Model Reference Control of Nonlinear Time-Delayed 1-st Order Plants

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***Abstract*—This paper deals with a new approach to the design of a two-degree-of-freedom controller for a first-order nonlinear plant with time-delays. There exist several traditional methods that can solve this problem (gain scheduling, exact linearization, generalized transfer functions for nonlinear systems, etc.). The advantage of the solution proposed in this paper consists in a simple and modular controller design that can be used for a broad class of nonlinear systems. In order to be able to comprise all required steps into a single conference paper, the overall design and the resulting performance of the proposed model reference control of nonlinear time-delayed systems will be documented by an example of a one-tank liquid level control in a real hydraulic plant. Starting from a basic P controller, its structure is gradually extended by a model reference control and finally by a nonlinear disturbance observer taking into account both the filtration of noisy signals and the time-delays always present in control circuits.**

I. INTRODUCTION

Although today's control theory offers very sophisticated control circuits that are suitable for complex control tasks there are still simple controller applications that can be improved significantly and these can influence the control quality of many simple plants that are very often present in practice [1]. In the case of nonlinear systems one can use different linearization techniques to deploy linear control system design. For instance the linearization in a fixed point can be improved by gain scheduling method to higher the dynamics of responses [2], [3], [4], [5]. The other way is to use the exact linearization method [6] or methods based on transfer functions generalized to nonlinear systems [7], [8], [9].

Different approach can be seen in combination of a nonlinear plant dynamics with use of parallel PI and PID algorithms [10], [11], [12], [13] though better results are achieved by introduction of an integral part in the form of nonlinear disturbance observers (see e.g. [14], [15], [16]). The proposed work is built on similar methods using disturbance observer PI and PID control [17], [18], [19], [20], [21], [22] that is extended by two-degree-of-freedom (2DOF) model reference control.

This paper will first present a design of a P controller applied to a nonlinear system after exact linearization. This first part will be later considered as a model because it produces an ideal reference signal for the model reference control structure. Therefore the first part fulfills the task of a dynamical feedforward controller that is later extended by the second part represented by a stabilizing controller. The established two-degree-of-freedom model reference control structure is finally completed by an integral part given by a nonlinear disturbance observer. At the end the influence of time-delays to the designed control structure is evaluated and compared with a PI controller without model reference control.

II. MODEL CONTROL: P-CONTROLLER APPLIED TO NONLINEAR FIRST ORDER DYNAMICS PLANT USING EXACT LINEARIZATION

Let us denote the output derivative of the model dy_m/dt that is proportional to the model control signal u_m, and the model gain $K_m(y_m) = g_m(y_m)$ that depends on the output y_m (state x_m) and the nonlinear feedback $f_m(y_m)$. Then the nonlinear plant model

$$\frac{dy_m}{dt} = g_m(y_m)\,u_m - f_m(y_m) \tag{1}$$

represents a direct generalization of a linear pole-zero plant model used in [22] to get a unified mathematical description of all stable and unstable linear systems with dominant first order dynamics for design of a high performance robust control with disturbance observer based integral action. The model control signal has to respect the constraints

$$U_{m1} \le u_m \le U_{m2} \tag{2}$$

Equations (1) may be simplified by introducing a new control signal

$$\mu = g_m(y_m)\,u_m \tag{3}$$

what leads to a simplified plant model

$$\frac{dy_m}{dt} = \mu - f_m(y_m) \tag{4}$$

with a feedback nonlinearity $f_m(y_m)$. The control signal transformation (3) enables to recalculate the original model control signal u_m from μ by means of

$$u_m = \mu / g_m(y_m) \tag{5}$$

By proposing for (4) the control

$$\mu = f_m(y_m) + u \tag{6}$$

the system could formally be transformed to a simple integrator

$$dy_m/dt = u \tag{7}$$

and treated as a linear one. This control eliminates the plant nonlinearities and with r being a piecewise constant reference

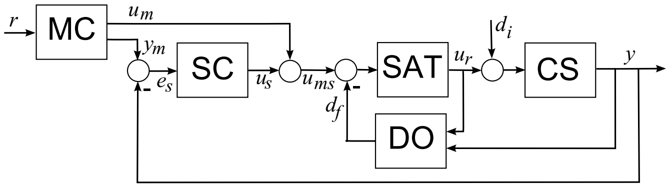

Figure 1. Model reference control block diagram - model control (MC), stabilizing controller (SC), saturation (SAT), disturbance observer (DO), controlled system (CS)

variable (setpoint), the remaining single integrator control u may be then calculated according to

$$u = K_m(r - y_m) \; ; \quad K_m = 1/T_c \tag{8}$$

Here, T_c represents the closed loop time constant. Thus, one gets for (1) from (6) the exact linearization controller [6] that can be written as

$$u_m = \frac{K_m e_m + f_m(y_m)}{g_m(y_m)} \tag{9}$$

where $e_m = r - y_m$ represents a model control error.

III. MODEL REFERENCE CONTROL

The principal block diagram of the model reference control (MRC) can be seen in the Fig. 1. The model with controller (MC block) produces two signals. First one is the output of the model controller u_m (9) and the second one is the output of the model itself y_m. The signal y_m is compared with the output of the real system y and the established difference between the model and the real system creates the stabilizer control error denoted as e_s

$$e_s = y_m - y \tag{10}$$

This is multiplied by the gain K_s as a parameter of the second P-controller and that produces the stabilizing control action

$$u_s = K_s \, e_s \tag{11}$$

In the block diagram this is represented by SC block. Then the control action u_{ms} resulting from the model reference control is given as a sum of u_m and u_s

$$u_{ms} = u_m + u_s \tag{12}$$

IV. NONLINEAR DISTURBANCE OBSERVER BASED PI CONTROL

To eliminate possible steady state error, nonlinear PI controller will be introduced based on a reconstruction and compensation of an input disturbance d_i. This will constitute a nonlinear disturbance observer represented by DO block in the Fig. 1. Such solutions may be based on inversion of the plant dynamics described by the differential equation

$$\frac{dy}{dt} = g(y)\left(u_r(t - T_d) + d_i\right) - f(y) \tag{13}$$

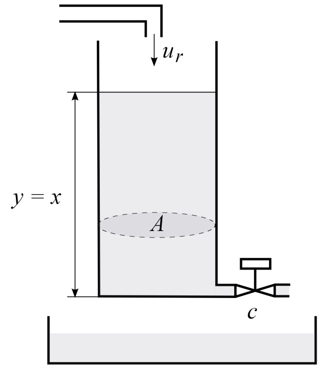

Figure 2. Nonlinear disturbance observer implementation for one-tank hydraulic system

where $u_r(t - T_d)$ represents a real control action applied to the plant with a time delay T_d. Then after solving (13) for the disturbance d_i denoted by its estimation $\hat{d}_i$ one gets

$$\hat{d}_i = \left[\frac{dy}{dt} + f(y)\right]\frac{1}{g(y)} - u_r(t - T_d) \tag{14}$$

In order to eliminate algebraic loops, to increase robustness and to eliminate measurement noise, at least a first order filter [19], [22] has to be added

$$d_f = \frac{1}{1 + T_f s}\hat{d}_i \tag{15}$$

For generating the usually not directly measurable output derivation, a first order low pass filter may be used

$$\frac{dy}{dt} \approx \frac{s}{1 + T_{fd}s}Y(s), T_{fd} << T_f \tag{16}$$

Then the same low pass filter should be applied to the signal $u_r(t - T_d)$.

Finally, to evaluate the real control law u_r resulting from the model reference control and the nonlinear disturbance observer it is necessary to subtract the filtered estimated disturbance d_f (15) from the model and stabilizing control action u_{ms} (12) and in the case of saturated control apply the real control constraints

$$u_r = sat(u_{ms} - d_f) \tag{17}$$

where

$$U_{r1} \leq u_r \leq U_{r2} \tag{18}$$

V. APPLICATION TO THE ONE-TANK HYDRAULIC SYSTEM

In this section the one-tank hydraulic system will be described and model reference control with nonlinear disturbance observer will be applied to it.

A. Description of one-tank hydraulic system

The model of a one-tank level control (Fig. 2) considers liquid in one tank that amount is controlled by an inflow generated by a pump pulling liquid from a reservoir. The tank has a drain through an output valve to the liquid reservoir that enables the outflow of the liquid.

The hight of the liquid's level in the first tank is denoted as x with the liquid's inflow u_r, tank's cross-sections A and the valve's outflow coefficient c. After a pump linearization the dominant dynamics of this hydraulic model can be described by the following differential equation acquired through application of the law of substance preservation

$$\begin{aligned} \frac{dx}{dt} &= \frac{1}{A}u_r(t-T_d) - c\sqrt{x} \\ y &= x \end{aligned} \tag{19}$$

where $y = x$ denotes the controlled output that is identical with the state variable of the tank. Comparing (19) with (13) gives $g(y) = 1/A$ and $f(y) = c\sqrt{y}$. Simple identification procedures described e.g. in [23] give the plant model parameters $A = 0.0015m^2,\ c = 0.032$.

B. Model reference control design

As it was written above model reference control u_{ms} (12) consists of two parts: model control u_m (9) and stabilizing control u_s (11). To design these it is necessary to evaluate two $P-$controllers, i.e. their gains K_m and K_s.

In the case of K_s the linear approximation of the nonlinear hydraulic plant (19) can be advantageous. To take into account also non-modeled dynamic the nonlinear system will be approximated by the first order linear system with dead time that can be expressed using the Laplace transform as

$$F(s) = \frac{K_{cs}}{s+a}e^{-T_d s} \tag{20}$$

where $K_{cs} = g(y) = 1/A$ represents the gain of the linearized system and T_d is an identified transport delay of the real hydraulic plant. Before evaluating a the working point of the linearization y_p has to be chosen. Then linearization of $f(y) = c\sqrt{y}$ at y_p gives $a = c/(2\sqrt{y_p})$.

After linearization the condition of the double real dominant pole of the closed loop characteristic polynomial can be applied to derive the optimal tunning of the P-controller in the form [22]

$$K_s = \frac{e^{-(1+aT_d)}}{K_{cs}T_d} \tag{21}$$

In the case of the model controller parameter K_m the nonlinear model has already been linearized by the exact linearization method and can be therefore expressed by the simple transfer function $F(s) = \frac{1}{s}$ according to (7). Choosing arbitrary small closed loop time constant T_c in (8) one gets very high K_m that will result in fast output responses of the model. To be realistic such fast responses could not be followed by the real hydraulic plant. Therefore it is recommended to choose values of K_m that will respect real conditions. Under assumption of non-modeled dynamic approximated by a time delay it is possible to use the expression (21) also for setting the model controller gain K_m where this time the simple transfer function ($F(s) = \frac{1}{s}$) causes the parameters $K_{cs} = 1$ and $a = 0$ should be applied that for K_m yields

$$K_m = \frac{1}{eT_d} \tag{22}$$

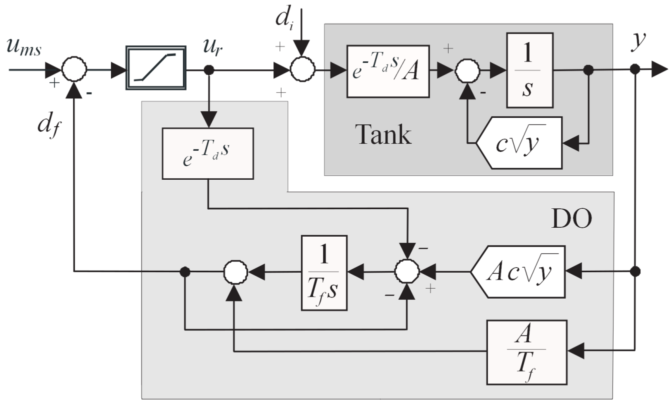

Figure 3. Nonlinear disturbance observer implementation for one-tank hydraulic system

C. Design of nonlinear disturbance observer

In the case of a one-tank hydraulic plant with $K_{cs} = g(y) = const$ it is possible to derive a nonlinear disturbance observer implementation with the first-order filter without an additional output derivation (16). From (15) it follows

$$\tfrac{dd_f}{dt} = \tfrac{1}{T_f}\left[\hat{d}_i - d_f\right] \tag{23}$$

By integrating this equation and using (14) with substitution $g\left(y\right) = K_{cs} = 1/A$, $f\left(y\right) = c\sqrt{y}$ one gets

$$\begin{aligned} d_f &= \textstyle\int \frac{1}{T_f}\left[\hat{d}_i - d_f\right]dt = \\ &= \tfrac{y(t)}{T_f K_{cs}} + \tfrac{1}{T_f}\textstyle\int\left[\frac{f(y)}{K_{cs}} - u_r\left(t-T_d\right) - d_f\right]dt = \\ &= \tfrac{A}{T_f}y(t) + \tfrac{1}{T_f}\textstyle\int\left[Ac\sqrt{y} - u_r\left(t-T_d\right) - d_f\right]dt \end{aligned} \tag{24}$$

A possible implementation scheme of the designed nonlinear disturbance observer can be seen in the Fig. 3.

In the design of the nonlinear disturbance observer the parameter T_f represents the time constant of a filter and its setting influences the dynamics of the disturbance reconstruction. To respect real conditions the method of the double real dominant pole of the closed loop [22] will be used again in order to evaluate an equivalent pole

$$\alpha_e = -\frac{1 + aT_d e^{1+aT_d}}{T_d e^{1+aT_d}} \tag{25}$$

and use for setting T_f as

$$T_f = -\frac{1.5}{\alpha_e} \tag{26}$$

VI. Experimental results

After identifying the real system, designing the model reference control and nonlinear disturbance observer, it has been possible to carry out different experiments. Each experiment consists of the same phases when the reference signal r has been changed from $0m$ to $0.1m$, later from $0.1m$ to $0.2m$ and finally from $0.2m$ to $0.1m$. After these changes of the reference signal r also the changes of the input disturbance d_i have been applied when first d_i has been changed from 0 to $0.5\,10^{-5}m^3s^{-1}$ and after some period it has been canceled.

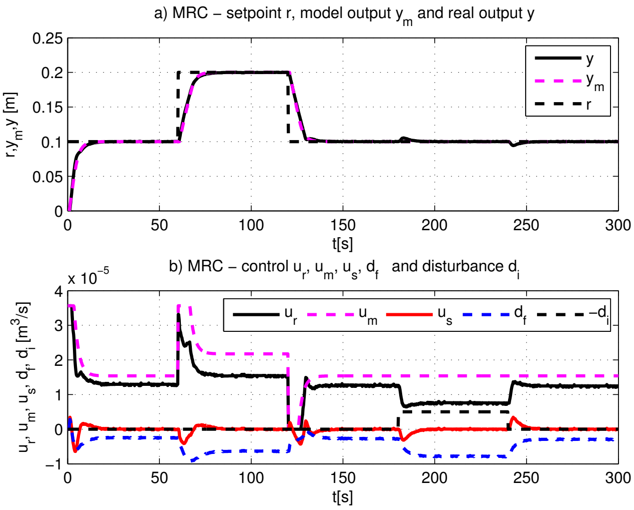

Figure 4. Model reference control of the real one-tank system without any additional delay - a) reference, model output and real output responses, b) control and disturbance responses

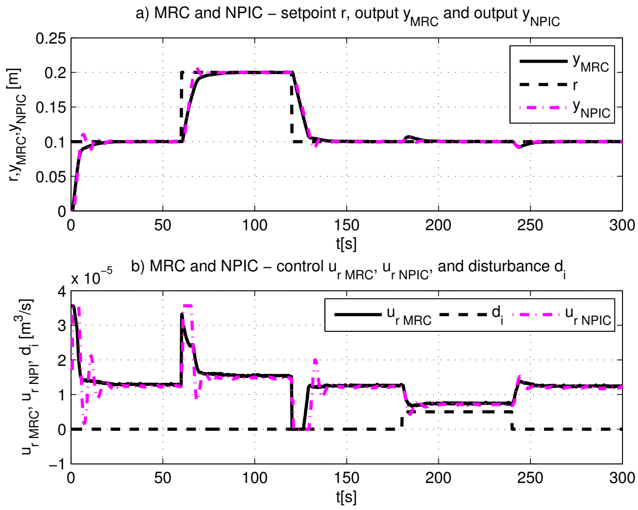

Figure 5. Comparison of model reference control (MRC) with nonlinear PI control (NPIC) of the real one-tank system without any additional delay - a) reference and output responses, b) control and disturbance responses

The times of the changes are chosen appropriately with respect to reach steady state behavior.

Transients of the first experiment are depicted in the Fig. 4. It represents the case when no additional delays have been applied. Only the natural delay of the hydraulic plant $T_d = 0.9s$ has to be considered. The parameters of the model controller, the stabilizing controller, and the nonlinear disturbance observer have been calculated as $K_m = 2.44$, $K_s = 5.93\,10^{-4}$, and $T_f = 3.4s$ according to (22), (21), and (26) respectively. The Fig. 4a shows the output responses where one can compare behavior of the model output y_m with the output of the real hydraulic plant y. The Fig. 4b displays the transients of the control signals where the real control signal u_r is the sum of the model control signal u_m, of the stabilizing control u_s and of a negative signal of the nonlinear disturbance observer $-d_f$. It can be noticed that the stabilizing control u_s slightly corrects the model control u_r that dominates. Nonlinear disturbance observer d_f correctly detects an input disturbance. However, its nonzero values during the times, when an input disturbance has not been present, shows differences between the model and the real system caused by a non-modeled dynamics and a simplified estimate of the plant nonlinearity in (1). In fact, experiments show that it should be approximated by a more complex function $f(y) = cy^d, d \neq 1/2$

Fig. 5 shows a comparison of the model reference control with its limit case when the dynamical feedforward control reduces to a static one. Such a control may then be denoted as a nonlinear PI control (NPIC). The NPIC has a similar structure as depicted in the Fig. 1 but instead of the model reference control block MC there is a simple static feedforward control and the input of the stabilizing controller SC is the control error $e = r - y$ instead of e_s, i.e. $y_m = r$. From the comparison it can be seen that the NPIC has a higher dynamics and therefore it also has an oscillatory behavior. The comparison has been made by the same stabilizing gain K_s. By decreasing K_s it is possible to suppress this oscillatory character of the NPIC transients but not in the case with longer time delays. One can take into account that decreased stabilizing controller gain values K_s can positively influence damping of the setpoint responses. But, in this way, the disturbance responses deteriorate.

The responses in the Fig. 6 display the influence of raising time delays, when a natural time delay $T_d = 0.9s$ has been increased by additional time delays $T_D = 2s$ and $T_D = 5s$. These have been realized artificially by inserting the transport delay block in the Simulink block diagram. The designed model reference controller has been able to cope with such delays although to reach monotonous responses the stabilization controller gains K_s have had to be decreased in comparison with the original values calculated according to (21) (by 20%, or by 50% of the original value of K_s evaluated for the sum of T_d and T_D, i.e. $T_d = 2.9s$ and $T_d = 5.9s$, respectively).

To document the performance of the compared controllers the additional delays have been increased to $T_D = 8s$. The corresponding transients are displayed in the Fig. 7. Both controller parameters are calculated according to (22), (21), and (26) where T_d has been substituted by $T_d + T_D$, i.e. by the value $8.9s$. In this case also the model reference controller produces a small overshoot in the first reaction to the setpoint. The oscillations of the NPIC are higher and may not be simply suppressed by changing its parameters K_s and T_f. As expected, the reactions to the disturbance steps are the same for both controllers, which results from the same values of the parameters K_s and T_f.

VII. Conclusion

The paper has presented a new design of a 2DOF model reference control and an evaluation of its performance carried out on a real one-tank hydraulic plant. The experimental results

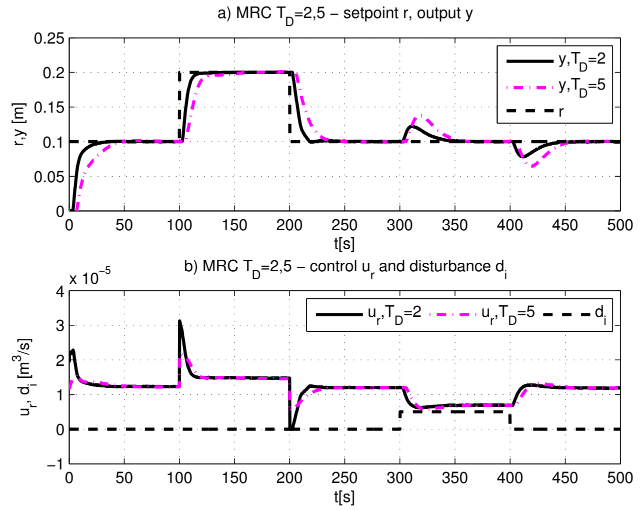

Figure 6. Model reference control of the real one-tank system with the additional delays $T_D = 2s$, $T_D = 5s$ - a) reference and real output responses, b) control and disturbance responses

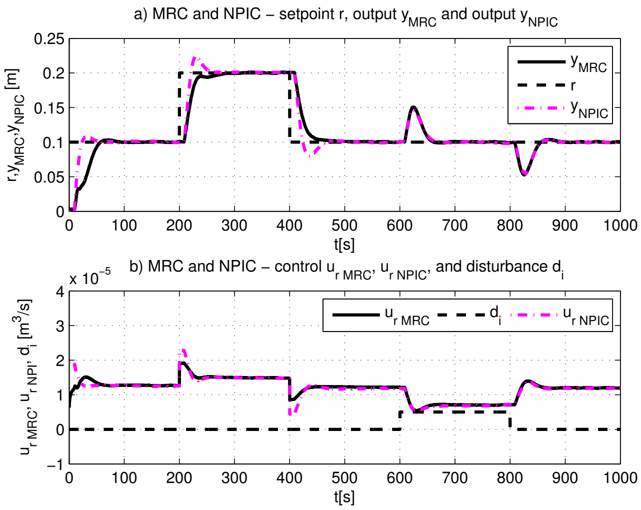

Figure 7. Comparison of model reference control (MRC) with nonlinear PI control (NPIC) of the real one-tank system with the additional delay $T_D = 8s$ - a) reference and output responses, b) control and disturbance responses

have confirmed an adequate performance of the designed controller also in the case of longer time delays. On the other hand, the model of the controlled system must be well known because the designed controller is sensitive to any plant-model mismatch. The two degree of freedom structure enables to tune the controller's parameters independently as concerning setpoint and disturbance reactions. The design structure can be used also for higher order nonlinear systems with time delays where its advantages against classical control structures should be more obvious.

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REFERENCES

[1] K. J. Åström and T. Hägglund, *Advanced PID Control*. NC: ISA, Research Triangle Park, 2006.

[2] W.J.Rough and J.F.Shamma, "Research on gain scheduling," *Automatica*, vol. 36, pp. 1401–1425, 2000.

[3] K. J. Åström and B. Wittenmark, *Adaptive Control - 2nd ed.* Reading: Addison-Wesley Publ. Comp., 1995.

[4] H. Khalil, *Nonlinear Systems, 2nd Ed.* London: Prentice Hall Int., 1996.

[5] K. Klatt and S. Engell, "Nichtlinearer reglerentwurf mit gain-scheduling techniken," in *Entwurf nichtlinearer Regelungen*, S. Engell, Ed. R.Oldenbourg Verlag München, 1995, pp. 93–121.

[6] A. Isidori, *Nonlinear Control Systems. 3rd edition*. New York: Springer, 1995.

[7] M. Halás, "An algebraic framework generalizing the concept of transfer functions to nonlinear systems." *Automatica*, vol. 44, no. 5, pp. 1181–1190, 2008.

[8] M. Halás and Ü. Kotta, "A transfer function approach to the realisation problem of nonlinear systems," *International Journal of Control*, vol. 85, pp. 320–331, 2012.

[9] M. Halás, "Nonlinear systems: A polynomial approach," in *EUROCAST*, 2009, pp. 595–602.

[10] M. Fliess and C. Join, "Intelligent PID Controllers," in *Proc. 16th Mediterrean Conference on Control and Automation*. Ajaccio, France: IEEE, 2008, pp. 326–331.

[11] M. Fliess, C. Join, and H. Sira-Ramirez, "Complex Continuous Nonlinear Systems: Their Black Box Identification And Their Control," in *Proc. 14th IFAC Symposium on System Identification (SYSID 2006)*, Newcastle, Australia, 2006.

[12] M. Fliess, C. Join, and H. Sira-Ramirez, "Non-linear estimation is easy," *Int. J. Modelling Identification and Control*, vol. 4, no. 1, pp. 12–27, 2008.

[13] M. Fliess, C. Join, and S. Riachy, "Revisiting some practical issues in the implementation of model-free control," in *18th IFAC World Congress*, vol. 18, 1, Milano, Italy, 2011, pp. 8589–8594.

[14] W. Chen, D. Ballance, P. Gawthrop, and J. O'Reilly, "A nonlinear disturbance observer for robotic manipulators," *IEEE Transactions on Industrial Electronics*, pp. 932–938, 2000.

[15] H. Park, D. Jung, and S. Won, "Based on nonlinear disturbance observer approach non-interactive control for hot rolling mill : Experimental validation," *13th Symposium on Automation in Mining, Mineral and Metal Processing*, 2010.

[16] M. Huba, "Gain Scheduled PI Level Control of a Tank with Variable Cross Section," in *2nd IFAC Conference on Control Systems Design*, Bratislava, Slovakia, 2003, pp. 53–58.

[17] M. Huba and P. Bisták, "Dynamic Classes in the PID Control," in *Proceedings of the 1999 American Control Conference*, San Diego, 1999.

[18] M. Huba, "Constrained pole assignment control," in *Current Trends in Nonlinear Systems and Control*, C. T. A. L. Menini, L. Zaccarian, Ed. Birkhäuser, Boston, 2006, pp. 163–183.

[19] M. Huba, "Modular disturbance observer based constrained PI ontroller design," in *Int. Conf. Advances in Motion Control*. Sarajevo, BIH: IEEE, 2012.

[20] M. Huba, "Performance measures, performance limits and optimal PI control for the IPDT plant," *Journal of Process Control*, vol. 23, 4, pp. 500–515, 2013.

[21] M. Huba, "Performance Portrait Method: a new CAD Tool," in *10th Symposium on Advances in Control Education (ACE)*. Sheffield, UK: IFAC, 2013.

[22] M. Huba, "Comparing 2DOF PI and Predictive Disturbance Observer Based Filtered PI Control," *Journal of Process Control*, vol. 23, 10, pp. 1379–1400, 2013.

[23] V. Zilka and M. Huba, "Laboratory model of coupled tanks," in *Workbook: Selected topics on constrained and nonlinear control*, M. Huba and S. Skogestad, Eds. STU Bratislava - NTNU Trondheim, 2011, pp. 229–246.