIDNN has been proposed and tested. Applying technology of GA in PID neural network system of ball and plate system is helpful to improve the dynamic properties and stability. Genetic algorithm is applied in training weighting factor of multilayered forward neural network, thus the disadvantage of BP algorithm that easily fall into partial extreme value can be overcome and at the same time, the advantage of PID neural network controller that has simple structure and good dynamic and static

performance is provided with. Design and simulation with MATLAB, Simulation results show that the proposed controller have the adaptability, strong robustness and satisfactory control performance in the ball and plate system, the training time of PIDNN is short and the final control result is very good.

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