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High Performance Single Tank Level Control as an Example for Control Teaching

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***Abstract*— A detailed analysis of a PI tank level control is performed with respect to both load disturbance attenuation and set-point change. Certain performance indices are given providing guidelines for achievable accuracy and controller settings. Exact formulas for extrema of time responses are derived, which use a novel parametrization of system poles. For transfer between equilibria under control signal limitations, set-point generators with a feed-forward from the reference signal are proposed. The effect of an anti-windup controller augmentation is examined. A method based on solution of the non-linear differential equation describing the tank draining is proposed for a tank model identification. Moreover, invariance of the control system properties with respect to the actual equilibrium is highlighted.**

I. Introduction

A model of a liquid tank depicted in Fig.1 can be found in almost every textbook on control engineering. Classical P and PI control of such system is the subject of many control teaching laboratory experiments, both physical [1]-[2], and virtual [11]. As a result, every year hundreds if not thousands of students all over the world get in touch with this problem. Unfortunately, despite its popularity and importance for control teaching, its theory is in our opinion surprisingly poor. The most advanced approach to be found in the literature, [1], [2], [16], based on pole placement which neglects the system zeros, does not allow to determine important control system properties. As a result, no appropriate tuning rules are available and the controller tuning is to be completed experimentally. Unfortunately, there is neither appropriate guidance on how to do it, nor it is known what is the performance limit. In consequence even poorly tuned controllers may appear to be tuned properly [2], [16], [11]. The aim of this paper is to fill this gap.

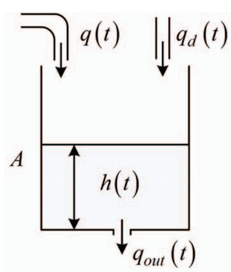

Fig. 1. Water tank, $q(t)$ - control input, $q_d(t)$ - disturbance

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II. Tank Model

A well known equation of the single tank has the following form

$$A\frac{dh\left(t\right)}{dt}=q\left(t\right)-\kappa\sqrt{h\left(t\right)}+q_d(t) \tag{1}$$

with

$$\kappa=c_d a\sqrt{2g}, \tag{2}$$

where A denotes a cross-sectional area of the tank, a is a cross-sectional area of an outlet orifice, c_d is a coefficent of draining[1], $h(t)$ liquid level and $q(t)$ input flow. Let us assume certain nominal values q_N and h_N[2] that fulfill the equation

$$q_N=\kappa\sqrt{h_N}. \tag{3}$$

Then we can define normalized variables $u(t)$ and $x(t)$

$$u(t)=q(t)/q_N,\ x(t)=h(t)/h_N,\ d(t)=q_d(t)/q_N \tag{4}$$

and the normalized equation becomes

$$T_N\frac{dx\left(t\right)}{dt}=u\left(t\right)-\sqrt{x\left(t\right)}+d(t), \tag{5}$$

where

$$T_N=\frac{Ah_N}{q_N}=\frac{A}{c_d a}\sqrt{\frac{h_N}{2g}}. \tag{6}$$

For a constant $u(t)=u$ and $d(t)=0$ the differential equation (5) has a solution

$$\sqrt{x_0}-\sqrt{x}+u\ln\left|\frac{u-\sqrt{x_0}}{u-\sqrt{x}}\right|=\frac{t}{2T_N},\ t\geq 0. \tag{7}$$

For $u=0$ equation in (7) becomes

$$\sqrt{x_0}-\sqrt{x}=\frac{t}{2T_N} \tag{8}$$

and can be used to estimate the value of T_N based on the time t to drain the tank from x_0 to x.

a) b)

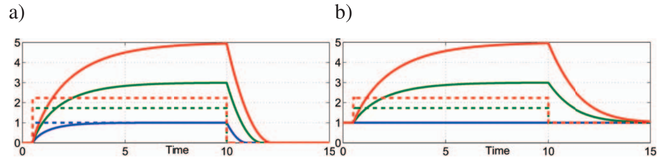

Fig. 2. a) Filling up from zero and complete emptying, b) Draining to nonzero level. Solid lines - level, dotted lines - flow

[1]$c_d=0.62$ for circular orifices
[2]A possible choice is e.g. $q_N=q_{min}$, or $h_N=h_{max}$

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Let us linearize equation in (5) around a working point (u_0, x_0) under the assumption $d(t) = 0$

$$u\left(t\right) = u_0 + \Delta u\left(t\right),\ x\left(t\right) = x_0 + \Delta x\left(t\right). \tag{9}$$

Then taking the linearized equation

$$T_N \frac{d}{dt} \Delta x\left(t\right) = \Delta u\left(t\right) - \frac{\Delta x\left(t\right)}{2\sqrt{x_0}} \tag{10}$$

into account, the transfer function becomes

$$\frac{\Delta X\left(s\right)}{\Delta U\left(s\right)} = \frac{2u_0}{2T_N u_0 s + 1}. \tag{11}$$

In further considerations we will use a normalized time variable $t' = t/2T_N$, which corresponds to the normalized Laplace variable $s' = 2T_N s$. However, in order not to introduce too many variables, we retain symbols t and s for normalized variables, i.e. we will use the following

$$\frac{\Delta X\left(s\right)}{\Delta U\left(s\right)} = \frac{2u_0}{u_0 s + 1}, \tag{12}$$

remembering that t and s denote normalized variables. A characteristic feature of the transfer function in (12) is that the value of its first Markov parameter equals to 2 in spite of the working point defined by u_0. This means that the high frequency gain does not depend on u_0. This property, illustrated in Fig. 3, will have positive influence on control systems robustness against changes of the working point.

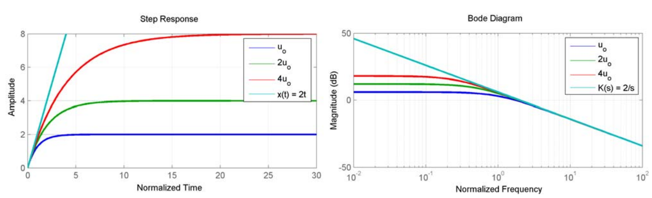

Fig. 3. Step responses and Bode plots for different working points; linearized model

III. Basic controller design

Consider a control system depicted in Fig.4 for the linearized model with the PI controller described by

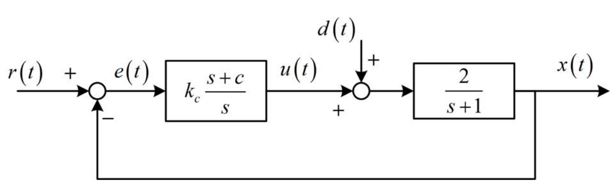

Fig. 4. Closed loop control system

$$K_c(s) = k_c\left(1 + \frac{c}{s}\right) = k_c \frac{s+c}{s}. \tag{13}$$

In order to retain notation simplicity we will omit the symbol Δ assuming that e.g. $x(t)$ means $\Delta x(t)$. Denote

$$G_{xr}(s) = X(s)/R(s), G_{xd}(s) = X(s)/D(s),$$
$$G_{ur}(s) = U(s)/R(s), G_{ud}(s) = U(s)/D(s).$$

The goal of the controller design is to find settings that assure high control quality characterized by fast set-point following, small overshoot and efficient disturbance attenuation. The simplest design decision is to take the proportional controller putting $c = 0$. Denote $k = 2k_c$. Then

$$G_{xr}(s) = \frac{k}{s+k+1},\ G_{xd}(s) = \frac{2}{s+k+1}. \tag{14}$$

Unfortunately, this control system exhibits steady-state error. However, choosing high value of the gain k_c one can made the control system arbitrary fast and the steady-state error arbitrary small. Moreover, the steady-state error from the constant reference value r can be eliminated using a nonlinear feedforward $u_{ff} = \sqrt{r}$.

In the case of zero steady-state disturbance error requirement a PI controller is necessary. The simplest design decision is to take $c = 1$. Then closed loop transfer functions take forms

$$G_{xr}(s) = \frac{k}{s+k},\ G_{xd}(s) = \frac{2s}{(s+k)(s+1)} \tag{15}$$

with time constant equal to the inverse of k. Unfortunately, the cancelled pole at $s = -1$ appears in the disturbance output making it slow. It will be shown that $c > 1$ gives better disturbance attenuation. Then, for arbitrary c, we have

$$G_{xr}(s) = \frac{k(s+c)}{s^2 + (k+1)s + kc} = \frac{k(s+c)}{(s+s_1)(s+s_2)}, \tag{16}$$

$$G_{ur}(s) = \frac{k_c(s+c)(s+1)}{s^2 + (k+1)s + kc} = \frac{k_c(s+c)(s+1)}{(s+s_1)(s+s_2)}, \tag{17}$$

$$G_{xd}(s) = \frac{2s}{s^2 + (k+1)s + kc} = \frac{2s}{(s+s_1)(s+s_2)}, \tag{18}$$

$$G_{ud}(s) = \frac{-k(s+c)}{s^2 + (k+1)s + kc} = \frac{-k(s+c)}{(s+s_1)(s+s_2)}. \tag{19}$$

Certain measures of quality can be defined based on (16)-(19). The area under the transient error caused by the disturbance $d(t) = d_0 1(t)$ is

$$I_1 = \int_0^\infty x(t)dt = \lim_{s\to 0} s\left[\frac{1}{s} G_{xd}(s) \frac{d_0}{s}\right] = \frac{d_0}{ck_c}. \tag{20}$$

Similarly, the steady state error caused by a set-point ramp $r(t) = \dot{r}_0 t 1(t)$ is

$$e_\infty = \lim_{s\to 0} s\left[1 - G_{xr}(s)\right] \frac{\dot{r}_0}{s^2} = \frac{\dot{r}_0}{2ck_c}. \tag{21}$$

From equations (16)-(19) it is clear that for every combination of $k > 0$ and $c > 0$ the closed loop system is asymptotically stable, both $I_1 \to 0$, $e_\infty \to 0$ as $k_c c \to \infty$, and there is no steady-state error under a constant set-point and constant disturbance. Further considerations can be very well explained using root loci of the control system presented in Fig. 5. The most interesting one is the locus for $c > 1$. Its complex part is defined [3] as a circle with the centre at $-c$ and the radius $\sqrt{c(c-1)}$

$$(\sigma + c)^2 + \omega^2 = c(c-1),\ \text{for } c > 1. \tag{22}$$

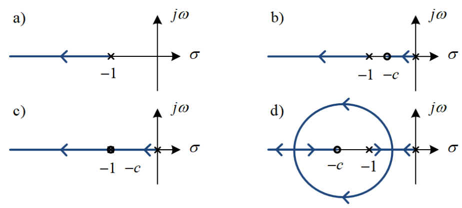

Fig. 5. Root loci: a) $c = 0$; b) $c < 1$; c) $c = 1$; d) $c > 1$

In the paper, we will consider complex roots on the circle after leaving the real axis for $k_c > c - 1/2 - \sqrt{c(c-1)}$, and the real roots after leaving the circle for values of $k_c > c - 1/2 + \sqrt{c(c-1)}$. Observe, that in spite of the case considered, one root tends to $-\infty$ and the other to $-c$ as $k \to \infty$. Since large values of k_c are desired, let us examine their influence on transients while assuming increasing values of k_c and constant c, see also [5], [14], [15]. From equation (27) it follows that $s_1 \to k, s_2 \to c$ as $k_c \to \infty$. As a result, for k_c large enough the transfer functions defined in (16)-(19) become

$$G_{xr}(s) \simeq \frac{k}{s+k}, \qquad G_{ur}(s) \simeq \frac{k_c(s+1)}{s+k} \tag{23}$$

$$G_{xd}(s) \simeq \frac{2s}{(s+k)(s+c)}, \qquad G_{ud}(s) \simeq \frac{-k}{s+k} \tag{24}$$

and the control system becomes very fast. From equation in (24) it follows that the value of c influences the disturbance attenuation ability. Indeed, from (24) it follows that for $d(t) = 1(t)$

$$x(t) = \frac{2}{k-c}(e^{-ct} - e^{-kt}). \tag{25}$$

Denote by x_m the maximum value of $x(t)$ in (25), and by t_m the instant such that $x_m = x(t_m)$. Then

$$x_m = \frac{2}{k}\left(\frac{c}{k}\right)^{\frac{c}{k-c}}, \; t_m = \frac{1}{k-c}\ln\frac{k}{c}. \tag{26}$$

From equations in (26) it is clear that the larger k the shorter time t_m is needed to reach the maximum, and the smaller value of the maximum x_m. Both of them tend to 0 as $k \to \infty$ regardless of c. From equation (25) it follows that for finite k, the larger c the faster the control error tends to zero. It appears that even $k_c = 10$ is great enough for one root almost to cancel the zero $c = -3$.

It should, however, be noticed that real systems always have some additional dynamics, like a certain inertia of the liquid supply system, which will become apparent at higher gains, where the performance will start to deteriorate. Another good example of such system is the Coupled Tank Apparatus, whose properties are described in [9], [10]

IV. Advanced PI control design

Control system characteristics depend on both poles and zeros of transfer functions in (16)-(19). In this subsection, exact formulas will be given that characterize transients for arbitrary, not necessarily large, values of k_c and c.

To this end, a novel parametrization of roots will be used with parameters θ and ϕ determining the shape of respective time responses, and σ that is mainly responsible for the time-scale.

A. Poles

The roots $-s_1$ and $-s_2$ depend on the value $\Delta = (k+1)^2 - 4kc$

$$s_{1,2} = \frac{1}{2}\left(k + 1 \pm \sqrt{(k+1)^2 - 4kc}\right). \tag{27}$$

For $\Delta > 0$ there are two real roots, and

$$s_{1,2} = \sigma \pm \delta = \sigma(1 \pm \phi), \; \phi = \frac{\delta}{\sigma} \le 1 \tag{28}$$

with

$$\sigma = \frac{1}{2}(k+1), \; \delta = \frac{1}{2}\sqrt{(k+1)^2 - 4kc} \tag{29}$$

and then

$$\chi(s) = (s + \sigma + \delta)(s + \sigma - \delta) \tag{30}$$
$$= s^2 + 2\sigma s + \sigma^2(1 - \phi^2). \tag{31}$$

For $\Delta < 0$ the roots are complex conjugate and

$$s_{1,2} = \sigma \pm j\omega = \sigma(1 \pm j\theta), \; \theta = \frac{\omega}{\sigma} \tag{32}$$

with

$$\sigma = \frac{1}{2}(k+1), \; \omega = \frac{1}{2}\sqrt{4kc - (k+1)^2} \tag{33}$$

and then

$$\chi(s) = (s + \sigma + j\omega)(s + \sigma - j\omega) \tag{34}$$
$$= s^2 + 2\sigma s + \sigma^2(1 + \theta^2). \tag{35}$$

For $\Delta = 0$ ($\theta = 0$ or $\phi = 0$)

$$s_{1,2} = \sigma = \frac{1}{2}(k+1) \tag{36}$$

and then

$$\chi(s) = (s + \sigma)^2 = s^2 + 2\sigma s + \sigma^2. \tag{37}$$

The relationship between θ, or ϕ, and the controller parameters is displayed in Fig.6.

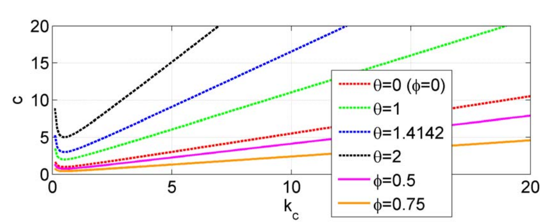

Fig. 6. Choice of c for given θ or ϕ as a function of k_c

In the next subsections we will examine influence of controller parameters on responses to stepwise reference and disturbance changes[4].

[3]Analytical expression in (22) results from $\Im\left\{ \frac{s+c}{s(s+1)}\Big|_{s=\sigma+j\omega} \right\} = 0$

[4]The problem of step response of systems with finite zeros is discussed in control literature e.g. [6], [7], [8], [12], [13], but only qualitatively, and no closed-form formulas similar to the ones derived here are provided.

B. Disturbance response

1) Real roots: For the transfer function

$$G_{xd}(s) = \frac{2s}{(s+s_1)(s+s_2)} \tag{38}$$

we have the following step response:

$$x(t) = \frac{2}{s_1 - s_2}\left(e^{-s_1 t} - e^{-s_2 t}\right). \tag{39}$$

By equating its derivative to 0 one gets

$$\sigma t_m = \frac{s_1+s_2}{2(s_1-s_2)} \ln\left(\frac{s_1}{s_2}\right) = \frac{1}{2\phi}\ln\left(\frac{1+\phi}{1-\phi}\right) \tag{40}$$

$$\sigma x_m = \frac{s_1+s_2}{s_1}\left(\frac{s_2}{s_1}\right)^{\frac{s_2}{s_1-s_2}} = \frac{2}{1+\phi}\left(\frac{1-\phi}{1+\phi}\right)^{\frac{1-\phi}{2\phi}}. \tag{41}$$

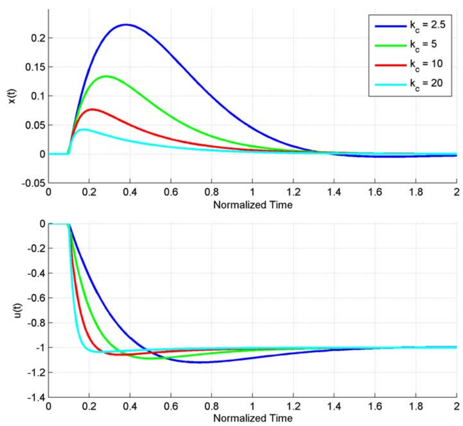

Fig. 7. Disturbance response; $c = 3$, $k = 2.5, 5, 10, 20$
$k_c = 2.5$ - complex roots, $k_c = 5, 10, 20$ - real roots

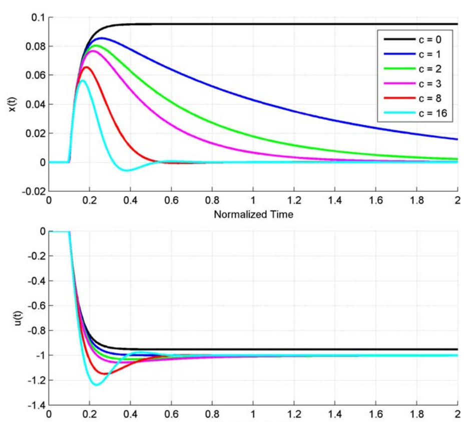

Fig. 8. Disturbance response; $k_c = 10$; $c = 0, 1, 2, 3, 8, 16$,
$c = 0, 1, 2, 3$ - real roots; $c = 8, 16$ - complex roots

2) Complex roots: Taking into account the disturbance transfer function

$$G_{xd}(s) = \frac{2s}{s^2 + 2\sigma s + \sigma^2 + \omega^2} \tag{42}$$

for the unit step disturbance $d(t) = 1(t)$ one gets the following

$$x(t) = \frac{2}{\omega} e^{-\sigma t} \sin(\omega t). \tag{43}$$

Taking its first derivative and equating the result to 0 we get time t_m of the extremum and its value $x_m = x(t_m)$

$$\sigma t_m = \frac{\arctan(\theta)}{\theta} \tag{44}$$

$$\sigma x_m = \frac{2}{\sqrt{1+\theta^2}} e^{-\sigma t_m}. \tag{45}$$

The *double root* case can be obtained from (44)-(45) or (40)-(41) letting $\theta \to 0$ or $\phi \to 0$ and putting $s_1 = s_2 = \sigma$. We have then

$$\sigma t_m = 1, \quad \sigma x_m = \frac{2}{e}. \tag{46}$$

From equations in (40)-(41) and (44)-(46) it follows that both t_m and x_m depend inversely proportionally on σ.

Maximum values of the disturbance responses for various controller settings are displayed in Fig.9.

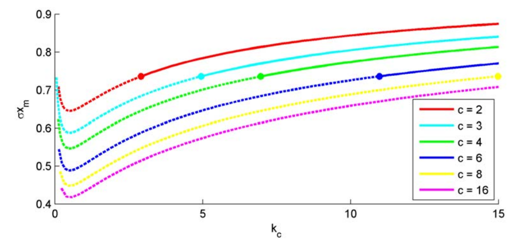

Fig. 9. Maximum value of the disturbance response: dotted line - complex roots, solid line - real roots

C. Reference response

Due to control signal limitations, the step response to reference change, as depicted in Fig.10, can be observed at high controller gains only for very small reference changes. For greater changes the control system saturates. Fortunately, due to the property $G_{ud}(s) = -G_{xr}(s)$, the results of this subsection characterize also the control signal response to the step-wise disturbance change, see e.g. Fig.7-Fig.8.

1) Real roots: Step response of the transfer function

$$G_{xr}(s) = \frac{s_1 s_2\left(\frac{s}{c}+1\right)}{(s+s_1)(s+s_2)} \tag{47}$$

has the form

$$x(t) = 1 - \frac{s_2(s_1-c)}{c(s_1-s_2)} e^{-s_1 t} + \frac{s_1(s_2-c)}{c(s_1-s_2)} e^{-s_2 t}. \tag{48}$$

Equating its derivative to 0 one gets (denoting $c = \alpha\sigma$)

$$\sigma t_m = \frac{s_1+s_2}{2(s_1-s_2)} \ln\left(\frac{s_1-c}{s_2-c}\right) = \frac{1}{2\phi}\ln\left(\frac{1+\phi-\alpha}{1-\phi-\alpha}\right) \tag{49}$$

$$\begin{aligned} x_m &= 1 + \frac{1}{c}(s_1-c)^{\frac{-s_2}{s_1-s_2}}(s_2-c)^{\frac{s_1}{s_1-s_2}} \\ &= 1 + \frac{1+\phi-\alpha}{\alpha}\left(\frac{1-\phi-\alpha}{1+\phi-\alpha}\right)^{\frac{1+\phi}{2\phi}}. \end{aligned} \tag{50}$$

The values of overshoot as a function of k_c for $c =$const is presented in Fig. 11. It is interesting to note that increasing the gain reduces the overshoot, which tends to 0 as $k_c \to \infty$.

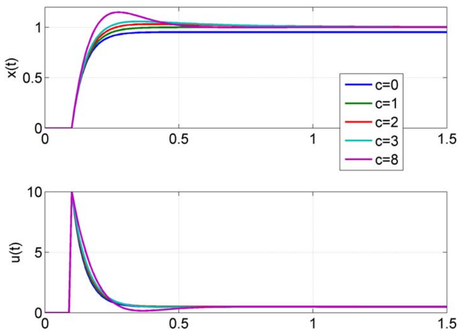

Fig. 10. Reference response for PI control, $k_c = 10$

2) Complex roots: Taking into account that $kc = \sigma^2 + \omega^2$ and denoting $c = \alpha\sigma$ equation (16) takes the form

$$G_{xr}(s) = \frac{(\sigma^2 + \omega^2)\left(\frac{s}{\alpha\sigma} + 1\right)}{s^2 + 2\sigma s + \sigma^2 + \omega^2}. \tag{51}$$

Then the step response is determined by

$$x(t) = 1 - e^{-\sigma t}\left[\cos(\omega t) + \frac{\alpha - (1+\theta^2)}{\alpha\theta}\sin(\omega t)\right], \tag{52}$$

with a maximum at $t = t_m$ such that

$$\sigma t_m = \begin{cases} \frac{1}{\theta}\arctan\frac{\theta}{1-\alpha}, & \alpha < 1 \\ \frac{\pi}{2\theta}, & \alpha = 1 \\ \frac{1}{\theta}\left(\pi + \arctan\frac{\theta}{1-\alpha}\right), & \alpha > 1 \end{cases} \tag{53}$$

and the maximum value $x_m = x(t_m)$ equals to

$$x_m = 1 + \frac{\sqrt{(1-\alpha)^2 + \theta^2}}{\alpha} e^{-\sigma t_m}. \tag{54}$$

The *double root* case is obtained by letting $\theta \to 0$ in (54) or $\phi \to 0$ in (50)

$$\sigma t_m = \frac{1}{1-\alpha}, \quad x_m = 1 + \frac{1-\alpha}{\alpha} e^{-\sigma t_m}. \tag{55}$$

Dependence of overshoot on various system parameters is depicted in Fig.11- Fig.12 It is interesting to notice that,

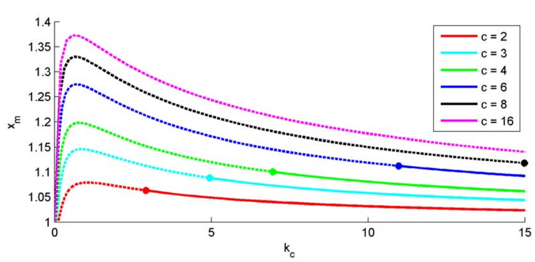

Fig. 11. Overshoot as a function of k_c: dotted line - complex roots, solid line - real roots

unlike the disturbance response, the overshoot does not depend on σ.

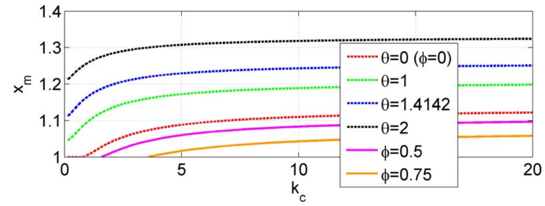

Fig. 12. Overshoot for selected values of θ or ϕ and variuos k_c

D. Influence of the working point

The property of the linearized system in (11) of its high frequency gain not depending on the working point defined by u_0, results in immunity of the properties of the closed loop system to the change of working point whenever the controller gain k_c is high enough. This is illustrated in Fig. 13.

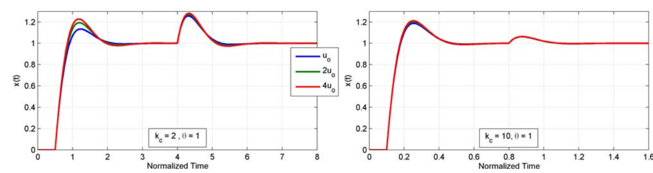

Fig. 13. Control system responses for $\theta = 1$ and different multiplies of u_0; reference response is followed by disturbance response. Notice also the change of the time scale, signals shape and magnitude when increasing σ by increasing the gain

E. Experimental PI controller tuning

One can start with $k_c \in \langle 5, 20\rangle$ and $c = 2$, which gives a controller with slight overshoot in control signal. If larger overshoot can be accepted, then c can be increased to get smaller maximum value and faster decay of disturbance response. Another reasonable option is to increase both, c and k_c, so as to retain a chosen value of θ, e.g. $\theta = 1$. The limiting value for k_c may result from the level of noise created by turbulent flow and air bubbles and/or additional fast dynamics.

V. Large reference changes: reference and feedforward signals generators

Larger values of k_c impose larger demand on control signal magnitude. In practice, the control signal magnitude is limited. Therefore, certain measures should be taken to deal with such restriction. A popular solution is the anti-windup augmentation of the PI controller, see e.g. Fig. 14. As an alternative, the reference signal can be designed so as to avoid control signal saturation. A popular solution is reference ramping, which can be augmented with a feedforward signal driving the system according to the demand. This can be accomplished in a combined feedback - feedforward system as in Fig. 15, see also [3]. In Fig. 17 - Fig. 18 results obtained using P and PI controllers with $k_c = 10, c = 2, k_a = 1.2$, both with and without feedforward, are displayed.

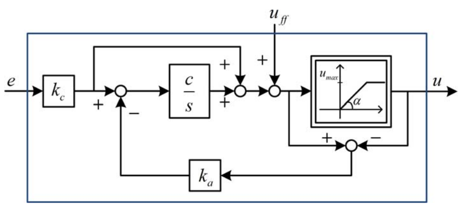

Fig. 14. PI with anti-windup and feedforward signal input, $\alpha = \pi/4$

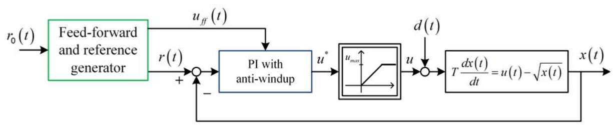

Fig. 15. Feedback and feedforward control system

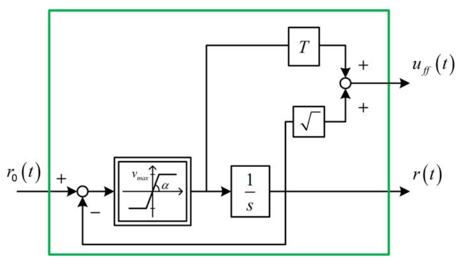

Fig. 16. Ramp set-point and feedforward generator, $\alpha \to \pi/2$

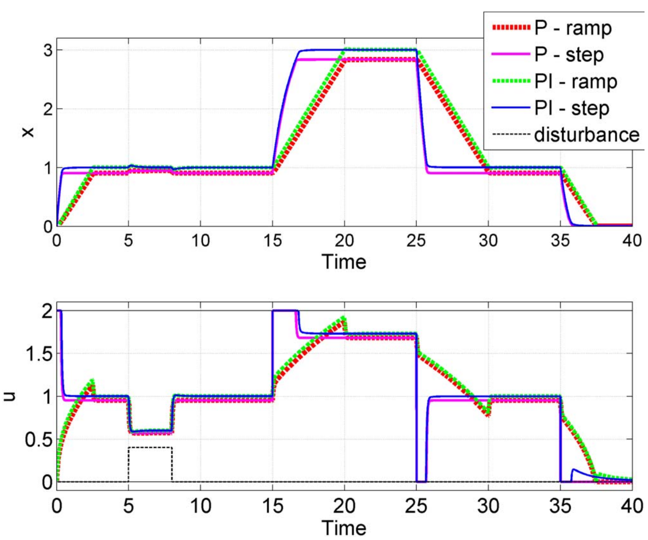

Fig. 17. P and PI controllers without feedforward. P control exhibits steady-state error, whose value can be decreased using larger controller gain.

VI. Concluding Remarks

Although being one of the simplest plants possible, the single tank system is an excellent example for teaching control at the introductory level using various design tools. In our opinion, the presented considerations are simple enough and give a solid engineering insight into the control problem, even in a virtual interactive environment. Better teaching results can be obtained when it is followed by a laboratory experiment [2], [1] which gives additional visual and accoustic sensations and reflects practical problems with

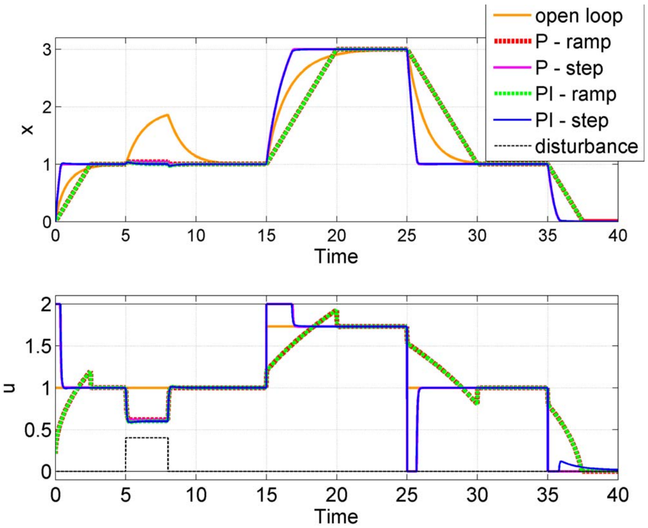

Fig. 18. P and PI controllers with feed-forward signal, and open loop control $u = \sqrt{r}$. Results of P and PI control are almost the same except for a certain small steady-state disturbance error in the case of P controller

regard to measurement noise and neglected dynamics [4].

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