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Modified Smith predictor based cascade control of unstable time delay processes

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ABSTRACT

An improved cascade control structure with a modified Smith predictor is proposed for controlling open-loop unstable time delay processes. The proposed structure has three controllers of which one is meant for servo response and the other two are for regulatory responses. An analytical design method is derived for the two disturbance rejection controllers by proposing the desired closed-loop complementary sensitivity functions. These two closed-loop controllers are considered in the form of proportional–integral-derivative (PID) controller cascaded with a second order lead/lag filter. The direct synthesis method is used to design the setpoint tracking controller. By virtue of the enhanced structure, the proposed control scheme decouples the servo response from the regulatory response in case of nominal systems i.e., the setpoint tracking controller and the disturbance rejection controller can be tuned independently. Internal stability of the proposed cascade structure is analyzed. Kharitonov's theorem is used for the robustness analysis. The disturbance rejection capability of the proposed scheme is superior as compared to existing methods. Examples are also included to illustrate the simplicity and usefulness of the proposed method.

1. Introduction

Cascade control is one of the popular multi-loop control methods commonly used in the process control industries for the control of physical parameters such as temperature, flow and pressure. Cascade controllers are effective when the single loop PID controller finds difficulty in regulating the output in the face of load disturbances. A cascade control structure utilizes two control loops: a secondary loop (inner or slave loop) embedded within a primary loop (outer or master loop). It is very important that the slave loop takes on a faster dynamic response as compared to the master loop, the main reason behind this is that the fast dynamics of the inner loop will provide faster disturbance attenuation and minimize the possible effect of the disturbances, before they affect the primary output.

Fig. 1 shows a typical example of cascade control, a stirred chemical reactor where cooling water flows through the reactor jacket to regulate the reactor temperature [1,2].

The reactor temperature is affected by changes in disturbance variables such as reactant feed temperature or feed composition. The simplest control strategy would handle such disturbances by adjusting a control valve on the cooling water inlet stream. However, an increase in the inlet cooling water temperature, an unmeasured disturbance, may cause unsatisfactory performance. The resulting increase in the reactor temperature, owing to a reduction in heat removal rate, may occur slowly. If appreciable dynamic lags occur in the jacket as well as in the reactor, the corrective action taken by the controller will be delayed. To overcome this disadvantage, a feedback controller for the jacket temperature, whose setpoint is determined by the reactor temperature controller, can be added to provide cascade control, as shown in Fig. 1. The reactor temperature controller is the primary controller; the jacket temperature controller is the secondary controller. The control system measures the jacket temperature, compares it to a setpoint, and uses the resulting error signal as the input to a controller for the cooling water makeup. The temperature setpoint and both measurements are used to adjust a single manipulated variable, the cooling water makeup rate. The principal advantage of the cascade control strategy is that a second measured variable is located close to a potential disturbance and its associated feedback loop can react quickly, thus improving the closed-loop response.

A cascade control scheme can be used to achieve better regulatory response than that existing in the slave loop; however, if a long time delay exists in the master loop, the cascade control may not give satisfactory closed-loop responses to setpoint changes. To overcome this problem, a Smith predictor scheme can be used in the outer loop of the cascade control system. Kaya [3] proposed a cascade control scheme combined with a Smith predictor for stable cascade processes with dominant time delay and achieved better control performance when compared

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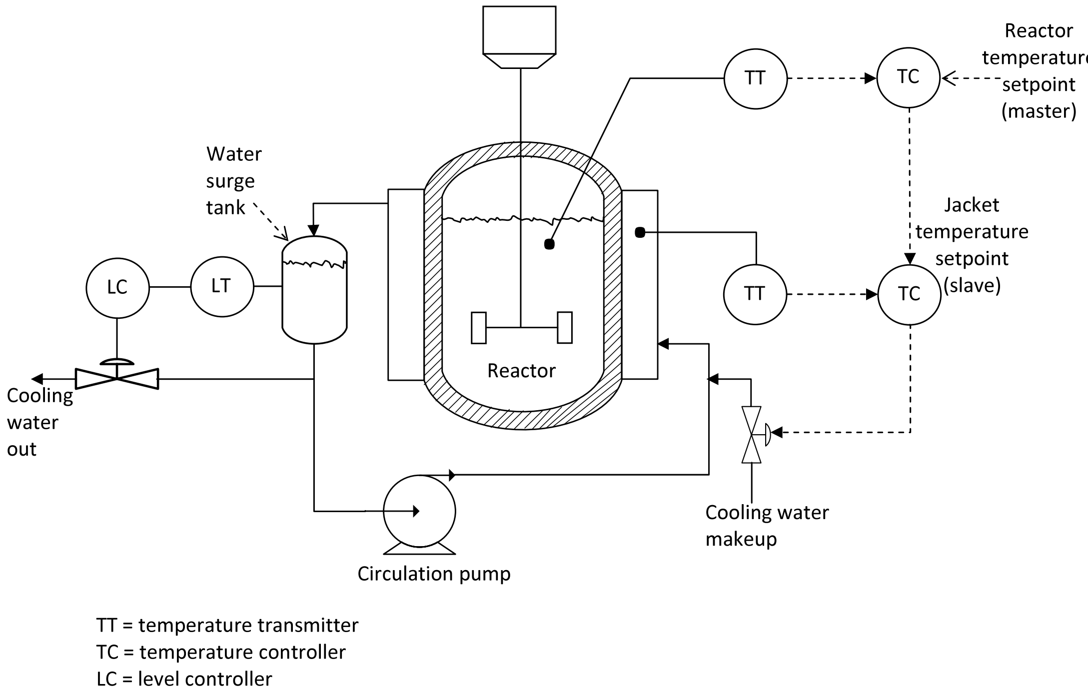


Fig. 1. A stirred chemical reactor with cascade control.

to some existing conventional PID tuning methods. Later on Kaya et al. [4] suggested an improved cascade control scheme for stable processes. The design and analysis of cascade control strategies for stable processes are addressed by many researchers [5–9]. However, limited research has been carried out for the design of cascade control strategies for unstable processes. The control of open-loop unstable processes are much more difficult than the control of stable processes. Liu et al. [10] proposed IMC based cascade control scheme for unstable processes with four controllers. Kaya and Atherton [11] proposed a cascade control structure for controlling unstable and integrating processes with four controllers. Very recently, Uma et al. [12] proposed an improved cascade control scheme for unstable processes using a modified Smith predictor with three controllers and one filter in the outer feedback path. In the work of Kaya and Atherton [11] and Liu et al. [10], one of the controllers is used only for stabilization and Uma et al.'s [12] scheme has one filter in the feedback path to improve the performance of the unstable processes. But in the proposed structure, there is no special controller for stabilization, rather the closed-loop controllers are used for rejecting the load disturbances as well as for stabilizing the unstable processes. Most of the methods discussed above involve many controllers with complex design methods. In practice, a simple control structure with fewer number of controllers is desirable.

This paper shows how to overcome the above deficiencies using a new cascade control structure with three controllers. Disturbance rejection in process industries is commonly much more important than setpoint tracking for many process control applications. This is because setpoint changes are often only made when the production rate is altered [13]. The proposed scheme leads to substantial control performance improvement, especially for the disturbance rejection. As the modified Smith predictor has been used in the outer loop of the cascade control structure, the proposed scheme takes advantage of the modified Smith predictor and the cascade control structure. Tuning rules are derived for the controllers used in the proposed structure for effective control of open-loop unstable plants. Internal stability, robustness and performance of the proposed method have been analyzed.

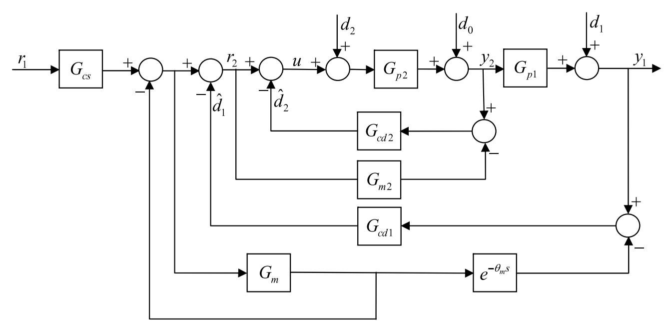


Fig. 2. Proposed cascade scheme with modified Smith predictor.

Simulation examples are provided to show how the proposed design method compares with recently reported methods.

For clear interpretation, the paper is organized as follows. Section 2 describes the proposed cascade control structure. In Section 3, the controller design methods are discussed followed by Section 4 in which the selection of tuning parameters is given. Section 5 deals with internal stability analysis. Robustness analysis and performance are discussed in Section 6. Simulation results are presented in Section 7 followed by the conclusions at the end.

2. Proposed cascade control scheme

A new cascade control structure is proposed for open-loop unstable processes as shown in Fig. 2 where a modified Smith predictor scheme is incorporated in the outer loop. $G_{p1}(=\tilde{G}_{p1}e^{-\theta_1 s})$ and $G_{p2}(=\tilde{G}_{p2}e^{-\theta_2 s})$ are the outer and the inner loop processes and $\tilde{G}_{p1}$ and $\tilde{G}_{p2}$ are the delay free parts and θ_1 and θ_2 are the time delays of G_{p1} and G_{p2}, respectively. $G_{m1}=\tilde{G}_{m1}e^{-\theta_{m1}s}$ is the primary process model and $G_{m2}=\tilde{G}_{m2}e^{-\theta_{m2}s}$ is the secondary process model in which $\tilde{G}_{m1}$ and $\tilde{G}_{m2}$ are the delay free parts. The overall process transfer function for the outer loop is

$$G_{po}=G_{p1}G_{p2}=\tilde{G}_{p1}\tilde{G}_{p2}e^{-(\theta_1+\theta_2)s}=G_pe^{-\theta_p s} \quad (1)$$

and the overall process model transfer function for the outer loop is given by

$$G_{mo} = G_{m1}G_{m2} = \tilde{G}_{m1}\tilde{G}_{m2}e^{-(\theta_{m1}+\theta_{m2})s} = G_m e^{-\theta_m s} \tag{2}$$

where $G_m = \tilde{G}_{m1}\tilde{G}_{m2}$ is the overall delay free part and $\theta_m = \theta_{m1} + \theta_{m2}$ is the overall time delay of the process model transfer function. The proposed scheme has three controllers: G_{cs}, G_{cd2} and G_{cd1}. The setpoint tracking controller G_{cs} is responsible for the overall servo performances, G_{cd2} is the inner loop disturbance rejection controller for rejecting the load disturbances entering into secondary process and G_{cd1} is the outer loop disturbance rejection controller for rejecting the load disturbances entering into the primary unstable process. Even though G_{cd2} and G_{cd1} are meant for load disturbance rejection, they also take part in stabilizing the unstable processes with time delay. It is to be noted that by virtue of the enhanced structure, the setpoint response is decoupled from the load disturbance response. The controllers G_{cd2} and G_{cd1} are assumed to have the forms (PID controller in series with first/second order lead/lag compensator)

$$G_{cd2} = K_{c2}\left(1 + \frac{1}{T_{i2}s} + T_{d2}s\right)\left(\frac{a_{f2}s^2 + a_{f1}s + 1}{b_{f2}s^2 + b_{f1}s + 1}\right) \tag{3}$$

and

$$G_{cd1} = K_{c1}\left(1 + \frac{1}{T_{i1}s} + T_{d1}s\right)\left(\frac{c_{f2}s^2 + c_{f1}s + 1}{d_{f2}s^2 + d_{f1}s + 1}\right) \tag{4}$$

where K_{c1} and K_{c2} are the proportional gains, T_{i1} and T_{i2} are the integral time constants and T_{d1} and T_{d2} are the derivative time constants of the PID controllers, and $a_{f1}, a_{f2}, b_{f1}, b_{f2}, c_{f1}, c_{f2}, d_{f1}$ and d_{f2} are the filter parameters.

The closed-loop transfer function of the master loop for servo response is given by

$$\frac{y_1}{r_1} = \frac{G_{cs}G_{p1}G_{p2}(1 + G_{m2}G_{cd2})(1 + G_m G_{cd1}e^{-\theta_m s})}{(1 + G_m)(1 + G_{p1}G_{p2}G_{cd1} + G_{p2}G_{cd2} + G_{m2}G_{cd1}G_{cd2}G_{p1}G_{p2})}. \tag{5}$$

Similarly, the closed-loop transfer function of the master loop for regulatory responses are given by

$$\frac{y_1}{d_2} = \frac{G_{p1}G_{p2}}{1 + G_{p2}G_{cd2} + G_{p1}G_{p2}G_{cd1} + G_{m2}G_{cd1}G_{cd2}G_{p1}G_{p2}} \tag{6}$$

$$\frac{y_1}{d_0} = \frac{G_{p1}}{1 + G_{p2}G_{cd2} + G_{p1}G_{p2}G_{cd1} + G_{m2}G_{cd1}G_{cd2}G_{p1}G_{p2}} \tag{7}$$

$$\frac{y_1}{d_1} = \frac{1 + G_{p2}G_{cd2}}{1 + G_{p2}G_{cd2} + G_{p1}G_{p2}G_{cd1} + G_{m2}G_{cd1}G_{cd2}G_{p1}G_{p2}}. \tag{8}$$

Assuming exact matching between the process and the model parameters, (5)–(8) reduce to

$$\frac{y_1}{r_1} = \frac{G_{cs}G_{p1}G_{p2}}{1 + G_m} \tag{9}$$

$$\frac{y_1}{d_2} = \frac{G_{p1}G_{p2}}{(1 + G_{p2}G_{cd2})(1 + G_{p1}G_{p2}G_{cd1})} \tag{10}$$

$$\frac{y_1}{d_0} = \frac{G_{p1}}{(1 + G_{p2}G_{cd2})(1 + G_{p1}G_{p2}G_{cd1})} \tag{11}$$

and

$$\frac{y_1}{d_1} = \frac{1}{1 + G_{p1}G_{p2}G_{cd1}}. \tag{12}$$

From (9)–(12), it is concluded that the new cascade structure has decoupled the regulatory response from the servo response for nominal systems which is an important feature of the proposed control scheme.

3. Controller design procedures

Generally, in process control the dynamics of inner loop process are stable while the outer loop process dynamics may be stable or unstable. Hence, the dynamics of the inner loop process and the corresponding process model can be represented by

$$G_{p2} = \frac{k_2 e^{-\theta_2 s}}{\tau_2 s + 1} \tag{13}$$

and

$$G_{m2} = \frac{k_{m2}e^{-\theta_{m2}s}}{\tau_{m2}s + 1} \tag{14}$$

where k_2 and k_{m2} are the steady state gains, θ_2 and θ_{m2} are the time delays and τ_2 and τ_{m2} are the time constants of the secondary loop process and the process model, respectively.

Similarly, the dynamics of the outer loop process and the corresponding process model can be represented by

$$G_{p1} = \frac{k_1 e^{-\theta_1 s}}{\tau_1 s - 1} \tag{15}$$

and

$$G_{m1} = \frac{k_{m1}e^{-\theta_{m1}s}}{\tau_{m1}s - 1} \tag{16}$$

where k_1 and k_{m1} are the steady state gains, θ_1 and θ_{m1} are the time delays and τ_1 and τ_{m1} are the time constants of the primary loop process and the process model, respectively. An analytical procedure is presented for determining parameters of the disturbance rejection controllers and the setpoint tracking controller is designed using the direct synthesis method. Design of the three controllers (G_{cs}, G_{cd2} and G_{cd1}) are explained in the following.

3.1. Design of slave loop controller G_{cd2}

In order to design the load disturbance rejection controllers analytically and independently, the inner loop and outer loop dynamics are considered separately. From Fig. 2, the nominal regulatory response transfer functions for the inner loop are

$$H_{d2}(s) = \frac{y_2}{d_2} = \frac{G_{p2}}{1 + G_{cd2}G_{p2}} \tag{17}$$

$$H_{d0}(s) = \frac{y_2}{d_0} = \frac{1}{1 + G_{cd2}G_{p2}}. \tag{18}$$

The nominal complementary sensitivity function of the inner loop for disturbance rejection can be obtained as

$$T_{d_{\text{slave}}} = \frac{\hat{d}_2}{d_2} = \frac{G_{cd2}G_{p2}}{1 + G_{cd2}G_{p2}}. \tag{19}$$

In order to reject step load disturbances injected into the secondary process input, the asymptotic constraint

$$\lim_{s \to 1/\tau_2}\left(1 - T_{d_{\text{slave}}}\right) = 0 \tag{20}$$

should be satisfied so that the closed-loop internal stability can be achieved. According to robust IMC theory [14], the desired closed-loop complementary sensitivity function is proposed as

$$T_{d_{\text{slave}}} = \frac{\alpha_2 s + 1}{(\lambda_2 s + 1)^2}e^{-\theta_2 s} \tag{21}$$

where λ_2 is a tuning parameter for obtaining the desirable regulatory performance of the inner loop. Now substituting (21) in (20), gives

$$\lim_{s\to 1/\tau_2}\left[1-\frac{\alpha_2 s+1}{(\lambda_2 s+1)^2}e^{-\theta_2 s}\right]=0. \quad (22)$$

By following a simple calculation, we get

$$\alpha_2=\tau_2\left[\left(1+\frac{\lambda_2}{\tau_2}\right)^2 e^{\theta_2/\tau_2}-1\right]. \quad (23)$$

Using (19), the inner loop controller G_{cd2} can be obtained as

$$G_{cd2}=\frac{T_{d_{\mathrm{slave}}}}{1-T_{d_{\mathrm{slave}}}}\frac{1}{G_{p2}}. \quad (24)$$

Substituting (13) and (21) into (24), we get

$$G_{cd2}=\frac{(\alpha_2 s+1)(\tau_2 s+1)}{k_2[(\lambda_2 s+1)^2-(\alpha_2 s+1)e^{-\theta_2 s}]}. \quad (25)$$

Due to the symmetrical pole-zero configuration, the phase of the standard Padé approximation ($R_{n,n}(s)$) goes to $-n\pi$ and its amplitude remains constant at all frequencies. On the other hand, the step-response of $R_{n,n}(s)$ exhibits a jump at $t=0$ which is not very desirable. To avoid this phenomenon the Padé approximation $R_{n-1,n}(s)$ where the numerator's degree is one less than that of the denominator has been suggested. This gives a better approximation of the step-response [15]. Using $R_{n-1,n}(s)$ (where $n=2$) Padé approximation for time delay term of (25) i.e. $e^{-\theta_2 s}=\frac{6-2\theta_2 s}{6+4\theta_2 s+\theta_2^2 s^2}$, now (25) becomes

$$G_{cd2}=\frac{(\alpha_2 s+1)(\tau_2 s+1)(6+4\theta_2 s+\theta_2^2 s^2)}{k_2[(\lambda_2 s+1)^2(6+4\theta_2 s+\theta_2^2 s^2)-(\alpha_2 s+1)(6-2\theta_2 s)]}. \quad (26)$$

Simplifying (26), we get

$$G_{cd2}=\frac{6+4\theta_2 s+\theta_2^2 s^2}{k_2 x_1 s}\times\frac{(\alpha_2 s+1)(\tau_2 s+1)}{1+p_1 s+p_2 s^2+p_3 s^3} \quad (27)$$

where

$x_1=6\theta_2+12\lambda_2-6\alpha_2,\quad x_2=6\lambda_2^2+8\lambda_2\theta_2+\theta_2^2+2\alpha_2\theta_2,$
$x_3=4\lambda_2^2\theta_2+2\lambda_2\theta_2^2,\quad x_4=\lambda_2^2\theta_2^2,\quad p_1=x_2/x_1,$
$p_2=x_3/x_1$ and $p_3=x_4/x_1$.

Comparing (27) and (3), the parameters of the slave loop controller G_{cd2} are expressed as

$$\begin{cases} K_{c2}=-4\theta_2/k_2x_1, & T_{i2}=2\theta_2/3, \quad T_{d2}=\theta_2/4 \\ a_{f1}=\tau_2+\alpha_2, & a_{f2}=\tau_2\alpha_2, \quad b_{f1}=-p_1, \\ b_{f2}=-p_2. \end{cases} \quad (28)$$

It is to be noted that all the parameters in (28) become positive since the expression of x_1 gives a negative value (see Appendix A).

3.2. *Design of master loop controller G_{cd1}*

From (12), the nominal regulatory response transfer function of the outer loop is given by

$$H_{d1}(s)=\frac{y_1}{d_1}=\frac{1}{1+G_{cd1}G_{p1}G_{p2}}. \quad (29)$$

The nominal complementary sensitivity function of the master loop for disturbance rejection is

$$T_{d_{\mathrm{master}}}=\frac{\hat{d}_1}{d_2}=\frac{G_{cd1}G_{p1}G_{p2}}{1+G_{cd1}G_{p1}G_{p2}}. \quad (30)$$

Like the previous subsection, an asymptotic constraint in order to reject the load disturbances injected into the primary process is

$$\lim_{s\to 1/\tau_1}(1-T_{d_{\mathrm{master}}})=0. \quad (31)$$

The desired closed-loop complementary sensitivity function is proposed as

$$T_{d_{\mathrm{master}}}=\frac{\alpha_1 s+1}{(\lambda_1 s+1)^3}e^{-\theta_m s} \quad (32)$$

where $\theta_m=\theta_1+\theta_2$ and λ_1 is a tuning parameter for obtaining the desirable regulatory performance of the outer loop. Substitution of (32) in (31) results in

$$\lim_{s\to 1/\tau_1}\left[1-\frac{\alpha_1 s+1}{(\lambda_1 s+1)^3}e^{-\theta_m s}\right]=0 \quad (33)$$

which upon simplification gives

$$\alpha_1=\tau_1\left[\left(\frac{\lambda_1}{\tau_1}+1\right)^3 e^{\theta_m/\tau_1}-1\right]. \quad (34)$$

Using (30), expression for the outer loop controller G_{cd1} can be written as

$$G_{cd1}=\frac{T_{d_{\mathrm{master}}}}{1-T_{d_{\mathrm{master}}}}\frac{1}{G_{p1}G_{p2}}. \quad (35)$$

Hence, from (13), (15), (32) and (35), the expression for the master loop controller G_{cd1} becomes

$$G_{cd1}=\frac{(\alpha_1 s+1)(\tau_1 s-1)(\tau_2 s+1)}{k_1k_2[(\lambda_1 s+1)^3-(\alpha_1 s+1)e^{-\theta_m s}]}. \quad (36)$$

Using Padé approximation $R_{n-1,n}(s)$ (where $n=2$) for the time delay term of (36),

$$G_{cd1}=\frac{(\alpha_1 s+1)(\tau_1 s-1)(\tau_2 s+1)(6+4\theta_m s+\theta_m^2 s^2)}{k_1k_2[(\lambda_1 s+1)^3(6+4\theta_m s+\theta_m^2 s^2)-(\alpha_1 s+1)(6-2\theta_m s)]}. \quad (37)$$

By following a simple calculation, we have

$$G_{cd1}=-\frac{6+4\theta_m s+\theta_m^2 s^2}{k_1k_2m_1 s}\times\frac{(\alpha_1 s+1)(\tau_2 s+1)}{(1+l_1 s+l_2 s^2+l_3 s^3+l_4 s^4)/(-\tau_1 s+1)} \quad (38)$$

where

$m_1=6\theta_m+18\lambda_1-6\alpha_1,$
$m_2=2\alpha_1\theta_m+\theta_m^2+12\lambda_1\theta_m+18\lambda_1^2,$
$m_3=3\lambda_1\theta_m^2+12\lambda_1^2\theta_m+6\lambda_1^3,\quad m_4=3\lambda_1^2\theta_m^2+4\lambda_1^3\theta_m,$
$m_5=\lambda_1^3\theta_m^2,\quad l_1=m_2/m_1,\quad l_2=m_3/m_1,$
$l_3=m_4/m_1$ and $l_4=m_5/m_1$.

Since the resulting controller does not have a standard PID controller form, the remaining issue is to design the PID controller that approximates the controller most closely. Therefore, the following procedure is employed to produce the desired disturbance estimator in a simple way. The filter parameters d_{f1} and d_{f2} can be determined from the second part of the denominator of (38) i.e. $(1+l_1s+l_2s^2+l_3s^3+l_4s^4)/(-\tau_1s+1)$. d_{f1} can be obtained by taking first derivative of the above term and setting $s=0$ i.e.

$$\frac{d}{ds}\left(d_{f2}s^2+d_{f1}s+1\right)\Big|_{s=0}=\frac{d}{ds}\left(\frac{1+l_1s+l_2s^2+l_3s^3+l_4s^4}{-\tau_1 s+1}\right)\Big|_{s=0}.$$

Similarly, taking second derivative and substituting $s = 0$ give the expression for d_{f2}. The filter parameters d_{f1} and d_{f2} can also be obtained using the Taylor series expansion as given in Appendix C.

(38) and (4) gives the parameters of master loop controller G_{cd1} as

$$\begin{cases} K_{c1} = -4\theta_m/k_1k_2m_1, \quad T_{i1} = 2\theta_m/3, \quad T_{d1} = \theta_m/4 \\ c_{f1} = \tau_2 + \alpha_1, \quad c_{f2} = \tau_2\alpha_1, \quad d_{f1} = l_1 + \tau_1, \\ d_{f2} = l_2 + \tau_1 d_{f1}. \end{cases} \tag{39}$$

It is to be noted that in (39) all the controller parameters become positive as the expression of m_1 gives a negative value (see Appendix B).

3.3. Design of setpoint tracking controller G_{cs}

In a feedback system with a stable/unstable process with time delay, often the setpoint response transfer function is chosen in the form of low pass filter with time delay for a unit step setpoint. In order to obtain good setpoint response for the nominal system, the setpoint tracking controller is designed to be rational and stable. G_{cs} is designed for the trajectory problem. The idea of direct synthesis is to specify the desired closed-loop response and solve for the corresponding controller. We use the following transfer function to obtain the desired response

$$H_{r1}(s) = \frac{1}{(\lambda_{cs}s + 1)^2} e^{-\theta_m s} \tag{40}$$

where $\theta_m = \theta_1 + \theta_2$, and λ_{cs} is the time constant of setpoint controller, or equivalently

$$h_{r1}(t) = \begin{cases} 0, & t < \theta_m \\ 1 - \left(1 + \dfrac{t}{\lambda_{cs}}\right) e^{-(t-\theta_m)/\lambda_{cs}}, & t \geq \theta_m. \end{cases} \tag{41}$$

(41) corresponds to a smooth setpoint response with no overshoot.

Using (9) and (40), the setpoint tracking controller can be obtained as

$$G_{cs}(s) = \frac{\tau_1\tau_2 s^2 + (\tau_1 - \tau_2)s + k_1k_2 - 1}{k_1k_2(\lambda_{cs}s + 1)^2}. \tag{42}$$

When the tuning parameter λ_{cs} tends to zero, $G_{cs}(s)$ becomes optimal. λ_{cs} is an adjustable closed-loop design parameter that is always positive. A fast speed of response is favored by a small value of λ_{cs}, and robustness is favored by a large value of λ_{cs}. Hence, there is a tradeoff in selecting the tuning parameter λ_{cs}. Thus, λ_{cs} should be selected such that there is a good compromise between the speed of response and the robustness. Guidelines for selection of the tuning parameters (λ_{cs}, λ_1 and λ_2) are presented in Section 4.

4. Selection of tuning parameters λ_{cs}, λ_2 and λ_1

The setpoint controller tuning parameter λ_{cs} is selected so as to achieve good performance and robustness. The response speed is determined by the parameter λ_{cs}. The choice of a higher value of λ_{cs} slows down the system and makes it more robust while small values of λ_{cs} may cause instability of the cascade control structure in the presence of unmodeled dynamics. $\lambda_{cs} > 0$ is an adjustable closed-loop design parameter whose initial value is typically chosen as half of the overall process model time delay. If good control performances are not achieved with this value, then λ_{cs} can be varied gradually from this value till good nominal and robust control performances are achieved. Quantitatively, the suggested range of λ_{cs} is $0.1\theta_m - \theta_m$. As the secondary controller mainly takes care of the regulatory performance of the inner loop and the primary controller takes care of the outer loop regulatory performance, λ_2 and λ_1 can be selected independently. The tuning of the control parameters λ_2 and λ_1 aims at the best trade-off between nominal performance of the closed-loop and its robust stability. That is, decreasing λ_2 and λ_1 improves the disturbance rejection performance of the closed-loop but degrades its robust stability in the presence of process uncertainty. In contrast, increasing λ_2 and λ_1 tends to strengthen the robust stability of the closed-loop but degrades its disturbance rejection performance. On the basis of extensive simulation studies based on the MATLAB toolbox, it is observed that the initial value of λ_2 is twice of the inner loop process model time delay and that of λ_1 is equal to overall process model time delay. If good control performances are not achieved with these values, then the tuning parameters can be varied gradually from these values till good nominal and robust control performances are achieved. The suggested range of tuning parameters are $\lambda_2 = 1.5\theta_{m2} - 3.6\theta_{m2}$ and $\lambda_1 = 0.5\theta_m - 1.5\theta_m$.

For clarity, the details of tuning procedure is summarized below.

I. With the known secondary process model (G_{m2}), select the tuning parameter λ_2 in the range of $1.5\theta_{m2} - 3.6\theta_{m2}$ and find the secondary loop disturbance rejection controller (G_{cd2}) parameters using (28).
II. Then from the known primary process (G_{p1}), obtain the overall primary loop process model (G_m) and overall primary loop time delay (θ_m) from (2).
III. Find the primary loop disturbance rejection controller (G_{cd1}) parameters from (39) after selecting the tuning parameter λ_1 in the range of $0.5\theta_m - 1.5\theta_m$.
IV. Then determine the setpoint tracking controller (G_{cs}) using (42) after selecting the tuning parameter λ_{cs} in the range of $0.1\theta_m - \theta_m$.

5. Internal stability analysis

A system with unstable pole-zero cancellations renders internally unstable closed-loop system. The input/output stability analysis may not be sufficient for the practical control system analysis and design. Therefore, the internal stability analysis is the basic requirement for any interconnected control systems. A linear time-invariant interconnected system consisting of single input and single output (SISO) plants is internally stable if and only if

$$p_c(s) = \Delta \prod_{i=1}^{n} p_i(s) \tag{43}$$

has all its roots in the left half of the complex plane [16]. In (43), Δ is the determinant of the systems as defined in Mason's gain formula for signal flow graph analysis i.e. $\Delta = 1 - \sum_i L_{1i} + \sum_j L_{2j} - \sum_k L_{3k} + \cdots$, ($L_{1i}$ is the loop gain, L_{2j} is the product of two non-touching loop gains and L_{3k} is the product of three non-touching loop gains), n is the number of subsystems, and $p_i(s)$ is the characteristic polynomial of the respective subsystem transfer function.

The proposed control scheme has eight subsystems such as $G_{cs}, G_{cd1}, G_{cd2}, G_{p1}, G_{p2}, G_{m2}, G_m$ and $e^{-\theta_m s}$. Let $G_{cs} = f_1(s)/g_1(s)$, $G_{cd1} = f_2(s)/g_2(s)$, $G_{cd2} = f_3(s)/g_3(s)$ and $G_{p1} = G_{p2} = G_{m2} = \delta(s)/\gamma(s)$ be coprime polynomial fractions. Their respective characteristic polynomials are $p_1 = g_1(s), p_2 = g_2(s), p_3 = g_3(s), p_4 = p_5 = p_6 = \gamma(s), p_7 = \gamma^2(s)$ and $p_8 = 1$.

Using (43), $p_c(s)$ can be expressed as

$$\begin{aligned} p_c(s) &= \Delta \prod_{i=1}^{8} p_i(s) \\ &= [\gamma^2(s) + \delta^2(s)][\gamma(s)g_3(s) + \delta(s)f_3(s)][\gamma^2(s)g_2(s) \\ &\quad + \delta^2(s)f_2(s)]g_1(s). \end{aligned} \tag{44}$$

The stabilization of G_{p2} depends on the polynomial $[\gamma(s)g_3(s) + \delta(s)f_3(s)]$. The slave loop controller G_{cd2} must be stable for the stability of $g_3(s)$ and is used to achieve best disturbance response. Similarly, the stabilization of the primary process G_{p1} depends on the polynomial $[\gamma^2(s)g_2(s) + \delta^2(s)f_2(s)]$. In order to achieve the best disturbance response, the master loop controller G_{cd1} must be stable for the stability of $g_2(s)$. The controller G_{cs} must be stable for the stability of $g_1(s)$ in order to achieve the best setpoint response. With these constraints, the overall system is internally stable if and only if G_{p1} and G_{p2} are stabilized by the controllers G_{cd1} and G_{cd2} respectively.

6. Robustness analysis and performance

6.1. Robustness analysis

Since no mathematical model of a system will be a perfect representation of the actual system, a study of robustness analysis is an important task in control system design. There are two most important adverse effects of modeling uncertainties such as degradation of the system performance and destabilization. In practical situations, there are at least two types of uncertainties. First, unmodeled dynamics which represent high frequency uncertainties, and second, parametric uncertainty representing lack of precise knowledge of the actual system parameters. Recent developments in the analysis of systems with parametric uncertainty have been inspired by Kharitonov's theorem. Kharitonov's theorem is probably the most well-known and simplest tool for robust stability analysis [17–21].

Interval polynomial may be written as

$$\Gamma = \left\{ p(s,q) | p(s,q) = \sum_{i=0}^{n} q_i s^i,\ q_i \in [q_i^-, q_i^+],\ i = 0, 1, \ldots, n \right\} \tag{45}$$

where q_i^- and q_i^+ represent lower and upper bounds for the continuous set q_i. Kharitonov's theorem states that $p(s,q) = 0$ is robustly stable if and only if the following four Kharitonov's polynomials are stable

$$\begin{aligned}
K_1(s) &= q_0^- + q_1^- s + q_2^+ s^2 + q_3^+ s^3 + q_4^- s^4 + q_5^- s^5 \\
&\quad + q_6^+ s^6 + q_7^+ s^7 + \cdots \\
K_2(s) &= q_0^+ + q_1^+ s + q_2^- s^2 + q_3^- s^3 + q_4^+ s^4 + q_5^+ s^5 \\
&\quad + q_6^- s^6 + q_7^- s^7 + \cdots \\
K_3(s) &= q_0^+ + q_1^- s + q_2^- s^2 + q_3^+ s^3 + q_4^+ s^4 + q_5^- s^5 \\
&\quad + q_6^- s^6 + q_7^+ s^7 + \cdots \\
K_4(s) &= q_0^- + q_1^+ s + q_2^+ s^2 + q_3^- s^3 + q_4^- s^4 + q_5^+ s^5 \\
&\quad + q_6^+ s^6 + q_7^- s^7 + \cdots.
\end{aligned}$$

The geometric meaning of this theorem is that if the origin point of the complex plane is outside the boundary of the Kharitonov rectangles then the system is robustly stable. If the family of nth-degree polynomials is robustly stable, the value sets will move in a counter-clockwise direction through n quadrants of the complex plane without passing through or touching the origin of the plane. The analytical meaning of this theorem is that the robust stability of an interval polynomial family can be determined by forming the four Kharitonov polynomials, factoring them, and examining the roots. If all the roots are in the left-half plane, then the family is robustly stable. If any of those polynomials has roots on the imaginary axis or in the right-half plane, the family is not robustly stable.

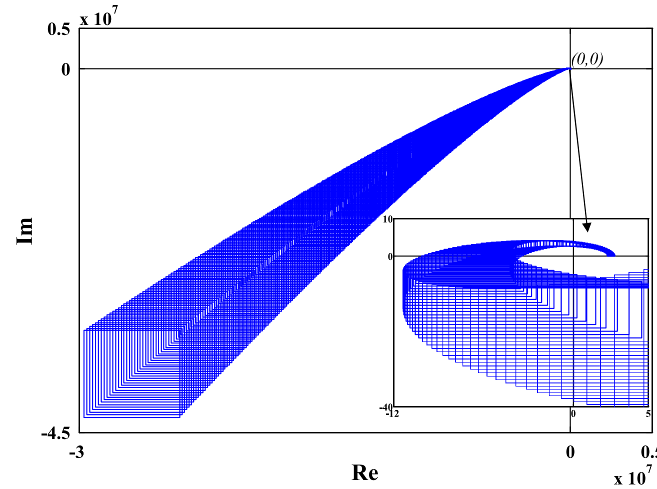


Fig. 3. Kharitonov's rectangle (nominal system) for $G_{p1} = e^{-0.339s}/(5s-1)$ and $G_{p2} = e^{-0.6s}/(2.07s+1)$.

For example, consider a system with the following outer and inner loop plant transfer functions $G_{p1} = \frac{e^{-0.339s}}{5s-1}$ and $G_{p2} = \frac{e^{-0.6s}}{2.07s+1}$.

From (29), the closed-loop characteristic equation of the outer loop is given by

$$1 + G_{cd1}G_{p1}G_{p2} = 0.$$

Now, the interval polynomial of this system is given by

$$\begin{aligned}
p(s,q) &= [2.2750\ 2.7806] + [14.3827\ 17.5789]s \\
&\quad + [36.9524\ 45.1640]s^2 + [50.9606\ 62.2852]s^3 \\
&\quad + [41.8061\ 51.0963]s^4 + [21.2444\ 25.9654]s^5 \\
&\quad + [5.8279\ 7.1229]s^6 + [0.7445\ 0.9099]s^7.
\end{aligned} \tag{46}$$

The four Kharitonov polynomials are obtained as

$$\begin{aligned}
K_1(s) &= 2.2750 + 14.3827s + 45.1640s^2 + 62.2852s^3 \\
&\quad + 41.8061s^4 + 21.2444s^5 + 7.1229s^6 + 0.9099s^7 \\
K_2(s) &= 2.7806 + 17.5789s + 36.9524s^2 + 50.9606s^3 \\
&\quad + 51.0963s^4 + 25.9654s^5 + 5.8279s^6 + 0.7445s^7 \\
K_3(s) &= 2.7806 + 14.3827s + 36.9524s^2 + 62.2852s^3 \\
&\quad + 51.0963s^4 + 21.2444s^5 + 5.8279s^6 + 0.9099s^7 \\
K_4(s) &= 2.2750 + 17.5789s + 45.1640s^2 + 50.9606s^3 \\
&\quad + 41.8061s^4 + 25.9654s^5 + 7.1229s^6 + 0.7445s^7.
\end{aligned}$$

The coefficients of Kharitonov polynomials ($K_1(s)$, $K_2(s)$, $K_3(s)$ and $K_4(s)$) are checked for the Hurwitz condition. It is observed that all the roots of the Kharitonov polynomials (Table 1) have negative real part i.e. all roots are in the left half of the complex plane. The Kharitonov rectangles of the closed-loop system for nominal and perturbed systems are shown in Figs. 3 and 4 respectively. From Figs. 3 and 4, it is clear that Kharitonov rectangles move about the origin in counterclockwise sense in order to have the monotonic phase increase property of Hurwitz polynomials. The graphs are zoomed to show what is happening in the neighborhood of the point $(0, 0)$. It is observed that the closed-loop characteristic equation of the system is stable in complex plane in Mikhailov's sense. Since the origin is excluded from the Kharitonov rectangles (Figs. 3 and 4) it is concluded that the closed-loop control system is robustly stable. By following a similar procedure as above the robustness analysis for other processes can be checked. Fig. 5 shows the Kharitonov rectangles for the closed-loop system for $G_{p1} = e^{-4s}/(20s-1)$ and $G_{p2} = 2e^{-2s}/(20s+1)$.

Table 1
The roots of Kharitonov polynomials.

$K_1(s)$	$K_2(s)$	$K_3(s)$	$K_4(s)$
-3.9185	$-2.4605 + j3.3704$	$-0.8300 + j2.7856$	$-3.8328 + j1.7493$
-2.3321	$-2.4605 - j3.3704$	$-0.8300 - j2.7856$	$-3.8328 - j1.7493$
$-0.1947 + j1.8466$	-1.6326	-2.6051	$-0.0634 + j1.2189$
$-0.1947 - j1.8466$	$-0.1122 + j0.7888$	-1.3674	$-0.0634 - j1.2189$
-0.7410	$-0.1122 - j0.7888$	$-0.1780 + j0.4606$	-1.1263
$-0.2236 + j0.2390$	-0.7869	$-0.1780 - j0.4606$	-0.3750
$-0.2236 - j0.2390$	-0.2630	-0.4165	-0.2735

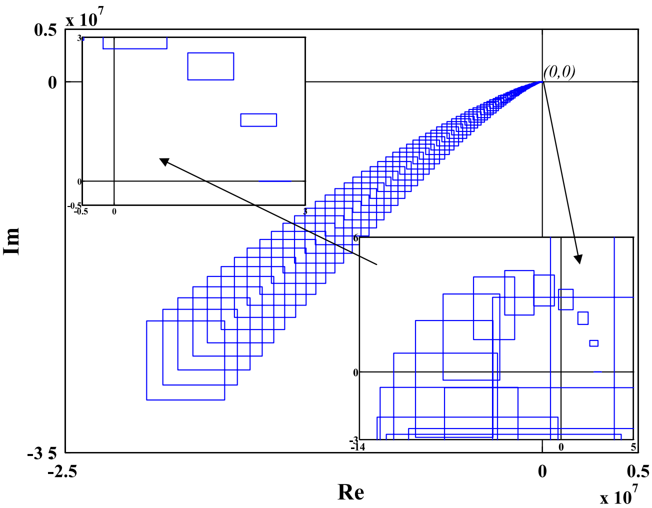


Fig. 4. Kharitonov's rectangle for $+10\%$ perturbation in both time delays and -10% in both time constants for $G_{p1} = e^{-0.339s}/(5s-1)$ and $G_{p2} = e^{-0.6s}/(2.07s+1)$.

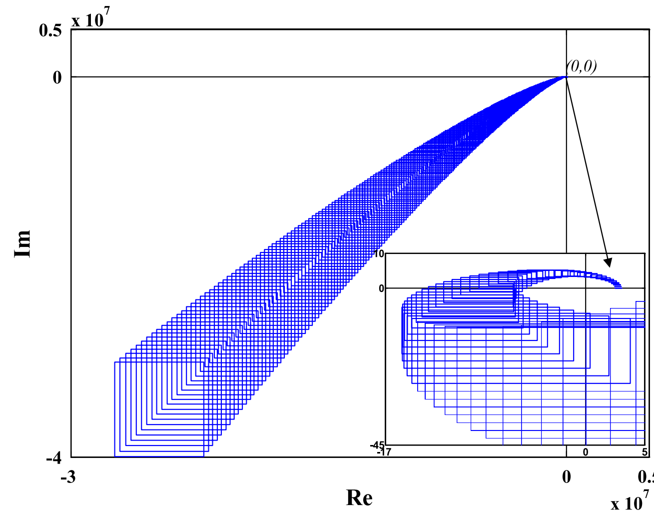


Fig. 5. Kharitonov's rectangle (nominal system) for $G_{p1} = e^{-4s}/(20s-1)$ and $G_{p2} = 2e^{-2s}/(20s+1)$.

6.2. Performance

To evaluate the closed-loop performance, we consider two popular performance specifications based on integral error such as the integral absolute error, IAE $= \int_0^\infty |e(t)|\,dt$ and the integral square error, ISE $= \int_0^\infty e(t)^2 dt$, where $e = r - y$.

To evaluate the manipulated input, we compute the total variation (TV) of the input $u(t)$ i.e.TV $= \sum_{i=1}^{\infty} |u_{i+1} - u_i|$, which should be as small as possible. The TV is a good measure of smoothness of a signal [22].

7. Simulation study

The results of simulations of three examples to illustrate the value of the proposed cascade control design method are given in this section.

Example 1. Consider the plant parameters of the master and slave loops studied by Liu et al. [10] where $k_1 = 1, \tau_1 = 5, \theta_1 = 0.339$ and $k_2 = 1, \tau_2 = 2.07, \theta_2 = 0.6$. Choosing the tuning parameter $\lambda_{cs} = 0.852\theta_m = 0.8$, the setpoint tracking controller becomes $G_{cs} = \frac{10.35s^2+2.93s}{0.64s^2+1.6s+1}$. The parameters of G_{cd2} are $a_{f1} = 3.9850, a_{f2} = 3.9641, b_{f1} = 1.9558, b_{f2} = 0.2294, K_{c2} = 0.5337, T_{i2} = 0.2260$ and $T_{d2} = 0.0848$ by selecting the tuning parameter $\lambda_2 = 1.7\theta_{m2} = 0.5763$. The parameters of G_{cd1} are obtained as $c_{f1} = 7.4950, c_{f2} = 11.2297, d_{f1} = 0.4757, d_{f2} = 0.1448, K_{c1} = 0.4213, T_{i1} = 0.6260$ and $T_{d1} = 0.2348$ by selecting $\lambda_1 = 1.065\theta_m = 1$. Uma et al. [12] have shown the superiority of their control scheme over Liu et al.'s [10] and Kaya and Atherton [11] schemes. In the case of Uma et al.'s method, the parameters for setpoint tracking controllers are $k_{cs} = 9.1554, k_{is} = 2.8953, k_{ds} = 4.4519, \beta_s = 0.2373$ and the tuning parameters are $\lambda_s = 0.7042$ and $\lambda_2 = 0.6$. The disturbance rejection controller parameters are $k_{cd} = 4.5867, k_{id} = 0.8488, k_{dd} = 2.4639, \alpha_d = 0.4695$ and $\beta_d = 0.022$. The setpoint weighting and filter parameters are $\varepsilon = 0.3$ and $\tau_f = 5.6340$, respectively. A unit-step setpoint is introduced at time $t = 0$, a unit negative step input load disturbance d_2 at time $t = 20$ s and a load disturbance $d_1 = -0.2$ at time $t = 40$ s.

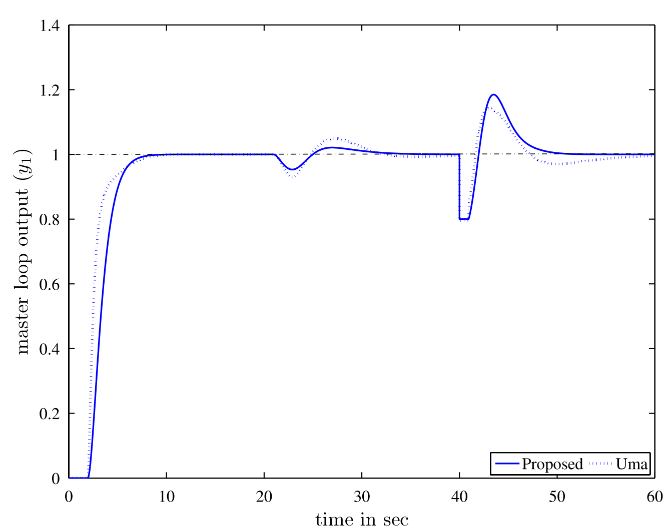


Fig. 6. Nominal responses for Example 1.

The closed-loop nominal responses for all the above controller settings are shown in Fig. 6. From the responses, it is observed that the proposed method gives almost similar setpoint response when compared with the other method. But particularly for disturbance rejection, the proposed method gives less settling time and comparatively superior performances to Uma et al.'s method.

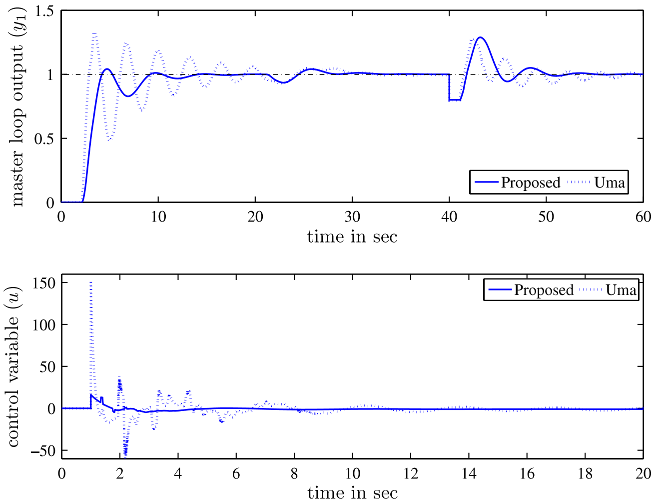


Fig. 7. +20% perturbation in both the time delays and −20% in both the time constants for Example 1.

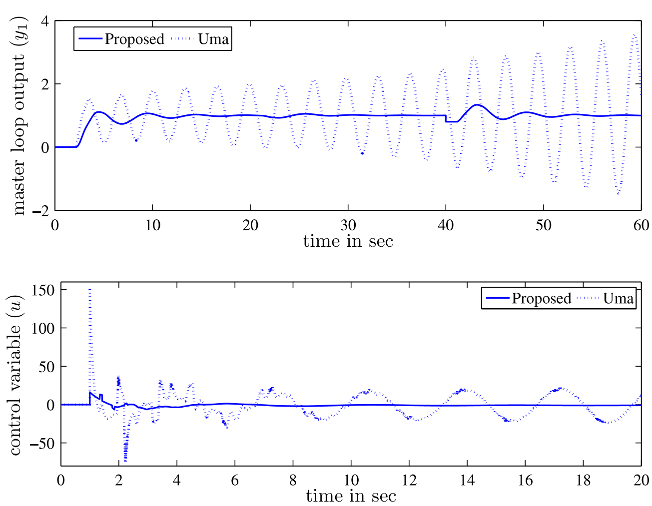


Fig. 8. +25% perturbation in both the time delays and −25% in both the time constants for Example 1.

In the present work, to illustrate the robustness to parameter variations, +20% perturbations in both the process time delays and −20% in both the process time constants are considered and corresponding responses are shown in Fig. 7. From Fig. 7, it is seen that the proposed cascade control structure gives improved responses both in setpoint as well as load disturbance rejection. Fig. 8 shows the responses for +25% perturbations in both the time delays and −25% in both the time constants. In Fig. 8, it is observed that Uma et al.'s method gives unbounded response. Also, it is seen from Figs. 7, 8 and Table 3 that the proposed method gives smooth control signal as compared to Uma et al.'s method. From Table 2, it is observed that the proposed control scheme gives low value of performance specifications.

Example 2. Consider a system with the following master and slave loop plant transfer functions $G_{p1} = \frac{e^{-4s}}{20s-1}$ and $G_{p2} = \frac{2e^{-2s}}{20s+1}$, respectively, studied by Liu et al. [10] and Uma et al. [12]. For the proposed method, taking $\lambda_{cs} = 0.334\theta_m = 2$ results in the setpoint controller of the form $G_{cs} = \frac{400s^2+1}{8s^2+8s+2}$ using (42). The parameters of G_{cd2} are $a_{f1} = 31.8289$, $a_{f2} = 236.5784$, $b_{f1} = 19.2568$, $b_{f2} = 14.5805$, $K_{c2} = 0.3645$, $T_{i2} = 1.3333$ and $T_{d2} = 0.5$ by selecting $\lambda_2 = 2\theta_{m2} = 4$. The parameters of G_{cd1} are obtained as $c_{f1} = 46.6511$, $c_{f2} = 533.0224$, $d_{f1} = 2.0483$, $d_{f2} = 3.0520$, $K_{c1} = 0.2312$, $T_{i1} = 4$ and $T_{d1} = 1.5$ by selecting $\lambda_1 = 0.667\theta_m = 4$. In Liu et al.'s method, the controllers are $P_c = 1 + s$, $C(s) = (400s^2 + 2s + 1)/2(6s + 1)^2$, $F_{10}(s) = (20s + 1)/2(0.5s + 1)$ and $F_2(s) = 1.9785 + (1/30.6256s) + 28.0736s$. In Uma et al.'s method, the settings are given by $k_{cs} = 4.6571$, $k_{is} = 0.1829$, $k_{ds} = 12.2857$, $\beta_s = 2.8571$, $k_{cd} = 3.1190$, $k_{id} = 0.0921$, $k_{dd} = 6.6156$, $\alpha_d = 3$, $\beta_d = 0.1440$, $\varepsilon = 0.3$ and $\tau_f = 36$.

Table 2
Performance specifications of regulatory responses.

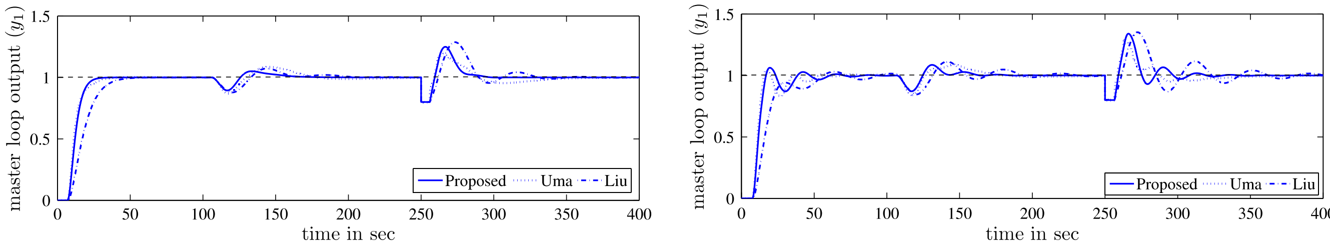
	Scheme	Nominal system		Perturbed system	
		IAE	ISE	IAE	ISE
Example 1	Proposed	1.115	0.1045	1.708	0.2581
	Uma	1.276	0.1105	4.141	0.6681
Example 2	Proposed	7.123	0.9748	8.751	1.213
	Uma	11.14	1.082	11.48	1.258
	Liu	11.83	1.689	16.68	2.477
Example 3	Proposed	6.258	1.277	7.236	2.102
	Uma	6.22	1.163	7.829	2.125
	Kaya	7.976	2.005	11	3.756

Table 3
Performance specifications of manipulated input.

	Scheme	Nominal system TV	Perturbed system TV
Example 1	Proposed	35.245	74.618
	Uma	90.66	1.388×10^3
Example 2	Proposed	50.568	87.958
	Uma	71.228	180.380
	Liu	12.742	19.861
Example 3	Proposed	3.000	4.585
	Uma	12.320	52.836
	Kaya	4.704	5.940

With these controller settings, the performances of the closed-loop system is evaluated by introducing a unit step input in the setpoint at time $t = 0$, a negative step load input (d_2) at $t = 100$ s and a negative disturbance (d_1) of magnitude 0.2 at $t = 250$ s. The step responses for no mismatch in the plant models are shown in Fig. 9(a). Now suppose that there exists a 10% error in estimating the process time delays and time constants i.e. both process time delays are 10% larger and both process time constants are 10% smaller, the perturbed system responses are shown in Fig. 9(b). From Table 2, it can be observed that the proposed method gives low IAE and ISE values for both nominal and perturbed systems. It is seen from Table 3, the proposed scheme gives smoother control signals compared to Uma et al.'s as the TV value of the proposed method is low. Often, the disturbance rejection is more important than setpoint tracking for industrial practice. It is evident that the proposed controller provides robust performance particularly in the load disturbance rejection.

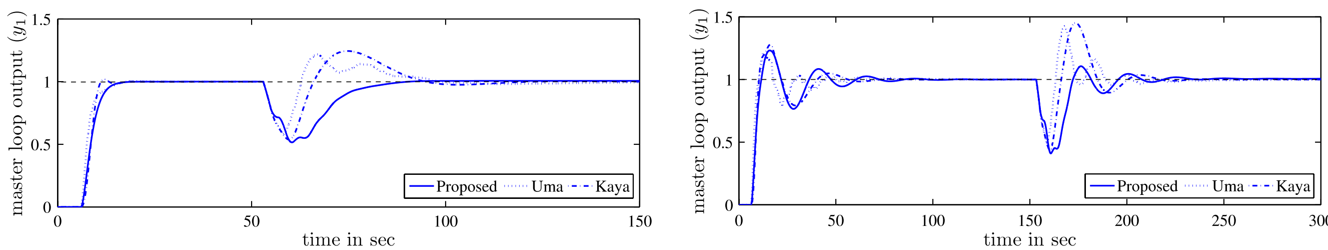
Example 3. Let the primary and secondary plant transfer functions be $G_{p1} = \frac{e^{-3s}}{10s-1}$ and $G_{p2} = \frac{2e^{-2s}}{s+1}$, respectively, studied by Kaya and Atherton [11] and Uma et al. [12]. For the proposed method, the setpoint tracking controller is obtained in the form $G_{cs} = \frac{10s^2+9s+1}{4.5s^2+6s+2}$ using (42) by selecting the tuning parameter $\lambda_{cs} = 0.3\theta_m = 1.5$. Using the design formulas the disturbance rejection controller parameters are obtained as $a_{f1} = 449.5502$, $b_{f1} = 0.8409$, $K_{c2} = 0.0015$, $T_{i2} = 1.3333$, $T_{d2} = 0.5$, $c_{f1} = 36.2409$, $c_{f2} = 35.2409$, $d_{f1} = 1.7273$, $d_{f2} = 2.2519$, $K_{c1} = 0.0914$, $T_{i1} = 3.3333$ and $T_{d1} = 1.25$ by selecting $\lambda_2 = 3.4\theta_{m2} = 6.8$ and $\lambda_1 = 0.8\theta_m = 4$. In Kaya et al.'s method, the controllers are $G_{c1} = 0.5(1+1/0.1s)$, $G_{c2} = (s+1)/(2s+2)$, $G_{c3} = 14.25+5.58s$ and $G_d = 1.414(1 + s)$. The controller settings of Uma et al.'s method are $k_{cs} = 7.7086$, $k_{is} = 1.1181$, $k_{ds} = 6.6604$, $\beta_s = 0.566$, $k_{cd} = 2.0907$, $k_{id} = 0.059$, $k_{dd} = 3.3963$, $\alpha_d = 2.5$,



(a) Nominal responses.

(b) Perturbed responses.

Fig. 9. Responses for Example 2.



(a) Nominal responses.

(b) Perturbed responses.

Fig. 10. Responses for Example 3.

$\beta_d = 0.1509$, $\varepsilon = 0.3$ and $\tau_f = 30$. A unit step input in the setpoint at $t = 0$ and a unit negative load disturbance (d_0) at $t = 50$ s are introduced. The closed-loop responses for the nominal system is shown in Fig. 10(a).

Now suppose that there exists a $+10\%$ perturbation in the primary process time delay and -10% in the primary time constant. The closed-loop performances of perturbed systems are evaluated by introducing a unit step input at $t = 0$ and a unit negative disturbance (d_0) at $t = 150$ s as shown in Fig. 10(b). From Tables 2 and 3, it is observed that the performance specifications are low for the proposed method. The proposed cascade control structure gives much improved performance especially for load disturbance rejection. It can be observed that the robustness of the proposed design technique toward assumed parameter perturbation is satisfactory. The performance of the proposed technique is found to be superior because of the modeling, the new cascade control structure and the design method used.

8. Conclusions

The problem of controlling an open-loop unstable process with time delay has been tackled by proposing an enhanced cascade control scheme. The cascade control structure improves regulatory response while the modified Smith predictor isolates and compensates the dead time. The proposed control structure combines the merits of the cascade control and the modified Smith predictor structure. The setpoint tracking is decoupled from the load disturbance rejection under ideal conditions which is an important feature of the proposed control scheme. It is demonstrated by Kharitonov's theorem that the controllers are robust and stable. The control method is very simple and has given improved results when compared with some recently published results.

Appendix A

In order to ensure the controller parameters in (28) to be positive, $x_1 < 0$.

The expression for x_1 is

$$x_1 = 6\theta_2 + 12\lambda_2 - 6\alpha_2. \tag{A.1}$$

Now, we can write

$$\frac{x_1}{6\theta_2} = \left(1 + \frac{2\lambda_2}{\theta_2} - \frac{\alpha_2}{\theta_2}\right) < 0 \tag{A.2}$$

or,

$$1 + \frac{2\lambda_2}{\theta_2} < \frac{\alpha_2}{\theta_2}. \tag{A.3}$$

The suggested range of the tuning parameter λ_2 is $1.5\theta_2 - 3.6\theta_2$. For the upper limit of λ_2, (A.3) gives $\alpha_2/\theta_2 > 8.2$. For every value of α_2/θ_2 to be greater than 8.2, x_1 is always negative.

(23) can be written as

$$\frac{\alpha_2}{\theta_2} = \frac{\tau_2}{\theta_2}\left[\left(1 + \frac{\lambda_2}{\tau_2}\right)^2 e^{\theta_2/\tau_2} - 1\right]. \tag{A.4}$$

For the smaller value of normalized time delay i.e. $\theta_2/\tau_2 = 0.01$, (A.4) gives $\alpha_2/\theta_2 = 8.4083$. Similarly, for the larger value of normalized time delay (i.e. $\theta_2/\tau_2 > 1$), say $\theta_2/\tau_2 = 10$, $\alpha_2/\theta_2 = 3\,015\,423.067$. From Fig. A.1, it is clear that the graph starts from the point (0.01, 8.4083) which indicates $\alpha_2/\theta_2 > 8.2$ and thus, x_1 is always negative irrespective of any value of normalized time delay. This is also true for the other values of λ_2.

Appendix B

In order to ensure the controller parameters in (39) to be positive, $m_1 < 0$.

The expression for m_1 is given by

$$m_1 = 6\theta_m + 18\lambda_1 - 6\alpha_1. \tag{B.1}$$

Now, we can write

$$\frac{m_1}{6\theta_m} = \left(1 + \frac{3\lambda_1}{\theta_m} - \frac{\alpha_1}{\theta_m}\right) < 0. \tag{B.2}$$

The above expression can also be written as

$$1 + \frac{3\lambda_1}{\theta_m} < \frac{\alpha_1}{\theta_m}. \tag{B.3}$$

The suggested range of the tuning parameter λ_1 is $0.5\theta_m - 1.5\theta_m$. For the upper limit of λ_1, (B.3) gives $\alpha_1/\theta_m > 5.5$. For every value of α_1/θ_m to be greater than 5.5, m_1 is always negative.

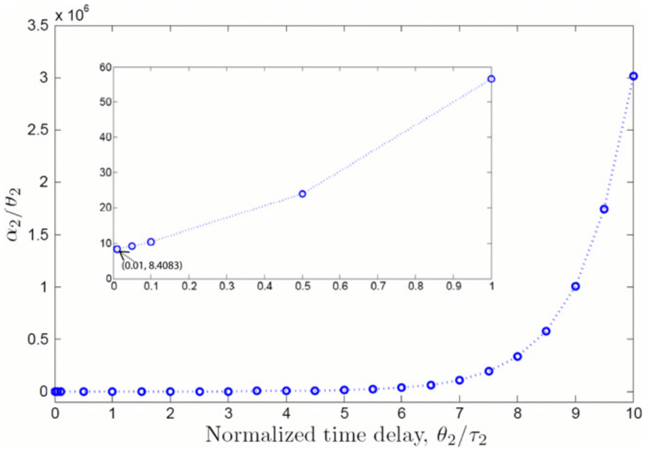


Fig. A.1. Plot for α_2/θ_2 versus θ_2/τ_2.

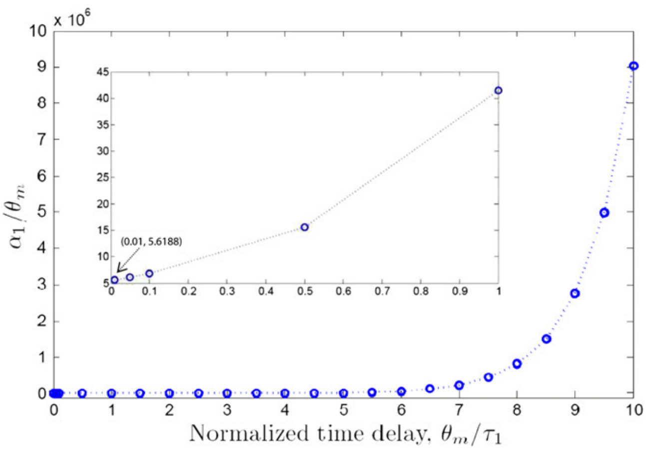


Fig. B.2. Plot for α_1/θ_m versus θ_m/τ_1.

(34) can be written as

$$\frac{\alpha_1}{\theta_m} = \frac{\tau_1}{\theta_m}\left[\left(1+\frac{\lambda_1}{\tau_1}\right)^3 e^{\theta_m/\tau_1} - 1\right]. \tag{B.4}$$

For the smaller value of normalized time delay i.e. $\theta_m/\tau_1 = 0.01$, (B.4) gives $\alpha_1/\theta_m = 5.6188$. Similarly, for the larger value of normalized time delay (i.e. $\theta_m/\tau_1 > 1$), say $\theta_m/\tau_1 = 10$, $\alpha_1/\theta_m = 9\,022\,040.29$. From Fig. B.2, it is clear that the graph starts from the point (0.01, 5.6188) i.e. $\alpha_1/\theta_m > 5.5$ which concludes that m_1 is always negative irrespective of any value of normalized time delay. This is also true for the other values of λ_1.

Appendix C

An alternate method to get the filter parameters (d_{f1} and d_{f2}) of the primary controller G_{cd1} is by using the Taylor series expansion. From (4) and (39), d_{f1} and d_{f2} can be obtained on equating $\left(1+d_{f1}s+d_{f2}s^2\right)$ and $\left(1+l_1s+l_2s^2+l_3s^3+l_4s^4\right)/(1-\tau_1 s)$. Thus,

$$\begin{aligned}\left(1+d_{f1}s+d_{f2}s^2\right) &= \left(1+l_1s+l_2s^2+l_3s^3+l_4s^4\right)/(1-\tau_1 s)\\ &= \left(1+l_1s+l_2s^2+l_3s^3+l_4s^4\right)(1-\tau_1 s)^{-1}.\end{aligned} \tag{C.1}$$

By expanding $(1-\tau_1 s)^{-1}$ using the Taylor series, (C.1) becomes

$$\left(1+d_{f1}s+d_{f2}s^2\right) = \left(1+l_1s+l_2s^2+l_3s^3+l_4s^4\right) \times \left(1+\tau_1 s+\tau_1^2s^2+\tau_1^3s^3+\cdots\right). \tag{C.2}$$

Neglecting the higher degrees of s, we obtain

$$1+d_{f1}s+d_{f2}s^2 = 1+(l_1+\tau_1)s+\left(l_2+\tau_1 l_1+\tau_1^2\right)s^2. \tag{C.3}$$

From (C.3), we get $d_{f1} = l_1+\tau_1$ and $d_{f2} = l_2+d_{f1}\tau_1$ which are exactly the same as that in (39).

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