PROCESS CONTROL TECHNOLOGY PCT-100

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Educational Technology

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PCT-100 CONTENTS

1.0. INTRODUCTION

1.1. Background.

Temperature, Level, Flow and Pressure are the four most common process variables. Similar to temperature, pressure is another key process variable because pressure provides a critical condition for boiling, chemical reaction, distillation, extrusion, vacuuming, and air conditioning. Poor pressure control can cause major safety, quality, and productivity problems. Overly high pressure inside a sealed vessel can cause an explosion. Therefore, it is highly desirable to keep pressure under control and maintained within its safety limits.

High productivity and consistent products reliability have become vital to industrial success. In particular, high reliability of the manufacturing process is important in the 'process industry' as any faults in the system can produce large volumes of a defective product in a very short time.

The development of microprocessor based control systems built around PLCs, industrial PCs, embedded micro-controllers etc. has been largely responsible for the huge increases in productivity seen in many plants over the last two decades. Micro-electronics has replaced manpower for monitoring and controlling both the product and the manufacturing process. The proliferation of modern micro-electronics based control systems in recent years has resulted in a great demand for high quality training in this field.

Production equipment could not be used because of the downtime cost, safety considerations and the difficulty students face in getting a good overview of a large plant. The PCT-100, Process Control Technology model is a bench-top system which implements several continuous fluid processes representative of many found in food and drink manufacturing, petrochemicals production, water purification, sewage processing and many other areas of industry. Software is supplied with the PCT-100 which provides facilities for teaching how 'three term' algorithms may be applied to control:

- Flow-rate
- Liquid level
- Temperature
- Pressure

Some of the provision of the unit are outlined below:

1. To provide a controlled process in a learning environment which reflects the control problems experienced in industry and on which students can carry out detailed analysis of alternative control techniques.

2. To illustrate simply and clearly, the fundamental control techniques of proportional, integral and derivative control.

Water is pumped around the system and heated or cooled as desired. Flow-rate, level, temperature and pressure measurements are fed back to the controller running the control software.

Some of the objectives which the PCT-100 may be used to achieve are outlined below:

- Illustrate the advantages offered by the application of microprocessor based controllers to process automation.
- Provide a small scale process which illustrates the problems found in industry to which students may apply different control techniques.
- Demonstrate simply and effectively the widely used 'three term' or 'proportional, integral and derivative' technique.
- Students may monitor digital and analogue signals in order to develop their understanding of the techniques involved.
- Provide facilities suitable for the practice of 'fault finding' techniques.
- Provide a realistic set of control problems which may be used as the basis of exercises for students, PLC programmers and software engineers.

1.2. System Overview.

The main elements of the PCT-100 are the process rig and control module. The PCT-100 rig is described in detail in section 3 but an overview is given here to provide a quick outline. The rig includes the following elements:

1. A sump from which the process fluid (water) can be pumped. The temperature of the water in the sump is measured with a platinum resistance thermometer (PRT), indicated on a digital display fitted to the rig and input to the controller via the control module.

2. A pump used to circulate the water around the system. The pump is driven by a d.c. motor, the speed of which may be varied through the controller allowing control of the flow rate sensor.

3. A turbine type flow meter which measures the water flow-rate. The flow-rate is indicated on a digital display fitted to the rig and input to the controller via the control module.

4. A process tank which contains:

    i.) A level sensor, for measuring level.
    ii.) A heating element, the power output of which may be varied through the control module, between zero and full power (400W with 48v.a.c.).
    iii.) A PRT for measuring the temperature of the water. The temperature is indicated on a digital display fitted to the rig and input to the controller via the control module.
    iv.) A vent pipe through which air can escape. The pipe is fitted with a manual valve that can be closed for performing pressure exercise.
    v.) A Needle drain valve used to add a disturbance to the system or to drain the process tank.
    vi.) A proportional drain valve to allow the flow of the water from the process tank into the sump tank.
    vii.) A pressure transducer for measuring the pressure in the process tank. The pressure is indicated on a digital display fitted to the rig and input to the controller via the control module.
    viii.) A float switch to cut the power to the pump should the water level reach the maximum height.
    ix.) A pressure relief valve. This valve will be activated if the pressure in the process tank exceeds the safety level.
    x.) A check valve to allow one directional flow of water into the process tank.

5. A forced air cooler. If the water needs to be cooled the flow is diverted via a 3-way valve to the cooler.

6. A proportional valve to control the flow rate in the system.

With the system outlined above there is scope for different 'process control' experiments. Initially simple computer activation of the various on/off elements on the rig and input of the signals from the sensors may be investigated. Subsequent experiments can range from comparison of flow, level, pressure and temperature processes, through to full three term control of flow, level, pressure temperature and batch processes.

The control module is described in detail in section 4. It includes all of the necessary signal conditioning and output driver circuitry etc. USB interface to link the PCT-100 to the computer and a sub D connector to allow connections of devices such as PLC or PID controller. The control module has been designed so that students have easy access for monitoring the various signals. The control module includes a mimic of the rig that has indicator lights to show status of the devices, test points and fault switches.

The software provided with the PCT-100 allows demonstrations and investigations of three term (PID) control applied to flow, level, temperature and pressure experiments. Open loop control of flow-rate is also available which allows students to investigate ideas such as frequency response (by driving the system with sine wave inputs at different frequencies) and open-loop step response.

2.0. GETTING STARTED

This section explains how to set up the PCT-100 before use. Detailed discussion of all aspects of the system, including the control software, is provided in subsequent sections.

2.1. Software:

To install the software, place the software CD in the CD drive, the CD should auto run. Select Install PCT-100 software from the menu.

Follow the on-screen instructions.
- Once the software is fully installed, connect and power on control module run it and select the USB interface from the configuration menu.
- The USB interface will be automatically detected and the software configured.

2.2. Connection Instructions

PLEASE READ THESE CONNECTION INSTRUCTIONS FULLY BEFORE ATTEMPTING TO OPERATE ANY PART OF THE SYSTEM.

- Connect the two ribbon and heater cables provided to the back of the control module and to the circuit board at the back of the process rig. It is impossible to connect them incorrectly.

- Connect the USB cable from the control module to a USB connection on the computer.

- Finally plug the power supply unit into the mains outlet.

- The maximum current requirement of the PCU is approximately 12A.

2.3. System Start Up

After connecting the PCT-100 to the PC and installing the software, as described in the previous section. Turn on the PCT-100 power. The computer should power up as usual and on the process rig the digital displays should illuminate. None of the system actuators should operate. If this is not so turn everything off, check all connections and retry.

Start the PCT-100 software in the usual manner (Start Menu, Programs etc.). The software is very easy to use but a full description of the controls etc. is provided in the Software chapter (section 5)

Before using the PCT-100:
Check the drain valve is closed. Fill the sump tank to the fill mark with distilled or deionised water and ensure the overflow/vent value on the top of the process tank is open. Close the needle value and run the pump using the control module. A smooth increase on the control should result in a linear response on the flow-rate display, up to approximately 3.2 litres/minute.

3.0. THE PROCESS RIG

3.1 Introduction

The PCT 100 provides flow level, temperature and pressure, processes which may be controlled by a computer, PLC or other device connected to the control module.

3.1.1 Description

For a diagram of the PCT- 100 refer to Figure 3.1. The PCT-100 includes the following elements:

1. Process tank
2. Sump tank
3. Cooler Unit
4. Sump tank Temperature sensor (PRT)
5. Variable speed pump with filter and pressure switch
6. 3/2 Diverter valve
7. 2/2 Proportional control valve
8. Flow rate sensor
9. One way check valve
10. 2/2 Proportional drain valve
11. Needle valve
12. Pressure relief valve
13. Heater
14. Level sensor
15. Pressure transducer
16. Float switch
17. Overflow/Vent valve
18. Digital LCD displays
19. Indicator lights

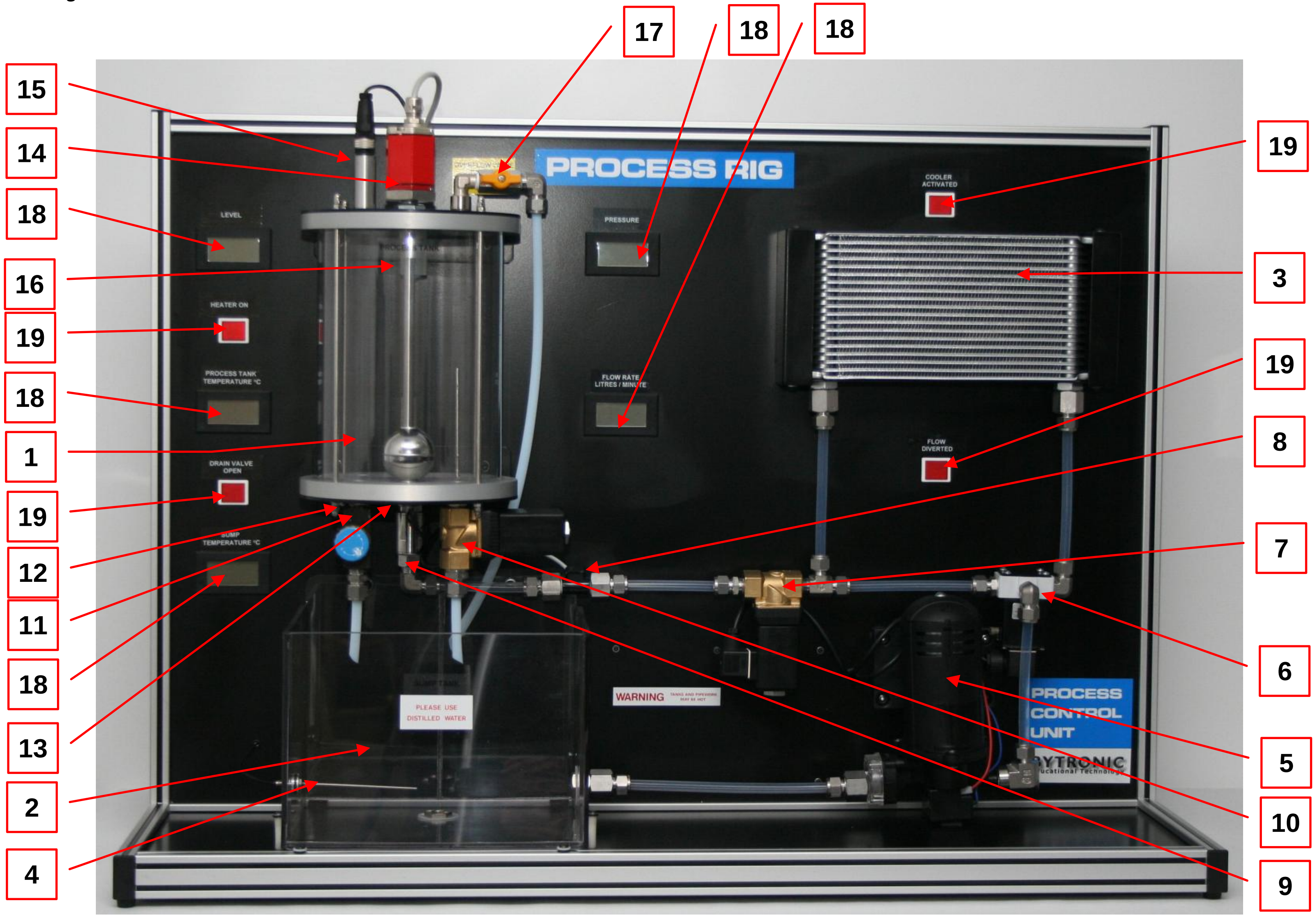
The sump contains a store of de-ionised or distilled water which may be pumped around the system at flow-rates up to about 3.2 litres/minute. Water may be pumped directly from the sump to the process tank or be diverted via the cooler. The fluid in the process tank may be drained via the manual or computer controlled valves below the tank, so completing the fluid cycle. The five digital displays are used to show the sump tank and process tank temperatures, flow-rate, pressure and level and the indicator lamps reveal the on/off status of the cooler fan, and computer controlled drain and diverter valves and the heater.

3.1.2 Feedback

For computer control of any plant feedback of the process variable values is essential. On the PCT-100 a turbine type flow-meter is used to measure the flow-rate and platinum resistance thermometers (PRTs) are used to measure the temperature at key points. A pressure sensor is used to measure the pressure and a level transducer to measure the liquid level in the process tank. All of the data from these sensors are fed back to the controlling computer via the control module.

3.1.3. Connections to the Control Module.

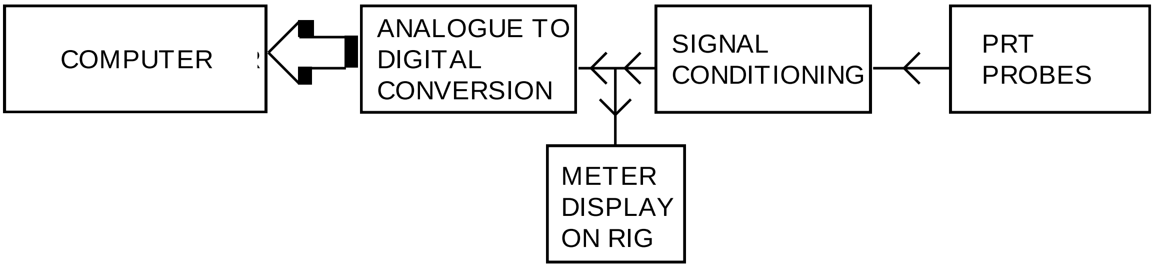
Figure 3.1 The Rig



3.2.1 Temperature Measurement

On the PCT 100 temperatures typically range between about 10°C and 60°C so platinum resistance thermometers (which can measure temperatures from -270°C to 660°C) have been used. The PRT is a transducer which operates on the basis that the electrical resistance of platinum wire (as all metals) changes with temperature. PRT's provide an absolute value with no reference point being necessary whereas other types of transducer frequently need a reference point.

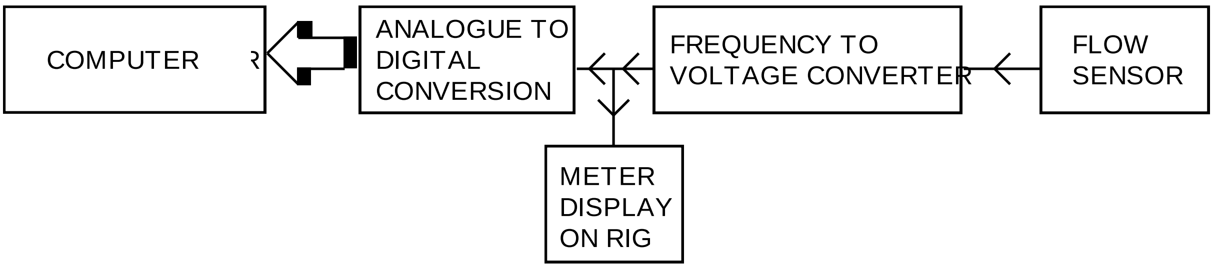
On the PCT-100 the PRT signals are conditioned in the control module to give a 0 to 10V analogue signal. These signals are then used to drive the digital temperature displays on the rig and converted to digital form by an ADC for input to the computer. This is all shown diagrammatically below.



3.2.2 Flow Measurement

The flow-rate of the water is measured by a turbine type flow-meter. The water flows through the meter and rotates an impeller. Mounted on either side of the impeller is an infra-red transmitter and receiver and the infra-red beam is repeatedly broken by the rotating impellor. The flow-meter produces a pulse train output the frequency of which is proportional to the flow-rate.

The approximate full scale output frequency of the flow-meter is 350Hz (pulses/sec). The flow-meter signal is converted to an analogue voltage by the signal conditioning circuit in the control module. This voltage is used to drive the digital flow-rate display on the rig and converted to digital form by an ADC for input to the controller. This is all shown diagrammatically below.



3.2.3 Level Measurement.

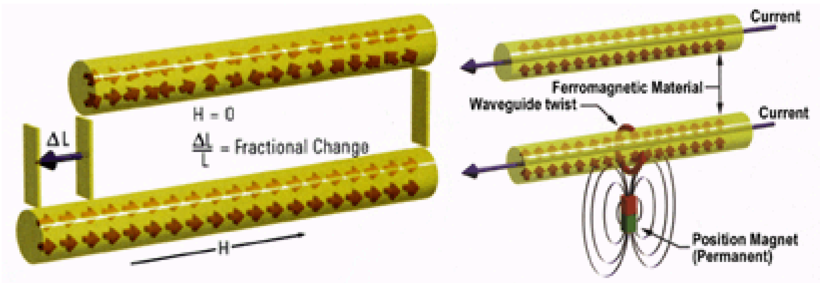


Figure 3.2.1

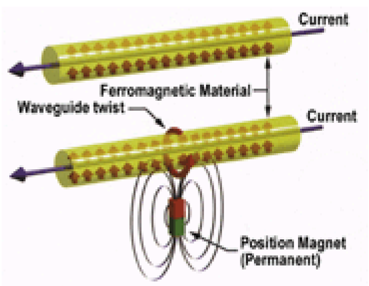
A magnetising force, H, causes a dimensional change due to the alignment of magnetic domains.

Magnetostriction is a property of ferromagnetic materials such as iron, nickel, and cobalt. When placed in a magnetic field, these materials change size and/or shape

The physical response of a ferromagnetic material is due to the presence of magnetic moments, and can be understood by considering the material as a collection of tiny permanent magnets, or domains. Each domain consists of many atoms. When a material is not magnetized, the domains arc randomly arranged. When the material is magnetized, the domains are oriented with their axes approximately parallel to one another. Interaction of an external magnetic field with the domains causes the magnetostrictive effect. This effect can be optimized by controlling the ordering of the domains through alloy selection, thermal annealing, cold working, and magnetic field strength.

The ferromagnetic materials used in magnetostrictive position sensors are transition metals such as iron, nickel, and cobalt. In these metals, the 3d electron shell is not completely filled, which allows the formation of a magnetic moment. (i.e., the shells closer to the nucleus than the 3d shell arc complete, and they do not contribute to the magnetic moment). As electron spins are rotated by a magnetic field, coupling between the electron spin and electron orbit causes electron energies to change. The crystal then strains so that electrons at the surface can relax to states of lower energy. When a material has positive magnetostriction, it enlarges when placed in a magnetic field; with negative magnetostriction, the material shrinks. The amount of magnetostriction in base elements and simple alloys is small, on the order of 10^{-6} m/m.

Since applying a magnetic field causes stress that changes the physical properties of a magnetostrictive material, it is interesting to note that the reverse is also true: applying stress to a magnetostrictive material changes its magnetic properties (e.g., magnetic permeability). This is called the Villari effect. Normal magnetostriction and the Villari effect are both used in producing a magnetostrictive position sensor.



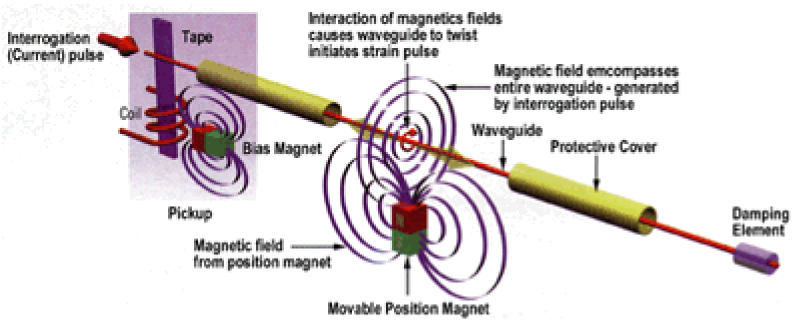
The Wiedemann effect describes the twisting due to an axial magnetic field applied to a ferromagnetic wire or tube that is carrying an electric current.

Figure 3.2.2

An important characteristic of a wire made of a magnetostrictive material is the Wiedemann effect (see Figure 3.2.2). When an axial magnetic field is applied to a magnetostrictive wire, and a current is passed through the wire, a twisting occurs at the location of the axial magnetic field. The twisting is caused by interaction of the axial magnetic field, usually from a permanent magnet, with the magnetic field along the magnetostrictive wire, which is present due to the current in the wire. The current is applied as a short-duration pulse, 1 or 2 µs; the minimum current density is along the centre of the wire and the maximum at the wire surface. This is due to the skin effect.

The magnetic field intensity is also greatest at the wire surface. This aids in developing the waveguide twist. Since the current is applied as a pulse, the mechanical twisting travels in the wire as an ultrasonic wave. The magnetostrictive wire is therefore called the waveguide. The wave travels at the speed of sound in the waveguide material, ~ 3000 m/s.

The operation of a magnetostrictive position sensor is shown in Figure 3.2.3.



The interaction of a current pulse with the position magnet generates a strain pulse that travels down the waveguide and is detected by the pickup element.

Figure 3.2.3.

The axial magnetic field is provided by a position magnet. The position magnet is attached to the machine tool, hydraulic cylinder, or whatever is being measured. The waveguide wire is enclosed within a protective cover and is attached to the stationary part of the machine, hydraulic cylinder, etc.

The location of the position magnet is determined by first applying a current pulse to the waveguide. At the same time, a timer is started. The current pulse causes a sonic wave to be generated at the location of the position magnet Wiedemann effect. The sonic wave travels along the waveguide until it is detected by the pickup. This stops the timer. The elapsed time indicated by the timer then represents the distance between the position magnet and the pickup.

The sonic wave also travels in the direction away from the pickup. In order to avoid an interfering signal from waves travelling in this direction, their energy is absorbed by a damping device (called the damp). The pickup makes use of the Villari effect. A small piece of magnetostrictive material, called the tape, is welded to the waveguide near one end of the waveguide. This tape passes through a coil and is magnetized by a small permanent magnet called the bias magnet. When a sonic wave propagates down the waveguide and then down the tape, the stress induced by the wave causes a wave of changed permeability (Villari effect) in the tape. This in turn causes a change in the tape magnetic flux density, and thus a voltage output pulse is produced from the coil (Faraday Effect). The voltage pulse is detected by the electronic circuitry and conditioned into the desired output.

3.2.4 Pressure Measurement

Pressure Measure is achieved using a pressure transducer:

0 to 10v; 0 – 5 bar;
Non-linearity, hysteresis (BSL) and repeatability – max $\pm$ 0.2
Response Time :- (10 to 90%) 10ms
Operating Temperature: - -40 to 100^{o}C

The Indicator and software output display the pressure as a percentage. 100% represents 2-Bar.

Pressure (*p*) is a derived unit with a own name. The name is Pascal (Pa), also known as N/m^2. in SI this will be kg·m^{-1}·s^{-2} (kg/m·s^2).

The unit of pressure used to be torr, also known as mmHG (millimetre mercury). Some mercury was put in a glass tube. If the pressure changes, the level of the mercury in the tube changes with it. The reason that mercury is used is that the thermal expansion of mercury is big and is mostly homogeneous. Mercury also doesn't stick to the glass. It was calibrated that on sea-level the height of the mercury was 760 mm. At sea-level the pressure is also defined as 1 atmosphere, that's why 1 atmosphere equals 760 torr. If you measure the pressure in Pascal at sea-level, you will find 101325 Pa. This also equals one atmosphere.

Pressure is related with temperature and volume. If the volume is constant and the temperature gets higher, the pressure also gets higher. This is because the molecules gets more energy and move faster. The relation is given with:

$$p * V = c*T$$

p in Pa
V in m^3
T in K
c is a constant value, we have taken one, but you can also take another value in its place

3.3 Displays

3.3.1 Digital LCD Displays

These are used on the rig to display the sump, and process tank temperatures in degrees Celsius, flow-rate in litres/minute, level in percentage and pressure is a percentage. A back light can be switched on or off from a switch on the front of the Control Module.

3.3.2 Indicators

These are sealed indicator lamp units which reveal the on/off status of the cooler fan, heater and drain and diverter valves.

3.4 Pump

The pump includes a safety shut off switch should the pressure in the system exceed 1.5bar (21.7557 PSI) (150000Pa) and a built in filter.

Voltage	: 24 V d.c.
Current	: 0.5A
Max Watt	: 7L/Min
Pressure cut out	: 1.5 bar

3.5 Cooler

If the temperature of the water is too high the flow may be diverted through the cooler. The cooler unit has an integral fan which may be activated by the controller through the control module.

3.6 Solenoid Valves

Diverter Solenoid Valve

The water is normally pumped directly to the process tank but the diverter valve may be used, when required, to divert the flow through the cooler. The solenoid is a 3 port 2 way type.

Drain and flow control Solenoid Valve

These are used to drain the process tank back to the sump, and to control the flow rate in the system.

3.7 Process Tank

When both the manual and computer controlled drain valves are closed the tank may be filled. A tank full sensor is provided from the level sensor which provides a digital input to the PC. When the tank is at the required level the water may be heated by the heater element. During temperature experiment the software reads the process tank temperature via the ADC. The water in the tank can be drained back to the sump for recycling.

3.8 Heater

This is a 400W 48 V cartridge type element. The computer controls the heating element, through the Control Module, with a pulse width modulated (PWM) signal which turns the power to the element on and off. The mark/space ratio of this signal is determined by a software PID algorithm. For a detailed explanation of PWM and the heater control circuitry refer to Section 4.6.2.

a) Tank Full

The software supplied with the PCT-100 incorporates a software interlock which prevents power being applied to the heater until the signal from the level sensor shows that the process tank is at the required level. If it is not the software displays a warning message and drives the pump at full speed to fill the tank to this level. When the tank is at the required level heating may begin.

If students are to write their own control software it is crucially important that they duplicate this software interlock in their program, before they use the heating element.

b) System Failure

If the computer fails for any reason, the circuitry on the control module detects this and shuts the heating element off.

N.B. THE PROCESS TANK CAN BE HEATED UP TO 60°C. THEREFORE DO NOT TOUCH THE TANK WHEN IN USE. THERE IS A PRESSURE RELIEF VALVE FITTED TO PREVENT THE PRESSURE IN THE PROCESS TANK REACHING A LEVEL THAT COULD CAUSE THE PROCESS TANK TO BE DAMAGED OR CAUSE INJURY TO THE OPERATOR. THE PRESSURE RELIEF VALVE IS SET TO OPERATE AT 2.5 BAR (36.2595PSI) (250000Pa).

4.0. THE CONTROL MODULE

4.1 General Arrangement

The control module incorporates all of the electronic circuitry required to link the process rig to the controller. The design of the circuits demonstrates the interfacing principles required in many process control situations where a mix of analogue, digital and frequency signals have to be processed.

The front of the Control Module has a schematic of the Process Rig, On/Off indicator, six illuminated fault switches, test points, indicators to show the operational status of the elements on the rig, and a backlight switch to turn on the backlights for the displays on the rig.

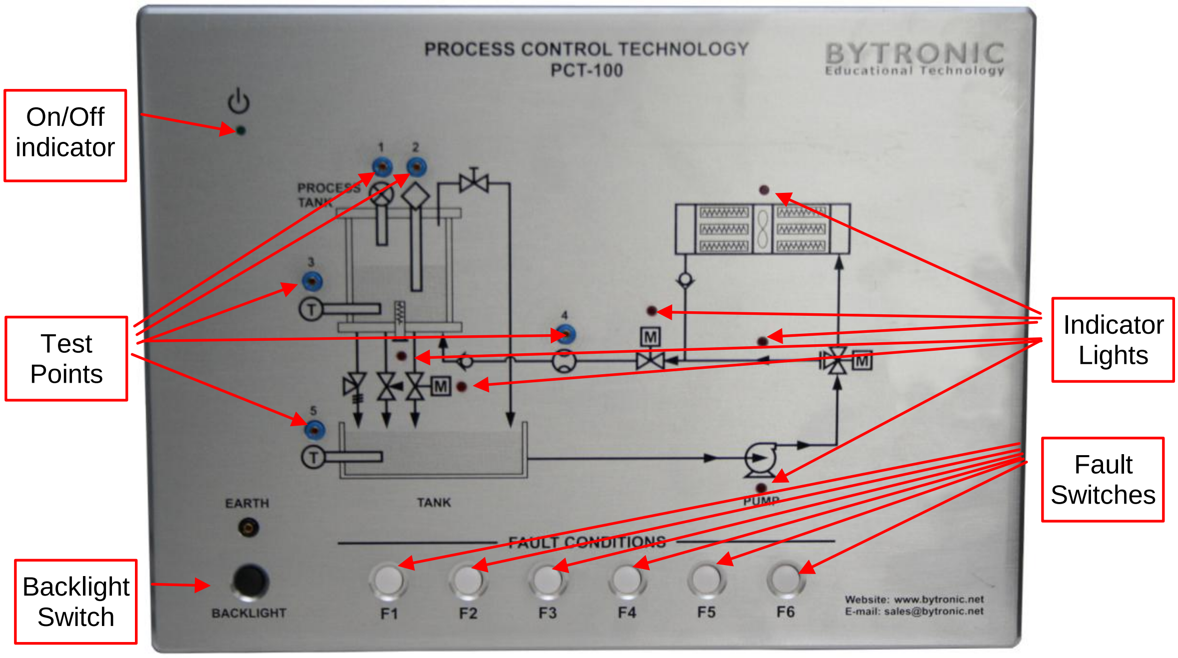


Figure 4.1

All connections to the process rig and power supply unit are made to the rear of the control module. The USB port on the PC is connected to the USB sockets on the control module.

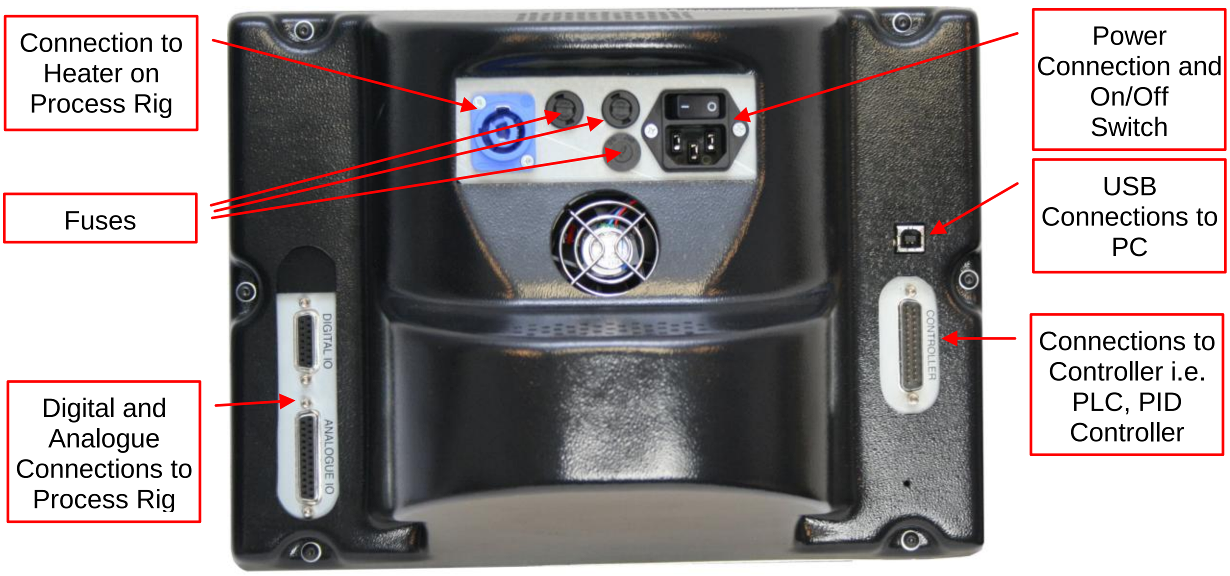


Figure 4.2

4.2 Heater Control - Safety Features

Since the heating element is capable of heating the water in the process tank to a high temperature or causing damage to the tank, several safety features have been incorporated into the PCT-100 system. The safety requirements are as follows:

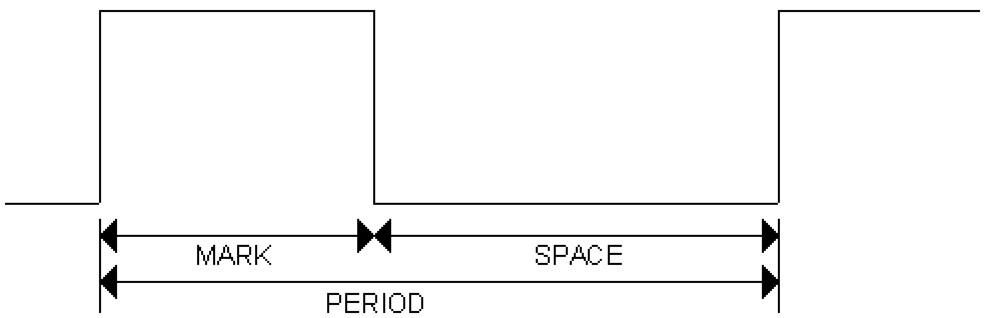
- The heating element must only be energised when the water level in the process tank is at the required level.
- If any connection between the computer and control module fails or the computer ‘crashes’, the heating element must be switched off automatically.
- Tank full signal. The software supplied incorporates an interlock to prevent power being applied to the heater until the signal from the level sensor shows that the process tank is the required level. If students are to write their own control software it is crucially important that they duplicate this software interlock in their program, before they use the heating element.
- Failures or crashes. If the computer-to-control module communication fails, the circuitry on the control module detects this and turns the heating element off.

4.2.1 Heater Control

The heating element contained in the process tank is controlled by the computer using a pulse width modulation (PWM) technique. The required ‘mark/space ratio’ is determined by a software algorithm in the case of computer control

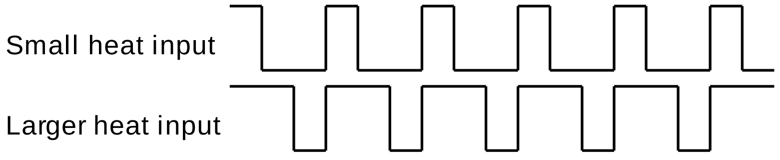
4.2.2 Pulse Width Modulation

A PWM signal is a rectangular wave of constant frequency but varying mark/space ratio as illustrated in the diagram below:



On the PCT-100 a PWM signal is used to switch power to the heating element. As the mark/space ratio increases the average rate at which electrical energy is dissipated by the heating element increases, increasing the rate at which the water is heated.

Example



Under computer control the software determines the difference between the required temperature and process tank measured temperature. The larger the difference, the greater the control algorithm sets the mark/space ratio and the faster the process tank is heated.

4.2.3 Pump Control

The software initially outputs zero and gradually increases the output to whatever value is necessary to achieve the required flow rate. A PID algorithm utilising feedback from the flow-meter re-calculates the output value at each sample interval.

4.2.4 Switched Faults

With the understanding of how systems operate within industrial control systems there is a great need for individuals with faultfinding skills. To help develop these skills the PCT-100 incorporates six individually selectable switched faults located on the front panel of the control module (see fig 4.1; front panel of the control module).

The switched faults are typical of those found in real industrial applications. Generally an electrical fault may be due to one or more of the following:

1. Component failure
2. Electrical short circuit to supply
3. Electrical short circuit to ground
4. Crossed signal wires either due to a short or incorrect commissioning.
5. Open circuit due to a broken wire, burnt circuit track or bad connection.

The student may be given fault finding tests following his observation of the correct operation of the PCT-100. The effect of each fault is shown in *Figure 4.3*. Using standard test equipment in conjunction with circuit diagrams, the faults may be successfully diagnosed.

FAULT NO.	FUNCTION	EFFECT
F1	A/D Fault	ADC LOCKS UP
F2	Pump	PUMP CANNOT BE ACTIVATED
F3	Temperature	PROCESS TEMPERATURE DISPLAYS FULL SCALE READING PERMANENTLY
F4	Cooler	COOLER PERMANENTLY ON
F5	Heater	HEAT DISPLAY GIVES A FAULTY READING
F6	Flow-meter	NO FLOW FEEDBACK

Figure 4.3 List of Switched Faults

5.0. SOFTWARE

The real-time control software supplied with the PCT-100 is compatible with Windows XP. With the USB driver from the CD-ROM

5.1.1 Hardware Requirements

The computer minimum specification is Pentium, 250MB RAM, 60GB hard-disk space available, CD-ROM drive and Windows XP or above.

5.1.2 Installation

This is very straightforward and described fully on the PCT-100 CD-ROM. See the earlier Getting started section of this manual for more information

5.1.3 Software Facilities Reference

As we have seen, the software is very straightforward to use. It has four main menus named File, Control, Setup and Help.

File:
New begins a new experiment.
Open opens and displays a (previously saved) trend file produced by an earlier experiment. (See Saving and Retrieval of Trends above).
Save saves the current experiment trend to disk for later retrieval.
Print prints the current experiment trend.
Exit exits from the software.

Control:
Manual provides manual control of the main PCT-100 features.
Flow allows control of the rate of flow of water from the sump to the process tank.
Temperature allows control of the temperature of the water in the process tank.
Pressure allows control of the pressurisation of the process tank.
Level allows control of the level of the water in the process tank.
Open Loop allows open loop control of the rate of flow of water from the sump to the process tank.

Setup:
Toolbar turns on the toolbar to reveal graphical command icons.
Configuration opens a window in which the start up preferences may be specified.

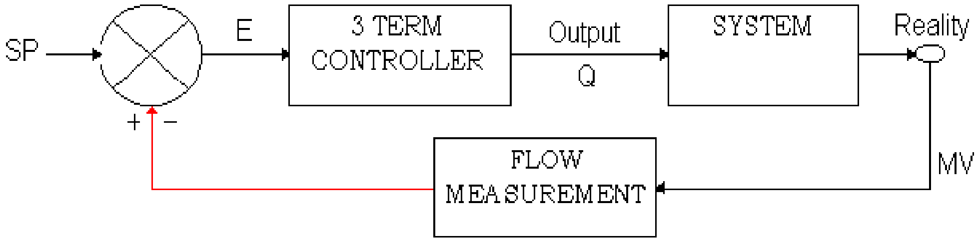
Help:
Contents gives entry to the help information for all features of the software. (Also accessible by hitting F1).
About gives the software version number.

The various menus and windows of which the software consists use certain phrases, terms and controls which are defined in the list below:

Start and **Stop** buttons start or stop the current experiment. Once the process has begun, the data is captured and displayed on the trend window. Viewing stored data is mouse controlled and highly intuitive. Once the experiment is stopped, graphical data may be saved or printed.

Control Mode for each experiment may be PID or Manual. This means that either the on screen PID parameters are used within a closed loop system or the user can control the process rig manually.

PID is a control technique used to calculate the output signal used to drive the pump or heating element. Essentially a three term controller takes a measured value from the sensor (flow-meter, temperature probe pressure transducer or level sensor) and compares it with the set point (desired value). The discrepancy between the measured and desired values, called the error, is used to determine the control output signal. This is all represented in the following block diagram, labelled for the flow loop, though do note that the same basic 'closed loop' arrangement applies to the control of flow rate, level temperature, and pressure and level within the software.



E : Error
SP : Desired flowrate
MV : Measured value

The line from the flow measurement block to the summing junction is the feedback signal and any process which contains such a signal is a closed loop system.

Set Point is the process variable desired value that the controller is trying to achieve and maintain. It can either be a fixed value or a square, sawtooth, ramp or sinusoidal waveform for the flow and level experiments. For the temperature and pressure experiments only fixed value set points are available, for obvious reasons.

Sample Time is an important factor which effects the performance of a computer based controller. It is the time interval between successive samples of the measured value. A long period between samples reduces the need for rapid analogue to digital conversion and reduces the computational load, but as the sample time is increased a number of degrading effects become significant. If the sampling frequency is too low then important high frequency information will be lost.

Trend windows show the results of the experiments. To display more data on the screen the user may adjust the slider to the top right of the trend window. This slider varies the timebase of the trend. The arrows to the left of the trend allow the data to be scrolled left or right. Direct numerical input equivalents for these two features are also provided.

Traffic Light displays are used to show the real-time state of the system. When red the process has been halted. When green the process is running in real-time. When amber the process is running slow, i.e. heavy processing of some sort is taking place e.g. moving windows, other programs, or processing by Windows, and the system is catching up a fraction each sample cycle. Recurrent amber can be solved by closing other programs, increasing the processing capacity of the computer or increasing the sample time.

Experiment Time for the batch volume experiment may be varied between 60 and 600 seconds. This is the period over which the target volume (set point) of water should be 'delivered' at the overflow outlet of the process tank.

Auto Drain is an optional function which automatically sets the drain valve open and closed in a repeating cycle if the measured level is greater than the set point in fluid level experiments.

Saving
The software is able to save the trend data recorded during an experiment to a comma delimited file which may be imported into various statistical packages such as Microsoft Excel or Mathworks MATLAB. You must complete an experiment before you can to save the data. The data may be re-examined by opening the file and viewing the trend on the screen.

Printing
When printing the software automatically selects the default printer and default settings. To change this click on the Setup… button next to the printer name. You can print all of the traces at once or single traces by selecting the tick boxes. You can choose to print the whole diagram or just a portion of it, in black and white or colour.

6.0. EXPERIMENTS USING PC SOFTWARE

IMPORTANT
BEFORE BEGINNING THESE EXPERIMENTS ENSURE THE VENT VALVE AT THE TOP OF THE PROCESS TANK IS OPEN.

The PCT-100 software allows students to investigate the following subject areas:

- Control of the rate of flow of water from the sump to the process tank.
- Control of the temperature of the body of water in the process tank.
- Control of the water level in the process tank.
- Control of the pressure in the process tank

Before using the software the start up preferences should be specified by selecting Configuration from the Setup menu. The Program Interface fields allow the user to select the USB interface

Manual Control
Select Manual Control (Control, Manual) and experiment by clicking upon the cooler fan, diverter valve drain valve slide, flow valve slide, heater slide. Click, hold down and slide the Pump Output control and observe the effects.

Flow Control
Select Flow Control (Control, Flow) and click START to run an experiment using the default PI controller. Click STOP once the traces have settled down to the 1.5 litres/minute set point value. Now click the ticked boxes adjacent to the legends: Flow SP (i.e. set point), Flow MV (i.e. measured value) and Pump Output. You will see that the traces may be hidden or revealed as required to help in your evaluation of the response.

Move the slider near the top right hand side of the screen across to the left to magnify the traces and position the tip of your mouse pointer at various positions on the traces to pop up the values recorded at specific times.

Move the slider back and start the experiment again, this time click the SP value after about ten seconds and change the set point to 1 litre/minute and then increase it to 3.6 litres/minute after a further ten seconds. You will see that whilst a little oscillatory the response is reasonable, i.e. the measured value reaches the set point quickly and without massive overshoot for the first two SP values. When the SP is 3.6 litres/minute however, saturation of the controller output occurs. i.e. Even with 100% controller output (white trace at the very top) the measured value does not reach the set point.

Change the SP to 1 litre/minute and the integral term in the controller to 999 seconds and then run the experiment again. (An integral action time of 999 seconds more or less eliminates any integral effect which means that we are now left with a proportional only controller. The proportional and integral terms etc. will be defined later). Once the proportional offset has been clearly established, i.e. the steady state gap between the set point and the flow measured value of about 0.5 litre/minute, reduce the integral action time to 1 second and observe how the integrating action, which takes account of the historical aspect of the error, brings the measured value smartly up to the set point.

Level Control
Select Fluid Level Control (Control, Fluid Level) and click START to see an experiment of level control in the process tank, using the default controller. Over the 120 second duration of the experiment vary the set point several times by clicking the SP box and observe the effects. In this experiment the process tank drain valve is normally closed and the controller opens it every time the set point is changed to a value which is *lower* than the current level. If the set point is increased however, the pump is turned on fully to drive water into the process tank as quickly as possible. As the water level approaches the new set point the control algorithm is used to progressively reduce the pump output so that there is no overshoot. The resulting graph may be manipulated as those looked at previously.

Pressure Control
For this experiment the vent valve should be closed. If a temperature experiment has been carried out cool fluid using cooler and flow to ambient temperature before proceeding, failure to do this may result in pressure build up on closing the vent valve and affect the results of the experiments. Select pressure control Pressure and click start. A disturbance can be added by opening the needle valve (small amount) to allow for a drain from the process tank and observe how the controller maintains the pressure value. Use a set point of 0.5.

Temperature Control
Select Temperature Control (Control, Temperature) and click START to run an experiment using the default PI controller. Click STOP after about ten minutes, enter “0” in the Start Time box at the top left hand side of the graph and move the slider near the top right hand side of the graph across to the right to show the whole trace. Position the tip of your mouse pointer at various positions on the traces to pop up the values recorded at specific times. Now click the ticked boxes adjacent to the legends; Temp SP (i.e. set point), Temp MV (i.e. measured value) and Temp Output. You will see that once again the traces may be hidden or revealed as required to help in your evaluation of the response.

Change the set point to a much higher temperature 40ºC by clicking on the Set Point box. Run the experiment and observe how much longer the measured value takes to reach the set point when the initial error is so large. Clearly the PCT-100 temperature control loop has larger time constants that the flow control loop. Control of temperature and flow are quite different since you cannot allow the temperature to go above the set point because the only way to then cool the specific body of water in the process tank is via natural methods. With the very short time constants of the flow loop it is quite acceptable, even desirable, to allow the flow to overshoot the set point once or twice in order to achieve the optimum settling time.

Open Loop Control
This section provides facilities to study the open loop response of the flow loop to manually selected step, ramp and sinusoidal inputs. Select Open Loop Control (Control, Open Loop) and click the sine-wave input option. Start the process and once a full wavelength of the traces has been drawn, experiment by changing the Max, Min and Period parameters to see how the input signal may be manipulated. In this open loop section the sinusoidal input function may be used to examine the frequency response of the flow loop by running several experiments with the same Max and Min values but with a range of different Periods.

Saving and Retrieval of Trends
On completion of any open or closed loop control experiment the graph which is produced may be saved onto the hard disk. As an example select Flow Control (Control, Flow) and click START to see the default experiment then click File, Save to save the graph to the hard disk. Now close down the software then restart it and click File, Open to retrieve this ‘trend’. Over a period of time a family of informative trends can be built up to illustrate key features such as ‘proportional offset’, the benefits of integral action, integral windup and the advantages and disadvantages of derivative action etc.

7.0. SYSTEM CONTROL

7.1 Introduction

In industry the word "process" refers to the set of operations required to manufacture a particular product. Whatever the process the end product must meet the specified design properties if it is to be of value to a potential customer. The product characteristics usually depend upon how well the process reactions and operations were controlled.

Initially in this chapter we consider three main categories: on/off control (sometimes called two step control), open loop control and closed loop control. In sections 7.5 and 7.6 we explain some basic principles that are used to assess a control system's performance mathematically. This is a big subject so the information provided can only serve as a pointer to the areas that may be studied. Section 7.7 which covers PID control, will be of greater practical interest to most users. Section 7.8 provides an introduction to the more complex subject of direct digital control using z transform techniques.

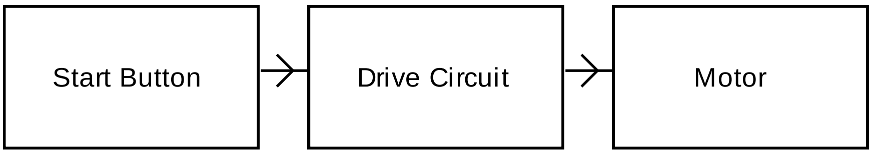
Some of the ideas in this chapter are reinforced by the labworks which follow in chapter 8.

7.2 On/Off Control

An on/off controller outputs digital signals to remote equipment to perform simple switching functions. On the PCT-100 all of the elements activated via the control module output drivers circuit are controlled in this manner. On/off control is often open loop as for example with the rig status indicators (their operation does not effect the process at all). It may also be used as part of a closed loop as for example when the cooler and diverter valve are activated to reduce the temperature of continuously circulating water. Here turning the cooler on and off will effect the process and this will be seen by feedback from the PRTs.

7.3 OPEN LOOP Control

Open loop control systems have no means of compensating for variations in their output. They do not employ feedback for error detection and so accurate control is sometimes not possible. An example is a simple motor drive circuit:

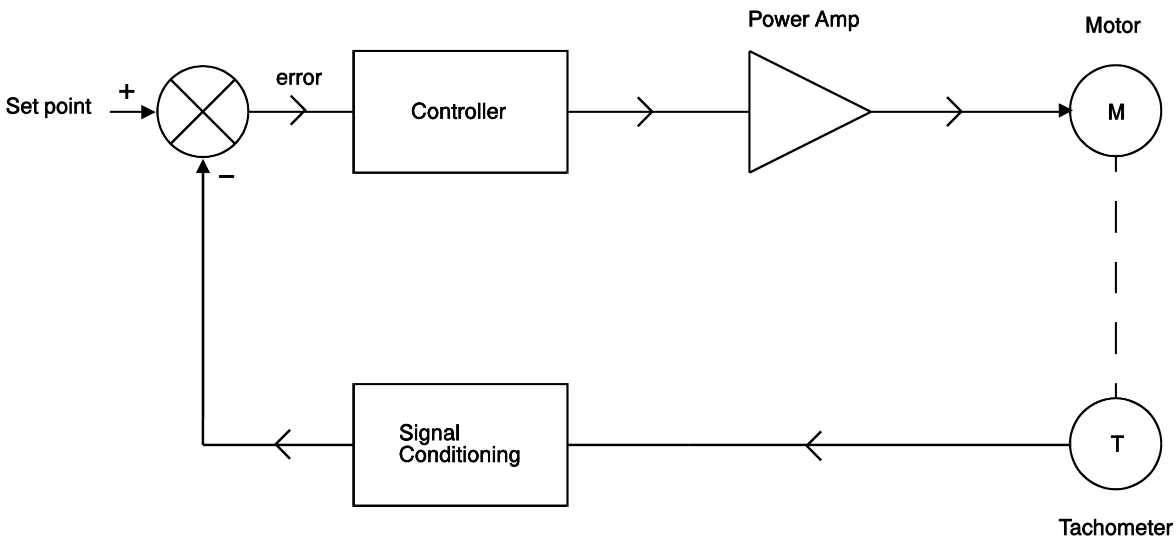


If the motor speed falls due to an increase in the load there is no method of automatically compensating for the error. However open loop control is cheap to implement and quite satisfactory in many cases.

7.4 CLOSED LOOP Control

Closed loop control systems employ feedback from output to input to compensate for variations in loading and other sources of error. The output value is measured and compared with the required value (set point). The size of the deviation (error) is used to determine the compensating control action. There are many ways in which the calculation may be done but in essence, the greater the error the greater will be the required control action. Closed loop systems are therefore said to be 'error activated'.

It is impossible to drive a system instantaneously to zero error however sophisticated the control algorithm because of the effects of inertia, friction and air resistance etc. This means that the output always lags the set point input and the output sometimes oscillates about a zero error position. System errors may also be introduced by the sensors themselves since their output signals are not always exactly proportioned to the variables which they are designed to measure. In this case the feedback is not a true replica of the signal being measured. An example of a typical closed loop control system is a motor speed controller:

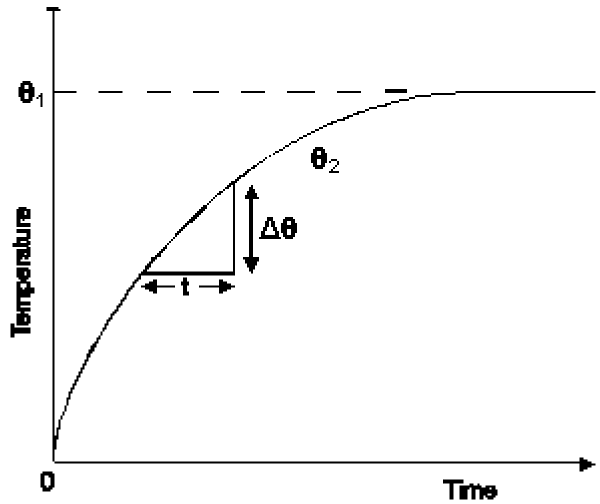


7.5 Basic Control Principles

To predict the way a system will respond to a particular disturbance or change of set point an attempt might be made to model it mathematically. This usually means representing physical device characteristics such as inertia, damping force, capacitance, and resistance etc. by terms in a differential equation. Although idealised conditions which cannot be achieved in practice often have to be assumed, valuable insights into a system's performance may be gained this way. The ideal system is linear in that it may be represented by a linear differential equation. Systems are often classified by the order of the equation which represents their behaviour e.g. 1^{st}. order, 2^{nd}. order etc.

7.5.1 1st. Order Systems

The behaviour of a 1^{st}. order system can be described by a 1^{st}. order linear differential equation. For example, consider a temperature probe with the following response:



Where:

θ_1	=	Temp of medium being measured	(ºC)
θ_2	=	Temp of probe	(ºC)
M	=	Mass of probe	(kg)
Cp	=	Specific heat capacity of probe	$(J.kg^{-1}.ºC^{-1})$
U	=	Overall heat transfer coefficient	$(J.m^{-2}.ºC^{-1}.s^{-1})$
A	=	Surface area	(m^2)
t	=	Time	(s)

Applying unsteady state "energy balance", the rate of heat transfer to the probe must equal the rate of heat accumulation at the probe. Therefore assuming no losses:

Heat transfer rate: $$Q = UA\Delta\theta = MCp\left(\frac{d\theta_2}{dt}\right)$$

Therefore: $$UA(\theta_1 - \theta_2) = MCp\left(\frac{d\theta_2}{dt}\right)$$

Where: $\frac{d\theta_2}{dt}$ = rate of change of the probe temperature

To comply with the standard format the forcing function must be on the right hand side and the response on the left hand side. Using the convention that operator D means d/dt we therefore have:

$$(UA + MCpD)\theta_2 = UA\theta_1$$

$$(1 + \frac{MCpD}{UA})\theta_2 = \theta_1$$

Comparing this with the standard form for a 1st. order system:

$$(1 + TD)\theta_2 = \theta_1 \qquad \text{(Equation 1)}$$

We see that for this example the time constant T is MCp/UA. Using K as a constant of integration, the solution for this particular differential equation is:

$$\theta_2 = \theta_1 + Ke^{-t/T}$$

In a similar manner it can be shown that a 2nd. order system may be represented by:

$$(MD^2 + fD + K)\theta_2 = K\theta_1$$

Laplace form: $(ms^2 + fs + K)\theta_2 = K\theta_1$

These linear differential equations provide a complete system description and for a given input the output may be determined by solving the equations with integrating factor or 'D' operator methods etc. However these methods can be cumbersome as well as difficult and therefore it is useful to consider the transfer function concept as described in section 7.5.2

7.5.2 Transfer Functions

The transfer function (TF) of a linear system is defined as the ratio of the Laplace transform of the output to the Laplace transform of the input, with all initial conditions assumed to be zero.

If differential equations are linear the Laplace transform may be regarded as a method of converting them into algebraic equations to allow more convenient manipulation. This method involves converting functions of time (t) in the differential equation to a function of the Laplace variable (s) by applying the transform:

$$L[f(t)] = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

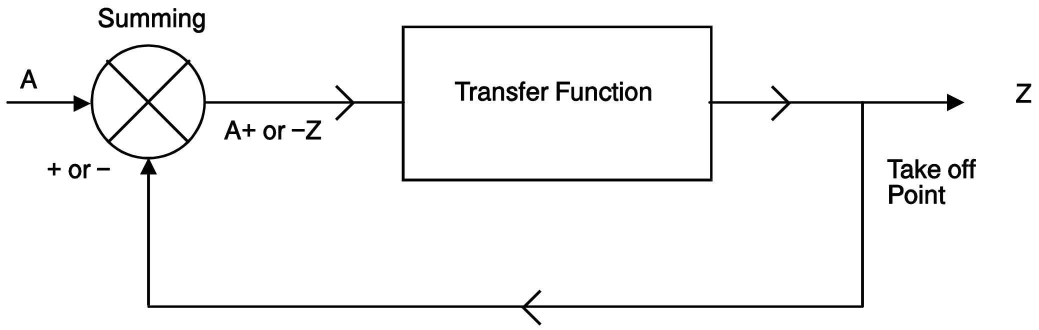
Extensive listings of these transforms have been compiled and it is rarely necessary to evaluate the integral. (See Appendix 1). The transformed equation may be manipulated with standard algebraic techniques to yield a solution in the variables. The final step is to inversely transform the solution in back into the time domain, again using the transform tables.

As control systems usually involve multiple ‘blocks’, s-plane manipulation can be quite complex. However it can be simplified by the use of block diagram reduction techniques (described in section 7.5.3) before applying the inverse transformation to the time domain.

Sometimes experimental tests are conducted to ascertain a system TF and this can be done with the PCT-100. In order to fully appreciate these tests a full understanding of sections 7.6 to 7.6.6 is necessary. Several experimental tests are referred to in section 7.6.7

7.5.3 Block Diagrams

In general these include blocks for the controller, plant and perhaps feedback signal processing plus take off points, summing junctions and directional arrows.



To ease algebraic manipulation a short hand notation for the transfer function G_n and feedback signals H_n may be used, where n is a suitable subscript. A typical closed loop control block diagram is shown in figure 7.1

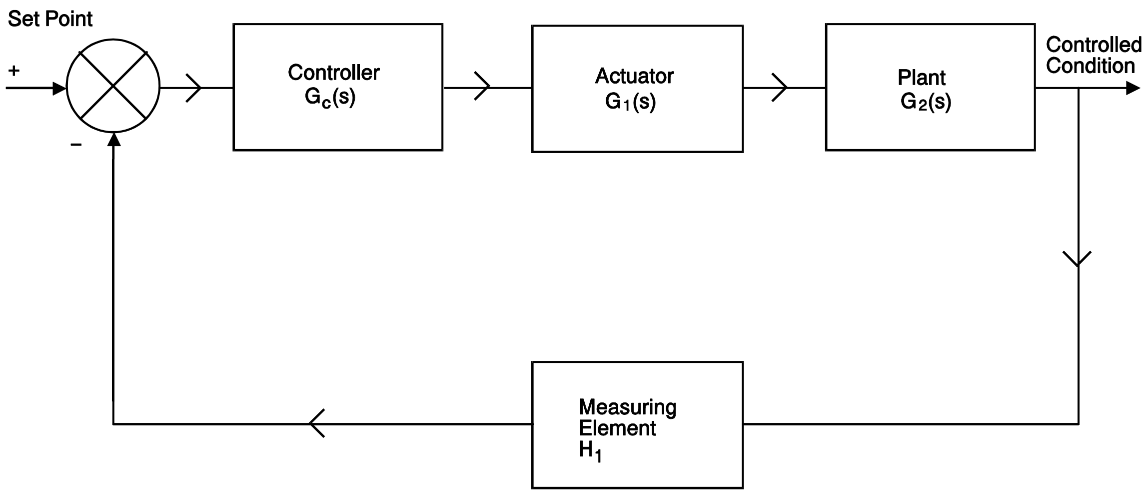


Figure 7.1 Typical Closed Loop Block Diagram

The block diagram of a practical system is often quite complicated but by applying systematic block diagram reductions, multiple loop systems may be simplified. A set of block diagram reduction rules are given in Appendix 3. An example is shown in figure 7.2.

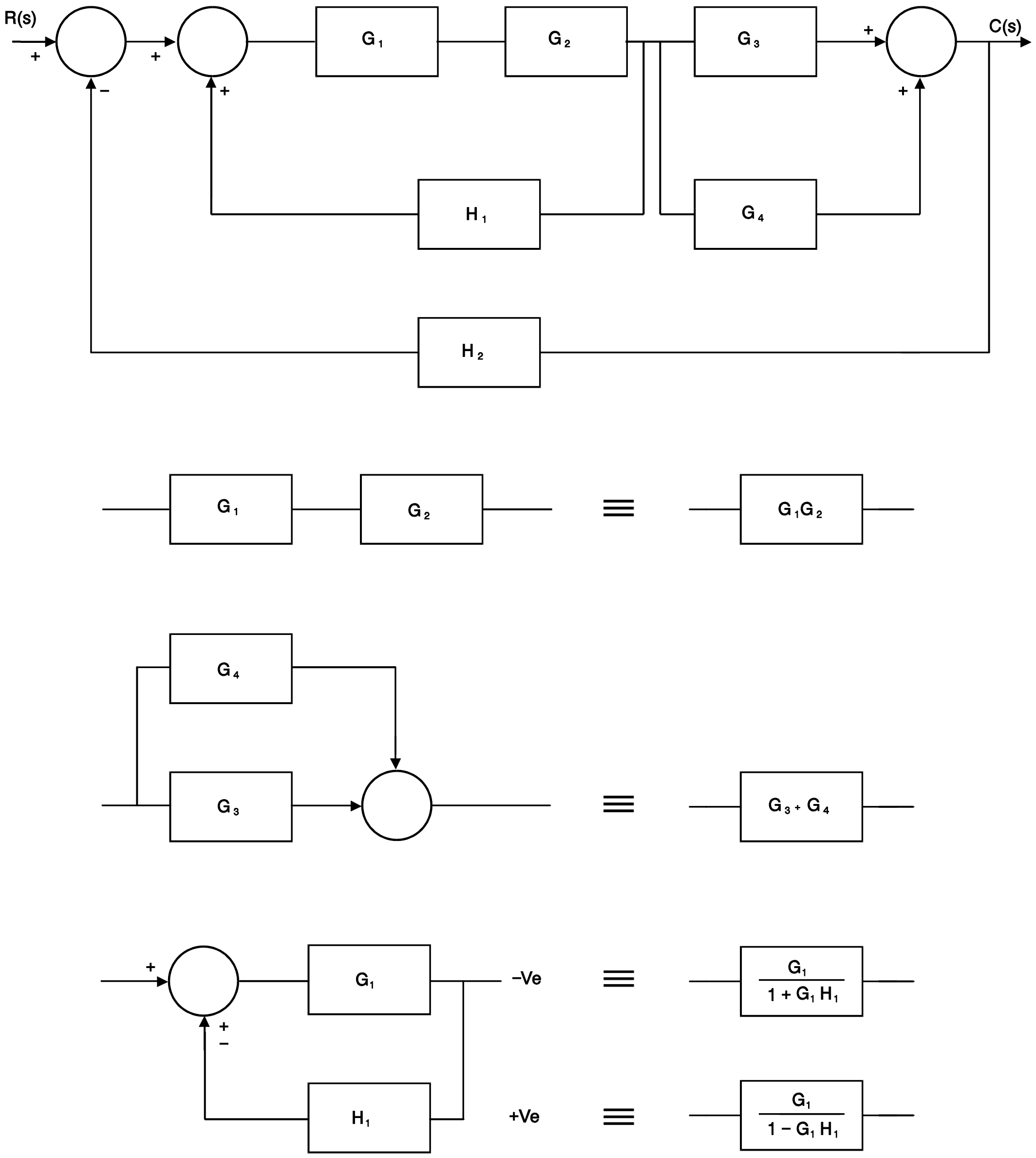
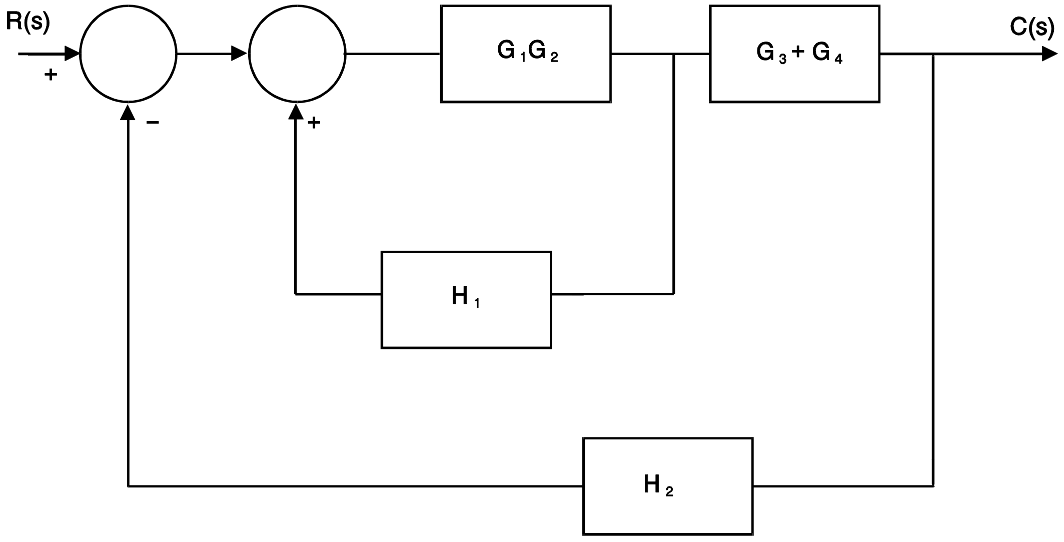


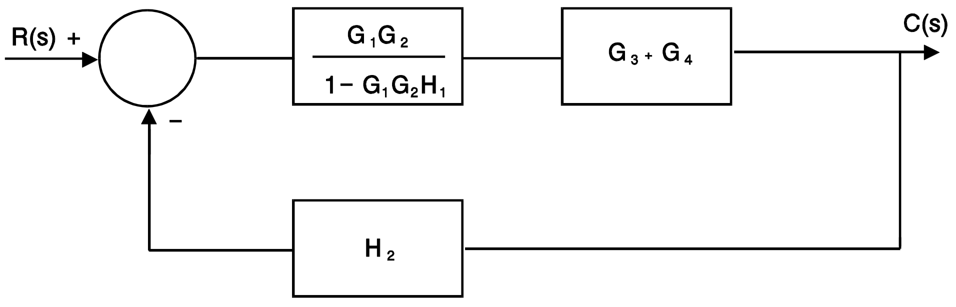
Figure 7.2 Example of Block Diagram Reductions

Figure 7.2 (continued)

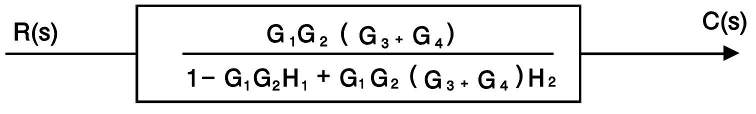
Step 1 -



Step 2 -



Answer



Having obtained the reduced block diagram the overall system TF may be determined by replacing the short hand notations G_n, H_n with the full forms of the individual block TFs. Inversely transforming the overall TF from the s-plane to the time domain will then allow the model's response to be calculated at specific times. For a list of transforms see Appendix 1.

As an example consider the servo control system shown diagrammatically in figure 7.3. The set point is determined by a single turn manual potentiometer and feedback is provided by another single turn potentiometer linked to the output shaft. The error between the set point and the measured value is amplified and used to control the motor to regulate the shaft position. When the feedback position equals the set point the error will be zero and the position will be maintained.

For example we will assume that the amplifier has a gain of K_1, the motor has a TF of K_2/s and the TF for the input potentiometer equals that of the feedback transducer.

For the potentiometer one complete rotation will cause the output voltage to vary from zero to its maximum value. Therefore:

$$\text{Output voltage} = \frac{AV}{2\pi} \qquad \text{(Equation 2)}$$

Where: A = Angle of rotation in radians (input)
V = Supply voltage

The potentiometer TF is therefore:

$$\frac{\text{Laplace Transform Output}}{\text{Laplace Transform Input}} = \frac{\text{Laplace Transform Equation 2}}{\text{Laplace Transform} \quad A} = \frac{1/s(AV/2\pi)}{1/s(A)} = \frac{V}{2\pi}$$

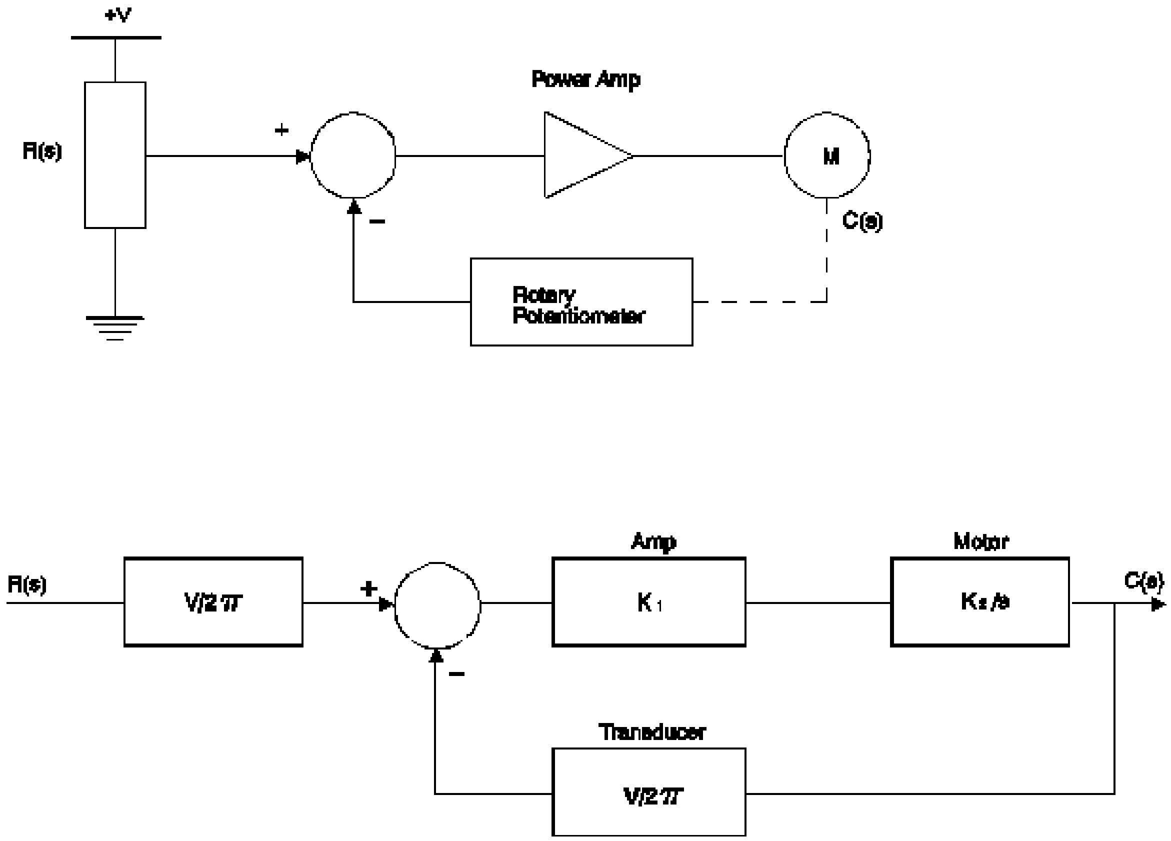


Figure 7.3 Servo Control Loop

Using the block diagram reduction techniques referred to earlier this reduces to:

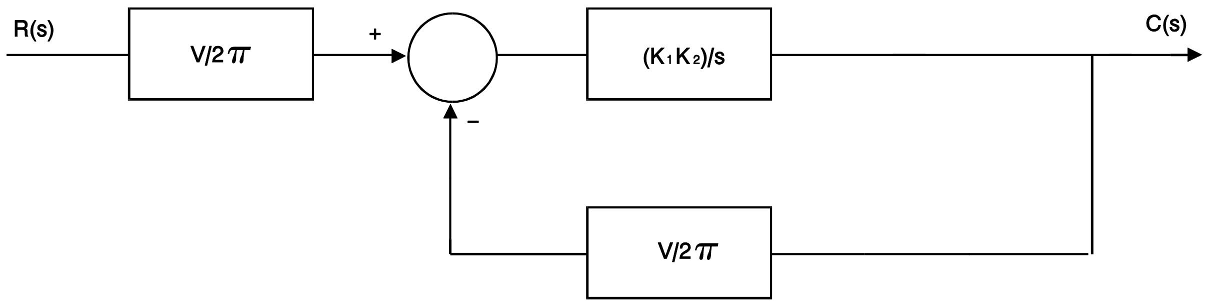


Figure 7.3.1 Reduced Block Diagram for the Servo Control System

The block diagram is shown in figure 7.3 and this can be reduced in several stages to that shown at the bottom of figure 7.3.2. Therefore the overall TF is:

$$\frac{C(s)}{R(s)} = \frac{(V/2\pi)K_1K_2}{s + K_1K_2V/2\pi}$$

If a step input is applied then from rule 2 of the Laplace transform table in Appendix 1, R(s) = A/s therefore the system output (given by R(s) multiplied by the TF) will be:

$$C(s) = \frac{A\ (V/2\pi)K_1K_2}{s\ (s + K_1K_2V/2\pi)}$$

We then use inverse transforms to transform from the s-plane back to the time domain. From rule 9 using laplace transform table inversely:

$$\frac{\alpha}{s(s+\alpha)} \rightarrow 1 - e^{-\alpha t}$$

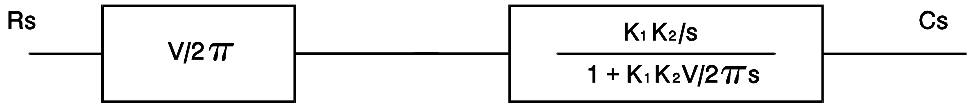
Therefore:

$$c(t) = A\{1 - e^{-(K_1K_2 V/2\pi)t}\}$$

The output response c(t) may be calculated by inserting the values K_1, K_2, V, A and the time (t). For example take K_1 = 2, K_2 = 5, V = 5V and the set point A = 0.5 radians. Substituting these values into the above equation produces the following results:

Time (t) Seconds	c(t) Response (Radians)
0.05	0.1641
0.10	0.2744
0.15	0.3484
0.20	0.3982
0.25	0.4316
0.30	0.4541
0.35	0.4691
0.40	0.4793
0.45	0.4860
0.50	0.4906

By plotting c(t) against time, the system response may be evaluated for a step input of 0.5 radians. The result may either be plotted by hand or the system may be simulated (open loop) using a control system design and simulation software package. The plot is shown in figure 7.3.3



Reduces to -

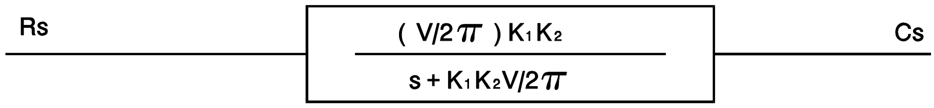


Figure 7.3.2 Reduced Block Diagram for the Servo Control System

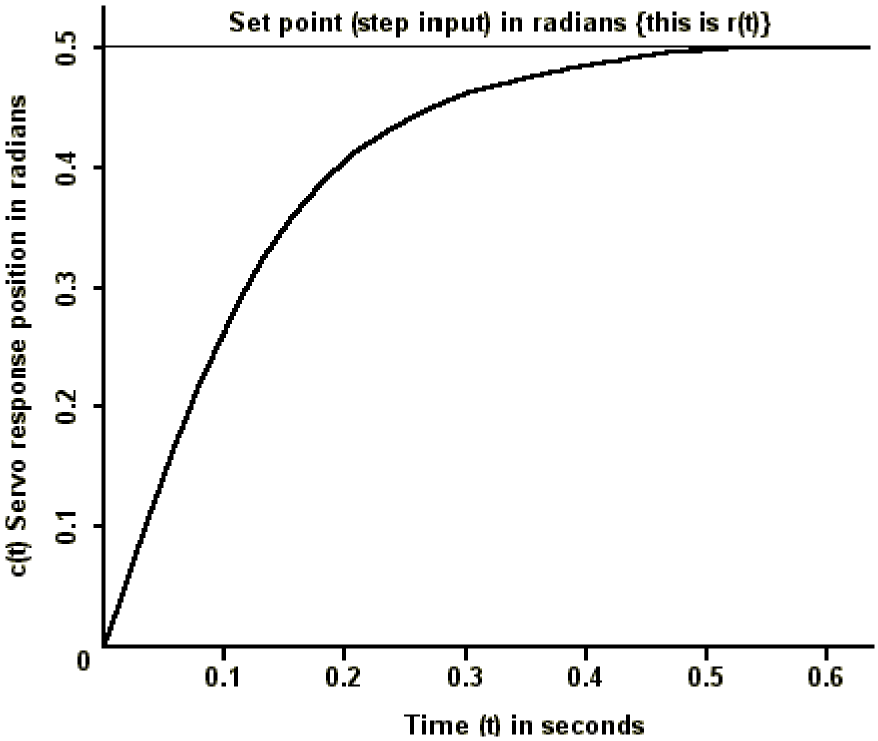


Figure 7.3.3 System Response of the Servo System

7.6 Assessment of System Performance

It is sometimes necessary to assess the performance of a process control loop, (plant, sensor, controller and actuator) in order to ascertain whether it may be improved. The best method is to obtain a response curve, (sometimes called a process reaction curve), for the type of input disturbance expected in normal operation. Typically the input disturbance might be a step change e.g. increase the flow-rate set point by 15% or reduce the temperature set point by 10%. In general a control loop's response to a disturbance includes both transient and steady-state components. Given enough time, the transient component will eventually die away completely and the difference between the measured value of the process variable before the disturbance and after the transient has faded, represents the steady-state element of the response. (Transient and steady-state responses, which are determined by alternative solutions to the underlying differential equation, are in fact superimposed upon each other).

7.6.1 Transient Responses

Transients are caused by sudden or discontinuous changes in a variable upon which the measured value depends. Depending upon the tuning of the controller, the transient response will be under damped, over damped or critically damped. Typical curves for these three are shown in figure 7.4.

In the under damped case the transient response of the system to a disturbance is oscillation about the set point. In the over damped case the transient response is an excessively long period in which the measured value increases until it reaches the set point. In the critically damped case the response is an optimally rapid increase to the set point without oscillation.

In order to compare system performance other parameters may be considered including peak overshoot, rise time, settling time, period and transport delay. (See Figure 7.4.1). The definitions that follow may be applied equally to open or closed loop systems.

- Peak overshoot is the maximum amount by which the response exceeds the final steady state value of the process variable. It is sometimes expressed as a percentage of the final steady state value.
- Rise time is the time taken for the response to increase from 10% of its final steady state value to 90% of its final steady state value.
- Settling time is the time taken for the response to reach its final steady state value, within some specified tolerance. Figure 7.4.1 shows the settling time for a 5% tolerance.
- Periodic time (or period) is the duration of one complete cycle of oscillation. It can therefore be measured as the interval between *alternate* crossings of the final steady state value or the interval between *successive* peaks or *successive* troughs on the response curve.
- Frequency is the reciprocal of the period, i.e. the number of cycles per second which is expressed in Hertz (Hz). Sometimes the frequency is expressed in radians per second and the relationship between the two units is that radians per second equals 2π times the frequency in Hertz.
- Transport delay is the period during which there is *no change* in the process variable after a step change has been made to the set point.

Completely separate from the transient, steady state error may also be present in the overall response, (see Figure 7.4.2). This is the deviation between the measured value and set point once the system has stabilised and the transient has faded. This may be evaluated mathematically for a system where the transfer function is known, using the final value theorem discussed in section 7.6.3

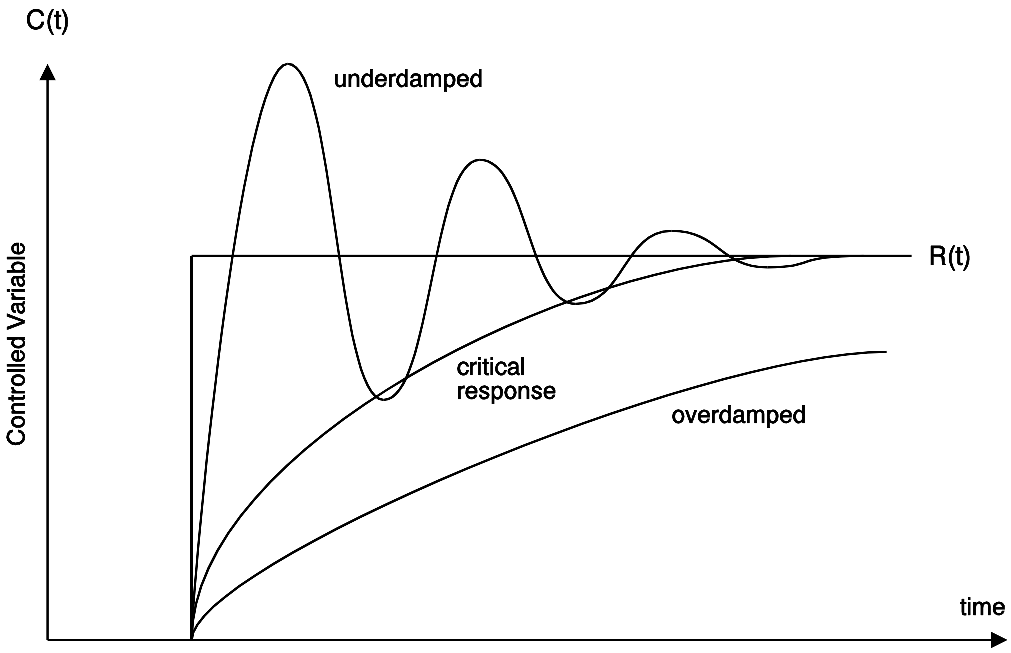


Figure 7.4 Typical System Response

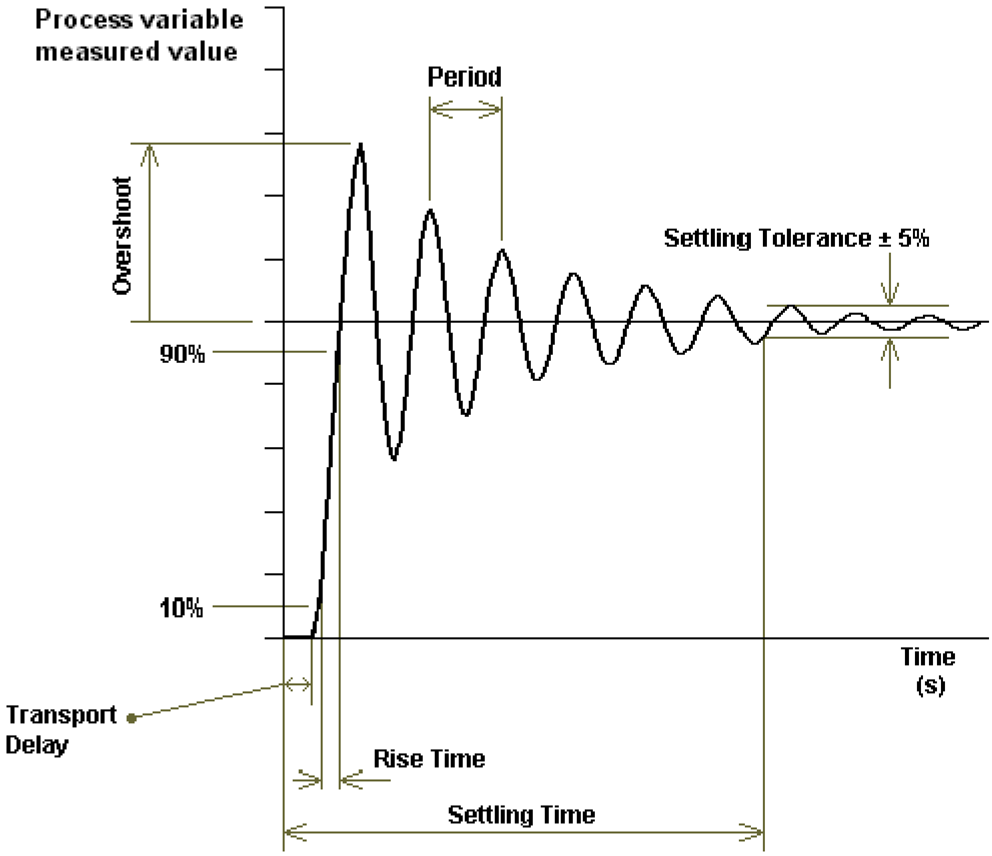
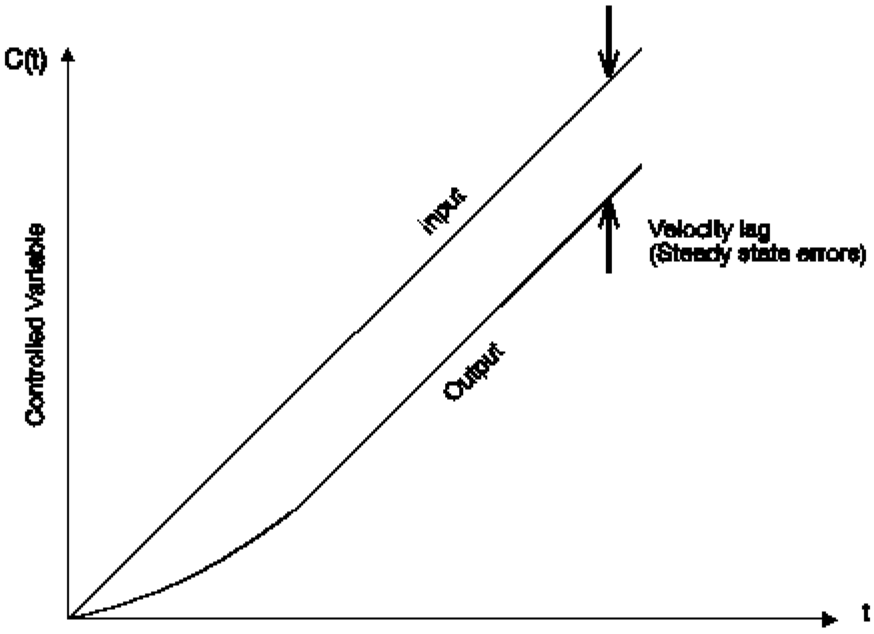


Figure 7.4.1 System Performance Parameters

Ramp Input



Step Input

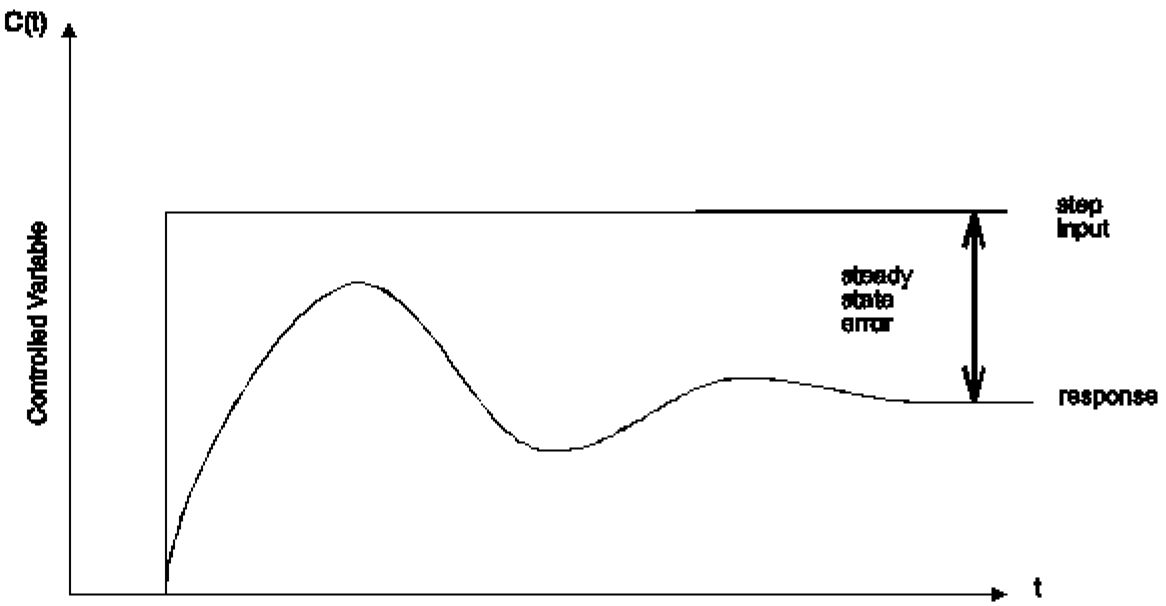
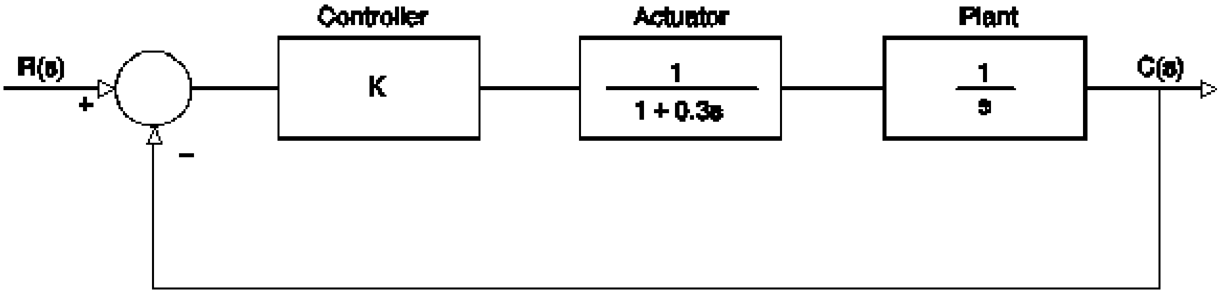
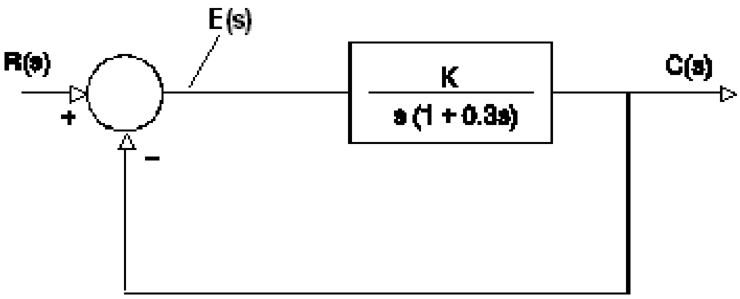


Figure 7.4.2 Steady State Errors



Which reduces to:



Which reduces to:

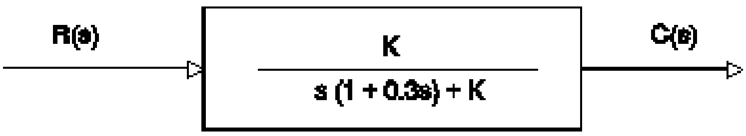


Figure 7.5 Final Value Theorem Example

NOTE - *Reduction Rule* $\frac{G}{1+GH}$ *where* $G = \frac{K}{s(1+0.3s)}$

7.6.2 Control System Instability

If the output of a system increases even though the input is not demanding such an increase the system is said to be unstable and is of no practical use. Closed loop control relies upon negative feedback to eliminate error i.e. measured value is subtracted from set point to determine error and this is used to calculate control action. If parameter values are such that positive feedback occurs then errors are amplified and the system becomes unstable. A control system should be designed to be rigidly stable within its intended operating range. Mathematical techniques involving Routh arrays, Bode plots or Nyquist plots may be used to assess system stability. These methods are discussed in sections 7.6.4 to 7.6.6 and are suitable for assessing stability of the process 6 control loops.

7.6.3 Final Value Theorem

The final value theorem allows the steady state value of a time domain function to be evaluated from its Laplace transform.

The final value theorem states that:

If: $L[f(t)] = E(s)$

Then: $$e_{ss} = \lim_{t \to \infty} f(t) = \lim_{s \to 0} sE(s)$$

Where: $f(t)$ is the error as a function of time, $E(s)$ is the error transfer function and e_{ss} is the steady state error.

For an example refer to figure 7.5. Using block diagram reduction techniques, the system transfer function is:

$$\frac{C(s)}{R(s)} = \frac{K}{s(1+0.3s)+K} \qquad (1)$$

We now wish to apply the final value theorem for a ramp input. Firstly it can be seen that:

$$E(s) = R(s) - C(s) \qquad (2)$$

We know the input R(s) and we are interested in the error E(s) so rearranging:

$$C(s) = R(s) - E(s) \qquad (3)$$

From equation (1): $C(s)(s(1 + 0.3s) + K) = R(s)K$

Substituting equation (3) for C(s):

$$(R(s) - E(s))(s(1 + 0.3s) + K) = R(s)K$$

$$R(s)\, s(1 + 0.3s) + R(s)K - E(s)\, s(1 + 0.3s) - E(s)K = R(s)K$$

Re-arranging:

$$E(s)\, s(1 + 0.3s) + E(s)K = R(s)\, s(1 + 0.3\, s)$$

$$E(s)(s(1 + 0.3s) + K) = R(s)\, s(1 + 0.3s)$$

Therefore:

$$E(s) = \frac{R(s)s(1+0.3s)}{s(1+0.3s)+K}$$

Applying the final value theorem: $e_{ss} = \lim_{t \to \infty} f(t) = \lim_{s \to 0} sE(s)$

The steady state error is:

$$e_{ss} = \frac{sR(s)\, s(1+0.3s)}{s(1+0.3s)+K}$$

However, R(s) is a ramp input which is $1/s^2$ (from rule 5 in Appendix 8) therefore:

$$e_{ss} = \frac{s^2(1+0.3s)}{s^2(s(1+0.3s)+K)}$$

As: $s \to 0$: $e_{ss} = 1/K$

Therefore the greater the value of K the lower will be the steady state error in this control loop.

7.6.4 The Routh-Hurwitz Test

A Routh array may be used to determine the stability of a linear system directly from its transfer function (TF), without any knowledge of the input signal. This is possible because stability is governed by the denominator of the closed loop transfer function. Equating the denominator to zero gives the 'characteristic equation' of the closed loop system. The nature of the roots of this equation indicates stability or instability.

For example, transfer function: $\dfrac{C(s)}{R(s)} = \dfrac{1}{s(1+0.3s)+K}$

Characteristic equation: $s(1+0.3s) + K = 0$

Firstly write down the characteristic equation in descending order of the powers of "s":

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} \text{..........} = 0$$

If any coefficient is missing or negative then the system is definitely unstable. If all of the coefficients are positive the next step is to construct the Routh array as shown below:

COLUMNS

		1	2	3	4
	1	a_0	a_2	a_4	a_6
	2	a_1	a_3	a_5	a_7
	3	b_1	b_2	b_3	
ROWS	4	c_1	c_2	c_3	
	5	d_1	d_2	d_3	
	6	e_1			
	7	f_1			

The first two rows are obtained from the descending order power characteristic equation. The remaining rows are calculated from the elements in the first two rows as follows:

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} \qquad c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1} \qquad d_1 = \frac{c_1 b_2 - b_1 c_2}{c_1}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} \qquad c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1} \qquad d_2 = \frac{c_1 b_3 - b_1 c_3}{c_1}$$

$$b_3 = \frac{a_1 a_6 - a_0 a_7}{a_1} \qquad c_3 = \frac{b_1 a_7 - a_1 b_4}{b_1} \qquad \text{etc.}$$

The construction of the array terminates only when zeros are obtained. The Routh-Hurwitz criterion is that the first column must have no sign changes in order for the system to be stable.

Example 1 $\dfrac{C(s)}{R(s)} = \dfrac{1}{10s^2 + 2s + 3}$

Characteristic Equation: $0 = 10s^2 + 2s + 3$

Routh Array:

$$\begin{array}{lll} 10 & 3 & 0 \\ 2 & 0 & 0 \\ \dfrac{6-0}{2} = 3 & 0 & \\ 0 & & \end{array}$$

There is no sign change in first column therefore the system is STABLE.

Example 2 $\dfrac{C(s)}{R(s)} = \dfrac{3K}{10s^4 + s(s+7) + K}$

Characteristic Equation: $0 = 10s^4 + s^2 + 7s + K$

There is a missing s^3 term therefore the system is UNSTABLE.

Example 3 $\dfrac{C(s)}{R(s)} = \dfrac{1}{5s^3 + 4s^2 + 10s + 20}$

Characteristic Equation: $0 = 5s^3 + 4s^2 + 10s + 20$

Routh Array:

$$\begin{array}{lll} 5 & 10 & 0 \\ 4 & 20 & 0 \\ -15 & 0 & \\ 20 & & \end{array}$$

There is a sign change in the first column therefore the system is UNSTABLE.

Example 4

The Routh array method may be used to find the limiting gain value for a controller (K), whilst avoiding instability. e.g.: Characteristic equation $s^4 + 6s^3 + 11s^2 + 6s + K = 0$:

Routh Array:

$$\begin{array}{lll} 1 & 11 & K \\ 6 & 6 & 0 \\ 10 & K & 0 \\ \dfrac{60-6K}{10} & 0 & \\ K & & \end{array}$$

For the system to be stable then 60 must be greater than 6K therefore K must be less than 10. Note that the Routh array method does not indicate the degree of stability, this may be examined using either Bode or Nyquist plots which are discussed in sections 7.6.5 and 7.6.6.

7.6.5 Bode Plots

The phrase 'frequency response' refers to the overall relationship between a system's input and output signals over a range of input signal frequencies. In general we are interested in the way the system's gain and phase lag (between input and output) vary with frequency. (Gain is the ratio of output signal amplitude to input signal amplitude). The Bode plot method of assessing stability analyses the open loop frequency response of a system in order to predict the degree of stability for the closed loop. A complete Bode plot comprises graphs of gain magnitude |G| and phase lag angle θ plotted against input signal frequency ω. Determination of the degree of stability is by a straightforward inspection of the graphs.

Bode plots may be drawn using experimentally determined open loop data which are often quite easy to obtain. This offers the great advantage that no knowledge of the differential equations underlying the system or its transfer function, are required. Alternatively, if the TF is known then an approximate Bode plot may be drawn without carrying out any experiments. (This would be desirable for certain industrial systems which might suffer damage if driven beyond their design limits with experimental test signals). However the open loop Bode plot is produced, if it is reasonably accurate it will give a good guide to the degree of closed loop stability. Before considering how to interpret a Bode plot it is worth digressing a little to discuss how to sketch one, given a known TF.

As an example consider the open loop transfer function = $\dfrac{10(1+0.1s)}{s(1+0.5s)(1+0.04s)}$

This may be considered as the <u>product</u> of several standard TF elements for which Bode plots are shown in Appendix 4. If the Bode plots for each element are plotted individually on <u>logarithmic</u> scales, the overall Bode plot for the whole TF may be obtained by simple <u>addition</u> and straight line approximation. (Since multiplication is equivalent to adding logarithms).

<u>Bode Gain versus Frequency Plot:</u>
With reference to the standard functions in Appendix 4, the transfer function may be broken down into the following elements:

type K	$20\log_{10}K = 20\log_{10}10 = 20\text{decibels}$
type 1 +Ts	$(1+0.1s)$ has a break point at $\frac{1}{T} = \frac{1}{0.1} = 10$ rad/s
type 1/s	- 20 decibels/decade
type $\frac{1}{1+Ts}$	$\frac{1}{1+0.5s}$ has a break point at $\frac{1}{T} = \frac{1}{0.5} = 2$ rad/s
type $\frac{1}{1+Ts}$	$\frac{1}{1+0.04s}$ has a break point at $\frac{1}{T} = \frac{1}{0.04} = 25$ rad/s

Gain |G| in decibels may be plotted against frequency (ω) in radians per second. Refer to figure 7.6.

NOTE: It is best to plot gain against frequency and phase angle against frequency on the same log paper since cross referencing is necessary to assess system stability.

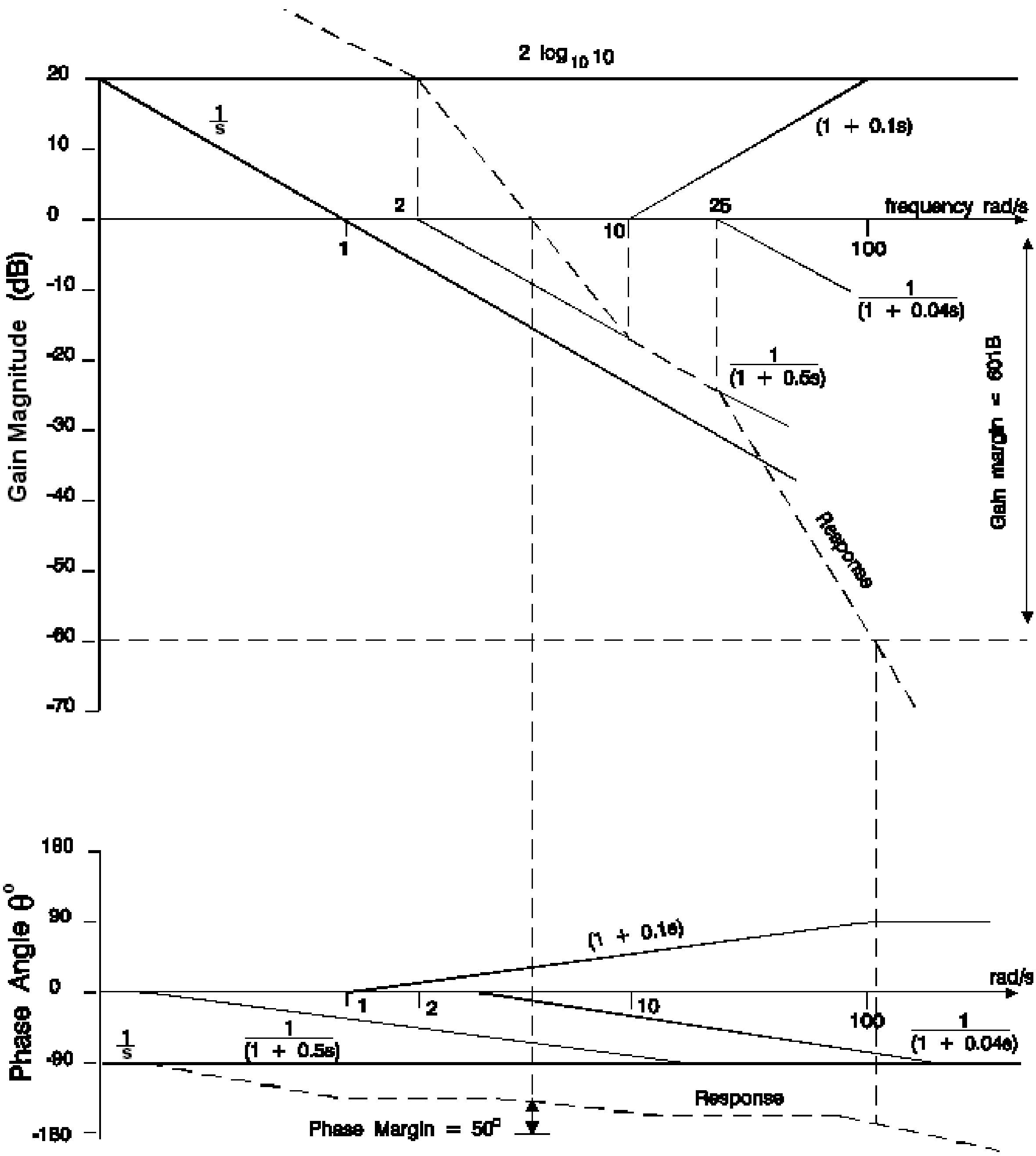
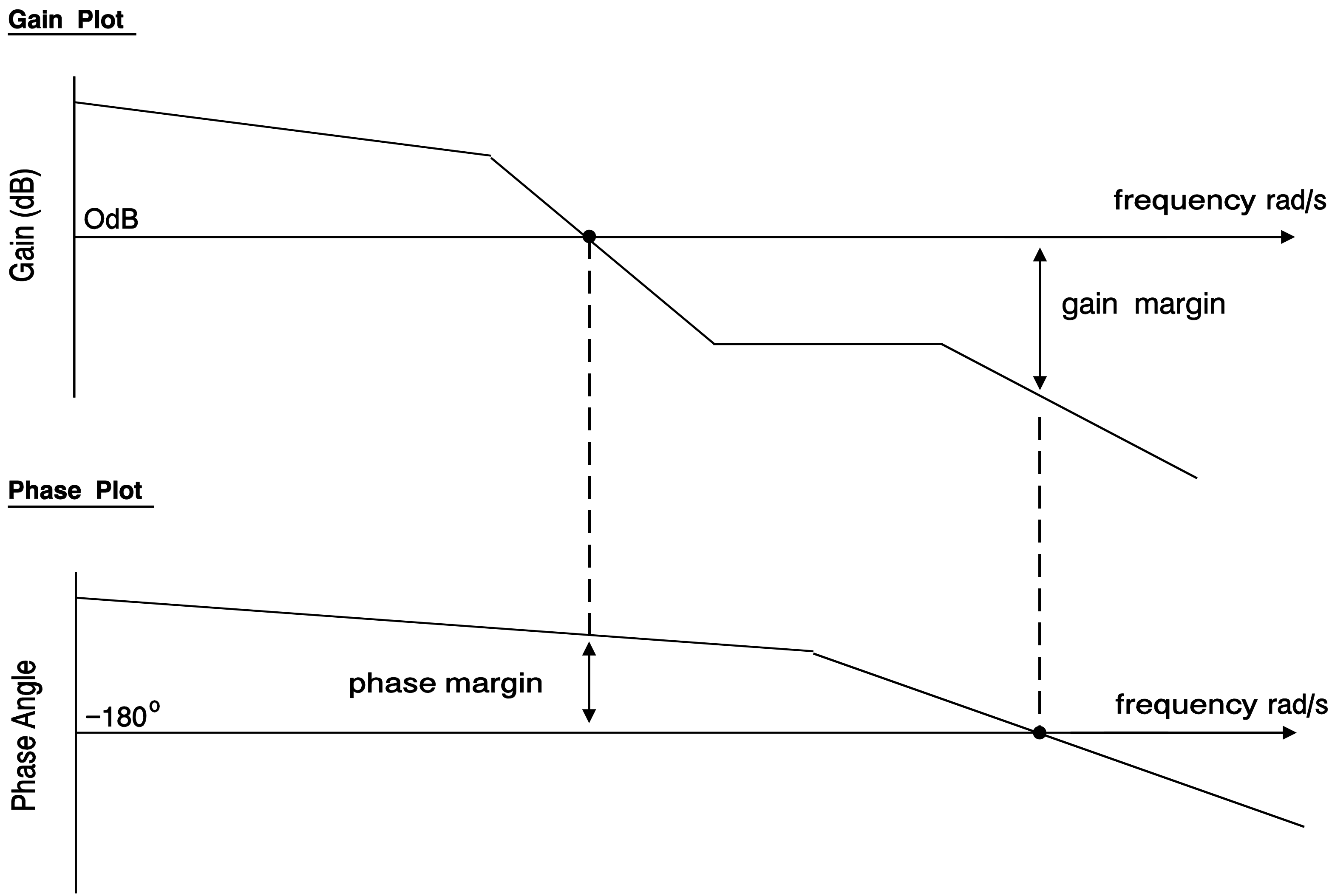


Figure 7.6 Bode Plot Example

Figure 7.6.1 Determining the Gain and Phase Margins from a Bode Plot



7.6.5.1 Bode Phase Lag versus Frequency Plot:

With reference to the standard functions, the transfer function may be broken down into the following elements:

type K	no effect
type 1+ Ts	(1 + 0.1s) has a break point at 10 rad/s
type 1/s	lag of -90^{o}
type $\frac{1}{1+Ts}$	has break points at 2 rad/s and 25 rad/s

Phase lag angle (θ) may be plotted against frequency (ω) in radians per second. Refer to figure 7.6.

For both graphs in figure 7.6 the line for the overall system response may be obtained by adding the values of the individual elements at each point along the frequency axis.

For closed loop feedback systems instability occurs if an open loop gain of 1 coincides with an open loop phase lag of -180º. (This leads to positive feedback because closing the loop introduces a further 180º phase lag when the feedback is subtracted from the input). The relative stability of a closed loop system is therefore defined using two criteria, see figure 7.6.1:

Gain Margin. This is the gain of the system in decibels (dB) when the phase lag is -180º. It expresses the factor by which the gain could be increased before the system becomes unstable (or more accurately, marginally unstable).

Space Margin. This is 180º less the phase lag angle at the frequency when the gain is 0 dB. It expresses the additional phase lag that would make the system unstable (or more accurately, marginally unstable).

Typical values for a stable system are a gain margin of more than 10 dB and a phase margin of more than 45º.

For our original Bode plot example (figure 7.6) the gain margin is 60 dB and the phase margin is 50º so this system would be stable in closed loop mode. The Bode plot method may be used to analyse the stability of the PCT100 flow loop - Labworks 9, 10 and 13.

7.6.6 Nyquist Plots

The Nyquist stability criterion is another method of assessing the closed loop stability of a system by reference to its open loop frequency response. A Nyquist plot represents the open loop frequency response as a polar plot of the gain magnitude |G| and the phase lag <G over a frequency range from zero to infinity. The information is plotted using vectors from the origin and an open loop system is represented by a series of such vectors, each one at a different frequency. By joining the extremities of the vectors a frequency locus may be obtained, see figure 7.7.

As with the Bode plot method, data for drawing the curve may be obtained experimentally or from analysis of a known transfer function. These days a simulation package would normally be used to display the curve for a particular TF but it could be done manually if necessary. In the latter case the first step is to evaluate the gain magnitude |G| and phase lag <G. This is done by subdividing the system's open loop transfer function into the standard elements shown in Appendix 12.

As an example consider the open loop transfer function = $\dfrac{5}{s(1+0.5s)(1+0.166s)}$.

The standard elements within this TF are:

$$K \qquad \frac{1}{s} \qquad \frac{1}{1+Ts} \qquad \frac{1}{1+Ts}$$

To evaluate |G| <u>multiply</u> the standard gain elements from Appendix 12:

$$|G| = 5 \times \frac{1}{\omega} \times \frac{1}{\sqrt{(1+(0.5\omega)^2)(1+(0.166\omega)^2)}} \tag{1}$$

To evaluate <G <u>add</u> the standard phase lag elements from Appendix 12:

$$<G = 0 + (-90°) - \tan^{-1}(0.5\omega) - \tan^{-1}(0.166\omega)$$

$$<G = -90° - \tan^{-1}(0.5\omega) - \tan^{-1}(0.166\omega) \tag{2}$$

Now substitute values for ω into (1) and (2) and calculate |G| and <G to obtain:

ω (rad/s)	|G| (dB)	<G (º)
1.5	2.58	-140.8
2	1.68	-153.4
3	0.74	-173
4	0.46	-187
8	0.09	-219
16	0.013	-242

Plot the Nyquist diagram
The assessment of closed loop system stability reduces to an observation of whether or not the open loop Nyquist plot encloses the point on the real axis where the gain is -1. If it does then the system is unstable. The relative stability of the closed loop system is again defined using the two criteria:

Gain Margin. This is the number of decibels (dB) by which the magnitude of the open loop gain falls short of unity when the phase angle is -180º.

Phase Margin. This is the angle by which the open loop phase lag falls short of -180º when the gain magnitude is unity.

As was stated earlier, typical values for a stable system are a gain margin of more than 10 dB and a phase margin of more than 45º. the gain margin in dB for our example is -20 $\log_{10}$ 0.6201 = 4.151dB and the phase margin just over 12º. Since these values are below those recommended for a stable closed loop system the system would be regarded as unstable, or at best, marginally stable.

The Nyquist stability criterion may be used to analyse the stability of the PCT-100 flow loop - Labwork 13.

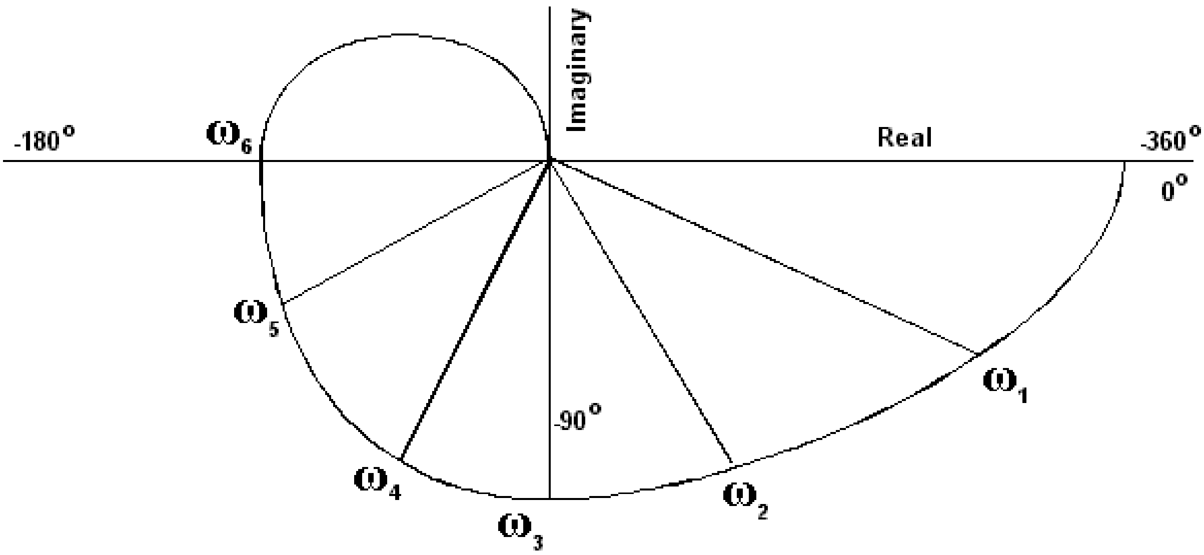


Figure 7.7 Principle of Nyquist Plot

7.6.7 Process Modelling

In the previous sections we have seen that given the open loop transfer function it's possible to determine whether the closed loop system will be unstable or not using the Routh test. It is also possible to determine the degree of stability with a Bode or Nyquist plot. However in many situations we do not have any idea of the transfer function for a particular open loop system. This section touches on different types of tests which may be used to collect data, from which an approximate transfer function may be derived. Labworks 11 and 12 apply two of these methods to the PCT-100 and show how to estimate an open loop TF for its flow loop.

There are three types of experimental approaches:

- Step Data Tests see Section 7.6.7.1
- Frequency Response Tests see Section 7.6.7.2
- Time Domain Test (Online Method) see Section 7.6.7.3

7.6.7.1 Process Models from Step Data Tests

This technique subjects a process operating under steady-state open loop conditions to a step change input and records the resulting transient response, which is called a process reaction curve. Data taken from this curve are used to estimate the mathematical model (i.e. transfer function). In the open loop context, the word 'input' refers to the actuator setting which directly effects the process variable. (Clearly the phrase 'set point' has no meaning in an open loop situation). For the example of the PCT-100 flow loop, a sudden increase in the voltage to the pump drive motor would constitute the step change input. In real world step data tests the following points should be observed:

- Step size must be carefully chosen so that the process reaction curve is distinguishable from plant noise without the system being driven beyond its linear range.
- The process should be free of load changes for the duration of the tests.
- Tests should be repeated several times and the average values of parameters should be used to estimate the transfer function.

From the process reaction curve several methods have been suggested for determining the parameters used in the transfer function. Three important approaches are:

- Ziegler, Nichols and Miller describe a simple method for finding the parameters of a first order plus dead time model.
- Caldwell, Oldenbourg and Sartorius et al suggest a procedure to determine the parameters of a second order model.
- Sundaresan et al developed a method for evaluating second order models with dead time.

An inherent feature of the PCT-100 is the 'transport delay' most evident between pump activity and flow-meter feedback on start up. This is due to the time delay between the pump starting to rotate and the water pressure beginning to turn the impeller inside the flow-meter. The delay depends partly upon the length of pipe-work between the pump and flow-meter. This type of lag, which is typical of industrial process systems, increases the difficulty of maintaining good quality control. Since the PCT-100 flow loop is at least second order, we would suggest the use of the second and third methods listed above, to estimate open loop TFs following step data tests. (See labworks 11 and 12). A typical process reaction curve generated from a flow loop step data test is shown in Figure 7.8. In order to model the response mathematically we suggest:

- Caldwell's method for estimating an open loop TF for 'continuous flow', (i.e. excluding steps from zero flow rate) - Labwork 11.
- Sundaresan's method, which makes allowance for dead time, for estimating an open loop TF for the transient behaviour when the flow rate is stepped up from zero - Labwork 12.

Once the two TFs have been estimated, flow loop stability may be analysed for both using Bode or Nyquist plots. It will be seen that the dead time causes a significant loss in stability on start up from zero flow-rate. The effect of dead time on a large industrial control system would depend upon the process. Since most industrial systems run continuously, the initial instability might be quite acceptable. However, if the instability risked a serious detrimental effect, for example a level control system on a tank containing dangerous chemicals, the problem would need to be corrected. This might entail:

- Redesign of the system to remove the system lag, this might not be possible of course.
- Design of a controller to include a method of soft-starting to prevent initial instability. This could take the form of a band-tuned controller where one set of control parameters is used for starting and another set, tuned for a faster response time, is used for continuous running.
- Implementation of an advanced control algorithm which takes the dead time into account such as the Smith Predictor Algorithm.

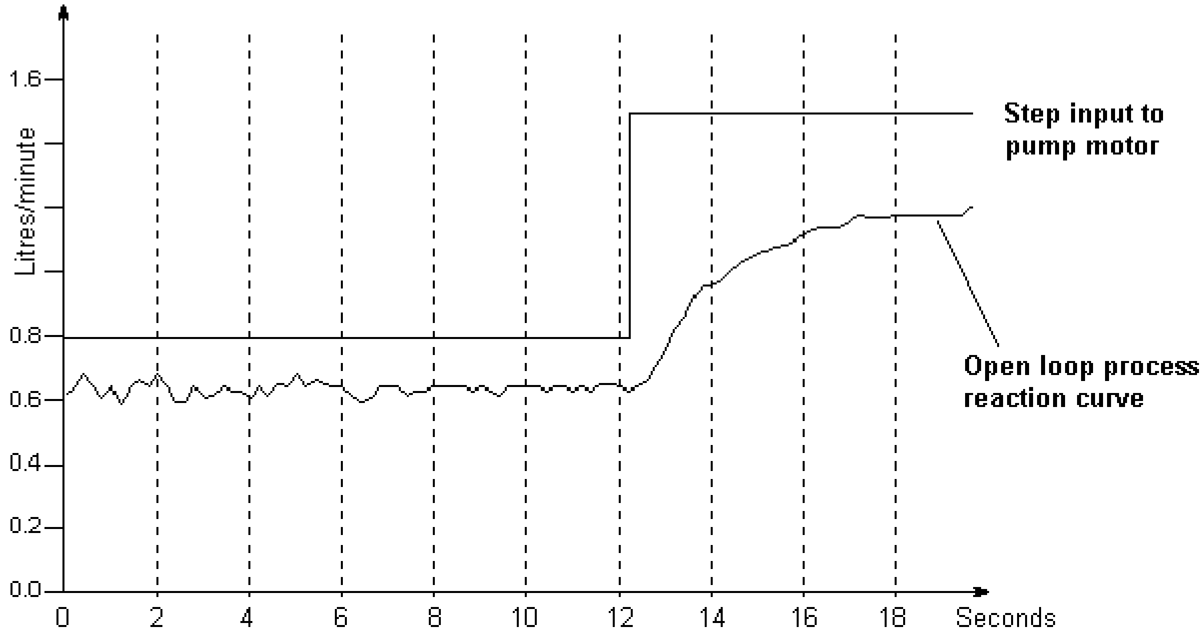


Figure 7.8 Typical Flow Rate Response to a Step Input

7.6.7.2 Process Models from Frequency Response Tests

Another way to estimate the TF of an open loop system is to examine its frequency response across a large range of frequencies. There are several ways to do this but the most straightforward is to drive the system with a series of sinusoidal input signals. (These are often referred to as forcing functions). These inputs should have the same amplitude but different frequencies and in each case the gain and phase lag (between input and output signals) should be measured. Once the frequency response tests have been completed a Bode plot should be drawn from the data. The standard functions shown in Appendix 11 should then be 'fitted' to the results allowing straight line approximation to be used to estimate the system's gain and time constant(s). Once these are known the open loop TF may be written down.

The use of frequency response tests to determine the transfer function for a process depends upon various factors:

- Is the production down time for the period required to conduct the tests, affordable?
- What type of driving signal is permissible? (Sinusoidal signals are ideal but they may be completely unacceptable for some industrial processes).
- What magnitude of driving signal is acceptable? (Components should not be stressed beyond the level for which they were designed).
- The degree of signal noise in the system.

Frequency response tests may be carried out on the PCT-100 flow loop by selecting Open Loop control in the control software, clicking on the sine wave input option and entering values for the Max, Min and Period parameters. Clicking Start will then inject this sine wave into the pump control circuit (on the control module) and display it (white trace) and the resulting flow measured value (cyan trace) on the screen. Input frequency (rad/s), gain (dB) and phase lag (º) may be tabulated in order to draw the Bode plot. (It is important to remember to scale the values for gain appropriately based on the fact that when the input frequency is zero the open loop gain is 0dB). Any distortion in the output trace indicates the presence of system non linearity.

The validity of any transfer function estimated using this technique may be checked by simulating the open loop response using a simulation package and comparing this to the system's actual response.

7.6.7.3 Process Models from Time Domain Tests

In many industrial processes the parameters vary with time, for example the build up of sediment in pipework or reaction vessels might increase the lag between input and output signals. In such cases it might be advisable to identify the transfer function from time to time and update the controller settings if necessary. This leads to the more complex approach of auto-tuning and on-line system identification both of which are beyond the scope of this manual. However the PCT-100 is an ideal system for introducing these sorts of control methods on courses.

7.7 PID Controllers

Once you have a reasonable idea of a system's transfer function it is often possible to design a control algorithm which uses measured value feedback to maintain the output at the set point. Normally an industrial controller would be designed to accept set point changes and drive the output to the new value smoothly and quickly without oscillation or dramatic overshoot. A good controller will also eradicate small to medium sized perturbations due to changes in other parameters, e.g. upstream pressure, fluid density etc. Even without a detailed mathematical model of the system it is often still possible to design such a controller, see Ziegler Nichols tuning in section 7.7.5. In many cases the controller of choice is the so called 'three term algorithm'. This takes its name from the fact that the control output is determined from the error signal (set point minus measured value) by means of a calculation involving up to three distinct mathematical operations.

A three term controller consists of elements which are proportional to the:

- Magnitude of the error signal – 'proportional term'
- Time integral of the error signal – 'integral term'
- Time derivative of the error signal – 'derivative term'

It has been found that such a controller can give excellent results, taking into account as it does the absolute magnitude of the error, the history of the error and the current rate of change of the error. The three terms are individually explained in sections 7.7.1 to 7.7.4.

7.7.1 Proportional Control Term

The output of a proportional controller (or the portion of the output of a two or three term controller contributed by its proportional term) is proportional to the error between the set point (SP) and the measured valve (MV). Proportional control may be expressed as either:

1. Proportional Gain (PG): Control output (M_P) equals proportional gain multiplied by the error.

Mathematically: $M_P = PG \times (SP - MV) + C = PG\, e(t) + C$

Where: C = Controller output with zero error

$e(t)$ = Error as a function of time

2. Proportional Band (PB): With proportional control saturation occurs at a certain value of error when the control output reaches 100%. Thereafter further increases in error do not produce further increases in control output. This same effect occurs when the output drops to 0%. The error band where the control output is between 0% and 100% is called the proportional band, thus the higher the gain, the smaller the proportional band.

Mathematically: $$M_P = \frac{100}{PB} \times (SP - MV) + C = \frac{100}{PB} e(t) + C$$

Unfortunately a simple proportional controller rarely produces adequate results. The main problem is a phenomenon called proportional offset which is a steady state error between SP and MV. A proportional controller cannot eliminate this offset - there must be some error in order for there to be an automatically determined component of the controller output. The only value of SP for which there is no offset is the value for which C was initially chosen. (C is often called the manual contribution because it is set from outside the control loop [usually by a human operator] and is not under the controller's influence). Proportional offset may be easily demonstrated with the PCT-100 flow loop using the following parameters:

Set Point	SP	= 1.0 litre/min	
Proportional Gain	PG	= 1.0	
Integral Action Time	IAT	= 0	(Or remove tick from box)
Derivative Action Time	DAT	= 0	(Or remove tick from box)

If the set point is increased or decreased it will be seen that the controller output also changes but the offset (though different for different set points) is never eliminated.

7.7.2 Integral Control Term

The integral control term is often used to remove proportional offset errors. It determines a component of controller output (M_I) based upon the history of the error. It is calculated by multiplying the net area under the error curve, e(t), by PG divided by the integral action time (IAT) in seconds.

Mathematically: $$M_I = \frac{PG}{IAT}\int e(t)\,dt$$

IAT is defined as the time taken for the integral action to duplicate the proportional action of the controller, if the error were to remain constant during the period. This may be demonstrated using the PCT-100 flow loop. To ensure that the error remains nearly constant use a low gain to prevent the pump operating initially. The following parameters would be suitable:

SP	= 1.0 litre/min	
PG	= 1.0	
IAT	= 64s	(Ensure box is ticked)
DAT	= 0	(Or remove tick from box)

The error will be a constant 1.0 litre/min. The control output will initially rise due to the proportional term and then build up over time due to the integral action.

If the same experiment is run again initially with IAT set to zero and then if IAT is changed to 2 after about 20 seconds, the elimination of proportional offset due to the integral action will be obvious.

7.7.3 Derivative Control Term

The derivative control term is often used to reduce the response time of the system. It determines a component of controller output (M_D) based upon the current rate of change of the error. It is calculated by multiplying the gradient of the error, e(t), by PG times the derivative action time (DAT) in seconds.

Mathematically: $$M_D = PG \times DAT\,\frac{de(t)}{dt}$$

DAT is defined as the time it would take the proportional action of the controller to duplicate the instantaneous output of the derivative term. The rate of change of error may be approximated by taking the difference between two values of error and dividing by the time between the two values.

Mathematically: $$M_D = PG \times DAT\left(\frac{e(t) - e(t_0)}{t - t_0}\right)$$

Where: $e(t)$ = error at time t
$e(t_0)$ = error at time (t_0)

The derivative control mode is never used alone since there is no controller output corresponding to a zero rate of change. An important point to remember about derivative action is that it can exaggerate any high frequency noise in the system so it should always be used cautiously. This may be demonstrated using the PCT-100 flow loop with the following parameters:

PG = 0.8
IAT = 4 seconds
DAT = 1 seconds

7.7.4 Multi Term Control

In many industrial control situations the three terms, P I and D discussed in sections 7.7.1 to 7.7.3 are combined to form two or three term controllers:

PI: proportional + integral controller:

$$M = PG\left[e(t) + \frac{1}{IAT}\int e(t)dt\right]$$

PD: proportional + derivative controller:

$$M = PG\left[e(t) + DAT\frac{d}{dt}e(t)\right]$$

PID: three term controller:

$$M = PG\left[e(t) + \frac{1}{IAT}\int e(t)dt + DAT\frac{d}{dt}e(t)\right]$$

Or in laplace format:

$$M = PG\left[1 + \frac{1}{IATs} + DATs\right]$$

The use of P, PI, PD and PID controllers may be fully investigated using the PCT-100, see Labworks 1 to 5. Labwork 13 covers the design, and evaluation of performance of, PCT-100 flow loop controllers.

Note that since the control algorithms applied to the PCT-100 are implemented on a computer, they are digital forms of theoretical analogue controllers. A discussion of digital control for analogue systems is provided in section 7.8.7.

7.7.5 Ziegler Nichols Tuning

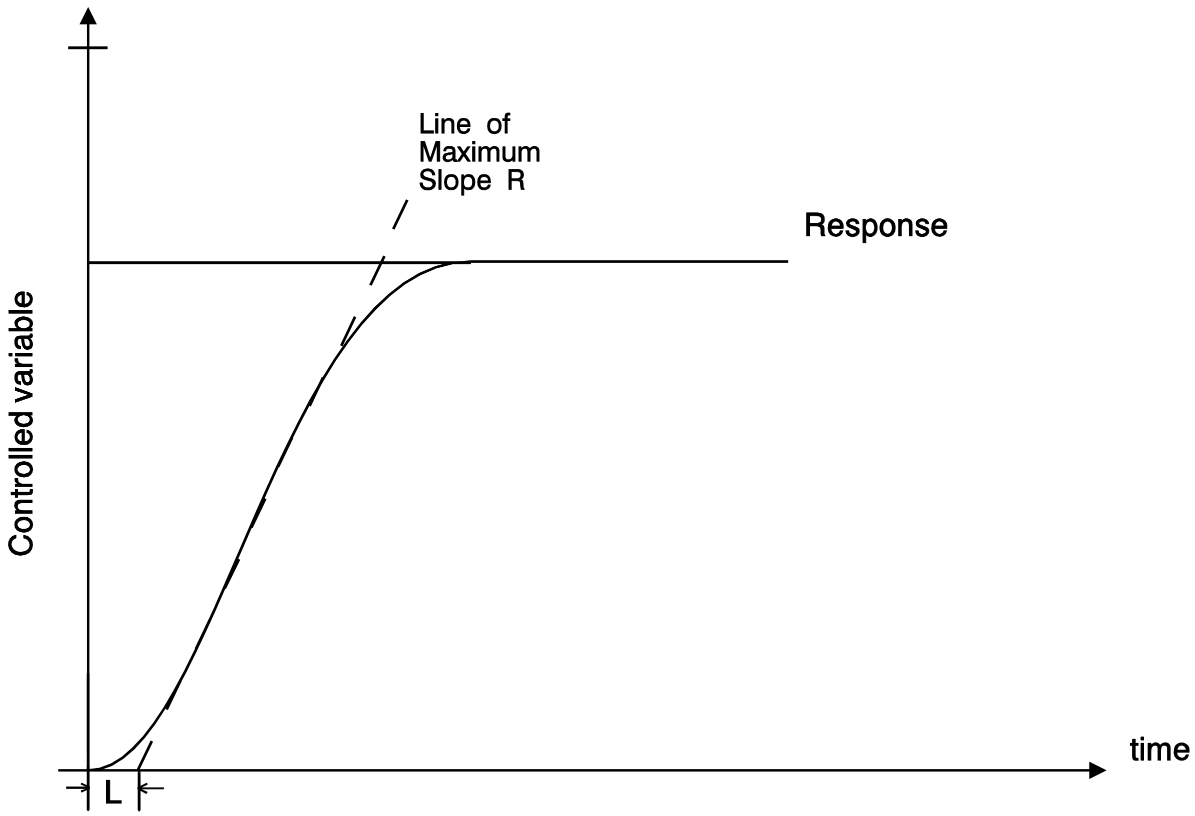
Ziegler Nichols tuning is a semi-empirical method of controller tuning that gives a reasonable guide to the parameters PG, IAT and DAT. Once the values have been determined it might be necessary to apply some fine tuning for optimum performance but in general, Ziegler Nicholls tuning gives acceptable results for many systems. There are two techniques and most industrial processes will be amenable to one or other.

- **Closed Loop Continuous Cycling Method**
- **Open Loop Process Reaction Curve Method**

In the case of the former, integral and derivative control actions are reduced to zero and the controller gain (PG) is gradually increased until the onset of permanent oscillations. At this point the gain (k_p) and the period of oscillation (T) are recorded. The recommended control parameter settings are then:

	PG	IAT	DAT
PI Controller:	0.45 k_p	0.83T	
PID Controller:	0.6 k_p	0.5T	0.125T

This closed loop method does assume that the system may be made unstable using only proportional control. For some processes this might not be true or it might be entirely unacceptable to push the process into instability because of the risk of equipment damage or operator injury. The second method uses parameters obtained from an open loop process reaction curve to calculate the required controller terms. (See also section 7.6.7.1). A typical process reaction curve is shown below:



For a process which reacts in this way the recommended control parameter settings are then:

	PG	IAT	DAT
P Controller:	$\frac{\Delta u}{RL}$		
PI Controller:	$0.9\frac{\Delta u}{RL}$	3.3L	
PID Controller:	$1.2\frac{\Delta u}{RL}$	2L	0.5L

Where: Δu represents the step input used to obtain the reaction curve, as a fraction of the total possible input range.
R is the maximum slope of the process reaction curve
L is as defined on the graph above

Here is an example of the calculation of Δu. On the PCT-100 the range of input to the pump is 0 to 255 (zero volts to the maximum voltage). If the initial (open-loop) input to the pump was 125 and this was stepped up to 175 to produce the process reaction curve then Δu would be approximately 0.2.

$$\Delta u = \frac{(175-125)}{255} \sim 0.2$$

7.8 Digital Control

In the 1960s mainframe computers were first used for supervisory control of industrial processes. The feedback loops still involved the same analogue electronic or pneumatic controllers dating from around the middle of the century. The main functions of the computer were to gather information on how the overall process was functioning, feed the data into a model of the process and periodically update the set points of the analogue controllers. A logical extension was direct digital control (DDC), where analogue controllers were no longer used and the central computer served as a single 'time shared' controller. This is the method used with the PCT-100, conventional PID control is still applied but a digital version of the control laws reside in the interactive software. A block diagram for the PCT-100 digital control loops is shown in figure 7.9. (This is the same for both the flow, level, pressure and temperature loops).

The process variable is measured using a feedback transducer and this signal is sampled every T seconds by the control module. A diagram of a sampled signal is shown in figure 7.9.1. When a continuous signal f(t) is sampled at intervals T, the discrete time signal f(n) is a stream of pulses as shown in figure 7.9.1. In the software the value of this discrete feedback signal is subtracted from a discrete form of the set point. This gives the error from which the next digital controller output value is calculated.

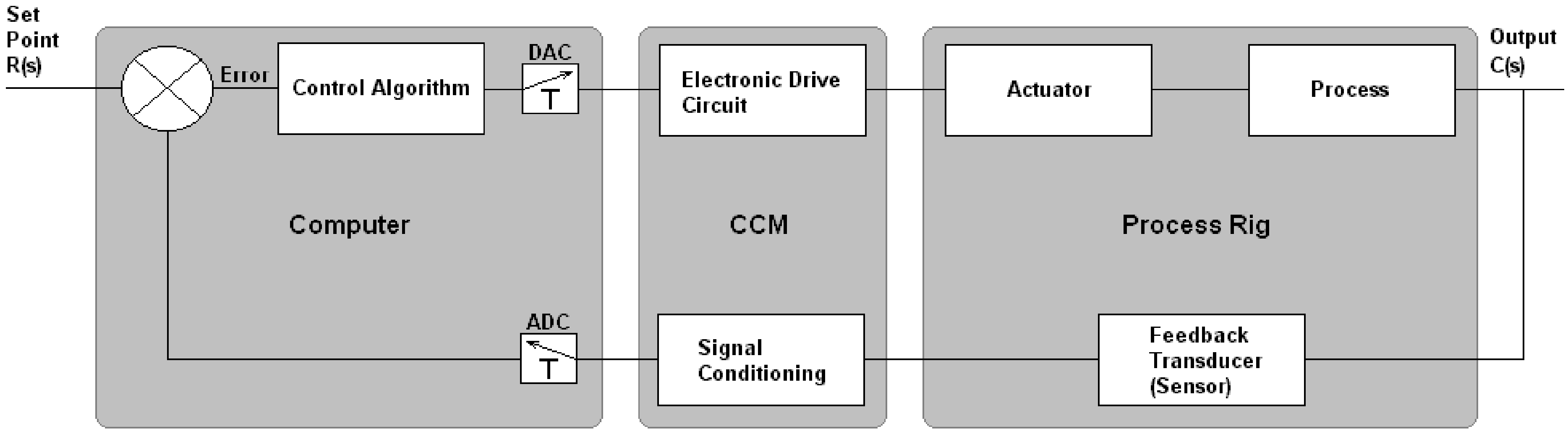
In all digital control loops the controller output is converted to an analogue signal with a DAC, usually followed by a zero order hold device. The hold device cannot be assumed to yield f(t) from an input of f(n) because its output is constant over the sampling interval. In the analysis of a digital control loop these effects must be taken into account by including a mathematical model of the holding device before the process model in the control loop.

The laplace transform for the unit discrete time impulse of the holding device is:

$$\frac{Y(s)}{X(s)} = \frac{1-e^{-Ts}}{s}$$

Where: T is the DAC zero order hold duration.

Figure 7.9 Block Diagram for the PCT-100 Digital Control Loops



7.8.1 The Analysis of Digital Control Systems

Section 7.5.2 introduced the use of laplace transforms for analysis and design of continuous linear control systems. Similarly, z transforms may used to represent and manipulate discrete time systems. The z transformation transforms functions of discrete time, f(n), into functions of the variable z. As for laplace transforms, tables of z transforms have been evaluated. See Appendix 2.

Mathematically: $Z\{f(n)\} = F(z) = \sum_{n=0}^{\infty} f(n)\, z^{-n}$

Where: The capital Z and braces indicate the z transformation operator

The subject of z transform techniques is very advanced and well beyond the scope of this documentation. A brief summary of some of the idea is given in the remainder of this section.

Several steps are required to analyse a digital control loop:

- Starting with the block diagram of the system use block diagram reduction techniques to reduce its complexity, see section 7.8.2.
- Determine the pulse transfer function of the reduced block diagram, see section in 7.8.3.
- The stability of the system may be assessed from the pulse transfer function, see section 7.8.5.
- The response of the system may be assessed in the time domain by determining the inverse z transform, see section 7.8.6.

The algorithm implemented in a digital control loop might be a digital form of a traditional three term controller (as with the PCT-100) or a specialised digital controller such as a Dahlin or Deadbeat type. These more advanced methods are discussed in section 7.8.7.

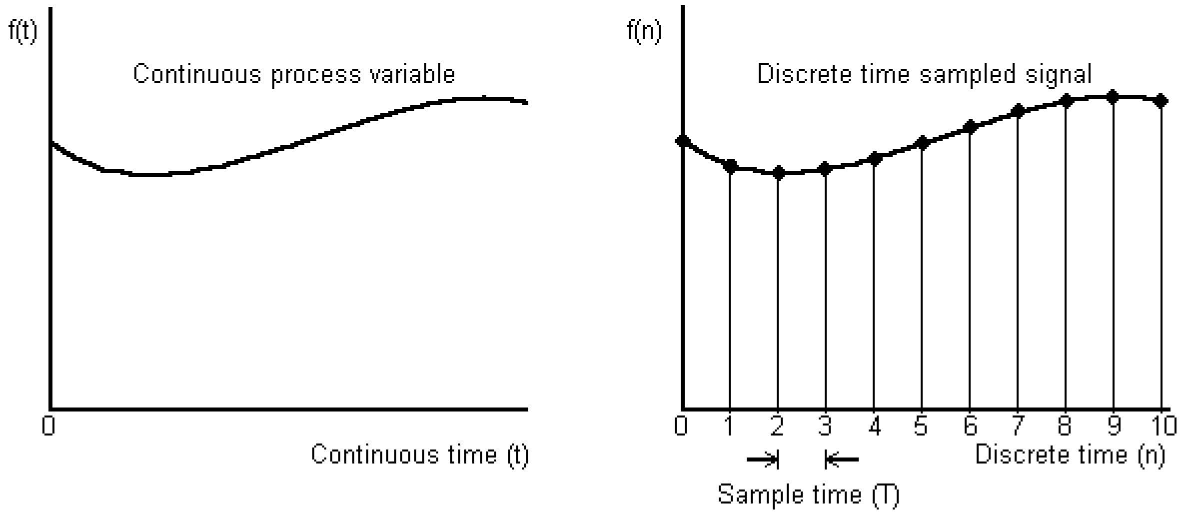


Figure 7.9.1 Diagram of a Sampled Signal

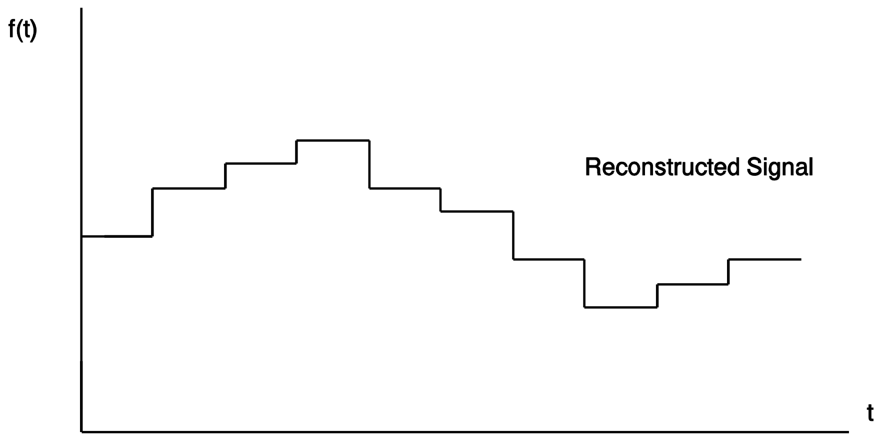
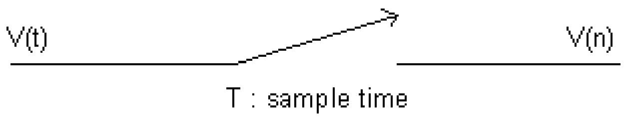


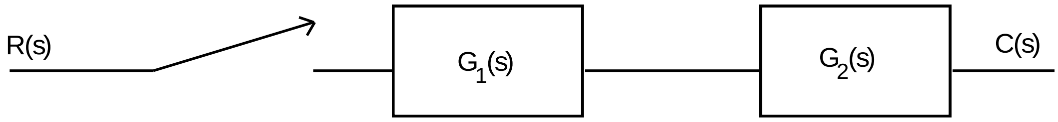
Figure 7.9.2 Signal Reconstruction using Digital-to-Analogue Conversion and a Zero-Order Hold Device

7.8.2 Block Diagrams for Digital Systems

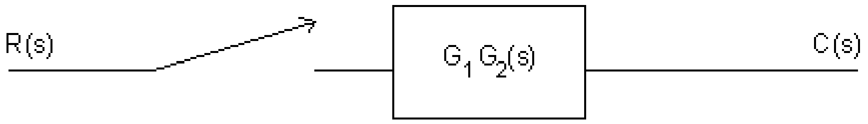
On a block diagram a sampled signal is denoted by a switch symbol with a directional arrow, as shown below.



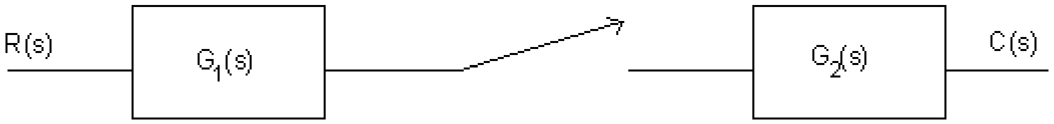
There is a significant difference between the reduction of continuous block diagrams and discrete block diagrams. If there is no sampling between two continuous elements then they may be combined. Thus:



Simplifies to:



However if sampling is present between two continuous elements they cannot be combined.

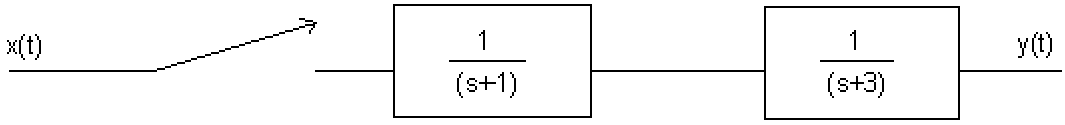


The block diagram above <u>cannot</u> be simplified, each block must be considered as an individual element.

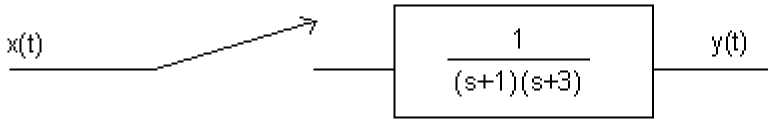
7.8.3 Pulse Transfer Functions

In continuous (analogue) control systems the laplace transform of the output function is related to the laplace transform of the input function by the transfer function of the system. Similarly, when dealing with digital systems the z transform of the pulsed output may be related to the z transform of the pulsed input by the Pulse Transfer Function of the system.

For example to find the pulse transfer function of the open loop system below:



Firstly simplify it to:



$$G(s) = \frac{1}{(s+1)(s+3)} = \frac{1}{(s+a)(s+b)}$$

From Appendix 2 the z transform of G(s) is therefore:

$$G(z) = \frac{1}{(b - a)} \left[\frac{z}{(z - e^{-aT})} - \frac{z}{(z - e^{-bT})} \right]$$

Where T is the sample time, substituting for a and b:

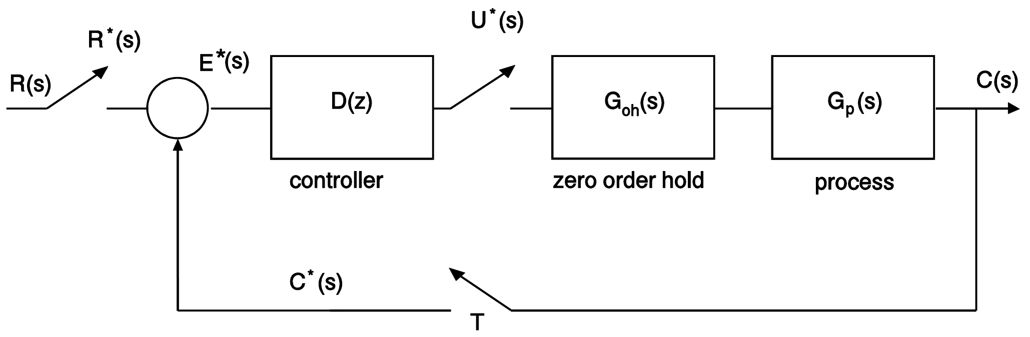
$$G(z) = \frac{1}{(3 - 1)} \left[\frac{z}{(z - e^{-T})} - \frac{z}{(z - e^{-3T})} \right]$$

$$G(z) = \frac{z(z - e^{-3T}) - z(z - e^{-T})}{2(z - e^{-T})(z - e^{-3T})}$$

Therefore the pulse transfer function is:

$$G(z) = \frac{z(e^{-T} - e^{-3T})}{2(z - e^{-T})(z - e^{-3T})}$$

With a closed loop system the analysis becomes slightly more complicated, for example:



Note that asterisks have been used to indicate the discrete time forms of continuous signals.

From the diagram: $C^*(s) = G_p(s).G_{0h}(s).U^*(s)$

Let: $G(s) = G_p(s).G_{0h}(s)$

Therefore: $C^*(s) = G(s).U^*(s)$ (1)

And: $U^*(s) = D(z).E^*(s)$ (2)

And: $E^*(s) = R^*(s) – C^*(s)$ (3)

Substitute (2) into (1): $C^*(s) = G(s).D(z).E^*(s)$ (4)

Substitute (3) into (4): $C^*(s) = G(s).D(z).[R^*(s) – C^*(s)]$

$C^*(s) = G(s).D(z).R^*(s) – G(s).D(z).C^*(s)$

$C^*(s).[1 + G(s).D(z)] = G(s).D(z).R^*(s)$

Therefore:
$$\frac{C^*(s)}{R^*(s)} = \frac{G(s).D(z)}{1 + G(s).D(z)}$$

The pulse transfer function is:

$$\frac{C(z)}{R(z)} = \frac{G(z).D(z)}{1+G(z).D(z)}$$

Where: $G(z) = G_p.G_{0h}(z)$

By introducing some values this may be further reduced. Taking D(z) as a pure gain controller with a gain value of 5; $G_p(s)$ as 1/(s+1); sampling time T as 0.2 seconds and $G_{0h}(s)$ as $(1-e^{-Ts})/s$ and using capital Z and square brackets to indicate the z transformation operator:

$$G(z) = Z\left[G_{0h}(s).Gp(s)\right]$$

$$G(z) = Z\left[\frac{1-e^{-Ts}}{s(s+1)}\right]$$

$$G(z) = Z\left[\frac{1}{s(s+1)}\right] - Z\left[\frac{e^{-Ts}}{s(s+1)}\right]$$

Since: $\quad Z\ e^{-Ts} \rightarrow z^{-1}$

Therefore: $$G(z) = Z\left[\frac{1}{s(s+1)}\right] - Z\left[\frac{1}{s(s+1)}\right]z^{-1}$$

$$G(z) = Z\left[\frac{1}{s(s+1)}\right](1-z^{-1}) \qquad (5)$$

Now to evaluate Z[1/s(s+1)] see Appendix 2:

$$Z\left[\frac{1}{s(s+a)}\right] \rightarrow \frac{(1-e^{-aT})z}{a(z-1)(z-e^{-aT})}$$

Therefore: $$Z\left[\frac{1}{s(s+1)}\right] = \frac{(1-e^{-T})z}{(z-1)(z-e^{-T})}$$

Substitute this into (5): $$G(z) = \frac{(1-e^{-T})z}{(z-1)(z-e^{-T})}\frac{(z-1)}{z}$$

$$G(z) = \frac{1-e^{-T}}{z-e^{-T}} \qquad (6)$$

Substitute (6) into: $$\frac{C(z)}{R(z)} = \frac{G(z).D(z)}{1+G(z).D(z)}$$

$$\frac{C(z)}{R(z)} = \frac{5(1-e^{-T})}{(z-e^{-T})+5(1-e^{-T})}$$

Since: T = 0.2 sec: $$\frac{C(z)}{R(z)} = \frac{0.906}{(z+0.088)}$$

7.8.4 Z Transform Initial and Final Value Theorems

These theorems provide a quick method of evaluating the initial and final values of a system represented in a z transform format, without having to calculate the inverse z transform.

The initial value theorem states that if the z transform of f(t) is F(z) then:

$$\lim_{t \to 0} f(t) = \lim_{z \to \infty} F(z)$$

The final value theorem states that if the z transform of f(t) is F(z) then:

$$\lim_{t \to \infty} f(t) = \lim_{z \to 1} [F(z)\,(1 - z^{-1})]$$

For example: $$f(z) = \frac{z^2}{(z-1)(z-0.5)}$$

Initial value theorem:

$$\lim_{t \to 0} f(t) = \lim_{z \to \infty} F(z) = \infty^2/\infty^2 = \mathbf{1}$$

Final value theorem:

$$\lim_{t \to \infty} f(t) = \lim_{z \to 1} [F(z)\,(1 - z^{-1})] = \lim_{z \to 1} \left[\frac{z^2}{(z-1)(z-0.5)} \frac{(z-1)}{z} \right]$$

$$= \frac{z}{(z-0.5)} = 1/0.5 = \mathbf{2}$$

7.8.5 Stability of Sampled Data Control Systems

A common method used to determine the stability of a digital system is to map the variable z to the W plane. This change of variable is known as the bi-linear transformation, mathematically it is described thus:

$$P(z) = \frac{1 + W}{1 - W}$$

This method is best explained in the form of an example. Consider the effect of the mapping upon the pulse transfer function:

$$\frac{C(z)}{R(z)} = \frac{3z^2 + z}{0.4z^2 - 0.25z - 0.1}$$

$$\frac{C(W)}{R(W)} = \frac{3\left[\frac{1+W}{1-W}\right]^2 + \left[\frac{1+W}{1-W}\right]}{0.4\left[\frac{1+W}{1-W}\right]^2 - 0.25\left[\frac{1+W}{1-W}\right] - 0.1}$$

By multiplying the numerator and denominator by $(1 - W)^2$ we can simplify this to:

$$\frac{C(W)}{R(W)} = \frac{20(2W^2 + 6W + 4)}{11W^2 + 20W + 1}$$

We now apply the Routh test to the characteristic equation to give the array:

	1	2
1	+11	+1
2	+20	0
3	+1	

Since there are no sign changes in column 1 the system is stable. If a sign change occurs in this column the system would be unstable. For stability, all of the poles of C(z)/R(z) (i.e. the roots of its denominator) must fall within the unit circle around the origin of the z plane.

7.8.6 Inverse Z Transformations

Once the response of a digital system to a given input has been evaluated using z transform techniques, we will generally wish to revert back to the time domain to determine the actual output. The inverse z transformation gives the discrete time function f(n), but not the continuous time function f(t). This is because a digital system has discrete output steps.

Mathematically: $f(n) = Z^{-1}\{F(z)\}$

Where: The capital Z^{-1} and braces indicate the inverse z transformation operator. (As opposed to the symbol z^{-1}, (z transform final value theorem), which means 1/z).

Common methods of determining the inverse transform are the use of inverse z transform tables, long division or partial fractions. The simplest method is the use of tables but since these are limited, the other methods are often required. Consider the z transform:

$$F(z) = \frac{z^2}{(z-1)(z-0.15)} = \frac{z^2}{z^2 - 1.15z + 0.15}$$

The use of inverse z transform tables:

A table of inverse z transforms will include the following entry:

$$Z^{-1}\left\{ \frac{z^2}{(z-a)(z-b)} \right\} = \frac{a^{n+1} - b^{n+1}}{a-b}$$

Where: n is the sample index of the discrete time function, a sequence of integers (0 to ∞).

In our example a = 1 and b = 0.15, therefore:

$$f(n) = \frac{1^{n+1} - 0.15^{n+1}}{1-0.15} = 1.1765 - (1.1765 \times 0.15^{n+1})$$

Clearly therefore f(n) has a steady state value of 1.1765. The actual values of f(n) at each discrete time interval may be calculated by substituting values for n in the expression above.

n = 0: $f(0) = 1.1765 - (1.1765 \times 0.15) = 1.0$
n = 1: $f(1) = 1.1765 - (1.1765 \times 0.15^2) = 1.15$
n = 2: $f(2) = 1.1765 - (1.1765 \times 0.15^3) = 1.1725$
n = 3: $f(3) = 1.1765 - (1.1765 \times 0.15^4) = 1.1759$
n = 4: $f(4) = 1.1765 - (1.1765 \times 0.15^5) = 1.1764$

Long Division:

By definition, the z transform of a discrete time sequence f(n) may be written as:

$$F(z) = f(0) + f(1)z^{-1} + f(2)z^{-2} + f(3)z^{-3} + \dots$$

Therefore if F(z) can be manipulated into this form, the values of f(n) may be determined for each value of n by inspection. Long division may be used to manipulate F(z) into the required form.

Considering our original example once again:

$$F(z) = \frac{z^2}{z^2 - 1.15z + 0.15}$$

Multiply the numerator and denominator by z^{-2} to give:

$$F(z) = \frac{1}{1 - 1.15z^{-1} + 0.15z^{-2}}$$

Apply long division to simplify this:

$$
\begin{array}{r l}
 & 1 + 1.15z^{-1} + 1.1725z^{-2} + 1.1759z^{-3} \\
1 - 1.15z^{-1} + 0.15^{-2} \;\big) & 1 \\
 & \underline{1 - 1.15z^{-1} + 0.15z^{-2}} \\
 & \quad 1.15z^{-1} - 0.15z^{-2} \\
 & \quad \underline{1.15z^{-1} - 1.3225z^{-2} + 0.1725z^{-3}} \\
 & \qquad 1.1725z^{-2} - 0.1725z^{-3} \\
 & \qquad \underline{1.1725z^{-2} - 1.3484z^{-3} - 0.1759z^{-4}} \\
 & \qquad\qquad 1.1759z^{-3} + 0.1759z^{-4}
\end{array}
$$

Therefore: $F(z) = 1 + 1.15z^{-1} + 1.1725z^{-2} + 1.1759z^{-3} + \ldots$

Therefore from inspection of F(z), we can say f(0) = 1; f(1) = 1.15; f(2) = 1.1725; f(3) = 1.1759.

<u>Partial Fractions:</u>

Considering our original example once again:

$$F(z) = \frac{z^2}{(z-1)(z-0.15)}$$

Use of partial fractions can yield terms that may be inversely transformed using the tables. To ensure the result is in the form of the tables it is necessary to calculate F(z)/z.

$$\frac{F(z)}{z} = \frac{z}{(z-1)(z-0.15)} = \frac{A}{(z-1)} + \frac{B}{(z-0.15)} \qquad (1)$$

Therefore: $z = A(z - 0.15) + B(z - 1)$
$z = Az - 0.15A + Bz - B$

Equating coefficients: z^0: $0 = -0.15A - B$ (2)
Z^1: $1 = A + B$ (3)

Therefore: A = 1.176 and B = -0.176

Substitute into (1):

$$\frac{F(z)}{z} = \frac{1.176}{(z-1)} - \frac{0.176}{(z-0.15)}$$

Therefore:

$$F(z) = \frac{1.176z}{(z-1)} - \frac{0.176z}{(z-0.15)} = \frac{1.176}{1-z^{-1}} - \frac{0.176}{1-0.15z^{-1}}$$

From inverse z transform tables:

$$f(n) = 1.176 - 0.176(0.15)^n$$

Substituting specific values for n: f(0) = 1; f(1) = 1.15; f(2) = 1.172; f(3) = 1.175; f(4) = 1.176.

7.8.7 Digital Controllers

In the following section we will discuss the digital 3 term controller implemented in the PCT-100 software. It is important to bear in mind however, that there are many alternative forms of digital controller. The PCT-100 may be used on programming courses as a target system for testing different types of algorithm. Programs may written in any suitable programming language capable of accessing the PCT-100. Algorithms which might be investigated include:

- Deadbeat Algorithm
- Dahlin Algorithm

Deadbeat control requires that the closed loop system response has finite settling time, minimum rise time and zero steady state error. The Dahlin approach specifies that the closed loop system behaves as though it were a continuous first order process with dead-time (D), i.e.:

$$\frac{C(s)}{R(s)} = \frac{e^{-Ds}}{(Ts+1)}$$

Both of these algorithms are specifically designed for one type of input signal, if any other type is applied then the system response might be unacceptable.

7.8.7.1 Digital Three Term Controller

The symbol e_n is used in this section to indicate the error at the $n^{th.}$ sample interval, the total controller output in that interval would be denoted by M_n. The three elements of the digital Proportional, Integral and Derivative controller are similar to those discussed in section 7.7, but they are calculated in a discrete interval of time:

<u>Proportional Component</u>
The equations used for the output of the proportional component of the controller are the same as those used for continuous systems:

Proportional Gain: $$M_P = PG \times (SP - MV) + C = PG\, e(t) + C$$

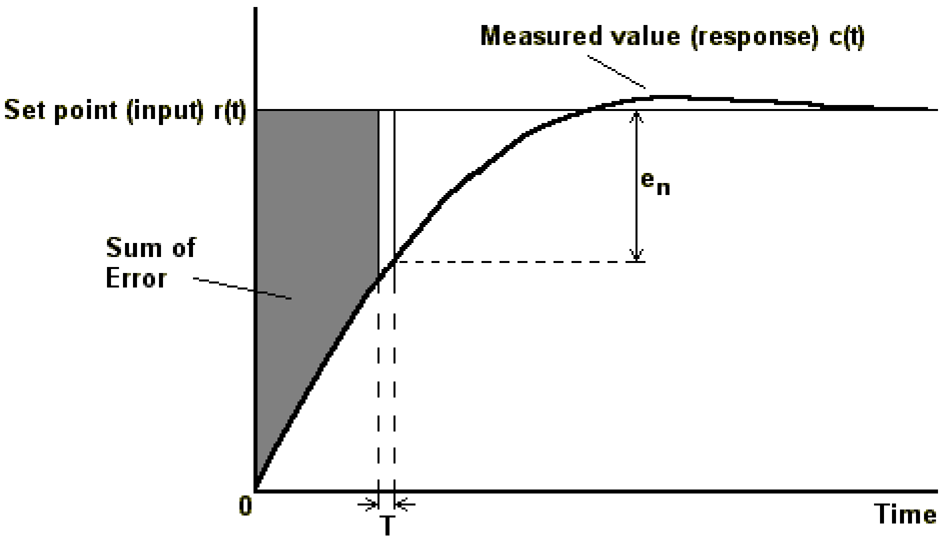
Proportional Band: $$M_P = \frac{100}{PB} \times (SP - MV) + C = \frac{100}{PB} e(t) + C$$

Where: C = Controller output with zero error
e(t) = Error as a function of time

Since digital systems are usually analysed in the z plane, these become:

$$M(z) = PG.E(z) + C(z) \quad \text{or} \quad M(z) = \frac{100}{PB} \cdot E(z) + C(z)$$

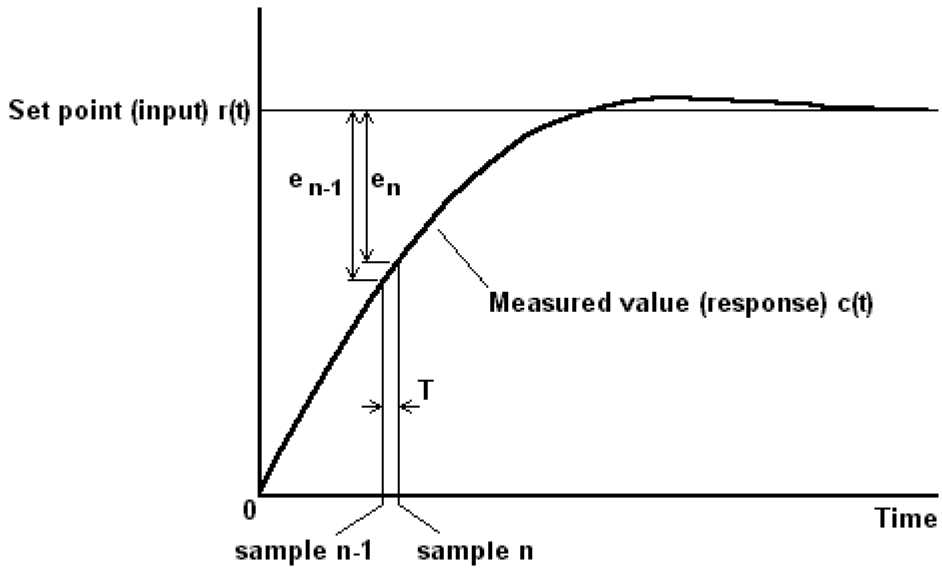
<u>Integral Component</u>
There are several ways of defining this since are several numerical integration techniques available. However, if the sampling time is small the simplest method is shown in the following diagram which shows a step input at time equals zero seconds.



Integral component of controller output is $$M_I = \frac{PG}{IAT}\left[\text{Sum of error} + e_n T\right]$$

Derivative Component

The derivative component is a constant (PG x DAT) multiplied by the rate of change of the error. This rate of change may be approximated by using the values of the error at the $n^{th.}$ and $(n - 1)^{th.}$ sampling instants, as shown below:



The approximate slope at the $n^{th.}$ sample interval is $= \dfrac{e_n - e_{n-1}}{T}$

$$M_D = \frac{PG.DAT(e_n - e_{n-1})}{T}$$

In z transform format: $M(z) = PG.DAT \dfrac{(E(z) - E(z)\, z^{-1})}{T}$

Therefore the transfer function of the derivative controller component is:

$$D(z) = \frac{M(z)}{E(z)} = \frac{PG.DAT}{T}\frac{(z-1)}{z}$$

7.8.7.2 The Effects of Sampling Time

When a digital control algorithm is designed a suitable sampling interval must first be chosen. However, as the sampling interval is increased some potentially degrading effects become significant.

Destabilising effect
Dead-time has a dramatic and destabilising effect on a closed loop system due to the phase shift that it can introduce.

Information loss
If the sampling frequency is too low (i.e. sample interval too long), vital high frequency information may be lost as shown in the figure 7.9.3. It is clear from the top section of the diagram that higher frequency information (shown circled) is missed by the computer because it occurs in between consecutive sampling events. The lower section shows a possible reconstruction of the sampled signal. It is quite accurate where the sampled signal varied slowly by hopelessly inaccurate where it varied at a higher frequency.

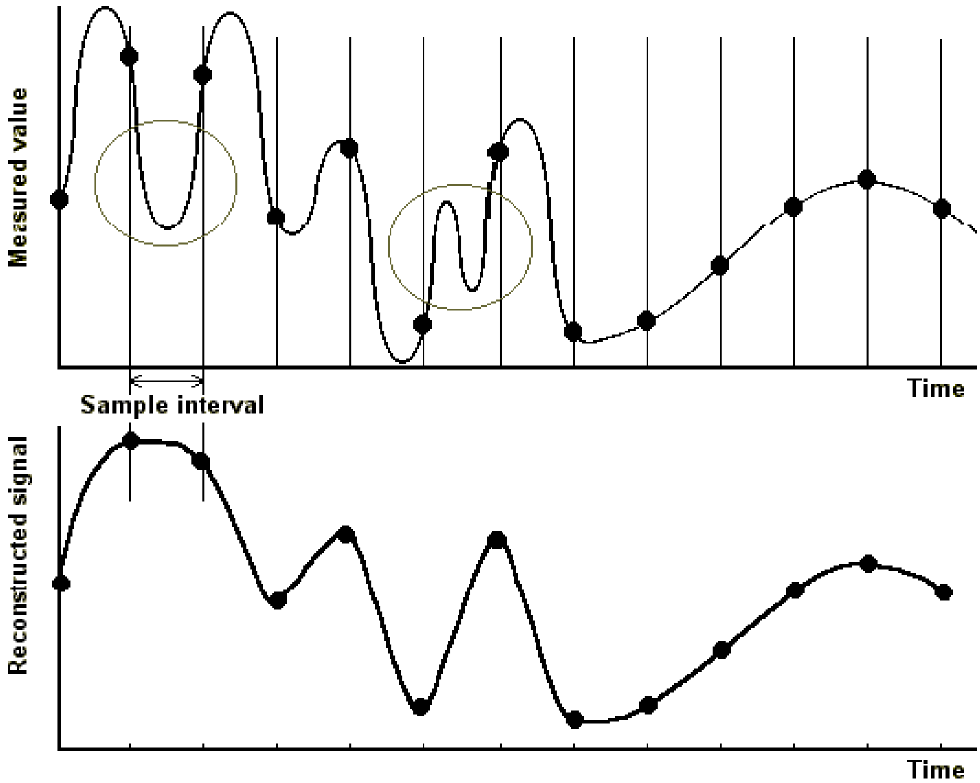


Figure 7.9.3 Information Loss - Sampling Frequency Too Low

Aliasing
Closely related to this type of information loss is the risk of aliasing when the sampling frequency is too low. The sampling frequency must be <u>at least</u> twice the highest frequency present in the sampled signal to prevent aliasing. (This principle is known as Shannon's sampling theorem). When the sampling rate complies with this principle, as in figure 7.9.4A, we can assume that the control algorithm will respond to the signal being sampled. However, if the sampling rate is less than twice the highest frequency of the measured value, the controller will respond to a lower frequency 'alias' of that signal. (In essence the computer will 'see' a lower frequency pseudo-signal rather than a sampled version of the true signal). Even in the case where the sampling rate is exactly twice the frequency of the measured value (figure 7.9.4B) it can be seen that square or triangular waves would give exactly the same samples. A worst case example would be where the measured value of a sinusoidally varying process variable is sampled at exactly the same frequency, as in figure 7.9.4C. The controller would effectively interpret the data as a constant (unchanging) signal because the samples would occur at corresponding points on subsequent wavelengths. The effects of the length of the sampling interval may be investigated using the PCT-100 flow loop

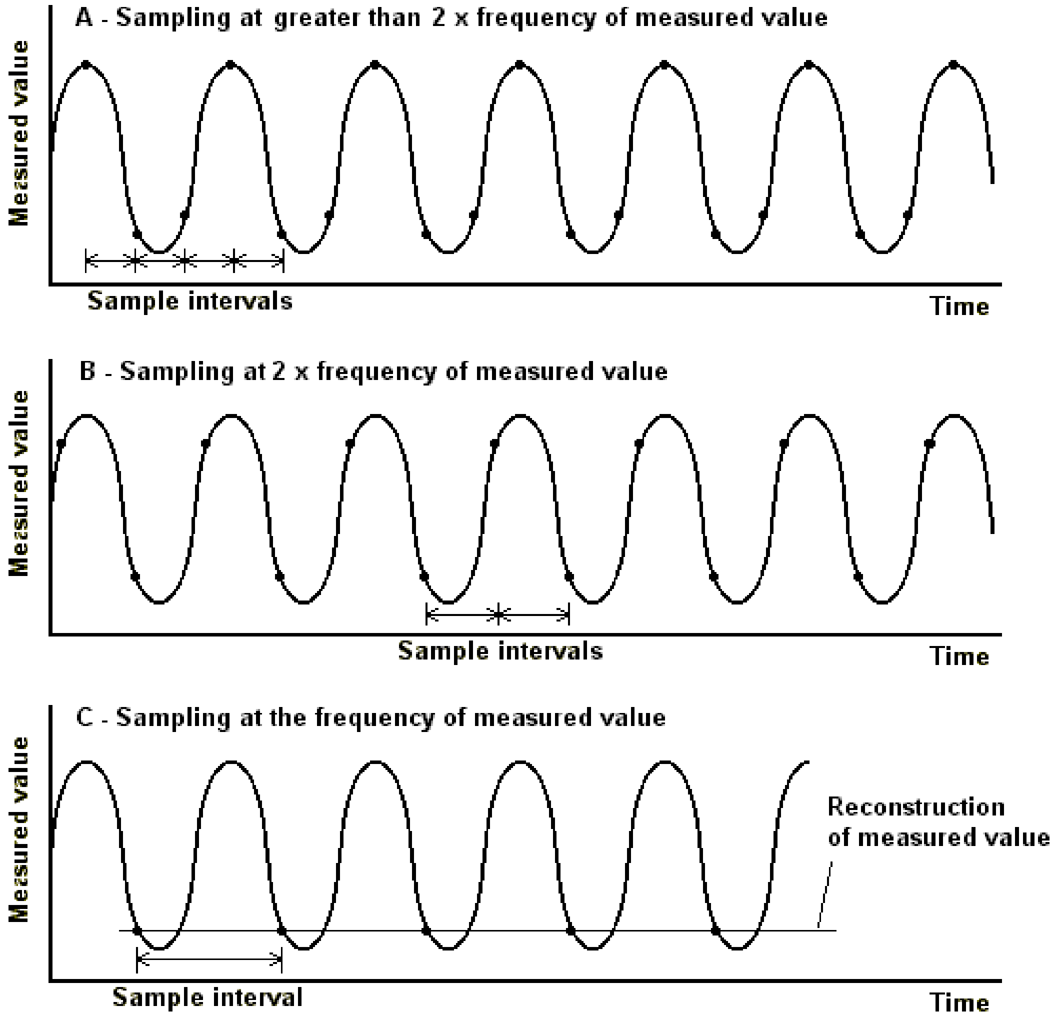


Figure 7.9.4 Aliasing - Sampling Frequency Too Low

7.9 Use of Simulation

Simulation can be an important element in the design process of control systems. It is also invaluable as a teaching aid for demonstrating ideas without confusing the student with a large amount of mathematics early on in a course.

7.10 More Advanced Areas of Work

The material in the preceding parts of section 7 has been designed to provide a basic introduction to both continuous and discrete time system control. However, more advanced work may be undertaken in conjunction with the PCT-100 and a range of ideas are suggested below:

- Phase lead compensation.
- Phase lag compensation.
- Dahlin algorithm
- Deadbeat algorithm
- Cascade controllers
- Use of specialist control algorithms for processes with dead-time
- Auto-tuning algorithms.
- On-line system identification
- Analysis of the flow and temperature loops using multi-variable techniques, such as state-space analysis.

8.0. LABWORKS

Since each PCT-100 will respond slightly differently, the values for set point, proportional gain, integral action time etc. quoted in this section, might need to be modified to produce the best results.

IMPORTANT
BEFORE BEGINNING THESE EXPERIMENTS ENSURE THE VENT VALVE AT THE TOP OF THE PROCESS TANK IS OPEN.

Labwork 1: Proportional Control

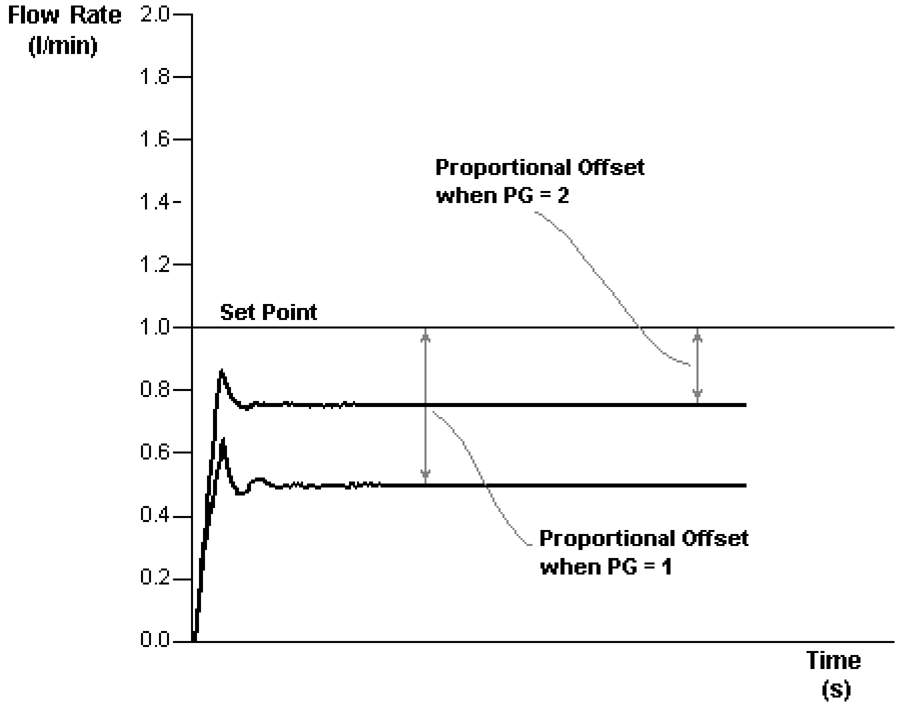
Use the PCT-100 software to implement 'proportional only control' of the flow loop with the following pairs of SP and PG values. Record the eventual 'steady state' flow rate values in litres/minute in the table below, once the initial oscillations have decayed.

SP (l/min)	PG	Steady State	SP (l/min)	PG	Steady State
2.0	0.5		0.6	1.0	
2.0	1.0		1.0	1.0	
2.0	1.5		1.4	1.0	
2.0	2.0		1.8	1.0	
2.0	4.0		2.2	1.0	
2.0	6.0		2.6	1.0	
2.0	8.0		3.0	1.0	
2.0	10.0		3.4	1.0	

What conclusions about the nature of 'proportional only control' may be drawn from your observations? You should be able to see that for a given SP the final 'steady state' value increases as PG is increased. However there is *no* value of PG for which the steady state value is exactly equal to the SP because with proportional control there must always be *some* error in order for there to be a controller output. As PG is increased there is also an increase in the magnitude and duration of the initial oscillations and if PG is too high many systems will oscillate continuously and never settle to a steady state.

For a proportional only controller with a given PG value although the set point is never reached, the resulting steady state value increases as the set point is increased. This shows that it is possible to use simpler proportional only control by setting SP a suitable amount *higher* than the truly desired value, effectively 'deceiving' the controller. This strategy might be acceptable in a situation where the set point is not going to change at all or where external disturbances are minimal but if either of these conditions are not true then a more sophisticated controller is generally required. To prove this run an experiment with PG = 1 and SP = 3 which means that we should actually obtain a steady state value of 1.8 litre/minute and if this is the truly desired value, and there are no disturbances, then all will be fine. Partially close the flow valve to apply a loading to the system. You should see that the flow rate immediately drops and then climbs back up *towards* the value of 1 litre/minute *but does not reach it*. You should also observe that the control output trace (white) is nowhere near the maximum value so there is plenty of capacity for the control output to be increased but the simple proportional only controller is not capable of doing this. If you reopen the valve completely, you will see that the flow rate climbs quickly back up to the value of 1 litre/minute. A few minutes spent experimenting in this manner will convince you that a proportional only controller is very deficient when external disturbances effect the process!

The following diagram illustrates the ‘proportional offset’ which is often (although not always) encountered when proportional only controllers are used.



Labwork 2: Proportional and Integral Control

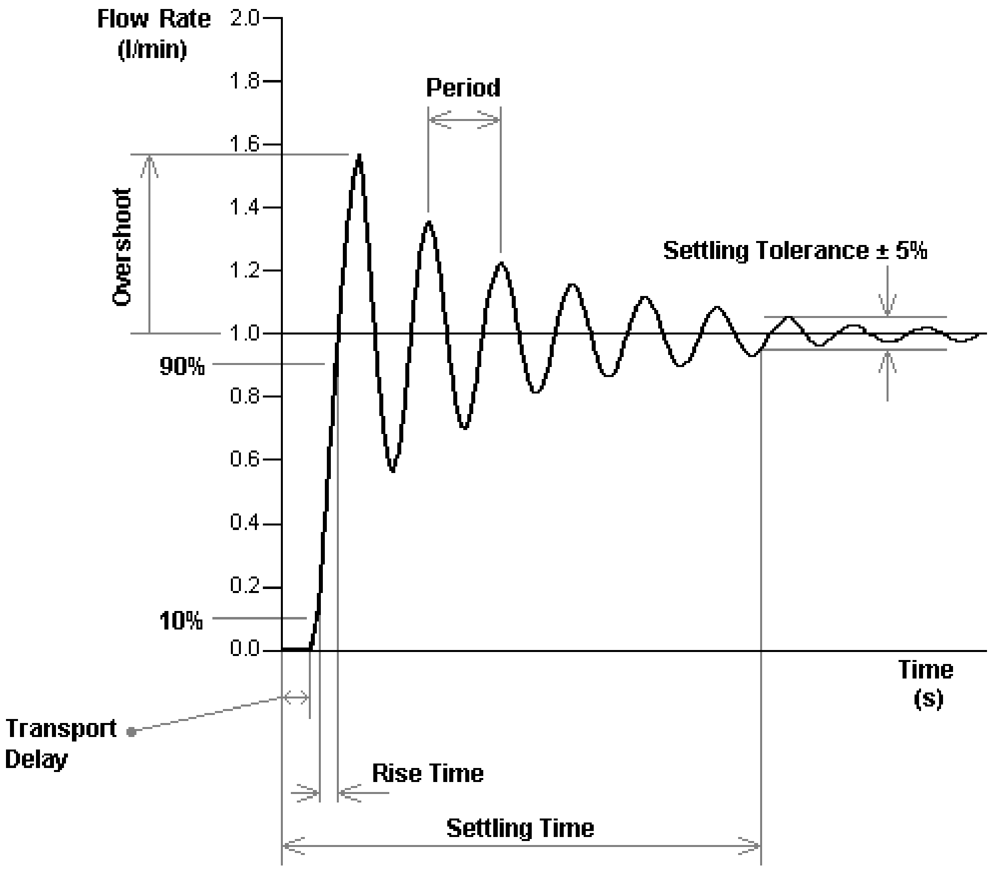
Start a flow loop experiment using only proportional control initially and then adding in an element of integral action after about ten seconds. Repeat this process several times and each time increase the amount of integral action according to the following table. In order to turn off the integral action completely you can either remove the tick from the white box to the left of the I term or you can leave this in and set the I term to 999. (Note that the *amount* of integral action is inversely proportional to the integral action *time* as it is specified in the software and on the table below). Record the final steady state flow value (if the flow actually does settle) and your main observations as to the nature of the response.

SP (l/min)	PG	I	Steady State	Observations
2.0	1.0	999		
2.0	1.0	100		
2.0	1.0	50		
2.0	1.0	10		
2.0	1.0	3		
2.0	1.0	1		
2.0	1.0	0.5		
2.0	1.0	0.2		
2.0	1.0	0.1		

What conclusions about the effects of integral action upon the nature of a 'PI controller' may be drawn from your observations? As in labwork 1, a controller with no integral action (I = 999) is characterised by a response which exhibits a constant offset between the SP and the steady state of the process variable. This is referred to as the 'proportional offset'. When a relatively small amount of integral action is added (I = 100, 50, 10) the flow rate increases gently and produces a trace which is similar to a capacitor charging curve. Clearly the integral action takes account of the recent *history* of the error whereas the proportional action only reacts to the *current value* of the error. PI control can be very effective if the terms are chosen appropriately and for the PCT-100 with I = 3, the response is rapid with little or no oscillation. Increasing the integral action above that which is suitable for a given system can lead to instability and the possibility of gross oscillations in the value of the process variable.

With the integral action time set to around 0.8 seconds the response is 'lightly damped' which means that whilst the control loop is not unstable, (i.e. continuous large amplitude and/or growing oscillations), the controller is not optimally 'tuned' for critical or near critical damping.

For the PCT-100 flow loop optimal tuning of the PI controller typically requires an integral action time of about 1 second if PG = 1. The lightly damped response is an interesting study and it is important because some systems are deliberately designed to be lightly damped. The following diagram illustrates several important terms which are routinely used to describe system response curves. The definitions that follow the diagram may equally be applied to open or closed loop systems.



The **Overshoot** is the maximum amount by which the response exceeds the final steady state value of the process variable. It is sometimes expressed as a percentage of the final steady state value.

The **Rise Time** is the time taken for the response to increase from 10% of its final steady state value to 90% of its final steady state value.

The **Settling Time** is the time taken for the response to reach its final steady state value, within some specified tolerance. The diagram above shows the settling time for a 5% tolerance.

The **Periodic Time** or **Period** is the duration of one complete cycle of oscillation. It can therefore be measured as the interval between *alternate* crossings of the final steady state value or the interval between *successive* peaks or *successive* troughs on the response curve.

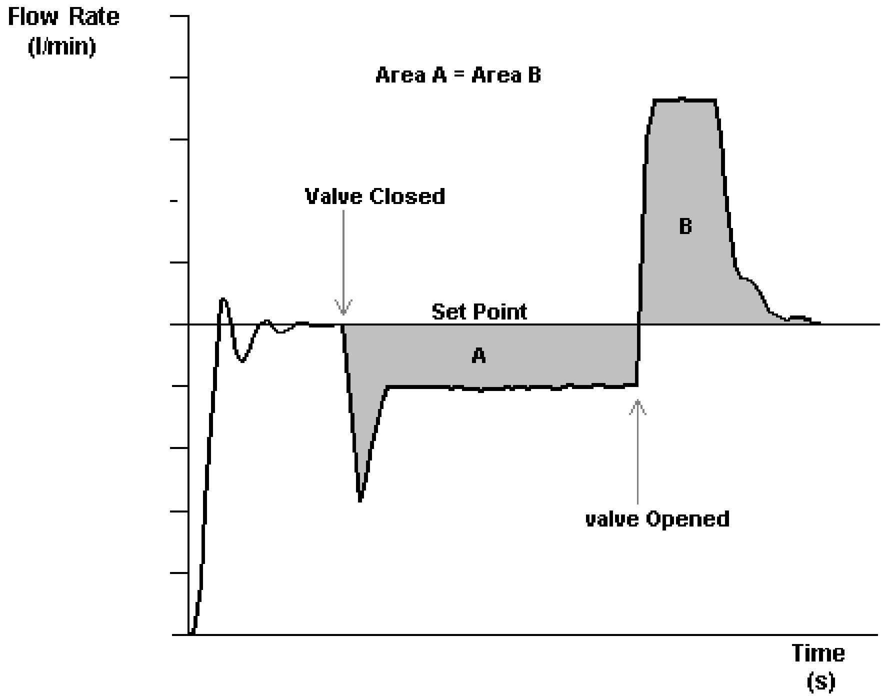
The **Frequency** is the reciprocal of the period, i.e. the number of cycles per second which is expressed in Hertz (Hz). Sometimes the frequency is expressed in radians per second and the relationship between the two units is that radians per second equals 2π times the frequency in Hertz.

The **Transport Delay** is the period during which there is *no change* in the process variable after a step change has been made to the set point.

Labwork 3: Saturation and Integral Windup

Run a flow loop experiment using PI control with P = 1 and I = 3. These are close to the optimum settings and if you make changes to SP, both up and down, you will see that the response is rapid and without excessive oscillation.

Once you have the flow running smoothly with a set point of 2 litres/minute, experiment by opening and closing the flow valve. You should see that the PI controller brings the flow smartly back to the set point without much overshoot in each case. Now close the valve 90% the pump may stop if pressure in the system exceeds 1.5 bar. The controller output (white trace) will shoot up to 100% as it tries to compensate but even with this maximum control effort the restriction is such that the flow rate is held below the SP. The system is now said to be 'saturated'. Allow this to continue for say twenty-five seconds and then suddenly open the valve completely. You will see that the flow rate immediately increases to a value *well above* the SP and stays there for between five and ten seconds before falling back to the SP. The following diagram summarises the nature of these changes.



Why does this effect occur? This phenomenon, which is called 'integral windup' (or 'reset windup'), is due to the fact that the integral term within the PI controller generates a component of the control output which is based upon the recent history of the error.

If you look closely at the graph you will see that the area between the cyan trace and SP (measured from when the valve was closed to when it was opened) is *equal* to the area between the cyan trace and SP (measured from when the valve was opened until the flow rate once again reached the SP).

In more elaborate industrial controllers there is often a feature called 'anti reset windup' which can be used to eliminate this problem so that after a saturation episode the process variable will be returned to the SP as soon as the physics of the hardware allows.

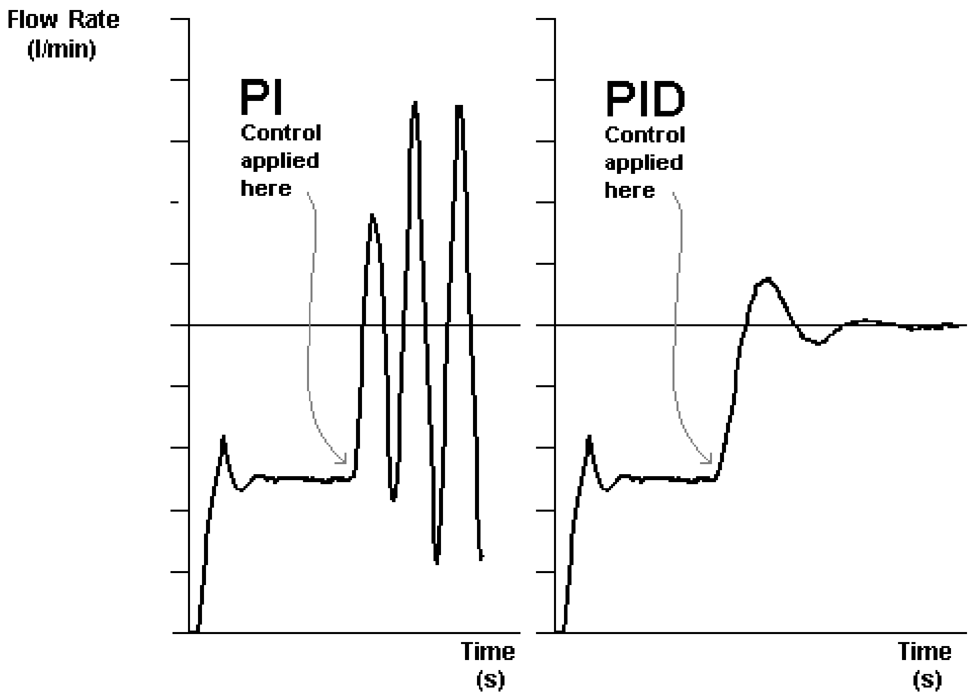
In industry, saturation might occur because an actuator (the pump in our case), is not powerful enough or the pipes are too small or accumulated detritus has reduced the effective diameter of the pipe at some point in the circuit.

Labwork 4: Three Term or PID Control

In this labwork you will investigate the effect of derivative action. Run a flow experiment with SP = 1, PG = 1 and the I and D terms turned off. After a few seconds set I = 0.35. The result will be similar to that seen in labwork 2, a proportional offset whilst there is no integral term followed by permanent oscillations once the integral term is added. Clearly too much integral action was added to the controller!

To eliminate the oscillation we could simply reduce the integral action by increasing the I term to 1 but it would be useful if we could retain the rapid response which the higher integral action confers without pushing the system into unstable oscillations. Whilst this is not always possible with a PI controller, a PID controller is usually capable of eliminating instability and providing a fast response.

Run another experiment with the same initial settings and after a few seconds add the same integral term of 0.35 and a derivative term of 1 second. After the initial sudden rise the flow trace does oscillate about six or seven times but there is a smooth decay of the amplitude and it soon settles at the SP. A little more derivative action will improve the response, try D = 1.9 second to prove this. The following diagram summarises the phenomena which you should have observed.

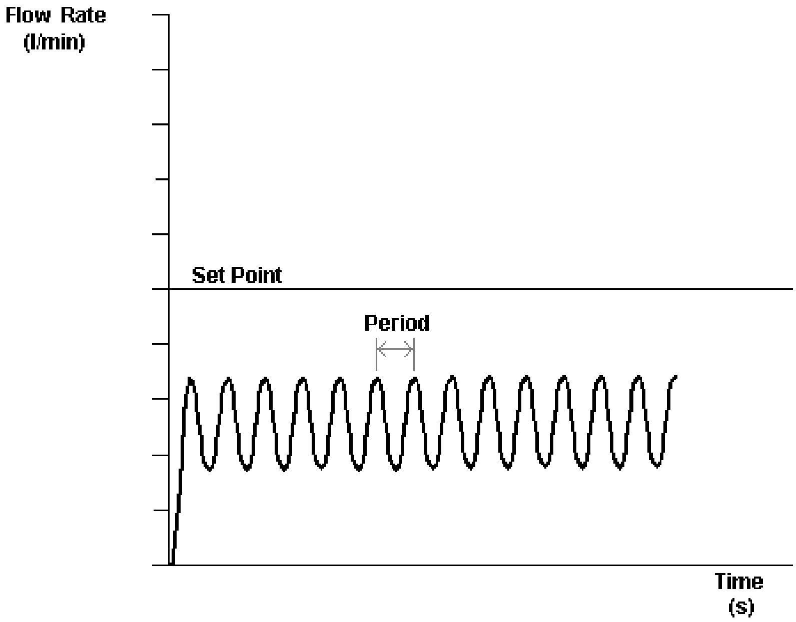


Beware that despite the benefits, derivative action may give rise to detrimental effects in some situations, particularly if there is a significant amount of high frequency 'noise' in the measured, and hence error, value. Derivative action can dramatically amplify this noise and degrade the performance of the controller.

Labwork 5: Ziegler / Nichols Tuning

How do we select appropriate proportional, integral and derivative values for any given process? Simply trying numbers in a PID controller might be acceptable for very small systems but if you are trying to establish a control algorithm for a full sized industrial process then this is not a very good strategy! This method could be time consuming which might bring about financial losses due to the plant downtime. This approach might also risk causing damage to actuators or sensors - if it was to drive them beyond their intended range of operating values. A more scientific approach to finding a reasonable set of PID terms is required.

Ziegler/Nichols tuning is a popular semi-empirical method of obtaining approximate PID values which can be applied successfully to many different types of processes. The method provides a reasonable starting point and the experienced control engineer might wish to adjust the calculated PID terms slightly to improve the nature of the response. There are two techniques the first called the ‘continuous cycling method’ assumes that the closed loop system can be made to oscillate permanently with a proportional only controller, as illustrated by the following graph.



The continuous cycling method requires that the gain of a proportional only controller is increased a small amount at a time until the onset of *permanent* oscillations occurs. At this point the value of the gain ($\mathbf{k_p}$) together with the period of the resultant oscillation (**T**) are noted. The recommended two and three term controllers are then given by:

PI: PG = $0.45\mathbf{k_p}$ IAT = $0.83\mathbf{T}$

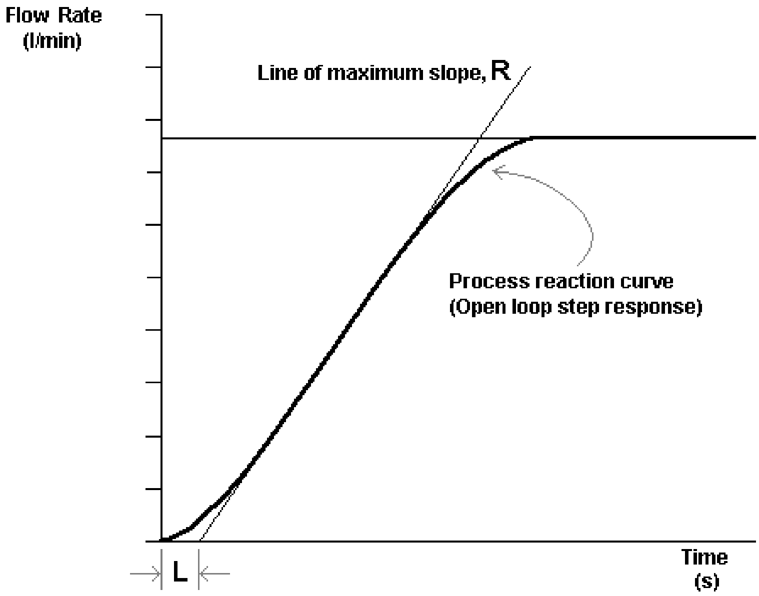
PID: PG = $0.6\mathbf{k_p}$ IAT = $0.5\mathbf{T}$ DAT = $0.125\mathbf{T}$

Run some flow experiments similar to those of labwork 1 with SP = 2. Increase the PG value from 1.0 to about 3.5, initially in increments of 0.5 or so. Try to estimate the *lowest* PG value which produces *permanent* oscillations in the flow rate. (You might need to adjust the PG value by 0.1 or 0.2 when you get close to this ‘ultimate proportional gain’). Record the PG value ($\mathbf{k_p}$) together with the period of the oscillation (**T**) and use them in the expressions quoted above for PI and PID algorithms. Apply these control algorithms to the flow simulation and note your observations in the table below.

Algorithm	Observations
PI	
PID	

Many systems cannot be made unstable (i.e. caused to oscillate) by means of a proportional only controller and for some industrial systems it might be undesirable to do this because of the risk of damage referred to above.

The second Ziegler/Nichols tuning technique called the ‘process reaction curve method’ requires that an open loop step response curve is produced showing a measurable ‘transport delay’ or ‘dead time’. (Transport delay is the period during which there is no change in the process variable after the controller output has been stepped up or down). The following graph shows a typical process reaction curve.



The process reaction curve method requires that an open loop step response of the system is obtained as shown above. From this graph the maximum slope (**R**) and the transport delay (**L**) are noted. The step input expressed as a fraction of the total range of the input (**Δu**) is also required. (In the open loop section of the PCT-100 software the step input fraction **Δu** is calculated very easily since the step input to the pump is specified as a percentage. When the input is 50%, **Δu** = 0.5, when the input is 70%, **Δu** = 0.7 and so on). The recommended two and three term controllers are then given by:

PI: PG = 0.9**Δu/RL** IAT = 3.3**L**
PID: PG = 1.2**Δu/RL** IAT = 2**L** DAT = 0.5**L**

Select the Open Loop option from the main menu and run an open loop simulation with a step input to the pump of 80%. Estimate the values of R and **L** from the resultant graph and record them with the **Δu** value (0.8 in this case). Use these values in the expressions quoted above for PI and PID algorithms.

Apply these control algorithms to the flow simulation and note your observations in the table below.

Algorithm	Observations
PI	
PID	

Comment if you think that the PI and PID algorithms derived from the continuous cycling method or those derived from the process reaction curve method gave the best results? In processes there are sometimes recommendations as to whether you should start with one technique or the other. Sometimes this will be because of concerns about safety, risk of damage and downtime as mentioned before.

Labwork 6: Temperature Control

Run several temperature loop experiments using PI control with the following parameters. In order to turn off the integral action completely either remove the tick from the white box to the left of the I term or leave this in and set the I term to 999. (Note that the *amount* of integral action is inversely proportional to the integral action *time* as it is specified in the software and on the table below). Record the eventual 'steady state' temperatures of the water in the process tank and your main observations as to the nature of the response in the table below. You will need to allow the traces to be drawn for about three minutes in some cases.

Starting Point	SP	PG	I	Steady State	Observations
20°C	30°C	10	999		
20°C	30°C	10	100		
20°C	30°C	10	10		
20°C	30°C	10	1		
20°C	30°C	100	999		
20°C	30°C	100	100		
20°C	30°C	100	10		
20°C	30°C	100	1		

What conclusions may be drawn about the nature of the temperature control loop and the responses produced?

The temperature loop presents a radically different scenario to the flow loop. It has a much larger lag, i.e. it is a much 'slower' process and its requirement for an integral component within the controller is also quite different from the flow loop. This temperature control loop has an element of 'built in' integral action by virtue of the heat capacity of the water in the process tank. This is why the temperature does eventually reach the set point, even without an integral term in the controller!

To gain an understanding of this, consider the following argument. What happens to the flow rate when the pump is turned off? Obviously the flow rate drops to zero straight away. What happens to the temperature of the water when the heater is switched off? Clearly its temperature does *not* immediately drop to zero; in fact the temperature of the water only reduces very slowly due to natural processes such as evaporation. (This is why it's not acceptable to allow any initial overshoot - once overheated; the water in the process tank cannot be forcibly cooled).

The heat capacity of the liquid in an industrial situation process tank will also provide a 'free' integral effect but usually needs to add some integral action to the controller to achieve the optimum response. (If the tank was poorly insulated and the temperature SP was very high then it would need more integral action than if the tank was well insulated and the SP was lower).

The temperature control loop on the PCT-100 cannot be made unstable (it won't oscillate). This means that it isn't possible to use the Ziegler/Nichols continuous cycling method to determine PID values. Try the process reaction curve method bearing in mind that the transport delay might not be particularly well defined on the resulting graph. You will need to estimate the delay as indicated in the second diagram of labwork 5.

Labwork 7: Batch Volume Control

Run a batch volume experiment with volume SP = '3 litres in 3 minutes' and temperature SP = 40°C using the default PID terms for the volume and temperature controllers. This will produce a graph with six traces the most important of which are the red and dark green ones. The red trace represents the measured temperature and this should follow the light green temperature SP line reasonably well. The dark green trace represents the volume of liquid which has been displaced from the process tank rather than the flow rate. This trace should meet the blue volume SP line at the 3 minute mark. (The flow rate is the *gradient* of the dark green line and you should see that this averages 1 litre per minute, i.e. 3 litres on the y-axis divided by 3 minutes on the x-axis).

With the default PI controllers the results are acceptable both for temperature and volume control although you might be able to improve the response a little by adjusting the parameters.

Experiment with different combinations of volume and temperature set points and try different sets of PI and PID controllers. Make observations and note your observations in the table below.

Observations

Labwork 8: Fluid Level Control

Run a set of fluid level control experiments beginning with an *empty* process tank and using the following table of parameters as a guide. Ensure that the Auto Drain feature is turned *off* in each case. In order to turn off the integral action completely you can either remove the tick from the white box to the left of the I term or you can leave this in and set the I term to 999. (Note that the *amount* of integral action is inversely proportional to the integral action *time* as it is specified in the software and on the table below). Record the eventual 'steady state' level of the water in the process tank and your main observations as to the nature of the response in the table below.

SP	PG	I	Steady State	Observations
30%	5	999		
30%	5	600		
30%	5	300		
30%	5	100		
30%	5	10		
30%	5	1		

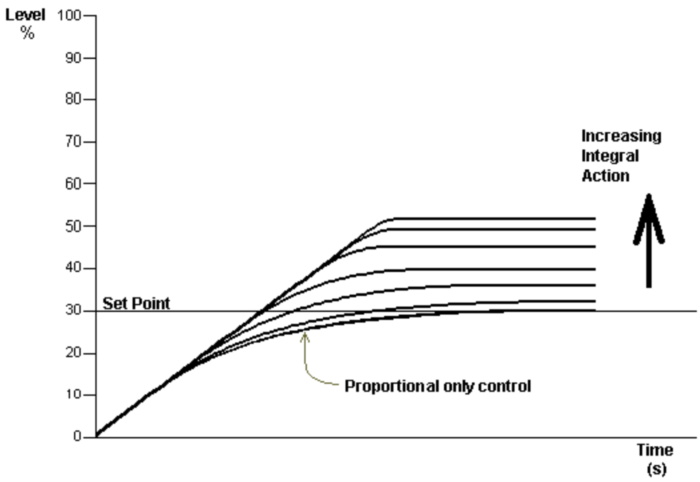
What conclusions about the nature of these fluid level experiments may be drawn from your observations?

The best response was achieved by the proportional only controller (i.e. the one with I = 999) and this fact seems to militate against what you learned from studying flow rate control in the earlier labworks. Normally we might expect to see a 'proportional offset' when using a proportional only controller, why is there no such offset seen in this particular example? Why is there apparently no need of an integral term within the algorithm which controls the pump being used to raise the liquid to the desired level?

These questions can lead to an insight into the fundamental nature of level control when it is implemented by pumping liquid into a closed tank.

This type of approach to level control is not unique, many industrial and other processes including the ubiquitous toilet cistern regulate level by controlling the rate at which liquid flows into a closed container.

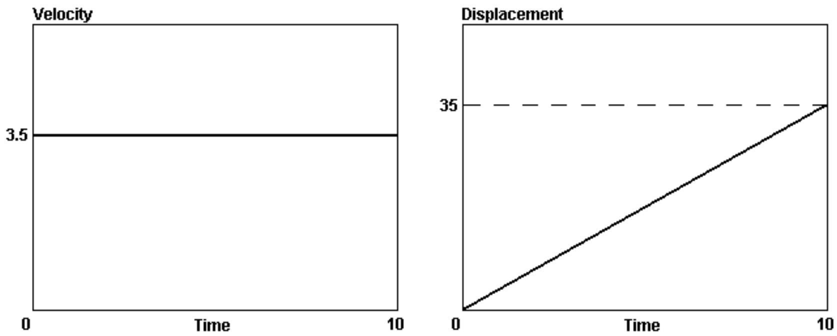
We can see the effectiveness of a simple proportional level controller at any time by lifting the cover off any convenient toilet cistern. The rate at which the water under mains pressure flows into the tank is determined by the choking action of the ball cock valve. The valve is progressively closed by the action of the rising water as the floating ball lifts the lever connected to it. The valve's 'degree of openness' is proportional to the difference between the desired level and the actual level. Once the inrush of water has come to an end the cistern is full to the desired level, (defined by the geometry of the ball/lever assembly) which should obviously be *below* the emergency overflow. On the PCT-100 the proportional controller brings the level up to the set point smoothly and then the pump cuts off altogether. (What is the effect of increasing PG in the proportional only controller?).



The graph above shows several fluid level responses taken from the PCT-100 (with PG = 5 in each case), drawn on the same axes. It is clear that there is a progressive increase in what we might call an integral offset effect as the magnitude of the integral action is increased. How can we account for this phenomenon? Why is there no proportional offset, why is an integral term in the controller unnecessary and why do integral terms give rise to offsets in this kind of level control?

The explanation is that by its very nature the process itself provides an element of integral action due to the volumetric capacity of the tank. (It is said to have a 'free integrator' in transfer function parlance, i.e. a 1/s term in its Laplace Transform). Consider flow control, *some* output to the pump is required if there is to be *any* fluid flow, if the pump is *off* then there can be *no* flow. With level control there is always a *certain* level of water when the pump is *off*, and if the pump is running then the level will *always* be rising.

Integral action in a PI (or PID) controller takes account of the recent history of the error by adding up the errors for a fixed number of 'recent' samples and contributing a component of controller output which is proportional to this sum. With the level control situation the volumetric capacity of the tank continuously 'integrates' the incoming flow rate which results in an increasing depth of liquid. This intrinsic characteristic is analogous to the familiar integration of velocity to obtain displacement thus:



In the level control experiment the actual water level is the *integral* of the flow rate into the tank. Therefore even with a proportional only flow rate controller, both proportional *and* integrating actions are present within the closed loop formed by the controller and the process. As a result of this intrinsic integral effect there will never be a proportional offset and an explicit integral term in the control algorithm would be entirely redundant. Any integral term which is added either produces no visible effect (if it is very small) or the aforementioned integral offset effect because it keeps the control output to the pump higher for longer than is required. This level control scenario is non-linear in that the water level may be *increased* by controlling the pump but, irrespective of the algorithm, the level *cannot be lowered through any use of the pump*. Once a particular level has been reached, even with the pump turned off completely it cannot be lowered. The only means of lowering the level is to open one of the drain valves.

Labwork 9: Open Loop Control

Run a set of open loop experiments using a sinusoidal input signal with Min = 30% and Max = 80% to drive the pump. Vary the Period between 0.5 and 20 seconds. Record your main observations as to the nature of the response in the table below.

Period (s)	Observations
0.5	
1	
2	
3	
4	
5	
7.5	
10	
15	
20	

What conclusions may be drawn about the nature of the open loop responses produced? Firstly you should see that the output *from* the system, i.e. the flow measured value (cyan trace) is a sine wave which always has the same period as the input *to* the system, i.e. the signal driving the pump (white trace). One of the characteristics of any *linear* system is that the resulting (output) signal always has the same *waveform* as the driving (input) signal. If, for the same period, the amplitude of the input was say doubled (or halved) then the amplitude of the output would be doubled (or halved) accordingly.

Analogy, is an alternating voltage applied across a resistor. An alternating current will pass through the resistor which will have exactly the same frequency (and hence period) as the voltage waveform. The amplitude of this alternating current will, in general, be different from that of the voltage but they will always have the same waveform and frequency. The waveforms will also be ‘in phase’, i.e. their peaks and troughs will always coincide. If the resistor was replaced by a capacitor then the alternating current would have the same waveform and frequency but this time it would be ‘out of phase’ with the alternating voltage as well as having a different amplitude. If the capacitor was replaced by a *non-linear* device such as a diode bridge then the current would no longer match the voltage in terms of its waveform or frequency. (i.e. in the case of full wave rectification).

You should also see that the ratio of the amplitude of the input to the amplitude of the output (which is referred to as the ‘gain’), varies between the experiments and that the peaks and troughs of the two traces never coincide exactly. The traces are ‘out of phase’ or separated by a ‘phase angle’. At lower frequencies the peaks and troughs tend towards coincidence but at higher frequencies they separate dramatically. At the higher frequencies (~ 1 Hertz) the input and output traces can be seen to be in ‘antiphase’, i.e. the peaks of the input occur at roughly the same time as the troughs of the output and vice versa. These observations lead to the concept of ‘frequency response’. The frequency response of a system is summarised in the form of specialised graphs of gain versus frequency and phase angle versus frequency. These are plotted logarithmically and are called ‘Bode gain and phase plots’.

Labwork 10: Bode Plots

Repeat the set of open loop experiments from labwork 9 and estimate the gain and phase angle in each case and record the results in the table below.

Period (s)	Frequency (Hz)	Gain	Gain ÷ by max. Gain	Phase Angle (°)
0.5	2			
1	1			
2	0.5			
3	0.333			
4	0.25			
5	0.2			
7.5	0.133			
10	0.1			
15	0.067			
20	0.05			
100	0.01			

It is necessary to minimise the period shown on the graph (using the slider at the top right) after the higher frequency experiments, in order to magnify the trace to estimate the phase angle. The gain can be calculated very easily in each case by obtaining the maximum and minimum flow values from the cyan trace, using the digitising cursor. The gain may then be calculated from the following equation:

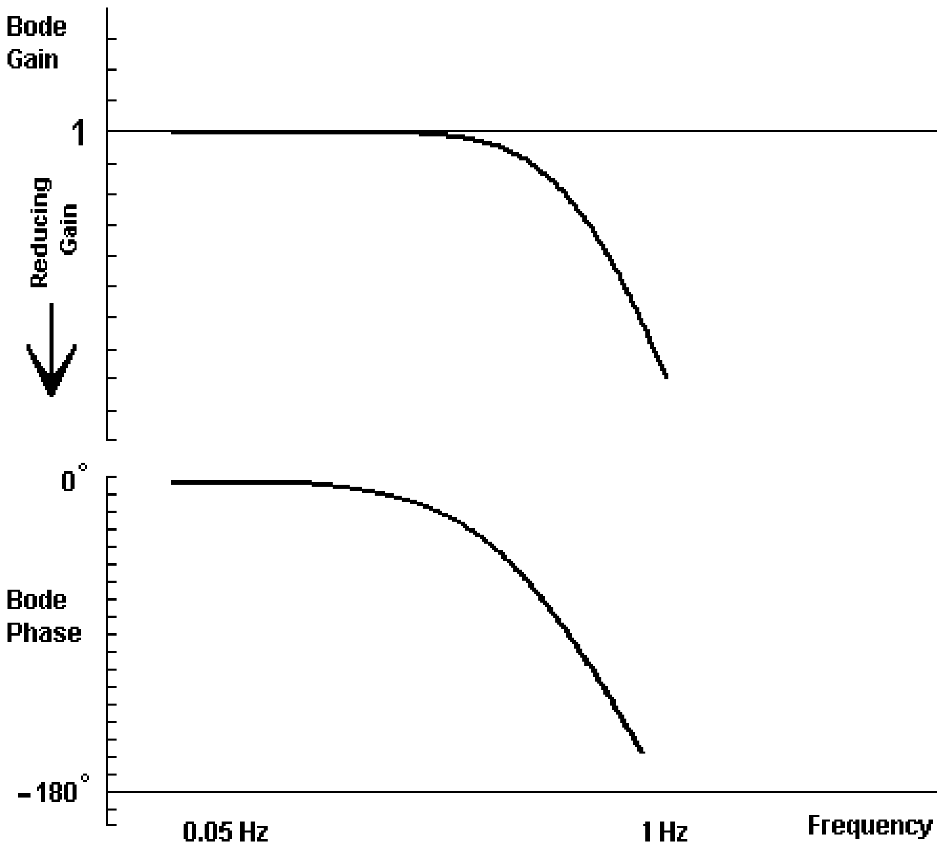
$$\text{Gain} = (\text{Maximum Flow Rate} - \text{Minimum Flow Rate}) / 50\%$$

The 50% represents the peak-to-peak value of the input, i.e. 80% - 30%. (The peak-to-peak ratio is identical to the amplitude ratio for the input and output signals). The problem with this is the input signal is quoted as a percentage and the output signal is measured in litres per minute. In order to normalise the calculated gain values in this situation we can divide *all* of the gain values by the maximum magnitude of the gain; which will be when the period is very large. For practical purposes it will be sufficient to estimate the gain when the period is 100 seconds or more and to divide all of the gain values by this figure. This explains the presence of the second gain column in the table above. The phase angle may be estimated in each case by observing how much time passes from the moment the input reaches a peak (or trough) until the output reaches the *corresponding* peak (or trough). If this time interval is called ΔT then the phase angle (in degrees) may be calculated from the following equation:

$$\text{Phase Angle} = (\Delta T/\text{Period}) \times 360^\circ$$

Estimates of the gain and phase angle will be inaccurate at the highest frequencies due to the difficulty of acquiring accurate data from the traces.

Once the table above has been completed use the data in the second, fourth and fifth columns to draw Bode Gain and Bode Phase plots for the flow loop. The graph below shows the sort of results this should achieve.



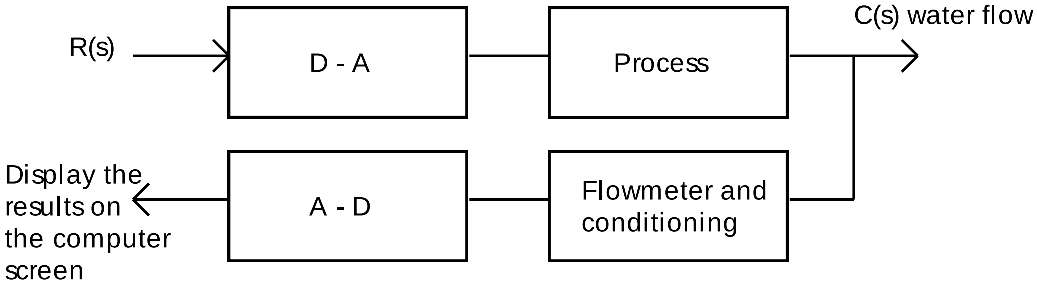
The open loop section within the software gives an opportunity to introduce the subject of 'system frequency response' and the Bode plot method of representing it. However this concept is more usually applied to servo control systems (as found in robotics, aero-engineering, vehicle suspension etc.,) rather than fluid process control systems. Normally Bode plots are drawn with a logarithmic frequency scale and the frequency is plotted in radians per second rather than hertz.

Labwork 11: Flow Loop Model Using Caldwell's Method

Caldwell's method estimates the time constants for a second order system using data taken from an open loop process reaction curve. Knowledge of the time constants allows the transfer function to be written down. We know that the PCT-100 flow loop can be made to oscillate easily we can be sure that it is at least a second order system. Even if the underlying mathematics were of a higher order we should still be able to model the loop response with a second order expression.

When we have obtained an approximate transfer function the flow loop may be simulated, using a simulation package. To adjust this to match the actual response. Using the refined TF, closed loop stability may be assessed using Bode or Nyquist plots. From these results the controller required to produce specified system performance may be designed as discussed in Labwork 13.

The procedure requires that an open loop response to a moderately sized step input is obtained, taking into account the guidelines discussed in section 7.6.7.1. To achieve this select the Open Loop section in the software and choose the step input function. The open loop is shown in the block diagram below.



Enter a value of around 40 in the SP box and then click the Start button. The pump will start the water begins to move. Wait until this highly unstable initial phase has ended and the flow has settled down to a steady state and then increase the pump output by entering a value of around 70 in the SP box. The pump will accelerate, the flow rate will increase and the software will draw the required process reaction curve. Once the flow rate has settled to the new steady state value click the Stop button. It is best to repeat this procedure several times until a trace is produced which seems a reasonable 'average' response. At this stage print out a hard copy of the graph.

The technique outlined above allows us to ignore the unstable transient response of the flow loop (caused by transport delay dead-time), when starting from a zero flow rate. This means that the second order model resulting from an application of Caldwell's method to the process reaction curve, will not describe the flow loop response when it changes from a zero flow rate. To model this initial phase we recommend Sundaresan's method - see labwork 12.

Caldwell's method estimates that the sum of the two time constants is equal to the time taken for the response to reach 73% of its final steady state value divided by 1.32. From the hard copy of the process reaction curve find the time taken to reach 73% of the final steady state value and calculate the sum of the time constants, $(T_1 + T_2)$. Next halve the sum of the time constants and find, from the process reaction curve, the fractional response at this time. (i.e. The fraction of the final steady state value). Use this fractional response to find $T_2/(T_1 + T_2)$ from figure 8.6. From the values for $(T_1 + T_2)$ and $T_2/(T_1 + T_2)$, calculate the individual time constants, T_1 and T_2. The Caldwell et al transfer function approximation is then:

$$Gp(s) = \frac{1}{(T_1 s + 1)(T_2 s + 1)}$$

Figure 8.6 Caldwell Reference Graph

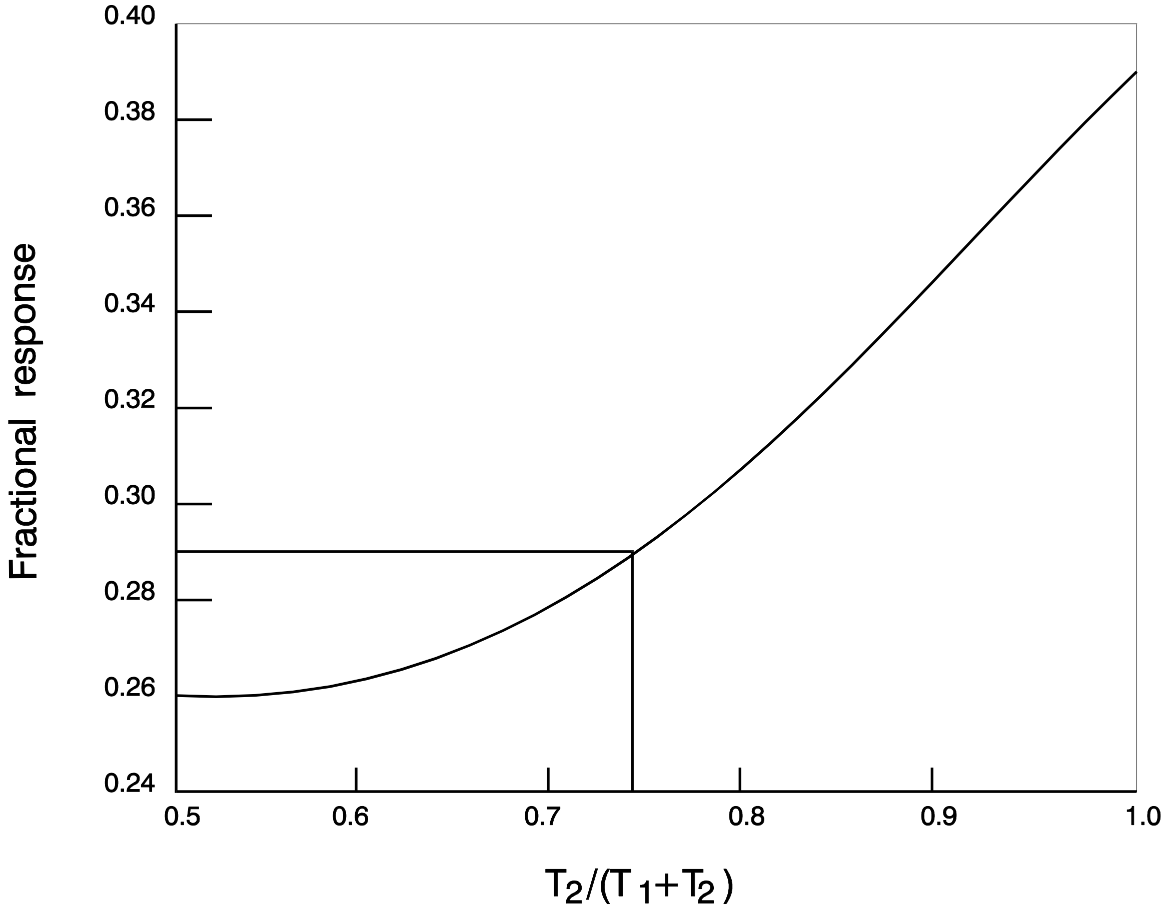
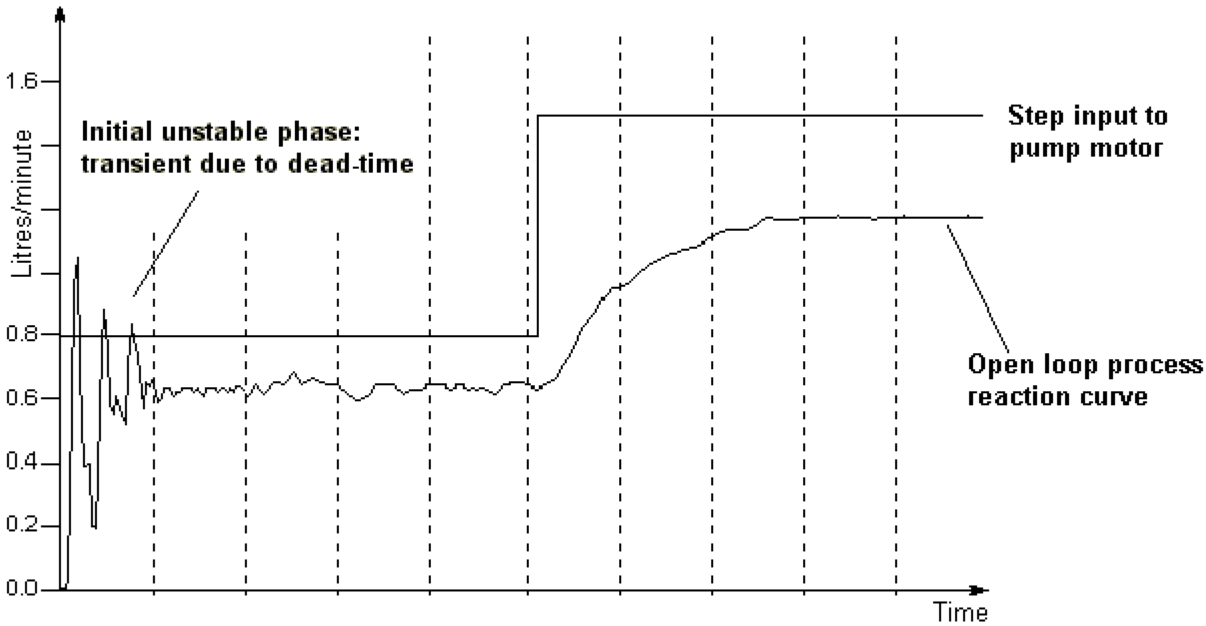


Figure 8.7 Typical PCT-100 Flow Loop Process Reaction Curve



For a more theoretical approach to the Caldwell method of system modelling

$$\text{Sum of time constants} = \frac{\text{Time to reach 73\% of final steady state value}}{1.32}$$

In test using the PCT-100, the time to reach 73% of the final steady state value was 0.8 seconds therefore:

$(T_1 + T_2) = 0.6$ seconds

$(T_1 + T_2)/2 = 0.3$ seconds

The fractional response at 0.3 seconds from the start of the step was estimated at 0.29. (From the process reaction curve). Therefore from figure 8.6:

$T_2/(T_1 + T_2) = 0.73$

Therefore: $T_2 = 0.6 \times 0.73 = 0.44$ seconds

Therefore: $T_1 = 0.6 - 0.44 = 0.16$ seconds

$$\text{Therefore: } Gp(s) = \frac{1}{(0.44\,s+1)(0.16\,s+1)}$$

This approximation may be fine tuned using a simulation package to match the actual process reaction curve obtained from the PCT-100 flow loop. The refined TF may be analysed for closed loop stability using Bode or Nyquist plots as described in section 7.6.5 and 7.6.6. A controller may then be designed as described in section 7.7.

Labwork 12: Flow Loop Model Using Sundaresan's Method

This is an alternative approach which approximates a system as a second order plus dead-time model. It is used where the process reaction curve exhibits a transport delay which is the case for the PCT-100 flow loop when starting from a zero flow rate. The TF of an overdamped second order system with dead-time is:

$$Gp(s) = \frac{e^{-\theta_d s}}{(T_1 s+1)(T_2 s+1)}$$

Alternatively the TF of an underdamped second order system with dead-time is:

$$Gp(s) = \frac{e^{-\theta_d s}}{(1/\omega_n^2)s^2+(2\varepsilon/\omega_n)s+1}$$

The objective of Sundaresan's method is to determine the three parameters θ_d, T_1 and T_2 or θ_d, ω_n and ε from a process reaction curve.

The procedure requires that an open loop response to a moderately sized step input is obtained, taking into account the guidelines discussed in section 7.6.7.1. To achieve this run the PCT-100 assignments software (which accompanies the main software) and select assignment number 5. This is used to produce a process reaction curve for the PCT-100 flow loop scaled to show fractional response. It will also automatically calculate the parameters required by Sundaresan's method.

The software requires the initial and final values for the step input to be entered before the Start button is clicked. These values represent the digital signal used to drive the pump circuit so the range of values is 0 – 255. Since we want to include the dead-time in this labwork we must start the step from 0, i.e. zero pump output and zero flow-rate). The software will drive the pump, draw the open loop process reaction curve and calculate three parameters: area m_1, gradient M_1 and time t_m. Figure 8.8 defines these three parameters. Repeat this procedure several times until a trace is produced which seems an 'average' response.

Select the Open Loop section in the software and choose the step input function. Enter a value of 70 in the SP box and click the Start button. Once the flow rate has settled to the steady state value click the Stop button. Repeat this procedure several times until a trace is produced which is an 'average' response. Print out a hard copy of the graph and calculate the required parameters [see Figure 8.8] manually).

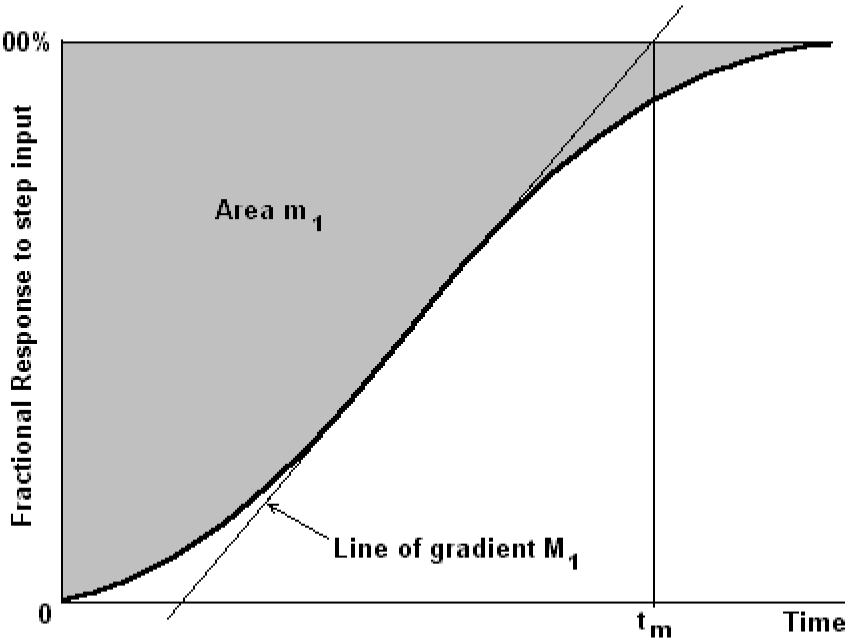


Figure 8.8 Parameters calculated within the PCT-100 Assignments Software

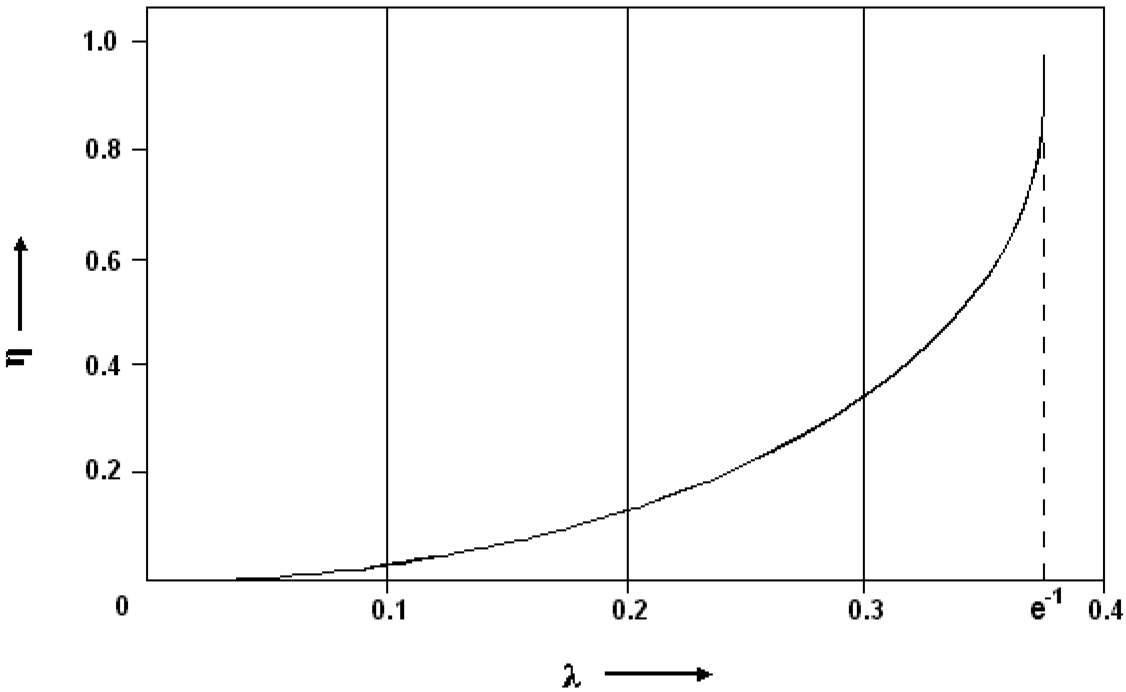


Figure 8.9 Reference graph for Sundaresan's method (overdamped)

Sundaresan's method varies depending upon whether the open loop response was overdamped or underdamped.

Overdamped response:

1) Calculate: $\lambda = M_1(t_m - m_1)$

2) If $\lambda > e^{-1}$ the system is underdamped so follow the underdamped procedure below.

3) From figure 8.9 find the value of η corresponding to the value calculated for λ.

4) Using:

$$T_1 = \frac{\eta^{\eta/(1-\eta)}}{M_1}, \quad T_2 = \frac{\eta^{1/(1-\eta)}}{M_1}, \quad \theta_d = m_1 - \frac{\eta^{1/(1-\eta)}}{M_1} \cdot \frac{(\eta+1)}{\eta}$$

5) Calculate: T_1, T_2 and θ_d.

6) Sundaresan's estimate for the transfer function is then:

$$Gp(s) = \frac{e^{-\theta_d s}}{(T_1 s+1)(T_2 s+1)}$$

Underdamped response:

1) Calculate: $\lambda = M_1(t_m - m_1)$

2) If $\lambda < e^{-1}$ the system is overdamped so follow the overdamped procedure above.

3) From Figure 8.10 find the value of ε corresponding to the value calculated for λ

4) Using: $\omega_n = \frac{\cos^{-1}\varepsilon}{\sqrt{1-\varepsilon^2}} \cdot \frac{1}{(t_m - m_1)}$ and $\theta_d = m_1 - \frac{2\varepsilon}{\omega_n}$

5) Calculate: ω_n and θ_d. Remember, angles are in radians rather than degrees!

6) Sundaresan's estimate for the transfer function is then:

$$Gp(s) = \frac{e^{-\theta_d s}}{(1/\omega_n^2)s^2 + (2\varepsilon/\omega_n)s + 1}$$

For a more theoretical approach to the Sundaresan method of system modelling

Experimental results, this time <u>including</u> the transport delay dead-time, were obtained with the PCT-100.

Area $m_1 = 1.089$
Gradient $M_1 = 1.157$
Time $t_m = 1.316$ seconds
Therefore $\lambda = 0.263$

Since $\lambda < e^{-1}$ the system is overdamped so Sundaresan's overdamped second order approximation technique must be used. From figure 8.9 η corresponding to $\lambda = 0.266$ is $\eta = 0.24$. Therefore $T_1 = 0.551$ seconds, $T_2 = 0.132$ seconds and $\theta_d = 0.406$ seconds.

$$\text{Therefore: } Gp(s) = \frac{e^{-0.41s}}{(0.55s+1)(0.13s+1)}$$

Comparing this result with that from the previous labwork, it can be seen to be similar except for the 0.41 second deadtime.

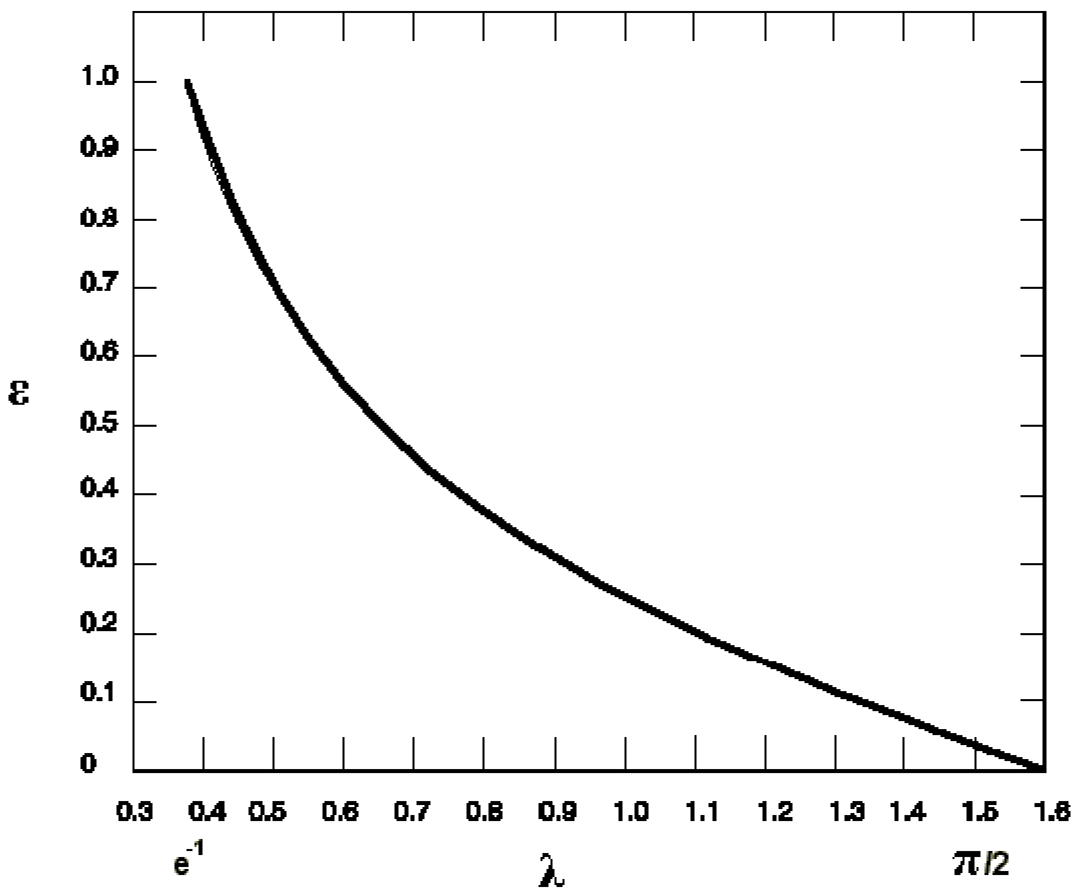


Figure 8.10 Reference graph for Sundaresan's method (underdamped)

Labwork 13: Design of Controller for PCT-100 Flow Loop.

The objective is to design and test PI and PID controllers that will control the PCT-100 flow loop using the software provided. Ideally controllers should give:

- Minimum settling time
- Maximum of 10% overshoot
- Zero steady state error
- A stable response to any step input
- Controller gain should be 0.5

Procedure based upon continuous (Analogue) approach:

Use the transfer functions for the PCT-100 flow loop obtained from labworks 11 and 12 or the approximations:

$$Gp(s) = \frac{1}{(0.44s+1)(0.17s+1)}$$

For the response excluding steps from zero flow-rate or, considering the dead-time when changing from zero flow-rate:

$$Gp(s) = \frac{e^{-0.41s}}{(0.55s+1)(0.13s+1)}$$

The controllers may be expressed in laplace format as:

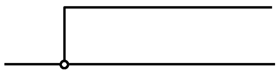
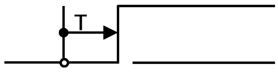
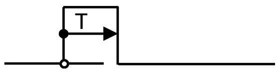
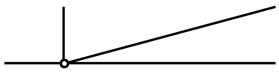
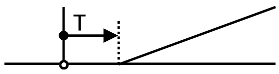
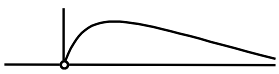
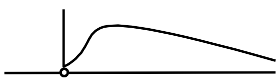
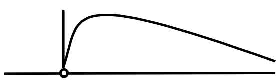
PI: Gc(s) = PG(1 + 1/IATs)
PID: Gc(s) = PG(1 + 1/IATs + DATs)

Using Laplace transform techniques evaluate:

1) Overall reduced system block diagrams incorporating Gc(s) and Gp(s)
2) Overall open loop transfer functions
3) Steady state values using final value theorem
4) Determine the limiting control parameters (PG, IAT and DAT) to ensure stability from a Routh array
5) Relative stability from Bode or Nyquist Plots
6) Test controllers using the PCT-100 software

APPENDIX 1

LIST OF LAPLACE TRANSFORMS

Description	f(t)	F(s)	Graph of f(t)
1. Unit impulse	$\delta(t)$	1	
2. Unit step	$H(t)$	$\frac{1}{s}$	
3. Delayed step	$H(t-T)$	e^{-sT}/s	
4. Rectangular pulse: duration T	$H(t) - H(t-T)$	$(1 - e^{-sT})/s$	
5. Unit ramp	t	$\frac{1}{s^2}$	
6. Delayed ramp	$(t-T)$	e^{-sT}/s^2	
7. n^{th}. order ramp	t^n	n/s^{n+1}	
8. Exponential decay	$e^{-\alpha t}$	$1/(s+\alpha)$	
9. Exponential rise	$1 - e^{-\alpha t}$	$\alpha/(s(s+\alpha))$	
10. Exponential αt	$te^{-\alpha t}$	$1/(s+\alpha)^2$	
11. Exponential αt^n	$t^n e^{-\alpha t}$	$n/(s+\alpha)^{n+1}$	
12. Difference of exponentials	$e^{-\alpha t} - e^{-\beta t}$	$\frac{(\beta - \alpha)}{(s+\alpha)(s+\beta)}$	

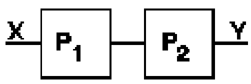
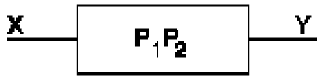
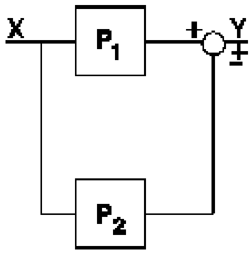
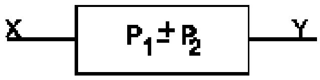
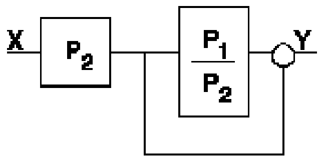
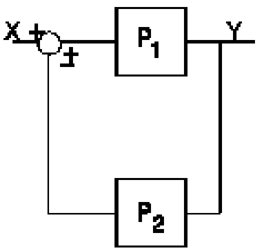
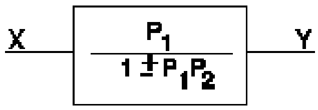
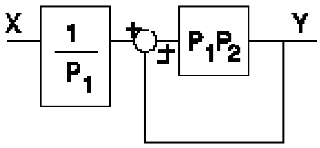
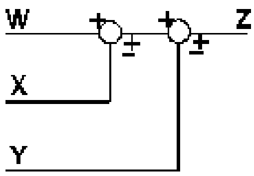
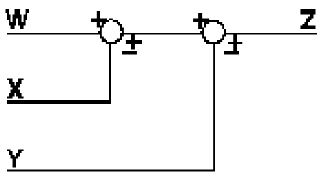
APPENDIX 2

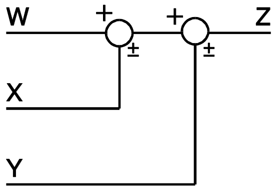
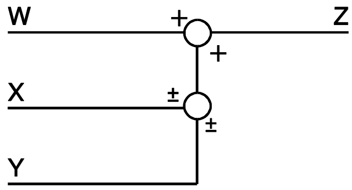
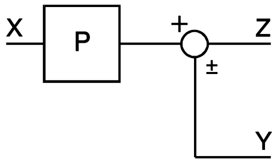
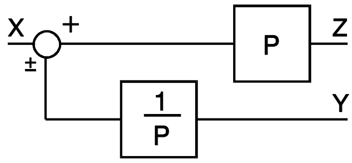
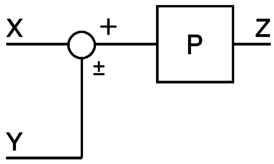
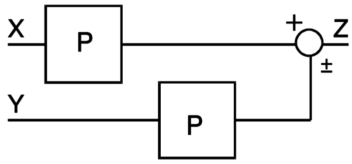
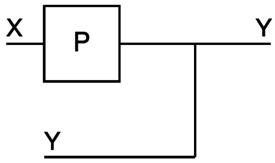
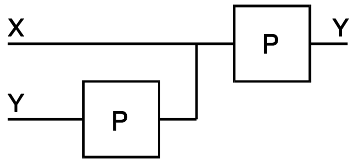
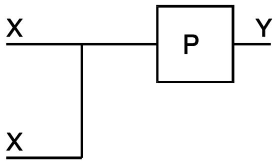
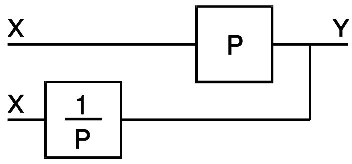
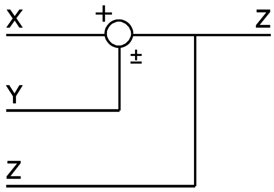
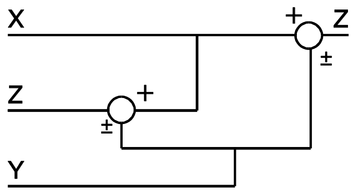
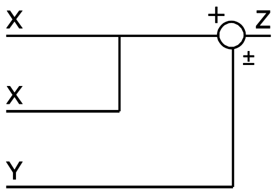
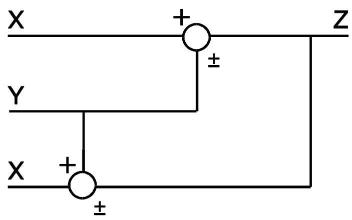
LIST OF Z TRANSFORMS

LAPLACE TRANSFORM E(s)	TIME FUNCTION e(t)	Z TRANSFORM E(z)
1	$\delta(t)$	1
e^{-nTs}	$\delta(t-nT)$	z^{-n}
$\frac{1}{s}$	$u(t)$	$\frac{z}{z-1}$
$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$
$\frac{1}{s+a}$	e^{-at}	$\frac{z}{z-e^{-aT}}$
$\frac{1}{(s+a)(s+b)}$	$\frac{1}{(b-a)}(e^{-at}-e^{-bt})$	$\frac{1}{b-a}\left(\frac{z}{z-e^{-aT}}-\frac{z}{z-e^{-bT}}\right)$
$\frac{1}{s(s+a)}$	$\frac{1}{a}\,(u(t)-e^{-at})$	$\frac{1}{a}\,\frac{(1-e^{-aT})z}{(z-1)(z-e^{-aT})}$
$\frac{1}{(s+a)^2}$	te^{-at}	$\frac{Tze^{-aT}}{(z-e^{-aT})^2}$

APPENDIX 3

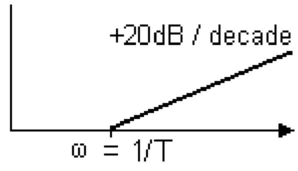
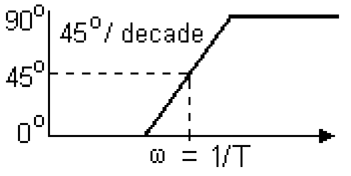
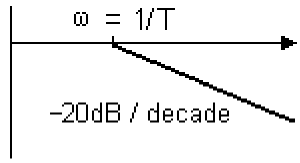
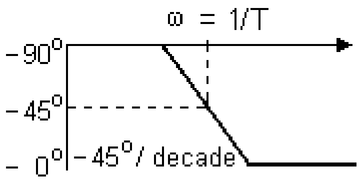
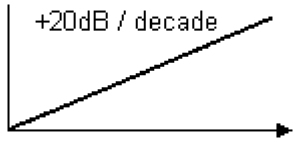
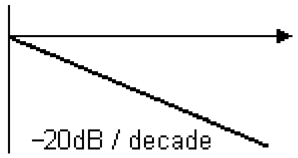
LIST OF STANDARD BLOCK DIAGRAM REDUCTIONS

TRANSFORMATION		EQUATION	BLOCK DIAGRAM	EQUIVALENT BLOCK DIAGRAM
1	Combining Blocks in Cascade	$Y=(P_1 P_2)X$		
2	Combining Blocks in parallel; or Eliminating a Forward Loop	$Y=P_1 X \pm P_2 X$		
3	Removing a Block from a Forward Path	$Y=P_1 X \pm P_2 X$		
4	Eliminating a Feedback Loop	$Y=P_1 (X \pm P_2 Y)$		
5	Removing a Block from a Feedback Loop	$Y=P_1 (X \pm P_2 Y)$		
6	Rearranging Summing Points	$Z=W+X+Y$		

TRANSFORMATION		EQUATION	BLOCK DIAGRAM	EQUIVALENT BLOCK DIAGRAM
6b	Rearranging Summing Points	$Z = W \pm X \pm Y$		
7	Moving a Summing Point Ahead of a Block	$Z = W \pm X \pm Y$		
8	Moving a Summing Point Beyond a Block	$Z = P(X \pm Y)$		
9	Moving a Takeoff Point Ahead of a Block	$Y = PX$		
10	Moving a Takeoff Point Beyond a Block	$Y = PX$		
11	Moving a Takeoff Point Ahead of a Summing Point	$Z = X \pm Y$		
12	Moving a Takeoff Point Beyond a Summing Point	$Z = X \pm Y$		

APPENDIX 4

STANDARD BODE PLOT FUNCTIONS

Transfer Function Element	Amplitude Plot	Phase Plot
K		No Change
1 + Ts		
$\frac{1}{1+Ts}$		
s		90°
$\frac{1}{s}$		-90°

APPENDIX 5

STANDARD NYQUIST PLOT FUNCTIONS

Function	Magnitude ($\lvert G \rvert$)	Phase Shift ($<G$)
K	K	0
1 + Ts	$\sqrt{1+(\omega T)^2}$	$\tan^{-1}(\omega T)$
$\frac{1}{1+Ts}$	$\frac{1}{\sqrt{1+(\omega T)^2}}$	$-\tan^{-1}(\omega T)$
s	ω	90°
$\frac{1}{s}$	$\frac{1}{\omega}$	-90°

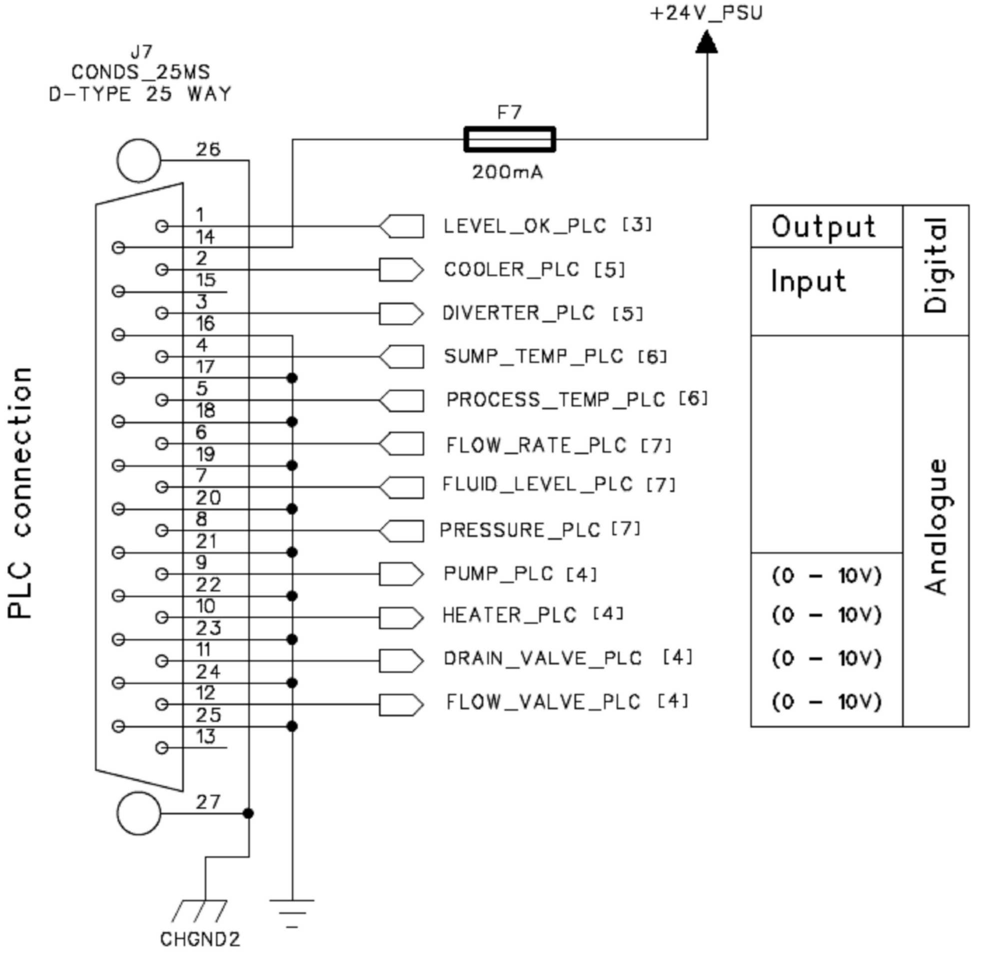
To evaluate $\lvert G \rvert$ for a complete transfer function MULTIPLY the separate $\lvert G \rvert$ elements.

To evaluate $<G$ for a complete transfer function ADD the separate $<G$ elements.

PCT-100 25 Way Connection

25 Pin D Socket	PLC Connection	24 Way Connector on Module	Function
1	Digital Input	Digital Output	Level Confirm (safety) OK
2	Digital Output	Digital Input	Cooler (Fan)
3	Digital Output	Digital Input	Diverter Valve
4	Analogue Input	Analogue Output	Sump Tank Temp PRT
5	Analogue Input	Analogue Output	Process Tank Temp PRT
6	Analogue Input	Analogue Output	Flow Sensor
7	Analogue Input	Analogue Output	Level Sensor
8	Analogue Input	Analogue Output	Pressure Sensor
9	Analogue Output	Analogue Input (0-10v)	Pump
10	Analogue Output	Analogue Input (0-10v)	Heater
11	Analogue Output	Analogue Input (0-10v)	Drain Valve
12	Analogue Output	Analogue Input (0-10v)	Flow Valve
13	N/A	N/A	N/A
14	24v DC Power	24v DC Power	24v Power Output
15	N/A	N/A	N/A
16	GND	GND	GND
17	GND	GND	GND
18	GND	GND	GND
19	GND	GND	GND
20	GND	GND	GND
21	GND	GND	GND
22	GND	GND	GND
23	GND	GND	GND
24	GND	GND	GND
25	GRD	GRD	GRD

Schematic for 25 way connection



Note Arrows on diagram above show direction of connection internally on the module and not connections to PLC or PID controller.

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