CONTROL SYSTEMS ENGINEERING

Second Revised Edition

U. A. Bakshi
S. C. Goyal

Technical Publications Pune™

Table of Contents :

Control Systems Engineering

Second Revised Edition : December 2003
Second Reprint : February 2007

ISBN 81 - 89411- 59 - 4

Printer :
Alert DTPrinters
Sr.no. 10/3,Sinhagad Road,
Pune - 411 041

Published by :
Technical Publications Pune®
#1, Amit Residency, 412, Shaniwar Peth, Pune - 411 030, India.

Head office
(Pune - Maharashtra)
1, Amit Residency, 412, Shaniwar Peth, Pune - 411030, India
+91 (020) 24495496, Tele/Fax : +91 (020) 24495497
Email : technical@vtubooks.com

Branches

Bangalore (Karnataka) : 1676-14th, Main Road, Prakash Nagar, Bangalore - 560021, India
Phone : (080) 23324037, Tele Fax : 2332 4437, Mobile : 98805 82331

Hyderabad (A.P.) : 22,Shivam Enclave, 4-5 - 947, Badi Chavadi, Hyderabad - 500095
India. Phone : (040) 65545313

Chennai (TN) : B - Ground floor, 'QUANTA ANNAM' 24th Street, H-965/966,
Opp. U C MAS Anna Nagar (W) Chennai - 600040,
Phone : (044) 26161903, Mobile : 9840324419

Malout (Punjab) : # 83, St. No. 6, Ward No. 5, Gurudwara Road, Malout,
Punjab - 152107 Phone : 01637 - 265853
Mobile : 9876877796, 9815900478

Wholesale Distribution

Pune
Pragati Books Pvt. Ltd.
119, Budhwar Peth, Jogeshwari Mandir lane,
Pune - 411 002, India
Phone : (020) 24452044

Mumbai
Pragati Books Pvt. Ltd.
385, S.V.P. Road, Rasdhara, Co-op. Society, Girgaum,
Mumbai - 400 004 India.
Phone : (022) 23869976, 23856339

Bangalore
Pragati Book House
1676-14th, Main Road, Prakash Nagar,
Bangalore - 560021, India
Phone : (080) 23324037, Telefax : 2332 4437,
Mobile : 98805 82331

Chennai
Pragati Books
9/1, Montieth Road, Behind Taas Mahal,
Egmore, Chennai - 600 008,
India. Ph. - (044) 6518 3535 Mobile - 94440 01782

Chennai
Technical Books Distributor
B - Ground floor, 'QUANTA ANNAM' 24th Street, H-965/966,
Opp. U C MAS Anna Nagar (W) Chennai - 600040,
Phone : (044) 26161903 Mobile : 9840324419

Hyderabad
Nirali Book House
22,Shivam Enclave, 4-5 - 947, Badi Chavadi,
Hyderabad - 500095
India. Mobile : 9440030608 Phone : (040) 65545313

Preface

Thanks to the professors, students and authors of many books, papers and articles for their overwhelming response to the earlier **seven editions** of the book **'Principles of Control Systems'**, This book covers the entire syllabus of the subject **'Control Systems Engineering'.**

The book uses a plain, lucid and everyday language to explain the subject, which many people consider as a complex technical subject. The book prepares very carefully a background of each topic with essential illustrations and practical examples and then step by step gives the complex derivations and explanations. Each chapter is supported with large number of solved problems. The theory of control systems can be digested through the working of many problems, solutions of which are known. From this point of view, at the end of each chapter the exercise including theory questions and the problems alongwith the answers are added. The exact and clear representation of complex diagrams like Root locus, Bode plot and Nyquist plot is the feature of this book. The stepwise methods given to solve the problems on various topics, greatly simplifies the analysis and the understanding of the problems.

Contents and Organization

The chapters in the book are arranged in a proper sequence that permits each topic to build upon earlier studies, which is important in understanding the complex subject like control systems.

The chapter one explains the basics of control systems, including classification of control systems and various types illustrations of control systems used in practice. The concept of transfer function plays an important role in the control systems engineering. The Laplace transform is the base of control systems analysis so the students are expected to go through the basics of Laplace transform included in the Appendix A before starting the study of control systems. The chapter two provides the knowledge of transfer function models and the impulse response models of the systems. The chapters three and four are dedicated to the two important representation techniques called Block diagram and Signal flow graph representations. The electrical systems are easy from the analysis point of view for an electronics engineer. The chapter five covers the concepts of analogous systems and mathematical models of the systems. It explains how to obtain models of various nonelectrical systems like mechanical systems, hydraulic systems, thermal systems etc. It includes the analysis of various practical systems using number of different control components like potentiometers, generators, a.c. and d.c. motors etc. Large number of practical systems, illustrations and solved problems are included in this chapter to inculcate the concepts of modeling, in the students. The chapter six includes the discussion on the steady state and transient response of the systems. The chapter seven starts with the explanation of fundamental ideas about the system stability and gives the famous Routh's method used for the stability analysis. The chapter eight gives the detail discussion of the Root locus method of analyzing stability. The simple approach and stepwise explanation made the Root locus topic very easy. The chapter nine introduces the basic concepts of the frequency domain stability analysis and co-relation between time domain and the frequency domain. The chapter ten explains the popular Bode plot method of analyzing stability. The problems solved accurately using the semilog papers are included as it is, to make the understanding of the method easy. The chapter eleven includes the discussion of the Nyquist plot method starting from the discussion of polar plot. The students find this method complicated but the steps used to discuss this method and to solve the problems on this method, included in this chapter, made the understanding of this method very easy. The chapter twelve includes the explanation of M and N circles and the Nichol's chart. The chapter thirteen includes the discussion of various control system components. The chapter fourteen introduces the various controller principles. It starts with the classification of controllers. Then it explains all the types of continuous, discontinuous and composite controllers. At the end, chapter explains the effect of composite controllers on the performance of the second order systems. The chapter fifteen includes the various miscellaneous questions and answers. The appendix A gives the basic theory of Laplace transform method.

In all, this book explains the philosophy of the subject Control Systems. Once again the book will be very much useful not only to the students but also to the subject teachers.

The students have to omit nothing and possibly, have to cover nothing more.

Acknowledgements

We wish to express our profound thanks to all those who helped in making this book a reality. Much needed moral support and encouragement is provided on numerous occasions by our whole family.

We wish to thank Prof. A .V. Bakshi and Prof. A . P. Godse for their valuable suggestions.

Without full support of Mrs. Goyal and Mrs. Varsha Bakshi, the book would not have been completed in time.

Finally, we wish to thank **Mr. Avinash Wani, Mr. Ravindra Wani** and the entire team of **Technical Publications** for bringing out, this book in a short time with quality printing.

Any suggestions for the improvement of the book will be acknowledged and appreciated.

Authors

Dedicated to Apurva and Gururaj

Table of Contents

Chapter-7 Stability of the System 471

Chapter-8 Root Locus 520

1
Basics of Control Systems

1.1 Introduction :

In recent years, concept of automatic control has achieved a very important position in advancement of modern science. Automatic control systems have played an important role in the advancement and improvement of engineering skills.

Practically, every activity in our day to day life is influenced by some sort of control system. Concept of control systems also plays an important role in the working of space vehicles, satellites, guided missiles etc. Such control systems are now integral part of the modern industrialization, industrial processes and home appliances. Control systems are found in number of practical applications like computerised control systems, transportation systems, power systems, temperature limiting systems, robotics etc.

Hence for an engineer it is absolutely necessary to get familiar with the analysis and designing methods of such control systems.

In this chapter we will try to get familiar with

1) What is system ?

2) What is control system ?

3) How control systems can be classified ?

4) Which are the basic components of control systems ?

1.2 Definitions :

To understand the meaning of the word control system, first we will define the word system and then we will try to define the word control system.

System : A system is a combination or an arrangement of different physical components which act together as a entire unit to achieve certain objective.

Every physical object is actually a system. A classroom is a good example of physical system. A room along with the combination of benches, blackboard, fans, lighting arrangement etc. can be called as a classroom which acts as elementary system.

Another example of a system is a lamp. A lamp made up of glass, filament is a physical system. Similarly a kite made up of paper and sticks is an example of a physical system.

Similarly system can be of any type i.e. physical, ecological, biological etc.

Control system : To control means to regulate, to direct or to command. Hence a control system is an arrangement of different physical elements connected in such a manner so as to regulate, direct or command itself or some other system.

For example if in a classroom, professor is delivering his lecture, the combination becomes a control system as; he tries to regulate, direct or command the students in order to achieve the objective which is to input good knowledge to the students. Similarly if lamp is switched ON or OFF using a switch, the entire system can be called as a control system. The concept of physical system and a control system is shown in the Fig.1.1 and Fig.1.2

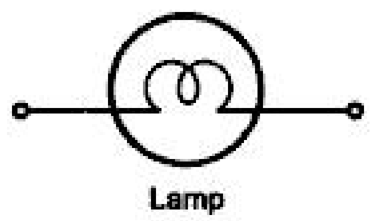


Fig. 1.1 Physical system

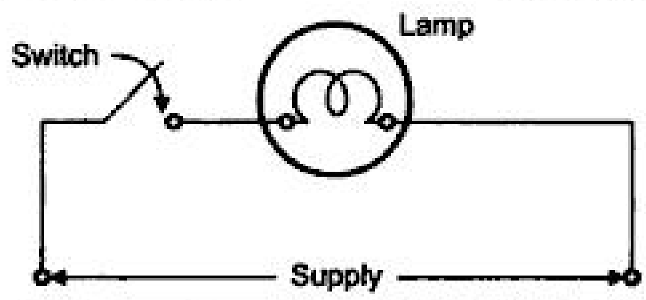


Fig. 1.2 Control system

When a child plays with the kite, he tries to control it with the help of string and entire system can be considered as a control system.

In short, a control system is in the broadest sense, an interconnection of the physical components to provide a desired function, involving some kind of controlling action in it.

Plant : The portion of a system which is to be controlled or regulated is called as the **plant or the Process.**

Controller : The element of the system itself or external to the system which controls the plant or the process is called as **controller.**

For each system, there must be excitation and system accepts it as an input. And for analyzing the behaviour of system for such input, it is necessary to define the output of a system.

Input : It is an applied signal or an excitation signal applied to control system from an external energy source in order to produce a specified output.

Output : It is the particular signal of interest or the actual response obtained from a control system when input is applied to it.

Disturbances : Disturbance is a signal which tends to adversely affect the value of the output of a system. If such a disturbance is generated within the system itself, it is called as **internal disturbance.** The disturbance generated outside the system acting as an extra input to the system in addition to its normal input, affecting the output adversely is called as an **external disturbance.**

Control systems may have more than one input or output. From the information regarding the system, it is possible to well define all the inputs and outputs of the systems.

The input variable is generally referred as the **Reference Input** and Output is generally referred as the **Controlled output**.

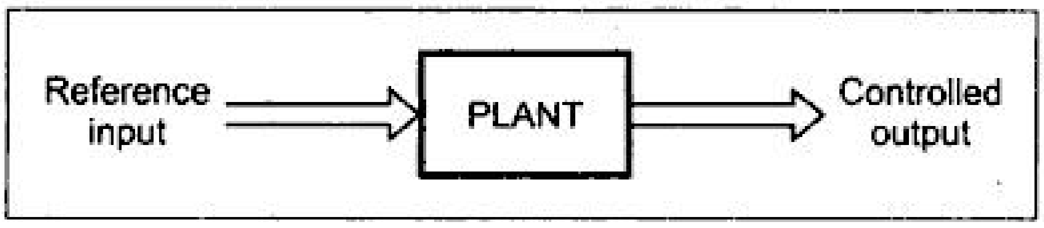


Fig. 1.3

Cause and effect relationship between input and output for a plant can be shown as in Fig. 1.3.

1.3 Classification of Control Systems :

Broadly control systems can be classified as,

1) **Natural Control Systems :** The Biological systems, systems inside human being are of natural type.

Ex. 1.1 : *The perspiration system inside the human being is a good example of natural control system. This system activates the secretion glands, secreating sweat and regulates the temperature of human body.*

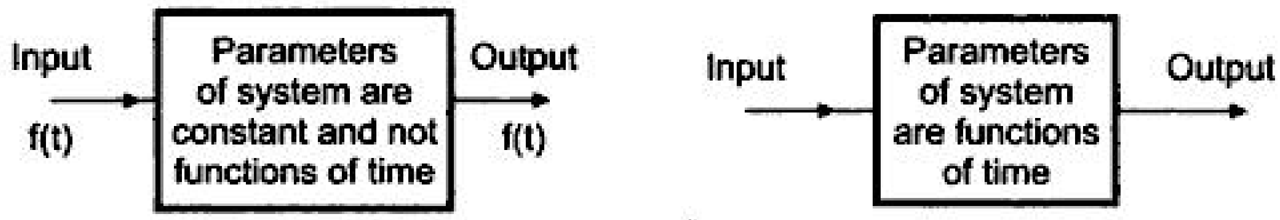
2) **Manmade Control Systems :** The various systems, we are using in our day to day life are designed and manufactured by human beings. Such systems like vehicles, switches, various controllers etc. are called as manmade control systems.

Ex. 1.2 : *An automobile system with gears, accelerator, braking system is a good example of manmade control system.*

3) **Combinational Control Systems :** Combinational control system is one, having combination of natural and manmade together: i.e. driver driving a vehicle. In such system, for successful operation of the system, it is necessary that natural systems of driver alongwith systems in vehicles which are manmade must be active.

But for the engineering analysis, control systems can be classified in many ways. Some of the classifications are given below.

4) **Time Varying and Time - Invariant Systems :** Time varying control systems are those in which parameters of the systems are varying with time. It is not dependent on whether input and output are functions of time or not. For example, space vehicle whose mass decreases with time, as it leaves earth. The mass is a parameter of space vehicle system. Similarly in case of a rocket, aerodynamic damping can change with time as the air density changes with the altitude. As against this if even though the inputs and outputs are functions of time but the parameters of system are independent of time, that is not varying with time and are constants, then system is said to be time invariant system. Different electrical networks consisting of the elements as resistances, inductances and capacitances are time invariant systems as the values of the elements of such system are constant and not the functions of time. The complexity of the control system design increases considerably if the control system is of the time varying type. This is shown in Fig.1.4.



(a) Time invariant system **(b) Time variant system**

Fig. 1.4

5) **Linear and Nonlinear Systems :** A control system is said to be linear if superposition principle applies to it. For linear systems the response to several forcing functions can be calculated by considering one forcing function at a time and adding the results.

The system is said to be linear if it satisfies following two properties.

i) Additive property that is for any x and y belonging to the domain of the function f, we have

$$f(x + y) = f(x) + f(y)$$

ii) Homogeneous property that is for any x belonging to the domain of the function f and for any scalar constant α, we have.

$$f(\alpha\, x) = \alpha \, . \, f(x)$$

These two properties together constitute a principle of superposition.

Hence the transformation, operation, function which satisfies above two properties is called as linear in nature.

The function $f(x) = x^2$ is nonlinear as

$$(x_1 + x_2)^2 \neq x_1^2 + x_2^2$$

and $(\alpha\, x)^2 \neq \alpha\, (x)^2$

It is very difficult to have a linear system satisfying the above two properties perfectly. All the physical systems are nonlinear to some extent. But if the presence of certain nonlinearity is not affecting the performances of system much, as per the above two properties and deviation of system from the principle of superposition is negligible, the presence of nonlinearity is neglected and the system can be assumed to be linear from the analysis point of view.

In practice most of the physical systems are nonlinear in nature because of different non-linearities present in the system i.e. saturation, friction, deadzone. etc. Such systems are nonlinear systems for which principle of superposition cannot be applied. Procedures for finding the solutions of nonlinear system problems are complicated and time consuming. Because of this difficulty generally nonlinear systems are treated as linear systems for a limited range of operation with some approximation. Then number of linear methods can be applied for analysis of such systems.

6) **Continuous Time and Discrete Time Control Systems :** In a continuous time control system all system variables are the functions of a continuous time variable 't'. The speed control of a d.c. motor using a tachogenerator feedback is an example of continuous data system. At any time 't' they are dependent on time. In discrete time systems one or more system variables are known only at certain discrete intervals of time. They are not continuously dependent on the time. Microprocessor or computer based systems use such discrete time signals. The reasons for using such signals in digital controllers are

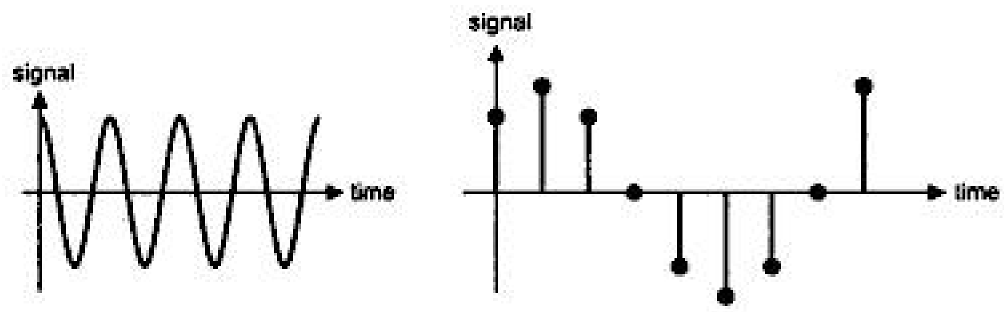
1) Such signals are less sensitive to noise.

2) Time sharing of one equipment with other channels is possible.

3) Advantageous from point of view of size, speed, memory, flexibility etc.

The systems using such digital controllers or sampled signals are called as sampled data systems.

Continuous time system uses the signals as shown in Fig. 1.5(a) which are continuous with time while discrete system uses the signals as shown in Fig. 1.5(b).



(a) Continuous signal **(b) Discrete signal**

Fig. 1.5

7) **Deterministic and Stochastic Control Systems :** A control system is said to be deterministic when its response to input as well as behaviour to external disturbances is predictable and repeatable. If such response is unpredictable, system is said to be stochastic in nature.

8) **Lumped Parameter and Distributed Parameter Control Systems :** Control system that can be described by ordinary differential equations is called as lumped parameter control system. For example electrical networks with different parameters as resistance, inductance, etc. are lumped parameter systems. Control systems that can be described by partial differential equations are called as distributed parameter control systems. For example, transmission line having its parameters resistance and inductance totally distributed along it. Hence description of transmission line characteristics is always by use of partial differential equations.

9) **Single Input Single Output (SISO) and Multiple Input Multiple Output (MIMO) Systems :** A system having only one input and one output is called as single input single output system. For example a position control system has only one input (desired position) and one output (actual output position). Some systems may have multiple type of inputs and multiple outputs, these are called as multiple input multiple output systems.

10) **Open loop and Closed Loop Systems :** This is another important classification. The features of both types are discussed in detail in coming sections.

1.4 Open Loop System :

Definition : A system in which output is dependent on input but controlling action or input is totally independent of the output or changes in output of the system, is called as Open Loop System.

In a broad manner it can be represented as in Fig. 1.6.

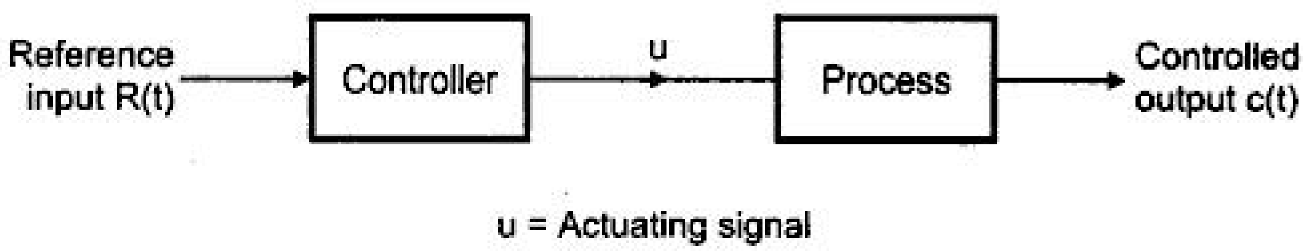


Fig. 1.6

Reference input [R(t)] is applied to the controller which generates the actuating signal (u) required to control the process which is to be controlled. Process is giving out the necessary desired controlled output C(t).

Advantages :

1) Such systems are simple in construction.
2) Very much convenient when output is difficult to measure.
3) Such systems are easy from maintenance point of view.
4) Generally these are not troubled with the problems of stability.
5) Such systems are simple to design and hence economical.

Disadvantages :

1) Such systems are inaccurate and unreliable because accuracy of such systems are totally dependent on the accurate precalibration of the controller.
2) Such systems give inaccurate results if there are variations in the external environment i.e. cannot sense environmental changes.
3) Similarly they cannot sense internal disturbances in the system, after the controller stage.
4) To maintain the quality and accuracy, recalibration of the controller is necessary, time to time.

To overcome all above disadvantages generally in practice closed loop systems are used.

For example, an electric switch. This is open loop because output is light and switch is controller of lamp. Any change in light has no effect on the ON-OFF position of the switch, i.e. its controlling action.

Similarly automatic washing machine. Here output is degree of cleanliness of clothes. But any change in this output will not affect the controlling action or will not decide the operation time or will not decide the amount of detergent which is to used. Some other examples are traffic signal, automatic toaster system etc.

Illustrations :

1.4.1 Sprinkler used to water a lawn :

The system is adjusted to water a given area by opening the water valve and observing the resulting pattern. When the pattern is considered satisfactory, the system is "calibrated" and no further valve adjustment is made.

1.4.2 Stepper motor positioning system :

The actual position in such system is usually not monitored. The motor controller commands a certain number of steps by the motor to drive the output to a previously determined location.

1.4.3 Automatic toaster system :

In this system, the quality of toast depends upon the time for which the toast is heated. Depending on the time setting, bread is simply heated in this system. The toast quality is to be judged by the user and has no effect on the inputs.

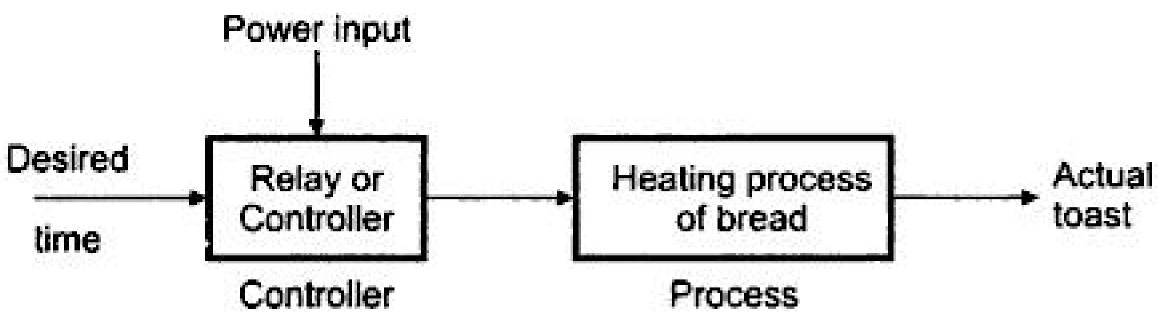


Fig. 1.7

1.4.4 Traffic light controller :

A traffic flow control system used on roads is time dependent. The traffic on the road becomes mobile or stationary depending on the duration and sequence of lamp glow. The sequence and duration are controlled by relays which are predetermined and not dependent on the rush on the road.

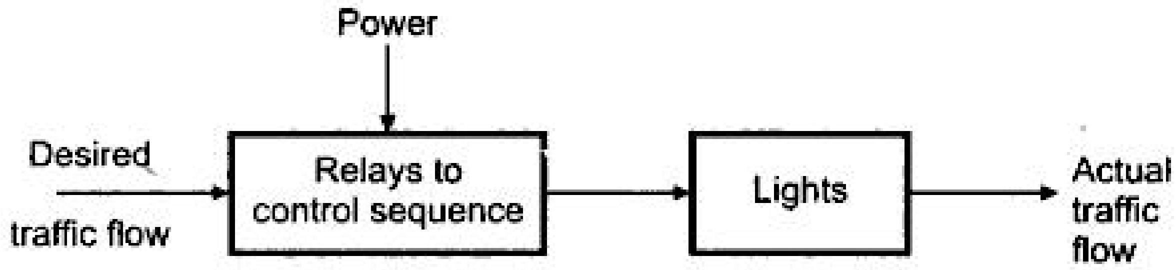


Fig. 1.8

1.4.5 Automatic door opening and closing system :

In this system, photo sensitive devices are used. When a person interrupts a light, photo device generates actuating signal which opens the door. When person passes through the door, light becomes continuous closing the door. The opening and closing of the door is the output which has nothing to do with the inputs, hence an open loop system.

The room heater, fan regulator, automatic coffee server, electric lift, theatre lamp dimmer, automatic dryer are another examples of open loop system.

1.5 Closed Loop System :

Definition : A system in which the controlling action or input is somehow dependent on the output or changes in output is called as closed loop system.

To have dependence of input on the output, such system uses the feedback property.

Feedback : Feedback is a property of the system by which it permits the output to be compared with the reference input so that appropriate controlling action can be decided.

In such system output or part of the output is fedback to the input for comparison with the reference input applied to it.

Closed loop system can be represented as in Fig. 1.9.

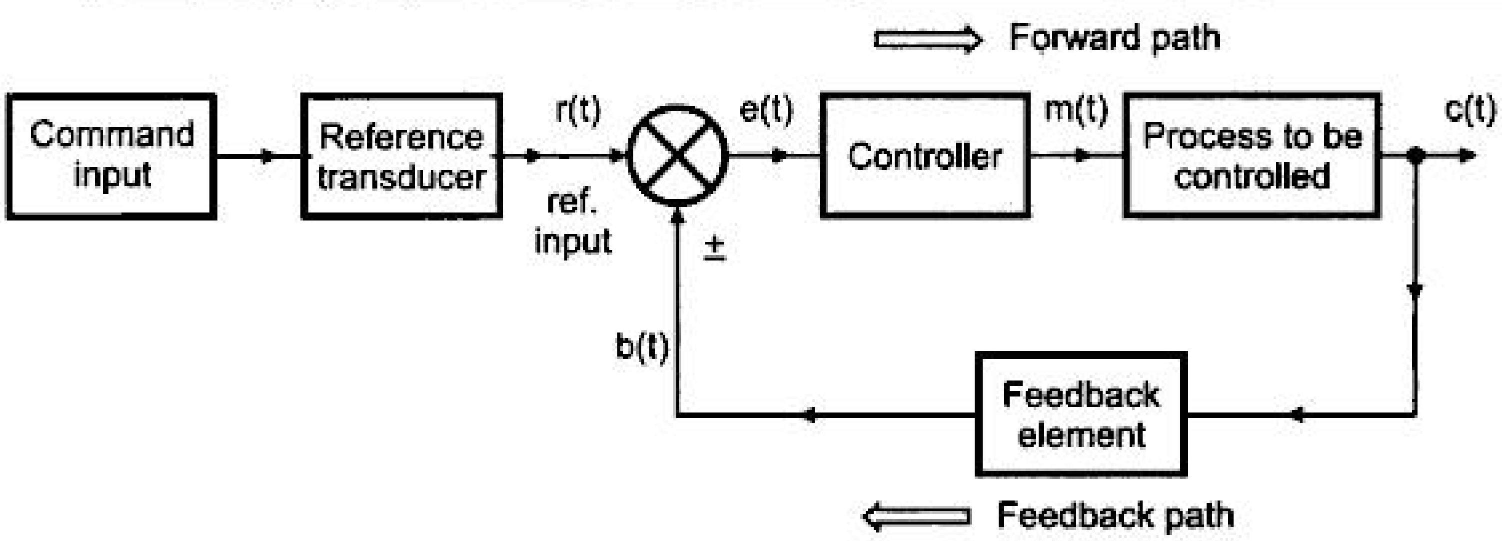


Fig. 1.9

$r(t)$ = Reference Input $e(t)$ = Error signal

$c(t)$ = Controlled output $m(t)$ = Manipulated signal $b(t)$ = Feedback signal

It is not possible in all the systems that available signal can be applied as input to the system. Depending upon nature of controller and plant it is required to reduce it or amplify it or to change its nature i.e. making it discrete from continuous type of signal etc. This changed input as per requirement is called as **reference input** which is to be generated by using reference transducer. The main excitation to the system is called as its **command input** which is then applied to the reference transducer to generate reference input.

The part of output, which is to be decided by feedback element is fed back to the reference input. The signal which is output of feedback element is called as 'feedback signal' b(t).

It is then compared with the reference input giving error signal $e(t) = r(t) \pm b(t)$

When feedback sign is positive, systems are called as positive feedback systems and if it is negative systems are called as negative feedback systems.

This error signal is then modified by controller and decides the proportional manipulated signal for the process to be controlled.

This manipulation is such that error will approach to zero. This signal then actuates the actual system and produces an output. As output is controlled one, it is called as controlled output c(t).

Advantages :

1) Accuracy of such system is always very high because controller modifies and manipulates the actuating signal such that error in the system will be zero.
2) Such system senses environmental changes, as well as internal disturbances and accordingly modifies the error.
3) In such system, there is reduced effect of nonlinearities and distortions.
4) Bandwidth of such system i.e. operating frequency zone for such system is very high.

Disadvantages :

1) Such systems are complicated and time consuming from design point of view and hence costlier.
2) Due to feedback, system tries to correct the error time to time. Tendency to overcorrect the error may cause oscillations without bound in the system. Hence system has to be designed taking into consideration problems of instability due to feedback.

Illustrations :

1.5.1 Human being :

The best example is human being. If a person wants to reach for a book on the table, closed loop system can be represented as in the Fig. 1.10.

Position of the book is given as the reference. Feedback signal from eyes, compares the actual position of hands with reference position. Error signal is given to brain. Brain manipulates this error and gives signal to the hands. This process continues till the position of the hands get achieved appropriately.

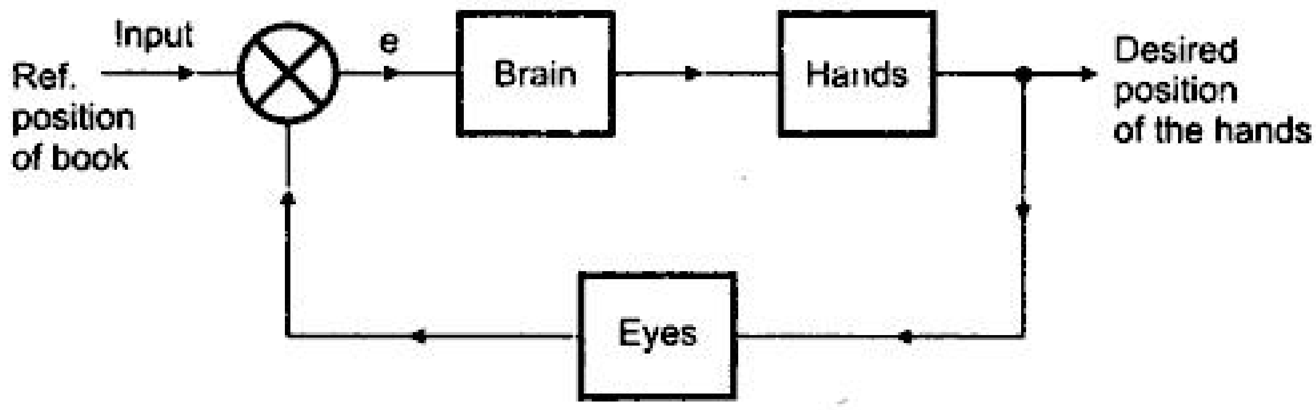


Fig. 1.10

1.5.2 Home heating system :

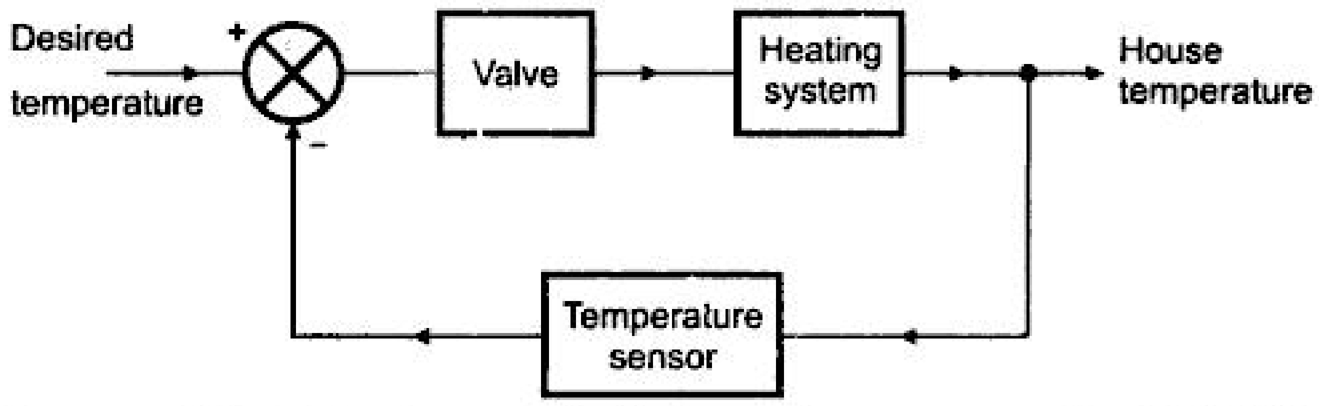


Fig. 1.11

In this system, the heating system is operated by a valve. The actual temperature is sensed by a thermal sensor and compared with the desired temperature. The difference between the two, actuates the valve mechanism to change the temperature as per the requirement.

1.5.3 Ship stabilization system :

In this system a roll sensor is used as a feedback element. The desired roll position is selected as θ_r while actual roll position is θ_c which is compared with θ_r to generate controlling signal. This activates fin actuator in proper way to stabilize the ship.

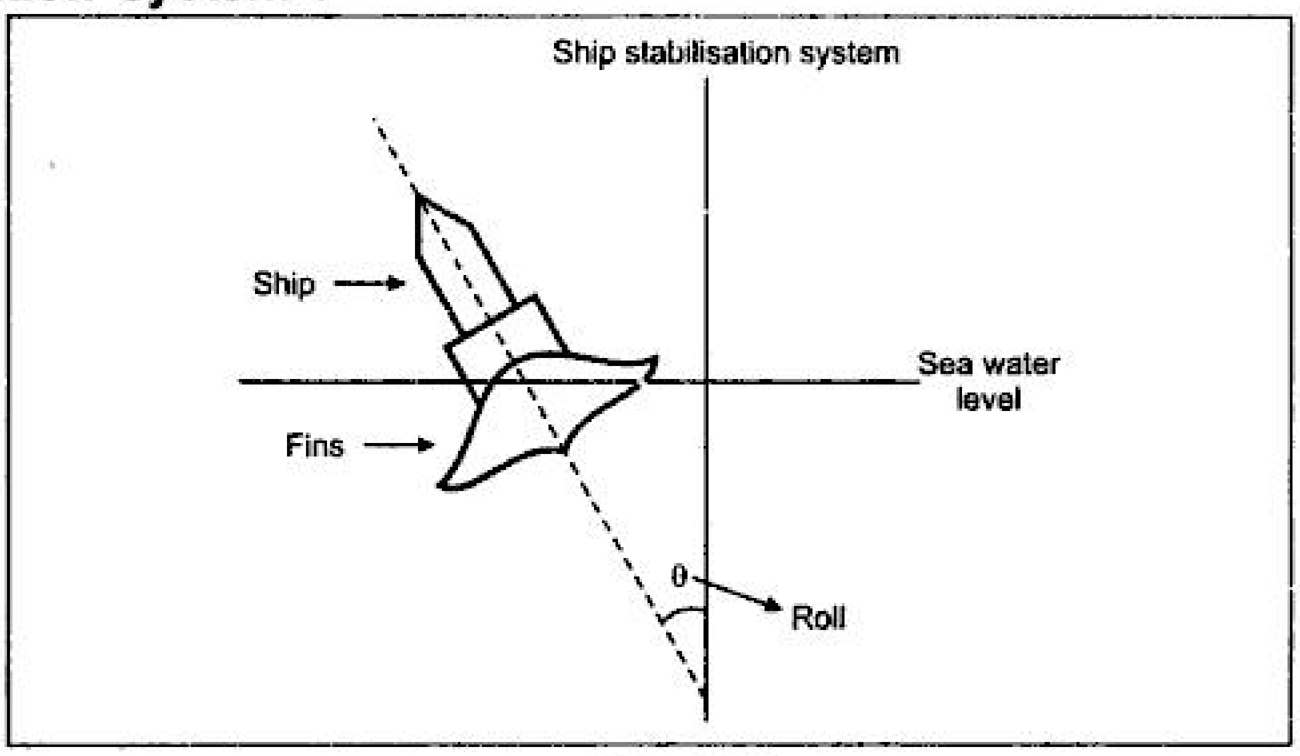


Fig. 1.12

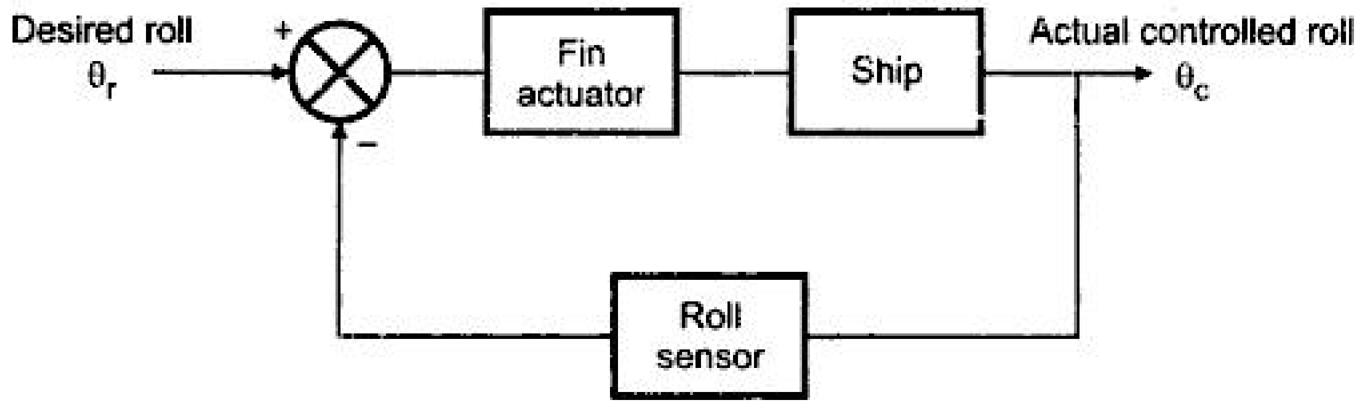


Fig. 1.13

1.5.4 Manual speed control system :

A locomotive operator driving a train is a good example of a manual speed control system. The objective is to maintain the speed equal to the speed limits set. The entire system is shown in the block diagram in the Fig. 1.14.

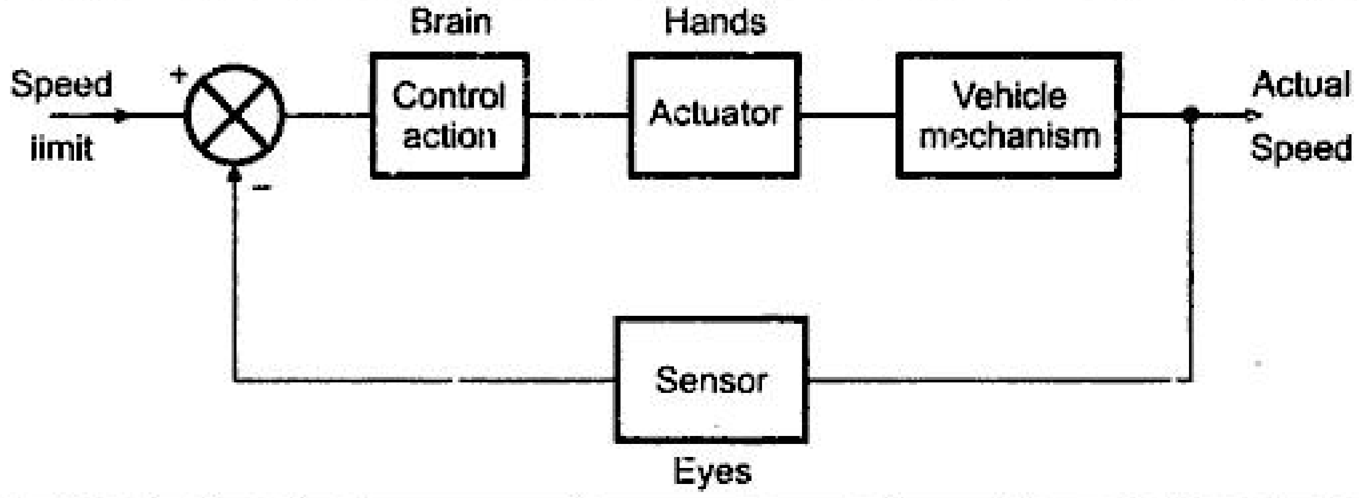


Fig. 1.14

1.5.5 D.C motor speed control :

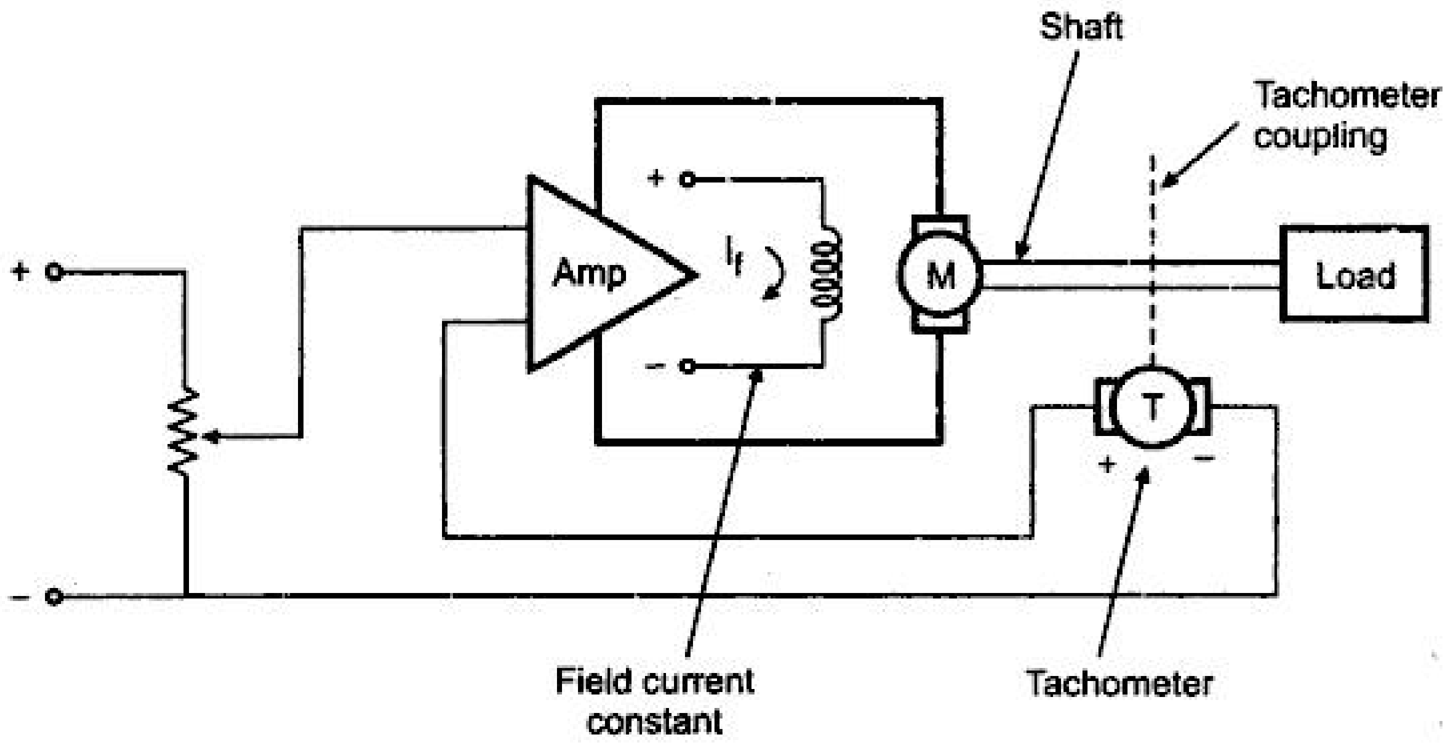


Fig. 1.15

The D.C. shunt motor is used where field current is kept constant and armature voltage is changed to obtain the desired speed. The feedback is taken by speed tachometer. This generates voltage proportional to speed which is compared with voltage required to the desired speed. This difference is used to change the input to controller which cumulatively changes the speed of the motor as required.

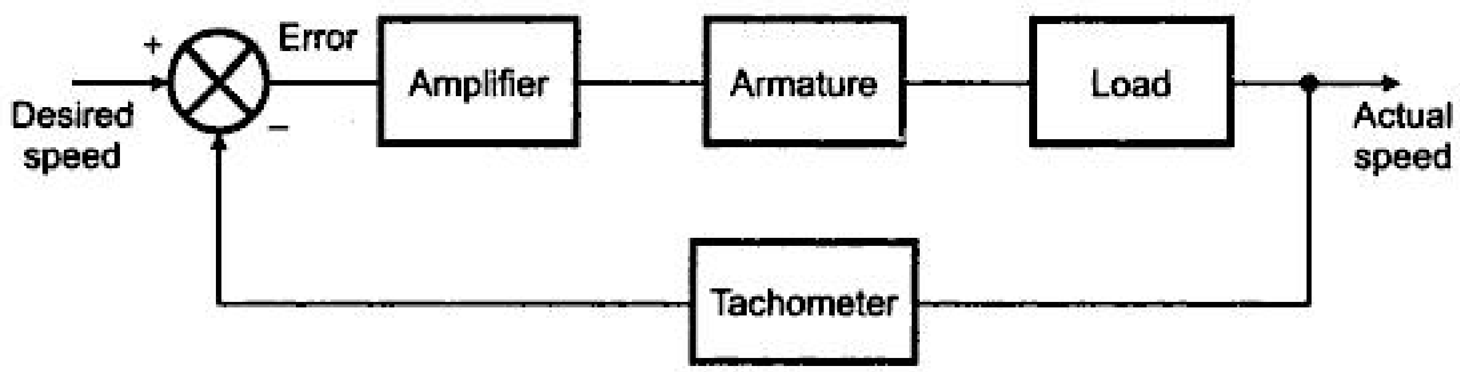


Fig. 1.16

1.5.6 Temperature control system :

The aim is to maintain hot water temperature constant. Water is coming with constant flow rate. Steam is coming from a valve. Pressure thermometer 'P' is used as a feedback element which sends a signal for comparison with the set point. This error actuates the valve which controls the rate of flow of steam, eventually controlling the temperature of the water.

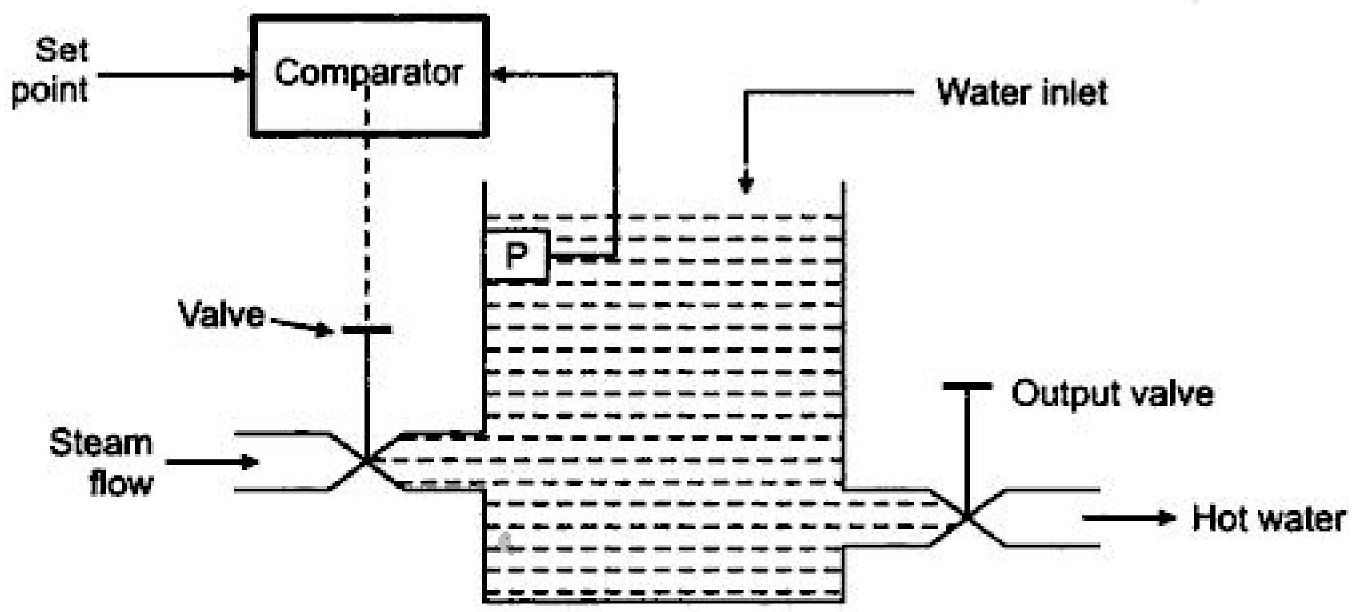


Fig. 1.17

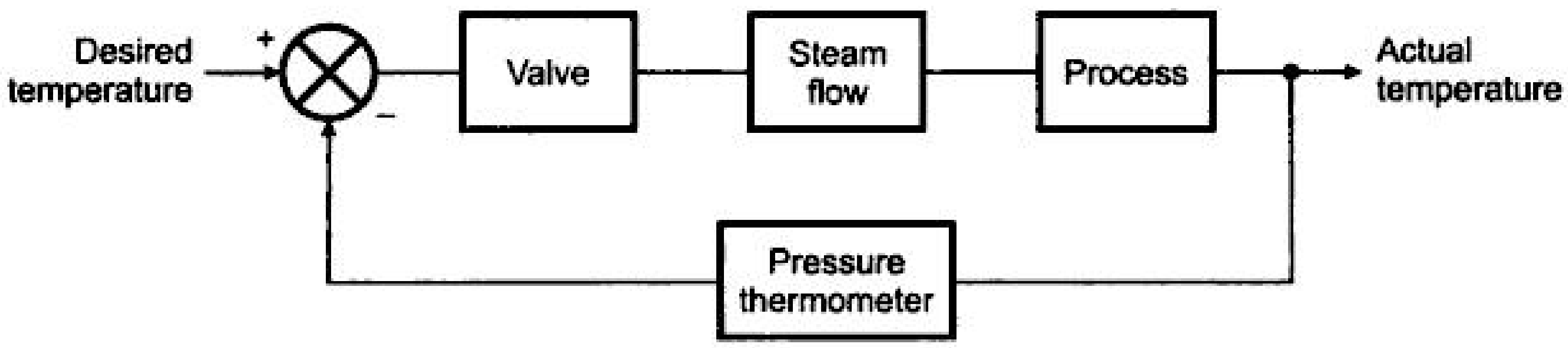


Fig. 1.18

1.5.7 Missile launching system :

This is sophisticated example of military applications of feedback control. The enemy plane is sighted by a radar which continuously tracks the path of the aeroplane. The launch computer calculates the firing angle in terms of launch command, which when amplified drives the launcher. The launcher angular position is the feedback to the launch computer and the missile is triggered when error between the command signal and missile firing angle becomes zero.

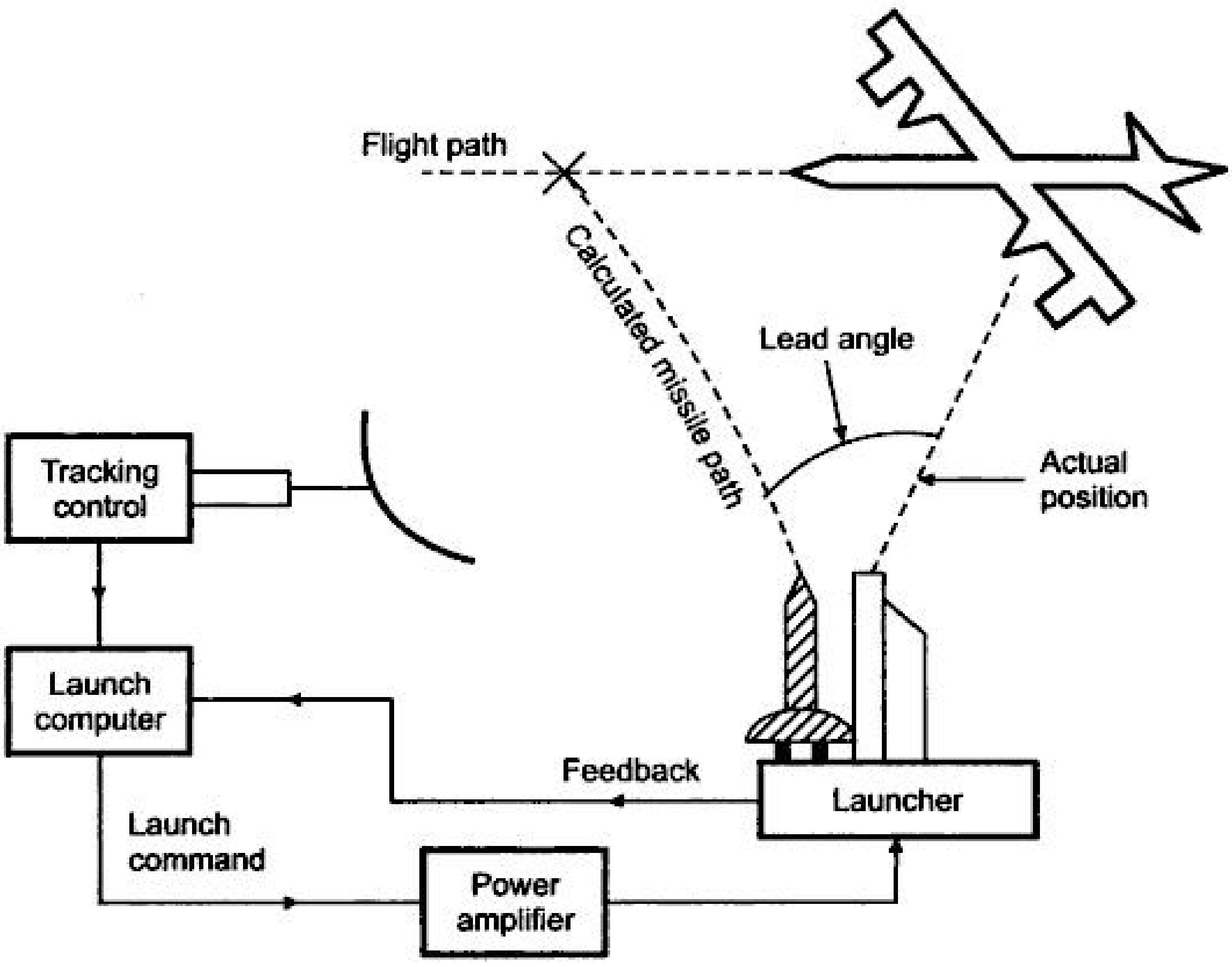


Fig. 1.19

1.5.8 Voltage stabilizer :

Supply voltage required for various single phase appliances must be constant and high fluctuations are generally not permitted. Voltage stabilizer is a device which accepts variable voltage and outputs a fixed voltage.

Principle of such stabilizer is based on controlling number of secondary turns as per requirement to increase or decrease the output voltage. The actual output voltage is sensed by a transformer and potential divider arrangement. The reference voltage is selected proportional to the desired output level. The actual output is compared with this to generate error which in turn is inputted to the controller. The controller takes the proper decision to increase or decrease the number of turns so as to adjust the output voltage. The scheme is shown in the Fig. 1.20.

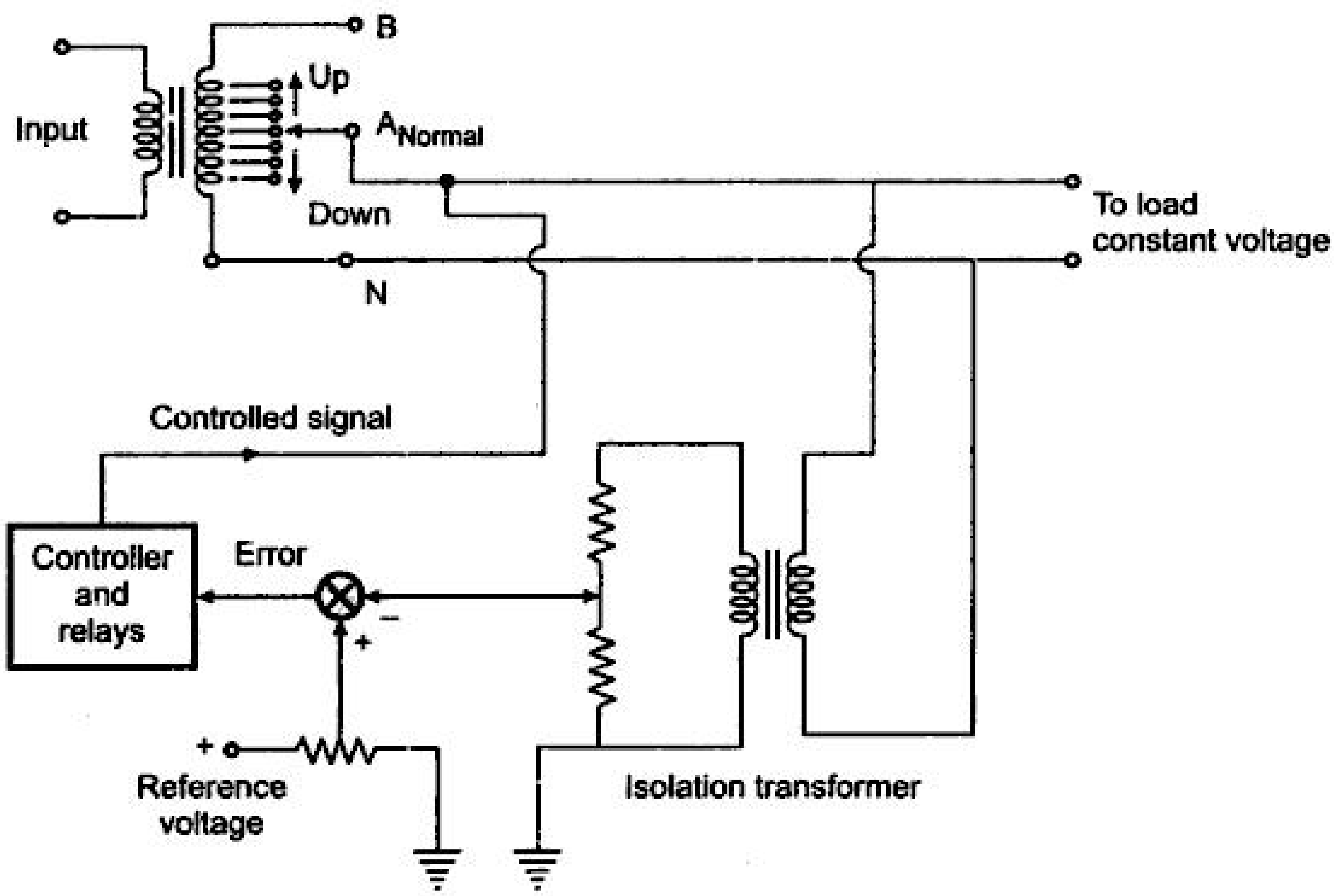


Fig. 1.20

The other examples of closed loop system are machine tool position control, positioning of radio and optical telescopes, auto pilots for aircrafts, inertial guidance system, automatic electric iron, railway reservation status display, sunseeker solar system, water level controllers, temperature control system. So in closed loop feedback control systems cause and effect relationship between input and output exists.

1.6 Comparison of Open Loop and Closed Loop control system :

	Open Loop		Closed Loop
1)	Any change in output has no effect on the input i.e. feedback does not exists.	1)	Changes in output, affects the input which is possible by use of feedback.
2)	Output measurement is not required for operation of system.	2)	Output measurement is necessary.
3)	Feedback element is absent.	3)	Feedback element is present.
4)	Error detector is absent.	4)	Error detector is necessary.
5)	It is inaccurate and unreliable.	5)	Highly accurate and reliable.
6)	Highly sensitive to the disturbances.	6)	Less sensitive to the disturbances.
7)	Highly sensitive to the environmental changes.	7)	Less sensitive to the environmental changes.
8)	Bandwidth is small.	8)	Bandwidth is large.
9)	Simple to construct and cheap.	9)	Complicated to design and hence costly.
10)	Generally are stable in nature.	10)	Stability is the major consideration while designing
11)	Highly affected by nonlinearities.	11)	Reduced effect of nonlinearities.

1.7 Feedback and Feed Forward System :

In the control systems considered uptill now, it is considered that the disturbance has affected the output adversely. Such an output is measured and compared with the reference input to generate an error. This error is fed to the controller which is successively operating the system to correct the output.

Thus such systems in which the effect of the disturbance must show up in the error before the controller can take proper corrective action are called as feedback systems.

If the disturbance is measurable, then the signal can be added to the controller output to modify the actuating signal. Thus, a corrective action is initiated without waiting for the effect of the disturbance to show up in the output i.e. cumulatively in the error. Thus the undesirable effects of measurable disturbances by approximately compensating for them before they affect the output. This is much more advantageous as in normal feedback system the corrective action starts only after the output has been affected. Such systems in which such corrective action is taken before disturbances affect the output are called as feed forward system.

A block diagram with feed forward concept is shown in the Fig. 1.21.

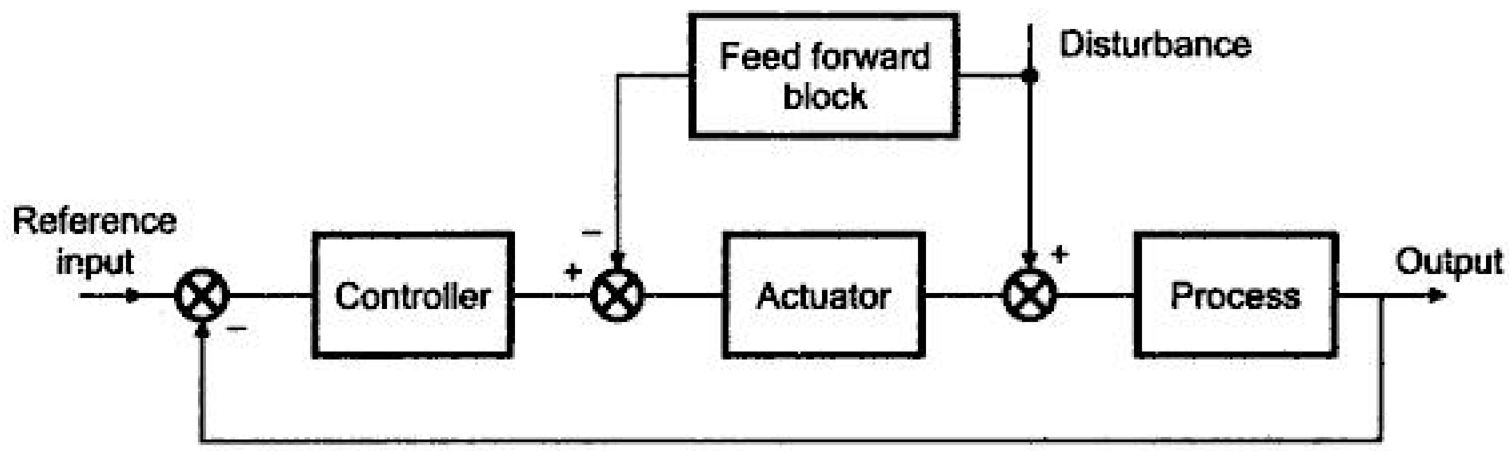


Fig. 1.21

The two difficulties associated with feed forward system are.

i) In some systems, the disturbance may not be measurable.

ii) The feedforward compensation is an open loop technique and if actuator transfer function is not known accurately, then such compensation cannot be achieved.

1.8 Servomechanisms :

Definition : It is a feedback control system in which the controlled variable or the output is a mechanical position or its time derivatives such as velocity or acceleration.

A simple example of servomechanism is a position control system. Consider a load which requires a constant position in its application. The position is sensed and converted to voltage using feedback potentiometer. It is compared with input potentiometer voltage to generate error signal. This is amplified and given to the controller. The controller in turn controls the voltage given to motor, due to which it changes its position.

The scheme is shown in the Fig. 1.22.

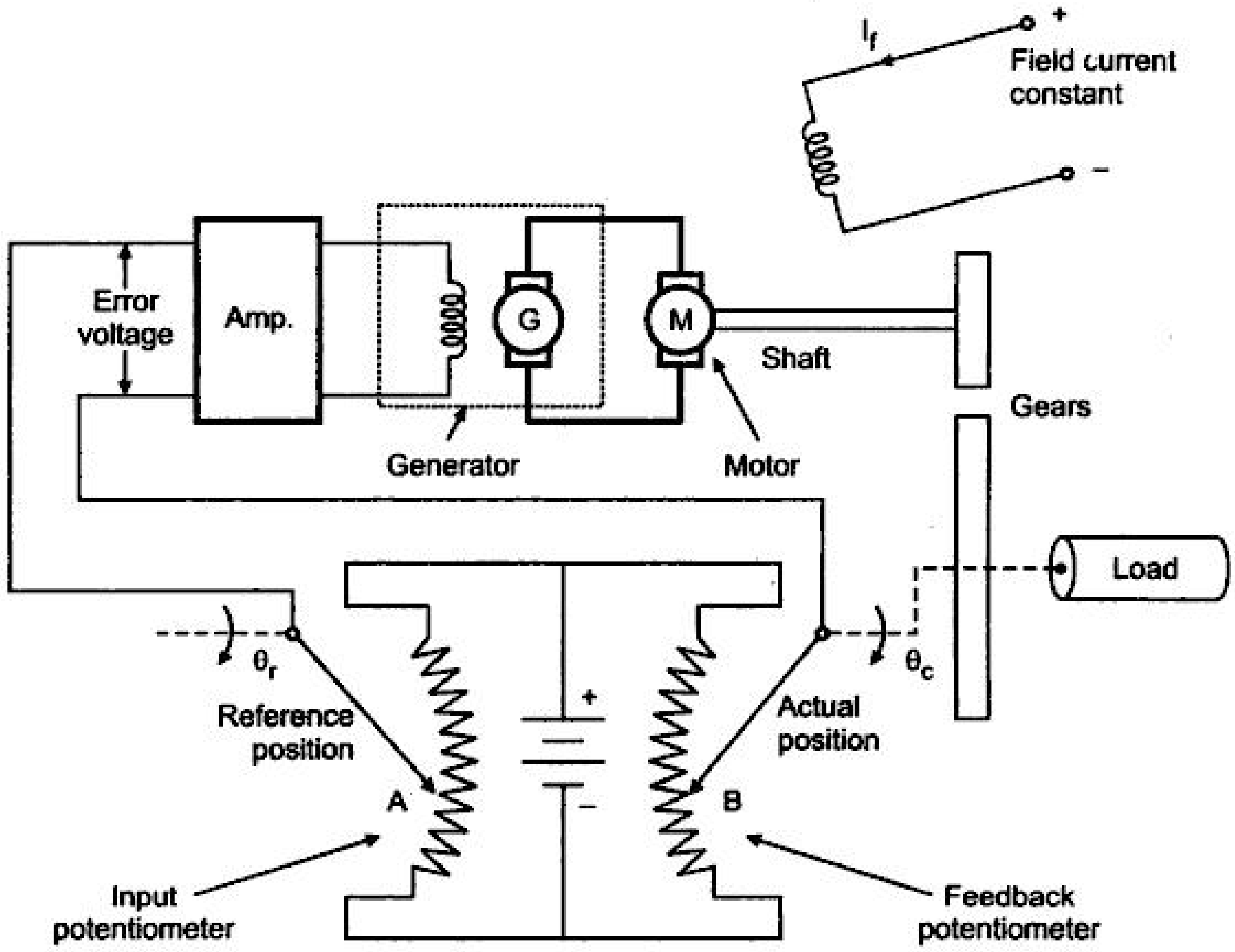


Fig. 1.22

Few other examples of servomechanisms are

1) Power steering apparatus for an automobile.
2) Machine tool position control.
3) Missile launchers.
4) Roll stabilization of ships.

1.9 Regulating Systems (Regulators) :

Definition : It is a feedback control system in which for a preset value of the reference input, the output is kept constant at its desired value.

In such systems reference input remains constant for long periods. Most of the times the reference input or the desired output is either constant or slowly varying with time. A regulator differs from a servomechanism in that the main function of a regulator is usually to maintain a constant output for a fixed input, while that of a servomechanism is mostly to cause the output of the system to follow a varying input.

A simple example of such regulator system is servostabilizer. We have seen earlier that in voltage stabilizer position of tap on secondary is adjusted by using relay controls. But instead of fixed tap, the entire secondary can be smoothly tapped using a servomotor drive. The servomotor drives the shaft and controls the position of tap on secondary as per the controller signal.

The actual scheme is shown in the Fig. 1.23.

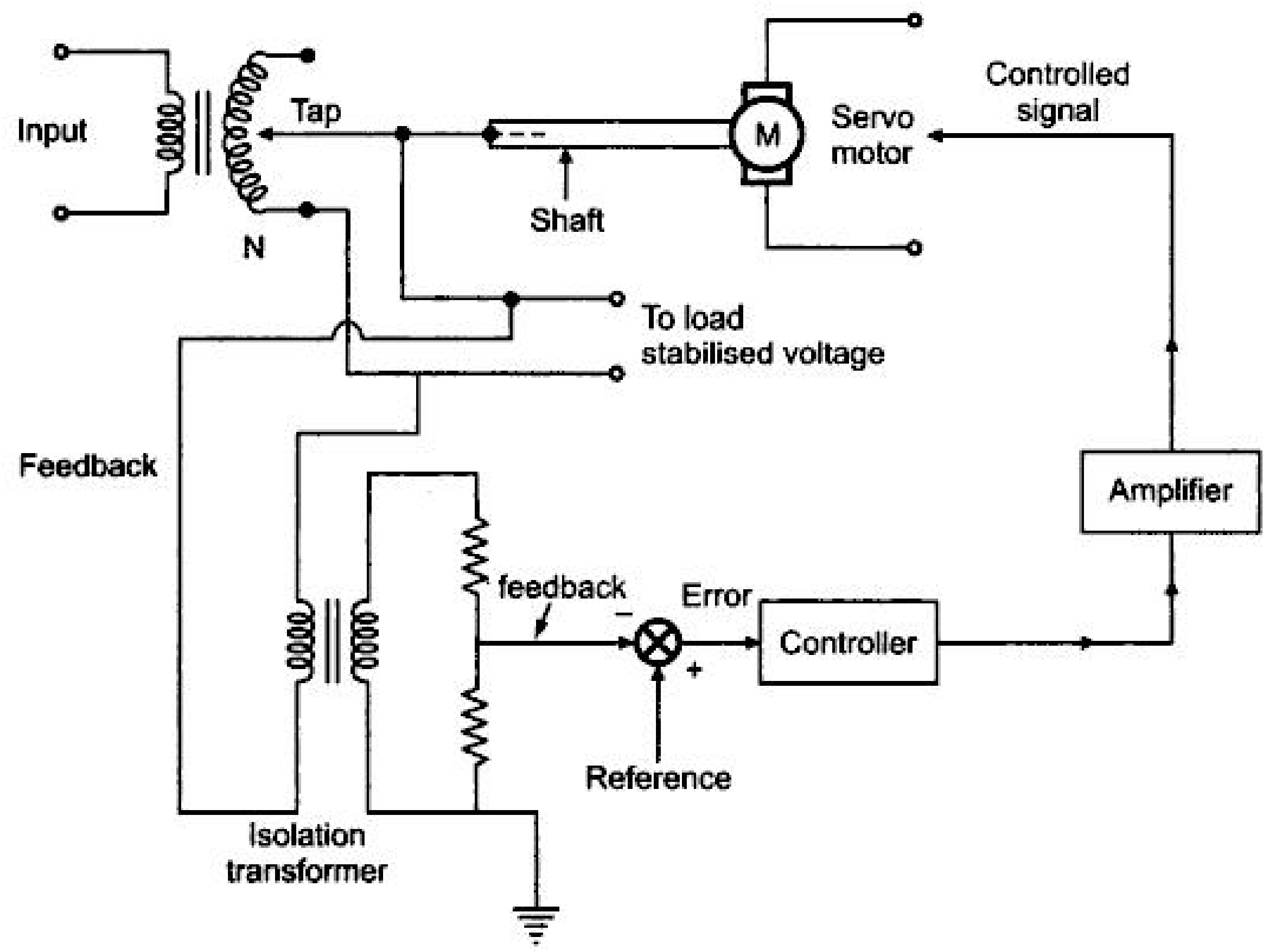


Fig. 1.23

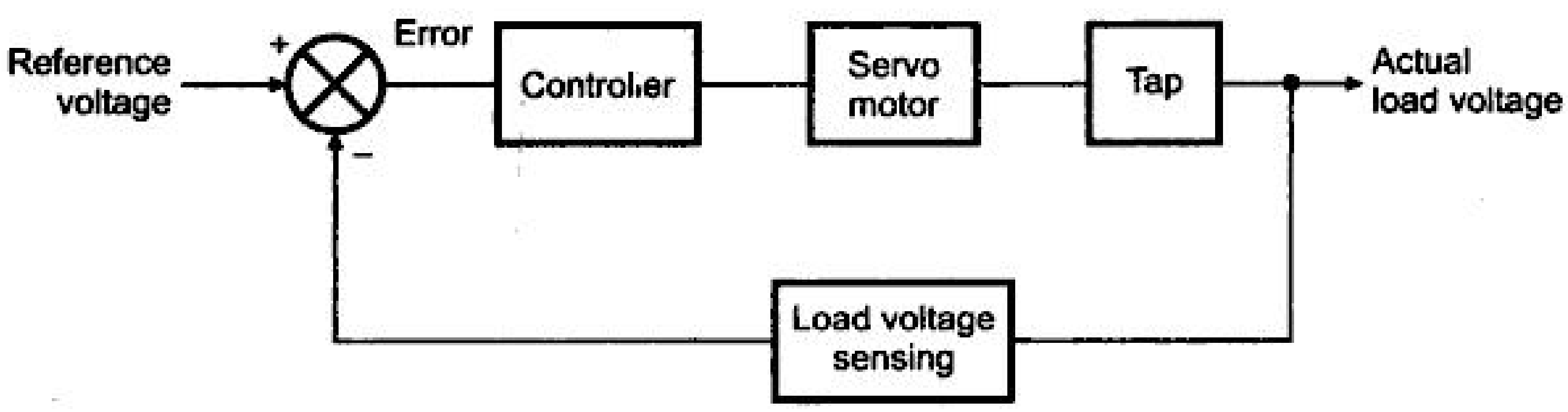


Fig. 1.24

Few other examples of regulating system are

1) Temperature regulators. 2) Frequency controllers. 3) Speed governors.

1.10 Multivariable Control Systems

The control system in which there is only one output of the interset is called single variable system. But in many practical applications more than one variables are involved. A control system with multiple inputs and multiple outputs in called a **multivariable system.**

The block diagram representation of a multivariable control system is shown in the Fig. 1.25. The part of the system which is required to be controlled is called plant. The controller provides proper controlling action depending on the reference inputs. There are n reference inputs r_1, r_2, ...r_n.

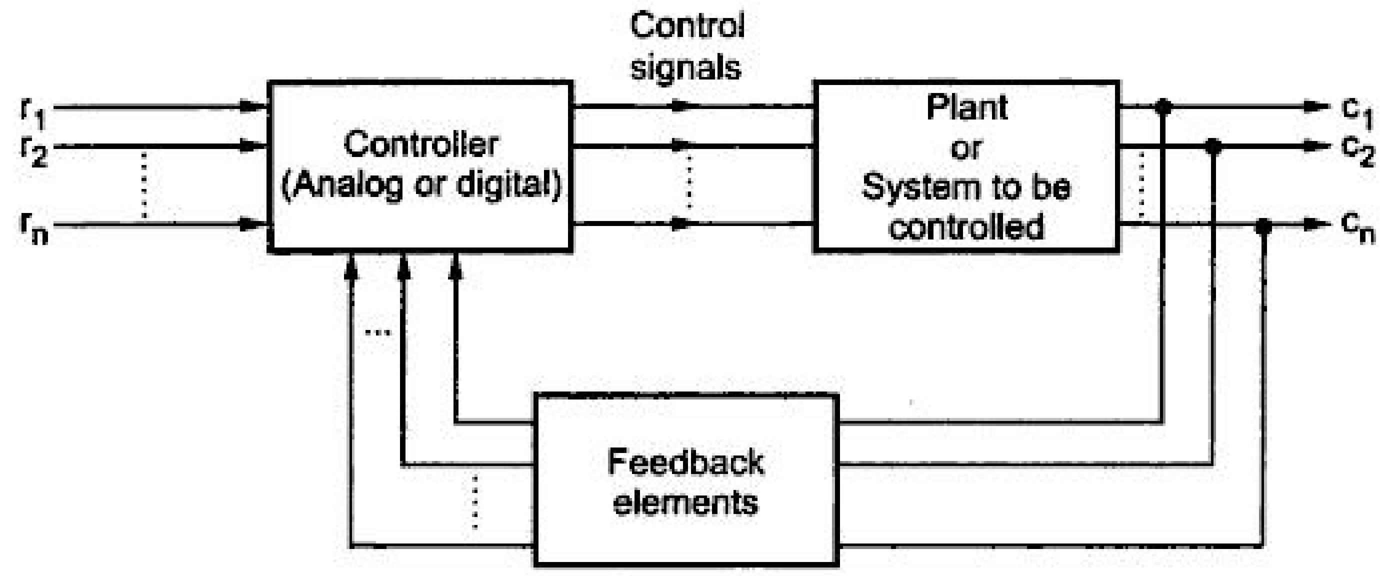


Fig. 1.25 Block diagram of multivariable control system

There are n output variables $c_1(t)$, $c_2(t)$,.........$c_n(t)$. The values of these variables represent the performance of the plant. The control signals produced by the controller are applied to the plant. With the help of feedback elements the closed loop control of the plant is also possible. Due to the feedback, the controller takes into account the actual output values to decide the control signals.

In case of multivariable systems, sometimes it is observed that a single input considerably affects more than one outputs. The system is said to be having strong **interactions** or **coupling.** This coupling is nothing but the disturbances for the separate systems. The interactions inherently present between inputs and outputs can be cancelled by designing a **decoupling controller.** Thus the resulting multivariable system is considered to have proper number of single input single output systems and the controller is designed for each system. The another way is to design a controller which will take care of all the inherent interactions present in the multivariable system.

In multivariable linear control system, each input is independently considered. Only one input and one output is considered and the total effect on any output because of all the inputs acting simultaneously is determined by addition of the outputs due to each input acting alone. Thus law of superposition is used to analyse multivariable linear control systems.

In many practical control systems, control is achieved by more than one input and the system may have many outputs. In chemical processes simultaneous control of pressure, temperature and concentration is required by commanding various inputs. Air crafts and space crafts are other examples where movement is controlled by various inputs. Power generators, atomic reactors and jet engines are some of other examples of multivariable systems.

Consider the block diagram of multivariable autopilot system shown in the Fig.1.26.

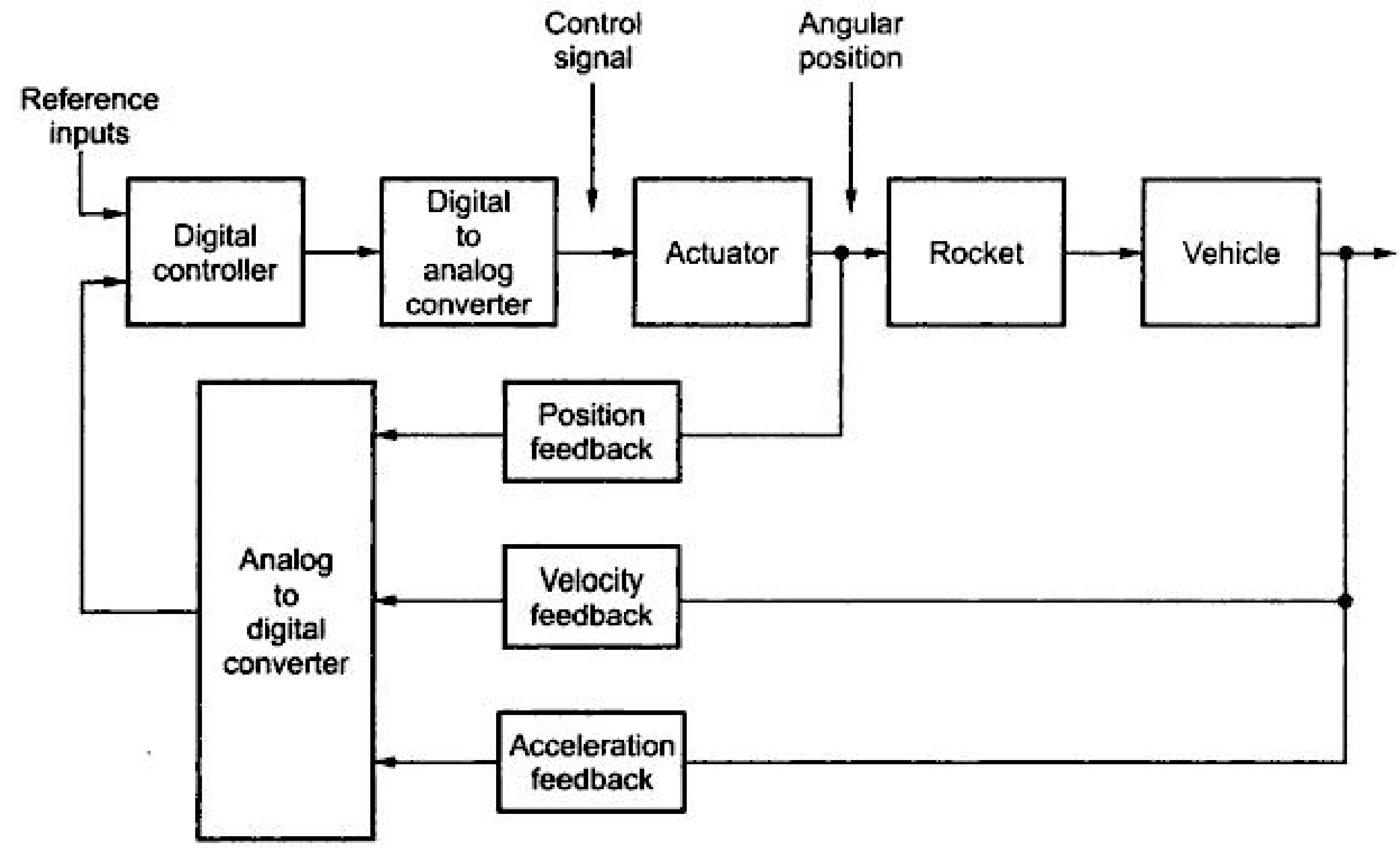


Fig. 1.26 Multivariable autopilot system

The system shown in the Fig. 1.26 keeps a track of rocket vehicle in response to reference inputs given to it. The position, velocity and acceleration of the vehicle are fed to the digital controller using motion sensors. The controller takes appropriate decision and sends a controlling signal which will drive the actuator, which will move the engine. Thus there are three output variables which are to be observed and controlled and there are corresponding reference inputs hence the system is multivariable system.

Summary

The major objective of this chapter has been the introduction of the terminology and different classifications of control system. A control system is an arrangement of different physical elements connected in such a manner so as to regulate, direct or command itself or some other system. Broadly, control systems are classified as natural, manmade and combinational control systems.

For the engineering analysis, the control systems are classified as

i) Time varying and time invariant

ii) Linear and Non-linear

iii) Continuous time and Discrete time

iv) Deterministic and Stochastic

v) Lumped parameter and Distributed parameter

vi) Single input single output and Multiple input multiple output.

vii) Open loop and Closed loop

viii) Feedback and Feedforward.

Open loop system is one in which feedback is absent and any changes in the output due to disturbances has no effect on the controller input. Feedback is an important feature of closed loop system. In closed loop system, output is compared with the input to generate an error which is then corrected by the controller to produce the required output.

In feedforward system, the disturbance is measured before it affects the output and compensating signal is added to the controller output. Servomechanism is a feedback control system in which the controlled variable is a mechanical position or its time derivatives such as velocity or acceleration. Regulator is a feedback control system in which for a preset value of the reference input, the output is kept constant at its desired value.

Review Questions

1. *Define the following terms*
   (i) System (ii) Control system (iii) Input (iv) Output (v) Disturbance.
2. *Explain how the control systems are classified*
3. *Define linear and nonlinear control systems.*
4. *What is time variant system? Give suitable example. How it is different than time in variant system?*
5. *Define open loop and closed loop system by giving suitable examples.*
6. *Differentiat between open loop and closed loop systems giving suitable examples.*
7. *With reference to feedback control system define the following terms*
   i) Command input (ii) Reference input (iii) Forward path (iv) Feedback path
8. *Explain the following terms giving suitable example*
   i) Servomechanism (ii) Regulator
9. *Distinguish between feedback control system and feed forward control system.*
10. *Differentiate between :*

    1. Linear and Nonlinear systems 2. Continuous and Discrete data systems
11. *Explain what is closed loop control system.*
12. *Write a note on multivariable control systems.*

❑❑❑

2

Transfer Function and Impulse Response

2.1 Introduction :

The indication of cause and effect relationship existing between input and output mathematically means to decide the transfer function of the given system. It is commonly used to characterize the input-output relationship of the system.

Transfer function explains mathematical function of the parameters of system performing on the applied input in order to produce the required output. Laplace transform plays an important role in making mathematical analysis easy. Laplace transform and its use in control system analysis is thoroughly discussed in Appendix-A.

2.2 Concept of Transfer Function :

In any system, first the system parameters are designed and their values are selected as per requirement. The input is selected next to see the performance of the system designed. This is shown in the Fig. 2.1

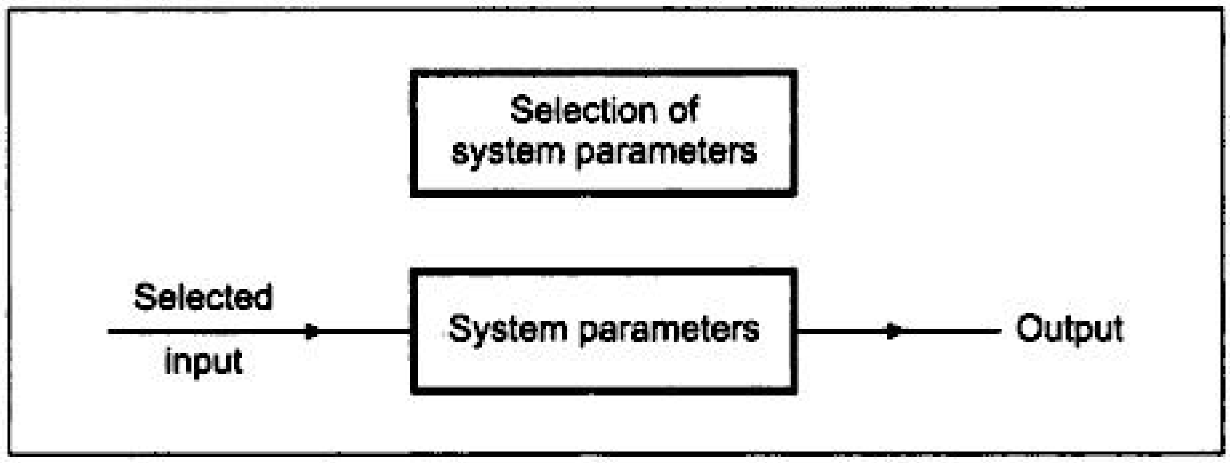


Fig. 2.1

Now performance of system can be expressed in terms of its output as,

Output = Effect of system parameters on the selected input

$\therefore$ Output = Input × Effect of system parameters.

$$\therefore \text{ Effect of system parameters} = \frac{\text{Output}}{\text{Input}}$$

This effect of system parameters, role of system parameters in the performance of system can be expressed as ratio of output to input. Mathematically such a function explaining the effect of system parameters on input to produce output is called as **transfer function**. Due to the own characteristics of the system parameters, the input gets transferred into output once applied to the system. This is the concept of transfer function. The exact definition of the transfer function is given in the next section.

2.3 Transfer Function :

2.3.1 Definition :

Mathematically it is defined as the ratio of Laplace transform of output (response) of the system to Laplace Transform of input (excitation or driving function), under the assumption that all initial conditions are zero.

Symbolically system can be represented as shown in Fig. 2.2(a). While the transfer function of system can be shown as in the Fig 2.2(b).

Fig. 2.2(a)

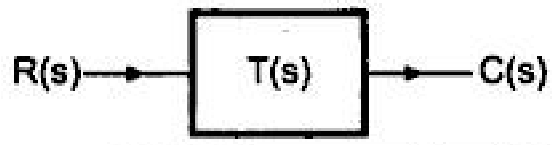


Fig. 2.2(b)

Transfer function of this system, $\frac{C(s)}{R(s)}$ where C(s) is Laplace of c(t) and R(s) is Laplace of r(t).

If T(s) is the transfer function of the system then

$$T(s) = \frac{\text{Laplace transform of output}}{\text{Laplace transform of input}} = \frac{C(s)}{R(s)}$$

Ex. 2.1 *For a system shown in Fig. 2.3, calculate its transfer function where $V_0(t)$ is output and $V_i(t)$ is input to the system.* **(Mumbai University May-99)**

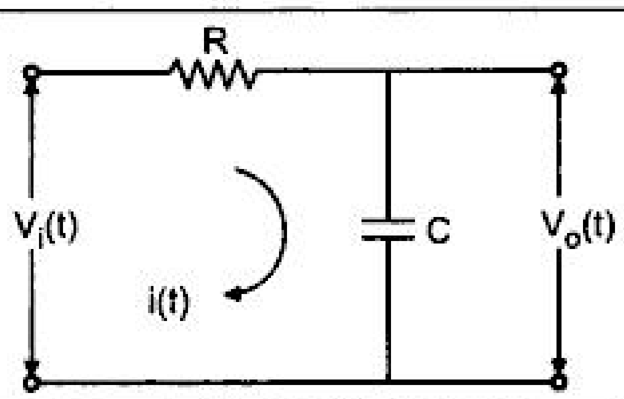


Fig. 2.3

Sol. : We can write for this system, equations by applying KVL as,

$$V_i(t) = R \cdot i(t) + \frac{1}{C}\int i(t)\,dt \qquad \text{... (1)}$$

and

$$V_0(t) = \frac{1}{C}\int i(t)\,dt \qquad \text{... (2)}$$

We are interested in $\frac{V_0(s)}{V_i(s)}$ where $V_0(s)$ is Laplace of $V_0(t)$ and $V_i(s)$ is Laplace of $V_i(t)$ and initial conditions are to be neglected.

So taking Laplace of above two equations and assuming initial conditions zero we can write

$$V_i(s) = RI(s) + \frac{1}{sC} I(s) \quad \text{... (3)}$$

$$V_o(s) = \frac{1}{sC} I(s) \quad \text{... (4)}$$

$$\therefore \quad I(s) = sCV_o(s)$$

Substituting in equation (3)

$$V_i(s) = sCV_o(s)\left[R + \frac{1}{sC}\right]$$

$$\therefore \quad V_i(s) = sCR\ V_o(s) + V_o(s) = V_o(s)[1 + sCR]$$

$$\therefore \quad \frac{V_o(s)}{V_i(s)} = \frac{1}{1 + sCR}$$

We can represent above system as in Fig. 2.4

Fig. 2.4

Ex. 2.2 *Find out the T.F. of the given network.*

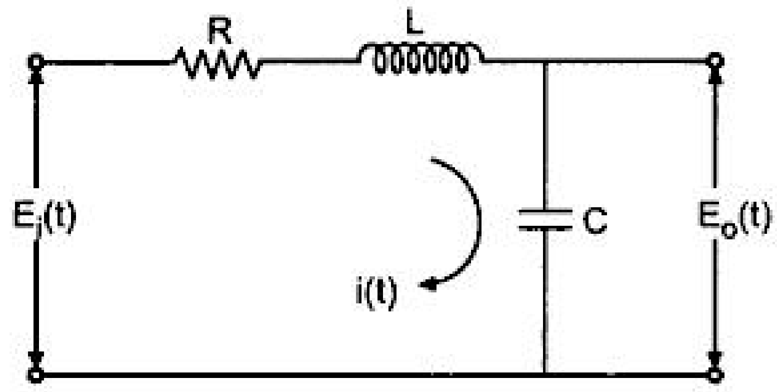


Fig. 2.5

Sol. : Applying KVL we get the equations as,

$$E_i = iR + L\frac{di}{dt} + \frac{1}{C}\int idt \quad \text{....(1)}$$

$$i/p = E_i \ ; \ o/p = E_o$$

Laplace transform of $\int F(t)\,dt = \frac{F(s)}{s}$, ... neglecting initial conditions

and laplace transform of $\frac{df(t)}{dt} = sF(s)$ ... neglecting initial conditions

Take Laplace transform,

$$\therefore \quad E_i(s) = I(s)\left[R + sL + \frac{1}{sC}\right]$$

$$\frac{I(s)}{E_i(s)} = \frac{1}{\left[R + sL + \frac{1}{sC}\right]} \quad \text{... (2)}$$

Now
$$E_o = \frac{1}{C}\int i\,dt \quad \text{... (3)}$$

$$\therefore \quad E_o(s) = \frac{1}{sC} I(s)$$

$$\therefore \quad I(s) = sC\ E_o(s) \quad \text{... (4)}$$

Substituting value of I(s) in equation (2)

$$\therefore \quad \frac{sCE_o(s)}{E_i(s)} = \frac{1}{\left[R + sL + \frac{1}{sC}\right]}$$

$$\therefore \quad \frac{E_o(s)}{E_i(s)} = \frac{1}{sC\left[R + sL + \frac{1}{sC}\right]} = \frac{1}{RsC + s^2\ LC + 1}$$

So we can represent the system as in Fig. 2.6

Fig. 2.6

2.3.2 Advantages and Features of Transfer Function :

i) It gives mathematical models of all system components and hence of the overall system. Individual analysis of various components is also possible by the transfer function approach.

ii) As it uses a Laplace approach, it converts integro-differential time domain equations to simple algebraic equations.

iii) It suggests operational method of expressing integro-differential equations which relate output to input.

iv) The transfer function is expressed only as a function of the complex variable 's'. It is not a function of the real variable, time or any other variable that is used as the independent variable.

v) It is the property and characteristics of the system itself. Its value is dependent on the parameters of the system and independent of the values of inputs. In the example 1, if the output i.e. focus of interest is selected as voltage across resistance R rather than the voltage across capacitor C, the transfer function will be different. So transfer function is to be obtained for a pair of input and output and then it remains constant for any selection of input as long as output variable is same. It helps in calculating the output for any type of input applied to the system.

vi) Once transfer function is known, output response for any type of reference input can be calculated.

vii) It helps in determining the important information about the system i.e. poles', zeros, characteristic equation etc..

viii) It helps in the stability analysis of the system.

ix) The system differential equation can be obtained by replacing variable 's' by d/dt.

2.3.3 Disadvantages :

i) Only applicable to linear time invariant systems.

ii) It does not provide any information concerning the physical structure of the system.

iii) Effects arising due to initial conditions are totally neglected. Hence initial conditions loose their importance.

2.3.4 Procedure to Determine the Transfer Function of a Control System :

The procedure used in Ex. 1 and Ex.2 can be generalised as below :

1) Write down the time domain equations for the system by introducing different variables in the system.

2) Take the Laplace transform of the system equations assuming all initial conditions to be zero.

3) Identify system input and output variables.

4) Eliminating introduced variables, get the resultant equation in terms of input and output variables.

5) Take the ratio of Laplace transform of output variable to Laplace transform of input variable to get the transfer function of the system.

Ex. 2.3 *Find out the T.F. of the given network*

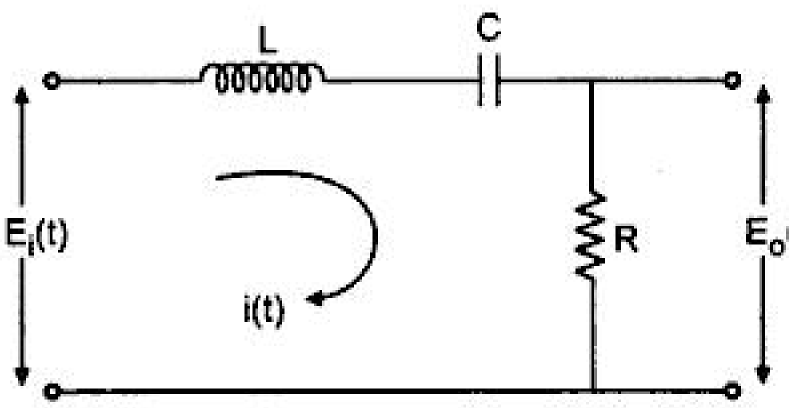


Fig. 2.7

Sol. : Applying KVL we can write,

$$E_i(t) = L\frac{d_i(t)}{dt} + \frac{1}{C}\int i(t)\,dt + i(t)R \qquad \text{... (1)}$$

While $\quad E_o(t) = i(t)R \qquad$ ... (2)

Where $\quad E_i(t) =$ input and $E_o(t) =$ output

Taking Laplace of equations (1) and (2), neglecting the initial conditions.

$$E_i(s) = sLI(s) + \frac{1}{C}\frac{I(s)}{s} + RI(s) \qquad \text{... (3)}$$

$$E_o(s) = I(s)R \qquad \text{... (4)}$$

$$\therefore \quad E_i(s) = I(s)\left[sL + \frac{1}{sC} + R\right] \text{ from (3)}$$

Substituting $\quad I(s) = \dfrac{E_o(s)}{R}$ from (4) in the above equation we get,

$$E_i(s) = \frac{E_o(s)}{R}\left[sL + \frac{1}{sC} + R\right]$$

$$\therefore \quad E_i(s) = \frac{E_o(s)}{R} \times \left[\frac{s^2LC + 1 + sCR}{sC}\right]$$

$$\therefore \quad \frac{E_o(s)}{E_i(s)} = \frac{sRC}{s^2\,LC + sRC + 1}$$

This is the required transfer function.

Note : The network in Ex. 2 and Ex. 3 is same but as focus of interest i.e. output is changed, the transfer function is changed. For a fixed output, transfer function is constant and independent of any type of input applied to the system.

2.4 Impulse Response and Transfer Function :

The impulse function is defined as

$$r(t) = A \qquad \text{for } t = 0$$
$$= 0 \qquad \text{for } t \neq 0$$

A unit impulse function $\delta(t)$ can be considered a narrow pulse (of any shape) occuring at zero time such that area under the pulse is unity and the time for which the pulse occurs tends to zero. In the limit $t \to 0$, the pulse reduces to a unit impulse $\delta(t)$. Consider a narrow rectangular pulse of width A and height $1/A$ units, so that the area under the pulse = 1, as shown in the Fig. 2.8(a).

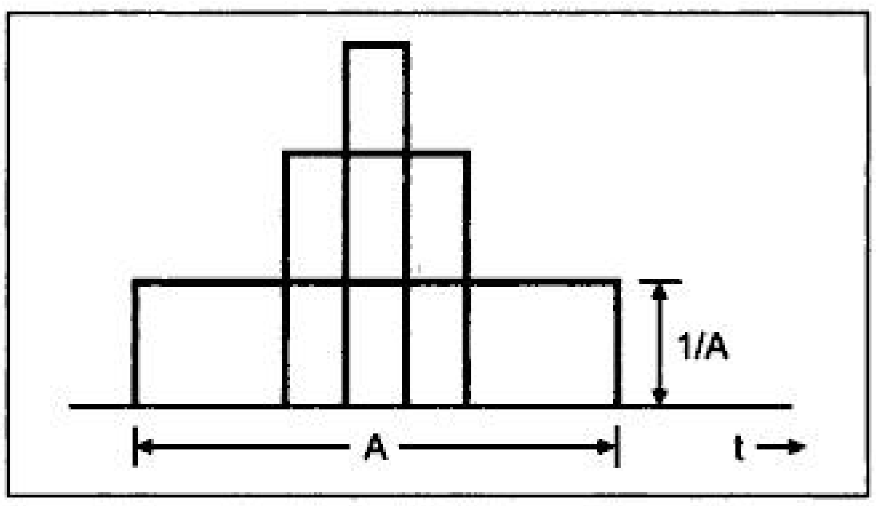


Fig. 2.8 (a)

Now if we go on reducing width A and maintain the area as unity then the height $1/A$ will go on increasing. Ultimately when $A \to 0$, $1/A \to \infty$ and the pulse is of infinite magnitude. It may then be called an **impulse of magnitude unity** and it is denoted by $\delta(t)$. It is not possible to draw an impulse function on paper, hence it is represented by a vertical arrow at t=0 as shown in the Fig. 2.8(b).

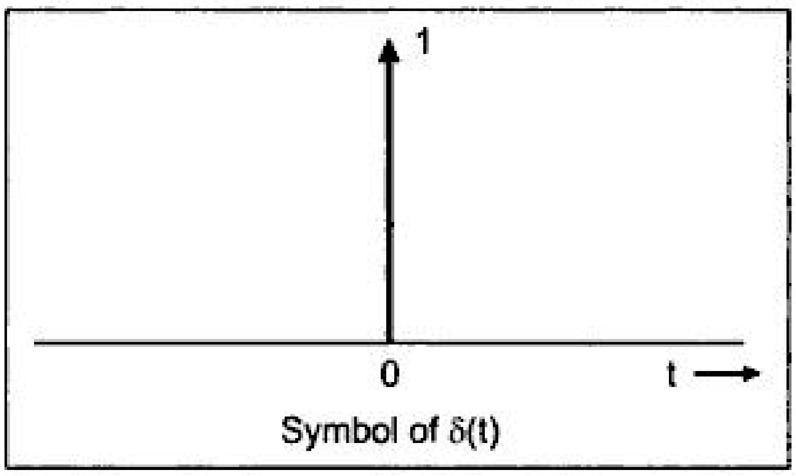


Fig. 2.8 (b)

So mathematically unit impulse is defined as,

$$\delta(t) = 1, \quad t = 0$$
$$= 0, \quad t \neq 0$$

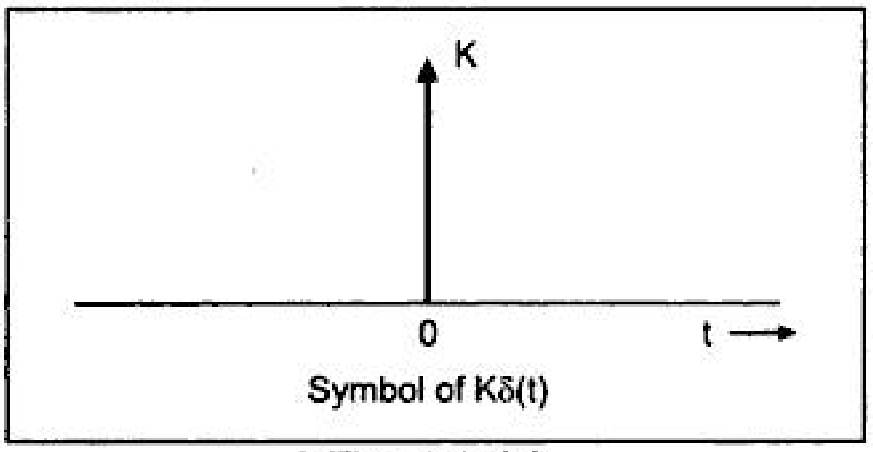


Fig. 2.8 (c)

If in the above example the area under the narrow pulse is maintained at K units while the period of pulse is reduced, it is called to be an impulse of magnitude 'K' and is denoted by $K\delta(t)$, as shown in the Fig. 2.8 (c).

An important property of impulse function is that if it is multiplied by any function and integrated then the result is the value of the function at t = 0

$$\text{Thus} \int_{-\infty}^{+\infty} f(t)\,\delta(t)\,dt = \int_{0^-}^{t} f(t)\,\delta(t)\,dt = \int_{0^-}^{0^+} f(t)\,\delta(t)\,dt = f(t)\big|_{t=0}.$$

This is called 'sampling' property of impulse. Hence if we define Laplace transform of $\delta(t)$ as

$$L[\delta(t)] = \int_0^{\infty} \delta(t)\,e^{-st}\,dt \quad \text{... by definition}$$
$$= e^{-st}\big|_{t=0} \quad \text{... by sampling property.}$$
$$= e^{-0} = 1$$
$$\therefore \quad L\{\delta(t)\} = 1$$

Thus Laplace Transform of impulse function $\delta(t) = 1$.

$$\text{Now} \quad T(s) = \frac{C(s)}{R(s)}$$
$$\therefore \quad C(s) = R(s) \cdot T(s)$$

So response C(s) can be determined for any input once T(s) is determined.

Note : The equation $[C(s) = R(s) \cdot T(s)]$ is applicable only in Laplace domain and cannot be used in time domain. The equation $[c(t) = r(t) \cdot t(t)]$ is **not at all valid** in time domain.

Now consider that input be unit impulse i.e.

$$r(t) = \delta(t) = \text{impulse input}$$

$$\therefore \quad R(s) = L\{\delta(t)\} = 1$$

Substituting in above

$$C(s) = 1 \cdot T(s) = T(s)$$

Now $$c(t) = L^{-1}\{C(s)\} = L^{-1}\{T(s)\} = T(t)$$

Thus we can say that for impulse input, impulse response C(s) equals the transfer function T(s). So impulse response is c(t) = T(t) as C(s) = T(s) hence we can conclude that,

Laplace transform of impulse response of a linear time invariant system is its transfer function with all the initial conditions asssumed to be zero.

Ex. 2.4 *The unit impulse response of a certain system is found to be* e^{-4t}. *Determine its transfer function.*

Sol. : Laplace of unit impulse response is the transfer function.

$$\therefore \quad L\left\{e^{-4t}\right\} = T(s)$$

$$\therefore \quad T(s) = \frac{1}{s+4}$$

Ex. 2.5 *The Laplace inverse of the transfer function in time domain of a certain system is* e^{-5t} *while its input is r(t) = 2. Determine its output c(t).*

Sol. : Let T(s) be the transfer function

$$L^{-1}[T(s)] = T(t) = e^{-5t} \qquad \text{given}$$

$$r(t) = 2$$

But $$c(t) \neq r(t) \times T(t),$$

it is mentioned earlier that $\frac{c(t)}{r(t)} = T(t)$ is not at all valid in time domain, so

$$c(t) \neq 2e^{-5t}$$

Hence the equation valid according to the definition of transfer function is,

$$T(s) = \frac{C(s)}{R(s)}$$

so $$T(s) = L\{T(t)\} = L\left\{e^{-5t}\right\}$$

$$= \frac{1}{s+5}$$

$$R(s) = \frac{2}{s}$$

$$\therefore \quad \frac{1}{s+5} = \frac{C(s)}{\left(\frac{2}{s}\right)}$$

$$\therefore \qquad C(s) = \frac{2}{s(s+5)} = \frac{a_1}{s} + \frac{a_2}{s+5}$$

$$\therefore \qquad C(s) = \frac{0.4}{s} - \frac{0.4}{s+5}$$

Taking Laplace inverse of this equation

$$c(t) = 0.4 - 0.4\, e^{-5t}$$

This is the required output expression.

2.5 Some Important Terminologies Related to T.F. :

As transfer function is a ratio of Laplace of output to input it can be expressed as a ratio of polynomials in 's'.

$$\text{T.F.} = \frac{P(s)}{Q(s)}$$

This can be further expressed as,

$$= \frac{a_0 s^m + a_1 s^{m-1} + a_2 s^{m-2} + \ldots + a_m}{b_0 s^n + b_1 s^{n-1} + b_2 s^{n-2} + \ldots + b_n}$$

The numerator and denominator can be factorised to get the factorised form of the transfer function.

$$\text{T.F.} = \frac{K(s-s_a)(s-s_b)\ldots\ldots(s-s_m)}{(s-s_1)(s-s_2)\ldots\ldots(s-s_n)}$$

Where K is called as system gain factor. Now if in the transfer function, values of 's' are substituted as $s_1, s_2, s_3 \ldots\ldots s_n$ in the denominator then value of T.F. will become infinity.

2.5.1 Poles of a Transfer Function :

Definition : The values of 's', which make the T.F. infinite after substitution in the denominator of a T.F. are called **'Poles'** of that T.F.

So values of $s_1, s_2, s_3 \ldots\ldots s_n$ are called poles of the T.F.

These poles are nothing but the roots of the equation obtained by equating denominator of a T.F. to zero.

For example, let the transfer function of a system be

$$T(s) = \frac{2(s+2)}{s(s+4)}$$

The equation obtained by equating denominator to zero is,

$$s(s+4) = 0$$

$$\therefore \quad s = 0 \quad \text{and} \quad s = -4$$

If these values are used in the denominator, the value of transfer function becomes infinity. Hence poles of this transfer function are s = 0 and – 4.

If the poles are like s = 0, –4, –2, +5, ... i.e. real and without repeated values, they are called as **simple poles**. A pole having same value twice or more than that is called as **repeated pole.** A pair of poles with complex conjugate values is called **complex conjugate poles.**

e.g. For $$T(s) = \frac{2(s+2)}{(s+4)^2\ (s^2+2s+2)\ (s+1)}$$

The poles are the roots of the equation $(s+4)^2\ (s^2+2s+2)\ (s+1) = 0$.

$\therefore$ Poles are $s = -4, -4, \ -1 \ \pm j1, -1$

so T(s) has simple pole at s = –1,

Repeated pole at s = –4, (two poles)

Complex conjugate poles at s = –1 ± j1

Poles are indicated by 'X' (cross) in s-plane.

2.5.2 Characteristic Equation of a Transfer Function :

Definition : The equation obtained by equating denominator of a T.F. to zero, whose roots are the poles of that T.F. is called as **characteristic equation** of that system.

$$F(s) = b_0 s^n + b_1 s^{n-1} + b_2 s^{n-2} + \ldots\ldots + b_n = 0$$

is called as the characteristic equation.

2.5.3 Zeros of a Transfer Function :

Similarly now if the values of 's' are substituted as $s_a, s_b \ldots\ldots + s_m$ in the numerator of a T.F., its value becomes zero.

Definition : The values of 's' which make the T.F. zero after substituting in the numerator are called as **'zeros'** of that T.F.

Such zeros are the roots of the equation obtained by equating numerator of a T.F. to zero. Such zeros are indicated by '0' (zero) in s-plane.

Poles and zeros may be real or complex-conjugates or combination of both types.

Poles and zeros may be located at the origin in s-plane.

Similar to the poles, the zeros also are called as simple zeros, repeated zeros and complex conjugate zeros depending upon their nature.

e.g. $$T(s) = \frac{2(s+1)^2\ (s+2)\ (s^2+2s+2)}{s^3\ (s+4)\ (s^2+6s+25)}$$

This transfer function has zeros which are roots of the equation,

$$2(s+1)^2\ (s+2)\ (s^2+2s+2) = 0$$

i.e. Simple zero at s = –2

Repeated zero at s = –1 (twice)

Complex conjugate zeros at $s = -1 \pm j1$.

The zeros are indicated by small circle 'O' in the s-plane.

2.5.4 Pole-Zero Plot :

Definition : Plot obtained by locating all poles and zeros of a T.F. in s-plane is called as **pole-zero** plot of a system.

2.5.5 Order of a Transfer Function :

Definition : The highest power of 's' present in the characteristic equation i.e. in the denominator polynomial of a closed loop transfer function of a system is called as 'Order' of a system.

For example consider Example 1 discussed earlier. The system T.F. is $\frac{1}{1+sRC}$

i.e. $1 + sRC = 0$ is its characteristic equation and system is first order system.

Then $s = -1/RC$ is a pole of that system and T.F. has no zeros.

The corresponding pole-zero plot can be shown as in Fig. 2.9.

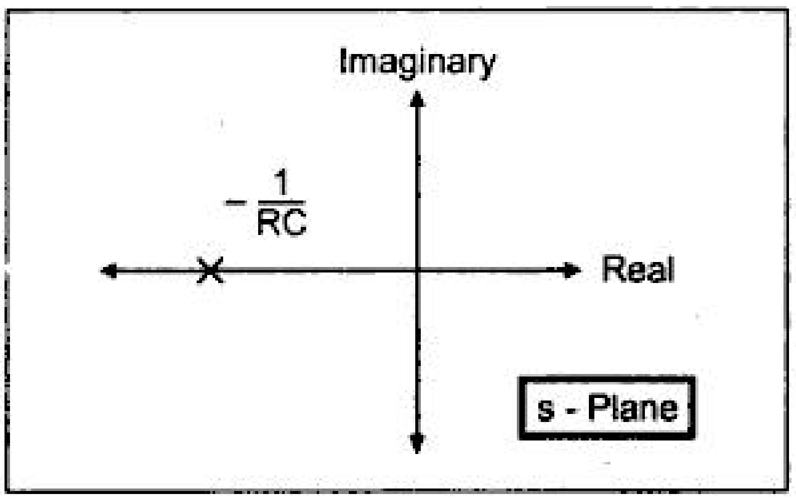


Fig. 2.9

Similarly for Example 2, the T.F. calculated is

$$\text{T.F.} = \frac{1}{s^2LC + sRC + 1} = \frac{1/LC}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

The characteristic equation is,

$$s^2 + s\frac{R}{L} + \frac{1}{LC} = 0$$

So system is 2nd order and the two poles are, $-\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$

T.F. has no zeros.

Now if values of R, L and C selected are such that both poles are real, unequal and negative the corresponding pole-zero plot can be shown as in Fig. 2.10

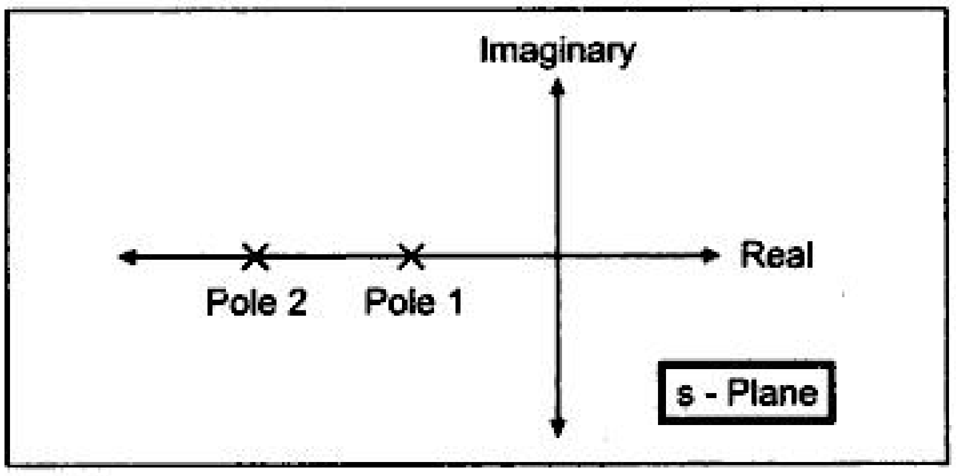


Fig. 2.10

For a system having T.F. as,

$$\frac{C(s)}{R(s)} = \frac{(s+2)}{s\left[s^2+2s+2\right]\left[s^2+7s+12\right]}$$

The characteristic equation is,

$$s(s^2+2s+2)(s^2+7s+12) = 0$$

i.e. system is 5th order and there are 5 poles. Poles are 0, $-1 \pm j$, –3, –4 while zero is located at '–2'.

The corresponding pole-zero plot can be drawn as shown in Fig. 2.11.

After getting familiar with introductory remarks about control system, now it is necessary to see how overall systems are represented and which are the methods to represent the given system based on the transfer function approach.

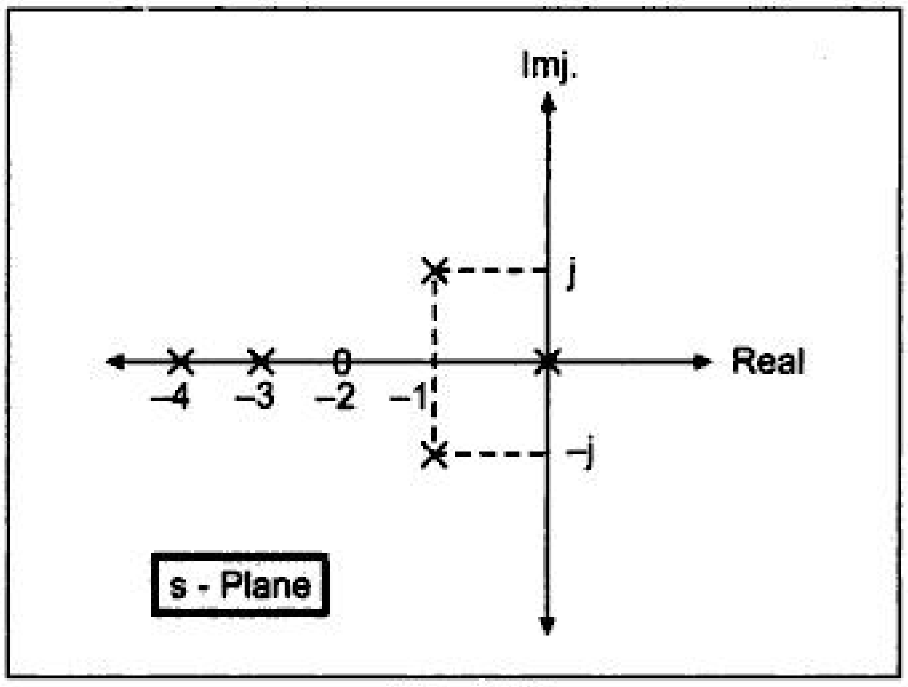


Fig. 2.11

2.6 Laplace Transform of Electrical Network :

In the use of Laplace in electrical systems, it is always easy to redraw the system by finding Laplace transform of the given network. Electrical network mostly consists of the parameters R, L and C. The various expressions related to these parameters in time domain and Laplace domain are given in the table below.

Element	Time domain expression	Laplace domain expression
Resistance R	$i(t) \times R$	$I(s)R$
Inductance L	$L\frac{di(t)}{dt}$	$sLI(s)$
Capacitance C	$\frac{1}{C}\int i(t)\,dt$	$\frac{1}{sC}I(s)$

Table 2.1

From the table it can be seen that after taking Laplace transform of time domain equations neglecting the initial conditions, the resistance R behaves as R, the inductance behaves as sL, while the capacitance behaves as $\frac{1}{sC}$ and all time domain functions get converted to Laplace domain like i(t) to I(s), V(t) to V(s) and so on.

By using these transformations, the parameters can be replaced by their Laplace transform to get Laplace transform of entire network. Once this is obtained, simple algebraic equations relating Laplace of various voltages and currents can be directly obtained. This eliminates the step of writing the integrodifferential equations and taking Laplace of them.

e.g. Consider a network shown below.

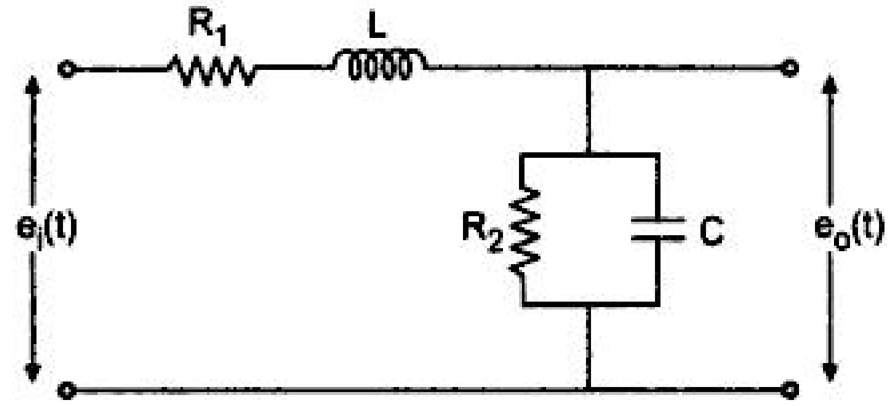


Fig. 2.12

The Laplace of the above network can be obtained by following replacements.

$$R_1 \rightarrow R_1$$
$$L \rightarrow sL$$
$$R_2 \rightarrow R_2$$
$$C \rightarrow \frac{1}{sC}$$
$$e_i(t) \rightarrow E_i(s)$$
$$e_o(t) \rightarrow E_o(s)$$

The other variables then can be introduced which will be directly Laplace variables to obtain the Laplace domain equations directly. Such Laplace of network is shown below in the Fig. 2.13.

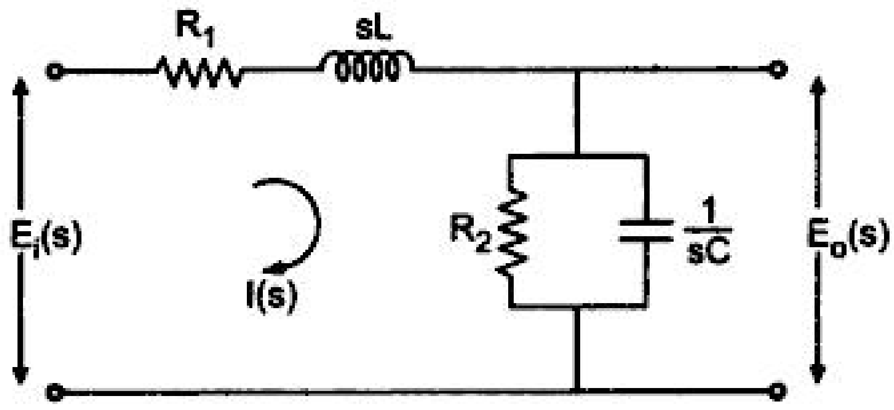


Fig. 2.13

Ex. 2.6 *The transfer function of a system is given by*

$$T(s) = \frac{K(s+6)}{s(s+2)(s+5)(s^2+7s+12)}$$

Determine i) Poles ii) Zeros iii) Characteristic equation and iv) Pole-zero plot in s-plane.

Sol. :

i) Poles are the roots of the equation obtained by equating denominator to zero i.e. roots of

$$s(s+2)(s+5)(s^2+7s+12) = 0$$

i.e. $s(s+2)(s+5)(s+3)(s+4) = 0$

So there are 5 poles located at

$s = 0, -2, -5, -3$ and -4

ii) Zeros are the roots of the equation obtained by equating numerator to zero i.e. roots of

$$K(s+6) = 0$$

i.e. $$s = -6$$

There is only one zero.

iii) Characteristic equation is one, whose roots are the poles of the transfer function. So it is

$$s(s+2)(s+5)(s^2+7s+12) = 0$$

i.e. $s(s^2+7s+10)(s^2+7s+12) = 0$

i.e $s^5+14s^4+71s^3+154s^2+120s = 0$

iv) Pole-zero plot

This is shown in the Fig. 2.14

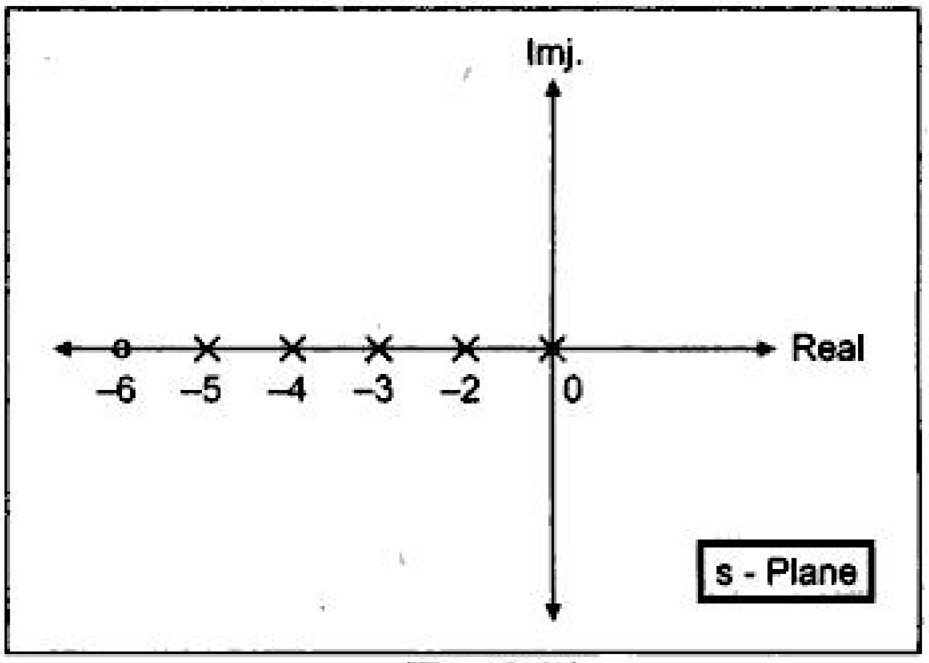


Fig. 2.14

Ex. 2.7 *The unit impulse response of a system is given by $T(t) = e^{-t}(1 - \cos 2t)$ Determine its transfer function*

Sol. : Laplace transform of the impulse response is the transfer function.

$$T(s) = L\{T(t)\}$$

$$= L\{e^{-t}(1-\cos 2t)\} = L\{e^{-t}\} - L\{e^{-t}\cos 2t\}$$

$$= \frac{1}{s+1} - \frac{(s+1)}{(s+1)^2+(2)^2} = \frac{1}{s+1} - \frac{(s+1)}{(s^2+2s+5)}$$

$$\therefore \quad T(s) = \frac{4}{(s+1)(s^2+2s+5)}$$

Ex. 2.8 *Obtain the transfer function of the lead network shown below.*

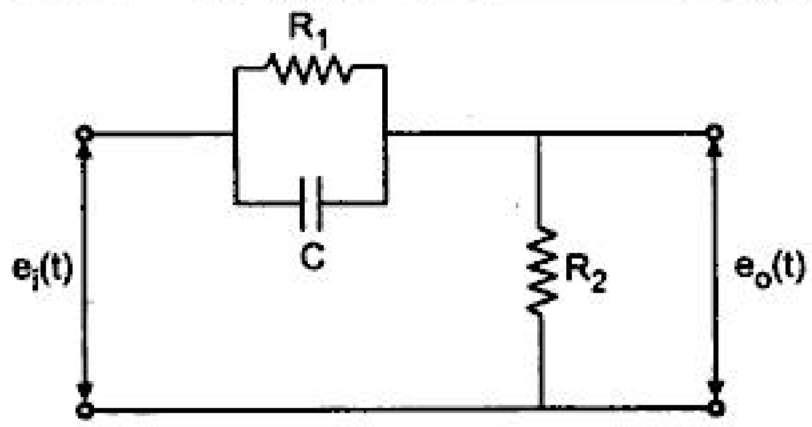


Fig. 2.15

Sol. : Taking Laplace transform of the network

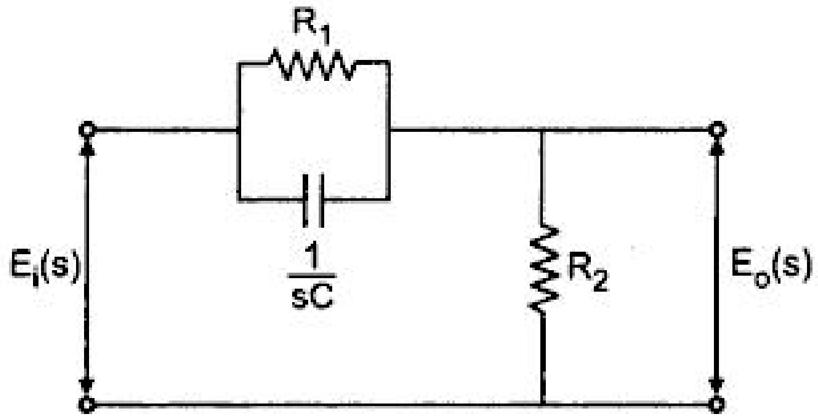


Fig. 2.16

The parallel combination of R_1 and $\frac{1}{sC}$ gives impedance of

$$Z = \frac{R_1 \times \frac{1}{sC}}{R_1 + \frac{1}{sC}} = \frac{R_1}{1 + sR_1C}$$

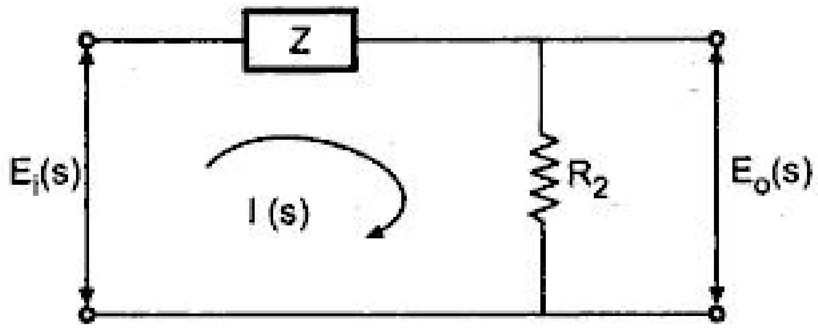


Fig. 2.17

Applying KVL to the circuit

$$E_i(s) = Z\,I(s) + I(s)\,R_2 \qquad \text{... (1)}$$

$$E_o(s) = I(s)\,R_2 \qquad \text{... (2)}$$

$$\therefore \quad I(s) = \frac{E_o(s)}{R_2} \qquad \text{from (2)}$$

Substituting in (1) we get

$$E_i(s) = I(s)\,[Z + R_2] = \frac{E_o(s)}{R_2}[Z + R_2]$$

$$\therefore \quad \frac{E_o(s)}{E_i(s)} = \frac{R_2}{Z + R_2}$$

Substituting Z, $\quad \text{T. F.} = \dfrac{R_2}{\dfrac{R_1}{1 + sR_1C} + R_2} = \dfrac{R_2(1 + sR_1C)}{R_1 + R_2(1 + sR_1C)}$

$$= \frac{s R_1 R_2 C + R_2}{R_1 + s R_1 R_2 C + R_2}$$

$$= \frac{s+\alpha}{s+\beta}$$

where $\alpha = \dfrac{1}{R_1 C}$,

$$\beta = \frac{(R_1 + R_2)}{R_1 R_2 C}$$

This circuit is also called as lead compensator.

Ex. 2.9 *Obtain the transfer function of the lag network shown.*

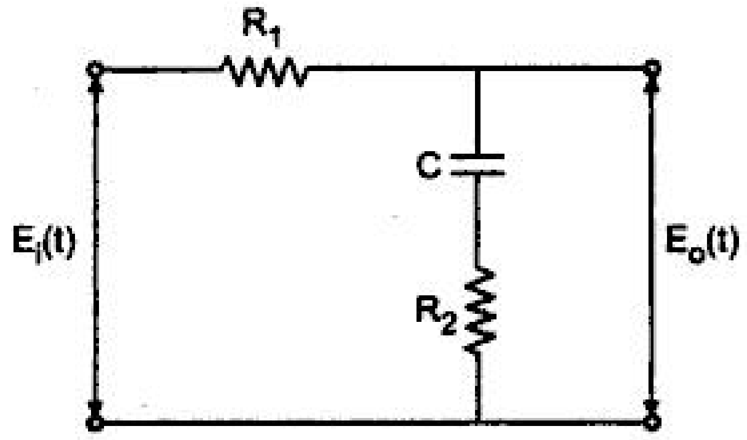


Fig. 2.18

Sol. : Taking Laplace transform of the network

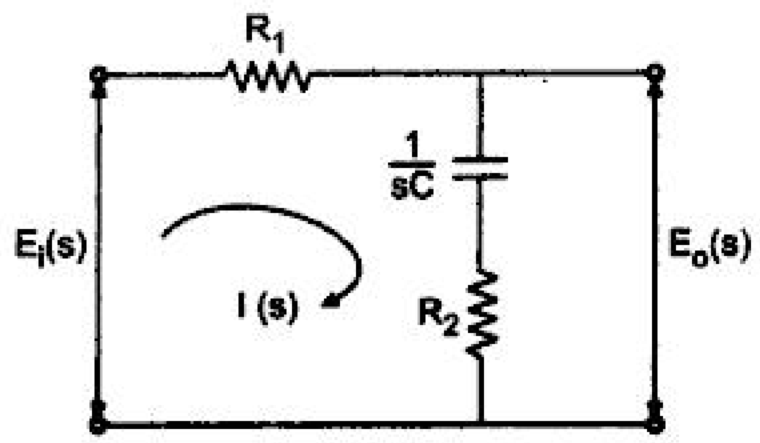


Fig. 2.19

Applying KVL to the network,

$$E_i\ (s) = R_1\ I(s) + \frac{1}{sC} I(s) + I(s)\ R_2 \qquad \text{... (1)}$$

$$E_o\ (s) = \frac{1}{sC} I(s) + R_2\ I(s) \qquad \text{... (2)}$$

$$\therefore \quad E_o\ (s) = I(s)\left[\frac{1}{sC} + R_2\right] = I(s)\left[\frac{1+sCR_2}{sC}\right]$$

$$\therefore \quad I(s) = \frac{sC\,E_0(s)}{1+sCR_2}$$

Substituting in (1)

$$E_i(s) = \frac{sCE_0(s)}{1+sCR_2}\left[\frac{R_1 sC + 1 + sCR_2}{sC}\right]$$

$$= E_0(s)\frac{1+sC[R_1+R_2]}{1+sCR_2}$$

$$\therefore \quad \frac{E_0(s)}{E_i(s)} = \frac{1+sCR_2}{1+sC[R_1+R_2]}$$

Ex. 2.10 *The dynamic behaviour of the system is described by the equation* $\frac{dC}{dt} + 10\,C = 40\,e$ *where e is the input and C is the output. Determine the transfer function of the system.*

Sol. : Take Laplace of the given differential equation and assume all initial conditions zero.

$$\therefore \quad s\,C(s) + 10C(s) = 40\,E(s)$$

$$\therefore \quad (s+10)\,C(s) = 40\,E(s)$$

$$\therefore \quad \frac{C(s)}{E(s)} = \frac{40}{s+10}$$

Ex. 2.11 *Find the transfer function* $\frac{C(s)}{R(s)}$ *of a system having differential equation given below.*

$$2\frac{d^2\,C(t)}{dt^2} + 2\frac{d\,C(t)}{dt} + C(t) = r(t) + 2r\,(t-1)$$

Sol. : Taking Laplace transform of the given equation and assuming all initial conditions zero we get

$$2s^2C(s) + 2s\,C(s) + C(s) = R(s) + 2e^{-s}\,R(s)$$

Laplace transform of delayed function is

$$L\,\{f\,(t-T)\} = e^{-sT}\,F(s) \qquad \text{(Refer Appendix A)}$$

$$\therefore \quad L\,\{r\,(t-1)\} = e^{-s.1}\,R(s)$$

Combining terms of C(s) and R(s) we get

$$(2s^2 + 2s + 1)\,C(s) = R(s)\,(1 + 2e^{-s})$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{1+2e^{-s}}{2s^2+2s+1}$$

Ex. 2.12 *Find $V_o(s) / V_i(s)$*

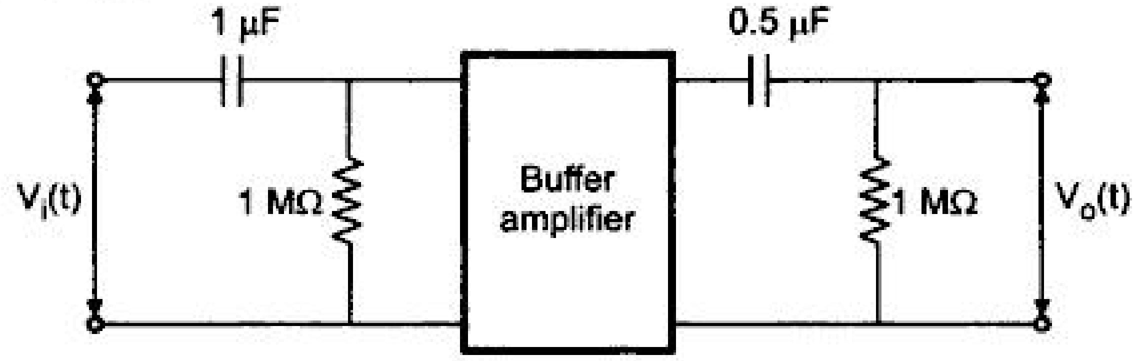


Fig. 2.20

Assume gain of buffer amplifier as 1.

Sol. : Taking Laplace transform of the network

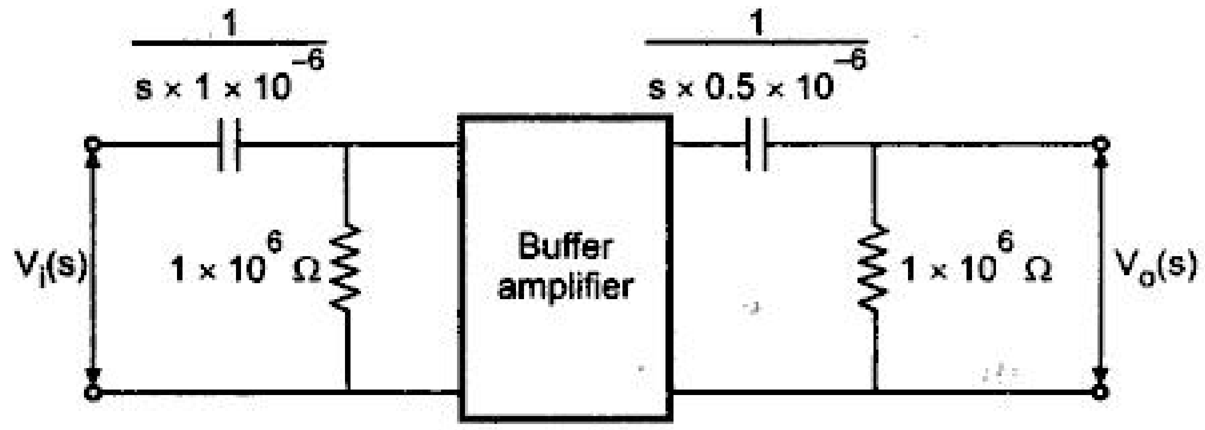


Fig. 2.21

Let us divide the network into two parts

Part 1)

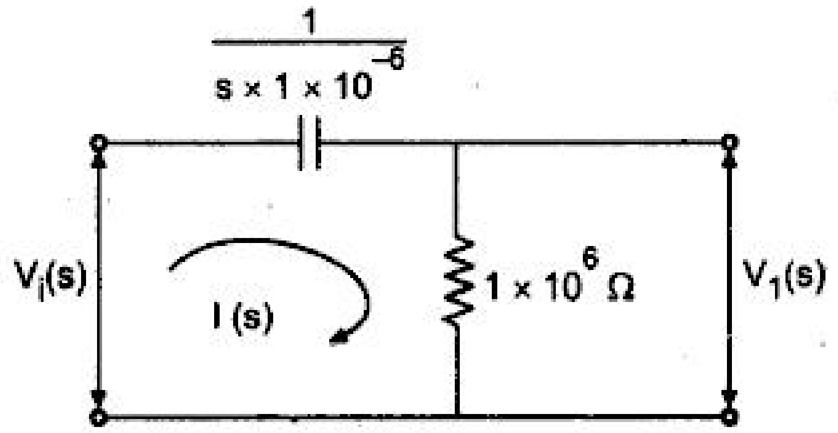


Fig. 2.22

Applying KVL

$$V_i(s) = \frac{1}{s \times 1 \times 10^{-6}} I(s) + 1 \times 10^6 I(s) \quad \text{... (1)}$$

$$V_1(s) = 1 \times 10^6 I(s) \quad \text{... (2)}$$

$$\therefore \quad I(s) = \frac{V_1(s)}{1 \times 10^6}$$

Substituting in (1) $$V_i(s) = \left[\frac{10^6}{s} + 10^6\right] I(s) = \left[\frac{10^6 + s10^6}{s}\right]\left[\frac{V_1(s)}{10^6}\right]$$

$$\therefore \quad \frac{V_1(s)}{V_i(s)} = \frac{s}{s+1}$$

Part 2)

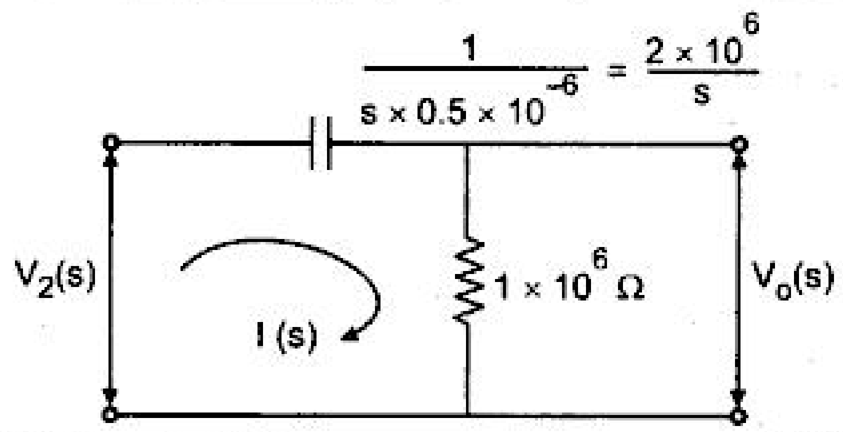


Fig. 2.23

$$\therefore \quad V_2(s) = I(s)\left[\frac{2\times 10^6}{s} + 1\times 10^6\right] \qquad \text{... (1)}$$

$$V_o(s) = I(s)\,1 \times 10^6 \qquad \text{... (2)}$$

$$\therefore \quad I(s) = \frac{V_o(s)}{10^6}$$

Substituting in (1) $V_2(s) = \frac{V_o(s)}{10^6}\left[\frac{2+s}{s}\right]10^6$

$$\therefore \quad \frac{V_o(s)}{V_2(s)} = \frac{s}{s+2}$$

Now gain of buffer amplifier is 1 (unity)

$$\therefore \quad V_1(s) = V_2(s)$$

$$\therefore \quad \left(\frac{s}{s+1}\right)V_i(s) = \frac{(s+2)}{s}\,V_o(s)$$

$$\therefore \quad \frac{V_o(s)}{V_i(s)} = \frac{s^2}{(s+1)(s+2)}$$

This is the required transfer function.

Ex. 2.13 *Determine $V_o(t)$, if $C_2 = 3\,C_1$ and $V_i(t) = 20\,e^{-3t}$ and switch is closed at $t = 0$*

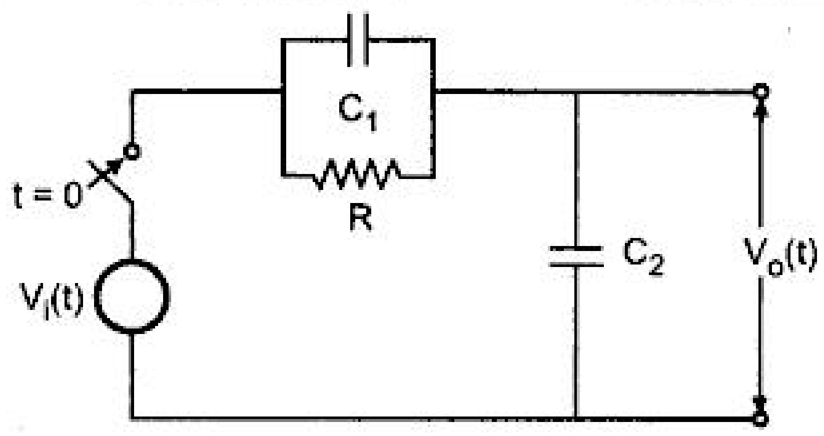


Fig. 2.24

Sol. : Let us find out transfer function of the network first. So taking Laplace of the network, neglecting all initial conditions we get,

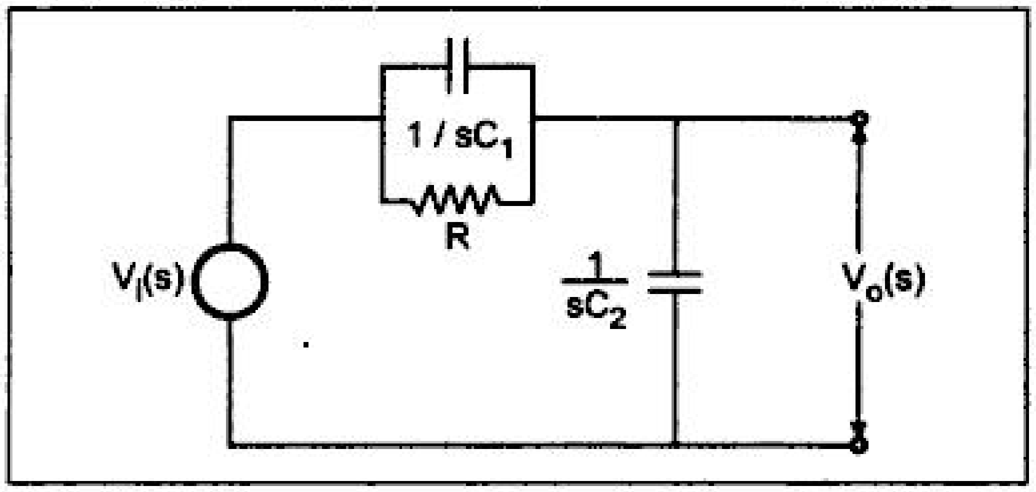


Fig. 2.25

Combining the parallel combination

$$Z = \frac{\frac{1}{sC_1} \times R}{\frac{1}{sC_1} + R} = \frac{R}{1 + sRC_1}$$

Hence network becomes as shown in the Fig. 2.26.

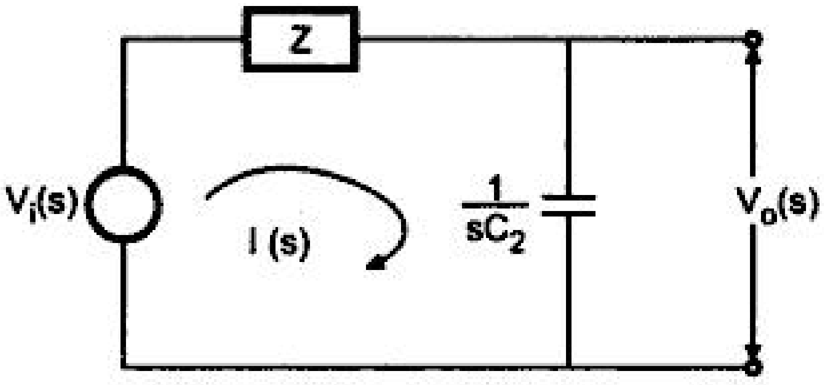


Fig. 2.26

Applying KVL

$$V_i(s) = I(s)\left[Z + \frac{1}{sC_2}\right] \quad \text{... (1)}$$

$$V_o(s) = I(s) \cdot \frac{1}{sC_2} \quad \text{... (2)}$$

From (2), $\quad I(s) = sC_2\, V_o(s)$

Substituting in (1)

$$V_i(s) = sC_2\, V_o(s)\left[Z + \frac{1}{sC_2}\right]$$

$$\therefore \quad \frac{V_o(s)}{V_i(s)} = \frac{1}{1 + ZsC_2}$$

Substituting $\quad Z = \dfrac{R}{1 + sRC_1}$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + \left(\frac{R}{1 + sRC_1}\right) sC_2} = \frac{1 + sRC_1}{1 + sRC_1 + sRC_2}$$

$$\therefore \quad \frac{V_o(s)}{V_i(s)} = \frac{1 + sRC_1}{1 + sR(C_1 + C_2)}$$

Now $$C_2 = 3C_1$$

$$\therefore \quad \frac{V_o(s)}{V_i(s)} = \frac{1+sRC_1}{1+4sRC_1}$$

Now $$V_i(t) = 20\,e^{-3t}$$

$$\therefore \quad V_i(s) = \frac{20}{(s+3)}$$

$$\therefore \quad V_o(s) = \frac{20}{(s+3)} \times \frac{(1+sRC_1)}{(1+4sRC_1)}$$

$$= \frac{20(1+sRC_1)}{(s+3)\times 4RC_1 \times \left(s+\dfrac{1}{4RC_1}\right)}$$

$$= \frac{\left(\dfrac{5}{RC_1}\right)(1+sRC_1)}{(s+3)\left(s+\dfrac{1}{4RC_1}\right)} = \frac{\left(\dfrac{5}{RC_1}+5s\right)}{(s+3)\left(s+\dfrac{1}{4RC_1}\right)}$$

$$= \frac{A}{(s+3)} + \frac{B}{\left(s+\dfrac{1}{4RC_1}\right)}$$ ... Partial fraction

$$\therefore \quad A\left(s+\frac{1}{4RC_1}\right) + B(s+3) = \frac{5}{RC_1} + 5s$$

$$\therefore \quad \left.\begin{aligned} A + B &= 5 \\ \frac{A}{4RC_1} + 3B &= \frac{5}{R(C)_1} \end{aligned}\right\} \text{equating coefficient of both sides for various powers of 's'}$$

Solving,

$$A = \frac{20(3RC_1-1)}{(12RC_1-1)}, \quad B = \left(\frac{15}{12RC_1-1}\right)$$

$$\therefore \quad V_o(s) = \frac{20(3RC_1-1)}{(12RC_1-1)(s+3)} + \frac{15}{(12RC_1-1)\left(s+\dfrac{1}{4RC_1}\right)}$$

$$\therefore \quad V_o(t) = \frac{20(3RC_1-1)}{(12RC_1-1)}\,e^{-3t} + \frac{15}{(12RC_1-1)}\,e^{-\frac{t}{4RC_1}}$$

Ex. 2.14 *Determine the transfer function if the d.c. gain is equal to 10 for the system whose pole-zero plot is shown below.*

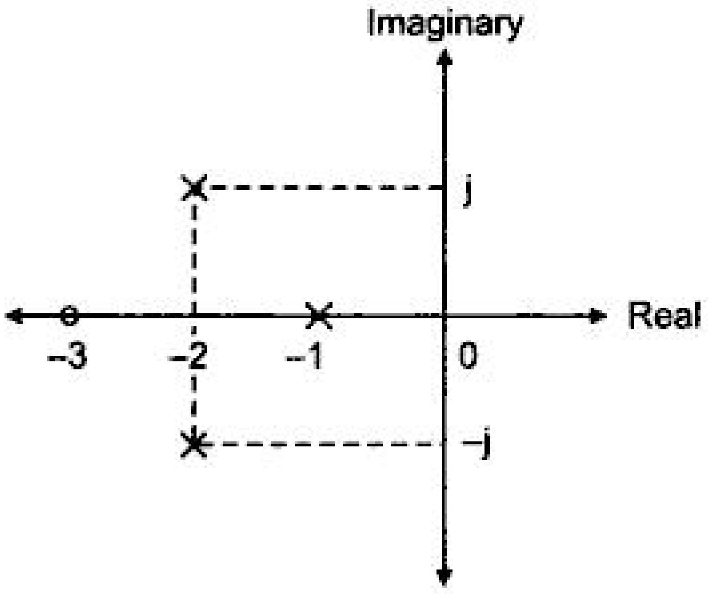


Fig. 2.27

Sol. : From pole-zero plot given the transfer function has 3 poles at s = 0, s = –1, –2+j and –2–j. And it has one zero at s = –3.

$$\therefore \quad T(s) = \frac{K(s+3)}{(s+1)(s+2+j)(s+2-j)}$$

$$= \frac{K(s+3)}{(s+1)\left[(s+2)^2-(j)^2\right]}$$

$$= \frac{K(s+3)}{(s+1)\left[s^2+4s+5\right]}$$

Now d.c. gain is value of T(s) at s = 0 which is given as 10.

$$\therefore \quad \text{d.c. gain} = T(s)|_{\text{at } s=0}$$

$$\therefore \quad 10 = \frac{K\times 3}{1\times 5}$$

$$\therefore \quad K = \frac{50}{3} = 16.667$$

$$\therefore \quad T(s) = \frac{16.667\,(s+3)}{(s+1)(s^2+4s+5)}$$

This is the required transfer function.

Ex. 2.15 *For a certain system c(t) is the output and r(t) is the input. It is represented by the differential equation.* $\frac{d^2\,c(t)}{dt^2} + 5\frac{dc(t)}{dt} + 8\;c(t) = \frac{2d\,r(t)}{dt} + r(t)$

Determine its transfer function.

Sol. : Finding Laplace of the given equation, neglecting the initial conditions.

$$\therefore \quad s^2C(s) + 5sC(s) + 8C(s) = 2sR(s) + R(s)$$

$$\therefore \quad C(s)\,[s^2 + 5s + 8] = R(s)\,[2s + 1]$$

$$\frac{C(s)}{R(s)} = \frac{2s+1}{s^2+5s+8}$$

This is the required transfer function.

Ex. 2.16 *If the system transfer function is*

$$\frac{Y(s)}{X(s)} = \frac{s+4}{s^2+2s+5}$$

Obtain the differential equation representing the system.

Sol. :

$$\frac{Y(s)}{X(s)} = \frac{s+4}{s^2+2s+5}$$

$$\therefore \quad (s^2+2s+5)\;Y(s) = (s+4)\;X(s)$$

$$\therefore \; s^2\,Y(s) + 2s\,Y(s) + 5Y(s) = 5X(s) + 4X(s)$$

Replacing variable s by $\frac{d}{dt}$ and Y(s) by y(t) and X(s) by x(t) we get,

$$\frac{d^2}{dt^2}\,y(t) + 2\frac{d}{dt}\,y(t) + 5y(t) = \frac{d}{dt}\,x(t) + 4x(t)$$

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5 = \frac{dx}{dt} + 4x$$

This is the required differential equation.

Summary

The transfer function is Laplace domain function which is the ratio of Laplace of output to Laplace of input, assuming all initial conditions zero. Transfer function depends on the output variable required. For fixed output and fixed values of system parameters, transfer function is constant. Then the output for any type of input applied to the system can be determined. The transfer function is the base of the conventional methods of analysis of feedback control systems. It gives very important information about poles, zeros, characteristic equation, order and stability of the system.

Laplace transform of unit impulse response of a linear time invariant system is its transfer function with all initial conditions assumed to be zero.

Poles are the values of s which when substituted in the denominator of a transfer function, make the transfer function value as infinity.

The poles are the roots of an equation obtained by equating denominator polynomial of a transfer function to zero which is called as characteristic equation.

Zeros are the values of s which when substituted in the numerator of a transfer function, make the transfer function value as zero.

The highest power s in the characteristic equation is called as order of system represented by the corresponding transfer function.

It is always convenient to obtain Laplace transform of electrical network by replacing R by R, L by sL and C by $\frac{1}{sC}$, while obtaining the transfer function. This helps in writing the simple algebraic equations in s domain directly without writing the integrodifferential equations for the system in the time domain.

Review Questions

1. *Define the transfer function of a system.*
2. *Explain the significance of a transfer function stating its advantages and features.*
3. *What are the limitations of transfer function approach?*
4. *How transfer function is related to unit impulse response of a system?*
5. *Define and explain the following terms related to the transfer function of a system.*
   (i) Poles (ii) Zeros (iii) Characteristic equation (iv) Pole-zero plot (v) Order.
6. *The unit impulse response of a system is* e^{-7t}. *Find its transfer function*

   Ans. : $\frac{1}{s+7}$

7. *A certain system is described by a differential equation*

   $$\frac{d^2 y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 11y(t) = 5x(t)$$

   where y(t) is the output and the x(t) is the input. Obtain the transfer function of the system.

   Ans.: $\frac{Y(s)}{X(s)} = \frac{5}{s^2 + 3s + 11}$

8. *A certain system has its transfer function as*

   $$\frac{C(s)}{R(s)} = \frac{2s+1}{s^2+s+1}$$

   Obtain its differential equation.

   Ans. : $\frac{d^2c(t)}{dt^2} + \frac{dc(t)}{dt} + c(t) = 2\frac{dr(t)}{dt} + r(t)$

9. *If a system equation is given as*

   $$3\frac{dc(t)}{dt} + 2c(t) = r(t-T)$$

   Where c(t) is output and r(t) is input shifted by T seconds. Obtain its transfer function.

   Ans. : $\frac{e^{-sT}}{3s+2}$

10. *A system when excited by unit step type of input gives following response*

    $$c(t) = 1 - 2e^{-t} + 4e^{-3t}$$

    Obtain its transfer function C(s)/R(s)

    Ans. : $\frac{3s^2+2s+3}{(s+1)(s+3)}$

11. *Derive the transfer function of the system shown in Fig 2.28. The amplifier gain is K.*

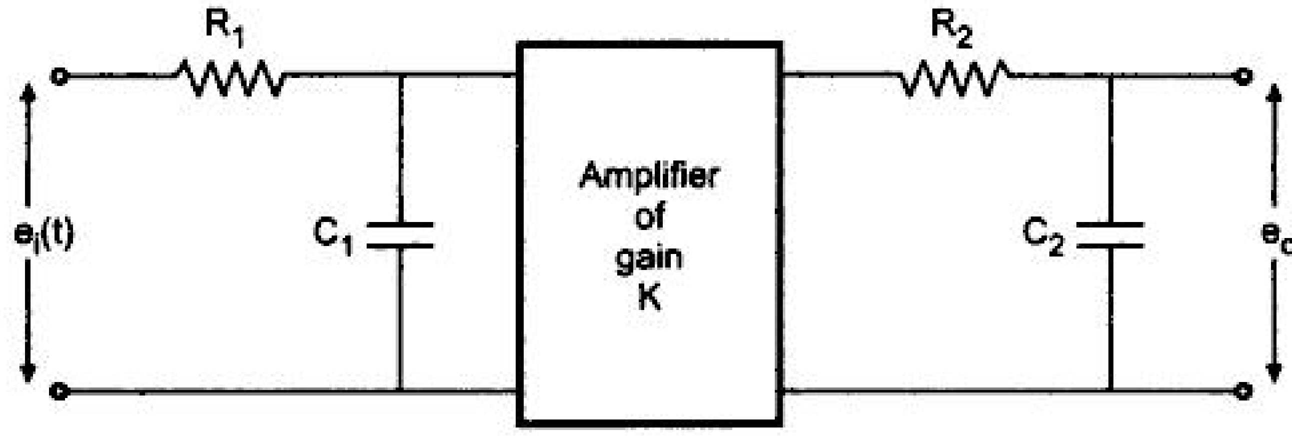


Fig. 2.28

Ans. : $\dfrac{E_o(s)}{E_i(s)} = \dfrac{K}{(1 + sR_1C_1)(1 + sR_2C_2)}$

12. *The transfer function of a system is given by*

$$T(s) = \frac{10(s+8)}{s(s+4)(s^2+6s+25)}$$

Obtain its (i) Poles (ii) Zeros (iii) Order

Sketch its pole zero plot. **Ans. : Poles at 0, –4, –3 ± j4, Zero at –8, Order 4**

13. *Obtain the transfer function of the network shown in the Fig. 2.29*

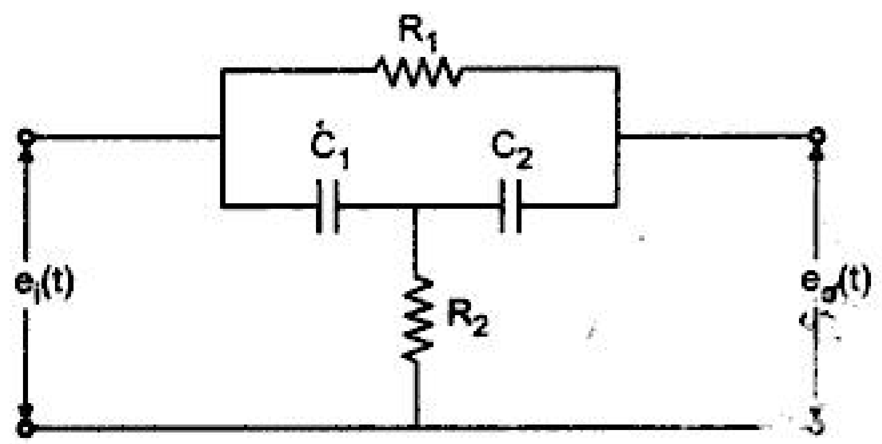


Fig. 2.29

Ans. : $\dfrac{E_o(s)}{E_i(s)} = \dfrac{R_1R_2C_1C_2s^2 + R_2(C_1 + C_2)s + 1}{R_1R_2C_1C_2s^2 + (R_1C_2 + R_2C_1 + R_2C_2)s + 1}$

14. Obtain the transfer function of the network shown in the Fig.2.30

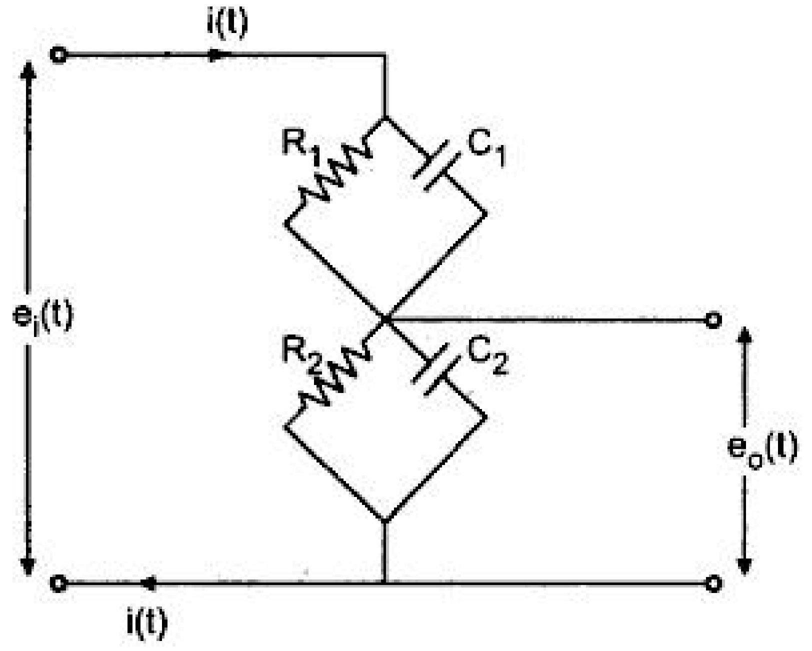


Fig. 2.30

Ans. : $\dfrac{E_o(s)}{E_i(s)} = \dfrac{K(1+T_1s)}{(1+T_2s)}$ where $K = \dfrac{R_2}{R_1+R_2}$

$T_1 = R_1C_1$, and $T_2 = \dfrac{R_1R_2C_1 + R_1R_2C_2}{R_1+R_2}$

❑❑❑

3
Block Diagram Representation

3.1 Introduction :

If the given system is complicated, it is very difficult to analyse it as a whole. With the help of transfer function approach, we can find transfer function of each and every element of the complicated system. And by showing connection between the elements, complete system can be splitted into different blocks and can be analysed conveniently.

Basically block diagram is a pictorial representation of the given system. It is a very simple way of representing the given complicated practical system. In block diagram, the interconnection of system components to form a system can be conveniently shown by the blocks arranged in proper sequence. It explains the cause and effect relationship existing between input and output of the system, through the blocks.

To draw the block diagram of a practical system, each element of practical system is represented by a block. The block is called as **functional block**. It means, block explains mathematical operation on the input by the element to produce the corresponding output. The actual mathematical function is indicated by inserting corresponding transfer function of the element inside the block. For a closed loop systems, the function of comparing the different signals is indicated by the **summing point** while a point from which signal is taken for the feedback purpose is indicated by **take off point** in block diagrams. All these summing points, blocks and take off points are then must be connected exactly as per actual elements connected in practical system. The connection between the blocks is shown by lines called as branches of the block diagram. An arrow is associated with each and every branch which indicates the direction of flow of signal along the branch. The signal can travel along the direction of an arrow only. It cannot pass against the direction of an arrow. Hence block diagram is a unilateral property of the system.

In short any block diagram has following five basic elements associated with it :

1) Blocks.

2) Transfer functions of elements shown inside the blocks.

3) Summing points.

4) Take off points.

5) Arrows.

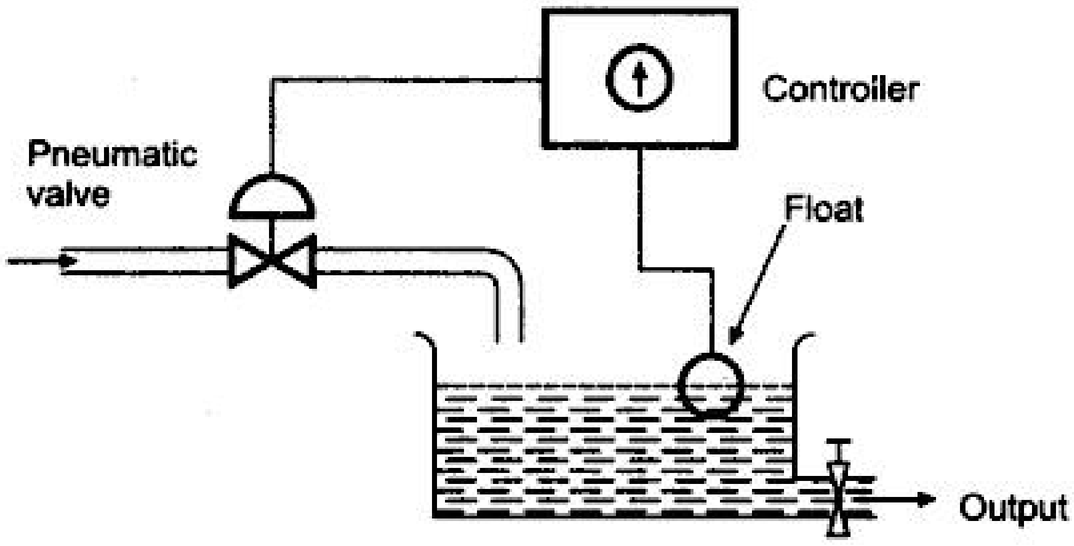


Fig. 3.1

For example consider the liquid level system as shown in Fig. 3.1. So to represent this by block diagram, identify the elements which are
i) Controller (ii) Pneumatic valve (iii) Tank (iv) Float.

Hence indicating them by blocks, the block diagram can be developed as in Fig.3.2.

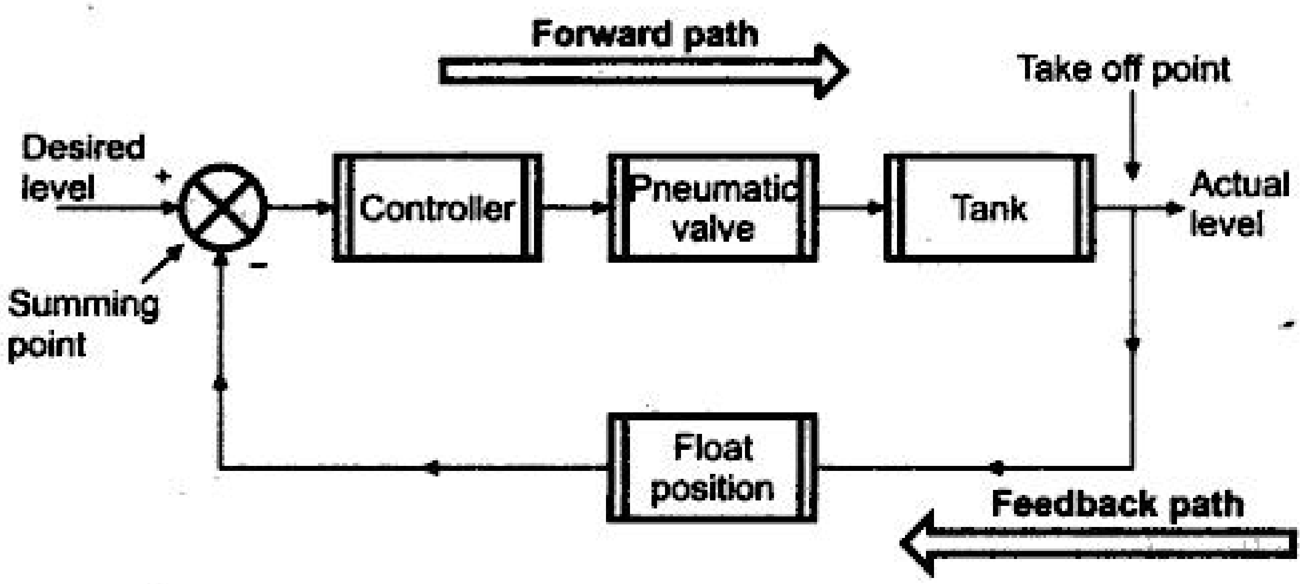


Fig. 3.2 Liquid level control

Consider another example of bottle filling mechanism. When bottle gets filled by the contents upto the required level it should get replaced by an empty bottle. This system can be made closed loop and hence can be shown as in Fig. 3.3 (See Fig. on next page)

In the system shown, conveyor belt is driven by the controller as well as valve position is also controlled by the controller.

When empty bottle comes at the specific position, weight sensor senses the weight and gives signal to controller. Controller stops conveyor movement and opens the valve so bottle starts getting filled. When required level is achieved, again weight sensor sensing the proper weight sends a signal to controller which sends signals to start movement of belt and also closing the valve position with proper time delay till next empty bottle comes at the proper position.

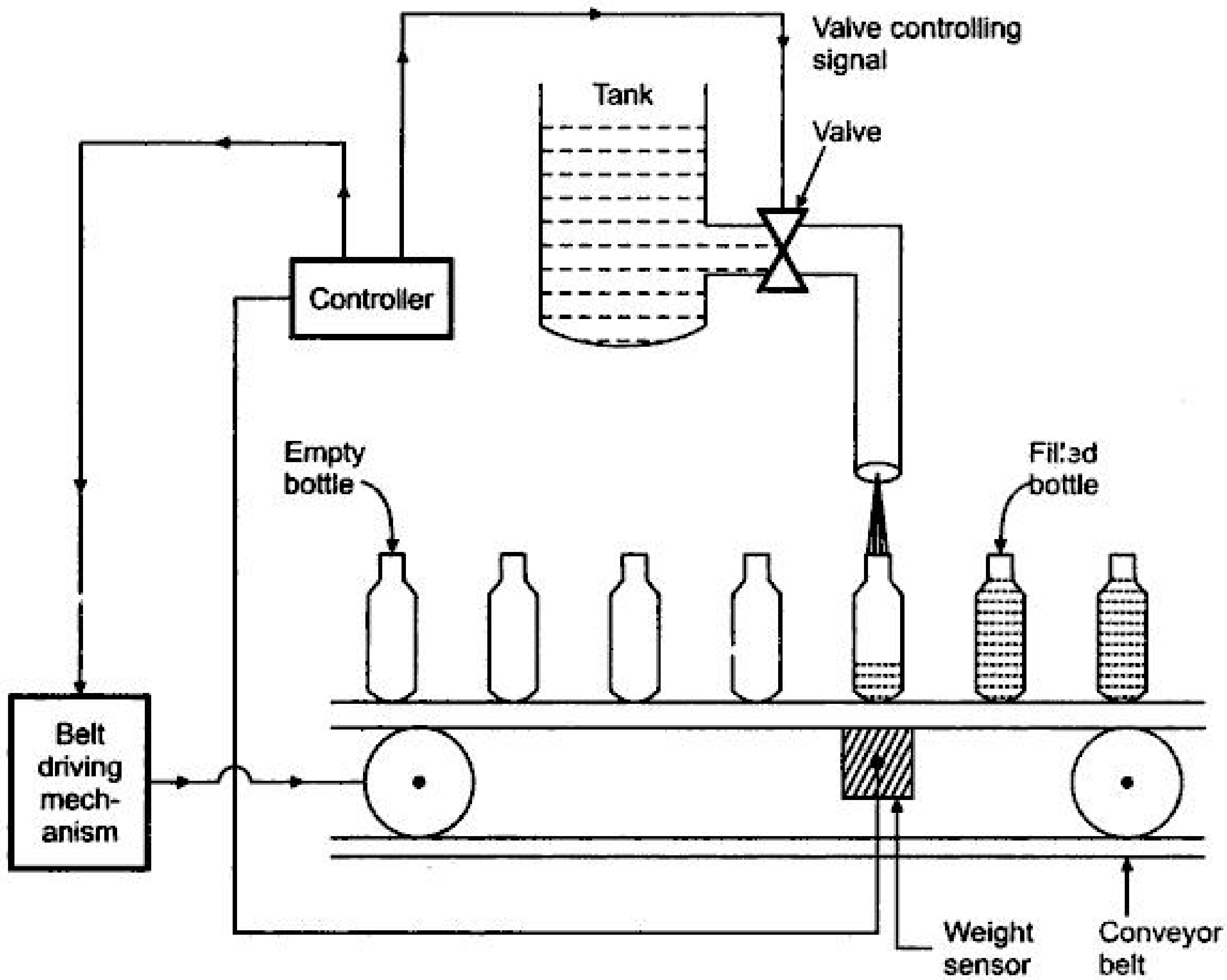


Fig. 3.3

This system can be represented as a block diagram as shown in Fig. 3.4

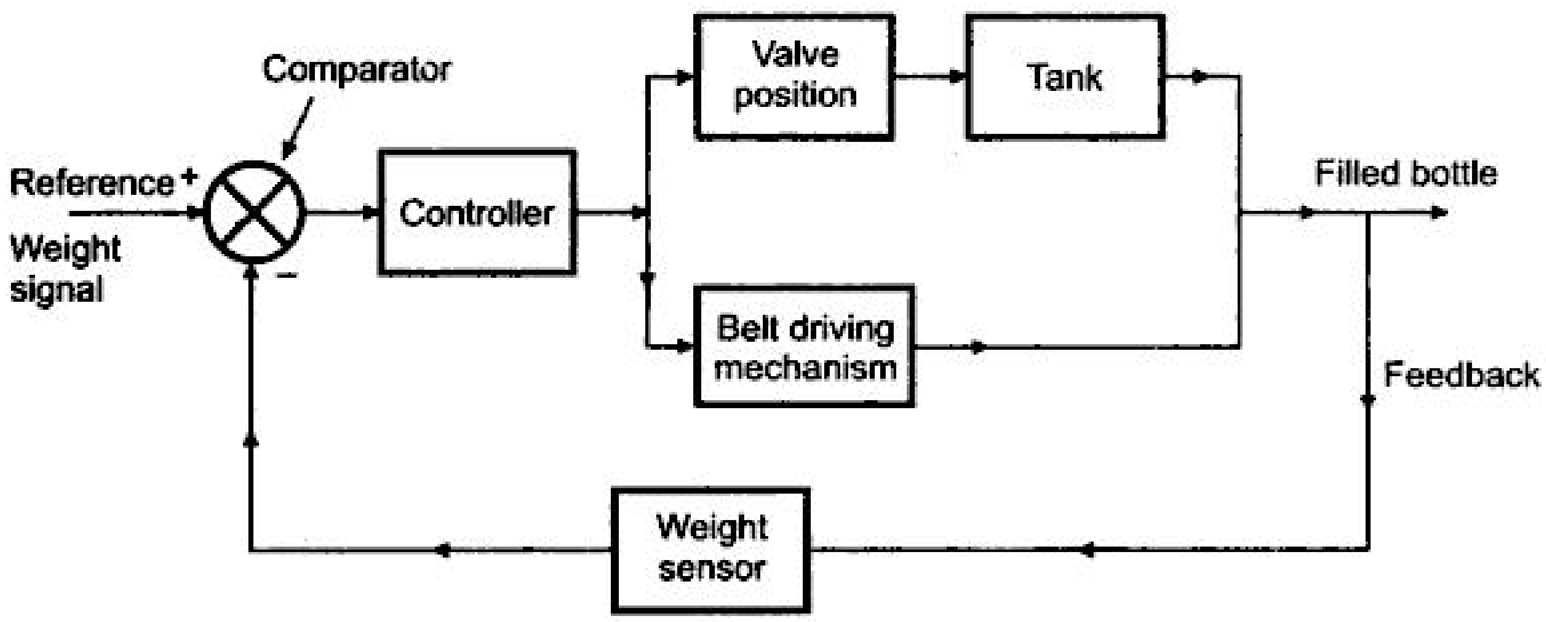


Fig. 3.4 Automatic bottle filling mechanism

3.1.1 Advantages of Block Diagram :

1) Very simple to construct the block diagram for complicated systems.

2) The function of individual element can be visualised from block diagram.

3) Individual as well as overall performance of the system can be studied by using transfer functions shown in the block diagram.

4) Overall closed loop T.F. can be easily calculated by using block diagram reduction rules.

3.1.2 Disadvantages :

1) Block diagram does not include any information about the physical construction of the system.

2) Source of energy is generally not shown in the block diagram. So number of different block diagrams can be drawn depending upon the point of view of analysis. So block diagram for given system is not unique.

3.2 Simple or Canonical Form of Closed Loop System :

A block diagram in which, forward path contains only one block, feedback path contains only one block, one summing point and one take off point represents **simple** or **canonical** form of a closed loop system. This can be achieved by using block diagram reduction rules without disturbing output of the system. This form is very useful as its closed loop transfer function can be easily calculated by using standard result. This result is derived in this section .

The simple form can be shown as in Fig. 3.5.

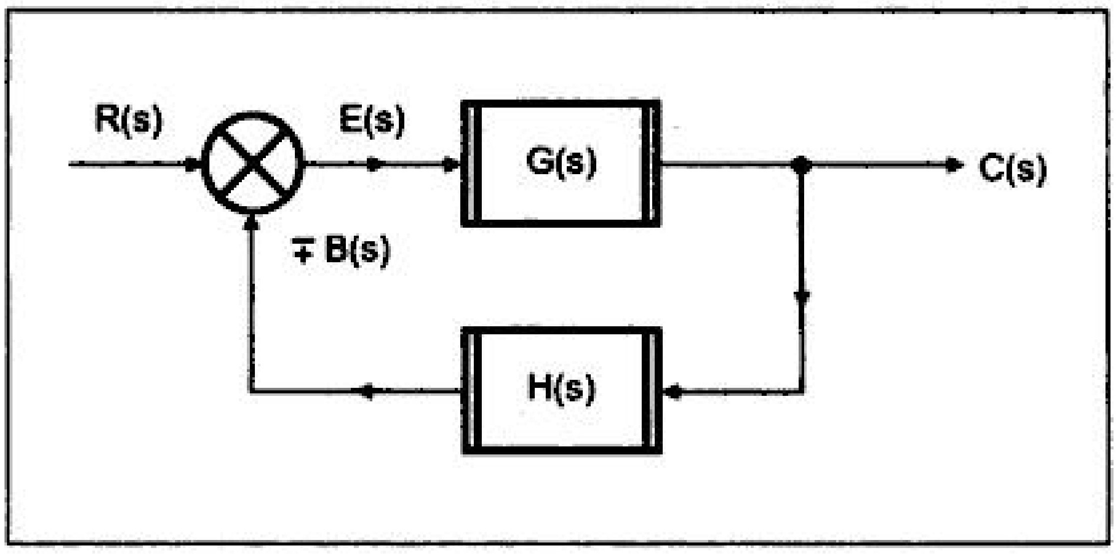


Fig. 3.5

where , R(s) $\rightarrow$ Laplace of reference input r(t)

C(s) $\rightarrow$ Laplace of controlled output c(t)

E(s) $\rightarrow$ Laplace of error signal e(t)

B(s) $\rightarrow$ Laplace of feedback signal b(t)

G(s) $\rightarrow$ Equivalent forward path transfer function .

H(s) $\rightarrow$ Equivalent feedback path transfer function .

G(s) and H(s) can be obtained by reducing complicated block diagram by using block diagram reduction rules.

3.2.1 Derivation of T.F. of Simple Closed Loop System :

Referring to Fig. 3.5, we can write following equations as,

$$E(s) = R(s) \pm B(s) \quad \text{... (1)}$$

$$B(s) = C(s)\,H(s) \qquad \text{... (2)}$$

$$C(s) = E(s)\,G(s) \qquad \text{... (3)}$$

$B(s) = C(s)\,H(s)$ and substituting in equation (1)

$$E(s) = R(s) \pm C(s)\,H(s)$$

$$E(s) = \frac{C(s)}{G(s)}$$

$$\frac{C(s)}{G(s)} = R(s) \pm C(s)\,H(s)$$

$$C(s) = R(s)\,G(s) \pm C(s)\,G(s)\,H(s)$$

$$\therefore \quad C(s)\,[1 \pm G(s)\,H(s)] = R(s)\,G(s)$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)\,H(s)}$$

+ sign → negative feedback

– sign → positive feedback.

This can be represented as in Fig. 3.6

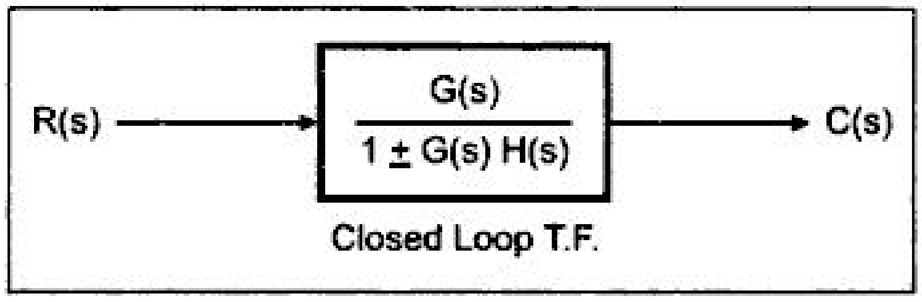


Fig. 3.6

This can be used as a standard result to eliminate such simple loop in a complicated system reduction procedure.

3.3 Rules for Block Diagram Reduction :

Any complicated system if brought into its simple form as shown in Fig. 3.5, its T.F. can be calculated by using the result derived eariler. To bring it into simple form it is necessary to reduce the block diagram but using proper logic such that output of that system and the value of any feedback signal should not get disturbed. This can be achieved by using following mathematical rules while block diagram reduction.

Rule 1 : Associative law : Consider two summing points as shown in Fig. 3.7.

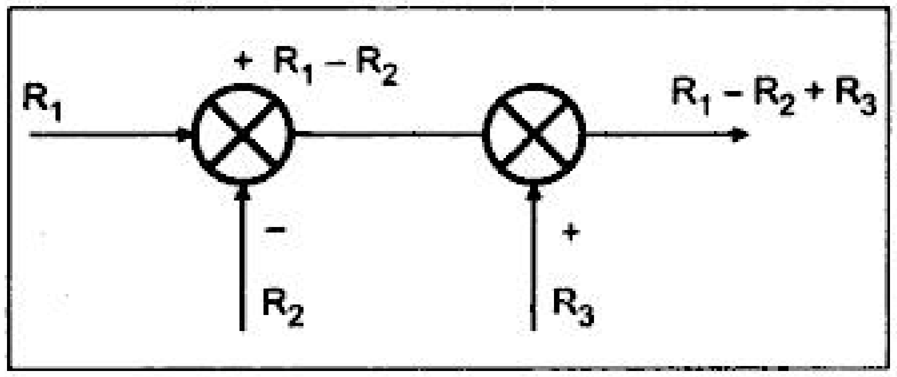


Fig. 3.7

Now change the position of two summing points. Output remains same.

So associative law holds good for summing points which are directly connected to each other (i.e. there is no intermediate block between two summing points).

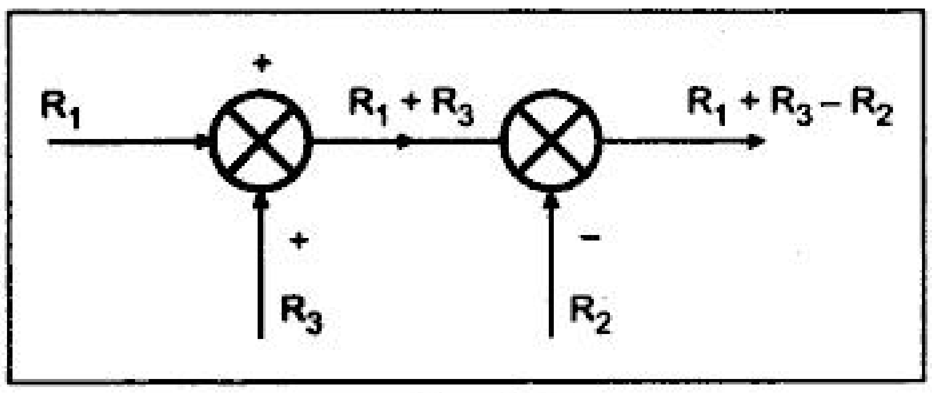


Fig. 3.8

Consider summing points with a block in between as shown in Fig. 3.9.

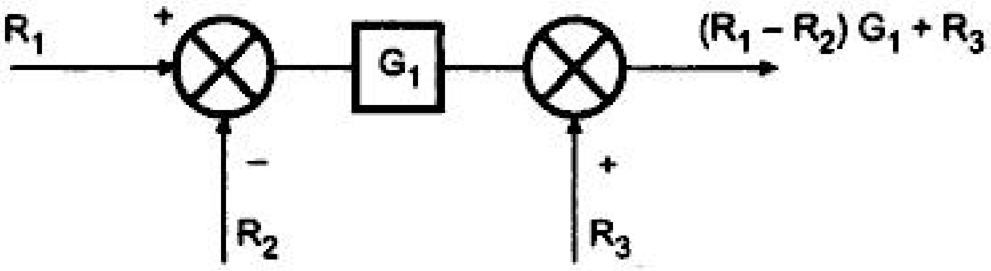


Fig. 3.9

Now interchange two summing points.

So the output does not remain same. So associative law is applicable to summing points which are directly connected to each other.

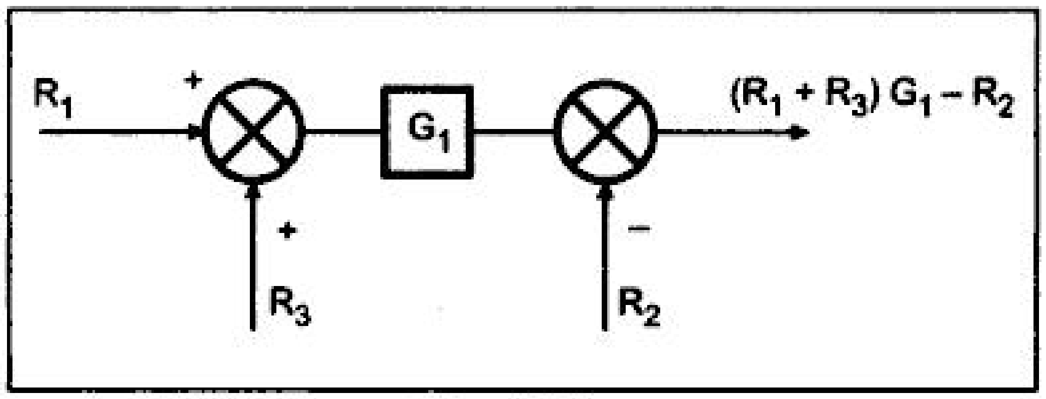


Fig. 3.10

Rule 2 : For blocks in series :

The transfer functions of the blocks which are connected in series get multiplied with each other.

Consider system as shown in Fig. 3.11

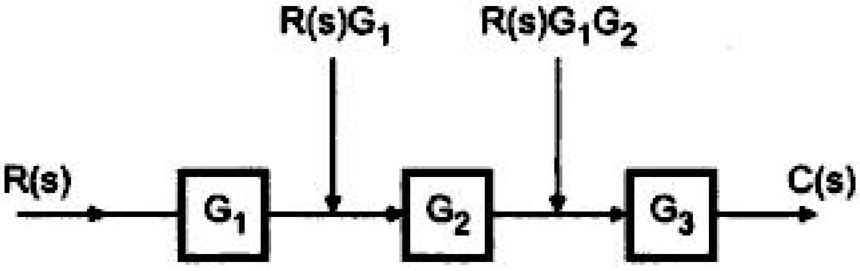


Fig. 3.11

$$C(s) = R(s)\,[G_1\,G_2\,G_3]$$

So instead of three different blocks, only one block with T.F. $[G_1\,G_2\,G_3]$ can be shown in system (Fig. 3.12)

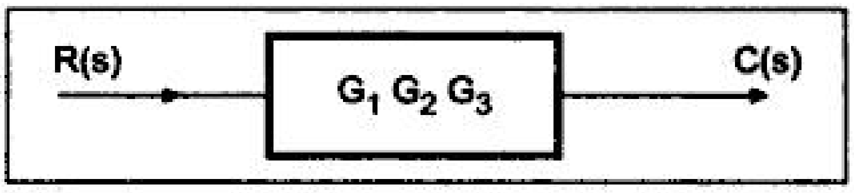


Fig. 3.12

Output in both cases is same.

It is important to note that if there is take off or summing point in between the blocks, the blocks cannot be said to be in series.

Consider the combination of the blocks as shown the Fig. 3.13

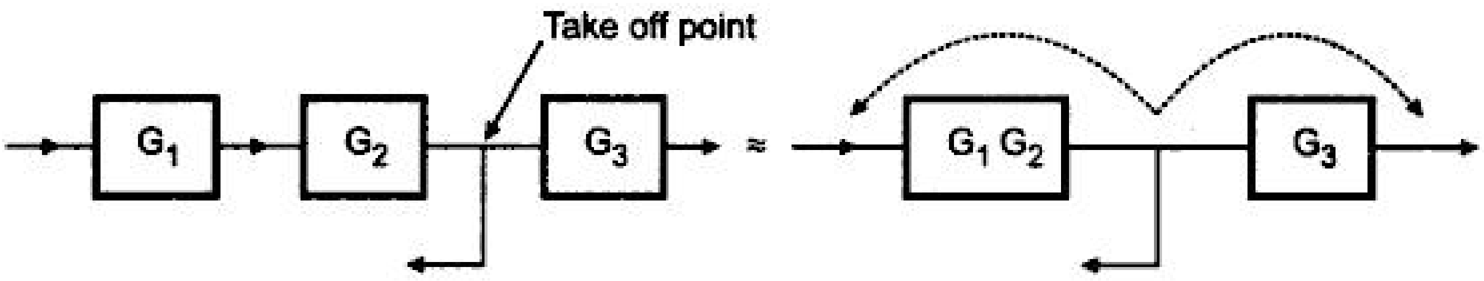


Fig. 3.13

In this combination G_1G_2 are in series and can be combined as G_1G_2 but G_3 is now not in series with G_1G_2 as there is take off point in between. To call G_3 to be in series with G_1G_2 it is necessary to shift the take off point before G_1G_2 or after G_3. The rules for such shifting are discussed later.

Rule 3 : For blocks in parallel. :

The transfer functions of the blocks which are connected in parallel get added algebraically (considering the sign).

Consider system as shown in Fig. 3.14.

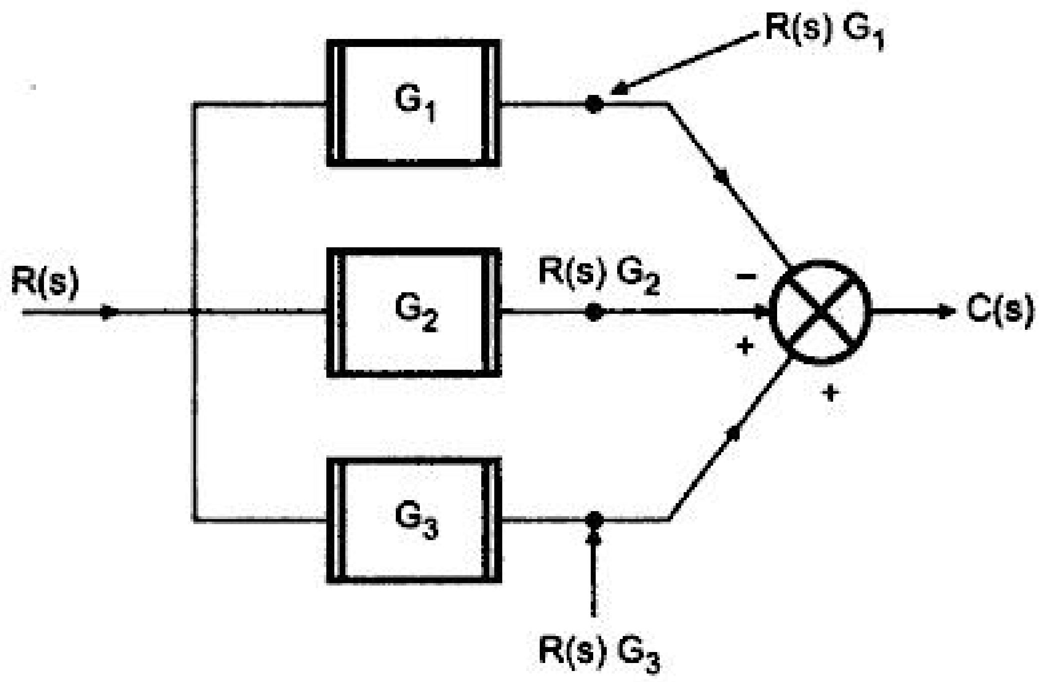


Fig. 3.14

$$C(s) = -R(s)\,G_1 + R(s)\,G_2 + R(s)\,G_3$$
$$= R(s)\,[G_2 + G_3 - G_1]$$

Now replace three block with only one block with T.F. $G_2 + G_3 - G_1$ (Fig. 3.15)

$$C(s) = R(s)\,[G_2 + G_3 - G_1]$$

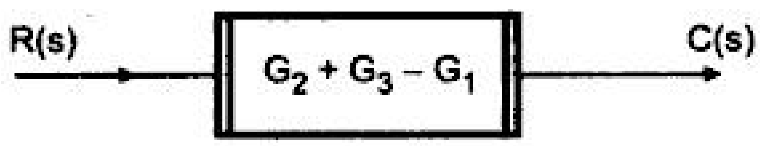
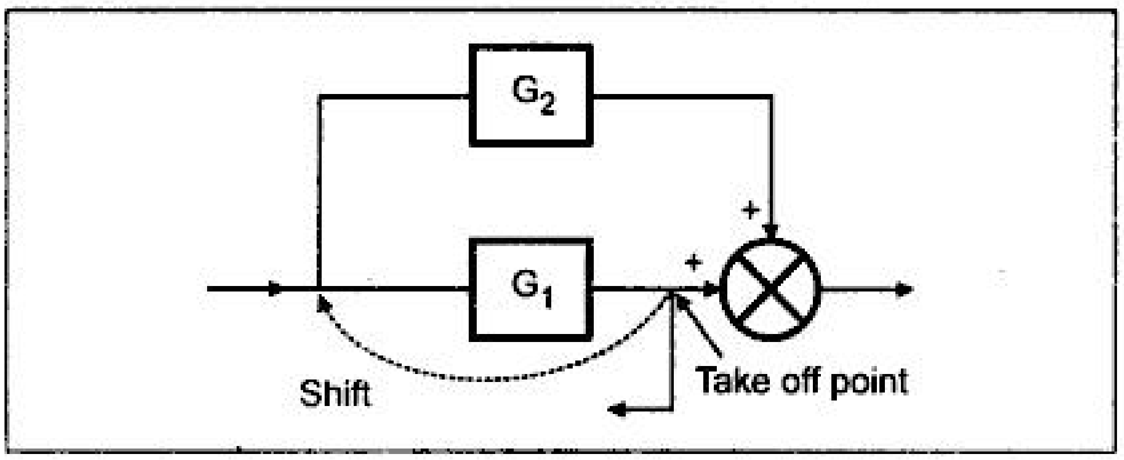


Fig. 3.15

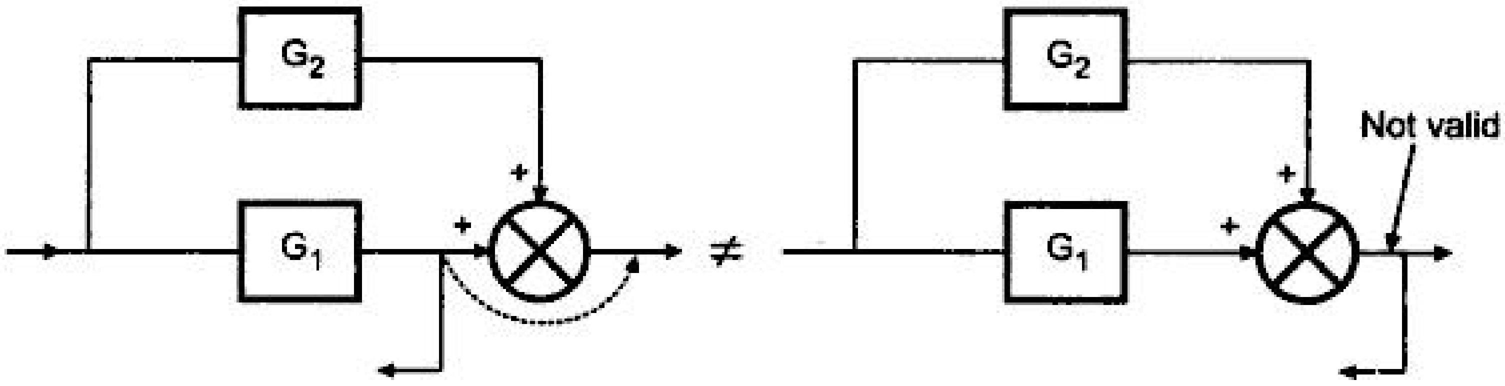
Output is same. So blocks which are in parallel get added algebraically.



The students may make mistakes while identifying blocks in parallel in following cases. If there exists a takeoff point as shown in the Fig. 3.16 along with blocks G_1, G_2 which appear to be in parallel.

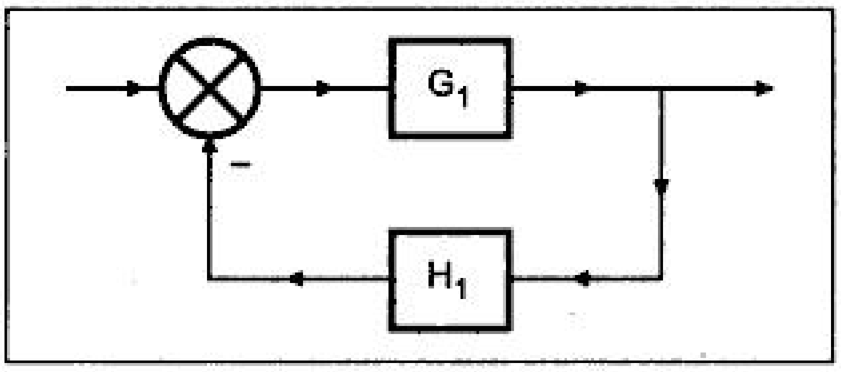
Fig. 3.16

But unless and untill this takeoff point is shifted before the block, blocks can not be said to be in parallel. Shifting of takeoff point is discussed next. Secondly the shifting a take off point after a summing point needs some adjustment to keep out put same. In above case the take off point can not be shown after summing point without any alteration. This type of shifting is discussed as critical rules later as such shifting makes the block diagram complicated and should be avoided as far as possible.



Avoid such shifting as far as possible
Fig. 3.17(a)

Without any alteration such shifting is invalid
Fig. 3.17(b)



Similarly consider a configuraion as shown in the Fig. 3.18.

This combination is not the parallel combination of G_1 and H_1. **For a parallel combination the direction of signals through the blocks in parallel must be same.**

Fig. 3.18

In this case direction of signal through G_1 and H_1 is opposite. Such a combination is called as **minor feedback** loop and reduction rule for this is discussed later.

Rule 4 : Shifting a summing point behind the block :

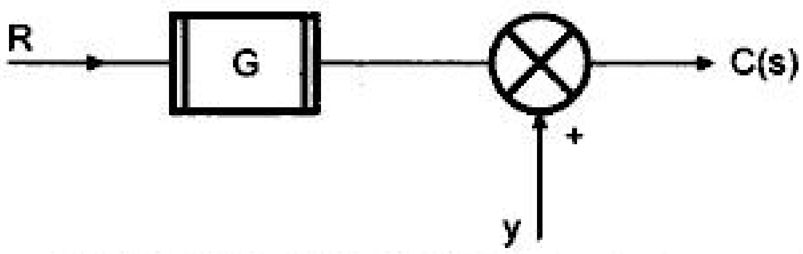


Fig. 3.19

$$C(s) = RG + y$$

Now we have to shift summing point behind the block.

Now output must remain same.

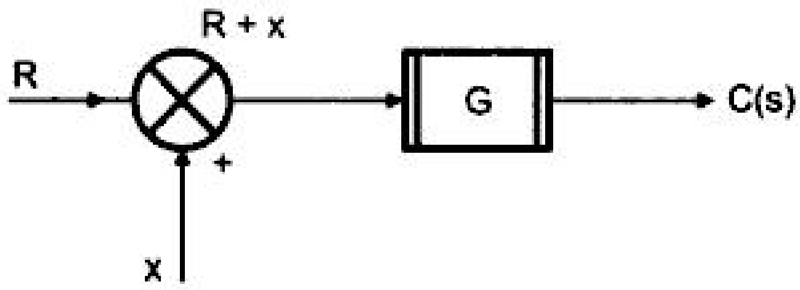


Fig. 3.20

$$\therefore \quad (R + x)G = C(s)$$

$$RG + xG = RG + y$$

$$\therefore \quad xG = y$$

$\therefore$ $x = \dfrac{y}{G}$ so signal y must be multiplied with $\dfrac{1}{G}$ to keep output same.

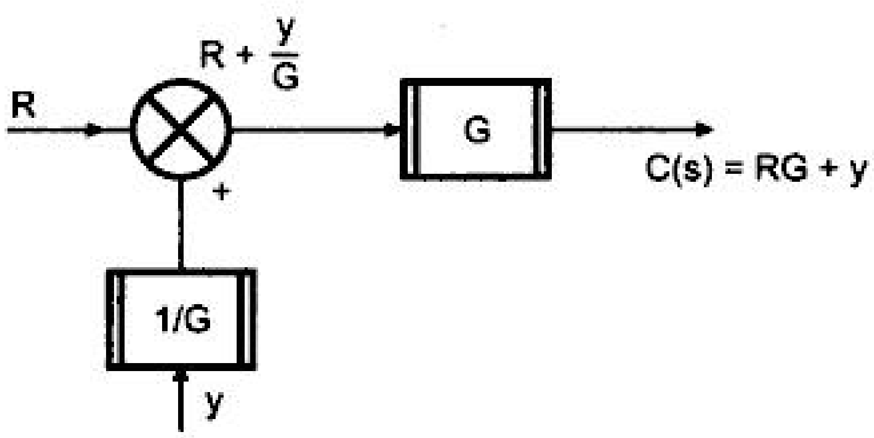


Fig. 3.21

Thus while shifting a summing point behind the block i.e. before the block, add a block having T.F. as reciprocal of the T.F. of the block before which summing point is to be shifted, in series with all the signals at that summing point.

Rule 5 : Shifting a summing point beyond the block.

Consider the combination shown in the Fig. 3.22.

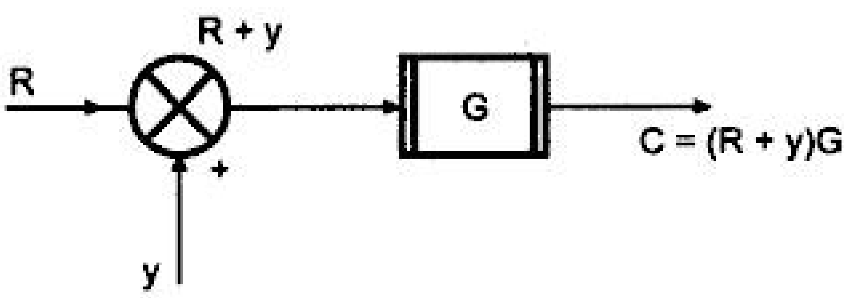


Fig. 3.22

Now to shift summing point after block keeping output same, consider the shifted summing point without any change.

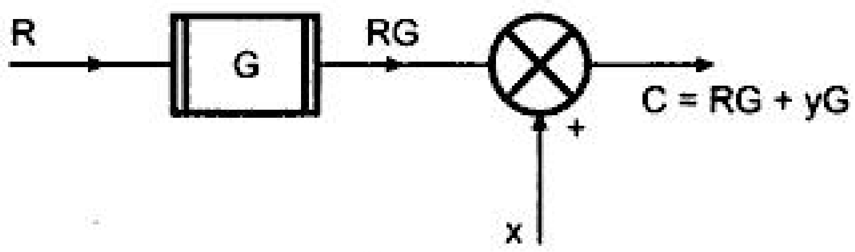


Fig. 3.23

$$\therefore \quad RG + x = RG + yG$$

$$\therefore \quad x = yG$$

i.e. signal y must get multiplied with T.F. of block beyond which summing point is to be shifted.

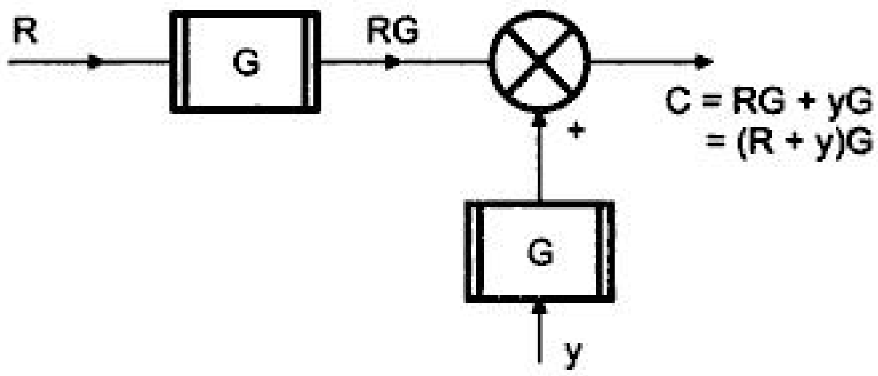


Fig. 3.24

Thus while shifting a summing point after a block, add a block having T.F. same as that of block after which summing point is to be shifted, in series with all the signals at that summing point.

Rule 6 : Shifting a take off point behind the blocks :

Consider the combination shown in the Fig. 3.25.

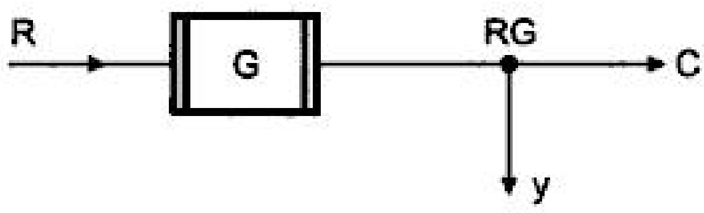


Fig. 3.25

$$C = RG$$

$$y = RG$$

To shift take off point behind block value of signal taking off must remain same.

Though shifting of take off point without any change does not affect output directly, the value of feedback signal which is changed affects the output indirectly which must be kept same. But without any change it is just R as shown in Fig. 3.26.

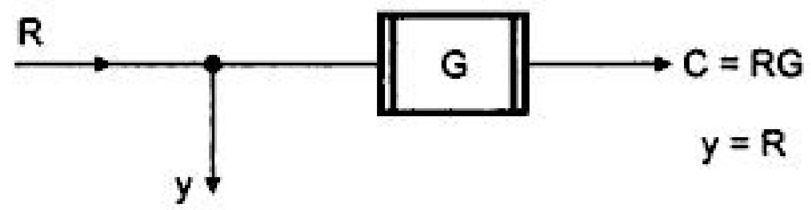


Fig. 3.26

But it must be equal to RG. So a block with T.F. G must be introduced i.e. signal taking off after the block must be multiplied with T.F. of that block while shifting behind the block.

This while shifting a take off point behind the block, add a block having T.F. same as that of the block behind which take off point is to be shifted, in series with all the signals taking off from that take off point.

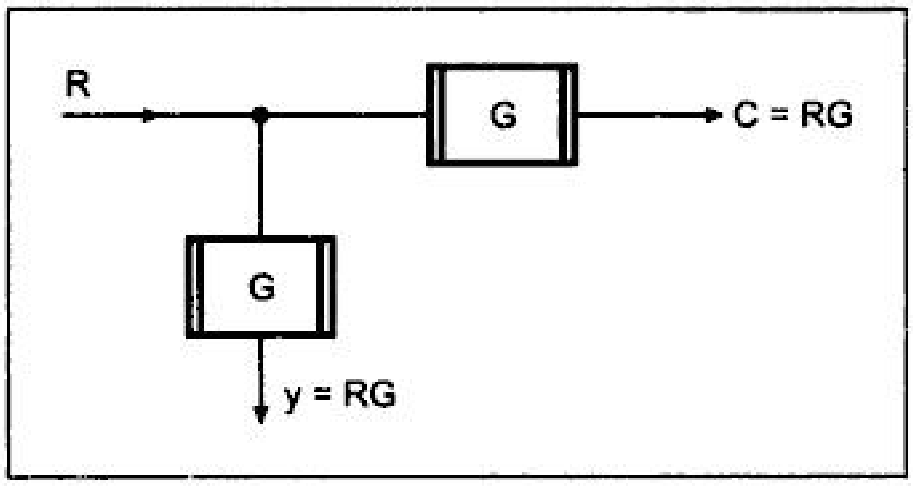
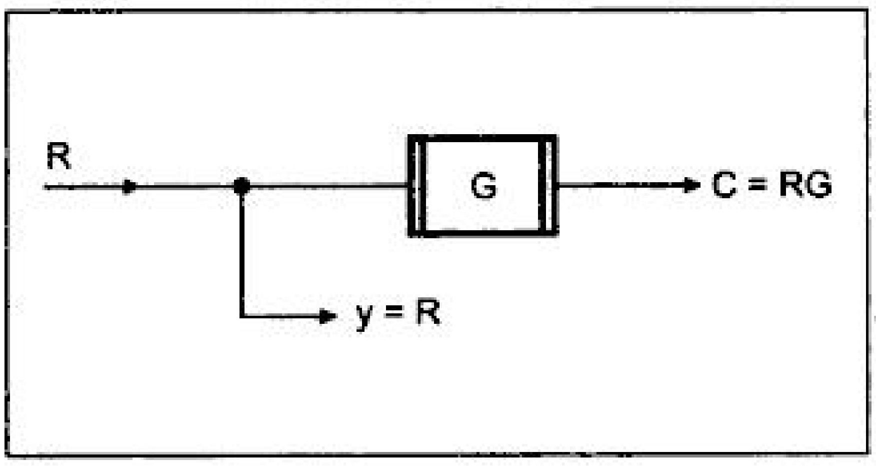


Fig. 3.27

Rule 7 : Shifting a take-off point beyond the block :

Consider the combination shown in the Fig. 3.28.



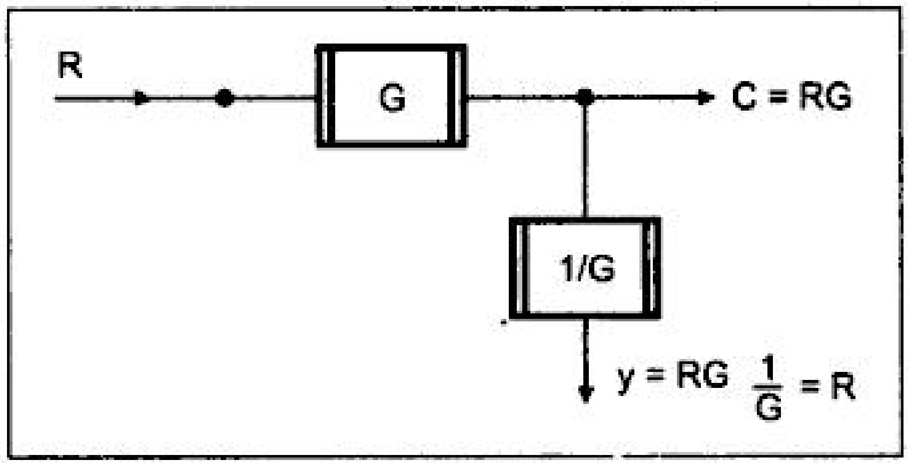


Fig. 3.28 Fig. 3.29

To shift take off point beyond the block, value of 'y' must remain same. To keep value of 'y' constant it must be multiplied by '1/G'. While shifting a take off point beyond the block, add a block in series with all the signals which are taking off from that point, having T.F. as reciprocal of the T.F. of the block beyond which take off point is to be shifted.

Rule 8 : Removing minor feedback loop :

This includes the removal of internal simple forms of the loops by using standard result derived earlier in section 3.2.

After eliminating such a minor loop if summing point carries only one signal input and one signal output, it should be removed from the block diagram to avoid further confusion.

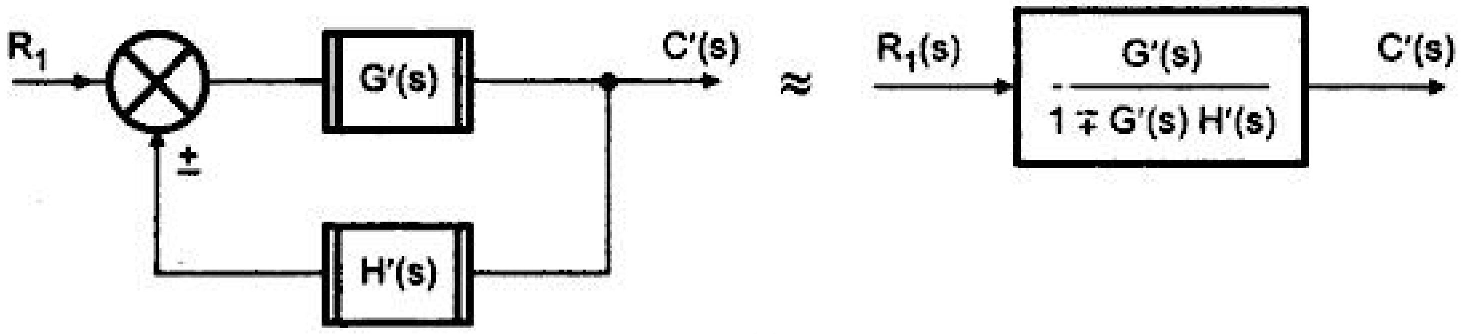


Fig. 3.30

Rule 9 : For multiple input system use superposition theorem :

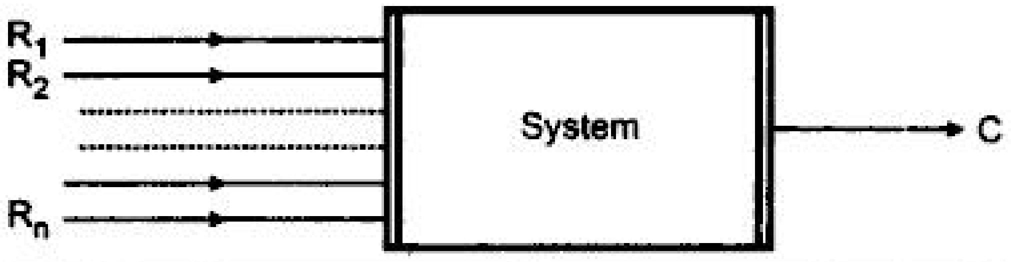


Fig. 3.31

Consider only one input at a time treating all other as zero

Consider R_1, $R_2 = R_3 = \ldots\ldots\ldots\ldots R_n = 0$ and find output C_1,

Then consider R_2, $R_1 = R_3 = \ldots\ldots\ldots\ldots R_n = 0$ and find output C_2

At the end when all inputs are covered take algebraic sum of all the outputs.

Total output $C = C_1 + C_2 + \ldots\ldots\ldots C_n$

Same logic can be extended to find the outputs if system is multiple input multiple output type. Separate ratio of each output with each input is to be calculated, assuming all other input and outputs zero. Then such components of outputs can be added to get resultant outputs of the system. In very few cases, it is not possible to reduce the block diagram to its simple form by use of above discussed nine rules. In such case there is a requirement to shift a summing point before or after a takeoff point to solve the problem. These rules are discussed below but reader should avoid to use these rules unless and until it is the requirement of the problem. Use of these rules in simple problems may complicate the block diagram. The use of these rules in actual problem solving is illustrated in solved problem no. 21.

3.3.1 Critical Rules :

Rule 10 : Shifting take off point after a summing point. Consider a situation as shown in Fig. 3.32.

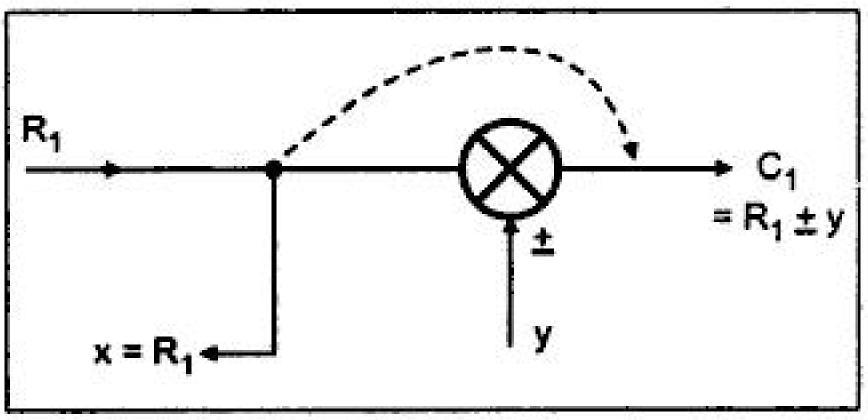


Fig. 3.32

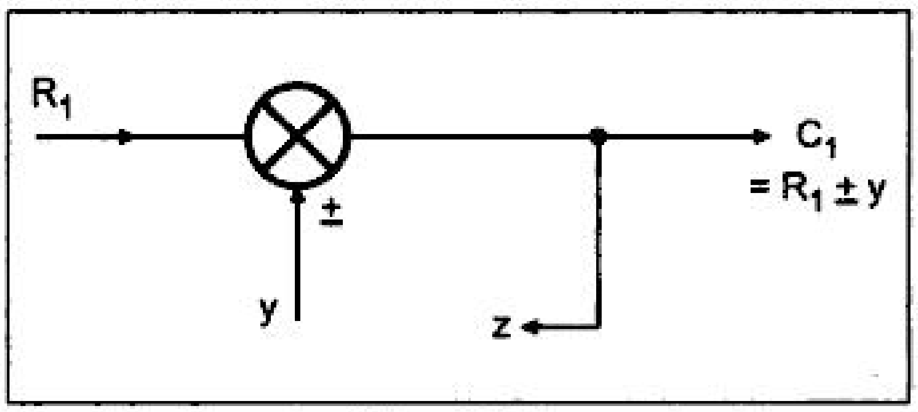


Fig. 3.33

Now after shifting the take off point , let signal taking off be 'z' as shown in Fig. 3.33.

Now $$z = R_1 \pm y$$

But we want feedback signal as $x = R_1$ only.

So signal 'y' must be inverted and added to C_1 to keep feedback signal value same. And to add the signal, summing point must be introduced in series with take off signal. So modified configuration becomes as shown in Fig. 3.34.

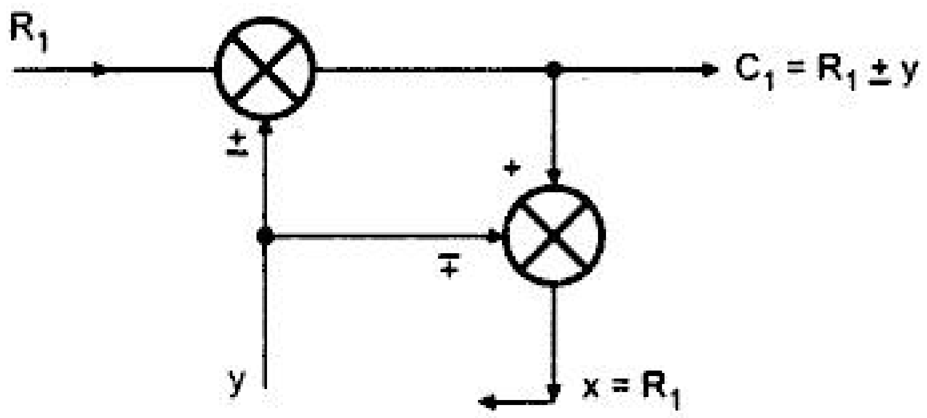


Fig. 3.34

Rule 11 : Shifting take off point before a summing point :

Consider a situation as shown in Fig. 3.35.

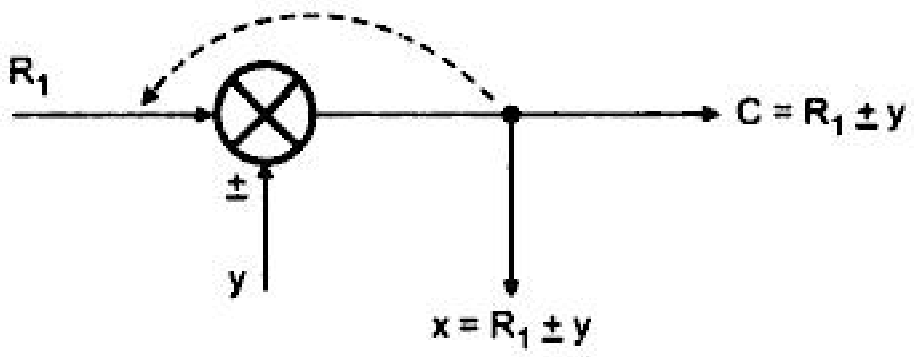


Fig. 3.35

. Now after shifting the take off point, let signal taking off be 'z' as shown in Fig. 3.36.

Now $z = R_1$ only because nothing is changed.

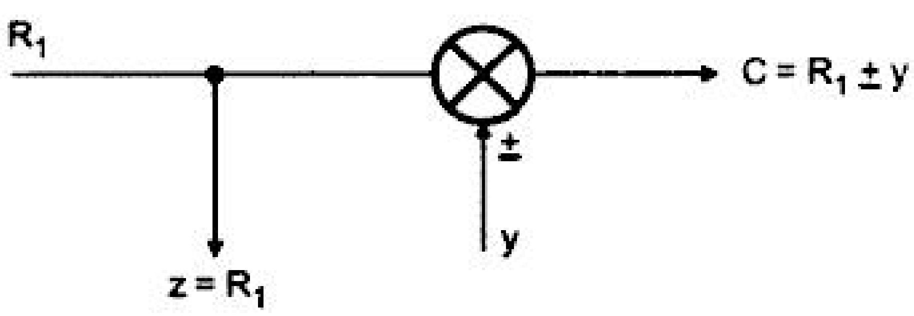


Fig. 3.36

But we want feedback signal x again which is $R_1 \pm y$. Hence to z, signal 'y' must be added with same sign as it is present at summing point which can be achieved by using summing point in series with take off signal as shown in Fig. 3.37.

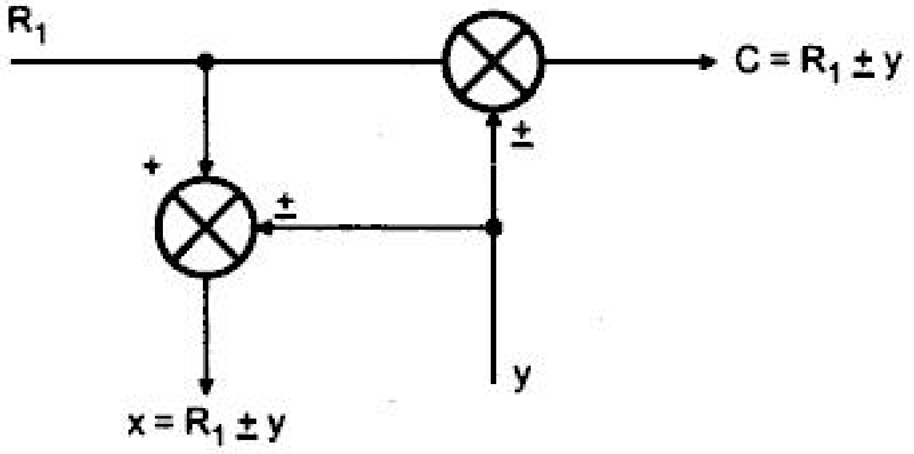


Fig. 3.37

Thus it can be noticed that shifting of take off point before or after a summing point adds an additional summing point in the block diagram and this complicates the block diagram. No doubt, in same rare cases, it is not possible to reduce the block diagram without such shifting of take off point before or after a summing point. Apart from such cases, students should not use such shifting which will complicate the simple block diagrams.

3.3.2 Procedure to solve block diagram reduction problems :

Step 1 : Reduce the blocks connected in series.

Step 2 : Reduce the blocks connected in parallel.

Step 3 : Reduce the minor internal feedback loops.

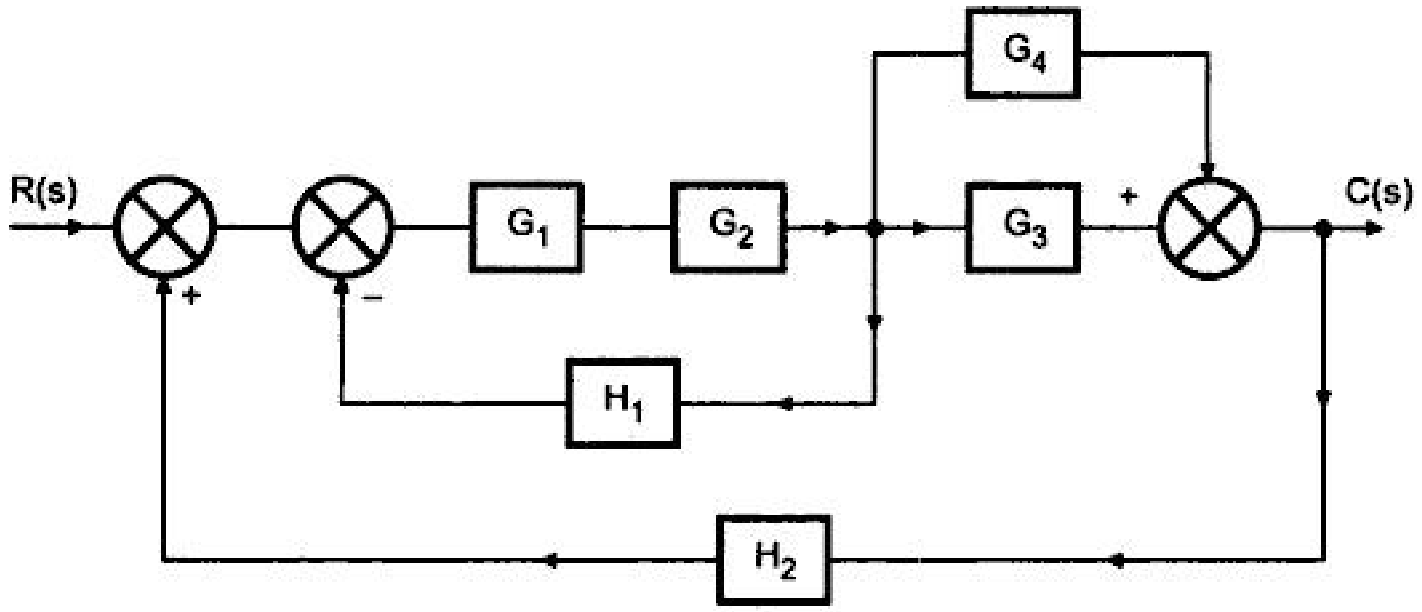
Step 4 : As far as possible **try to shift take off point towards right** and **summing points to the left**. Unless and until it is the requirement of problem do not use rule 10 and 11.

Step 5 : Repeat step 1 to 4 till simple form is obtained.

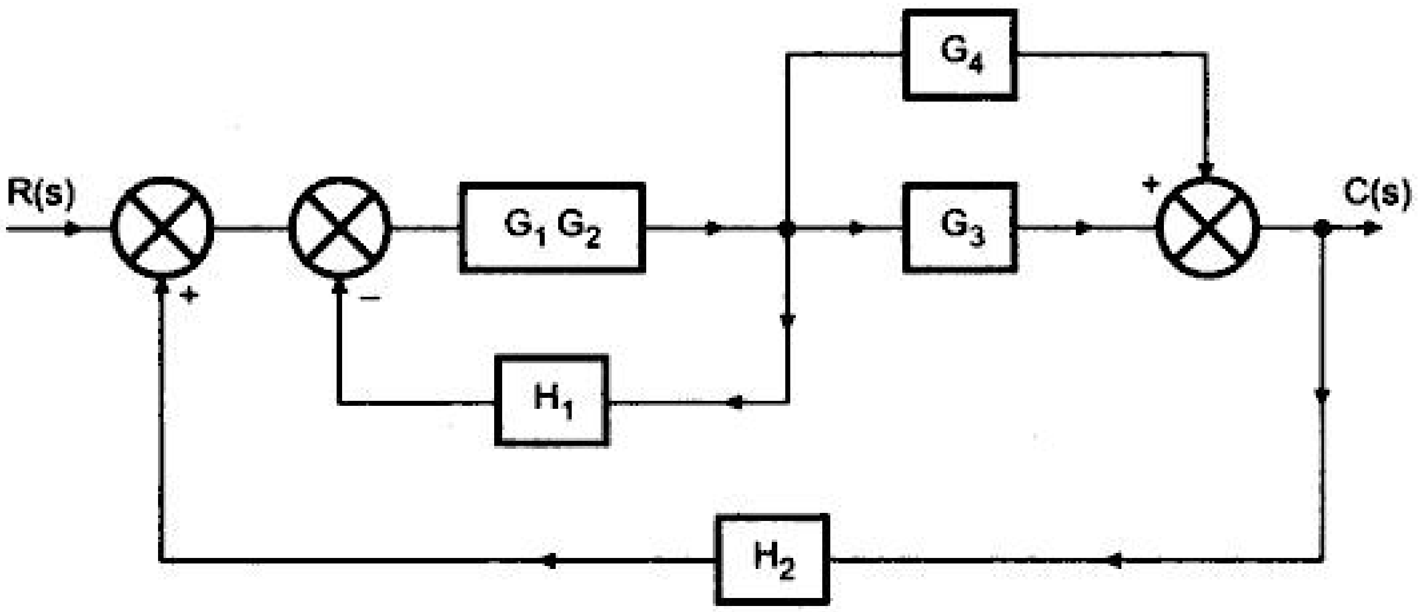
Step 6 : Using standard T.F. of simple closed loop system obtain the closed loop T.F. $\frac{C(s)}{R(s)}$ of the overall system.

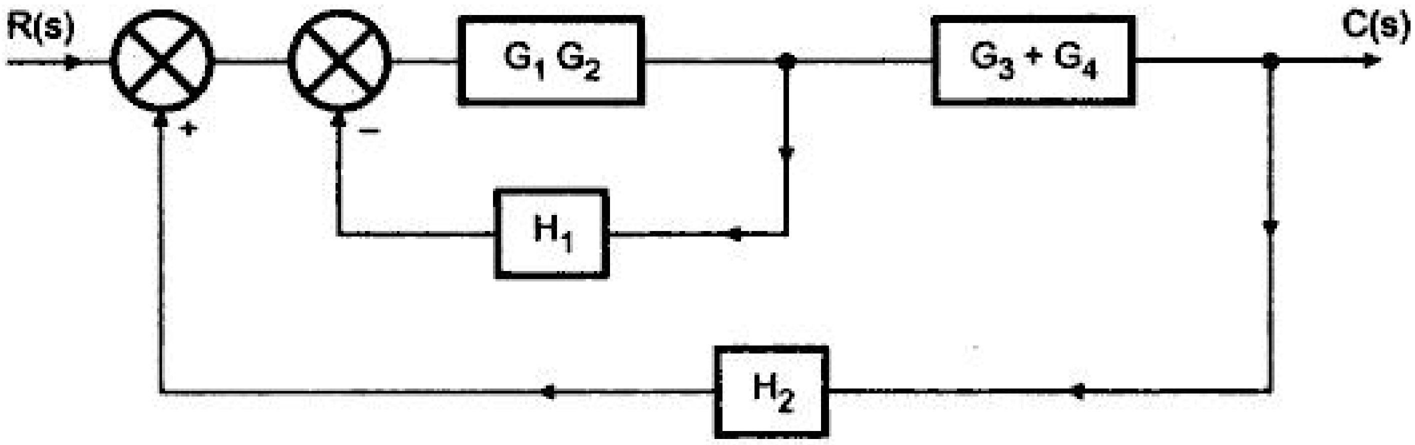
Solved Problems on Block Diagram Reduction

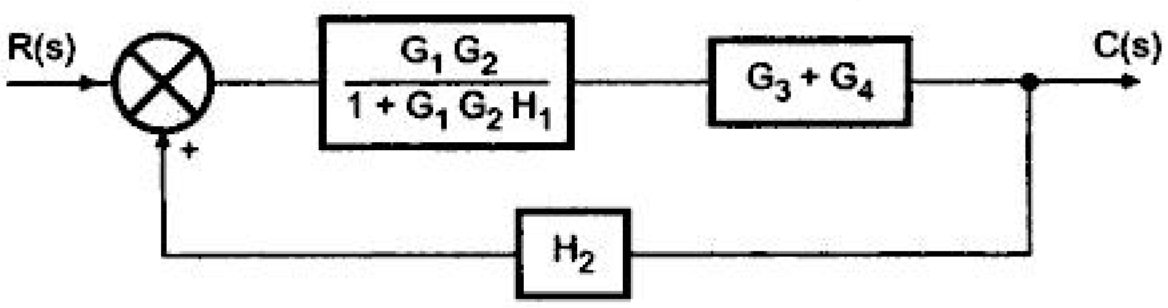
Ex. 3.1 *Reduce the given block diagram to its canonical (simple) form and hence obtain the equivalent transfer function* $\frac{C(s)}{R(s)}$.

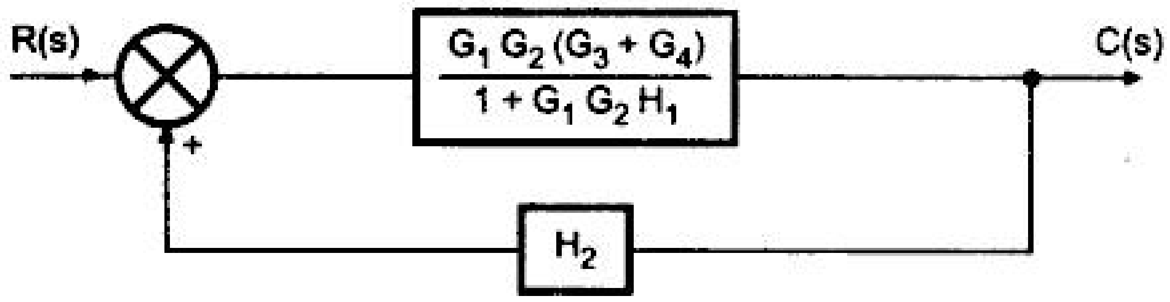

R(s)
C(s)
G_1
G_2
G_3
G_4
H_1
H_2
+
−
+

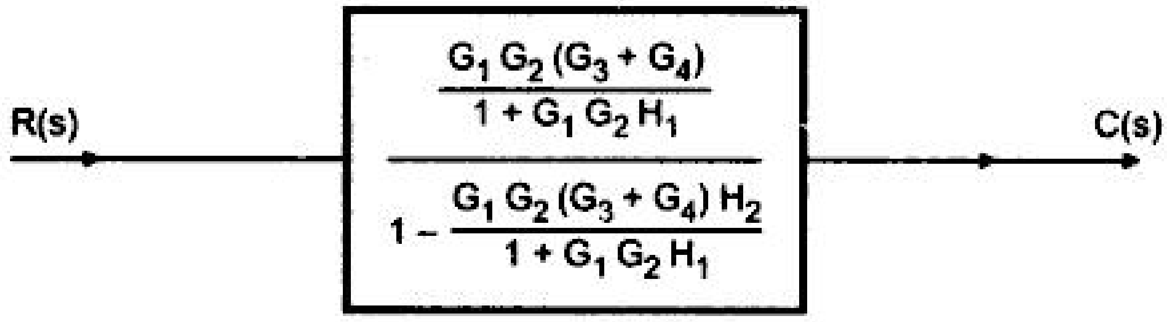
Sol. :


R(s)
C(s)
$G_1 G_2$
G_3
G_4
H_1
H_2
+
−
+


R(s)
C(s)
$G_1 G_2$
$G_3 + G_4$
H_1
H_2
+
−

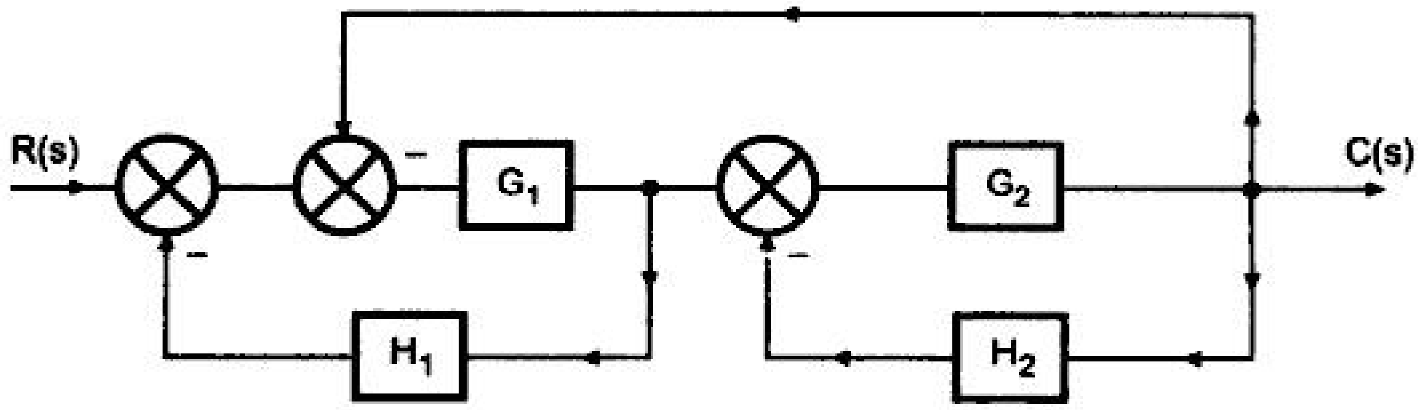

R(s)
C(s)
$\frac{G_1 G_2}{1 + G_1 G_2 H_1}$
$G_3 + G_4$
H_2
+



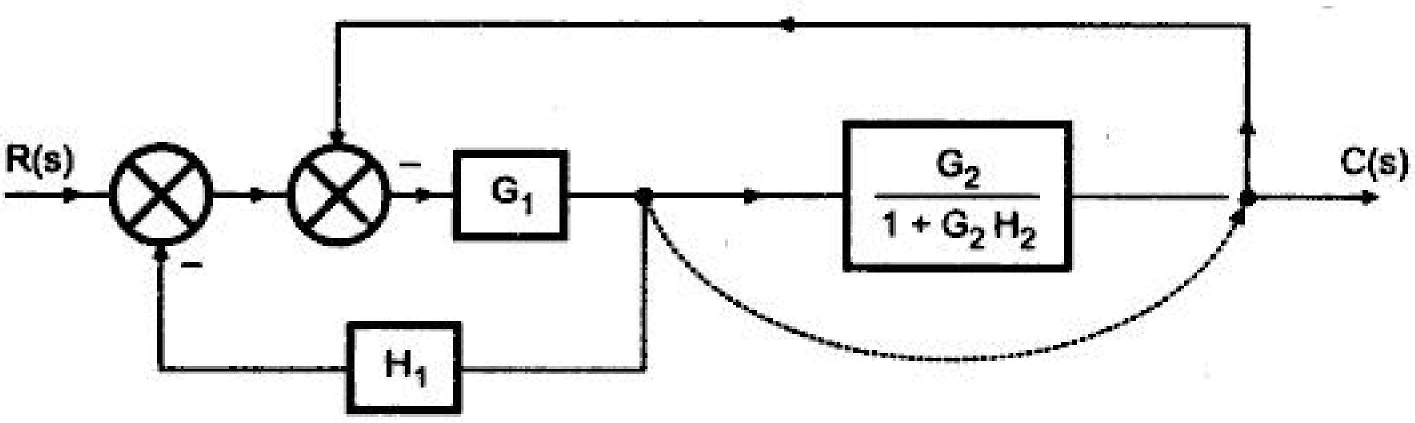


$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G_1 \, G_2 \, (G_3 + G_4)}{1 + G_1 G_2 H_1 - G_1 G_2 (G_3 + G_4) H_2}$$

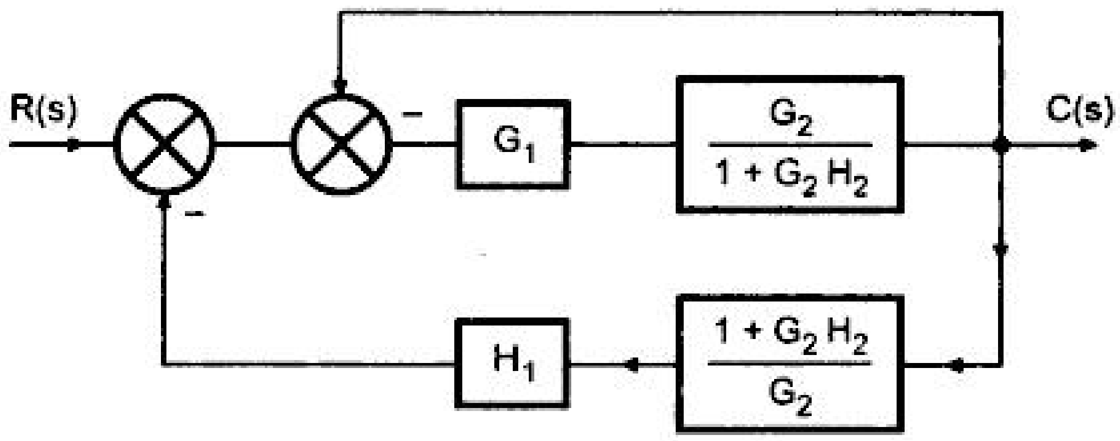
Ex. 3.2

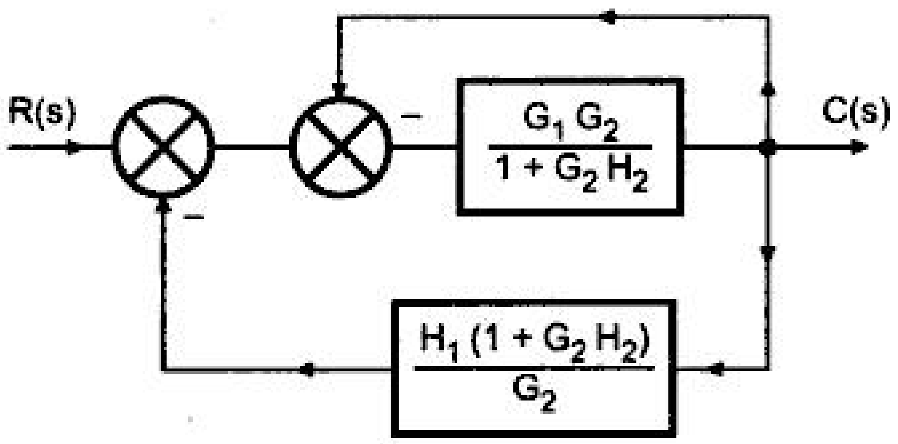


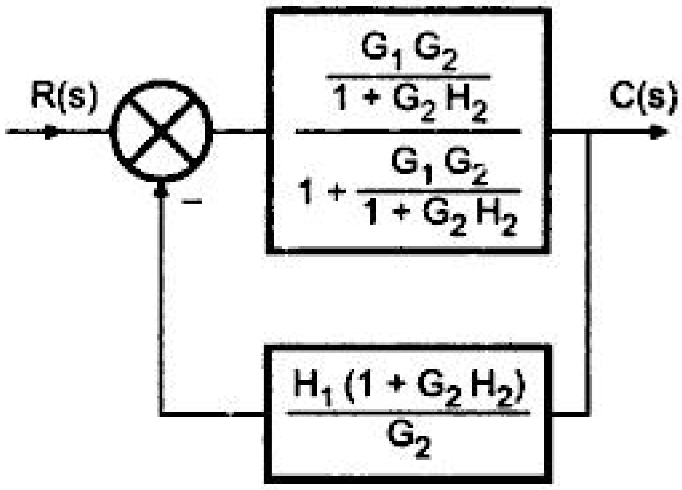
Sol. : No blocks are connected in series or parallel. Blocks having transfer functions G_2 and H_2 form minor feedback loop so eliminating that loop we get,

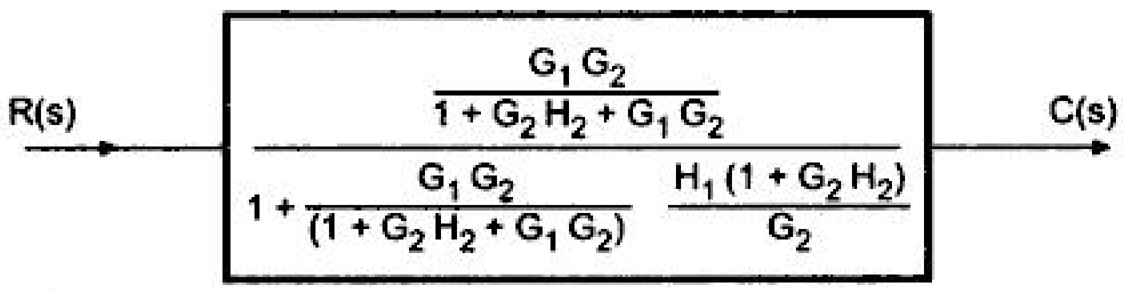


Always try to shift take off point towards right i.e. output and summing point towards left i.e. input.



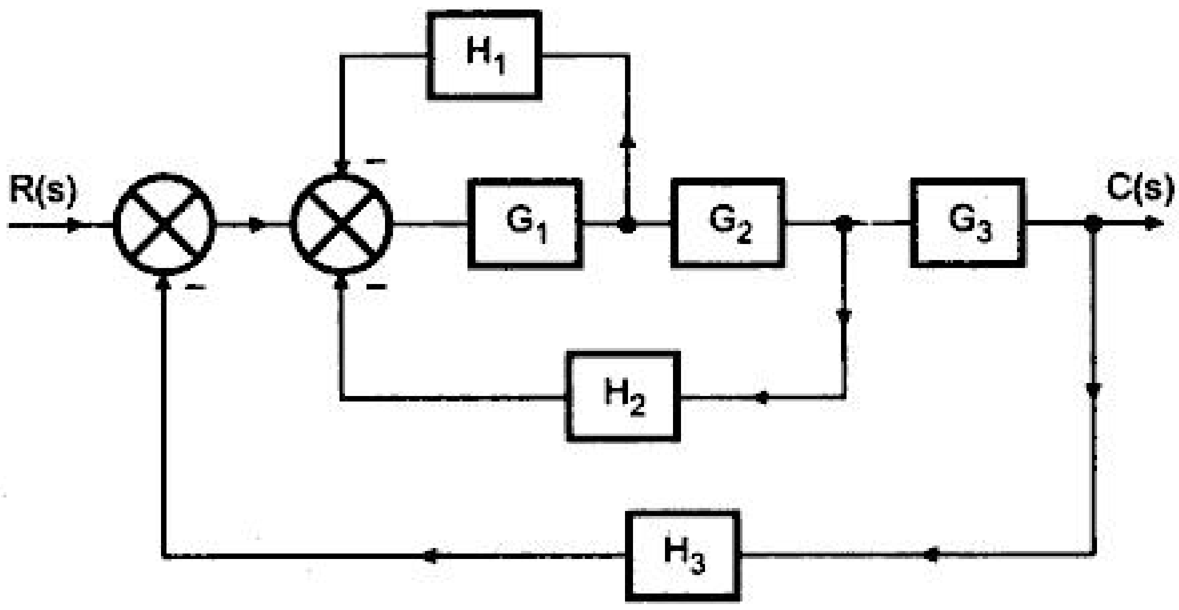




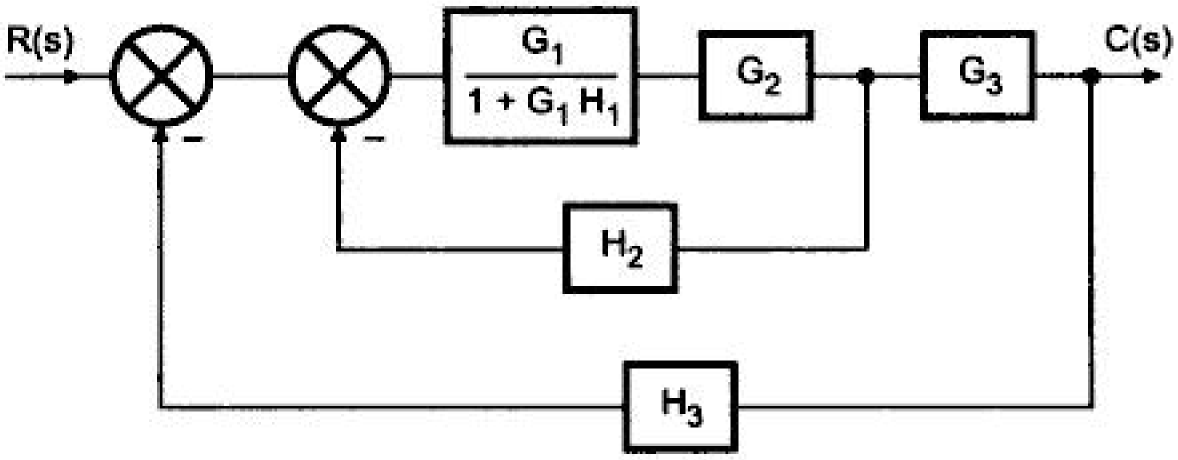


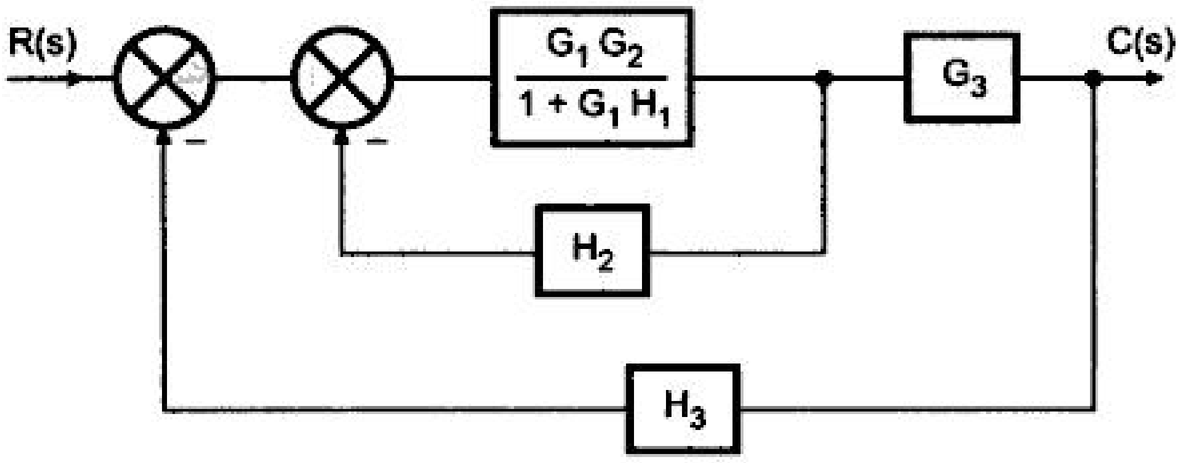
Simplifying $$\frac{C(s)}{R(s)} = \frac{G_1 G_2}{1 + G_1 G_2 + G_2 H_2 + G_1 H_1 + G_1 G_2 H_1 H_2}$$

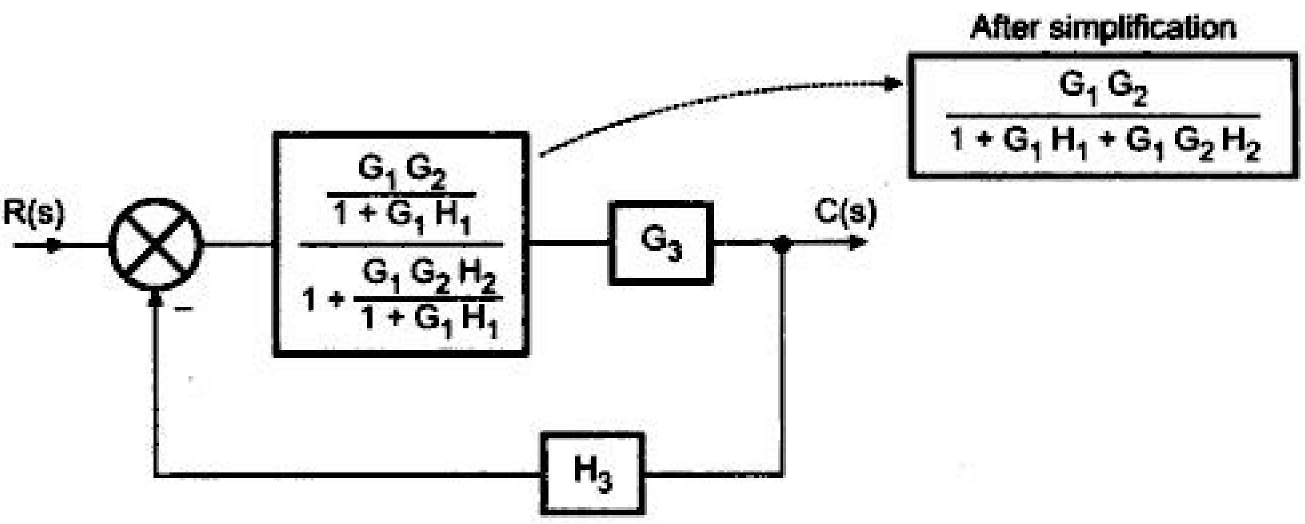
Ex. 3.3

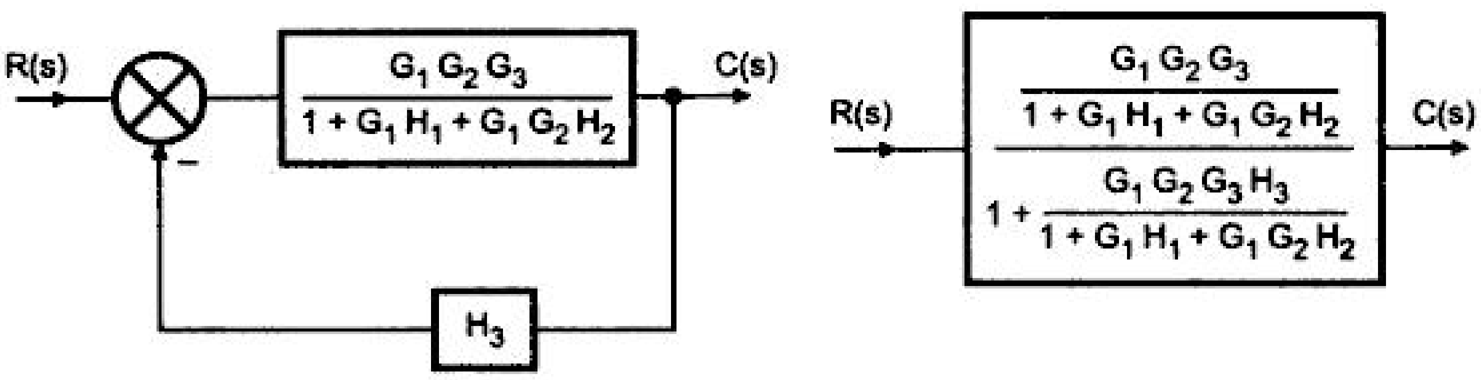


Sol. : No blocks are connected in series or parallel so reducing minor feedback loop formed by blocks with transfer function G_1 and H_1.





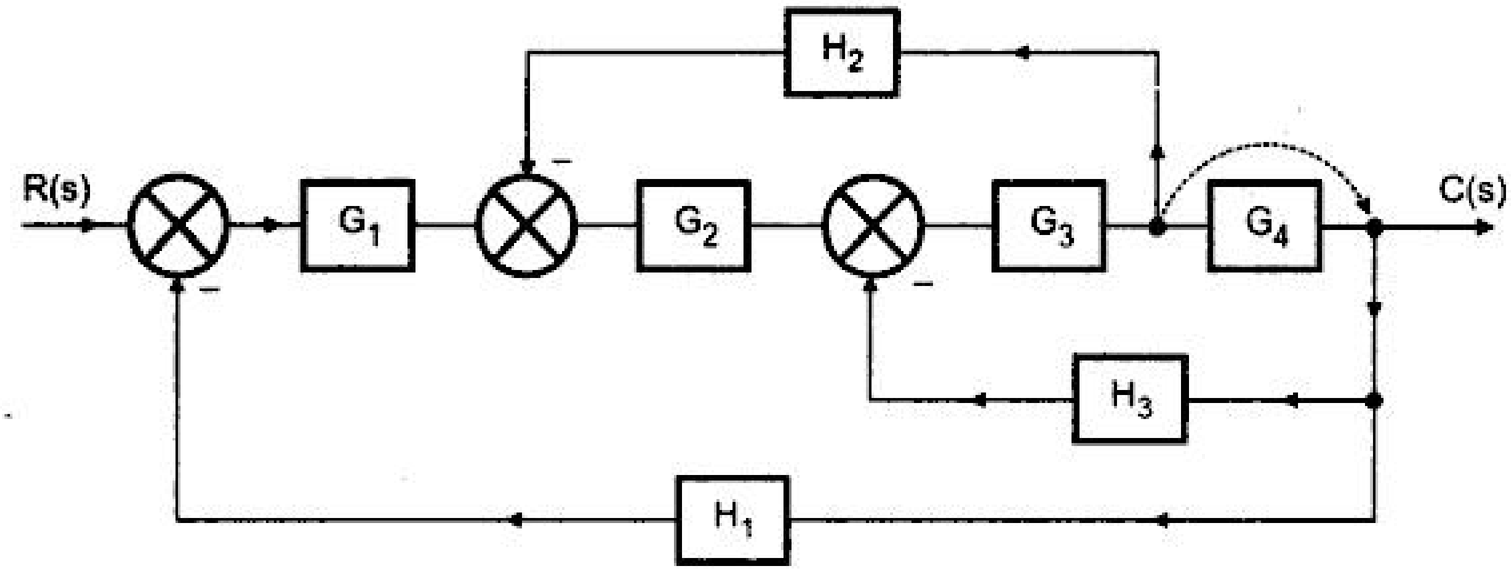




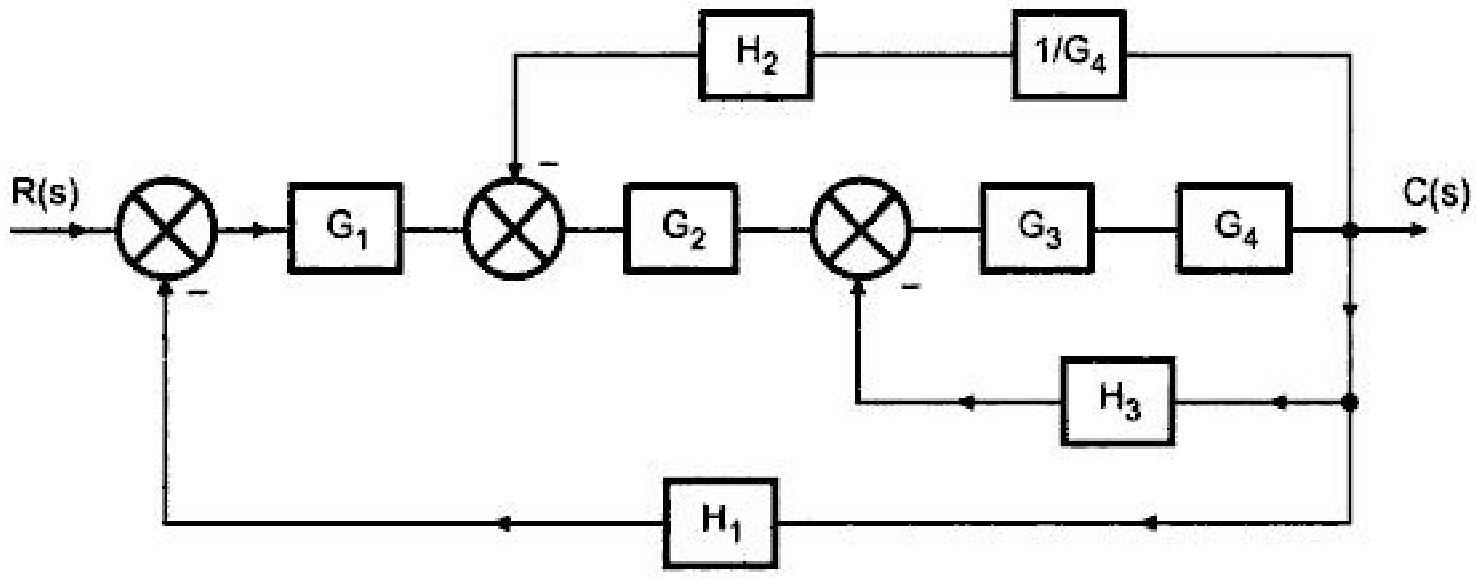
After simplification,

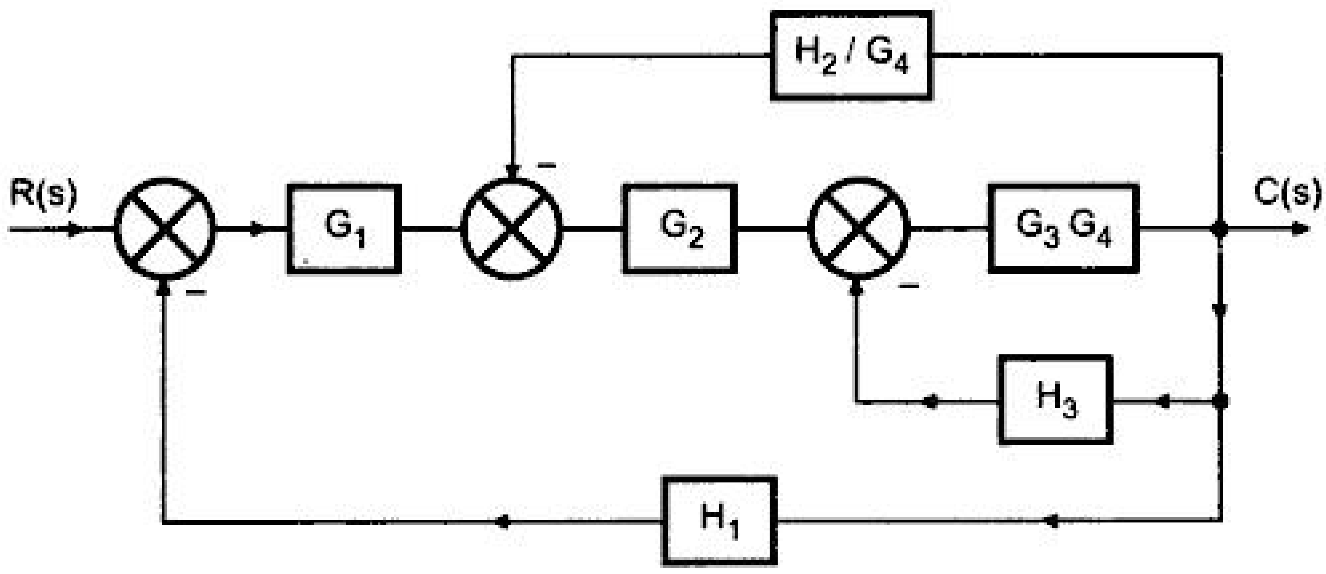
$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3}{1+G_1 H_1+G_1 G_2 H_2+G_1 G_2 G_3 H_3}$$

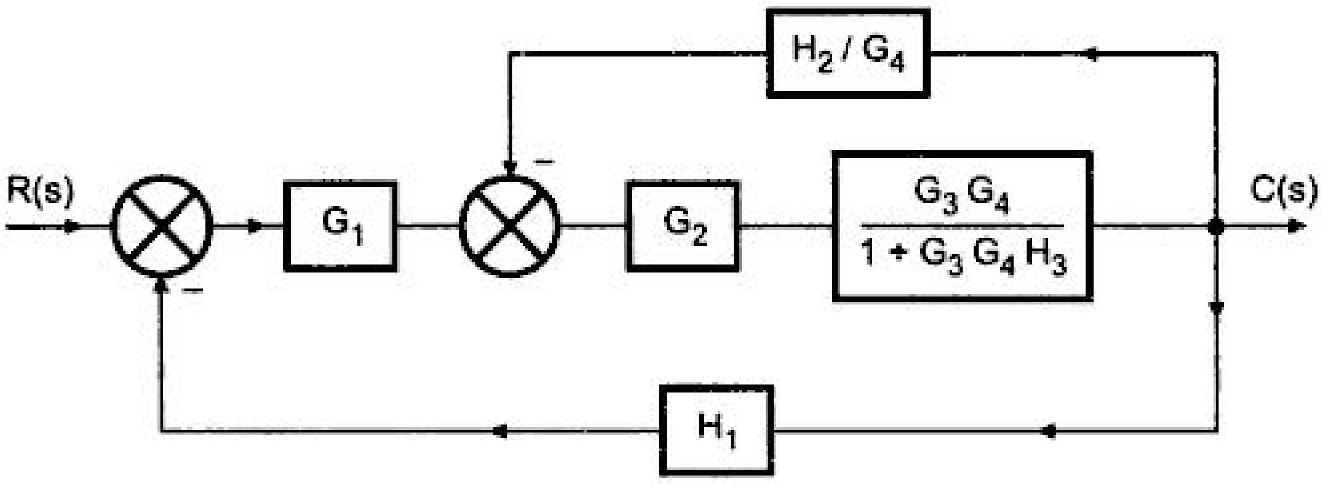
Ex. 3.4

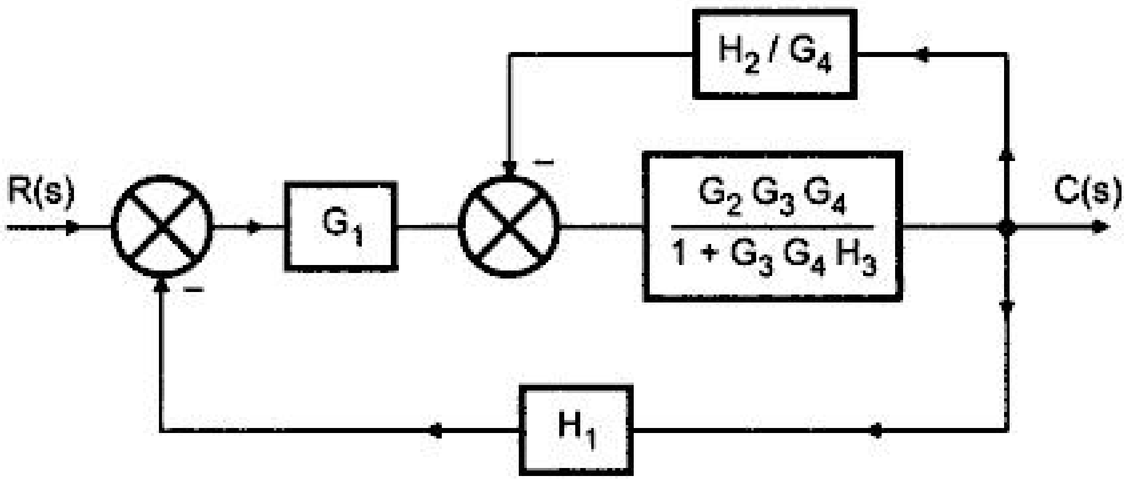


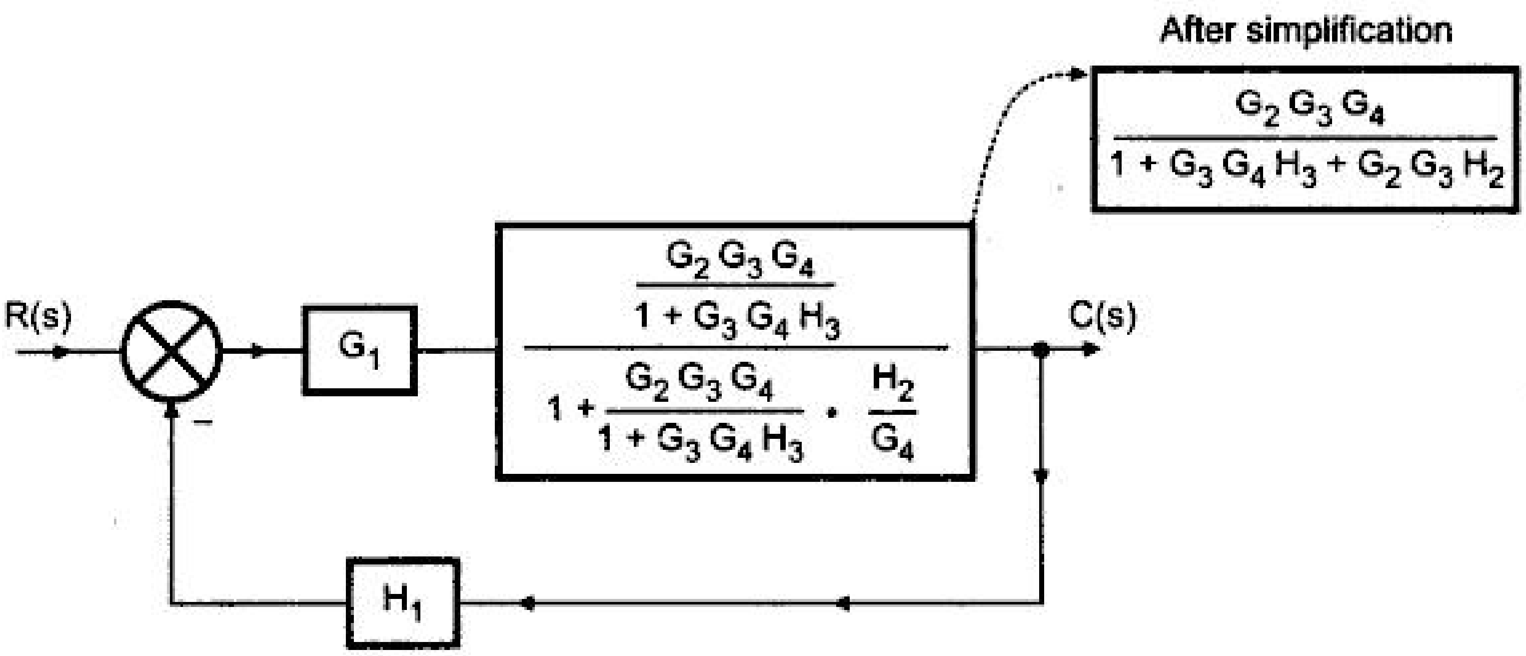
Sol. : No blocks are in series or parallel, similarly there is no minor feedback loop existing. Hence shifting takeoff point towards right as shown we get,

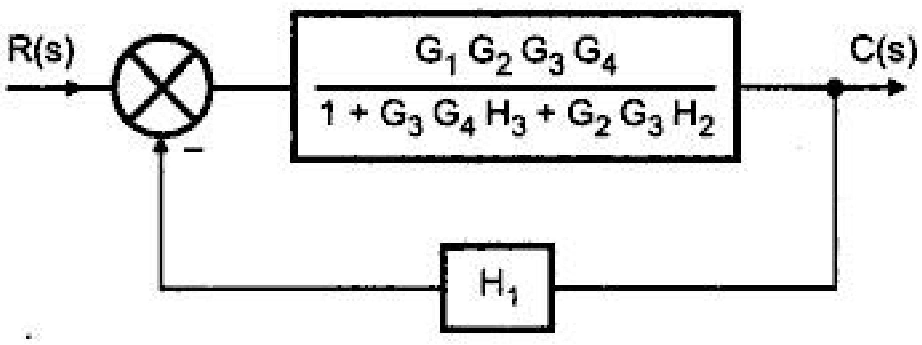


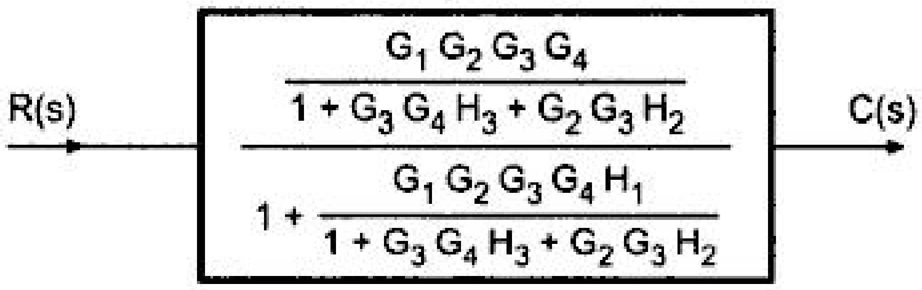



H2 / G4
R(s)
G1
G2
G3 G4 / 1 + G3 G4 H3
C(s)
H1


H2 / G4
R(s)
G1
G2 G3 G4 / 1 + G3 G4 H3
C(s)
H1

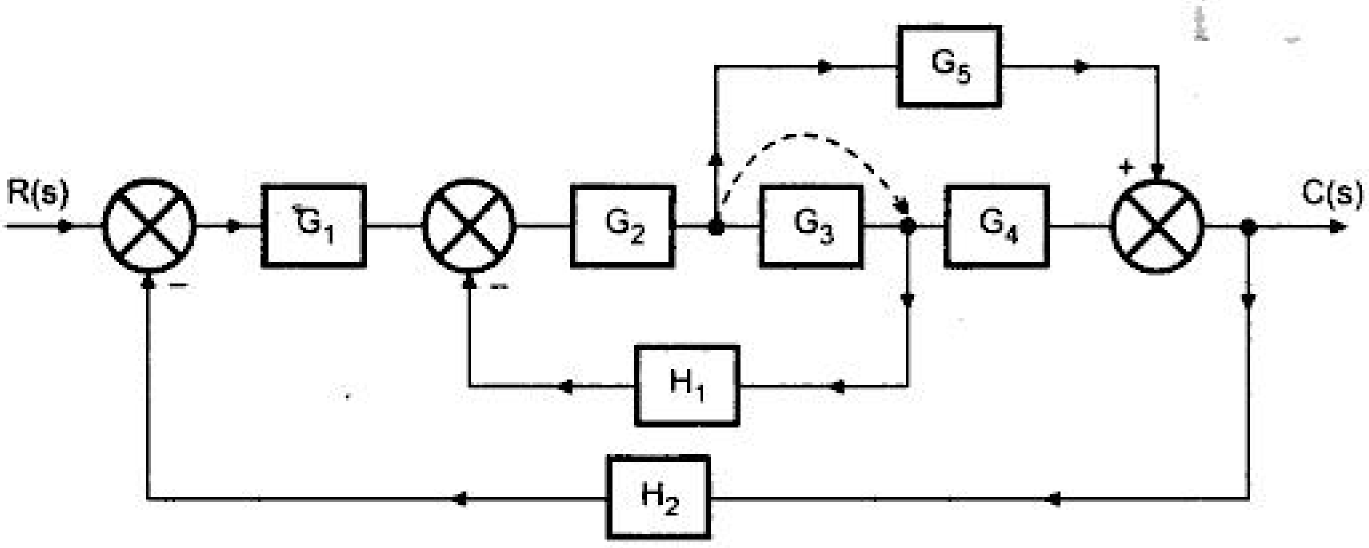

After simplification
G2 G3 G4 / 1 + G3 G4 H3 + G2 G3 H2
R(s)
G1
C(s)
H1


R(s)
G1 G2 G3 G4 / 1 + G3 G4 H3 + G2 G3 H2
C(s)
H1

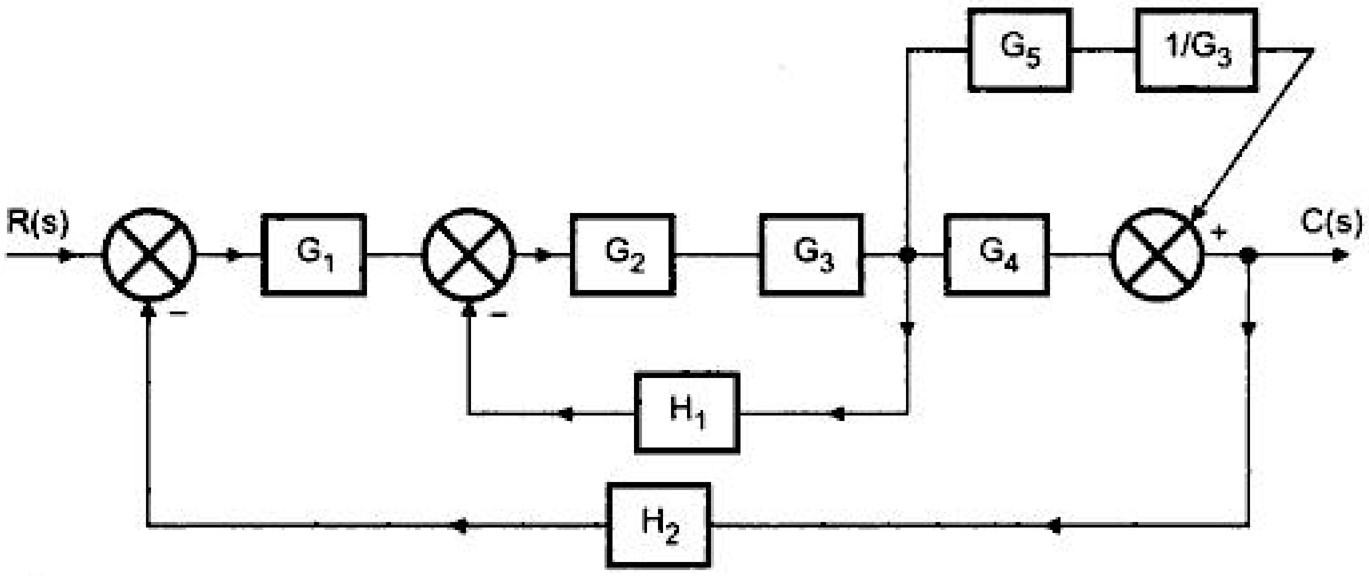


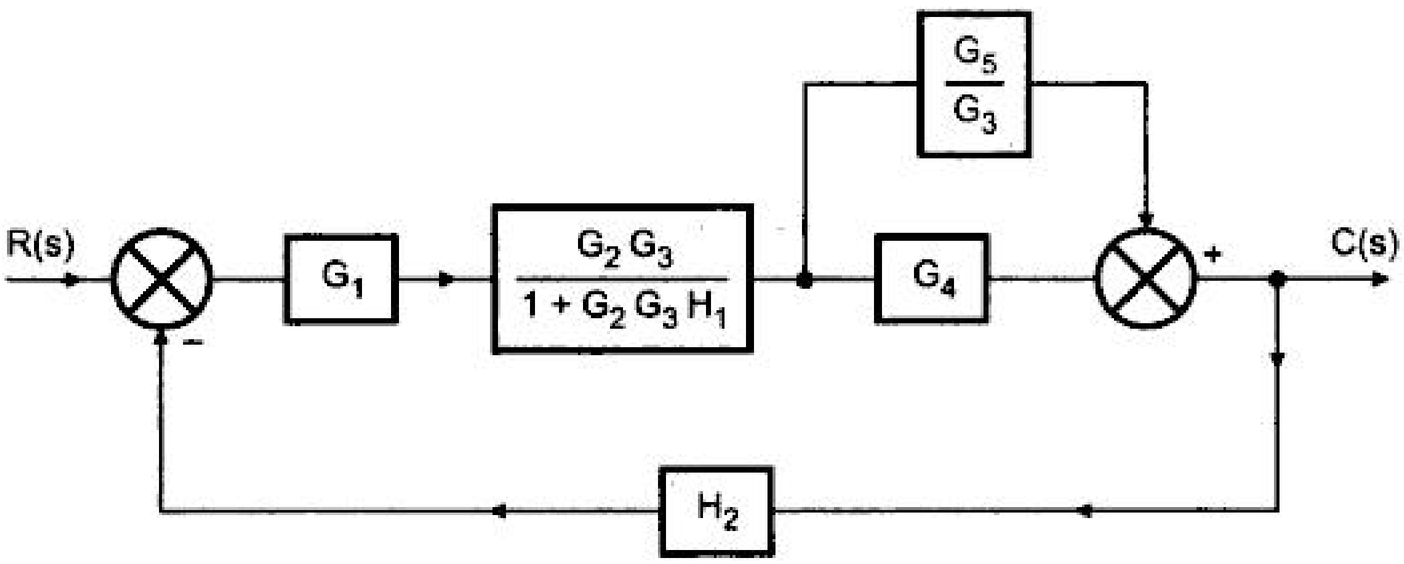
After simplification, $\dfrac{C(s)}{R(s)} = \dfrac{G_1 G_2 G_3 G_4}{1 + G_3 G_4 H_3 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_1}$

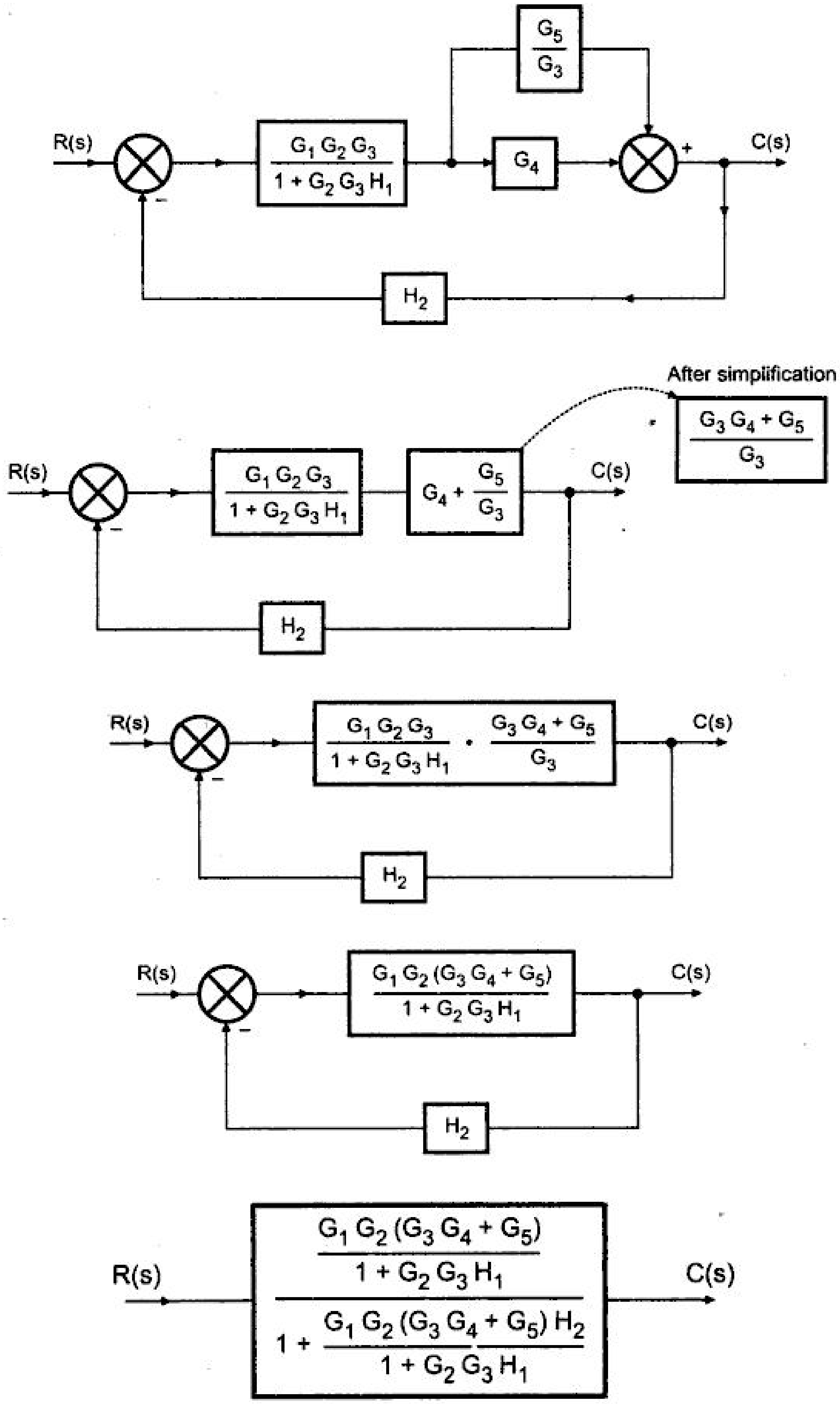
Ex. 3.5



Sol. : No blocks are in series or parallel, similarly there is no minor feedback loop so shifting takeoff point towards right as shown by dotted line we get,





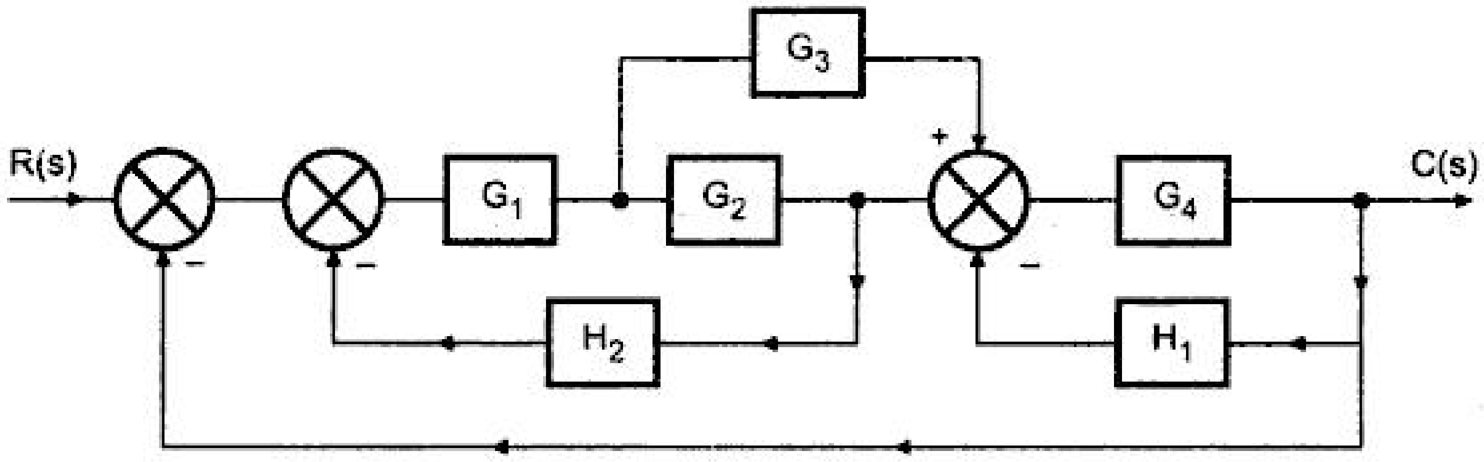


G5 / G3
R(s)
G1 G2 G3 / 1 + G2 G3 H1
G4
+
C(s)
–
H2
After simplification
G3 G4 + G5 / G3
R(s)
G1 G2 G3 / 1 + G2 G3 H1
G4 + G5 / G3
C(s)
–
H2
R(s)
G1 G2 G3 / 1 + G2 G3 H1 · G3 G4 + G5 / G3
C(s)
–
H2
R(s)
G1 G2 (G3 G4 + G5) / 1 + G2 G3 H1
C(s)
–
H2
R(s)
G1 G2 (G3 G4 + G5) / 1 + G2 G3 H1
1 + G1 G2 (G3 G4 + G5) H2 / 1 + G2 G3 H1
C(s)

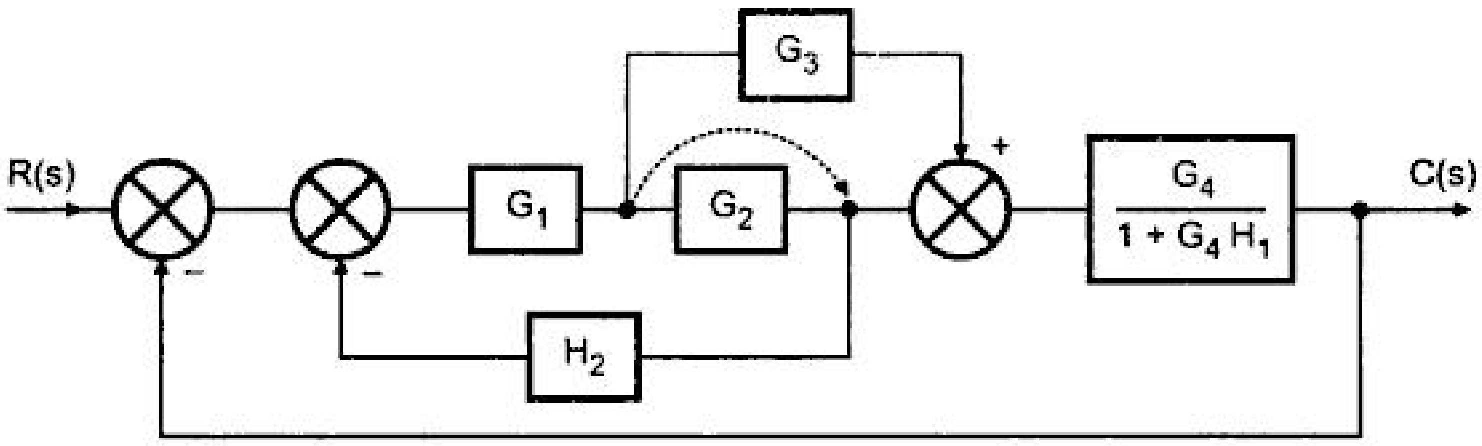
After simplification,

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 (G_3 G_4 + G_5)}{1 + G_2 G_3 H_1 + G_1 G_2 (G_3 G_4 + G_5) H_2}$$

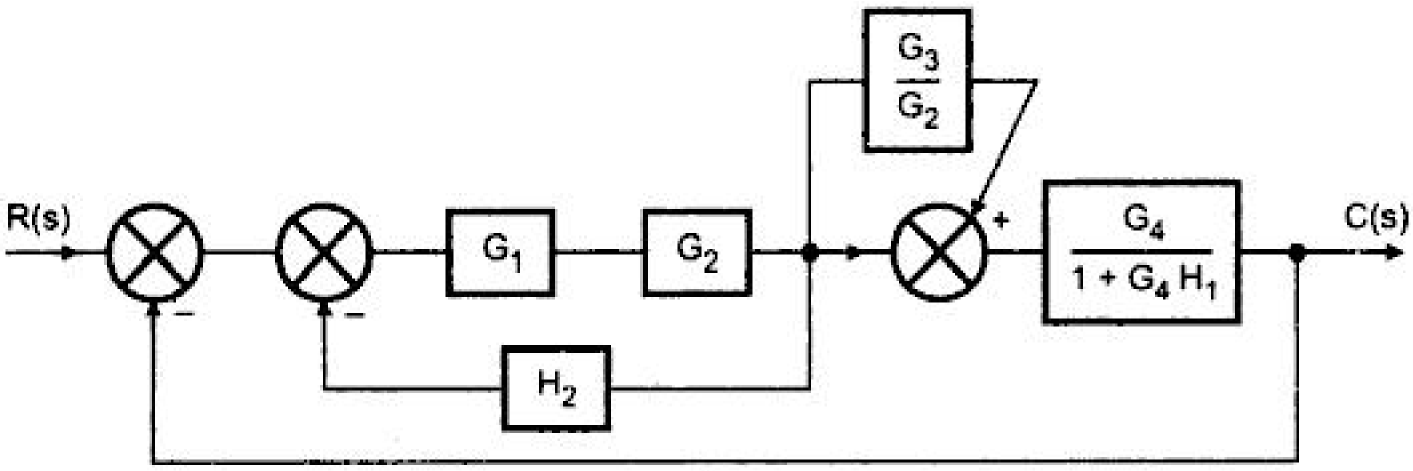
Ex. 3.6

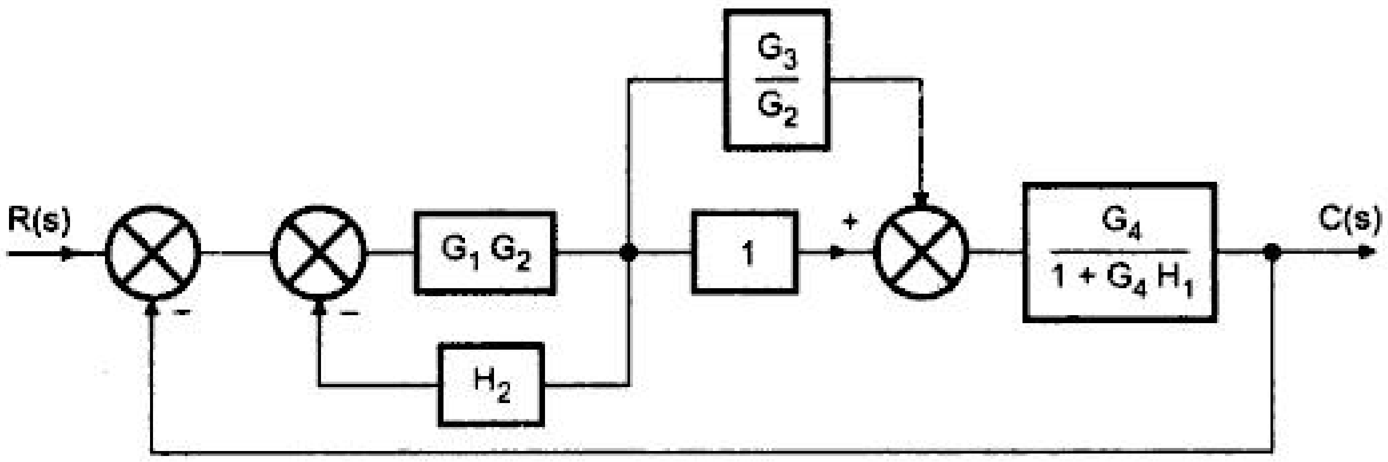


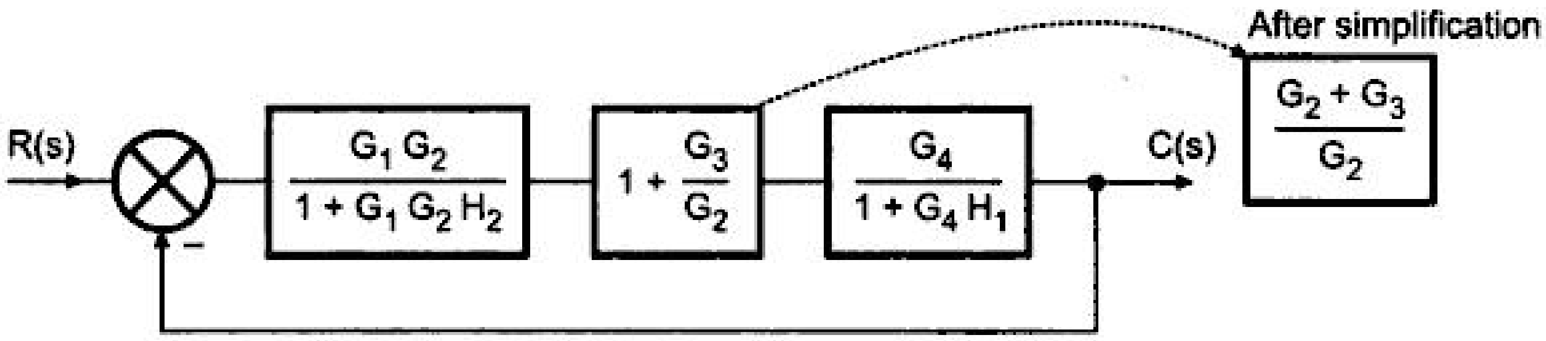
Sol. :

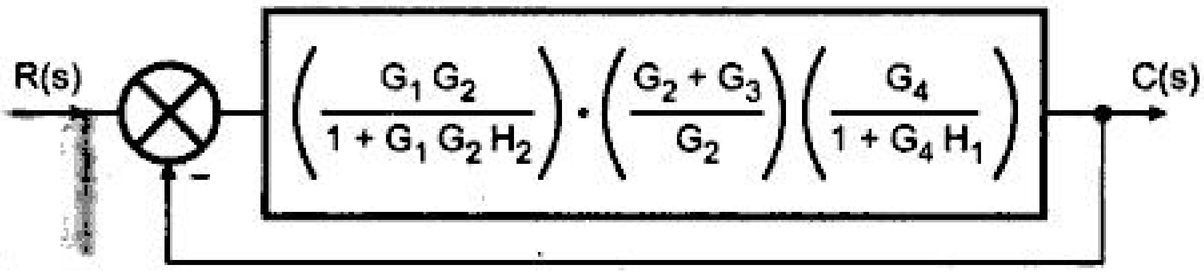


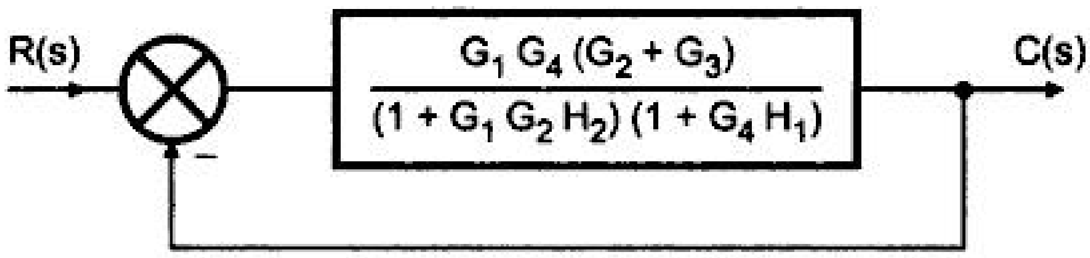
Shifting takeoff point as shown :

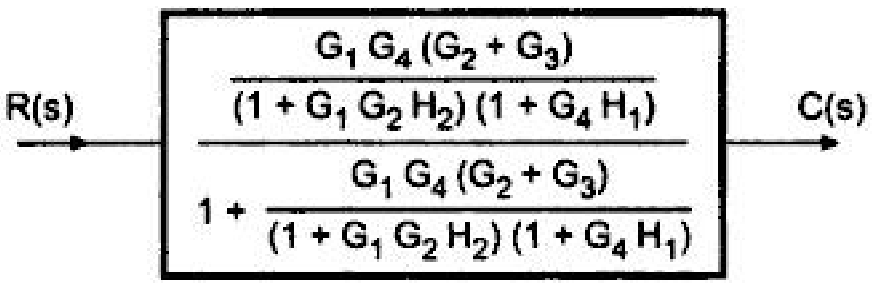






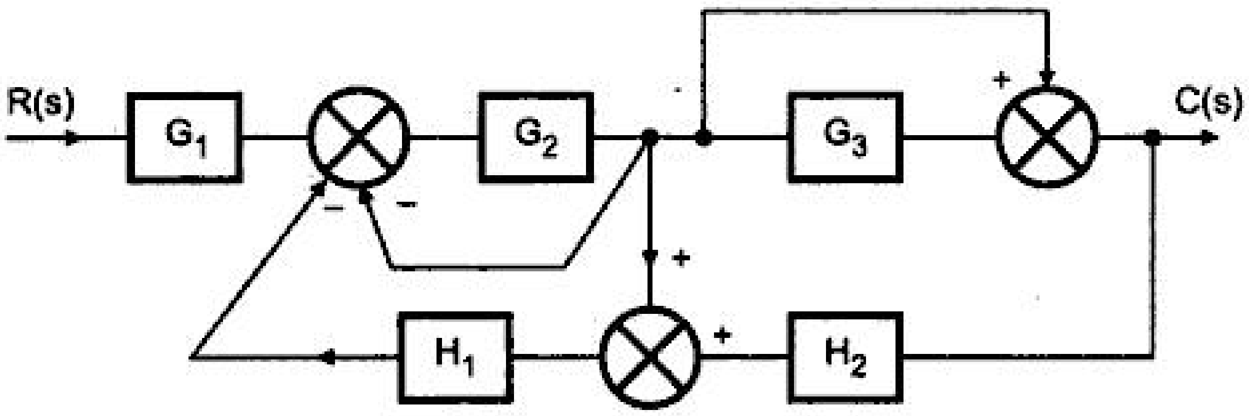




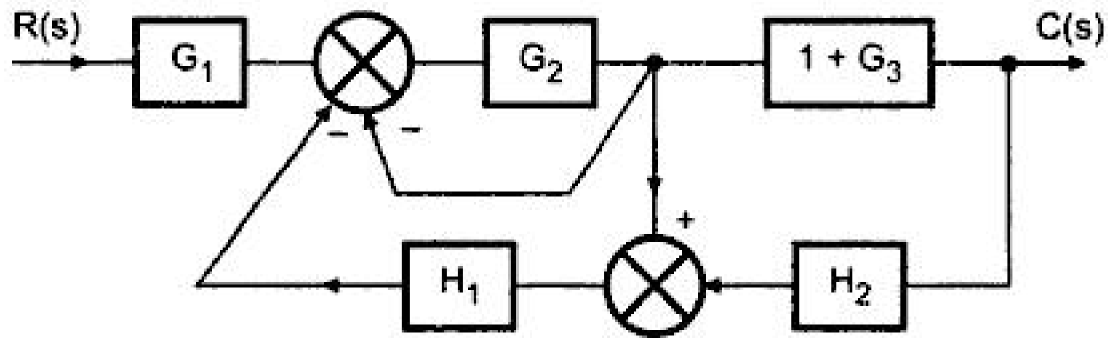


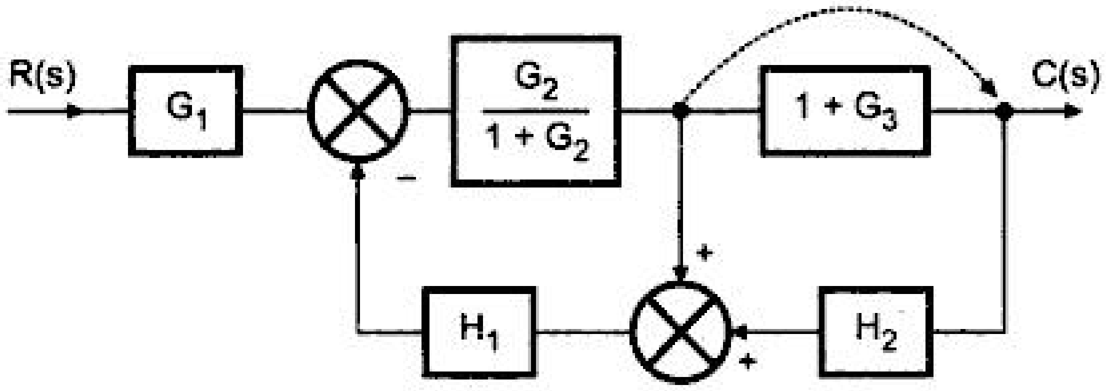
$$\frac{C(s)}{R(s)} = \frac{G_1 G_4 (G_2 + G_3)}{1 + G_1 G_2 H_2 + G_4 H_1 + G_1 G_2 G_4 H_1 H_2 + G_1 G_4 (G_2 + G_3)}$$

Ex. 3.7

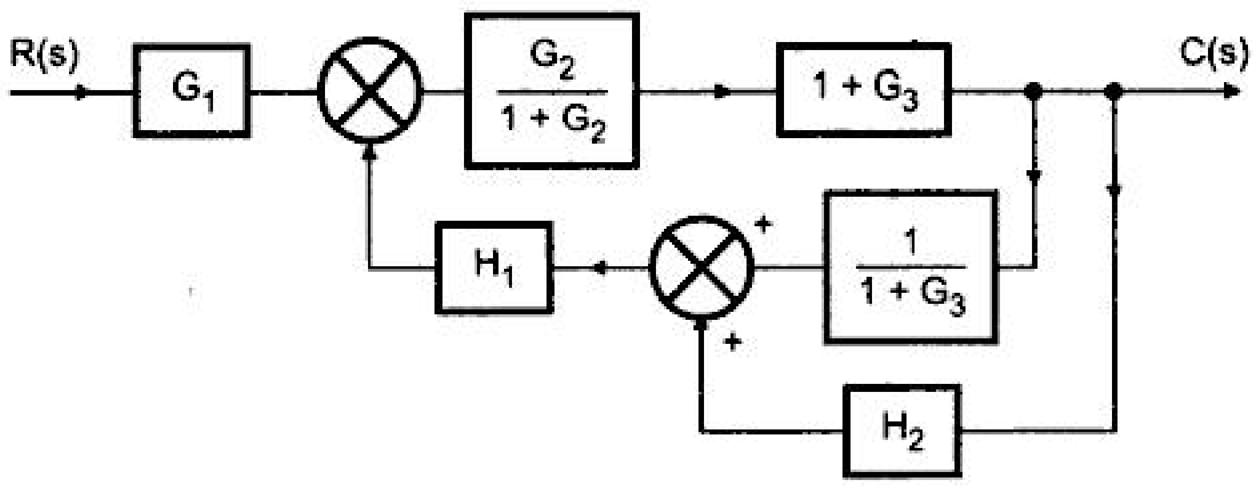


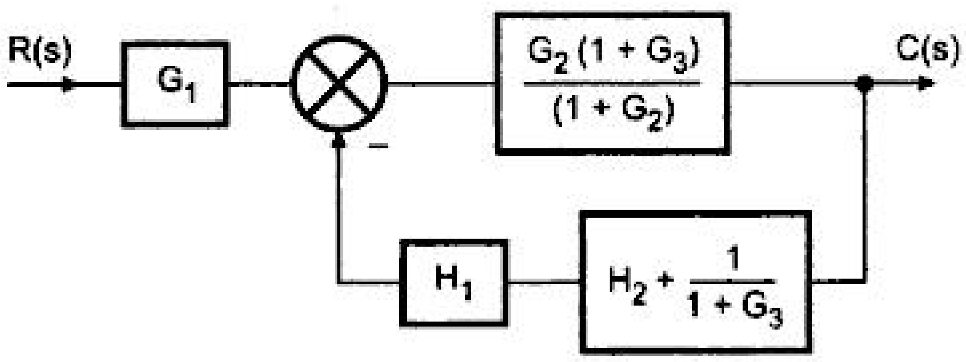
Sol. : Block with T.F. G_3 and unity gain block are in parallel so combining them we get,





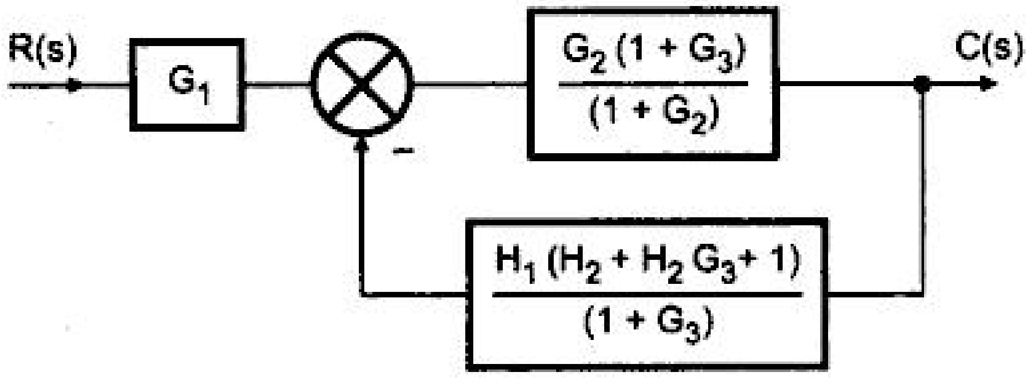
Shifting take off point

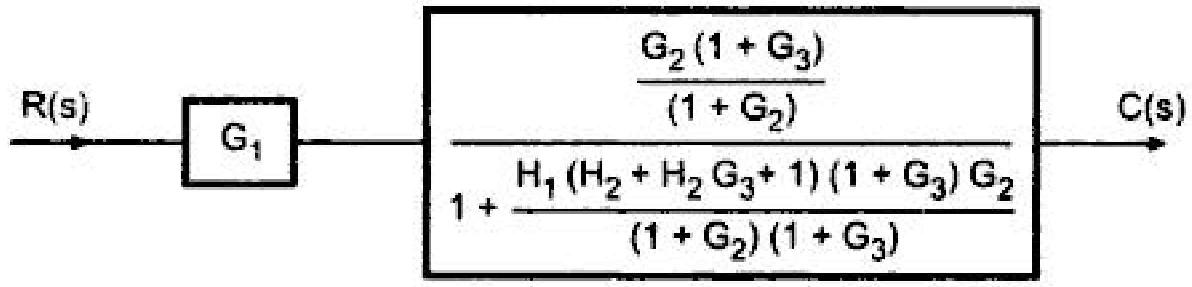


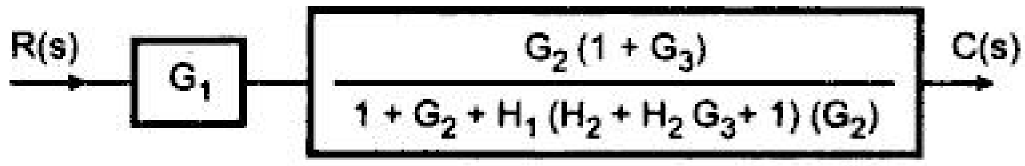


After simplification,

$$\frac{H_2\,(1+G_3)+1}{(1+G_3)} \quad \text{i.e.} \quad \frac{H_2+H_2\,G_3+1}{1+G_3}$$

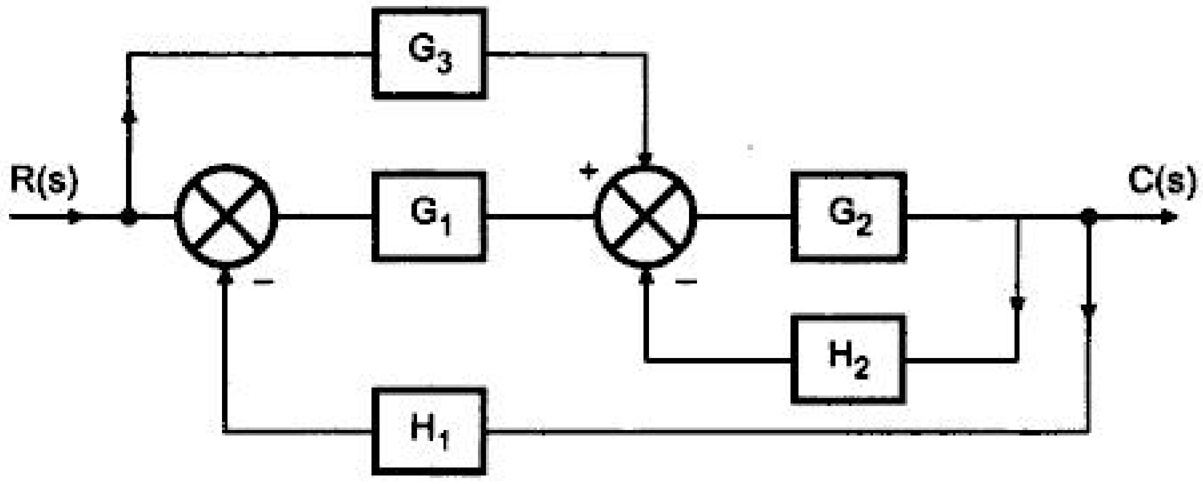




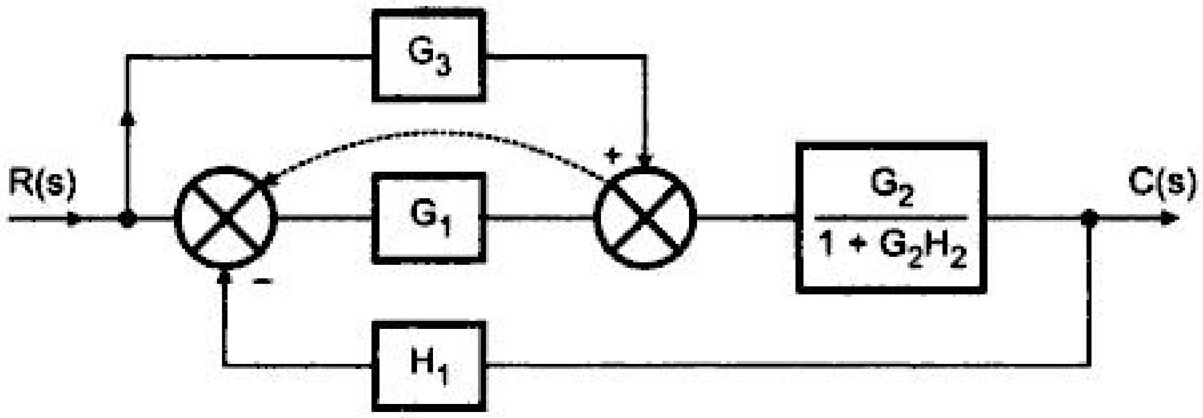


$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G_1 G_2 (1+G_3)}{1+G_2+H_1 G_2+H_1 H_2 G_2+H_1 H_2 G_2 G_3}$$

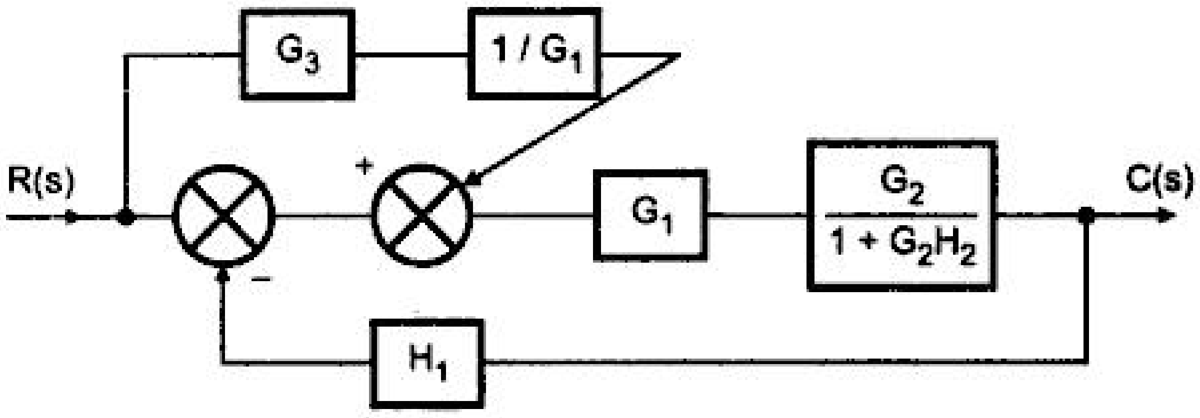
Ex. 3.8



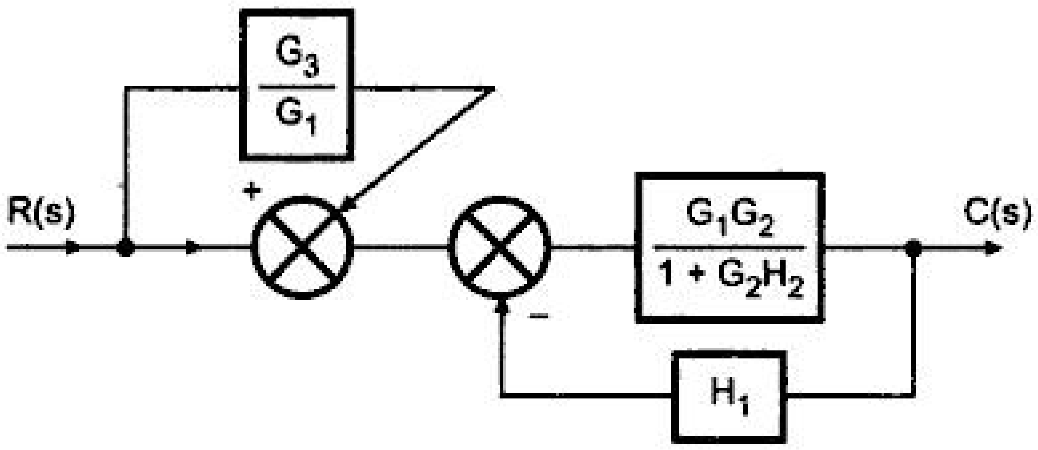
Sol. :

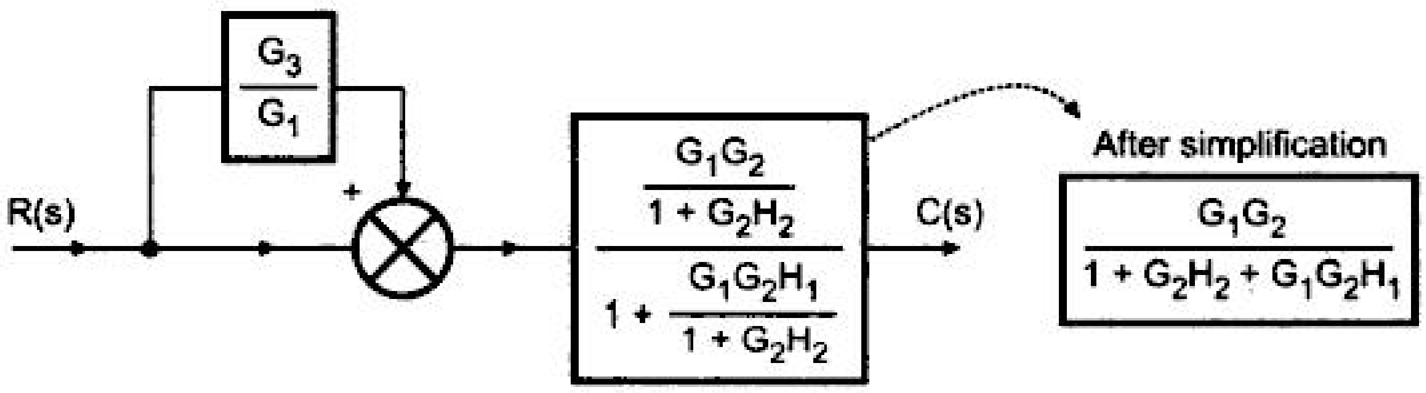


No blocks are in series or parallel so shifting summing point towards left i.e. before the block having transfer function G_1 as shown in Figure.

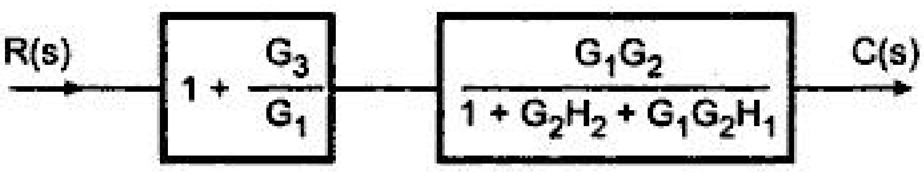


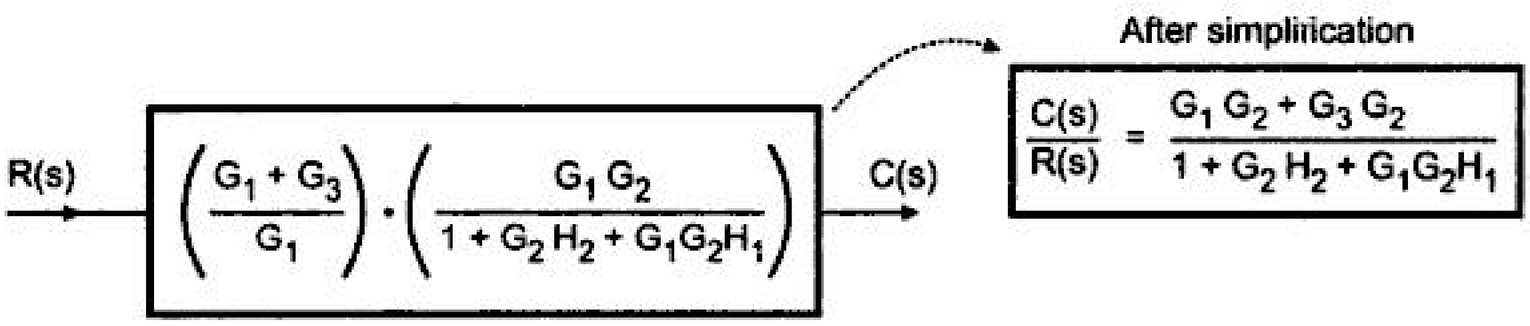
Using Associative law for two summing points we get,



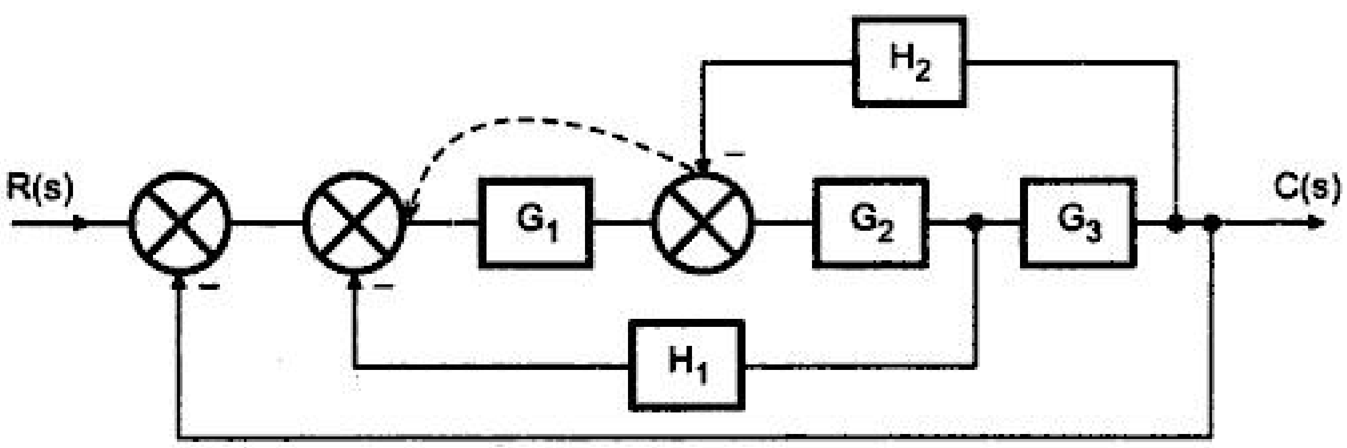


The two signals with transfer function 1 and with transfer function $\frac{G_3}{G_1}$ are in parallel. So they will add to each other so we have,

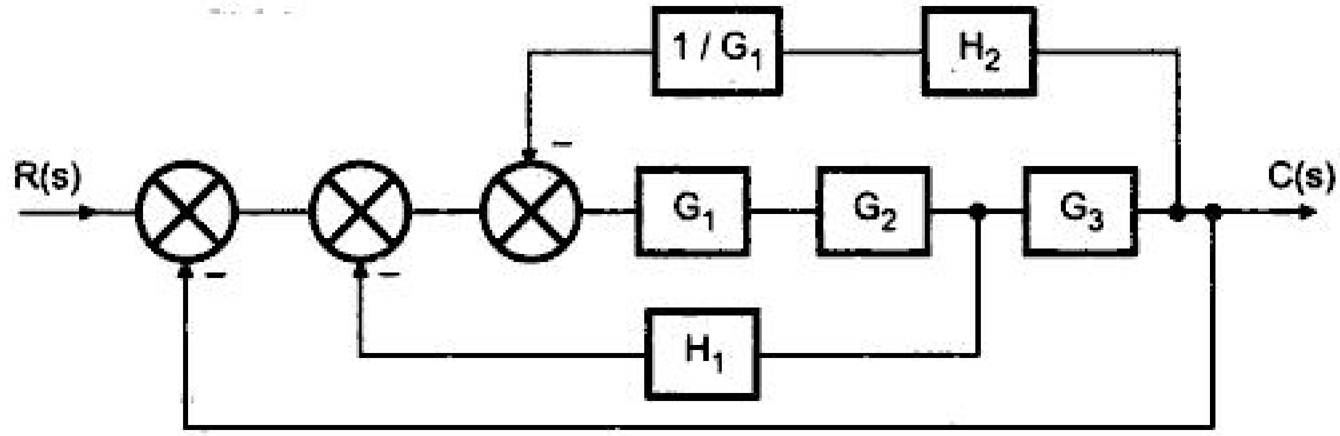


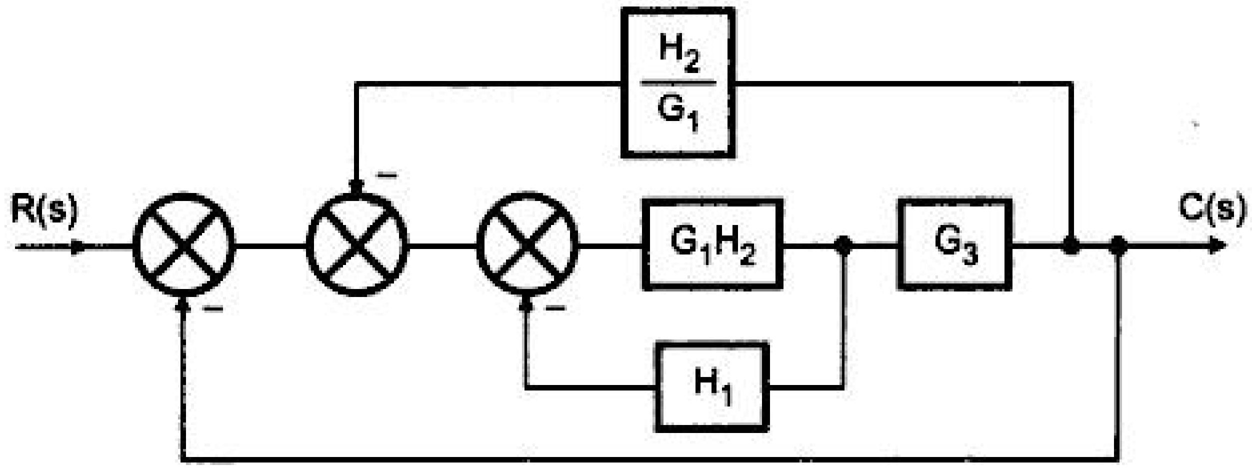


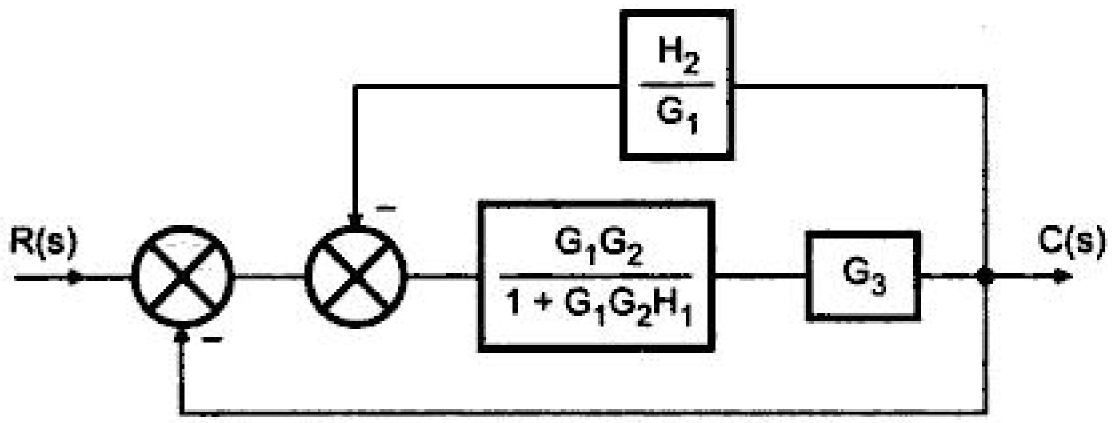
Ex. 3.9

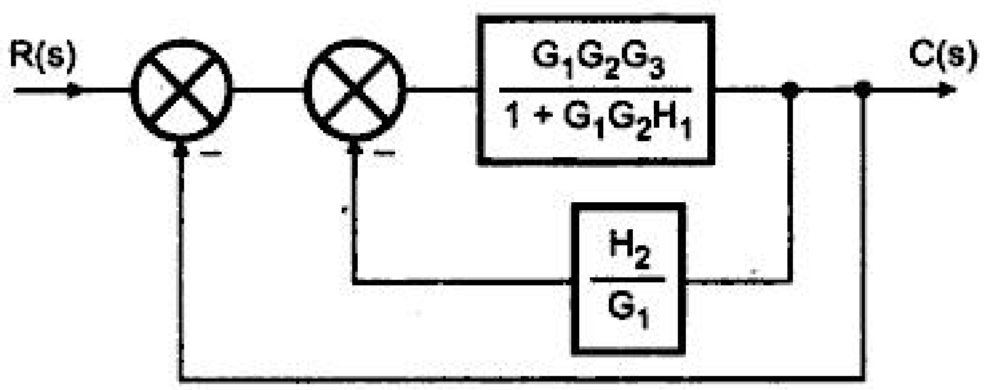


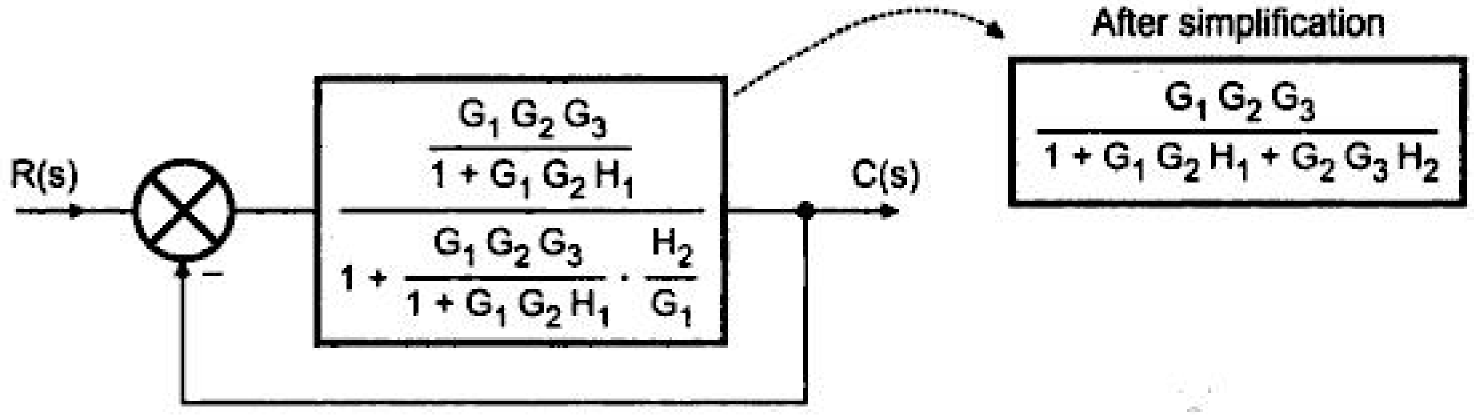
Sol. : No blocks are in series or parallel and no separate minor feedback loop is existing so shifting summing point towards left, before the block with transfer function G_1 as shown,

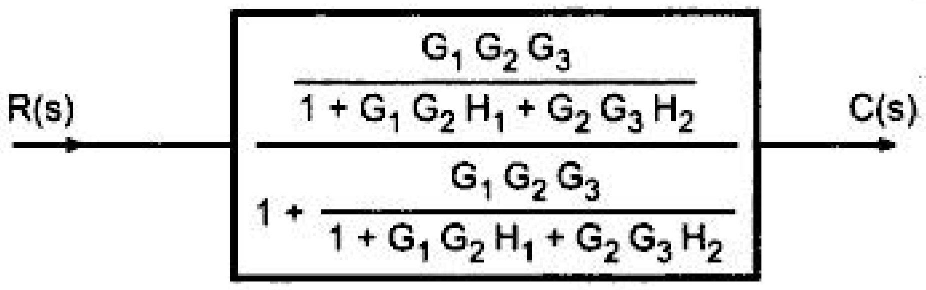






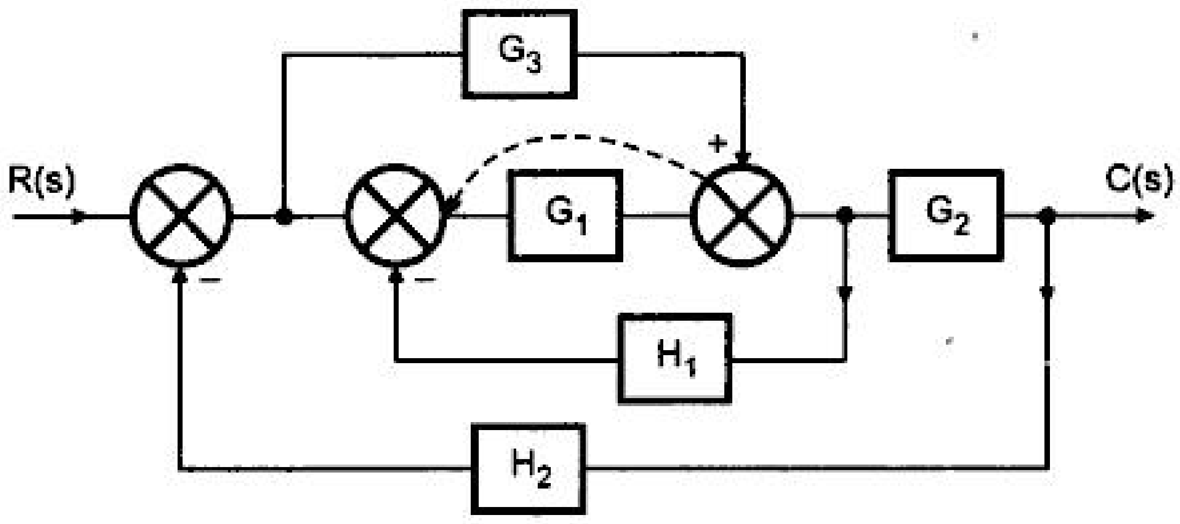




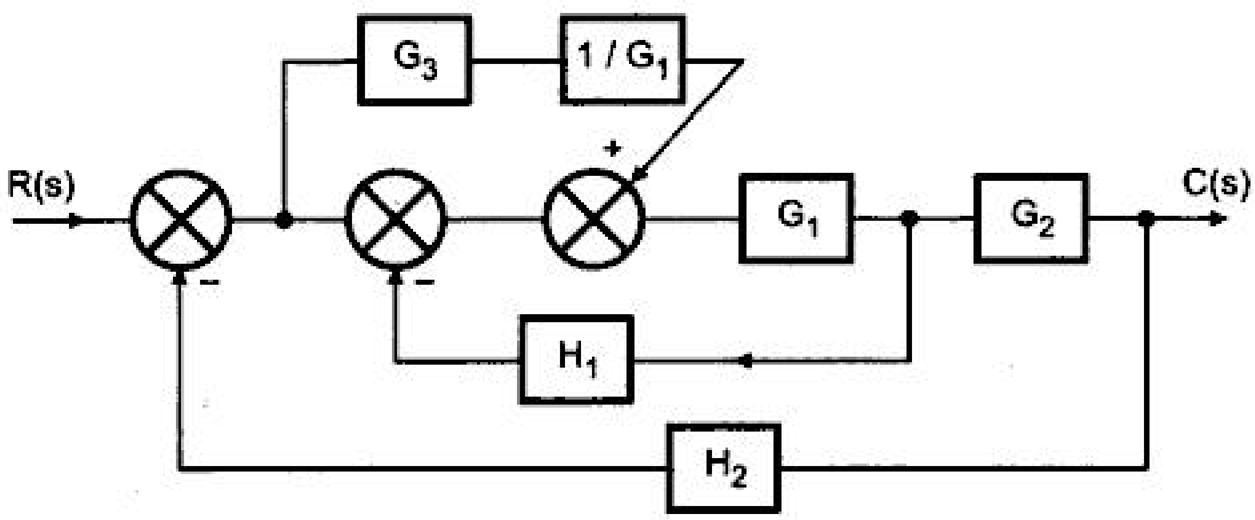


$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3}{1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3}$$

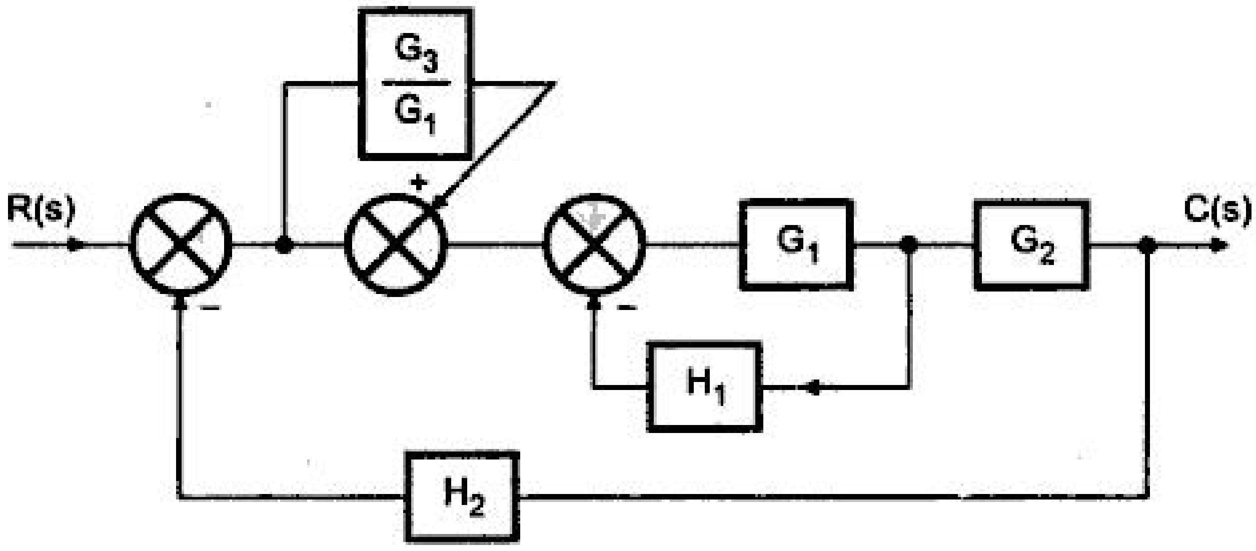
Ex. 3.10



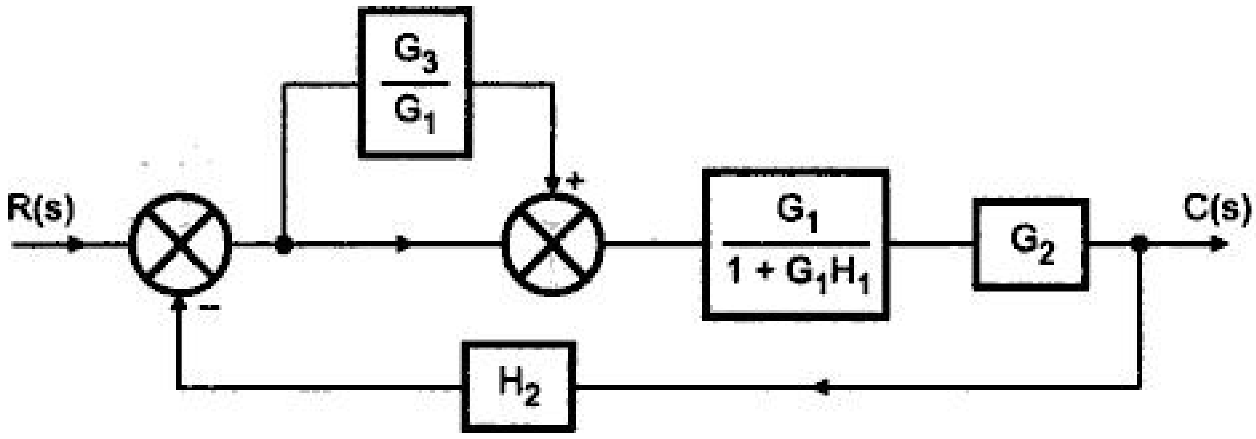
Sol. : No blocks are in series or parallel and no minor feedback loop is existing so shifting summing point towards left i.e. behind block with transfer function G_1 as shown, we get,



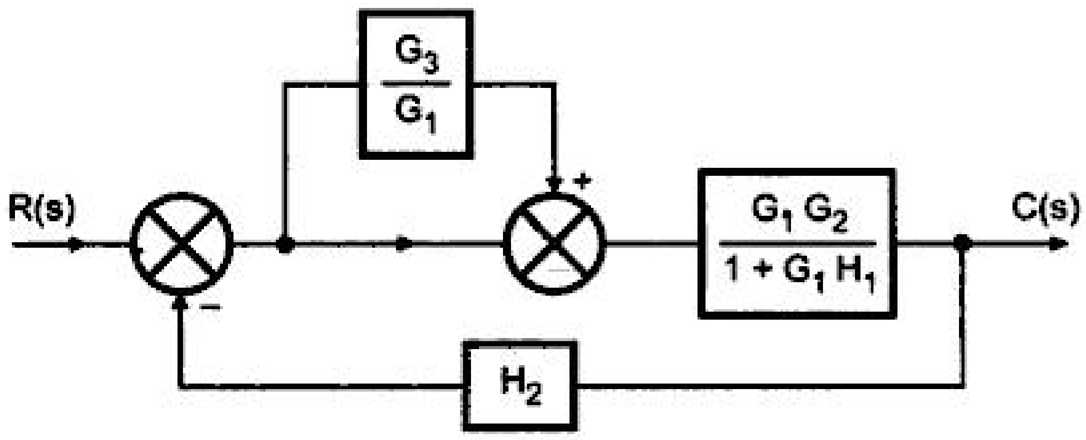
Use Associative Law for the summing points, we get,



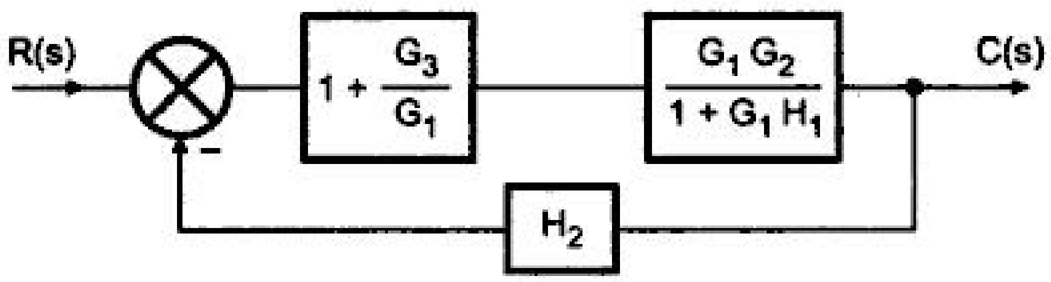
R(s)
$\frac{G_3}{G_1}$
G_1
G_2
C(s)
H_1
H_2



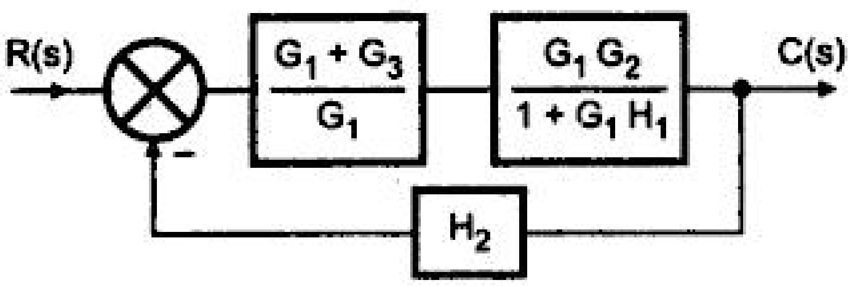
R(s)
$\frac{G_3}{G_1}$
$\frac{G_1}{1 + G_1H_1}$
G_2
C(s)
H_2



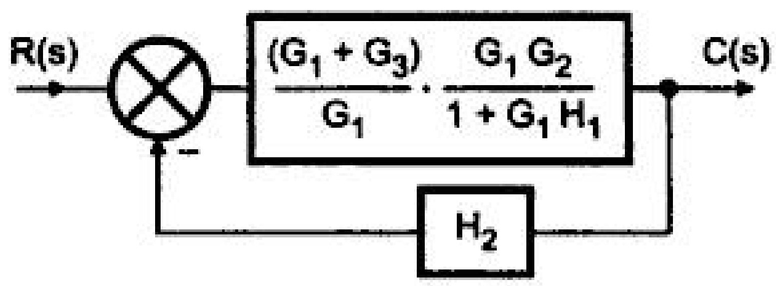
R(s)
$\frac{G_3}{G_1}$
$\frac{G_1 G_2}{1 + G_1 H_1}$
C(s)
H_2



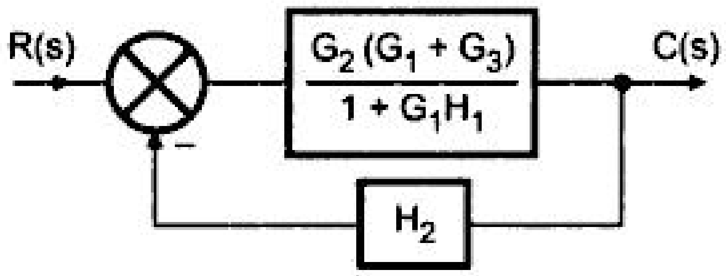
R(s)
$1 + \frac{G_3}{G_1}$
$\frac{G_1 G_2}{1 + G_1 H_1}$
C(s)
H_2

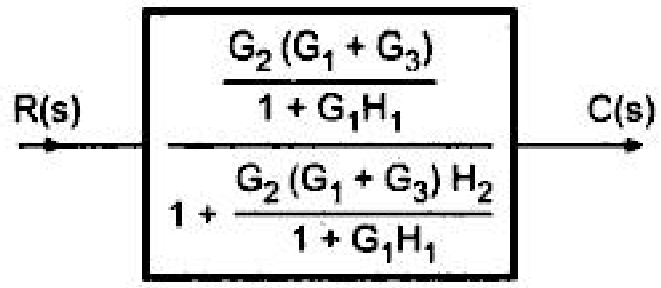


R(s)
$\frac{G_1 + G_3}{G_1}$
$\frac{G_1 G_2}{1 + G_1 H_1}$
C(s)
H_2



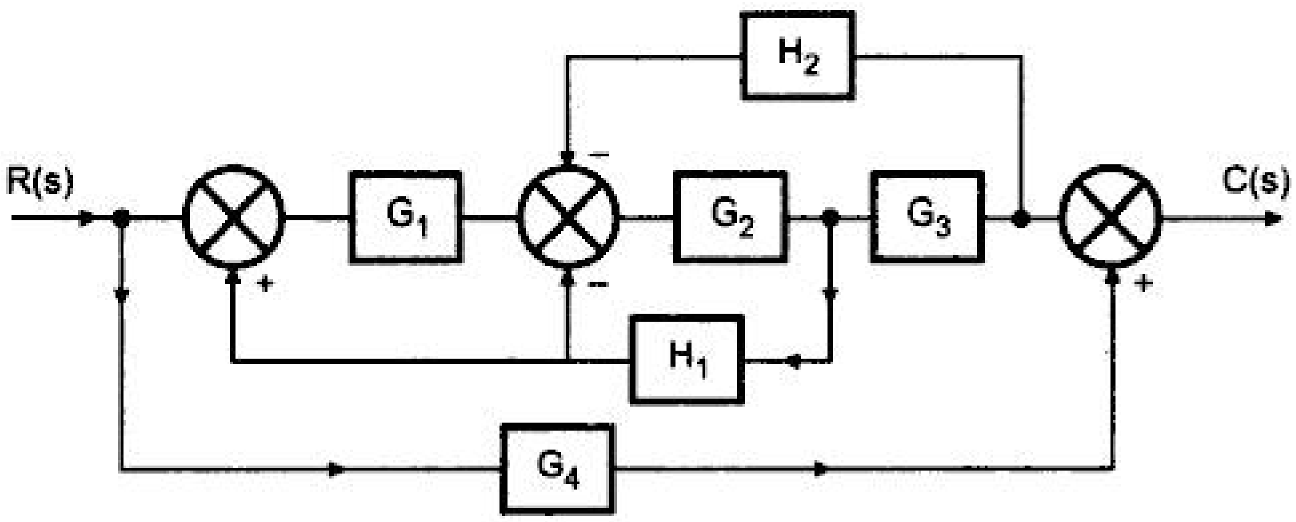
R(s)
$\frac{(G_1 + G_3)}{G_1} \cdot \frac{G_1 G_2}{1 + G_1 H_1}$
C(s)
H_2





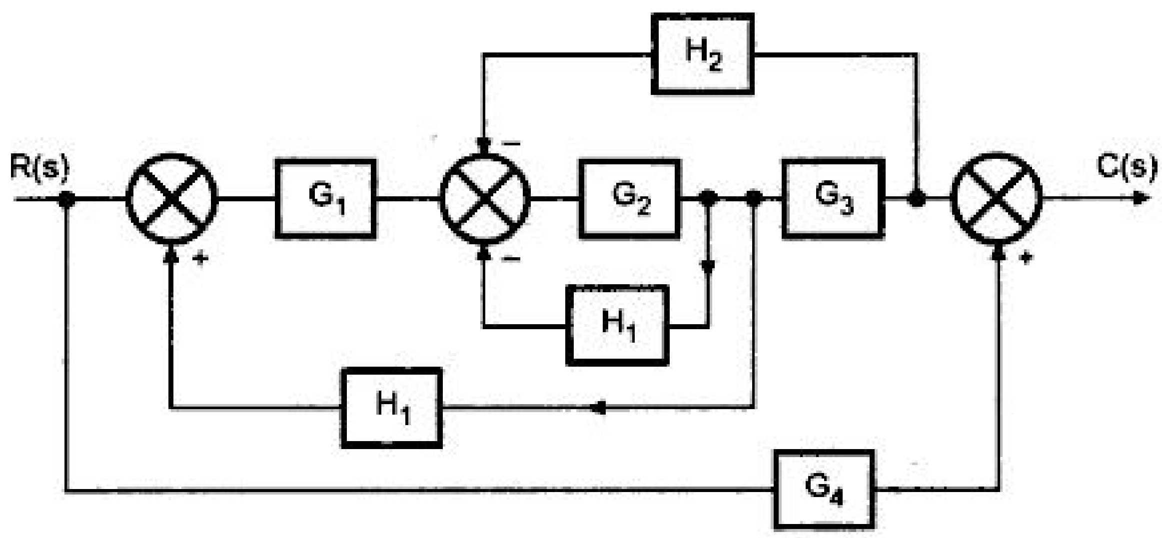
$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G_1G_2+G_2G_3}{1+G_1H_1+G_1G_2H_2+G_2H_2G_3}$$

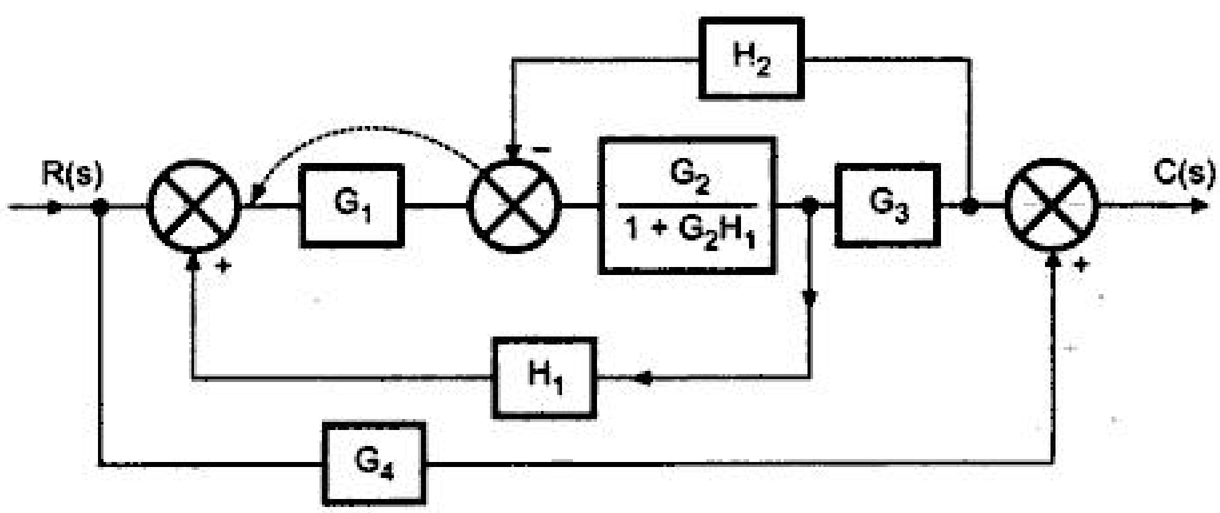
Ex. 3.11



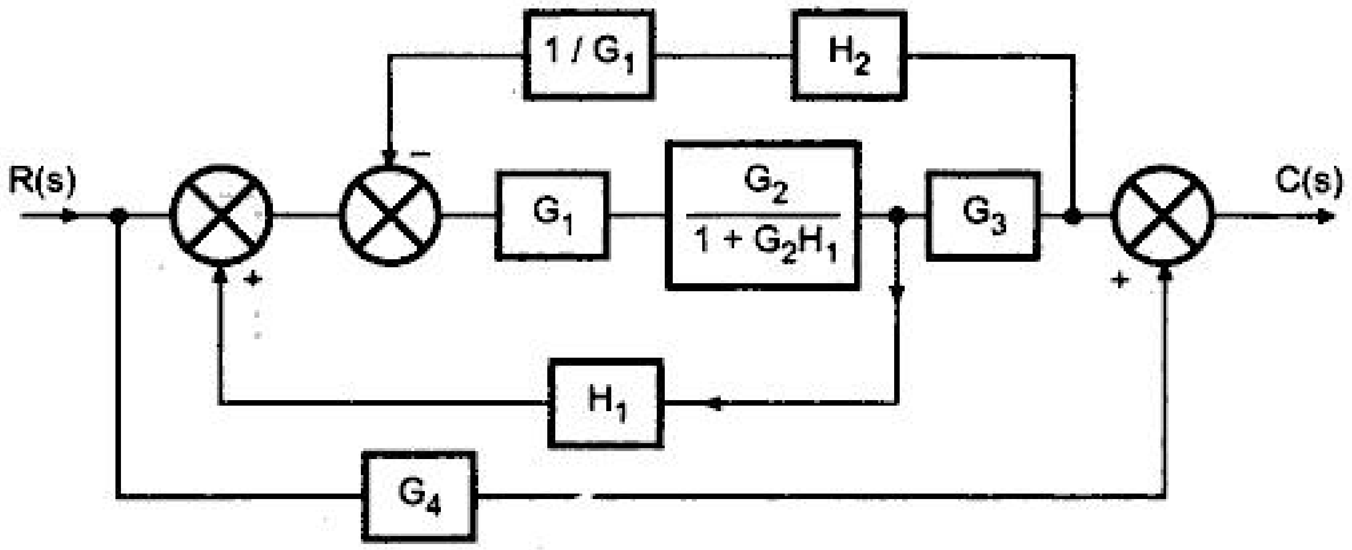
(Mumbai university Dec. 97)

Sol. : Separating two feedback from second takeoff point which is after block having transfer function G_2 as shown, we get,

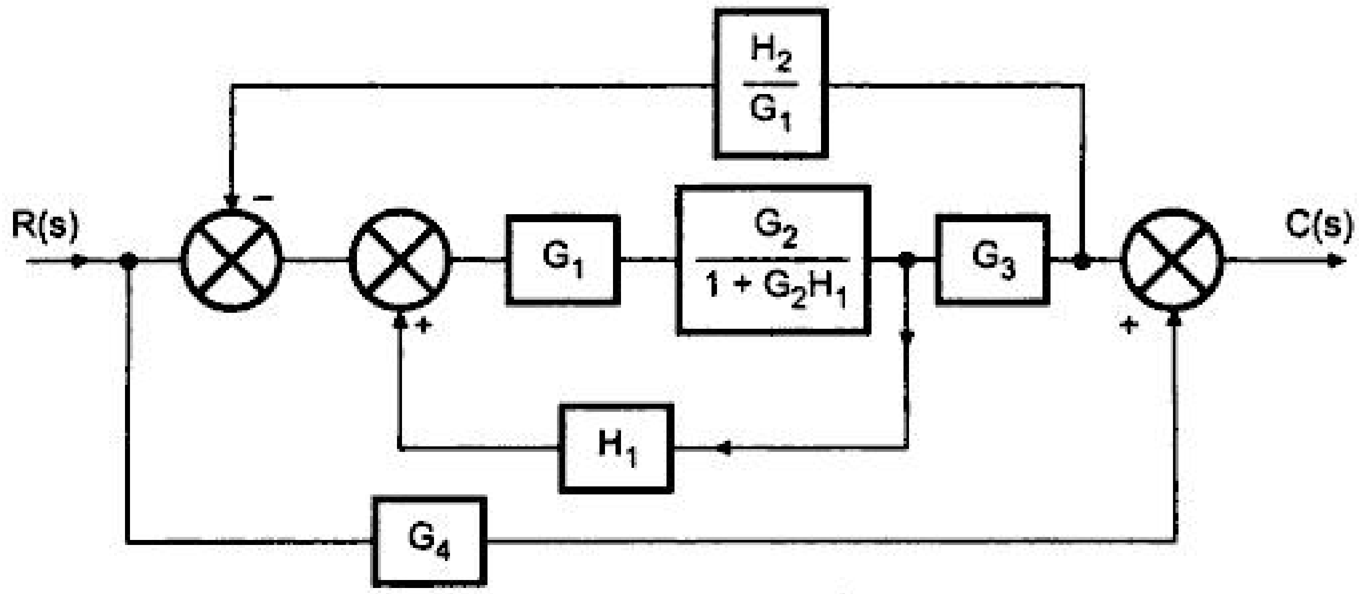


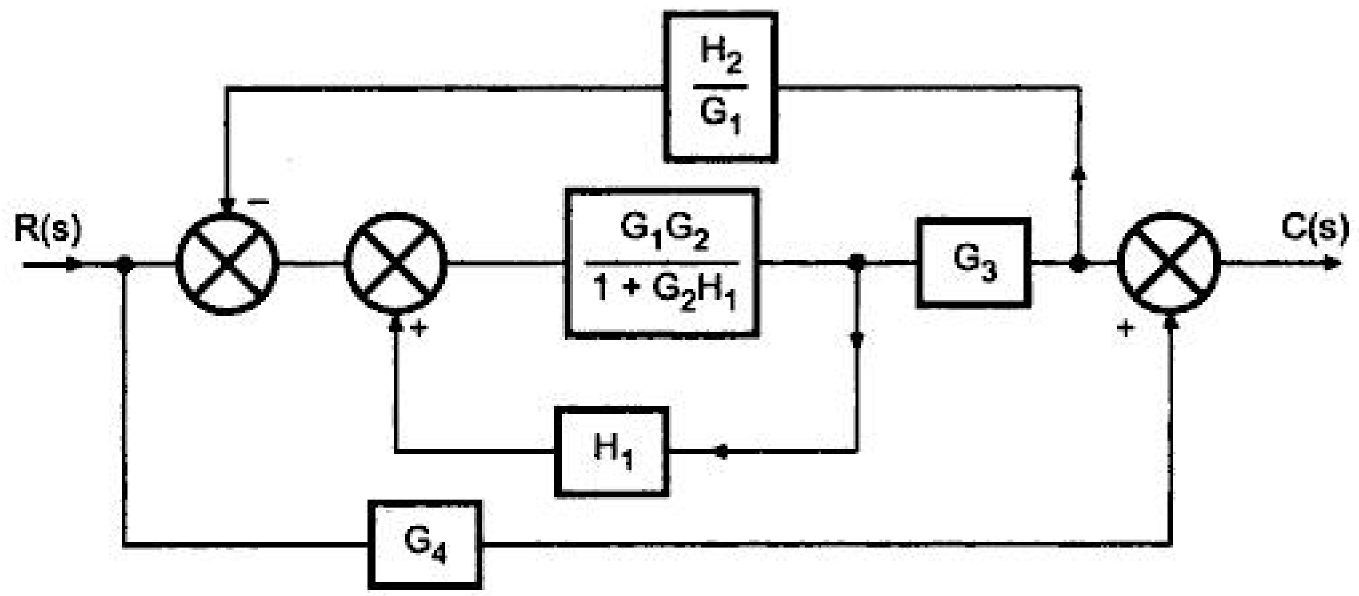


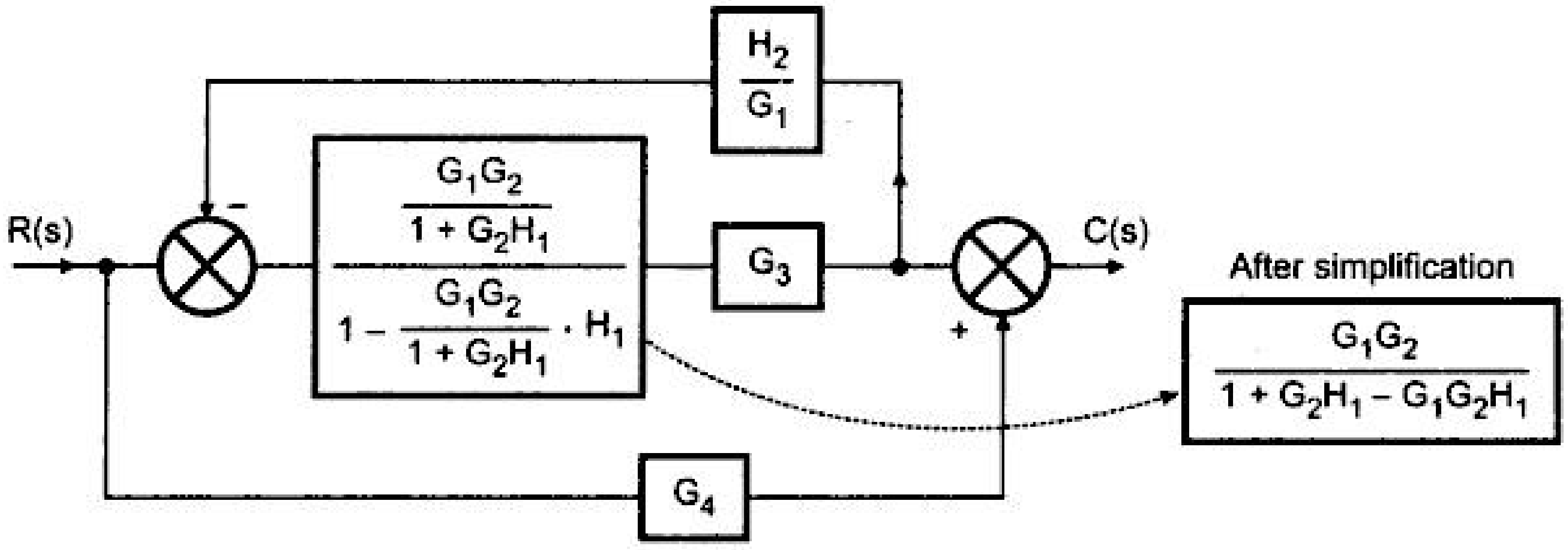
Shifting summing point behind the block having transfer function 'G_1' as shown we get,

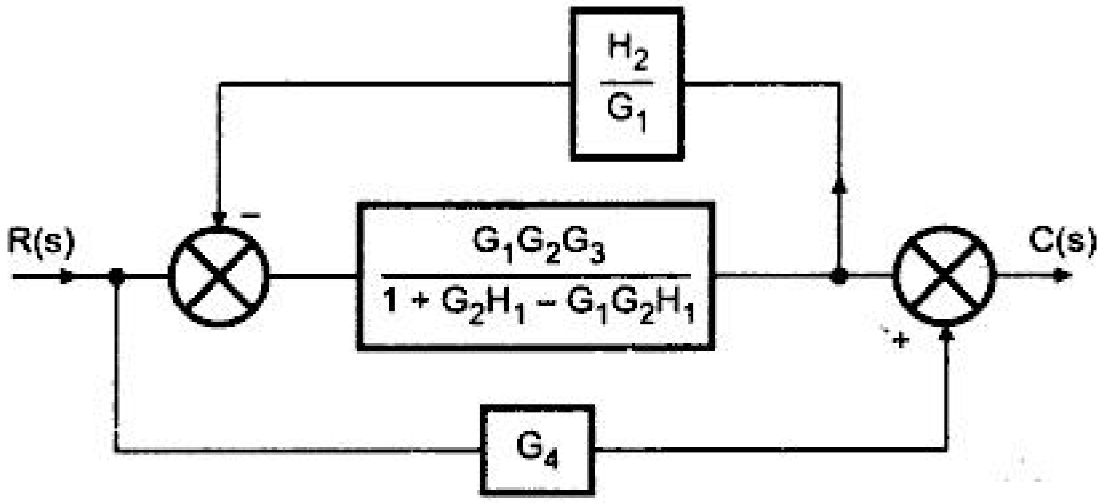


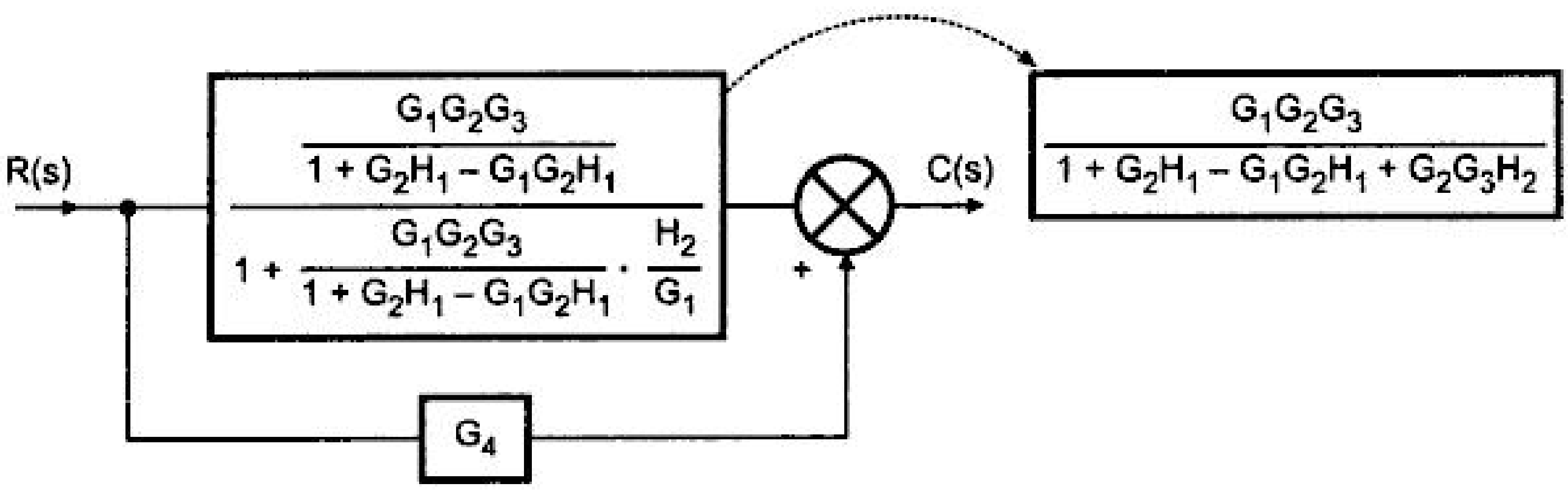
Use Associative Law for the two summing points and interchange their positions, we get,

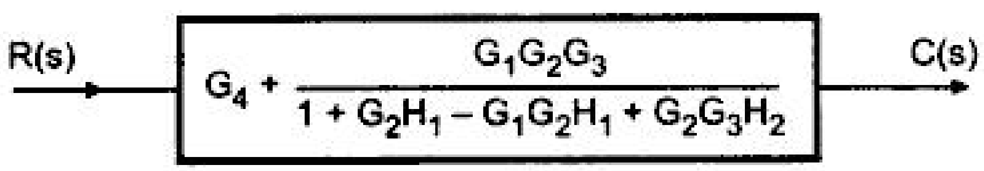






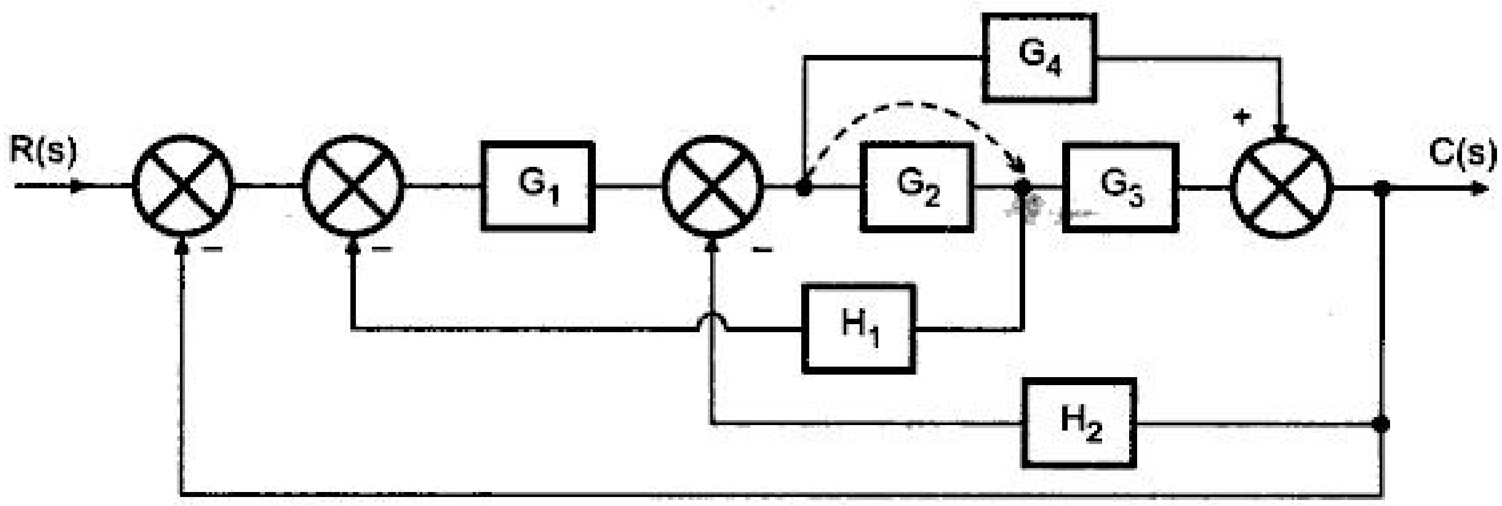




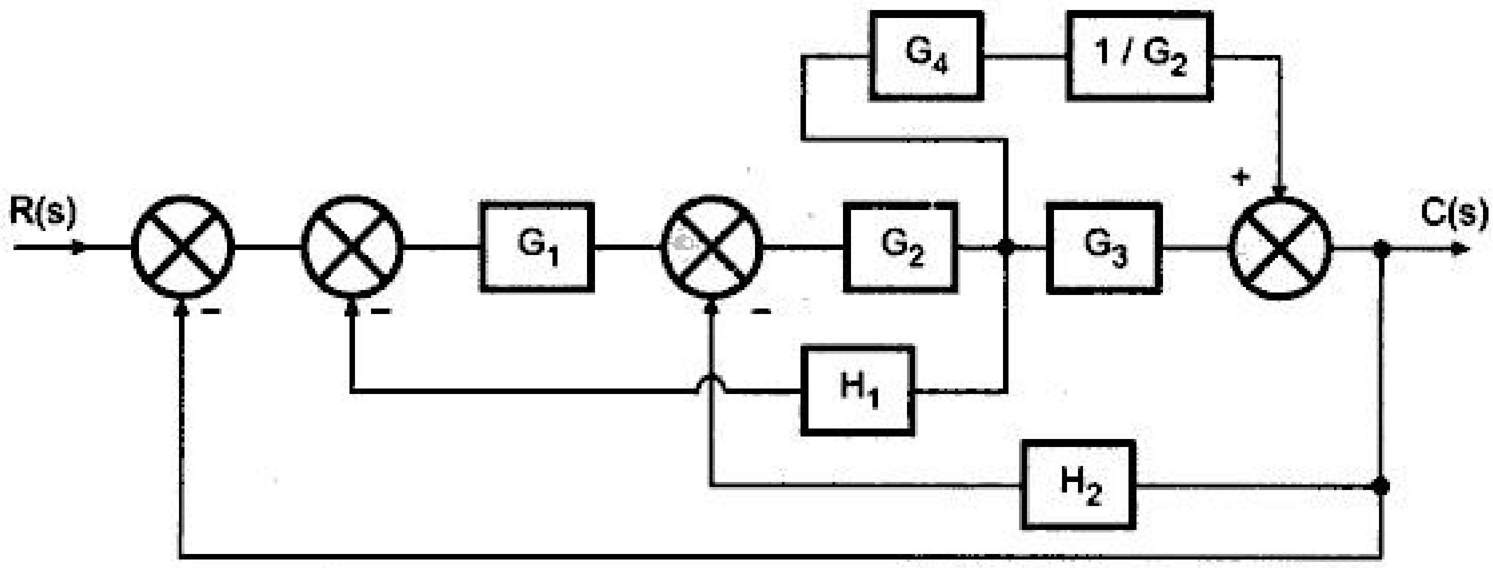


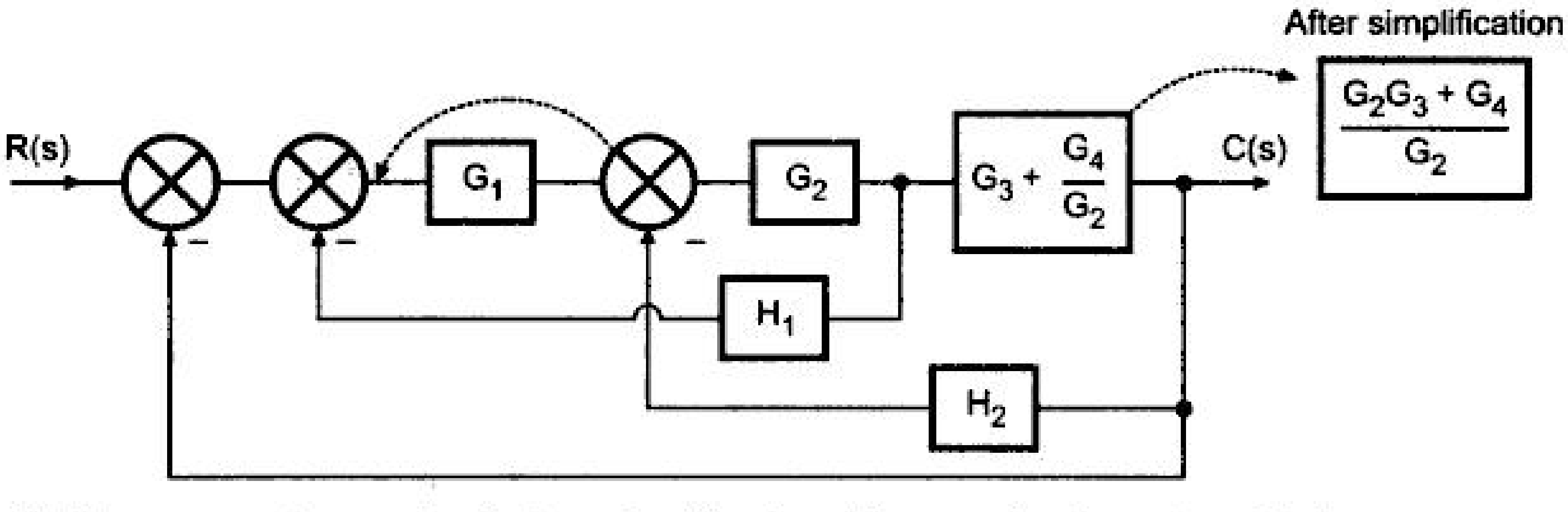
$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G_4 + G_4 G_2 H_1 - G_4 G_1 G_2 H_1 + G_2 G_3 G_4 H_2 + G_1 G_2 G_3}{1 + G_2 H_1 - G_1 G_2 H_1 + G_2 G_3 H_2}$$

Ex. 3.12

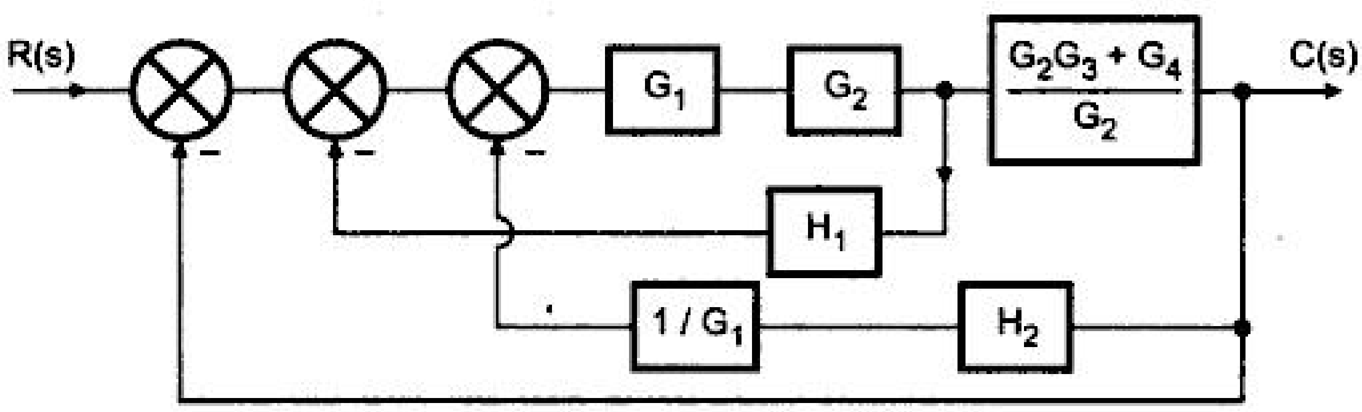


Sol. : Shifting take off point after the block having transfer function G_2 we get,

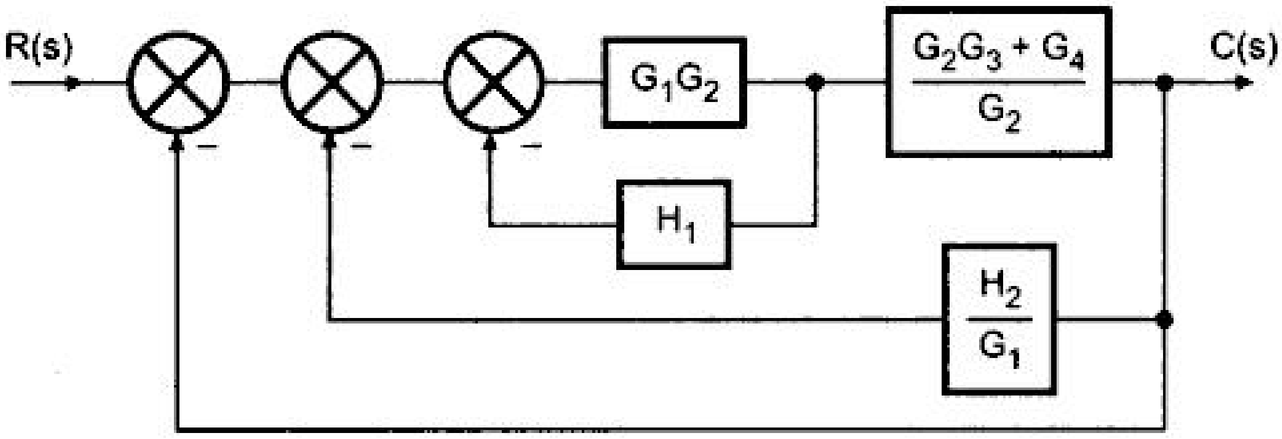


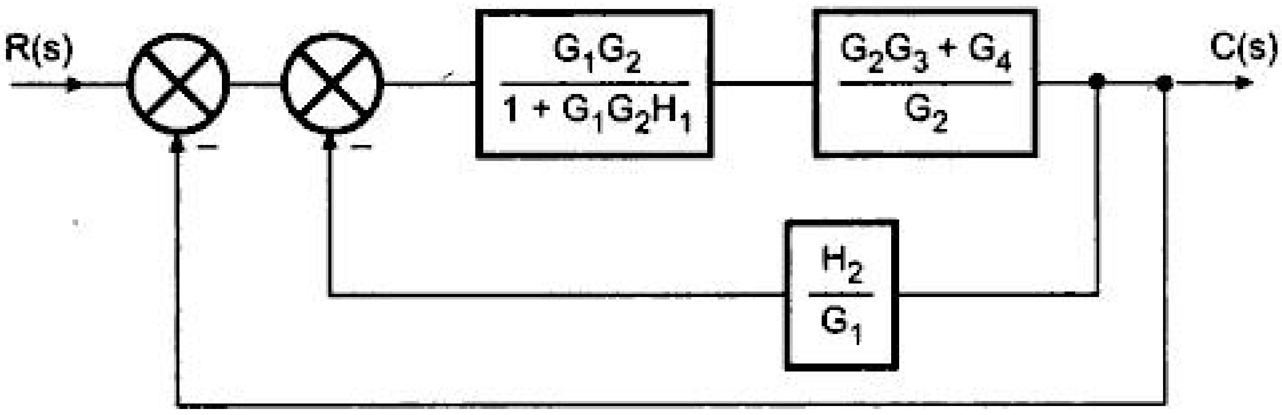


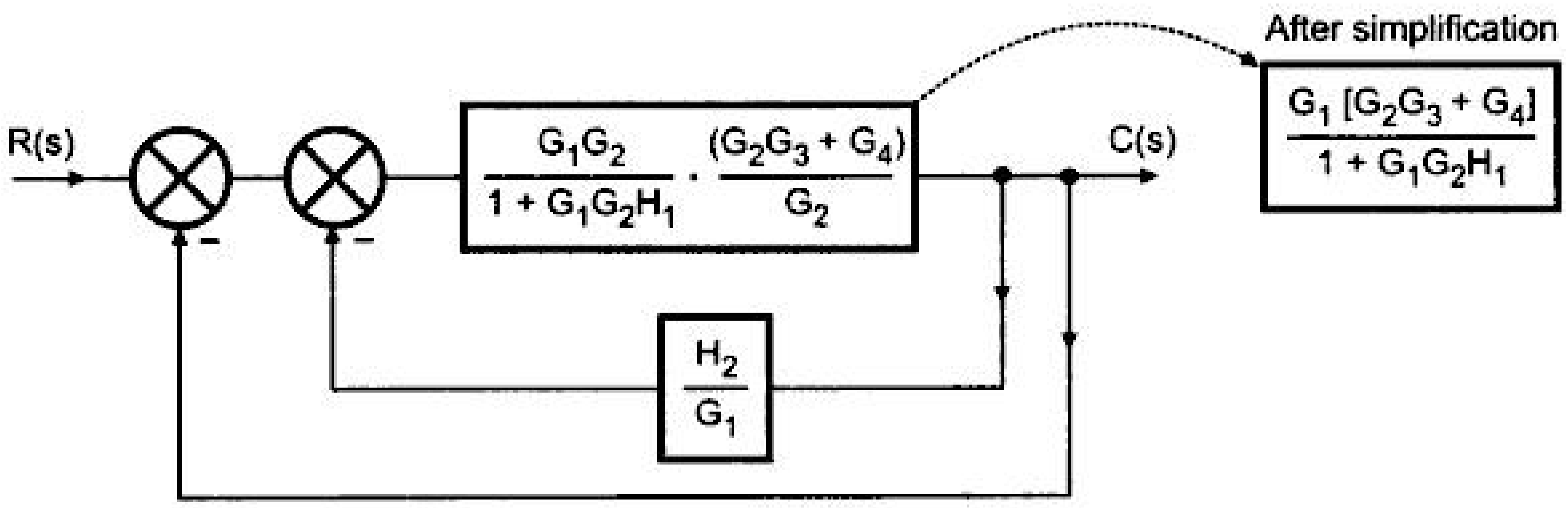
Shifting summing point before the block with transfer function 'G_1', we get,

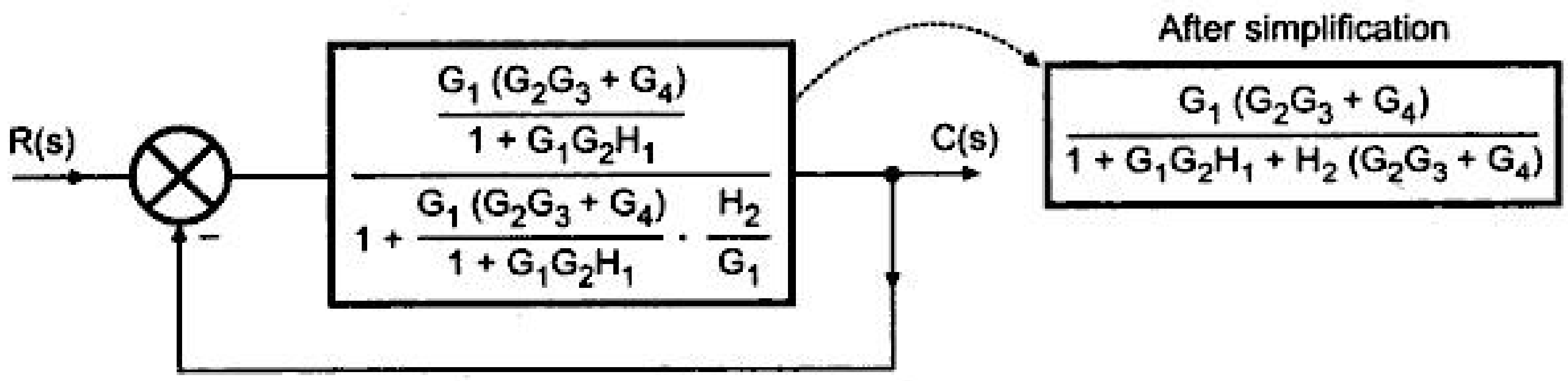


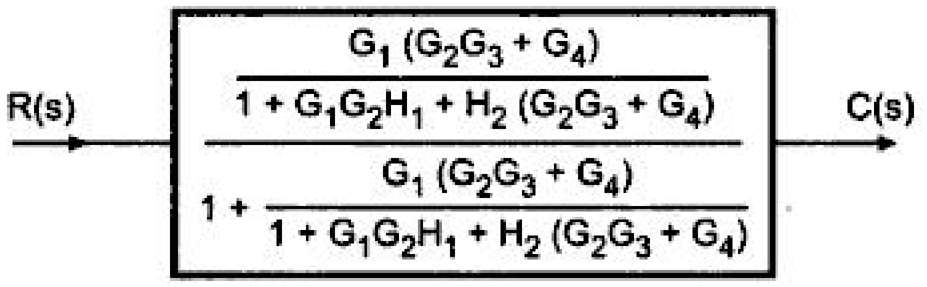
Using associative law for the summing points and interchanging their positions we get,





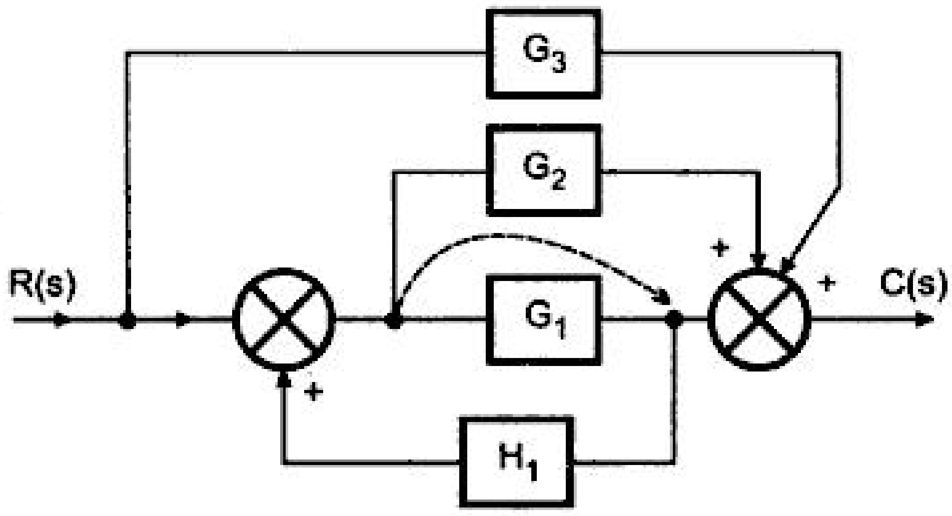




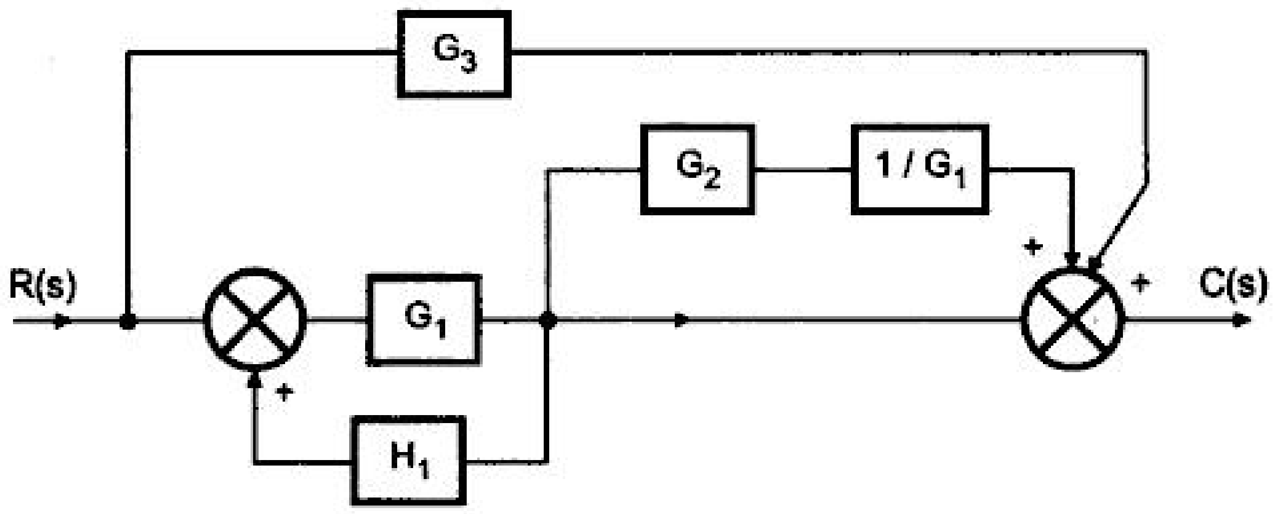


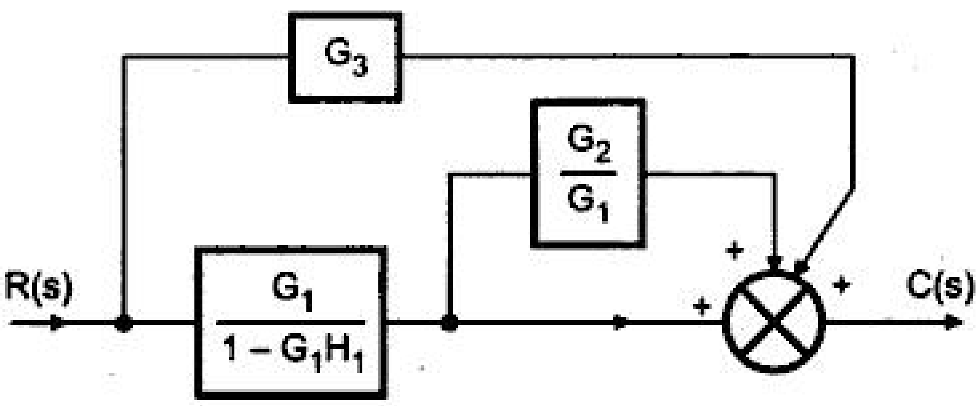
$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 G_2 H_1 + H_2 G_2 G_3 + H_2 G_4 + G_1 G_2 G_3 + G_1 G_4}$$

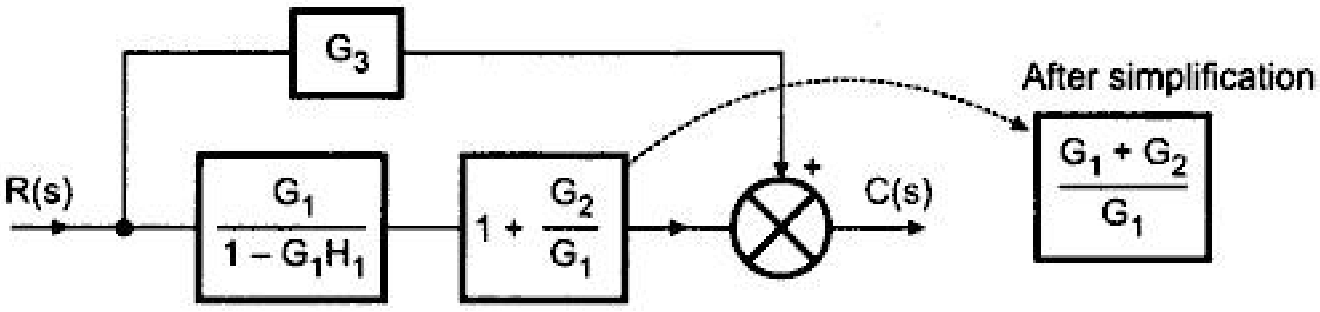
Ex. 3.13

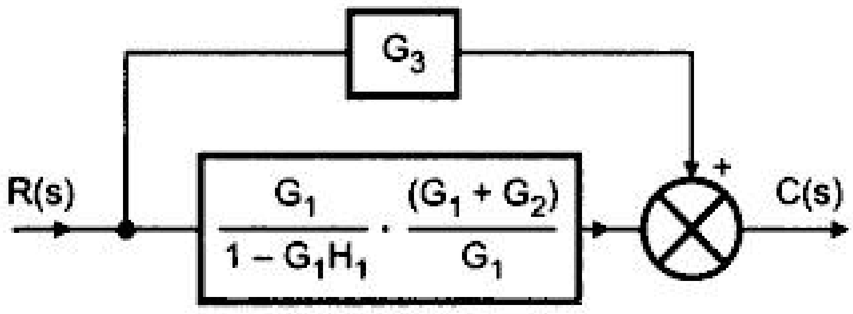


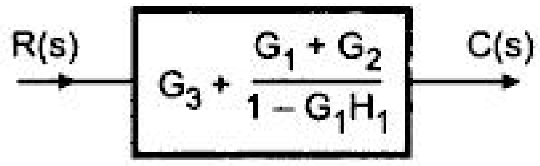
Sol. : Shifting takeoff point beyond the block having transfer function 'G_1'





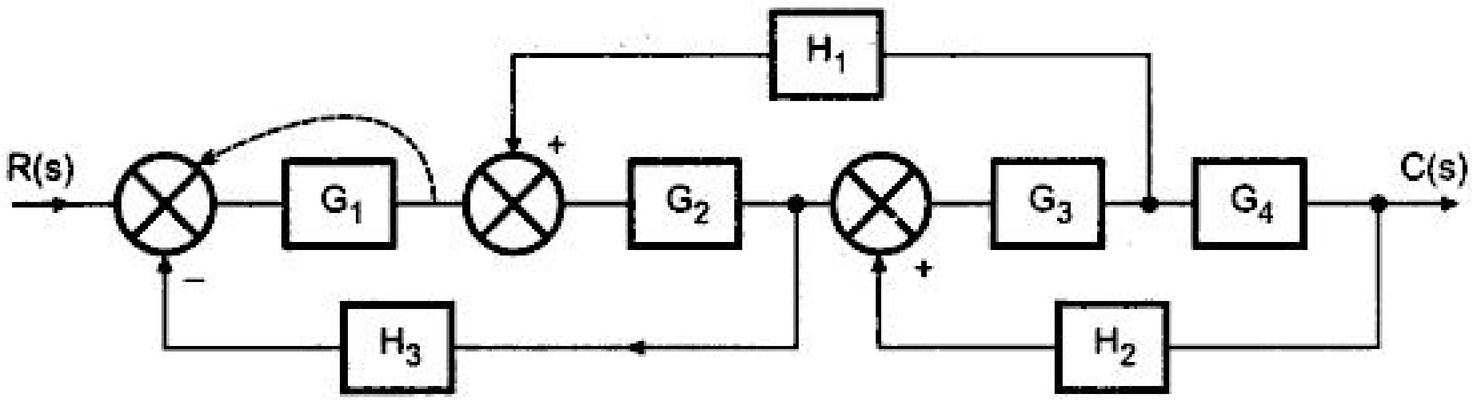






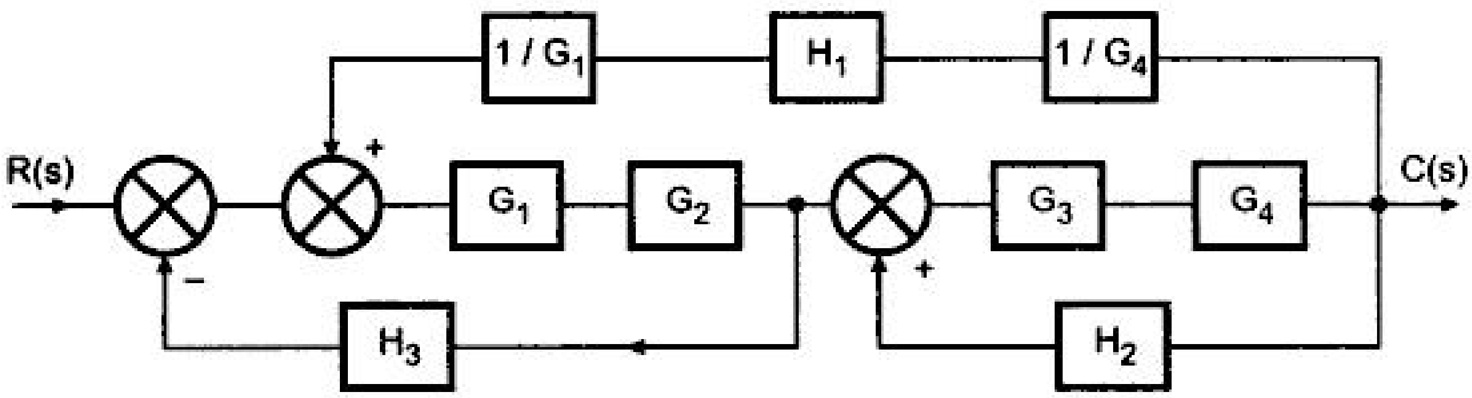
$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G_3 - G_1 G_3 H_1 + G_1 + G_2}{1 - G_1 H_1}$$

Ex. 3.14

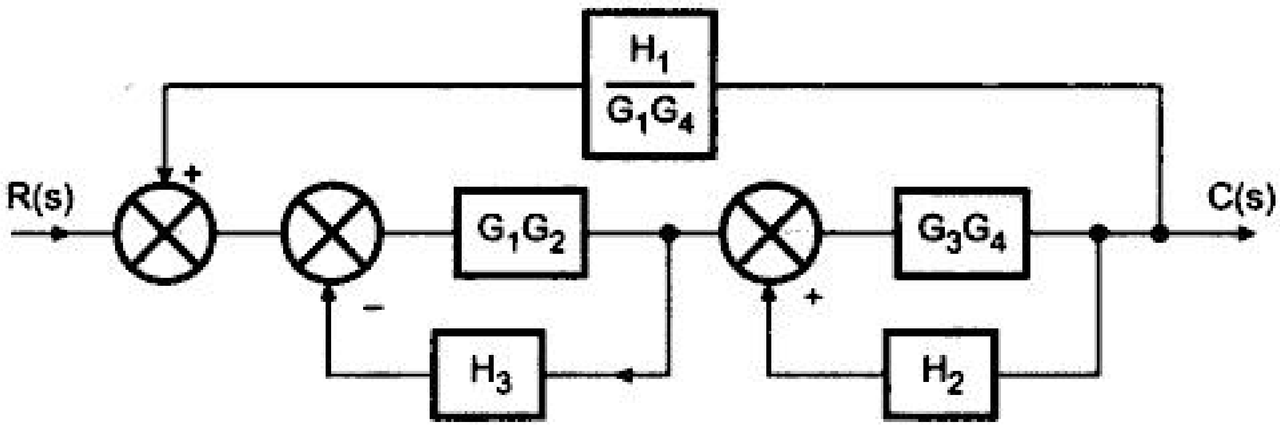


(Mumbai University Nov. 94, Dec. 98)

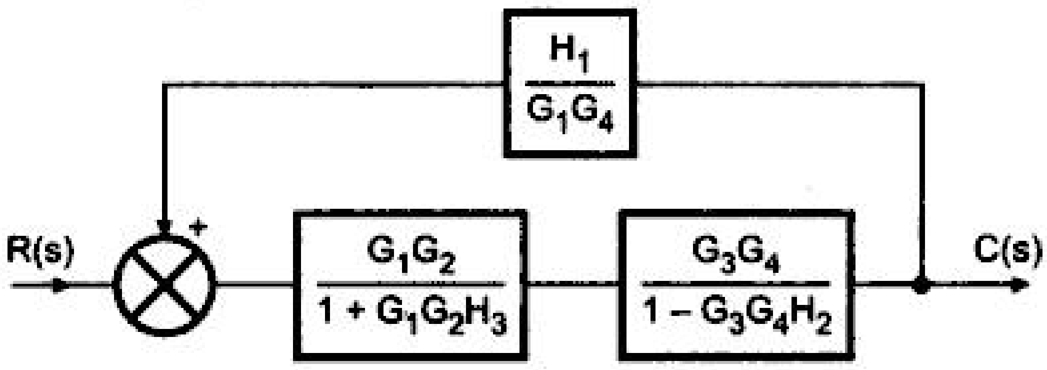
Sol. : Shifting take off point after the block 'G_4' and summing point behind the block having transfer function G_1 simultaneously, we get,

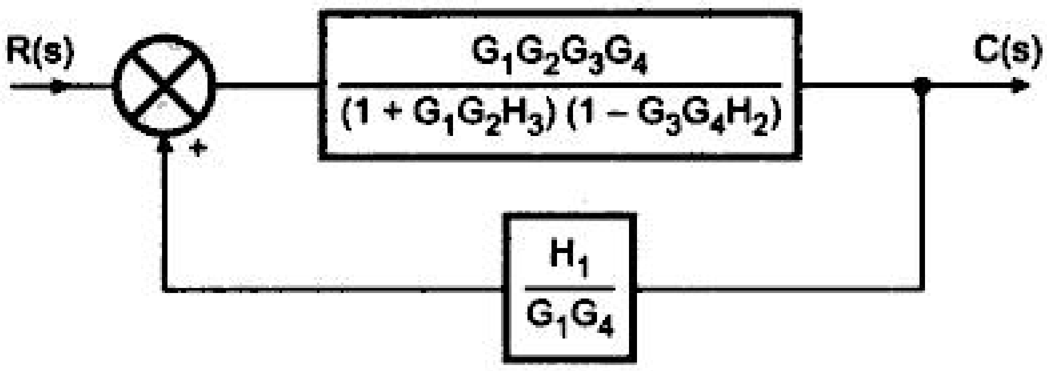


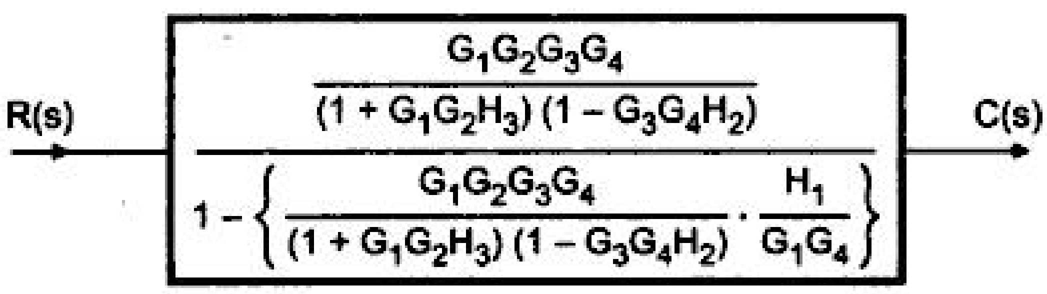
Using Associative Law for the first two summing points.



Solving both minor feedback loops we get,



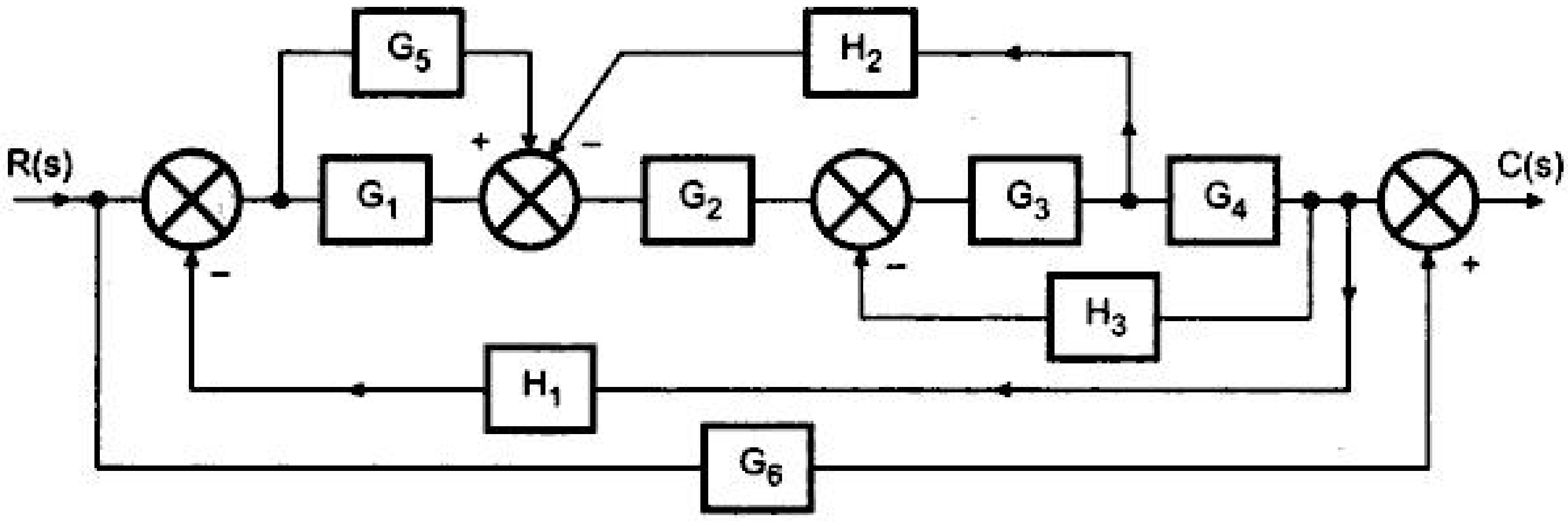




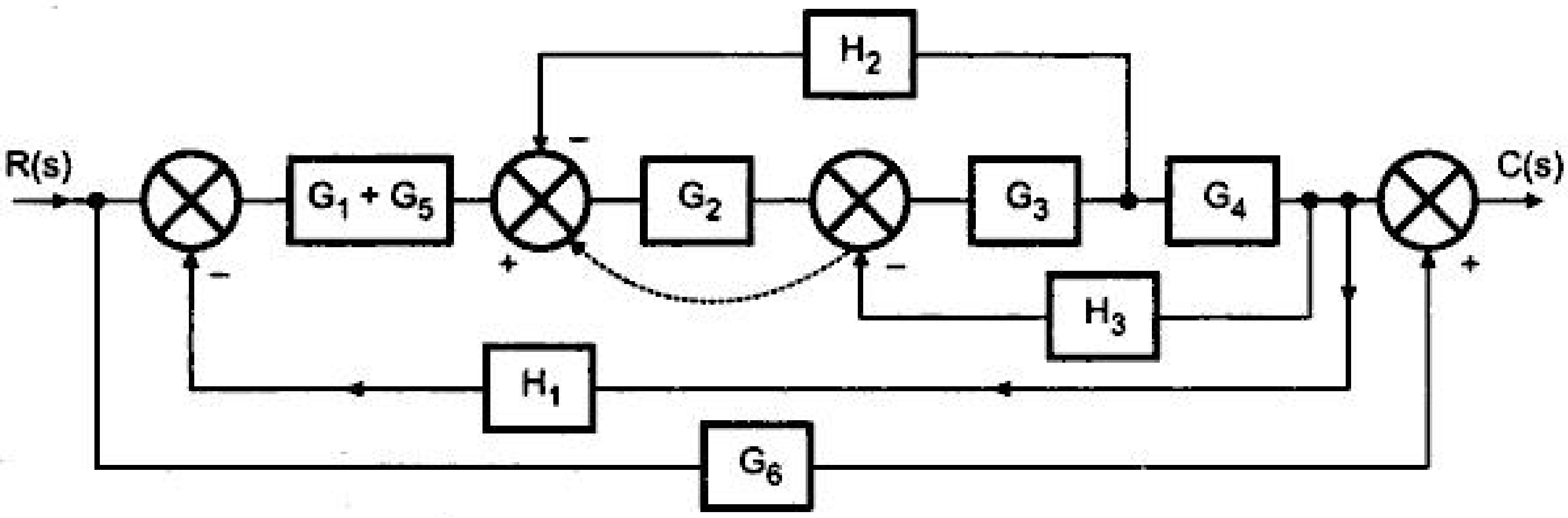
After simplification

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{(1+G_1 G_2 H_3)(1-G_3 G_4 H_2)-G_2 G_3 H_1}$$

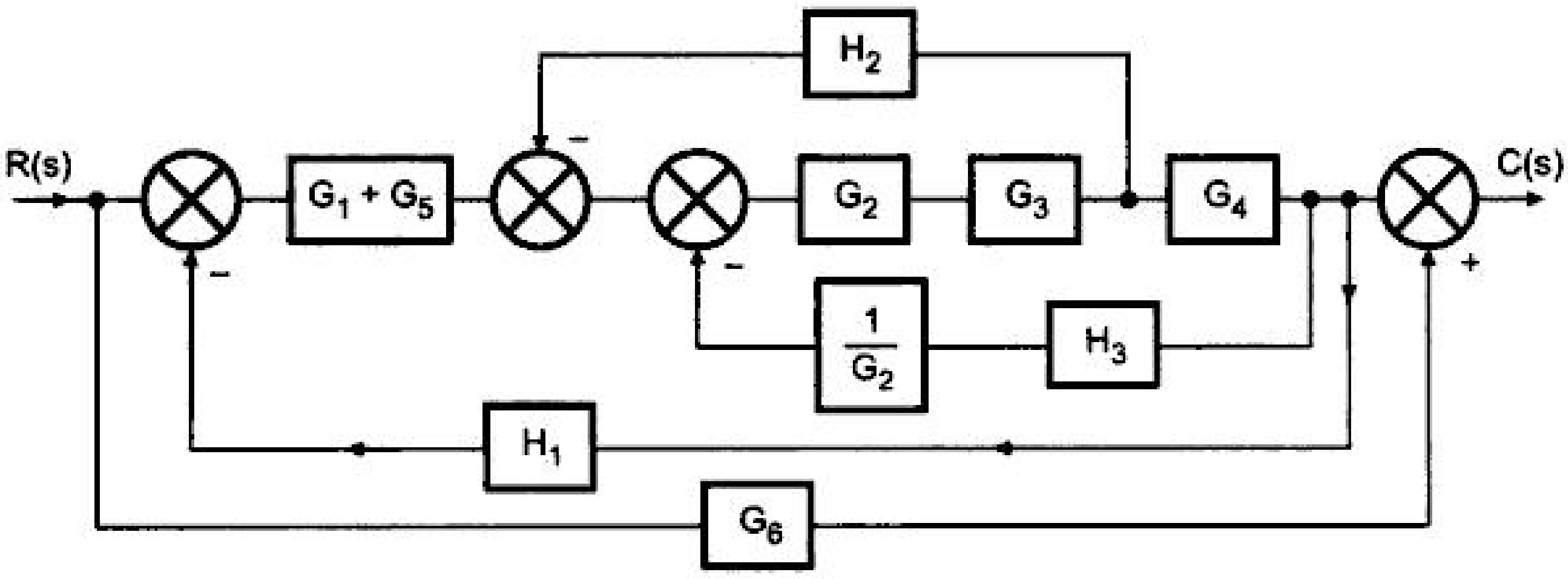
Ex. 3.15



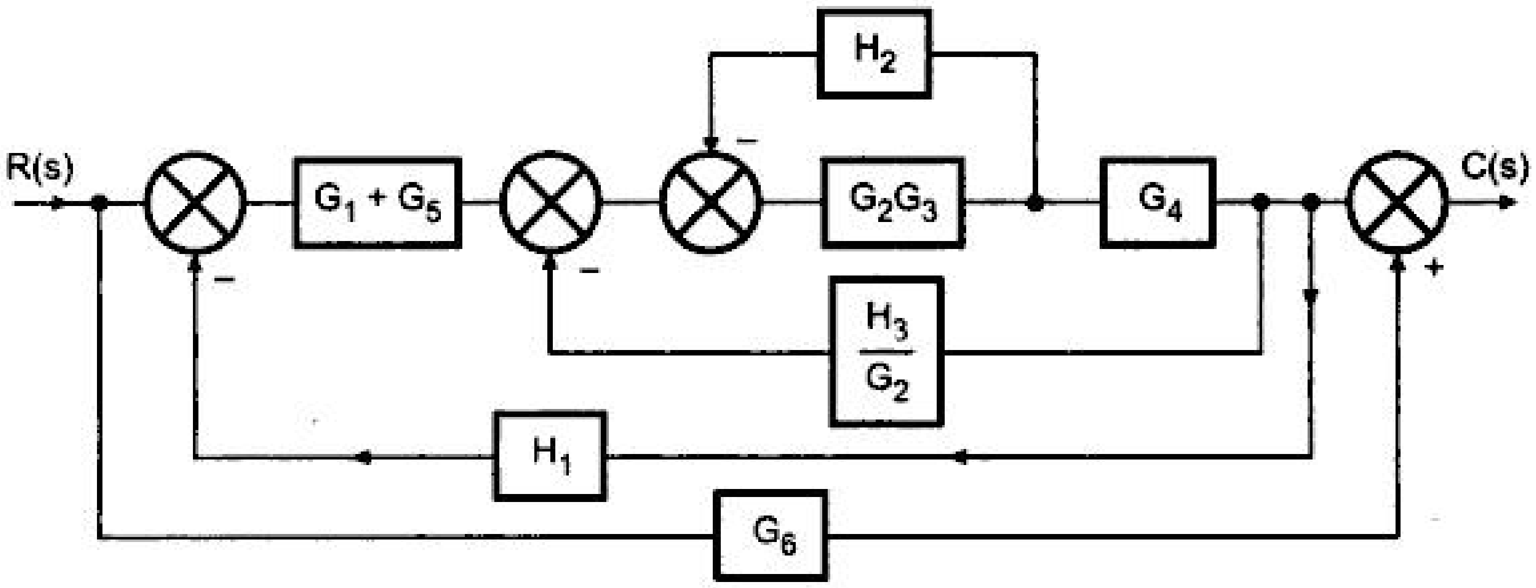
Sol. :

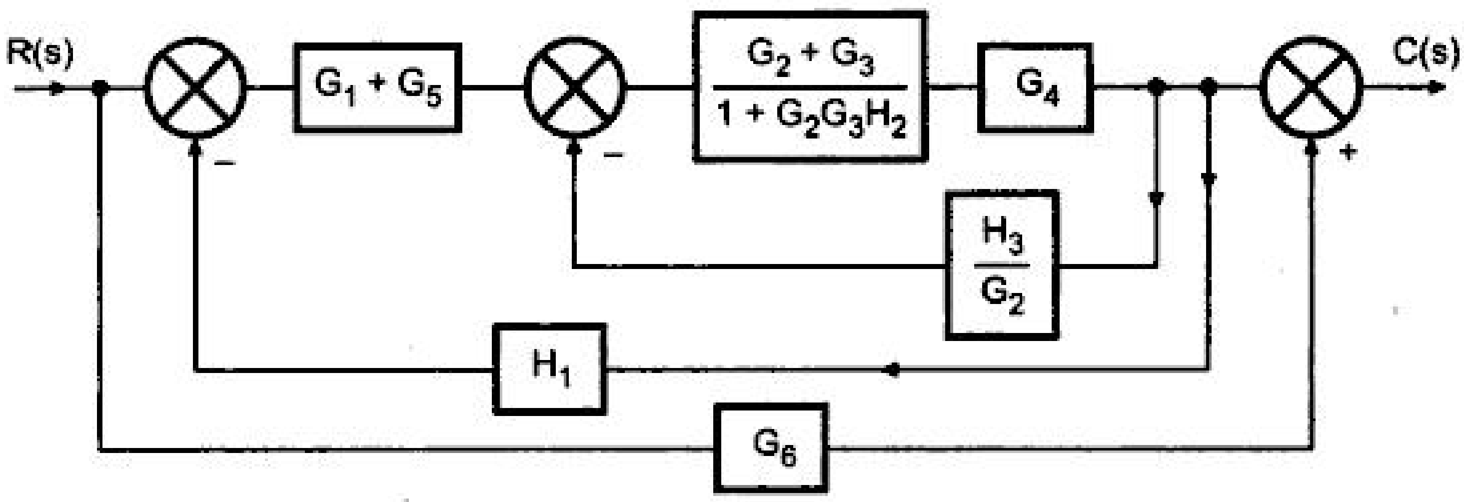


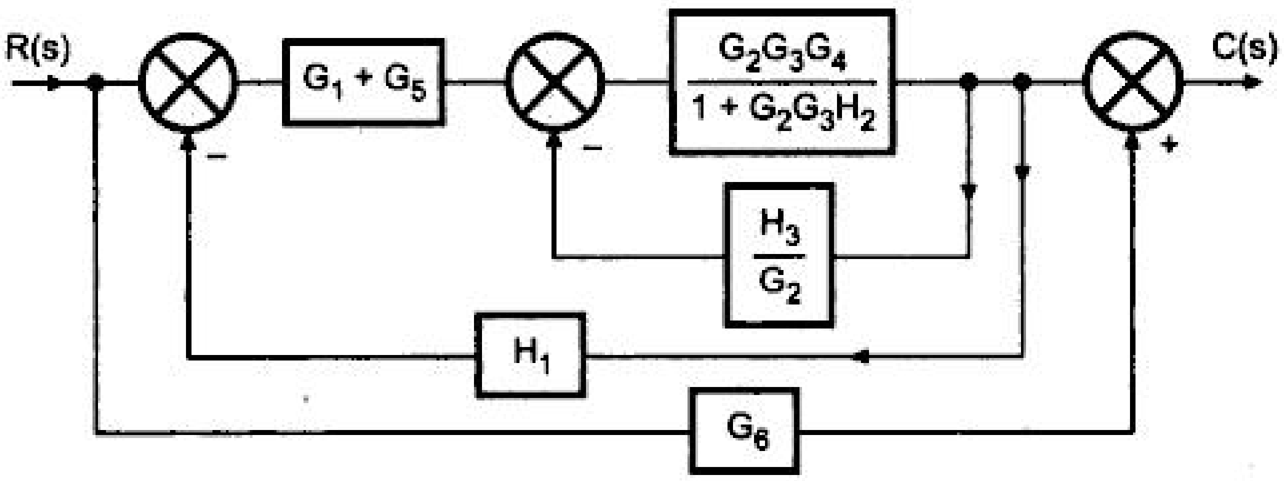
Shifting summing point behind the block 'G_2', towards left as shown we get,

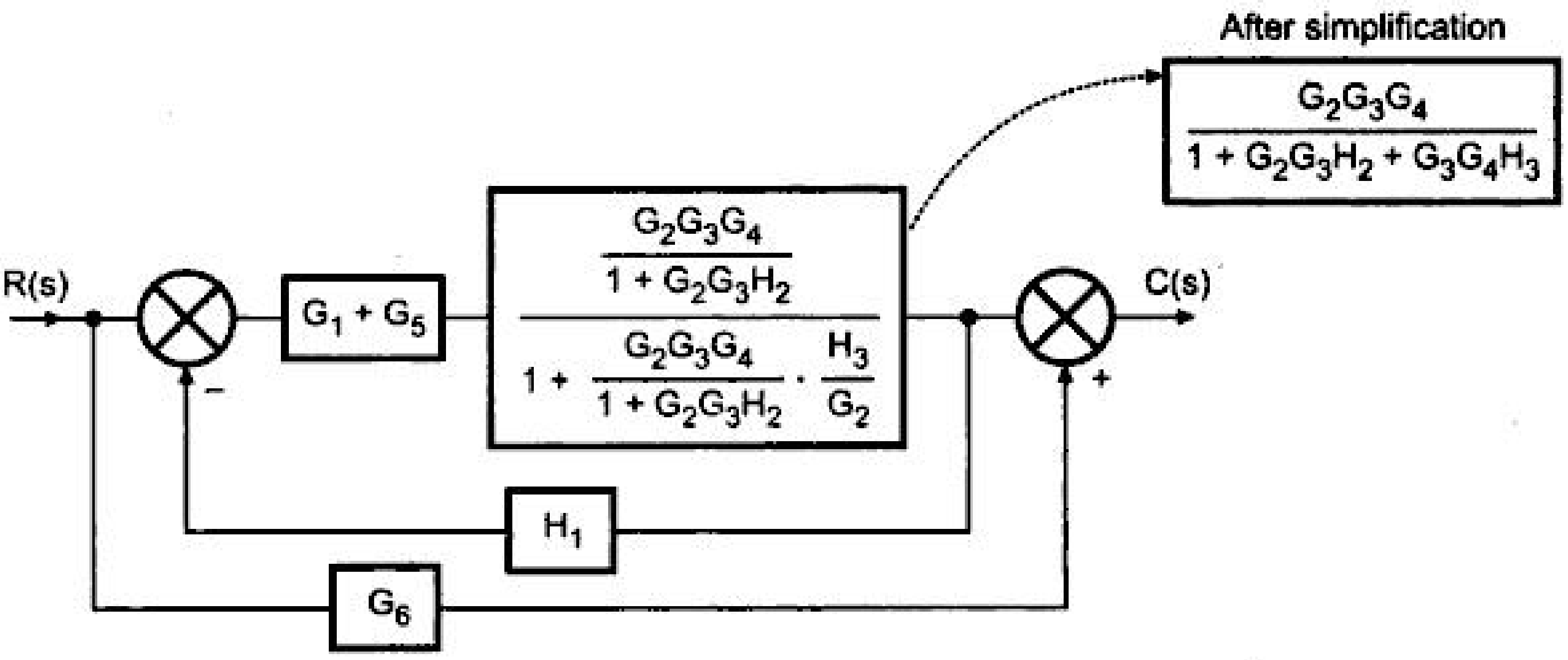


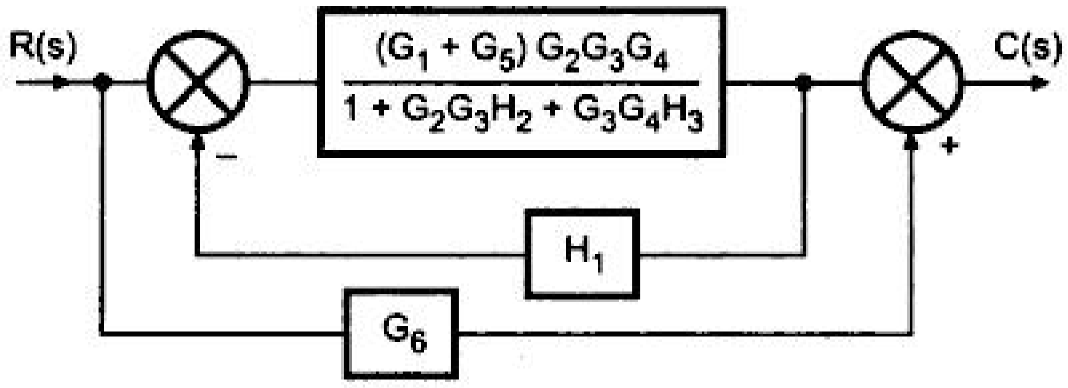
Using Associative law for the two summing points in between and interchanging their position we get,

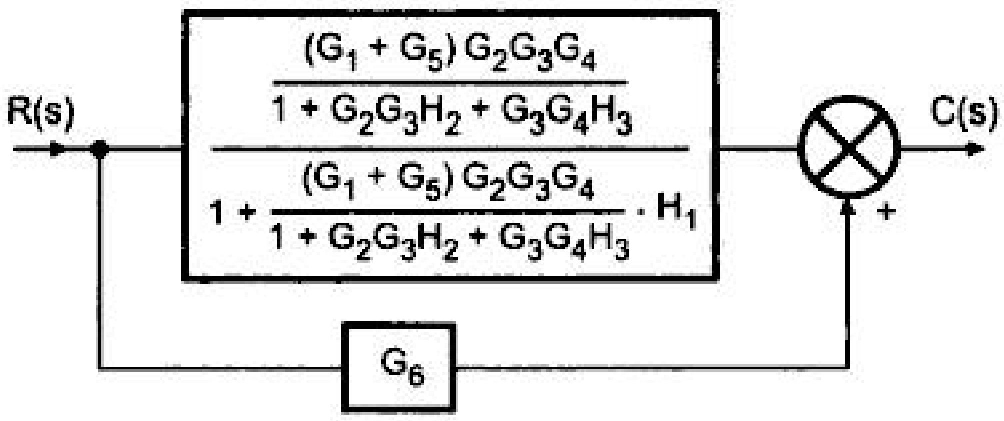

H2
R(s)
G1 + G5
G2G3
G4
C(s)
H3
G2
H1
G6

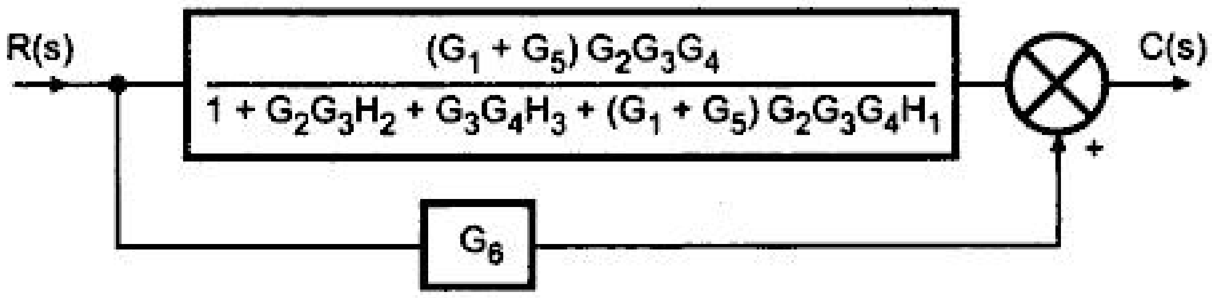

R(s)
G1 + G5
G2 + G3
1 + G2G3H2
G4
C(s)
H3
G2
H1
G6

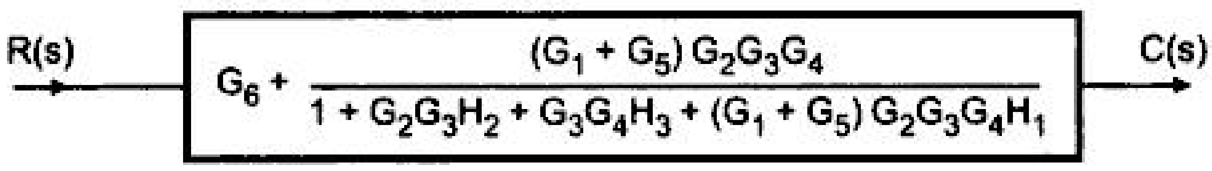

R(s)
G1 + G5
G2G3G4
1 + G2G3H2
C(s)
H3
G2
H1
G6


After simplification
G2G3G4
1 + G2G3H2 + G3G4H3
R(s)
G1 + G5
G2G3G4
1 + G2G3H2
1 +
G2G3G4
1 + G2G3H2
H3
G2
C(s)
H1
G6



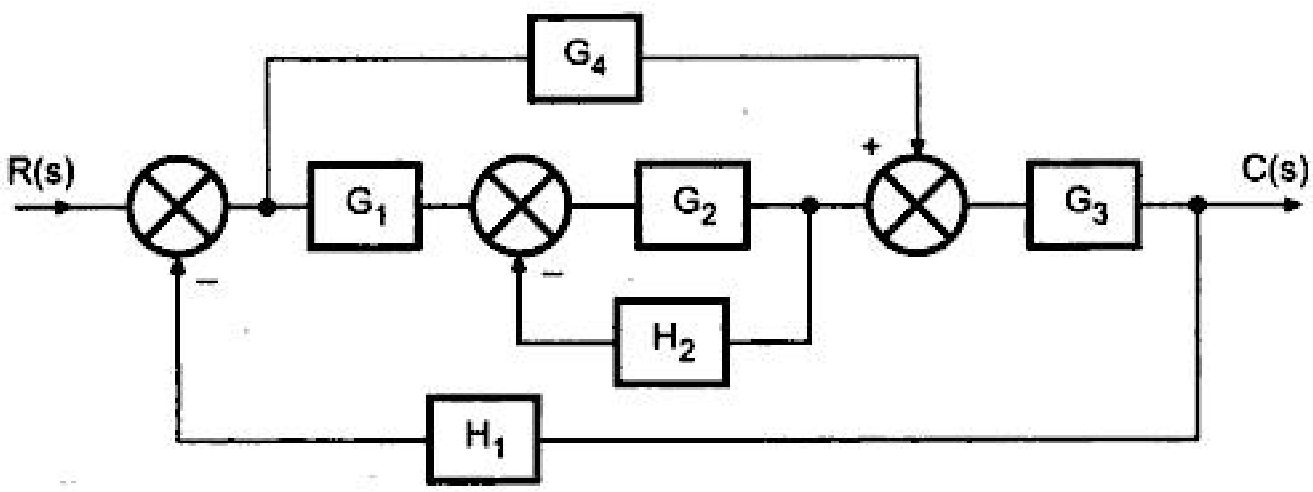






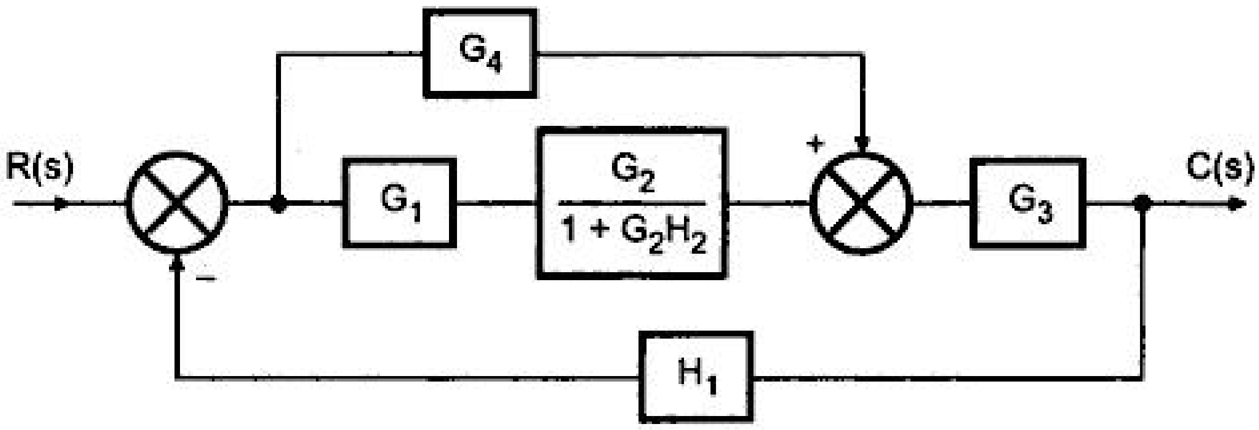
$$\therefore \frac{C(s)}{R(s)} = \frac{G_6 + G_2\,G_3\,G_6\,H_2 + G_3\,G_4\,G_6\,H_3 + (G_1 + G_5)\,G_2\,G_3\,G_4\,G_6\,H_1 + (G_1 + G_5)\,G_2\,G_3\,G_4}{1 + G_2\,G_3\,H_2 + G_3\,G_4\,H_3 + (G_1 + G_5)\,G_2\,G_3\,G_4\,H_1}$$

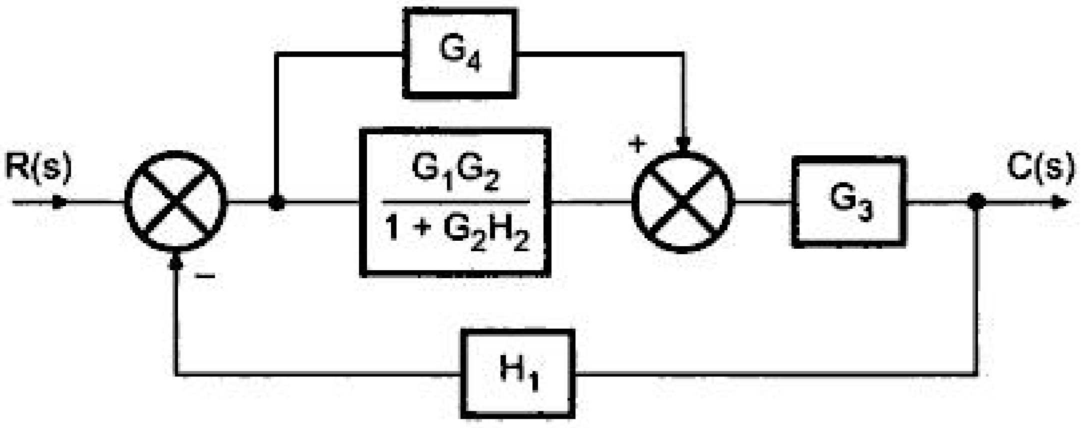
Ex. 3.16

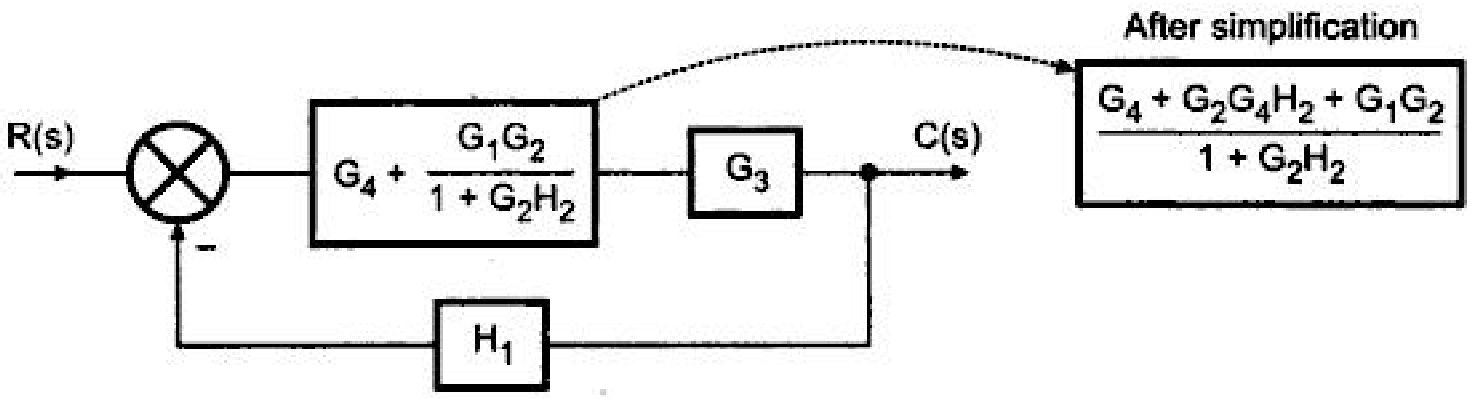


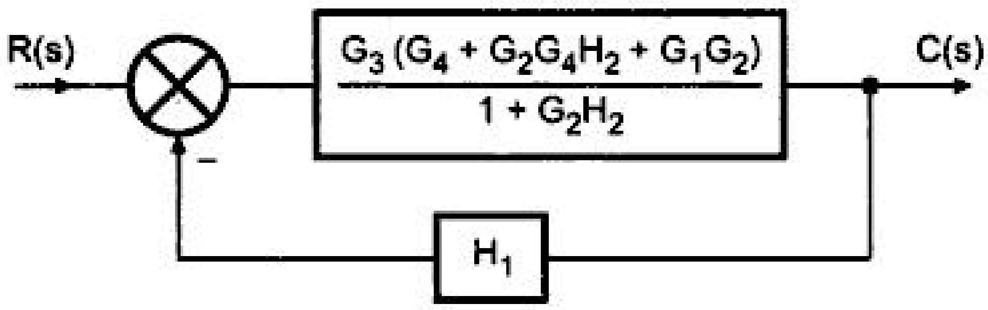
(Mumbai University May 97)

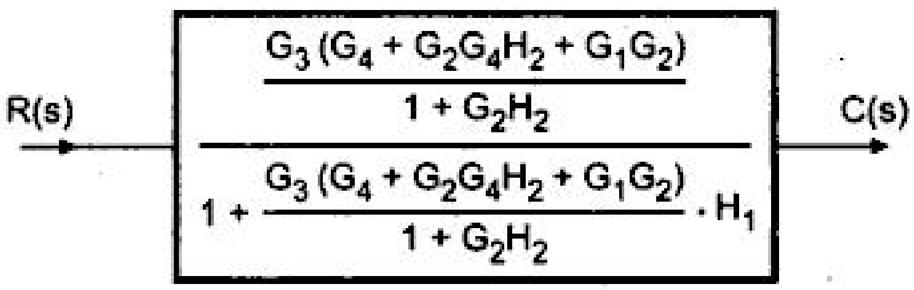
Sol. :





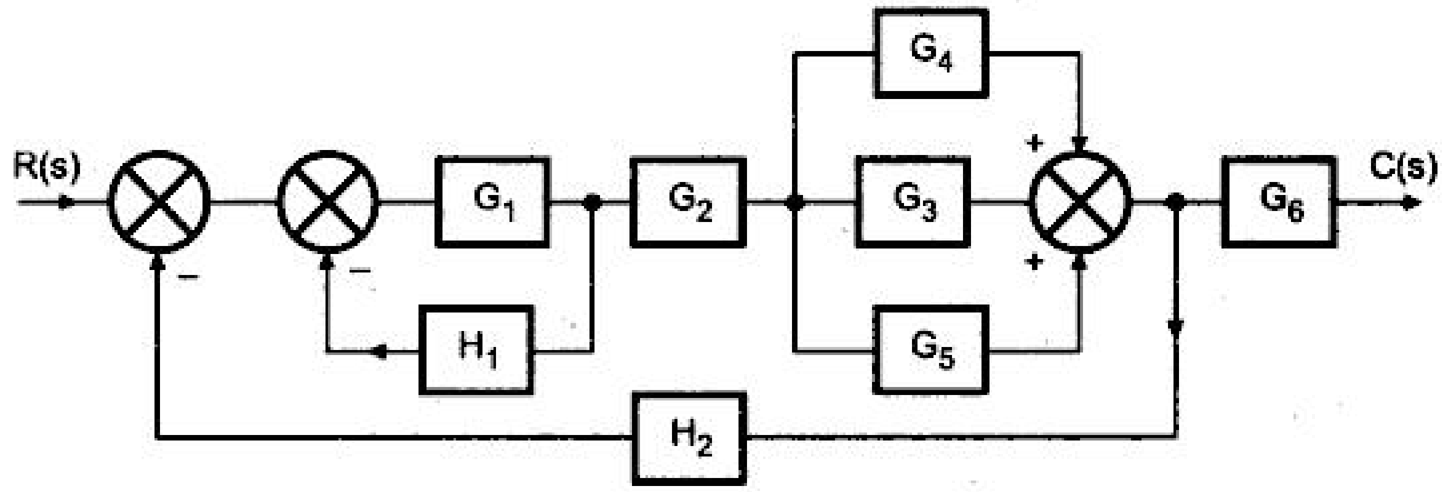




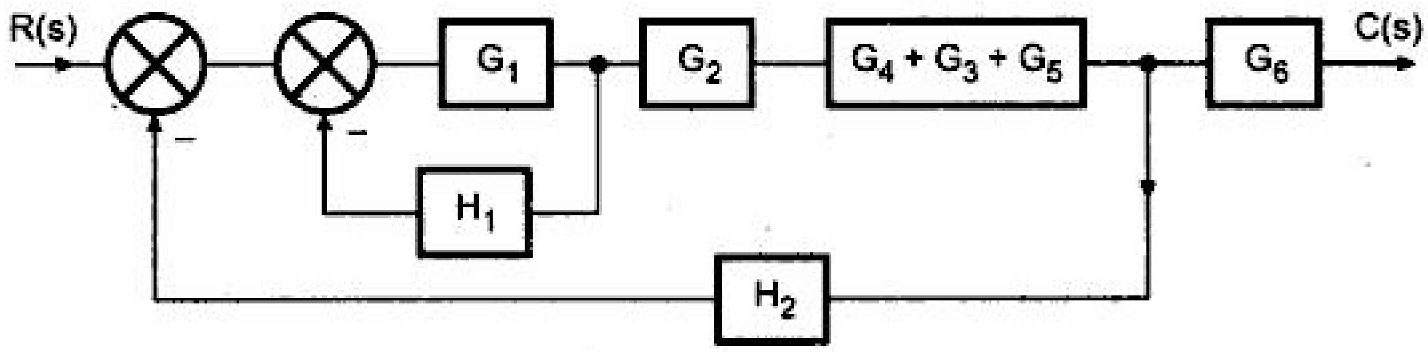


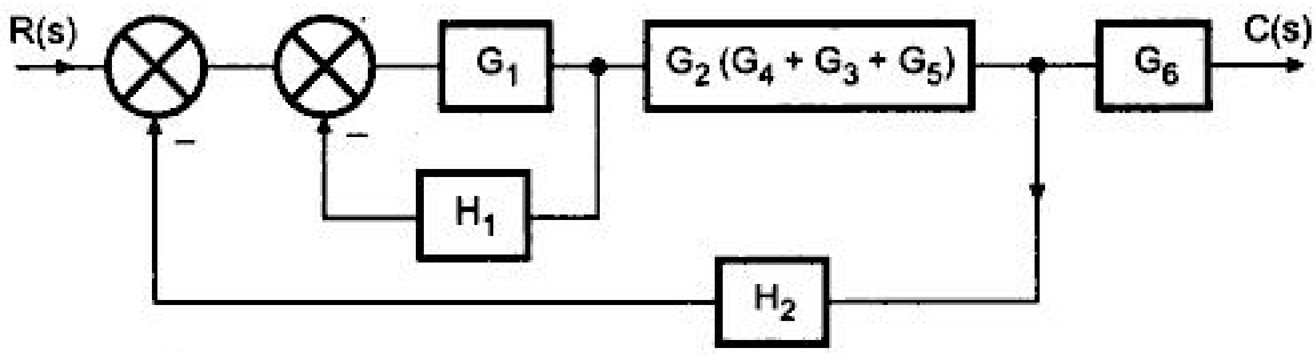
$$\therefore \qquad \frac{C(s)}{R(s)} = \frac{G_3 [G_4 + G_2 G_4 H_2 + G_1 G_2]}{1 + G_2 H_2 + G_3 H_1 [G_4 + G_2 G_4 H_2 + G_1 G_2]}$$

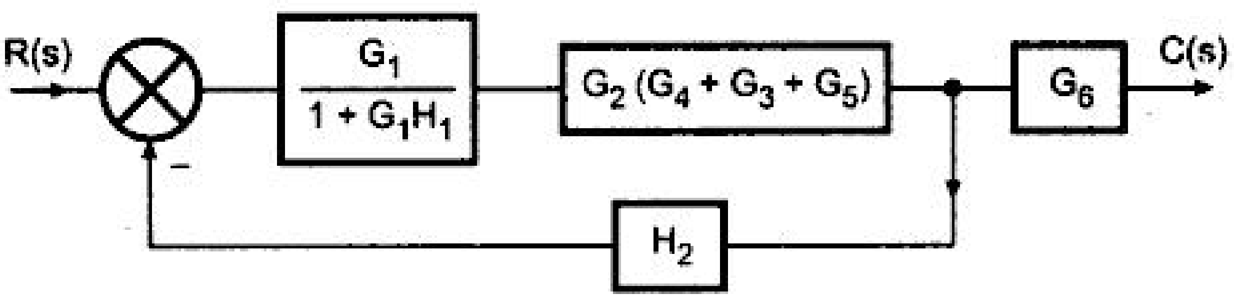
Ex. 3.17

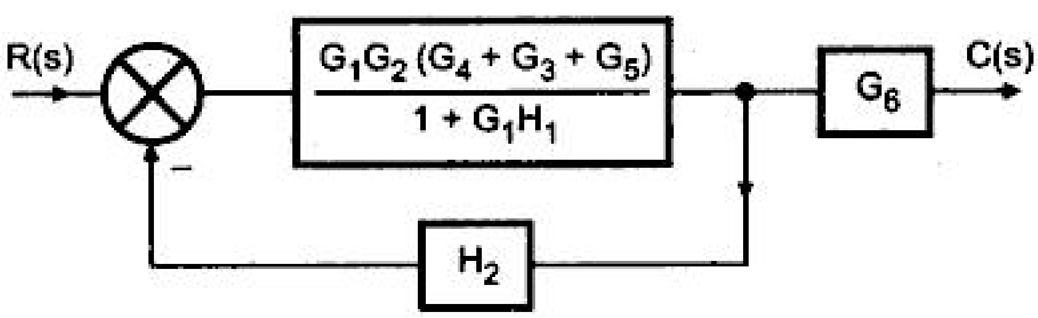

R(s)
G1
G2
G3
G4
G5
G6
H1
H2
C(s)

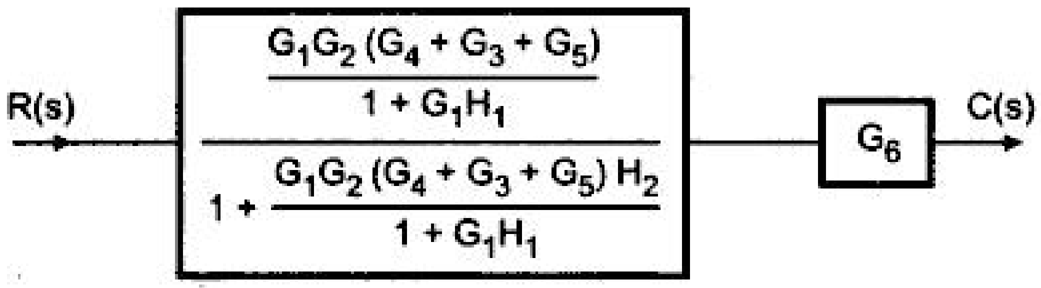
Sol. :

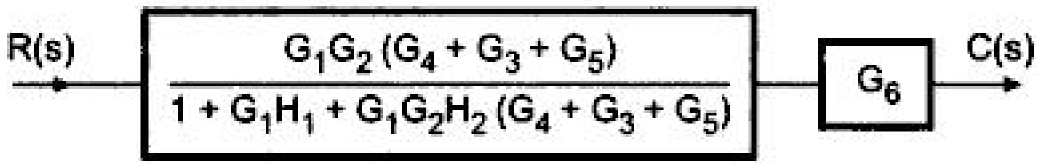

R(s)
G1
G2
G4 + G3 + G5
G6
H1
H2
C(s)


R(s)
G1
G2 (G4 + G3 + G5)
G6
H1
H2
C(s)


R(s)
G1 / 1 + G1H1
G2 (G4 + G3 + G5)
G6
H2
C(s)

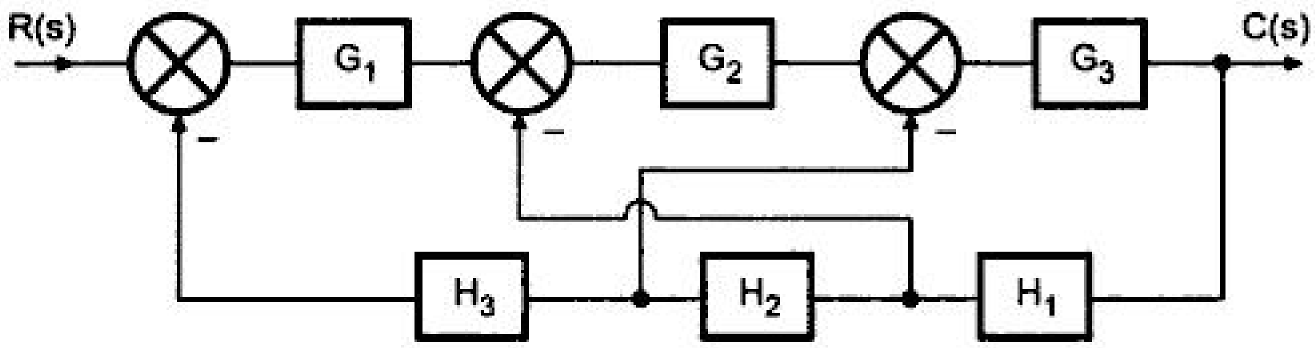

R(s)
G1G2 (G4 + G3 + G5) / 1 + G1H1
G6
H2
C(s)



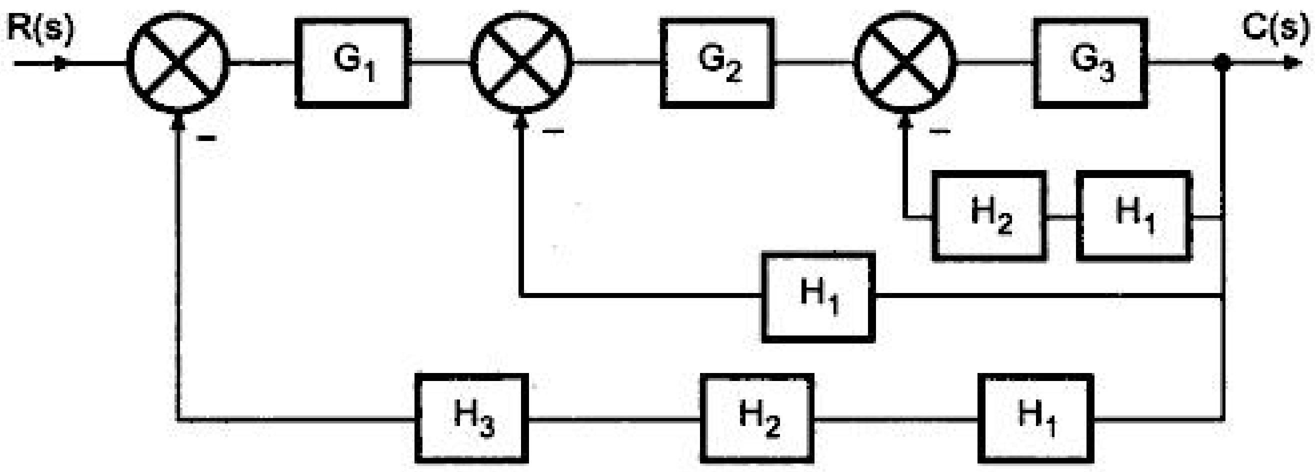


$$\therefore \qquad \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_6 (G_3 + G_4 + G_5)}{1 + G_1 H_1 + G_1 G_2 H_2 \ (G_3 + G_4 + G_5)}$$

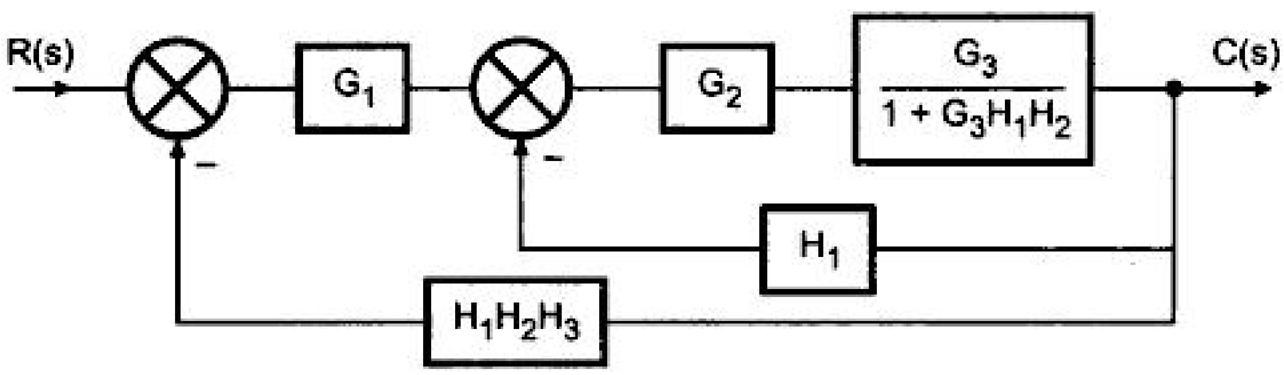
Ex. 3.18

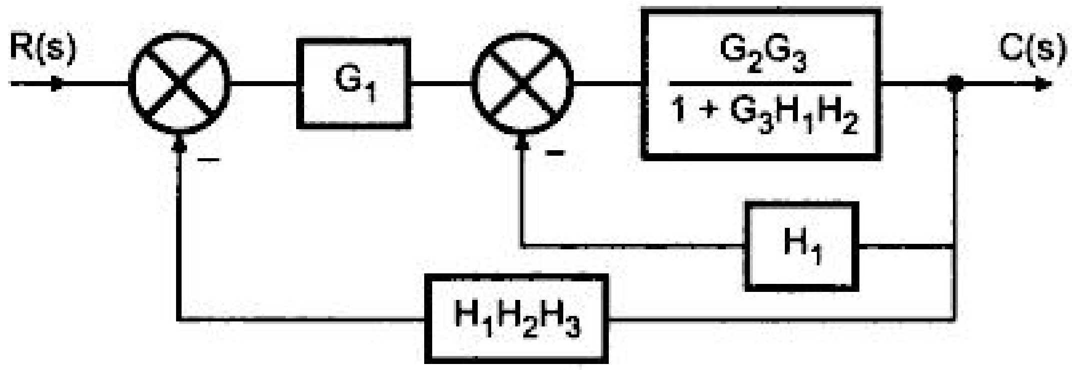


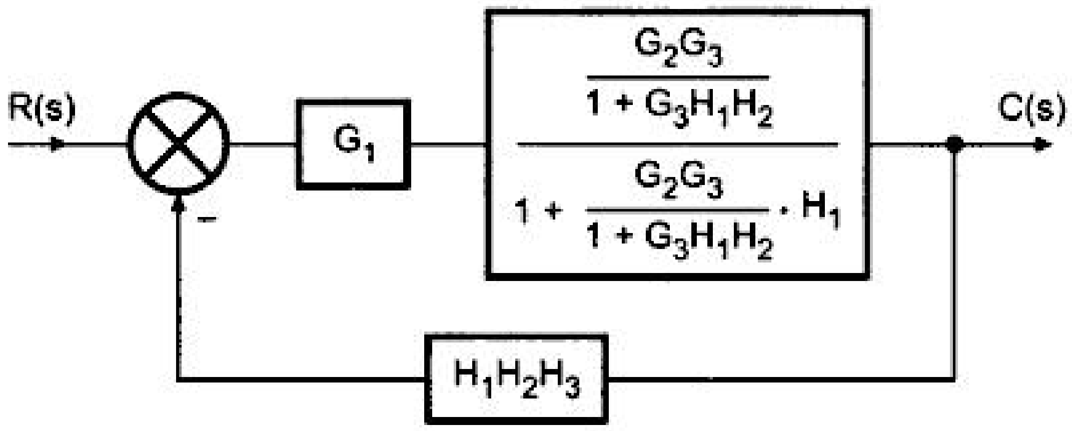
Sol. : Separating out the feedbacks at different summing points, we can rearrange the above block diagram as below :

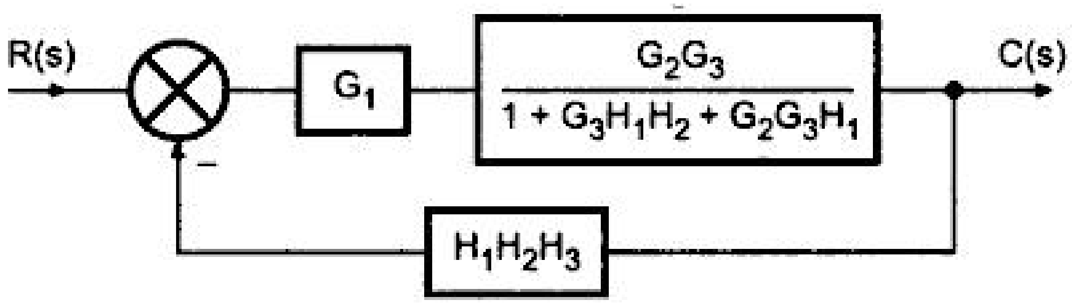


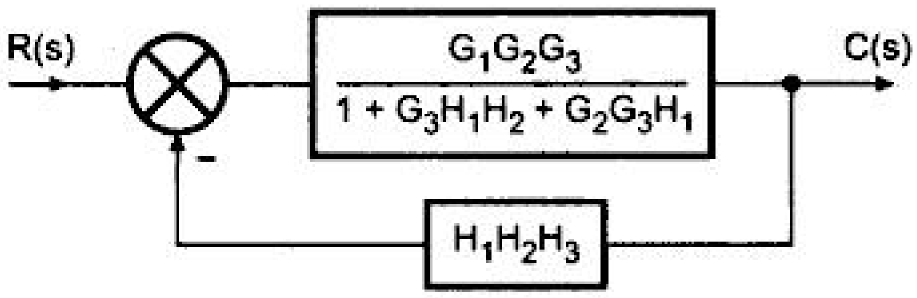
Solving minor feedback loop:

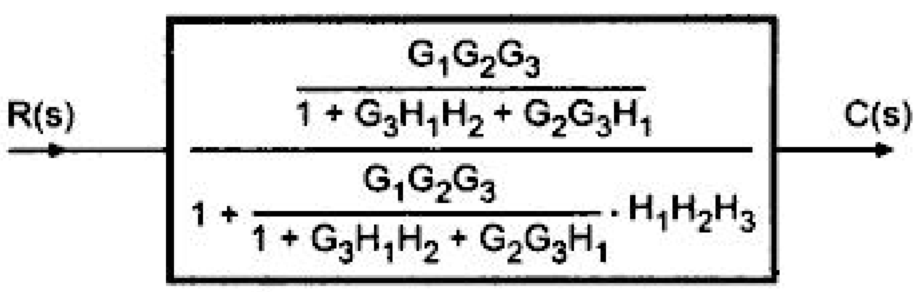






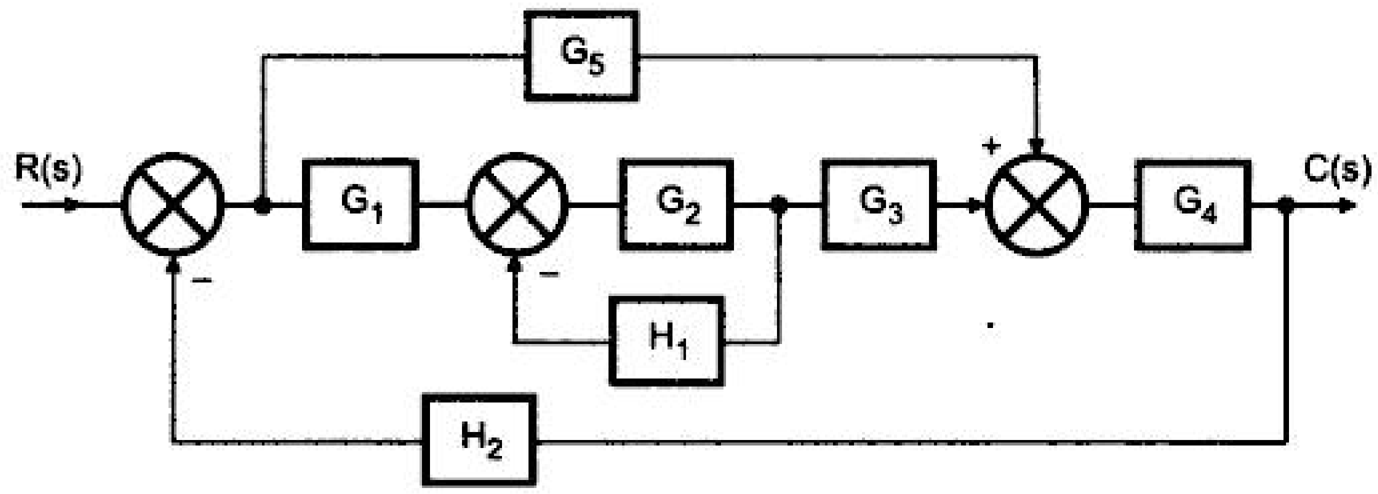




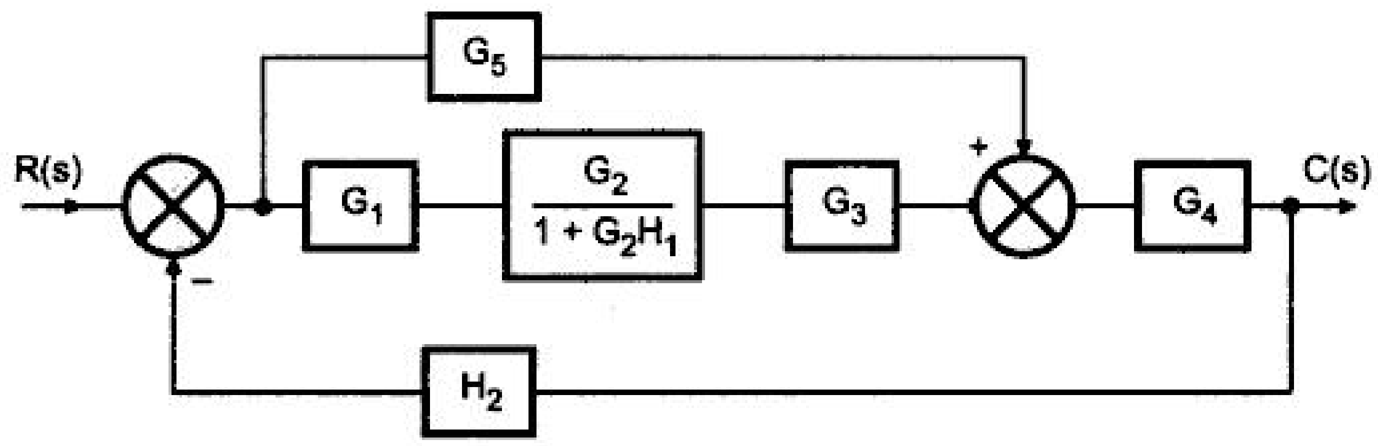


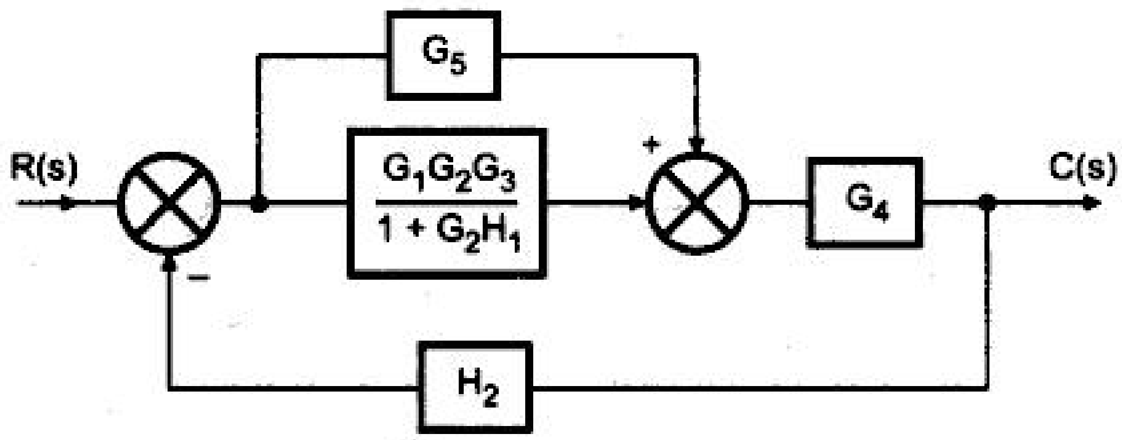
$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3}{1 + G_3 H_1 H_2 + G_2 G_3 H_1 + G_1 G_2 G_3 H_1 H_2 H_3}$$

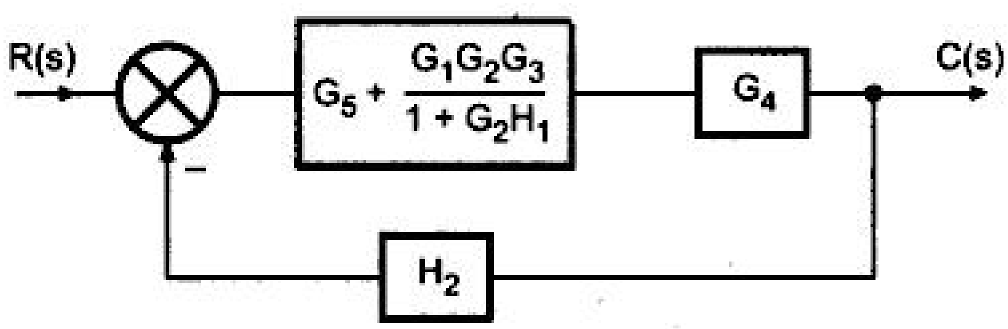
Ex. 3.19

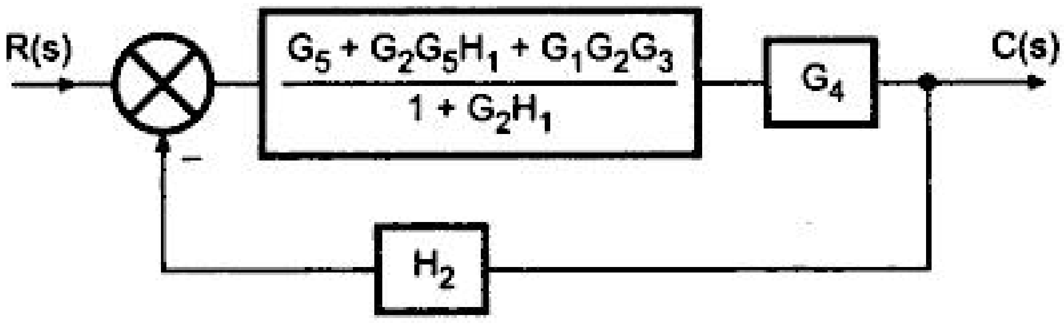

G5
R(s)
G1
G2
G3
+
G4
C(s)
−
−
H1
H2

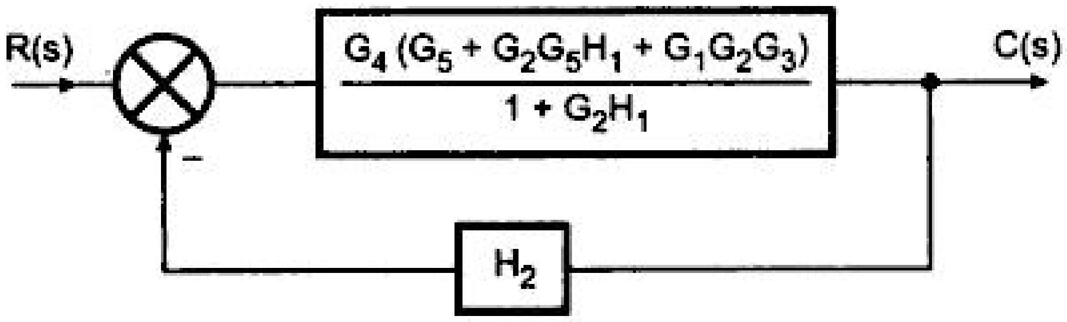
Sol. :

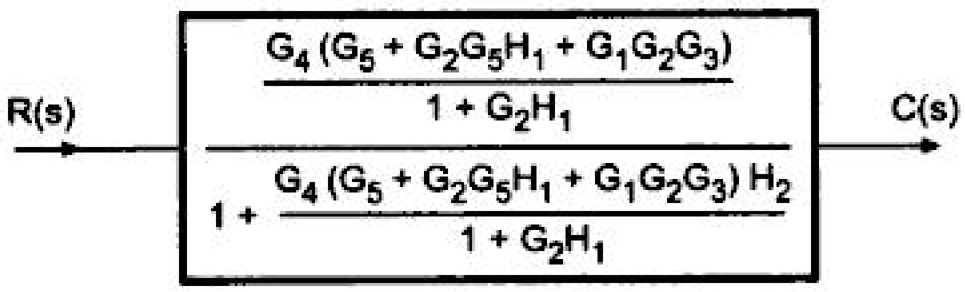

G5
R(s)
G1
G2
1 + G2H1
G3
+
G4
C(s)
−
H2


G5
R(s)
G1G2G3
1 + G2H1
+
G4
C(s)
−
H2


R(s)
G5 +
G1G2G3
1 + G2H1
G4
C(s)
−
H2

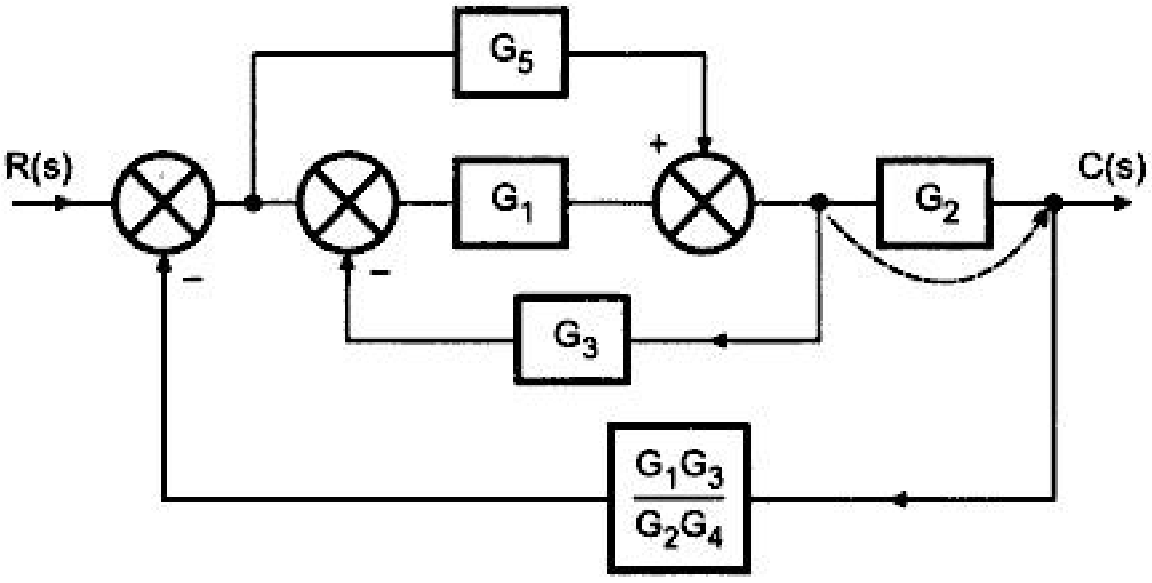




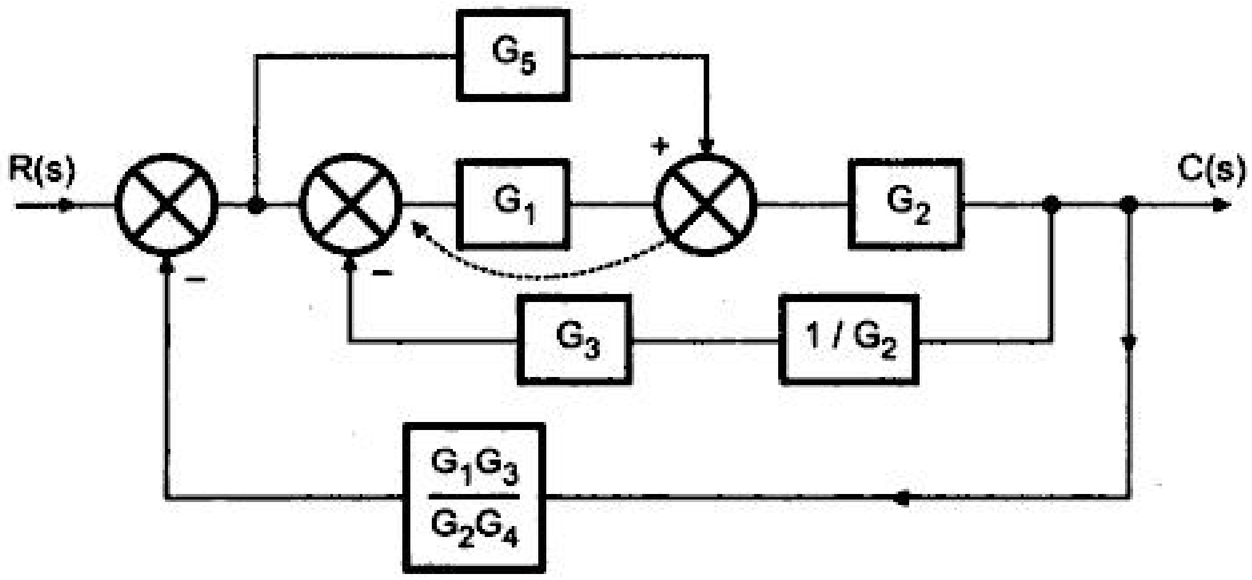


$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4 + G_4 G_5 + G_2 G_4 G_5 H_1}{1 + G_2 H_1 + H_2[G_1 G_2 G_3 G_4 + G_4 G_5 + G_2 G_4 G_5 H_1]}$$

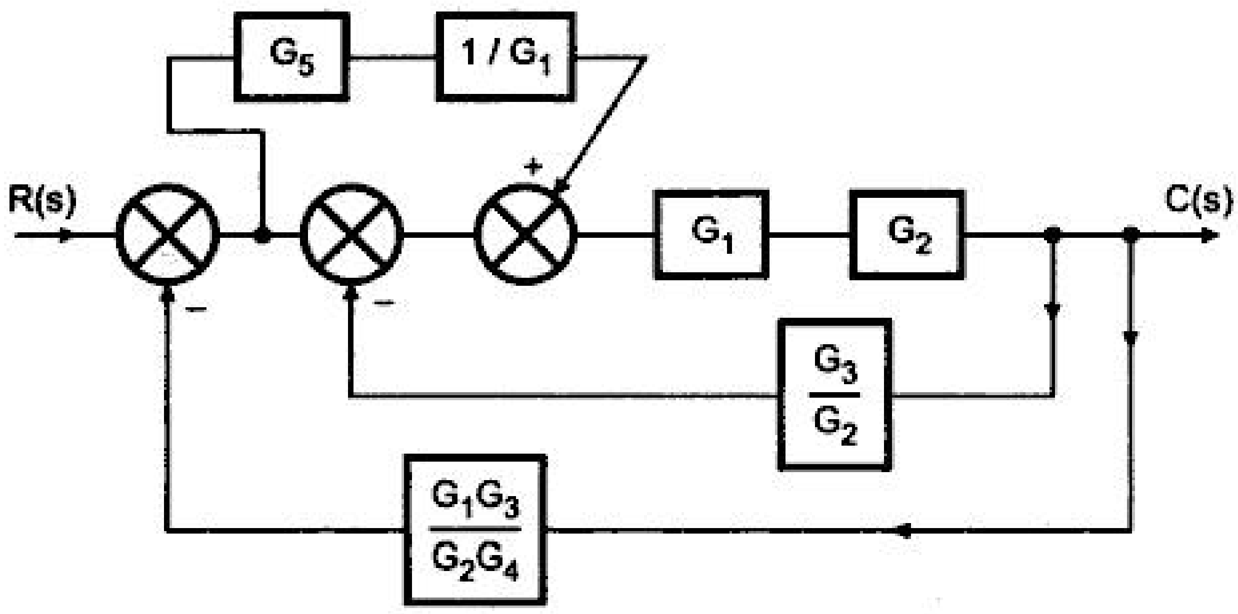
Ex. 3.20



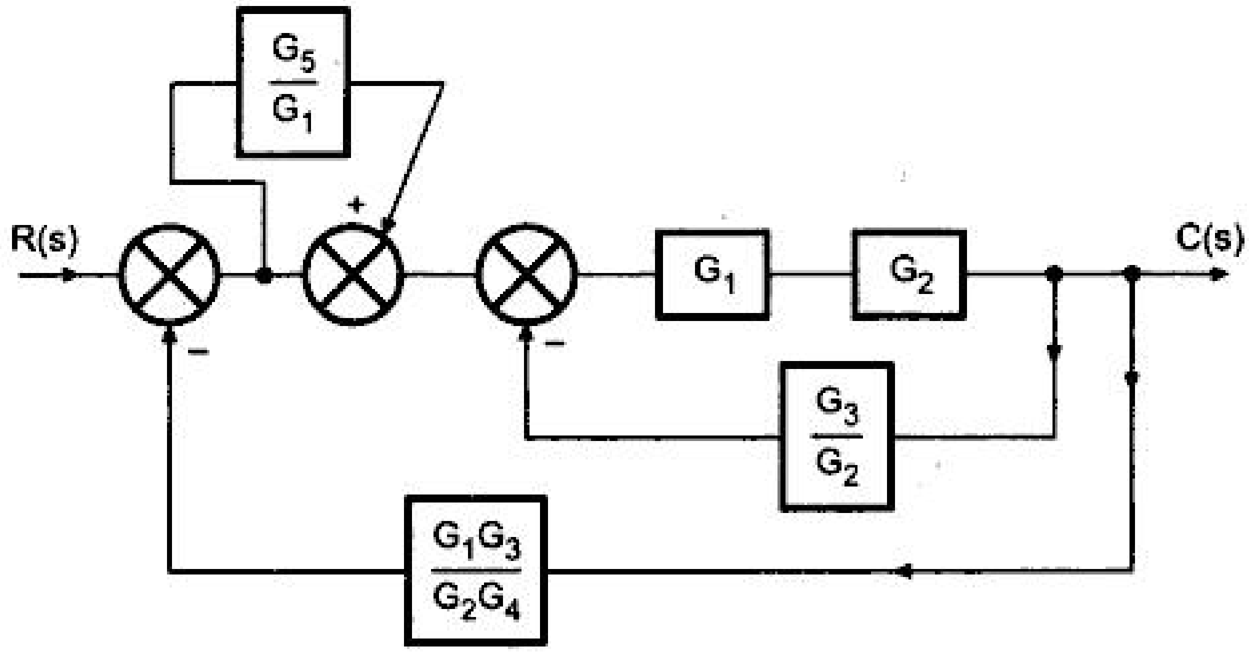
Sol. : Shifting takeoff point after the block having T.F. G_2



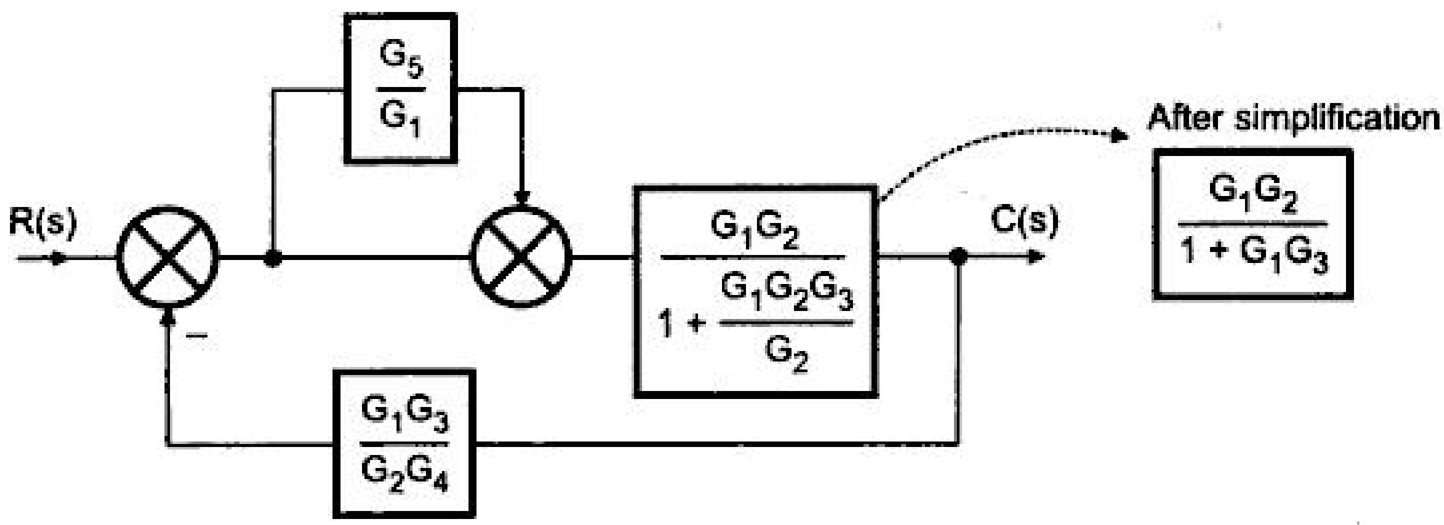
Shifting summing point before the block 'G_1', we get,



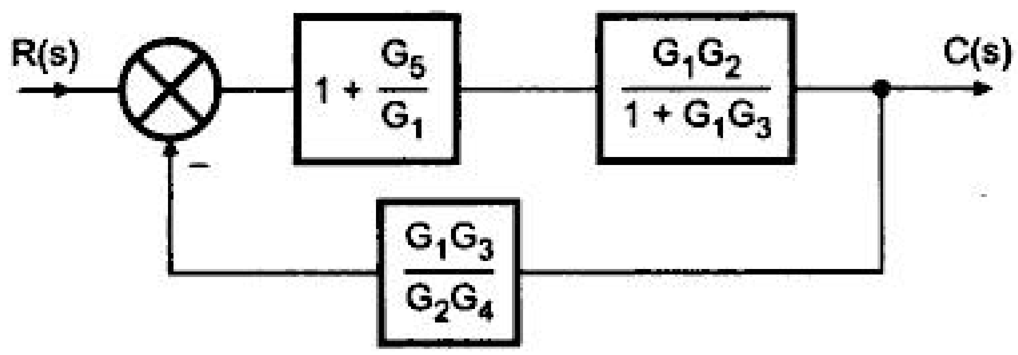
Interchanging the summing points by using Associative Law, we get,

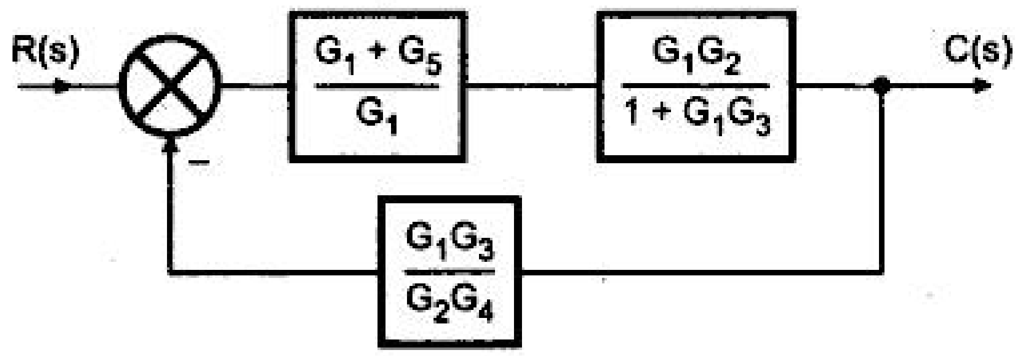


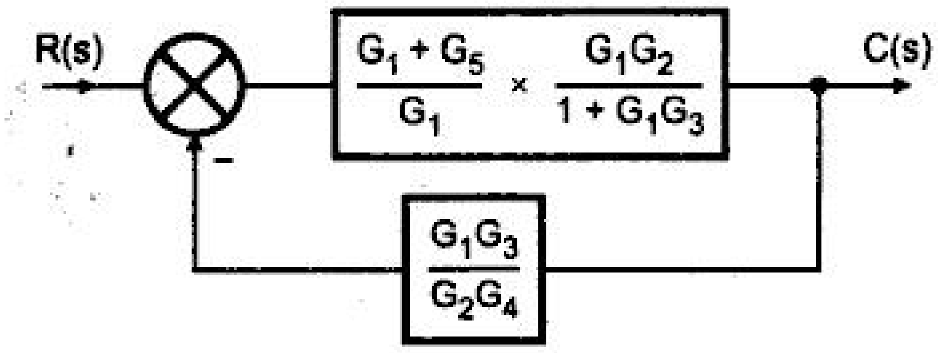
Solving the minor feedback loop.

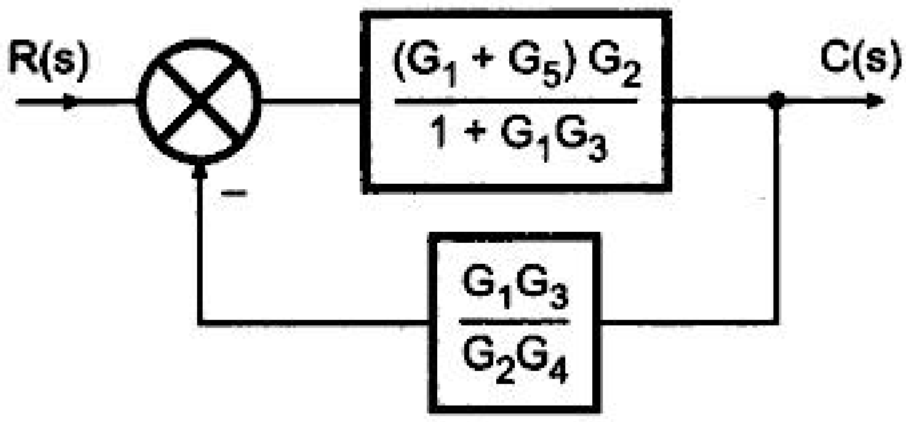


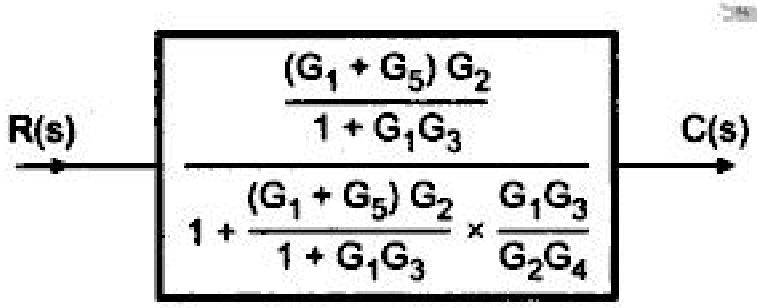
Combining two parallel blocks i.e. $\frac{G_5}{G_1}$ and '1' together we get,







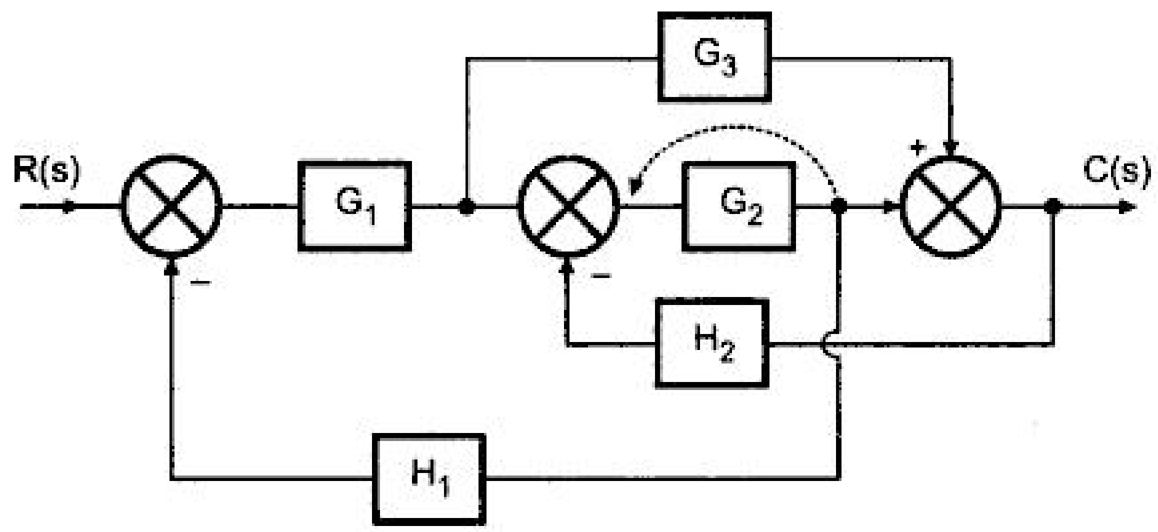




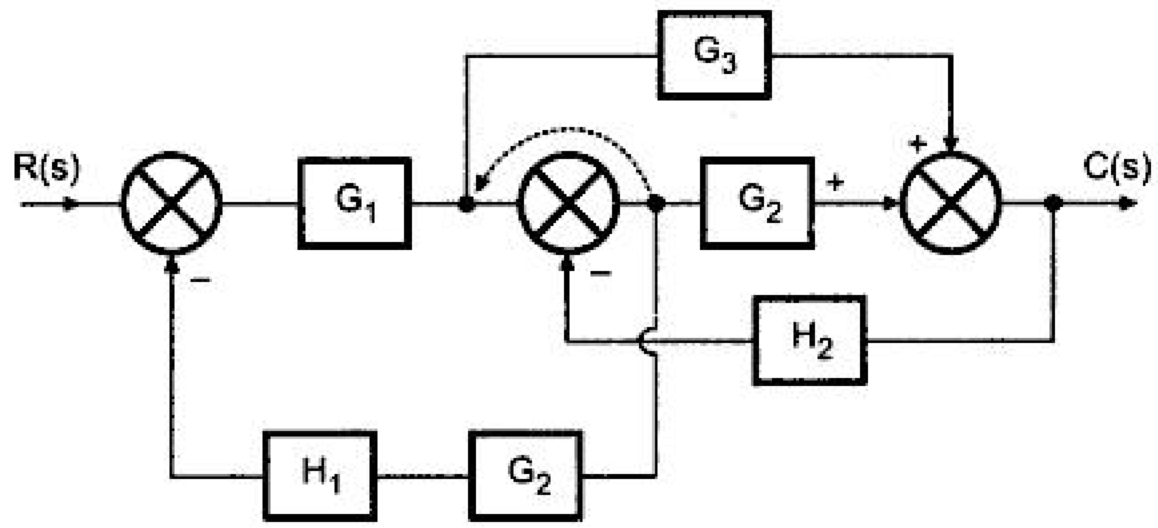
$$\therefore \quad \frac{C(s)}{R(s)} = \frac{(G_1 + G_5)G_2}{(1 + G_1 G_3)} \; \frac{(1 + G_1 G_3)G_4}{[(1 + G_1 G_3) G_4 + G_1G_3 (G_1 + G_5)]}$$

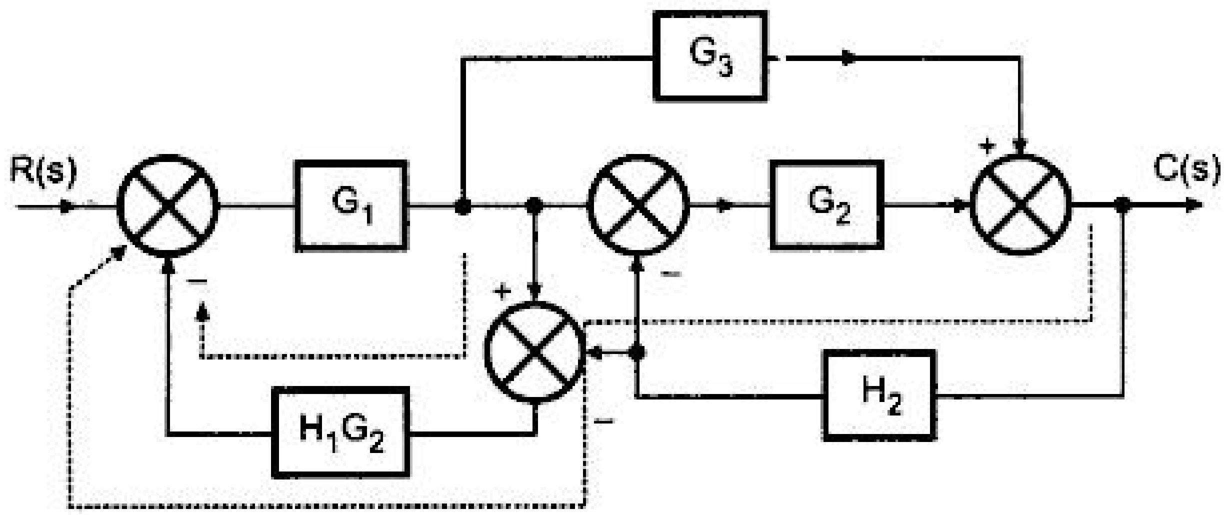
$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G_2 G_4 (G_1 + G_5)}{G_4 + G_1 G_3 G_4 + G_1 G_3 (G_1 + G_5)}$$

Ex. 3.21 *Use of Rule No. 10, critical rule illustration.*

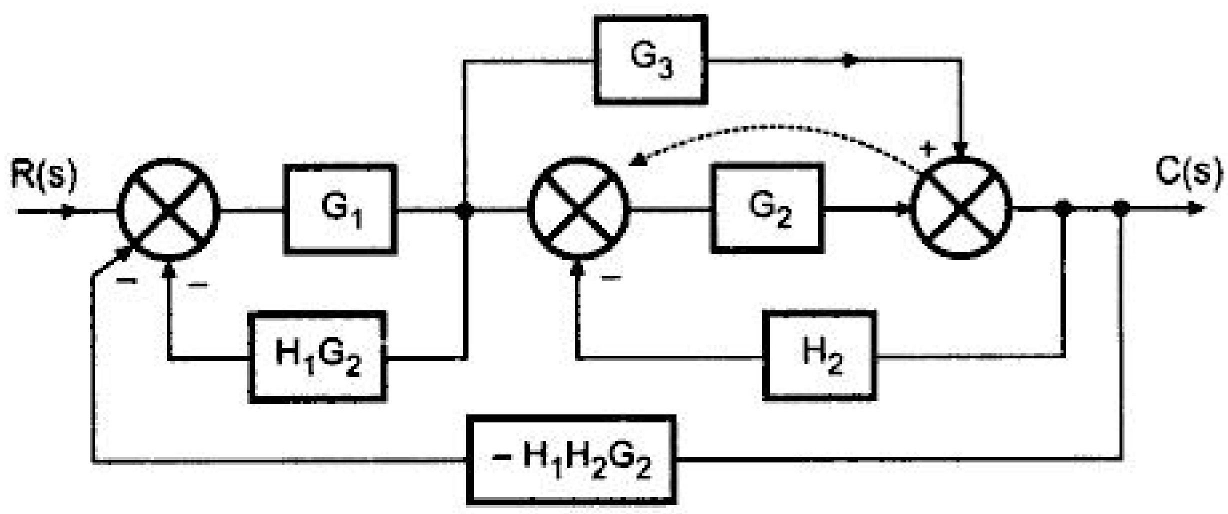


Sol. :

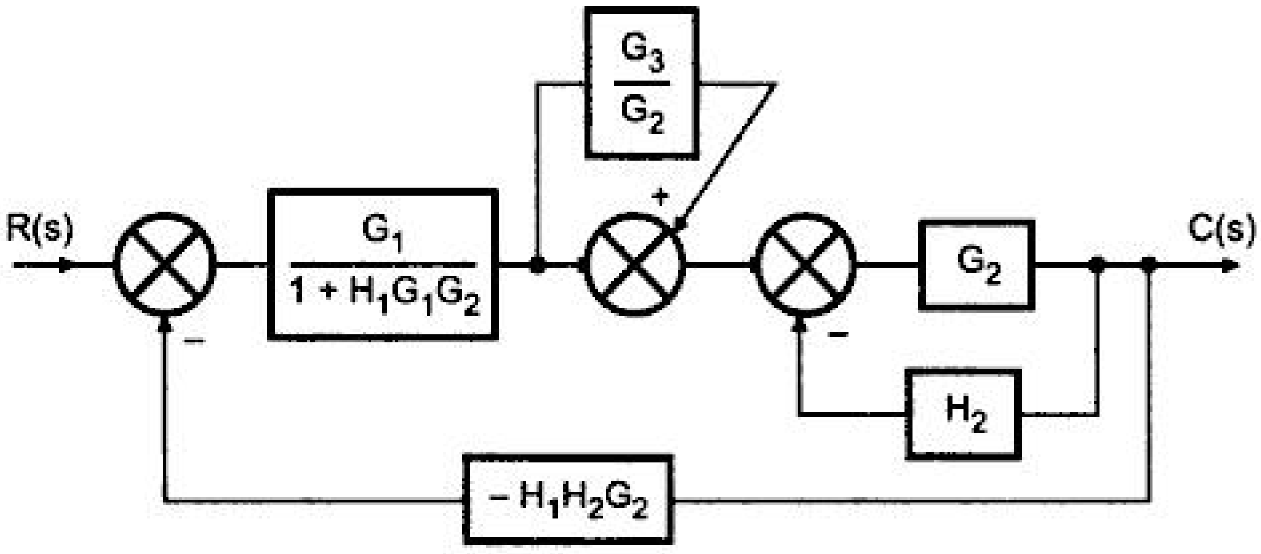


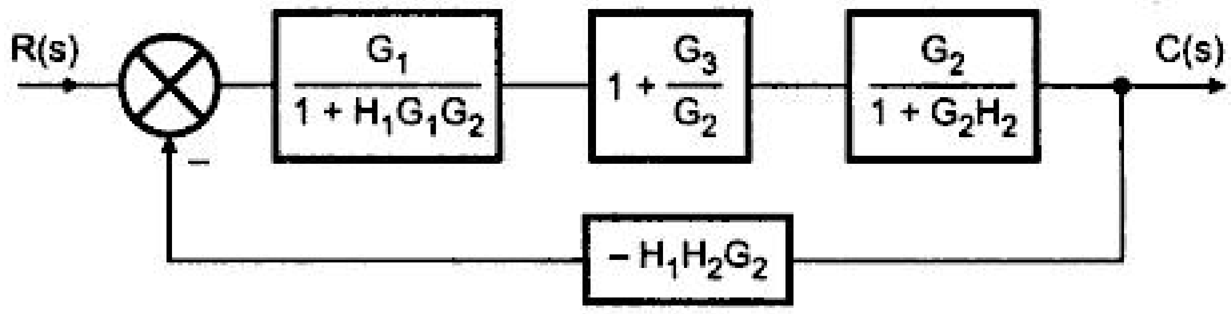


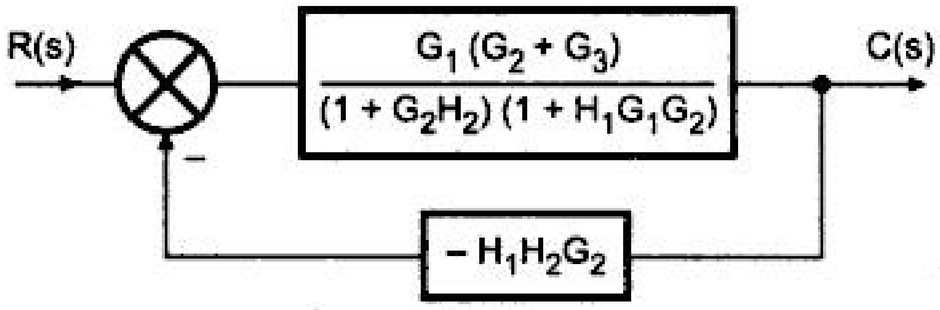
Separating the paths in the feedback path as shown.



Shifting summing point as shown and then interchanging the two summing points using Associative Law we get,







$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1(G_2+G_3)}{(1+G_2H_2)(1+H_1G_1G_2)}}{\dfrac{G_1(G_2+G_3)(-H_1H_2G_2)}{(1+G_2H_2)(1+H_1G_1G_2)}+1}$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G_1(G_2+G_3)}{1+G_2H_2+H_1G_1G_2-G_1G_2G_3H_1H_2}$$

3.4 Analysis of Multiple Input Multiple Output Systems

In these problems, the law of superposition is to be used, considering each input separately. While assuming the other inputs as zero, most of the times if only input is applied to the summing point, summing point is to be removed if not necessary. While removing summing point if sign of the signal present at that summing point which is to be removed is negative must be carried forward in the further analysis. This can be achieved by **introducing a block of transfer function –1 in series with that signal. This is the important step to be remembered while solving problems on multiple input multiple output systems.**

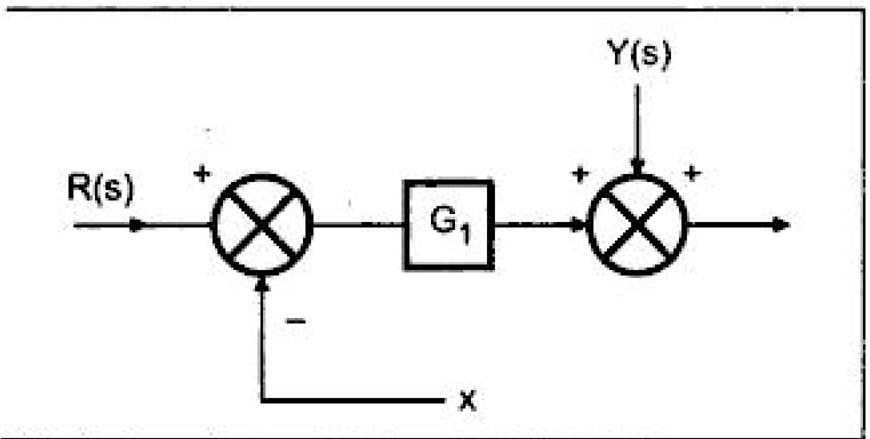


Fig. 3.38

e.g. consider a part of system showing two inputs R(s) and Y(s).

Other details are not shown for simplicity.

When R(s) is considered alone, Y(s) must be assumed zero and summing point at Y(s) can be removed as with Y(s) = 0 there remains only a single signal present at that point so system gets modified as shown in the Fig. 3.39.

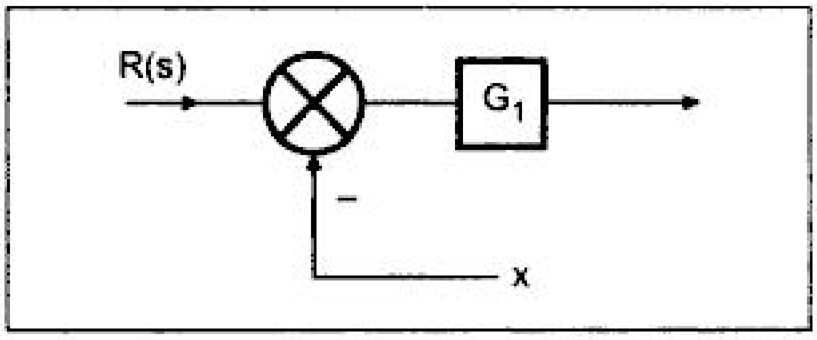


Fig. 3.39

Now sign of signal from block G_1 is positive at the summing point which is removed, hence there is no need of adding any other block.

Now when R(s) = 0 with Y(s) active, the summing point at R(s) also can be removed. But now sign of the signal 'x' at that suming point is negative which must be considered and carried forward for further analysis. This is possible by adding a block of –1 in series with x without altering any other sign. This avoids the confusion and problem can be solved without any error.

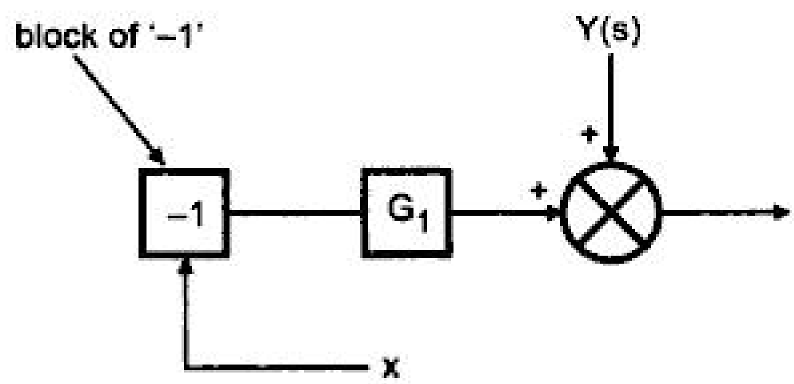
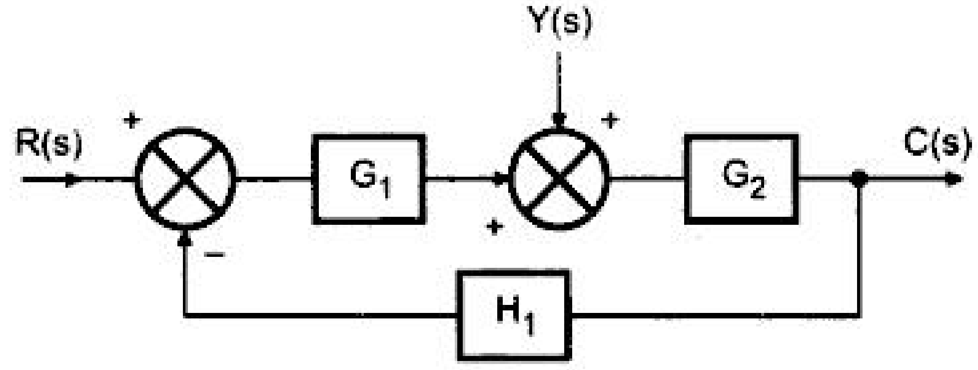
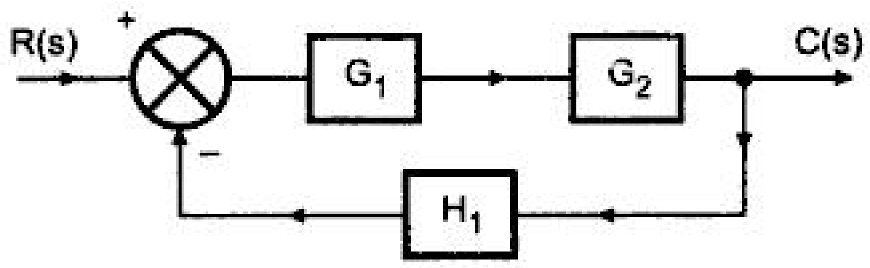


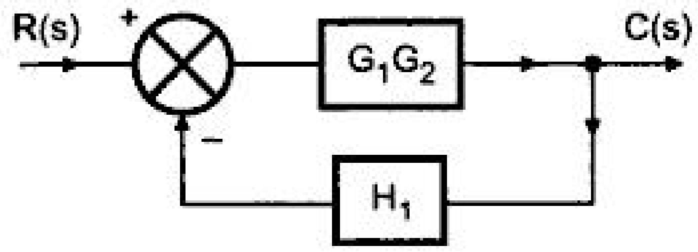
Fig. 3.40

Ex. 3.22 *Obtain the resultant output C(s) in terms of the inputs R(s) and Y(s).*



Sol. : As there are two inputs, consider each input separately. Consider R(s), assuming Y(s) = 0.





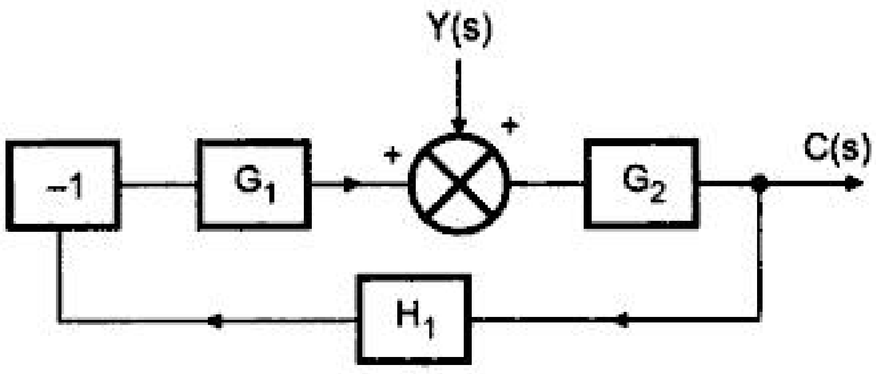
$$\frac{C(s)}{R(s)} = \frac{G_1 G_2}{1+G_1 G_2 H_1}$$

So part of C(s), due to R(s) alone is,

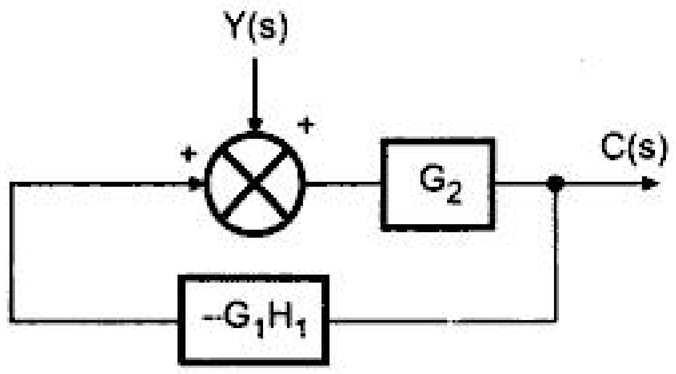
$$C(s) = R(s)\left[\frac{G_1 G_2}{1+G_1 G_2 H_1}\right]$$

Now consider Y(s) acting with R(s) = 0.

Now sign of signal obtained from H_1 is negative which must be carried forward, though summing point at R(s) is removed, as R(s) = 0, so we get,



Combining the blocks G_1 H_1 and −1 as in series,



Now equivalent G = G_2, tracing forward path from input summing point to output.

Equivalent H = $-G_1 H_1$ tracing feedback path from output to input summing point.

While sign of the final feedback is positive at the input summing point.

$$\therefore \quad \frac{C(s)}{Y(s)} = \frac{G}{1-GH} = \frac{G_2}{1-G_2(-G_1 H_1)}$$

H itself is negative.

$$\therefore \quad \frac{C(s)}{Y(s)} = \frac{G_2}{1+G_1 G_2 H_1}$$

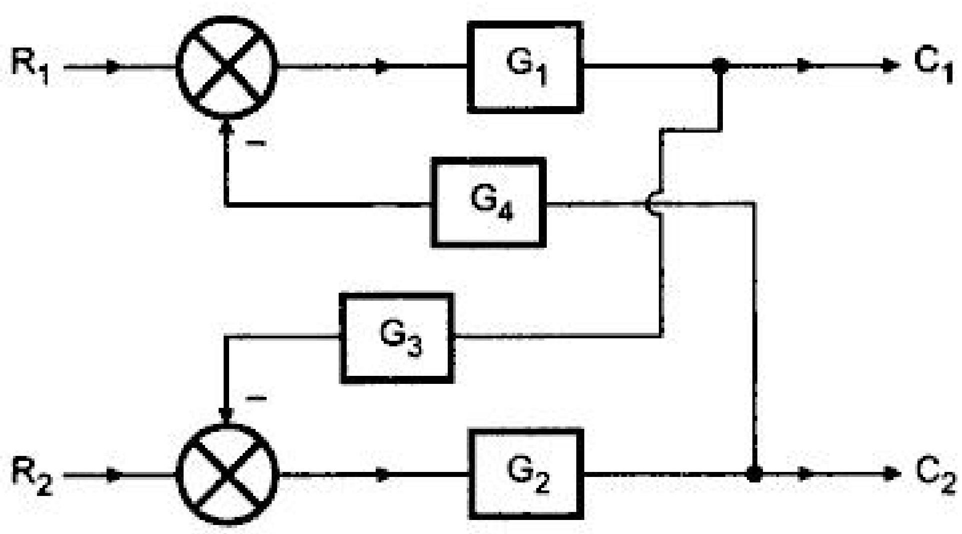
So part of C(s) due to Y(s) alone is,

$$C(s) = Y(s)\left[\frac{G_2}{1+G_1 G_2 H_1}\right]$$

Hence the net output C(s) is given by algebraically adding its two components,

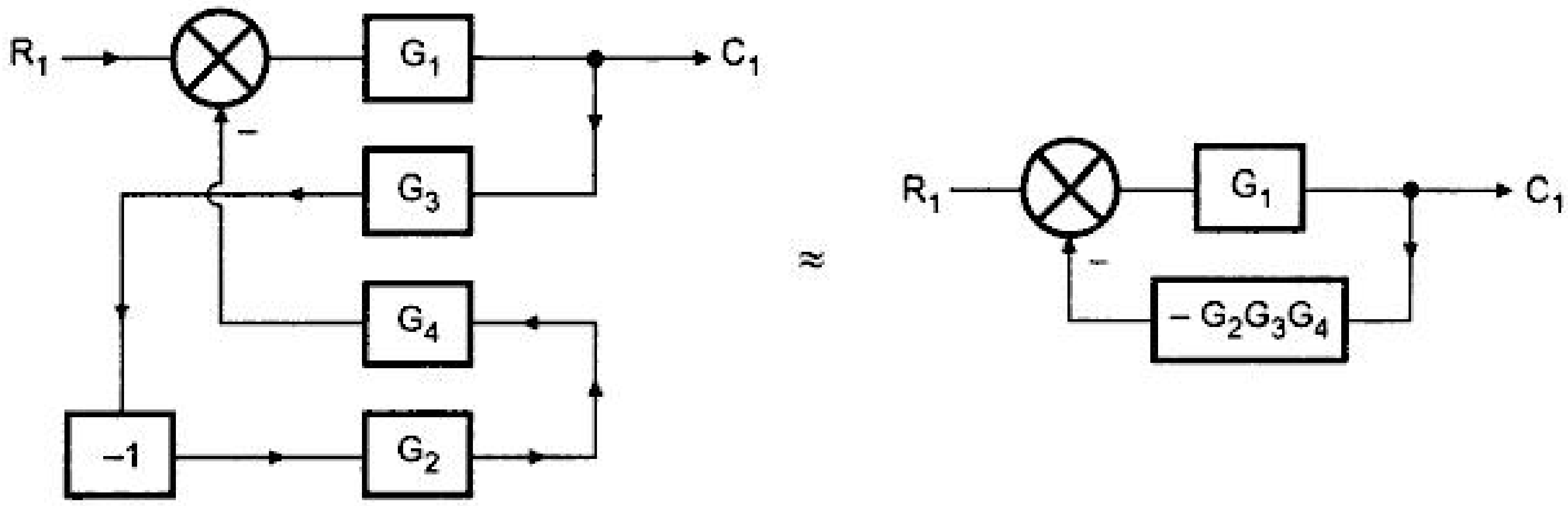
$$C(s) = \frac{G_1 G_2 R(s) + G_2 Y(s)}{1+G_1 G_2 H_1}$$

Ex. 3.23 *Obtain the expression for C_1 and C_2 for the given multiple input multiple output system.*



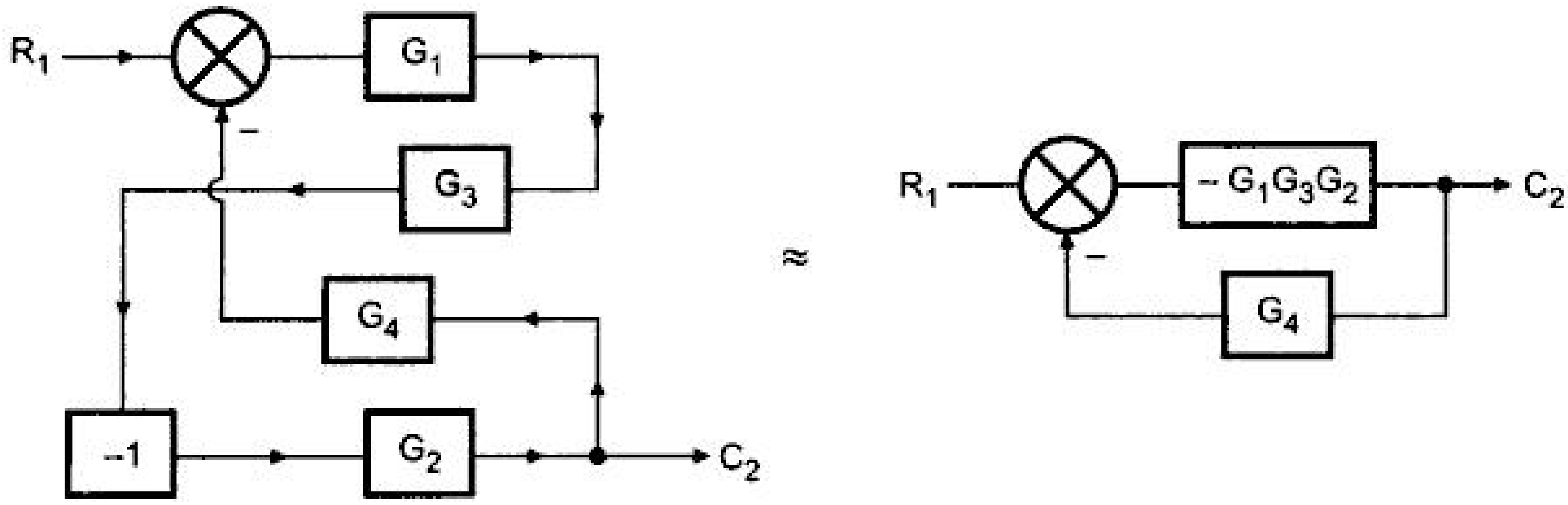
Sol. : In this case there are two inputs and two outputs. Consider one input at a time assuming other zero and one output at a time. Consider R_1 acting, $R_2 = 0$ and C_2 not considered R_1, $R_2 = 0$ and C_2 is suppressed (not considered). C_2 suppressed does not mean that $C_2 = 0$. Only it is not the focus of interest while C_1 is considered. As

$R_2=0$, summing point at R_2 can be removed but block of '–1 must be introduced in series with the signal which is shown negative at that summing point.



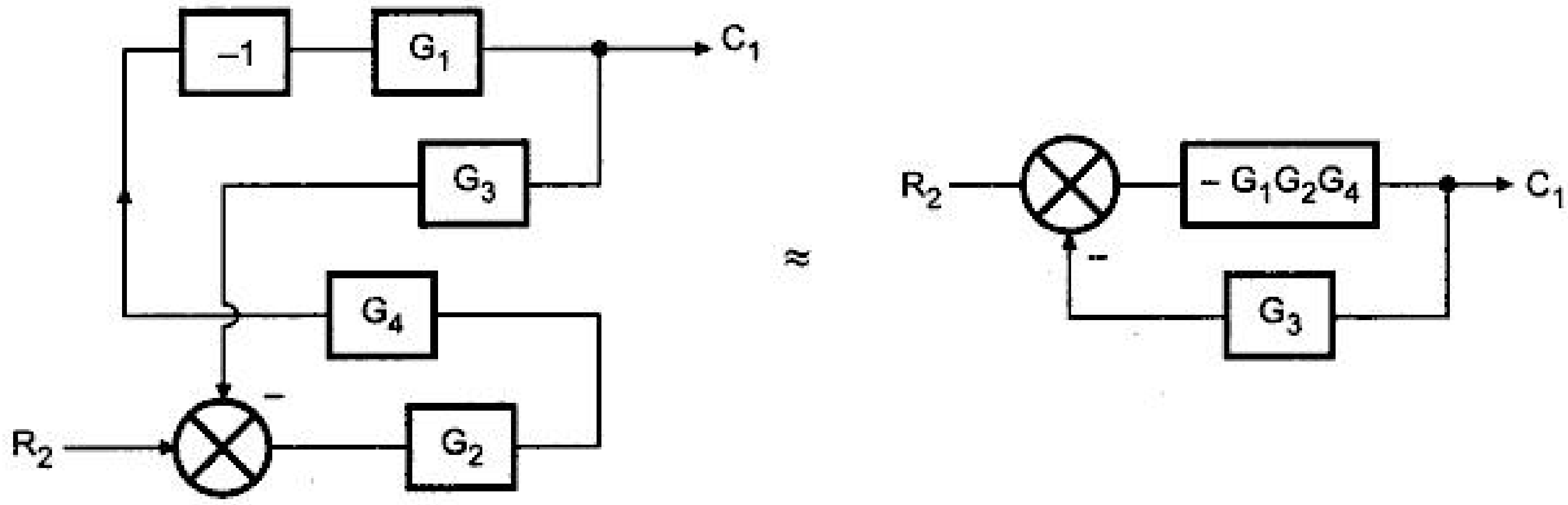
$$\frac{C_1}{R_1} = \frac{G_1}{1+[G_1][-G_2G_3G_4]} = \frac{G_1}{1-G_1G_2G_3G_4}$$

For $\frac{C_2}{R_1}$, assume C_1 suppressed.



$$\therefore \quad \frac{C_2}{R_1} = \frac{-G_1G_3G_2}{1+[-G_1G_3G_2][G_4]} = \frac{-G_1G_2G_3}{1-G_1G_2G_3G_4}$$

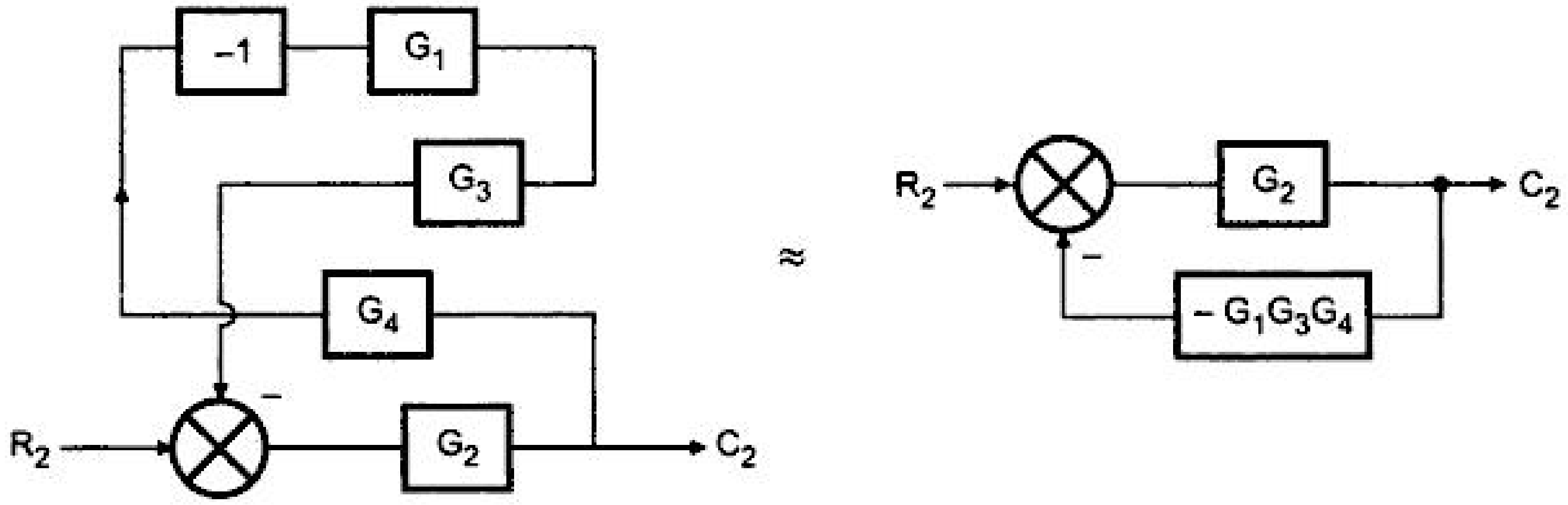
For $\frac{C_1}{R_2}$, $R_1 = 0$ and C_2 is suppressed.



$$\therefore \quad \frac{C_1}{R_2} = \frac{-G_1 G_2 G_4}{1+[-G_1 G_2 G_4][G_3]}$$

$$= \frac{-G_1 G_2 G_4}{1-G_1 G_2 G_3 G_4}$$

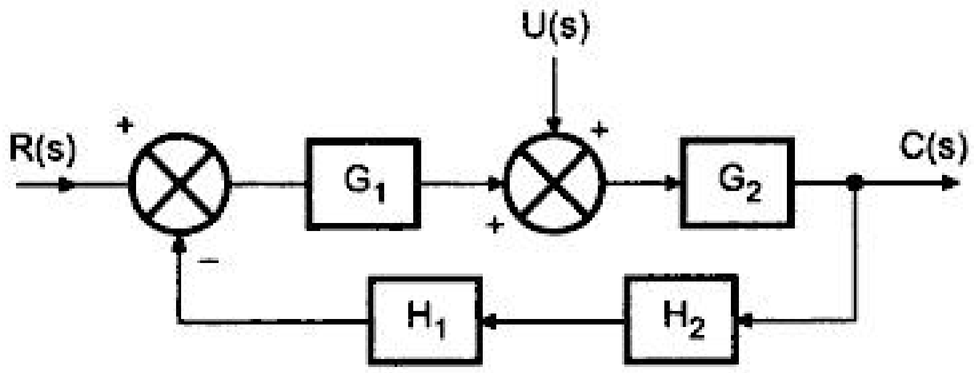
For $\frac{C_2}{R_2}$, $R_1 = 0$ and C_1 is suppressed.



$$\frac{C_2}{R_2} = \frac{G_2}{1+[G_2][-G_1 G_3 G_4]}$$

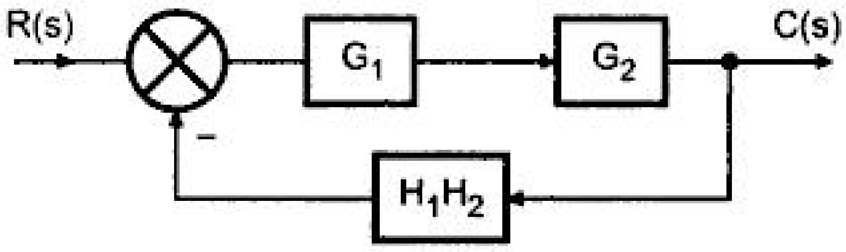
$$= \frac{G_2}{1-G_1 G_2 G_3 G_4}$$

Ex. 3.24 *Obtain the output in terms of the inputs for the system shown in Fig.*



Sol. : The system is multiple input single output system. Consider R(s) acting, U(s) = 0.

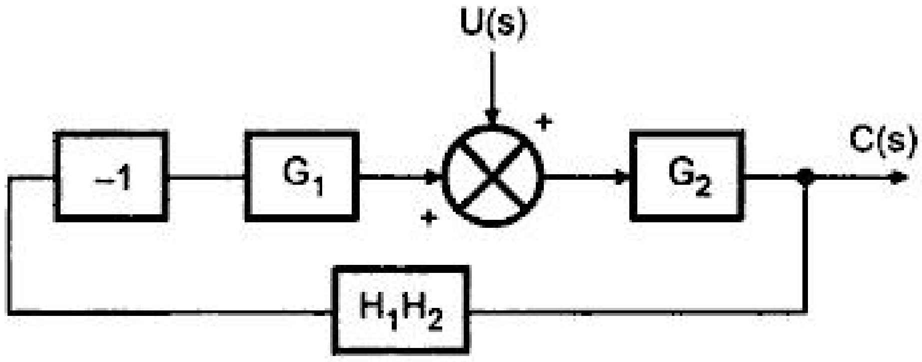
So summing point at U(s) can be removed and signs of all the signals at that point are positive, so there is no need of adding a block in series with any of the signals.

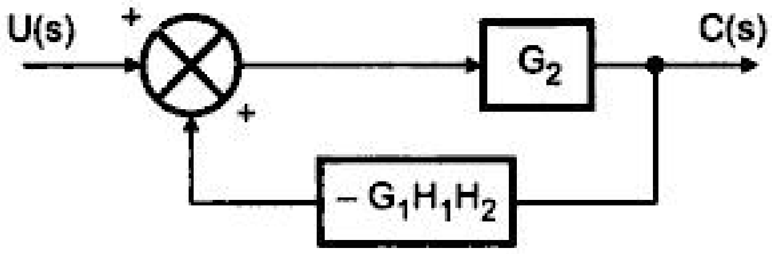


$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G_1G_2}{1+G_1\,G_2\,H_1\,H_2}$$

$$\therefore \quad C(s) = \frac{G_1G_2\,R(s)}{1+G_1\,G_2\,H_1\,H_2} \qquad \text{... (1)}$$

Consider U(s) acting alone, R(s)=0. So summing point at R(s) can be removed, but sign of the signal of the output of H_1 is negative at this summing point. This must be carried forward and hence block of '−1' must be introduced in series with that signal. Redrawing the diagram we get,





$$\therefore \quad \frac{C(s)}{U(s)} = \frac{G_2}{1-[G_2]\,[-G_1\,H_1\,H_2]}$$

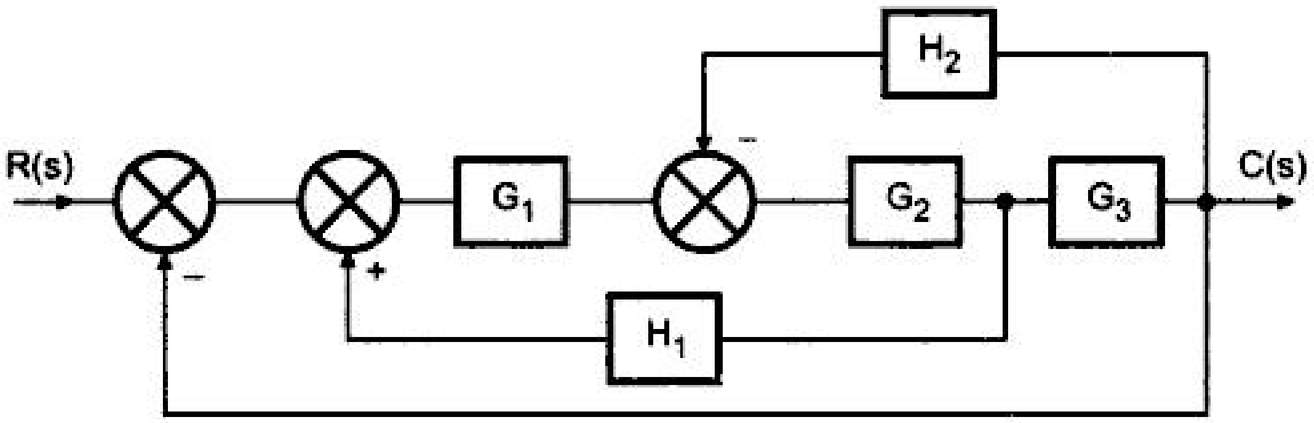
$$= \frac{G_2}{1+G_1\,G_2\,H_1\,H_2}$$

$$C(s) = \frac{G_2\,U(s)}{1+G_1\,G_2\,H_1\,H_2} \qquad \text{... (2)}$$

Total output can be obtained by adding (1) and (2)

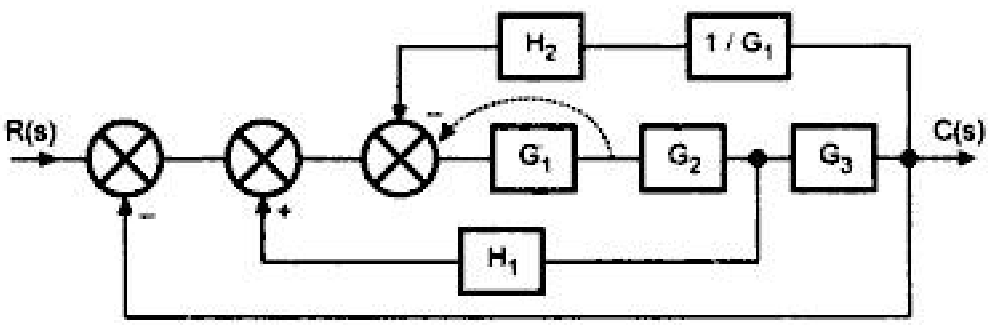
$$\therefore \quad C(s) = \frac{G_1\,G_2\,R(s)+G_2\,U(s)}{1+G_1\,G_2\,H_1\,H_2}$$

Ex. 3.25 *For the system shown, obtain the closed loop transfer function by block diagram reduction*

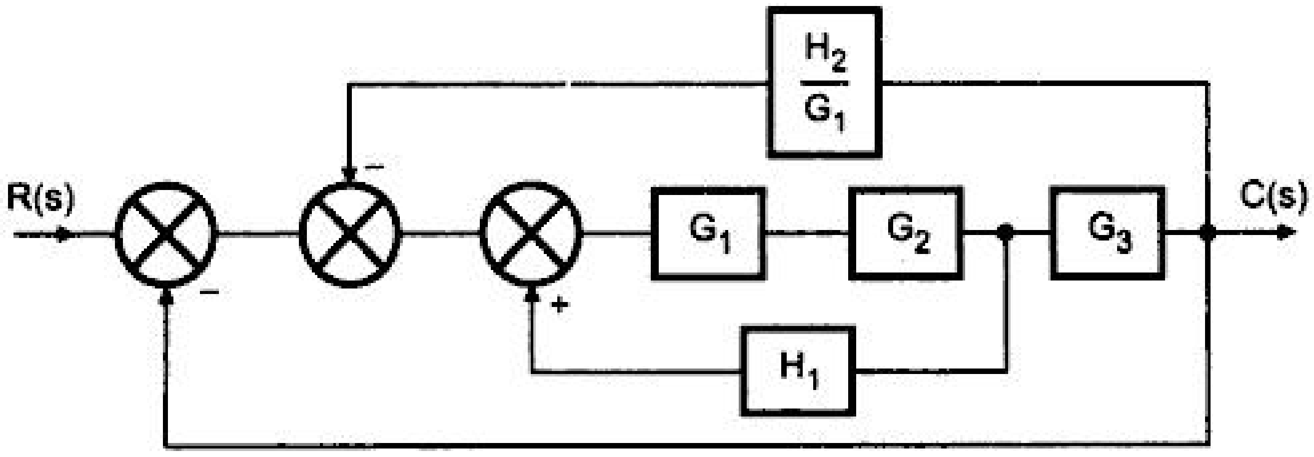


(Mumbai University)

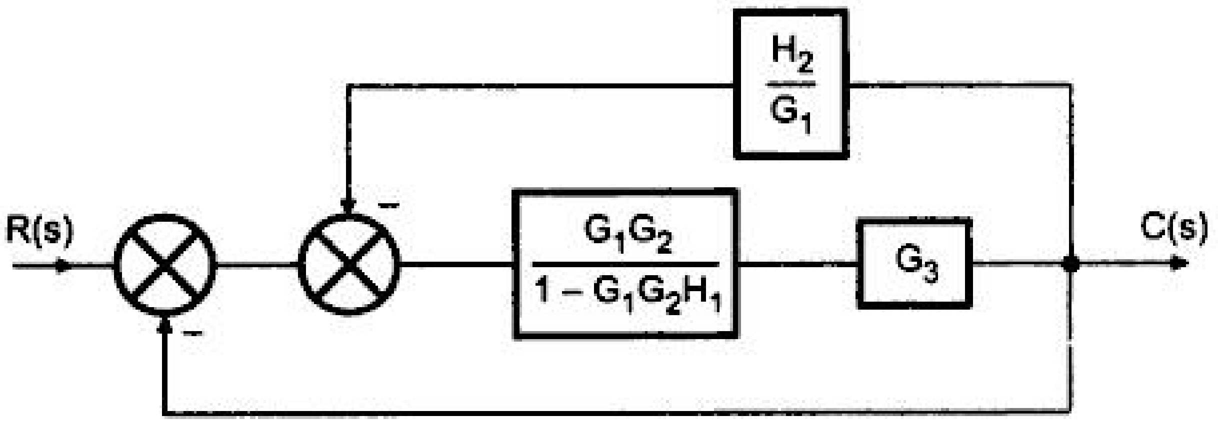
Sol. : No blocks are connected in series or parallel, no minor feedback loop existing so shifting summing point to the left of block with T.F. G_1 i.e. before the block we get



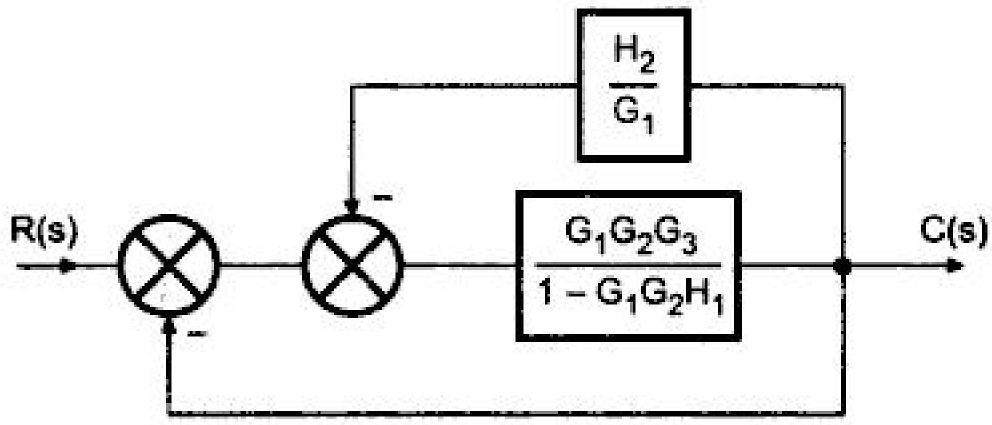
Using associative law, interchanging the positions of the summing points.



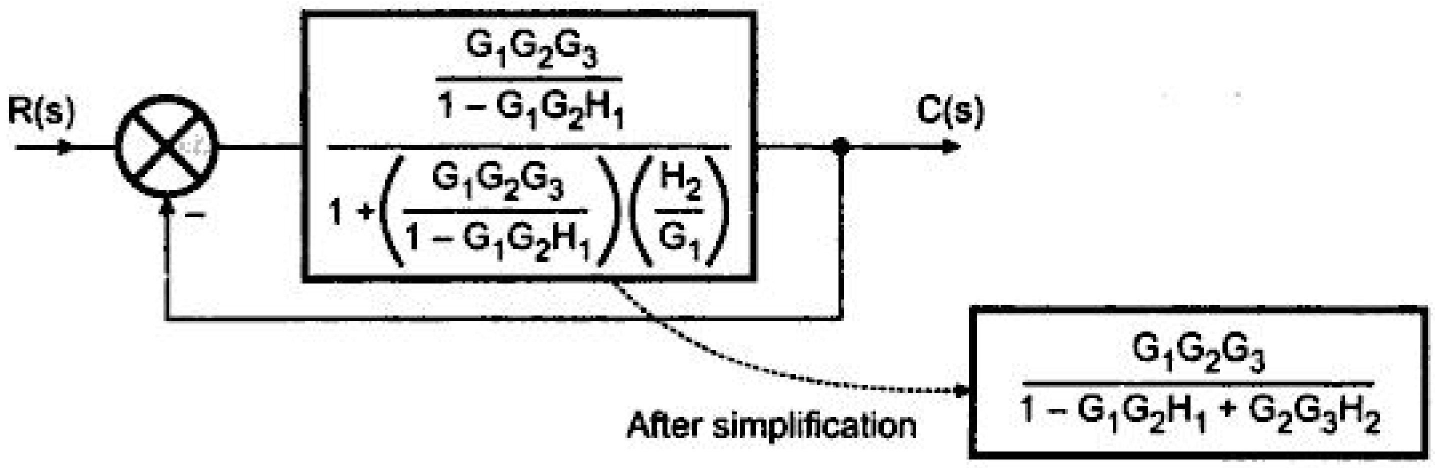
Eliminating the minor feedback loop.

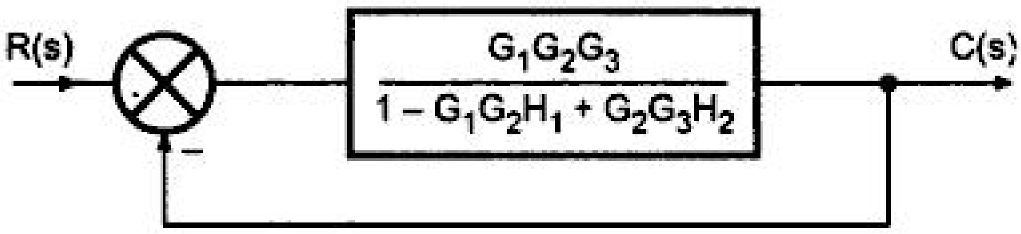


Combining the series blocks

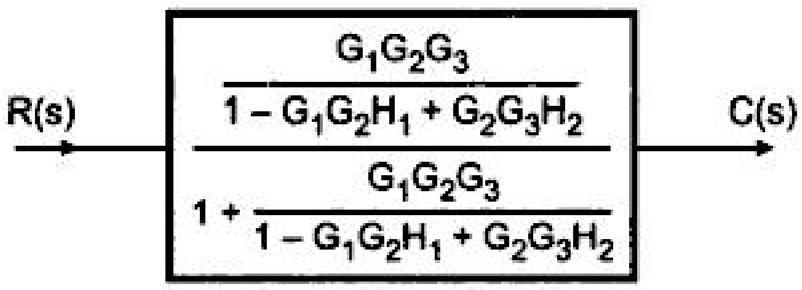


Eliminating minor feedback loop





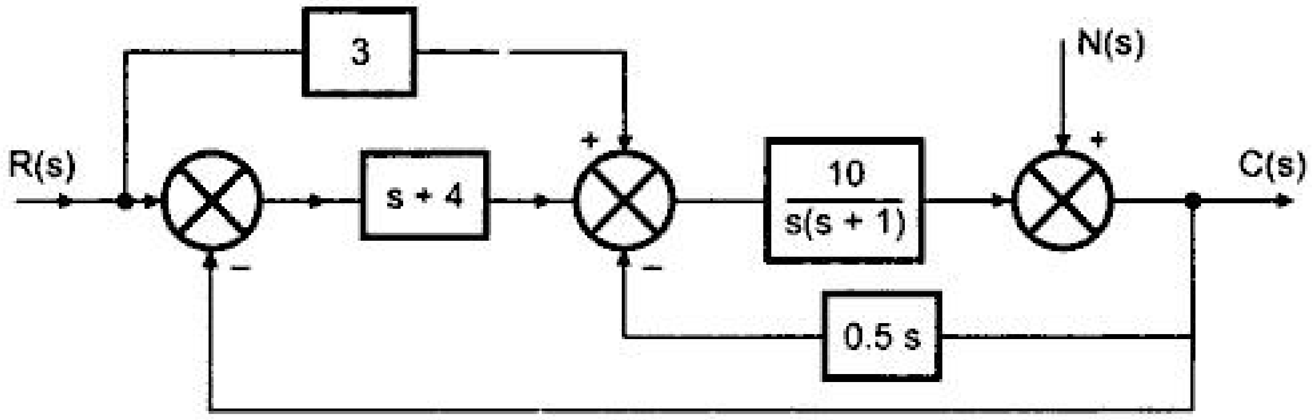
Eliminating minor feedback loop



$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3}{1 - G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3}$$

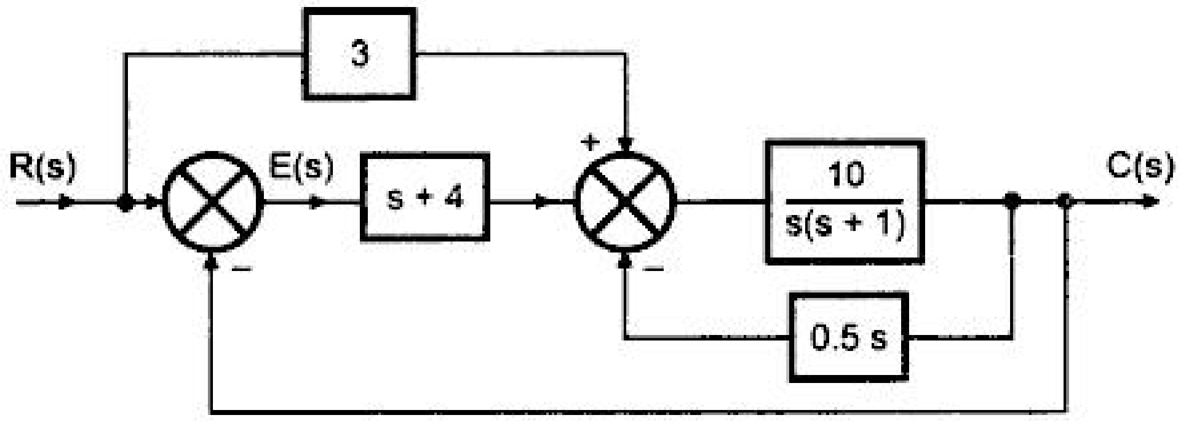
Ex. 3.26 *The system block diagram is given below*

Find i) $\frac{C(s)}{E(s)}$ *if* $N(s) = 0$ ii) $\frac{C(s)}{R(s)}$ *if* $N(s) = 0$ iii) $\frac{C(s)}{N(s)}$ *if* $R(s) = 0$



(Mumbai University May 95)

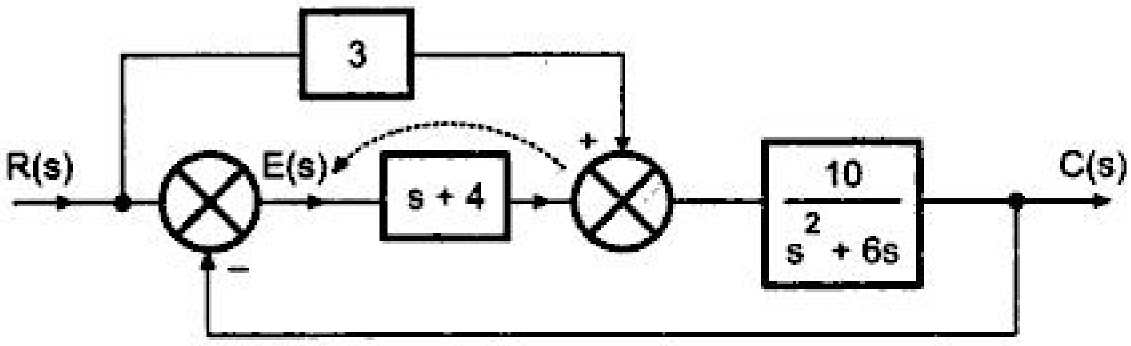
Sol. : i) With N(s) = 0 block diagram becomes



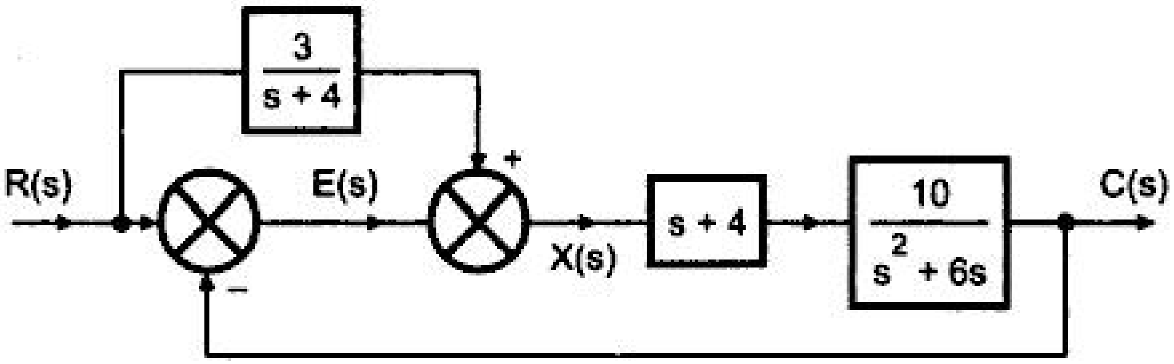
Solving minor feedback loop, we get

$$\text{Feedback loop} = \frac{\frac{10}{s(s+1)}}{1+\frac{10}{s(s+1)}0.5s}$$

$$= \frac{10}{s^2+s+5s} = \frac{10}{s^2+6s}$$



Shifting summing point to the left



Assume output of second summing points as X(s),

Hence
$$E(s) = R(s) - C(s) \qquad \text{... (i)}$$
$$C(s) = X(s)\,\frac{10\,(s+4)}{s^2+6s} \qquad \text{... (ii)}$$
$$X(s) = E(s) + \frac{3}{s+4}\,R(s) \qquad \text{... (iii)}$$

Substituting value of X(s) and R(s) from (i) & (ii) in (iii) we get,

$$\frac{s^2+6s}{10\,(s+4)}\,C(s) = E(s) + \frac{3}{s+4}\,E(s) + \frac{3}{s+4}\,C(s)$$

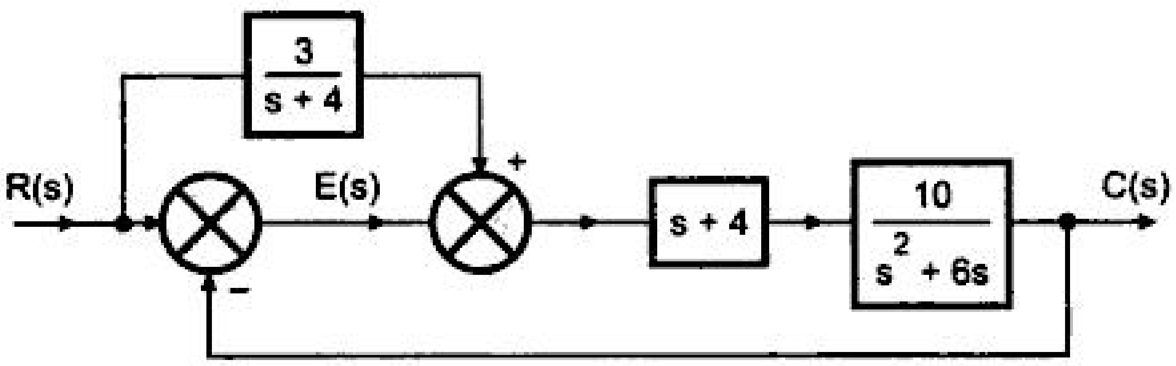
$$\left[\frac{s^2+6s}{10(s+4)} - \frac{3}{(s+4)}\right]C(s) = \left(1+\frac{3}{s+4}\right)E(s)$$

$$\frac{(s^2+6s-30)}{10\,(s+4)}\,C(s) = \frac{(s+7)}{(s+4)}\,E(s)$$

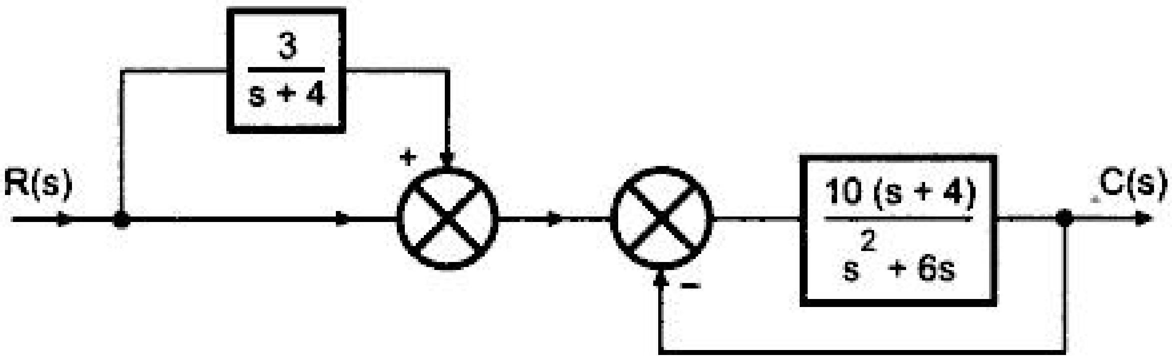
$$\therefore \qquad \frac{C(s)}{E(s)} = \frac{10\,(s+7)}{s^2+6s-30} \qquad \text{When } N(s) = 0$$

ii) To find $\frac{C(s)}{R(s)}$, we have to reduce block diagram solving minor feedback loop and shifting summing point to the left as shown earlier in (i).

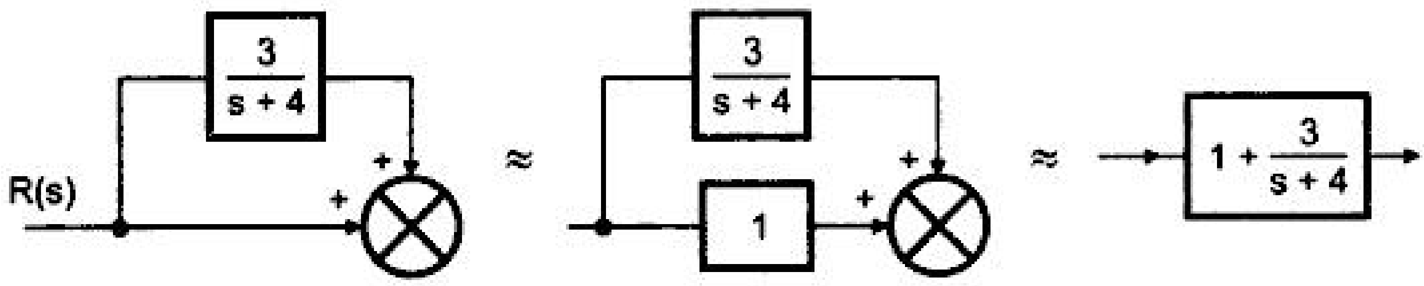
So referring to block diagram after these two steps i.e. Fig.



Exchanging two summing points using associative law,



Students should note that the block $\frac{3}{s+4}$ is in parallel with a branch of unity gain so its reduction is

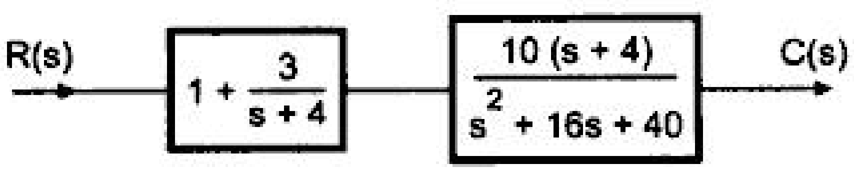


Solving other minor feedback loop

$$= \frac{\dfrac{10(s+4)}{s^2+6s}}{1+\dfrac{10(s+4)}{s^2+6s}}$$

$$= \frac{10(s+4)}{s^2+16s+40}$$

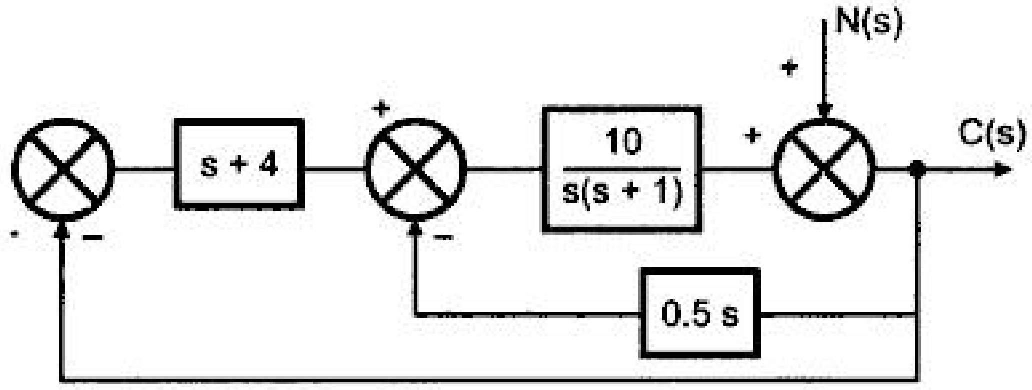
$\therefore$ Block diagram becomes,



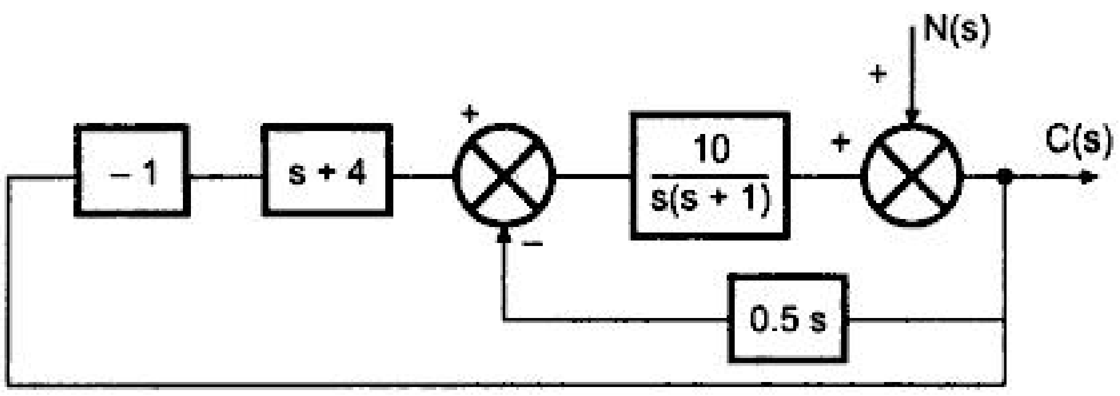
$$\therefore \quad \frac{C(s)}{R(s)} = \left(\frac{s+7}{s+4}\right) \times \left(\frac{10(s+4)}{s^2+16s+40}\right)$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{10(s+7)}{s^2+16s+40}$$

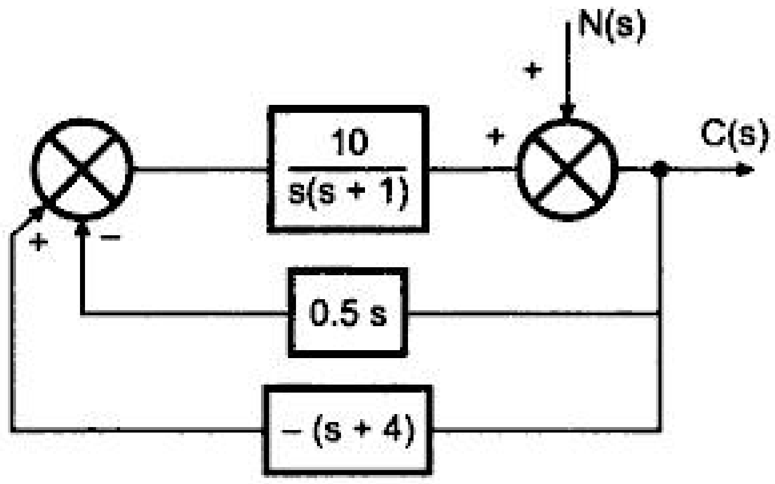
iii) With R(s) = 0 block diagram becomes



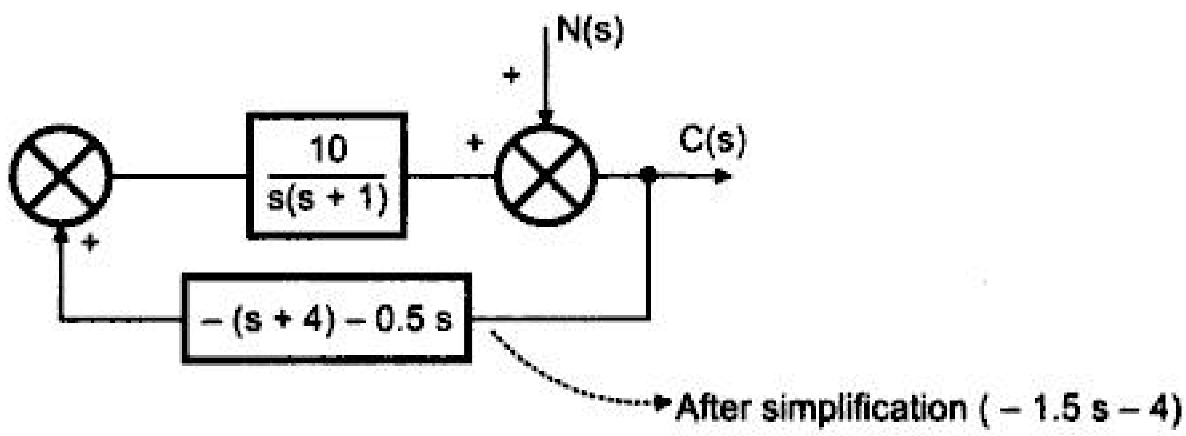
The block of '3' will not exist as R (s) = 0. Similarly first summing point will also vanish but student should note that negative sign of feedback must be considered as it is though summing point gets deleted.



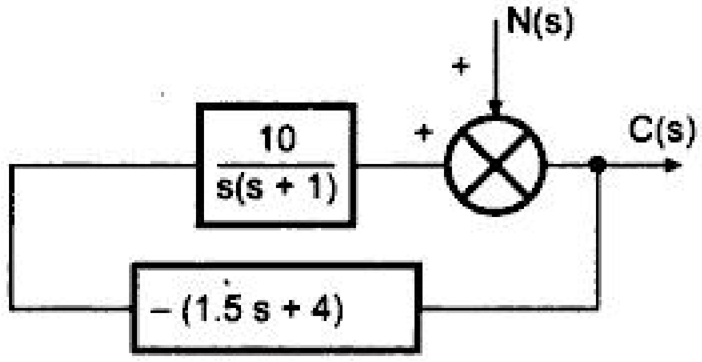
In general while deleting summing point, it is necessary to consider the signs of the different signals at that summing points and should not be disturbed. So introducing block of '–1' to consider negative sign.



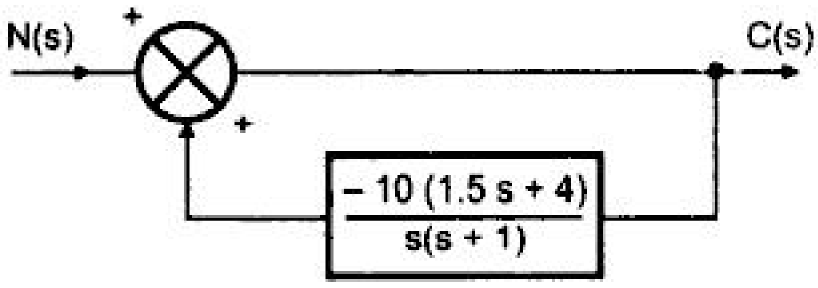
Two blocks are in parallel, adding them with signs.



Removing summing point, as sign is positive no need of adding a block



Redrawing the Figure



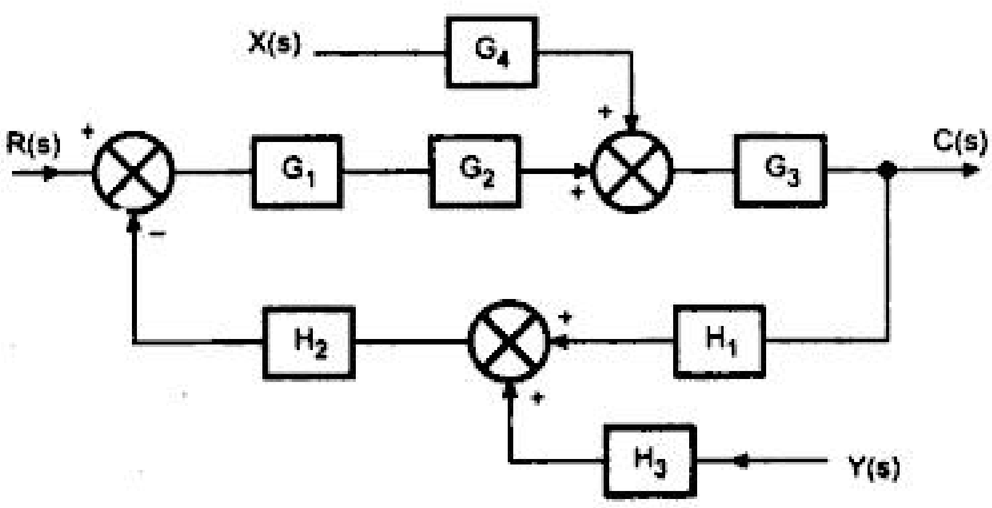
$$\therefore \quad \frac{C(s)}{N(s)} = \frac{1}{1-\left[\frac{-10(1.5s+4)}{s(s+1)}\right]}$$

$$\frac{C(s)}{N(s)} = \frac{1}{1+\frac{15s+40}{s(s+1)}}$$

$$\therefore \quad \frac{C(s)}{N(s)} = \frac{s(s+1)}{s^2+s+15s+40}$$

$$\therefore \quad \frac{C(s)}{N(s)} = \frac{s(s+1)}{s^2+16s+40}$$

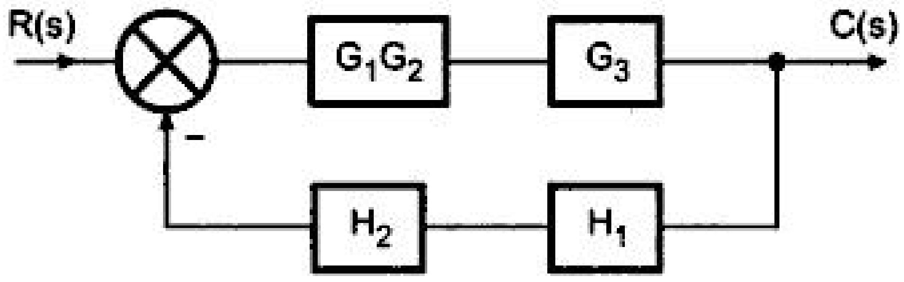
Ex. 3.27 *Use block diagram reduction technique and obtain the transfer functions*
i) $\frac{C(s)}{R(s)}$, *ii)* $\frac{C(s)}{X(s)}$ *iiii)* $\frac{C(s)}{Y(s)}$
Also find total output of the system. **(Mumbai University Nov. 95)**



Sol. : In such multiple input systems, it is necessary to use superposition principle And it must be noted that while removing summing point, it is necessary to consider signs of various signals at that summing point.

i) Consider R(s) acting alone, X(s) = Y(s) = 0

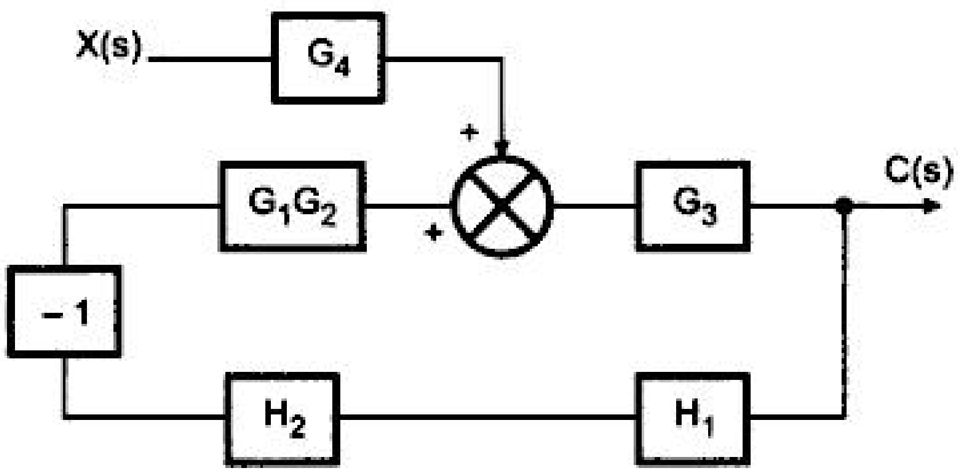
So blocks G_4 and H_3 will vanish along with the summing points and signs of all signals are positive.



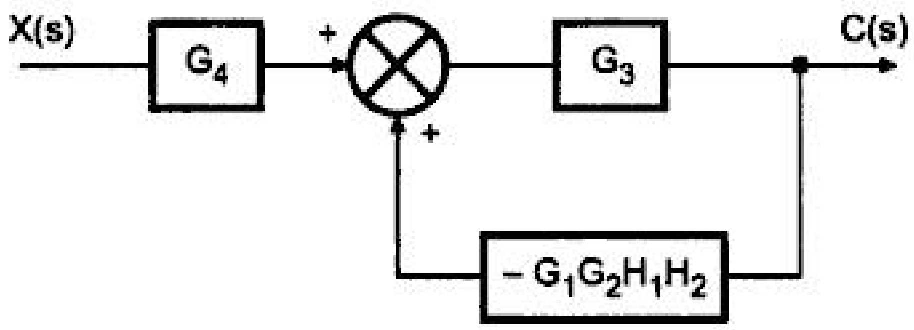
$$\therefore \qquad \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3}{1 + G_1 G_2 G_3 H_1 H_2} \qquad \text{... (i)}$$

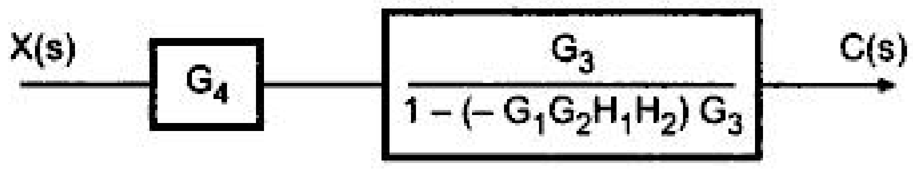
ii) Consider X(s) acting alonge, R (s) = Y(s) = 0

When R(s) = 0 , Summing point at R (s) will vanish, but sign of feedback signal at that summing point is negative. So it is necessary to carry on that sign by adding a block of '–1' in series with that signal.



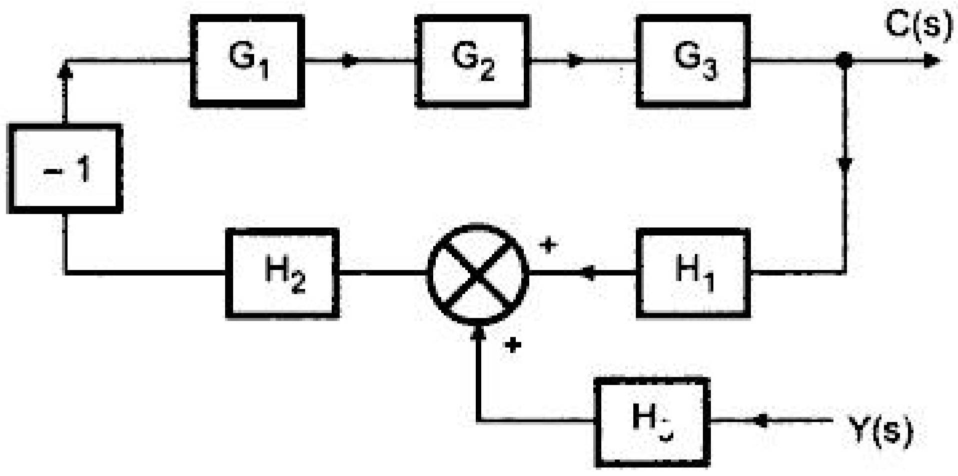
This may be drawn as below.



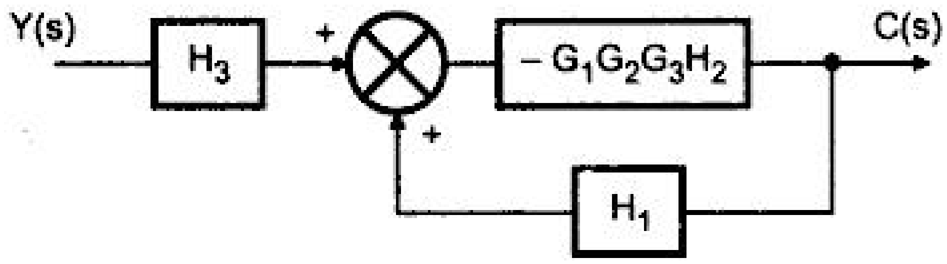


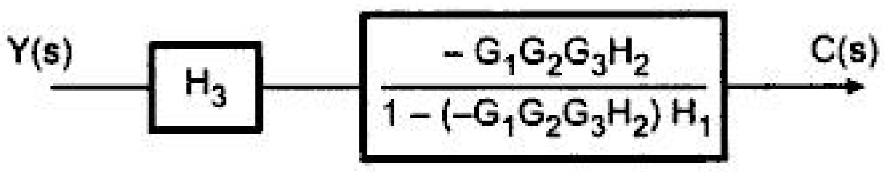
$$\therefore \quad \frac{C(s)}{X(s)} = \frac{G_3 G_4}{1+G_1 G_2 G_3 H_1 H_2} \qquad \text{... (ii)}$$

iii) Consider Y(s) acting alone, R (s) = X(s) = 0



Redrawing the block diagram and combining blocks in series. i.e.



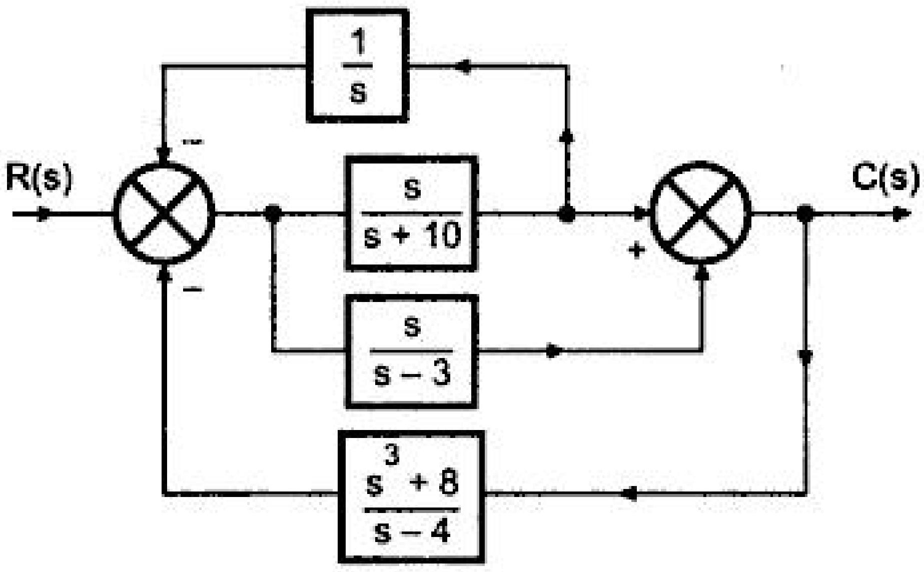


$$\therefore \quad \frac{C(s)}{Y(s)} = \frac{-G_1 G_2 G_3 H_2 H_3}{1+G_1 G_2 G_3 H_1 H_2} \qquad \text{.... (iii)}$$

From (i), (ii) and (iii) we can write the total output as

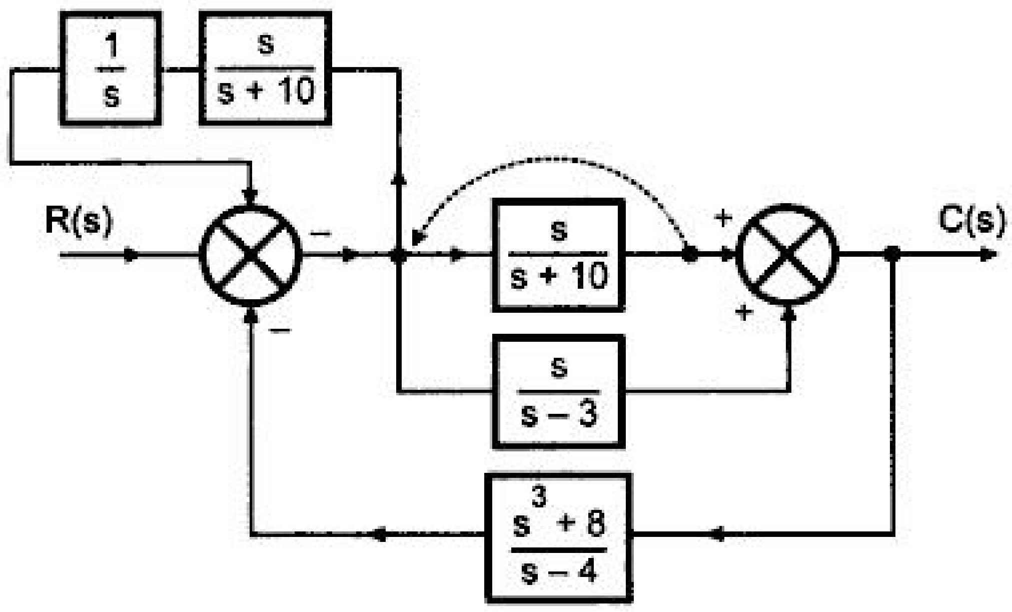
$$C(s) = \frac{G_1 G_2 G_3 R(s) + G_3 G_4 X(s) - G_1 G_2 G_3 H_2 H_3 Y(s)}{1 + G_1 G_2 G_3 H_1 H_2}$$

Ex. 3.28 *Reduce the block diagram and obtain the transfer function C(s)/R(s).*

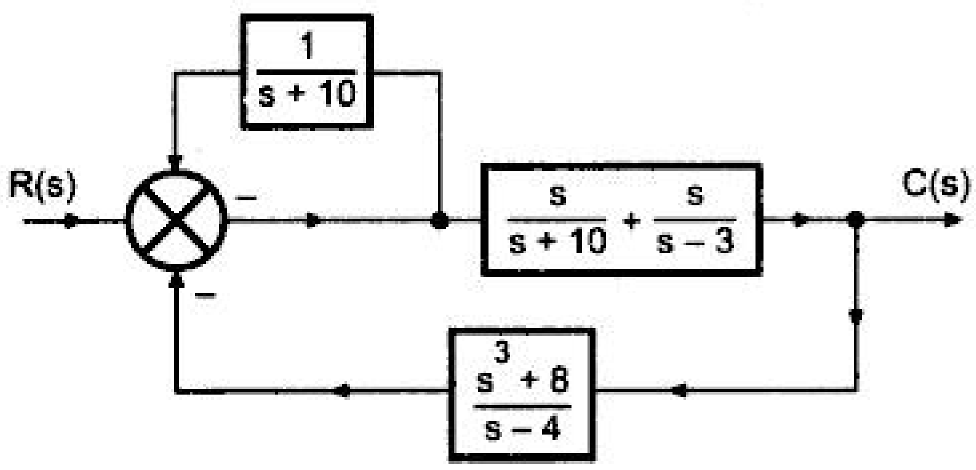


(Mumbai University, May 96)

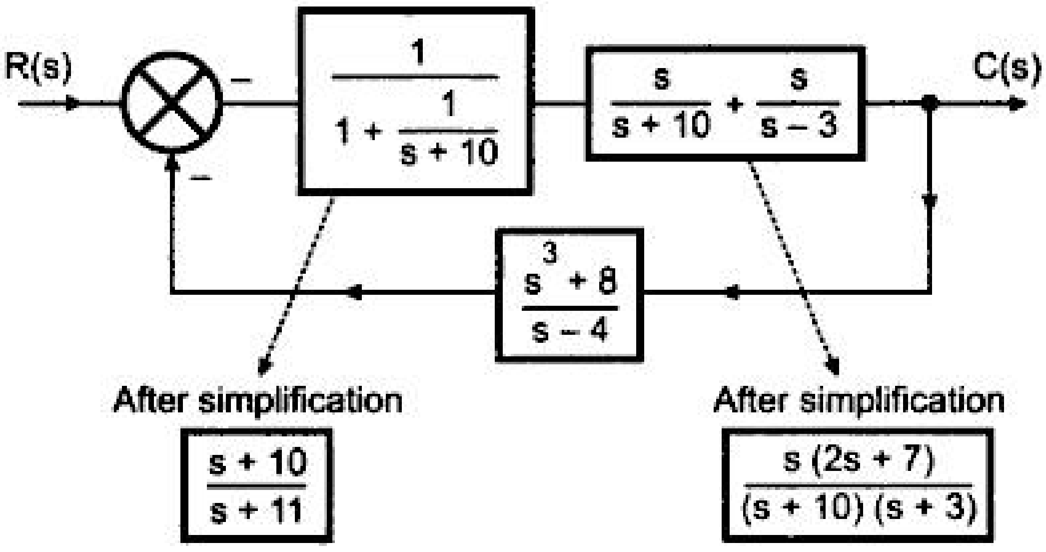
Sol. : No series, parallel combination and no minor feedback loop exists. So shifting take off point before the block of $\left(\frac{s}{s+10}\right)$.

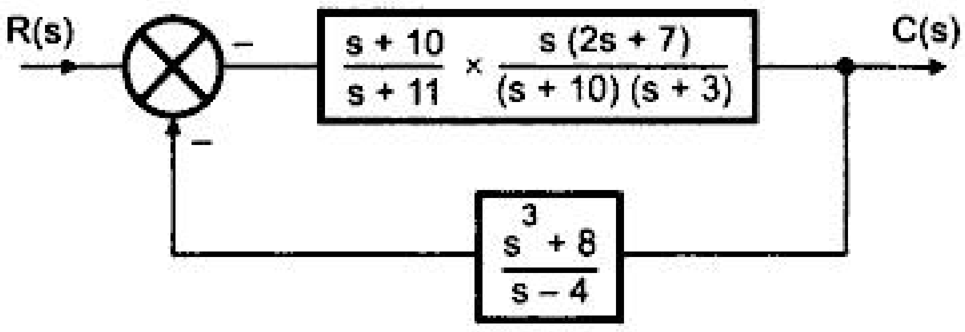


There exists a parallel combination of blocks $\frac{s}{s+10}$ and $\frac{s}{s-3}$ so adding them ,



There exists a minor feedback loop with forward path as unity and feedback transfer function as $\left(\frac{1}{s+10}\right)$.



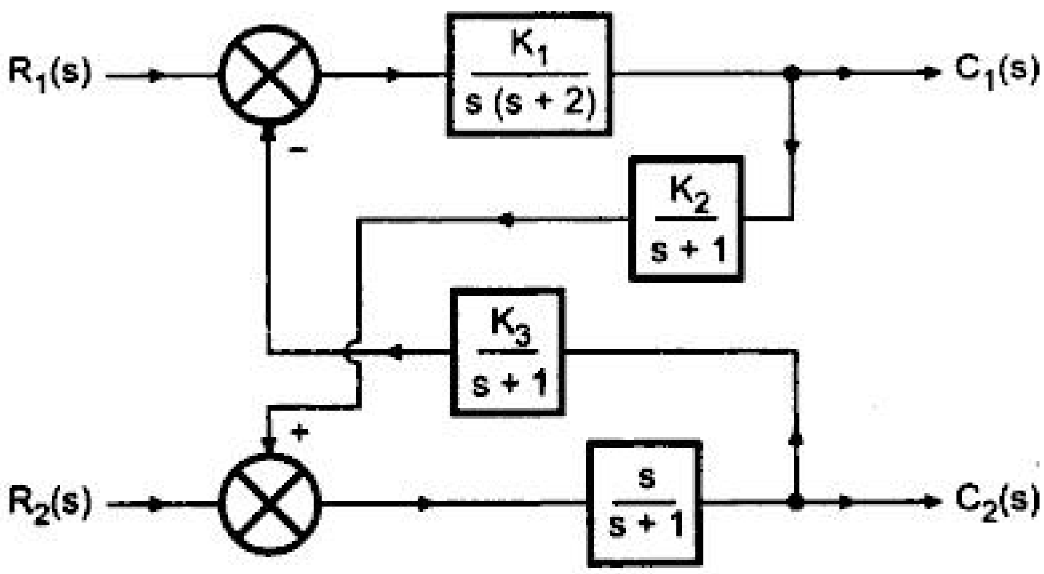


$$\therefore \quad \frac{C(s)}{R(s)} = \frac{\dfrac{s(2s+7)}{(s+11)(s-3)}}{1+\dfrac{s(2s+7)}{(s+11)(s-3)} \cdot \dfrac{(s^3+8)}{(s-4)}}$$

$$= \frac{s(2s+7)(s-4)}{(s+11)(s-3)(s-4)+s(2s+7)(s^3+8)}$$

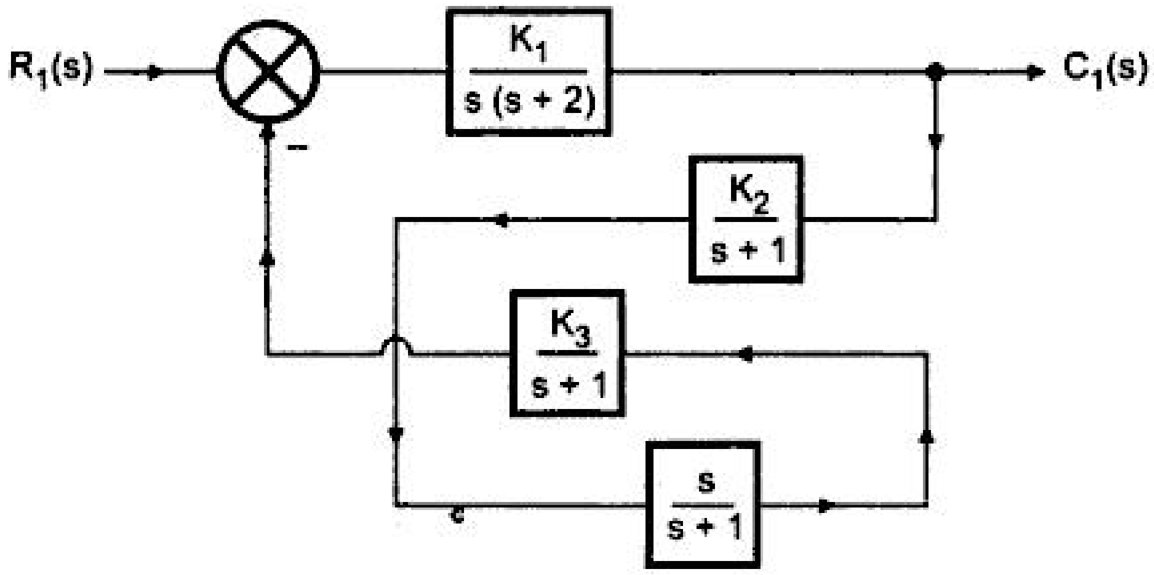
$$\therefore \quad \frac{C(s)}{R(s)} = \frac{s(2s^2-s-28)}{2s^5+7s^4+s^3+20s^2-9s+132}$$

Ex. 3.29 *Determine* $\frac{C_1(s)}{R_1(s)}$ *and* $\frac{C_2(s)}{R_2(s)}$ *for multiple input system shown below.*

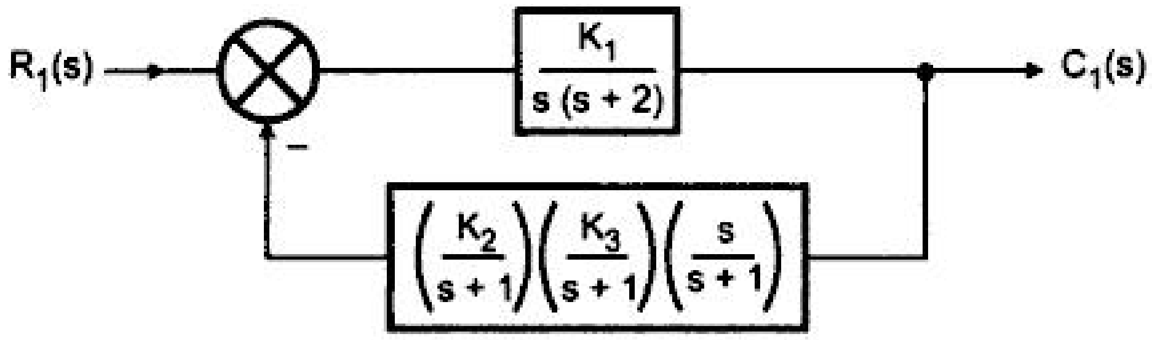


Sol. : Use Superposition principle and while removing summing point consider signs of all the signals at that summing point.

i) For $\dfrac{C_1(s)}{R_1(s)}$, Consider R_2 (s) = 0 and C_2 (s) suppressed, block diagram becomes

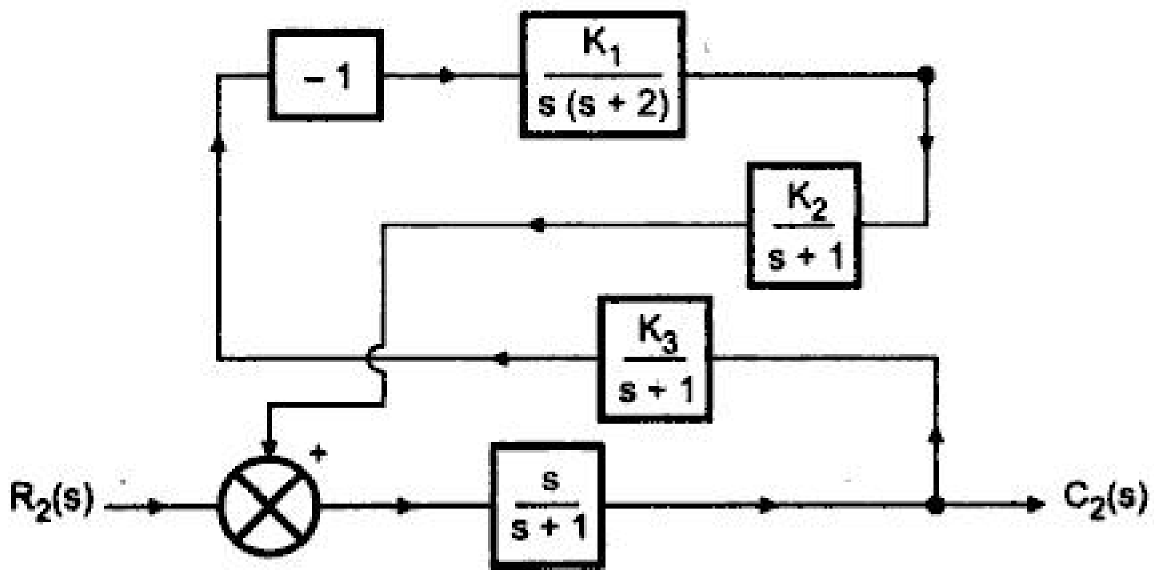


Combining blocks in series



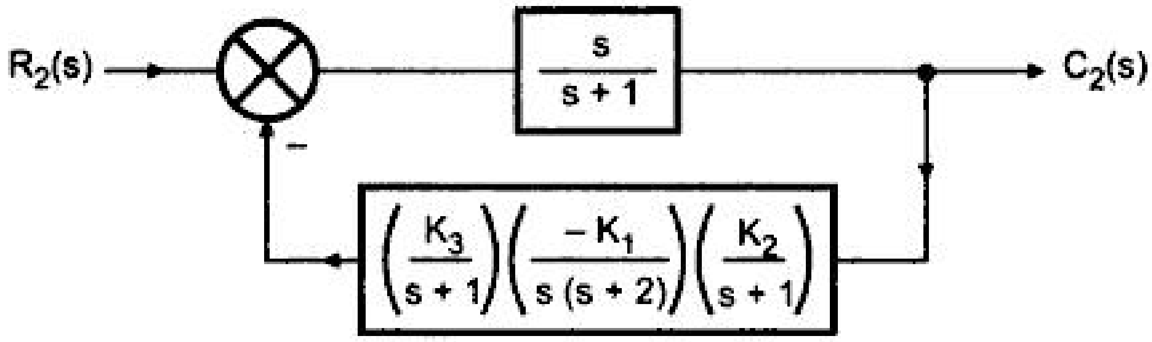
$$\therefore \qquad \frac{C_1(s)}{R_1(s)} = \frac{\dfrac{K_1}{s(s+2)}}{1+\dfrac{s K_2 K_3}{(s+1)^3}\cdot\dfrac{K_1}{s(s+2)}} = \frac{K_1 (s+1)^3}{s(s+2)(s+1)^3 + s K_1 K_2 K_3}$$

ii) For $\dfrac{C_2(s)}{R_2(s)}$, consider $R_1(s)$ = 0 and $C_1(s)$ suppressed block diagram becomes,



* Block of '−1' is added to consider negative sign of signal while removing summing point.

Combining all blocks in series.

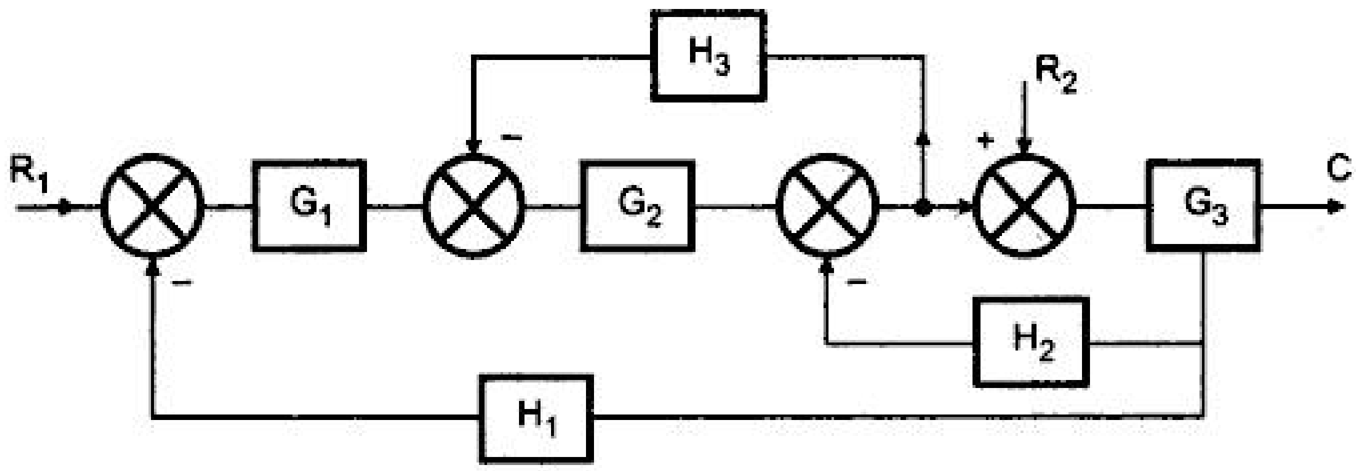


$$\therefore \quad \frac{C_2(s)}{R_2(s)} = \frac{\dfrac{s}{s+1}}{1-\left(\dfrac{K_3}{s+1}\right)\left(\dfrac{-K_1}{s(s+2)}\right)\left(\dfrac{K_2}{s+1}\right)\left(\dfrac{s}{s+1}\right)}$$

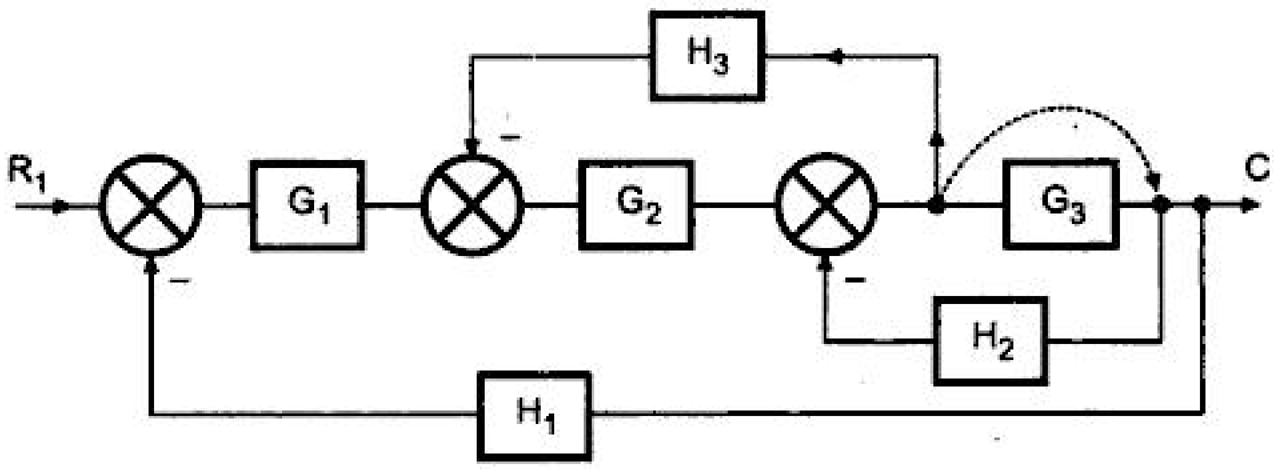
$$\therefore \quad \frac{C_2(s)}{R_2(s)} = \frac{s^2\,(s+2)\,(s+1)^2}{s(s+2)\,(s+1)^3+sK_1\,K_2\,K_3}$$

Ex. 3.30 *For the system shown in the Figure determine C/R_1 and C/R_2. Assume $R_2 = 0$ when R_1 is applied and $R_1 = 0$ when R_2 is applied use block diagram reduction.*

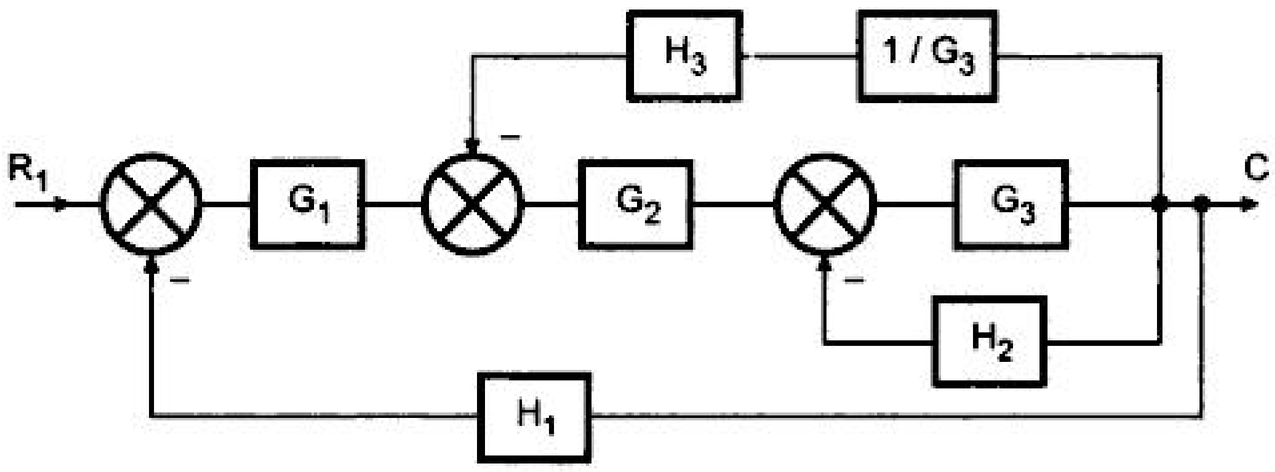
(Gate)



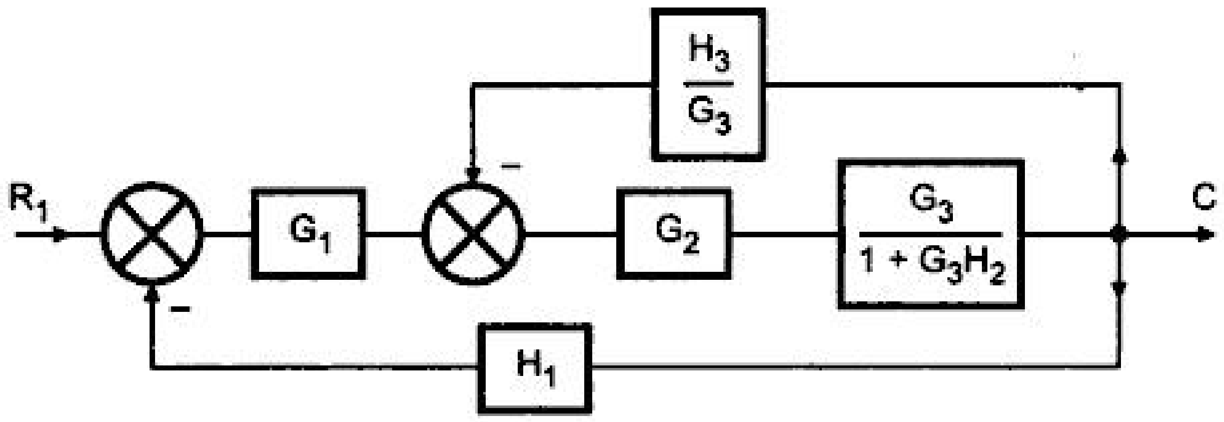
Sol. : Finding C/R_1, treat $R_2 = 0$, Block diagram becomes



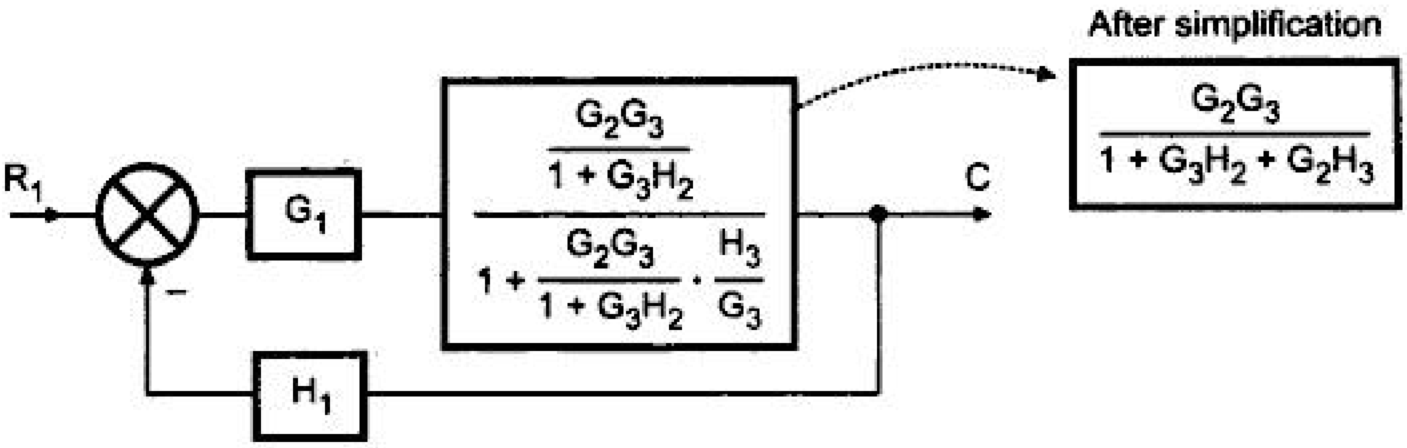
Shift take off point after the block as shown

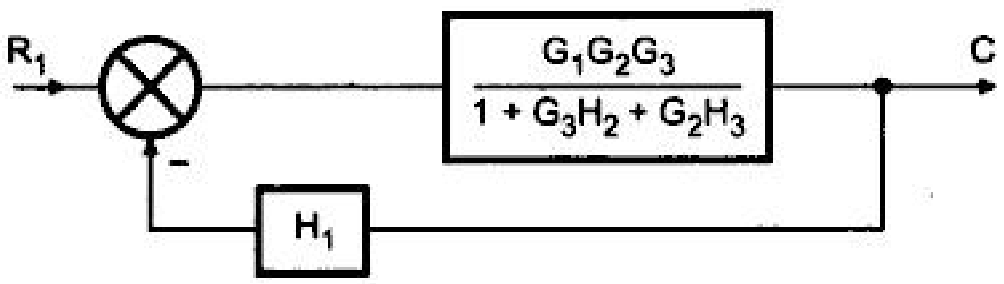


Eliminating minor feedback loop



Eliminating minor feedback loop



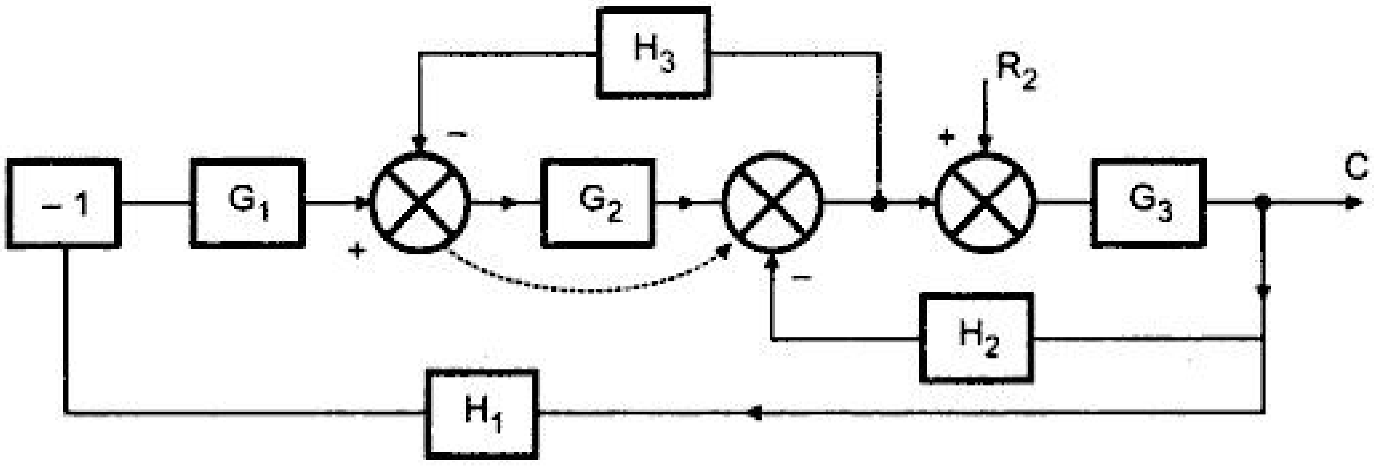


$$\frac{C}{R_1} = \frac{\dfrac{G_1 G_2 G_3}{1+G_3 H_2+G_2 H_3}}{1+\dfrac{G_1 G_2 G_3}{1+G_3 H_2+G_2 H_3}\cdot H_1}$$

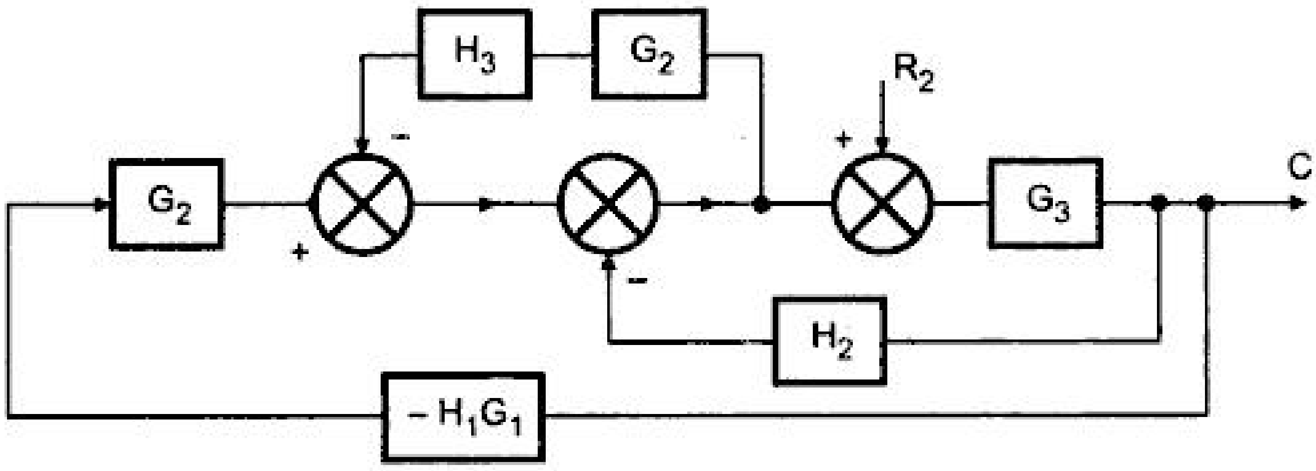
$$= \frac{G_1 G_2 G_3}{1 + G_3 H_2 + G_2 H_3 + G_1 G_2 G_3 H_1}$$

ii) Finding $\frac{C}{R_2}$, treat $R_1 = 0$

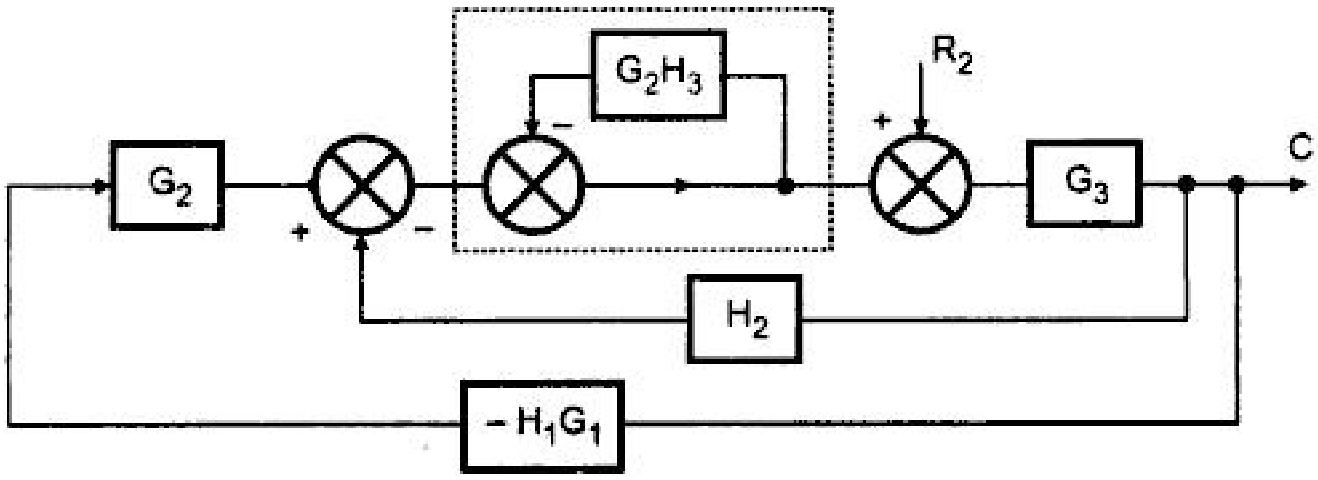
While removing summing point at R_1, consider the negative sign of the signal present at that summing point.



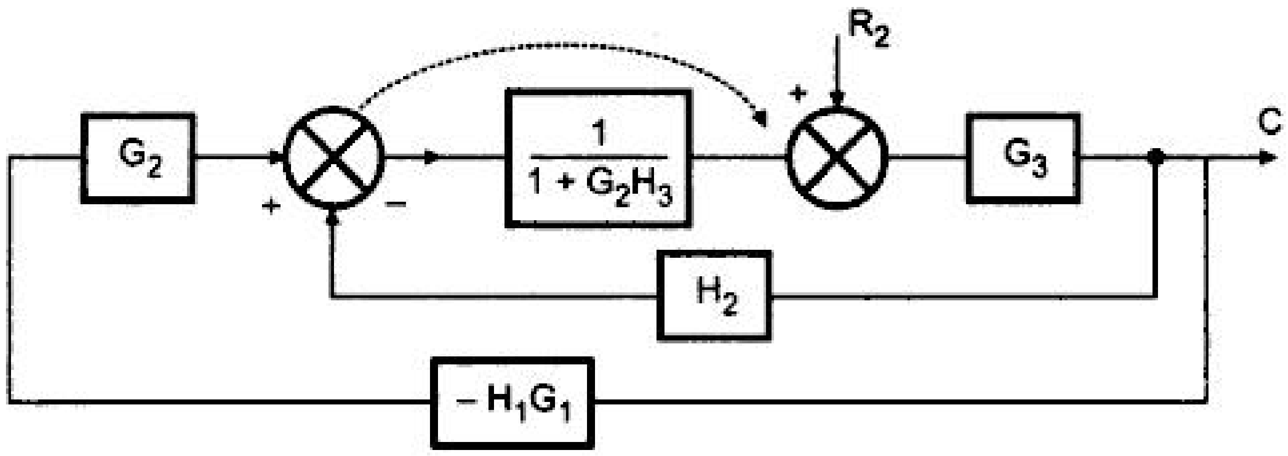
Shifting summing point after the block, and combining all blocks in series.



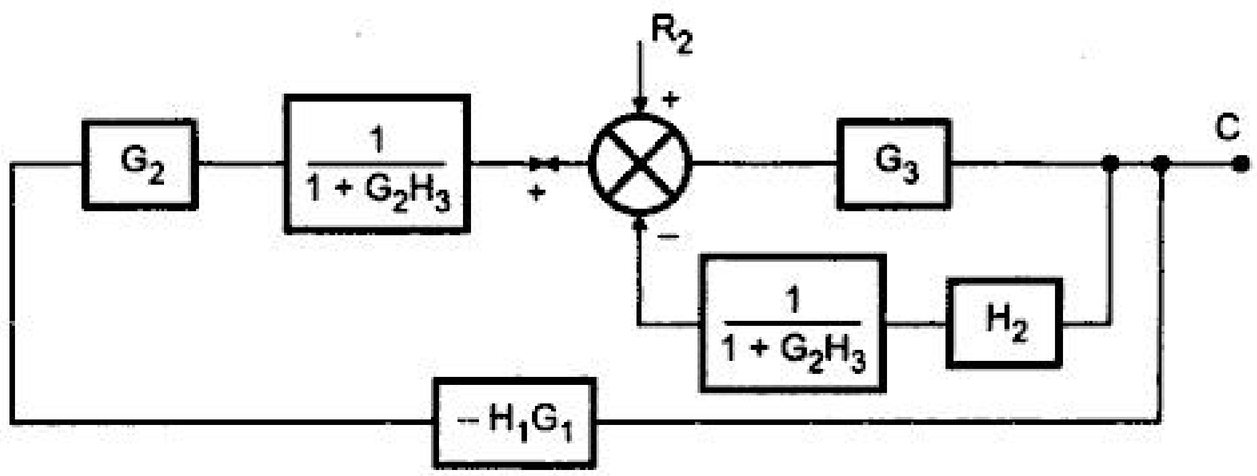
Interchanging the summing points using the associative law.



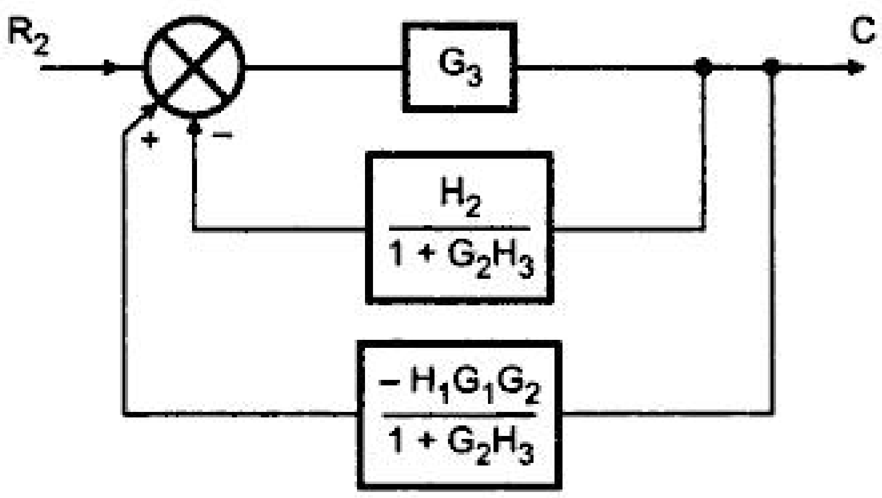
Solving minor feedback loop, shown dotted, we get,



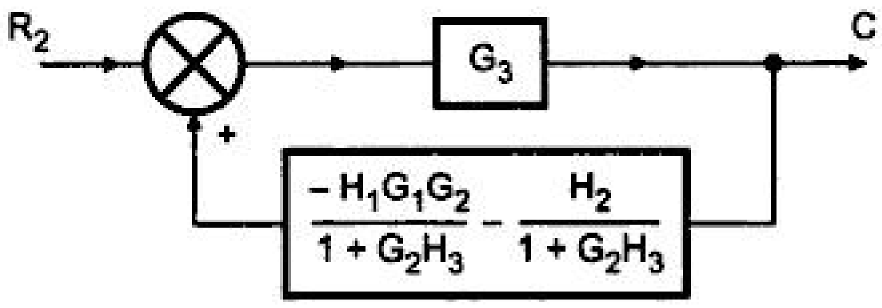
Shifting summing point after the block and combining it with summing point.

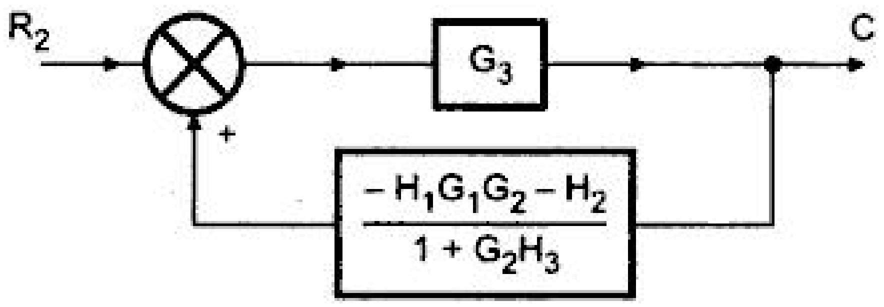


Redrawing the block diagram



Combining blocks in parallel along with signs.

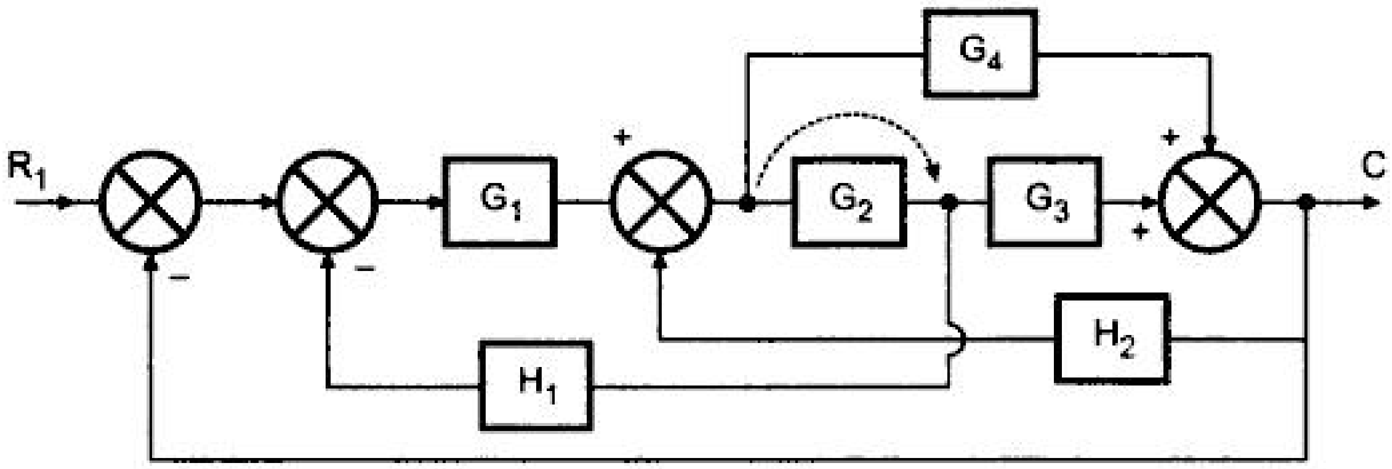




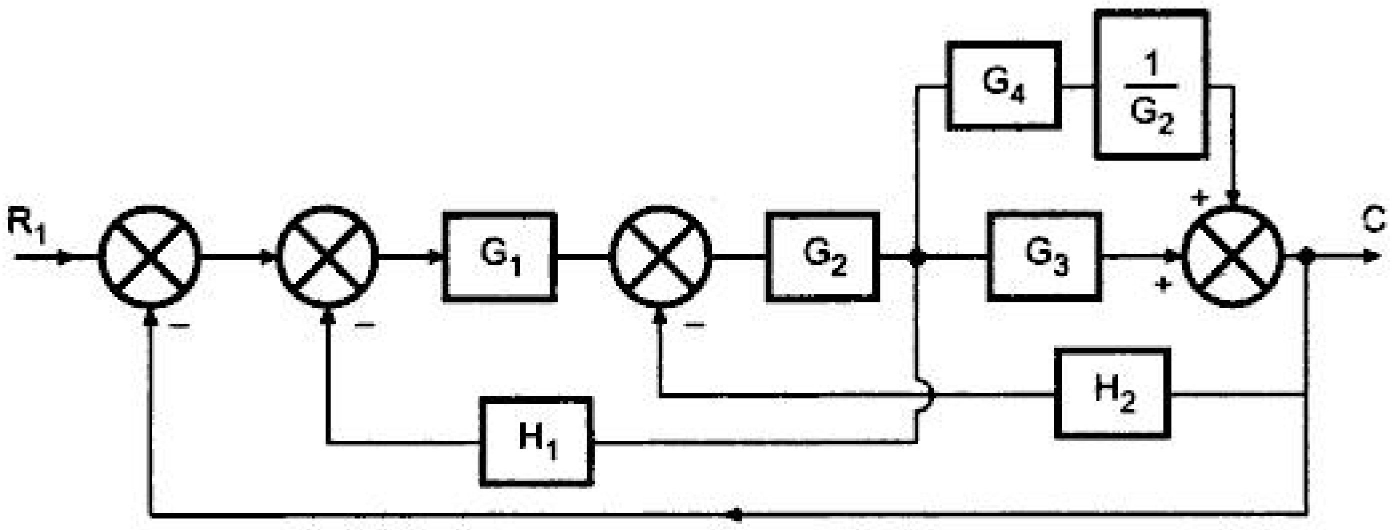
$$\therefore \quad \frac{C}{R_2} = \frac{G_3}{1-\left[\dfrac{-H_1 G_1 G_2 - H_2}{1+G_2 H_3}\right] G_3}$$

$$\frac{C}{R_2} = \frac{G_3(1+G_2 H_3)}{1+G_2 H_3 + G_3 H_2 + G_1 G_2 G_3 H_1}$$

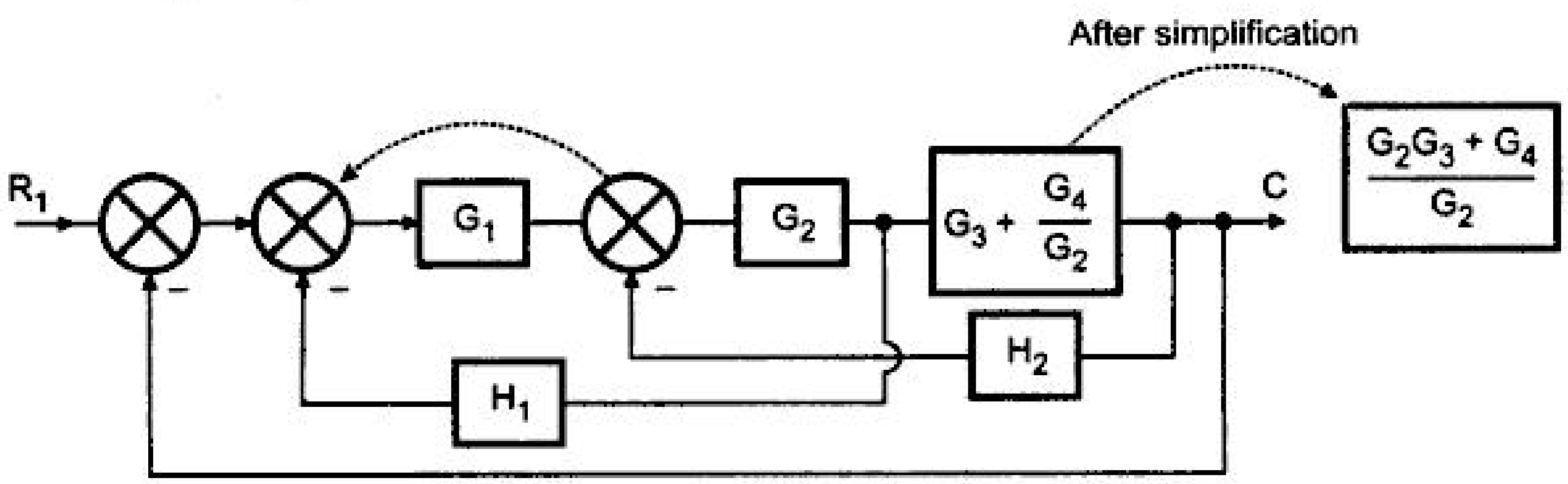
Ex. 3.31 *Determine C/R ratio for the system shown below* **(May 95 Mumbai University)**



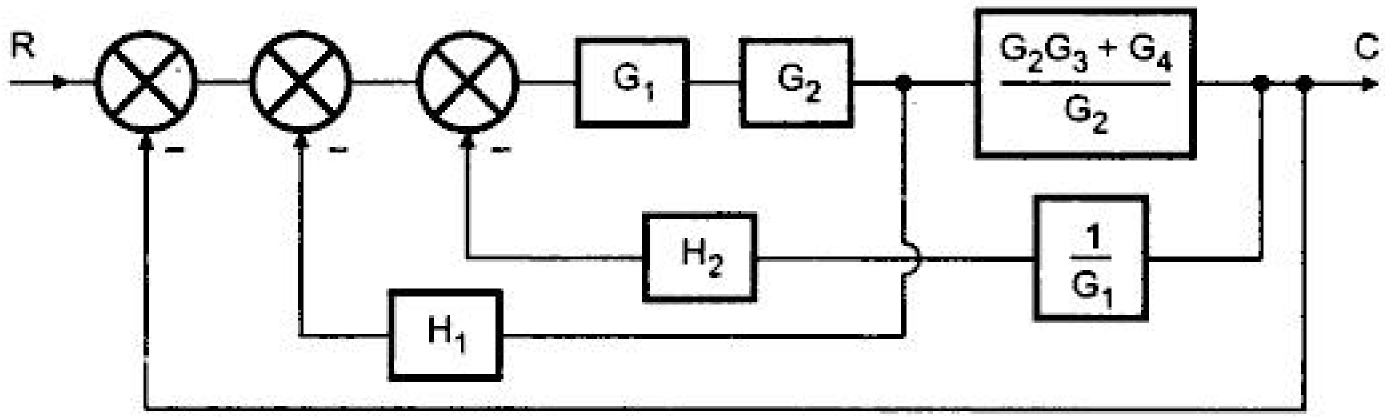
Sol. : Shifting take off point which is before G_2, after G_2.



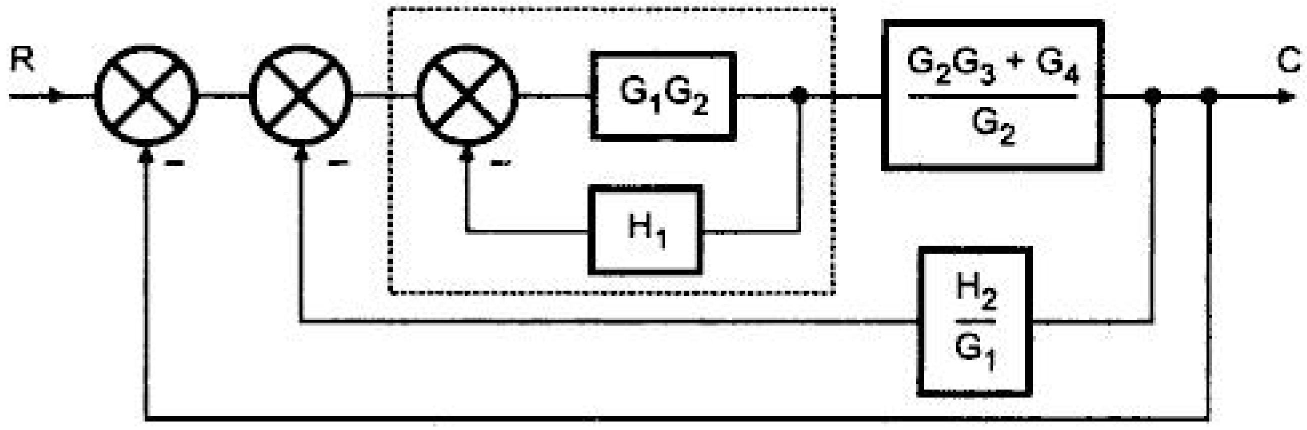
Combining the parallel blocks



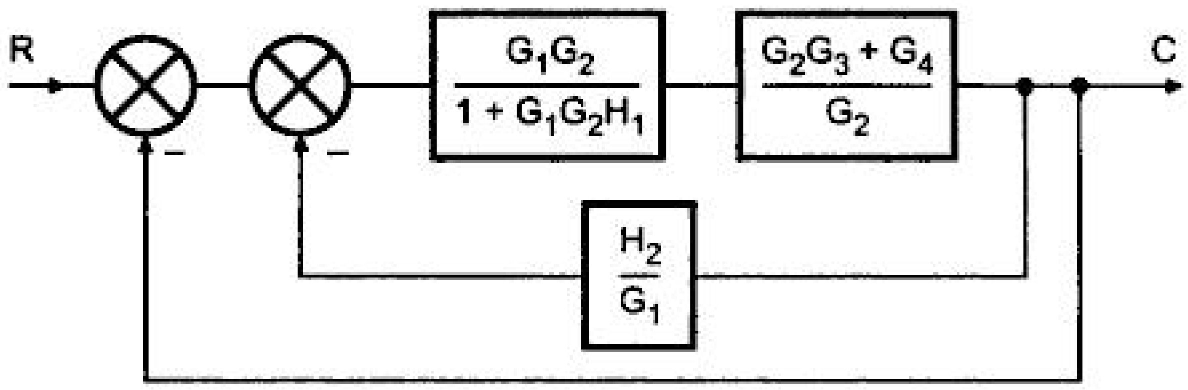
Shifting summing point before the block 'G1'

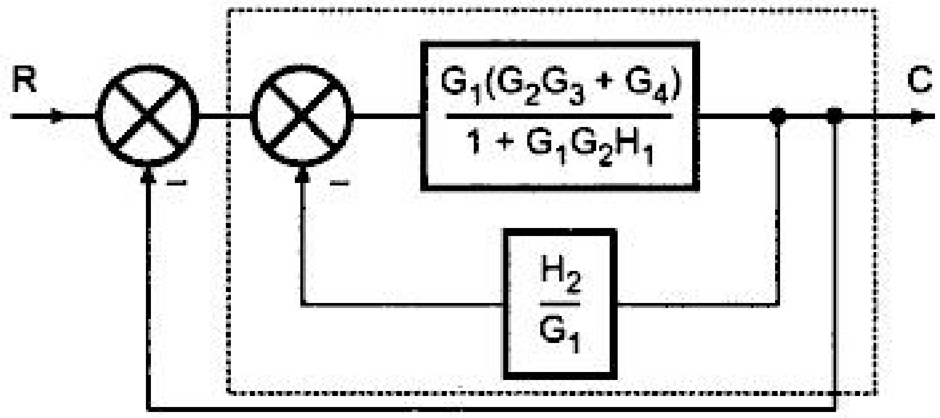


Interchanging the summing points using associative law,

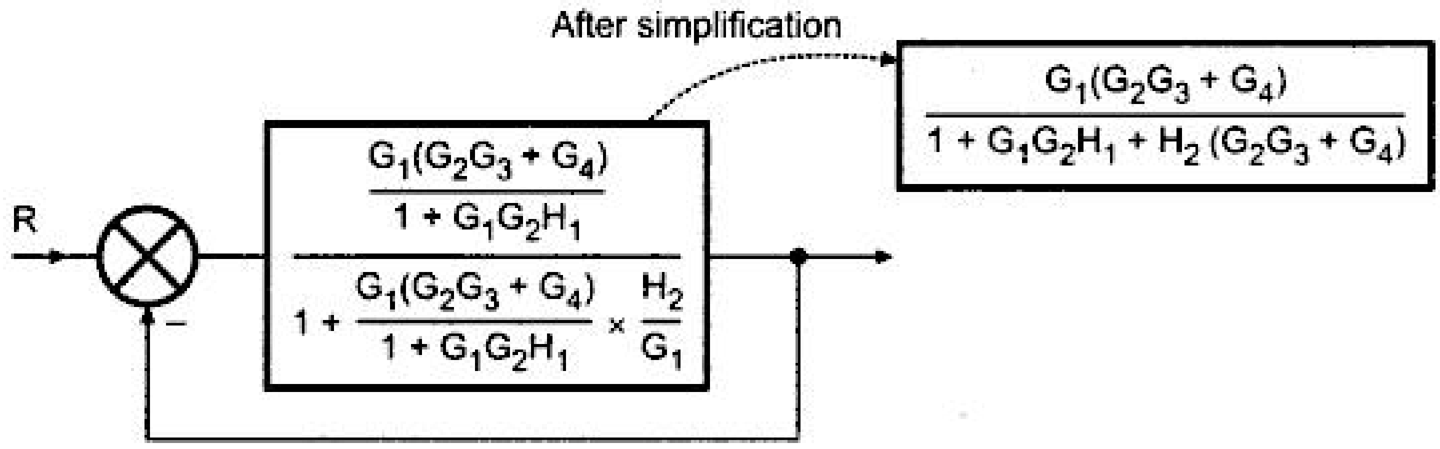


Eliminating minor feedback loop





Eliminating minor feedback loop

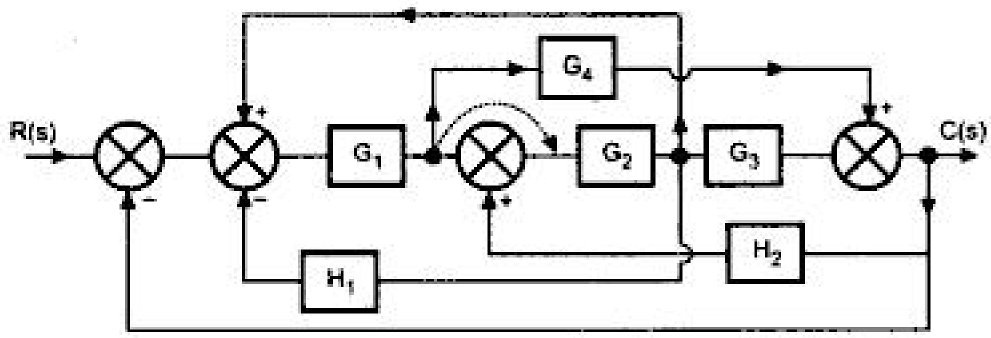


$$\therefore \quad \frac{C}{R} = \frac{\dfrac{G_1(G_2 G_3+G_4)}{1+G_1 G_2 H_1+H_2(G_2 G_3+C_4)}}{1+\dfrac{G_1(G_2 G_3+G_4)}{1+G_1 G_2 H_1+H_2(G_2 G_3+G_4)}}$$

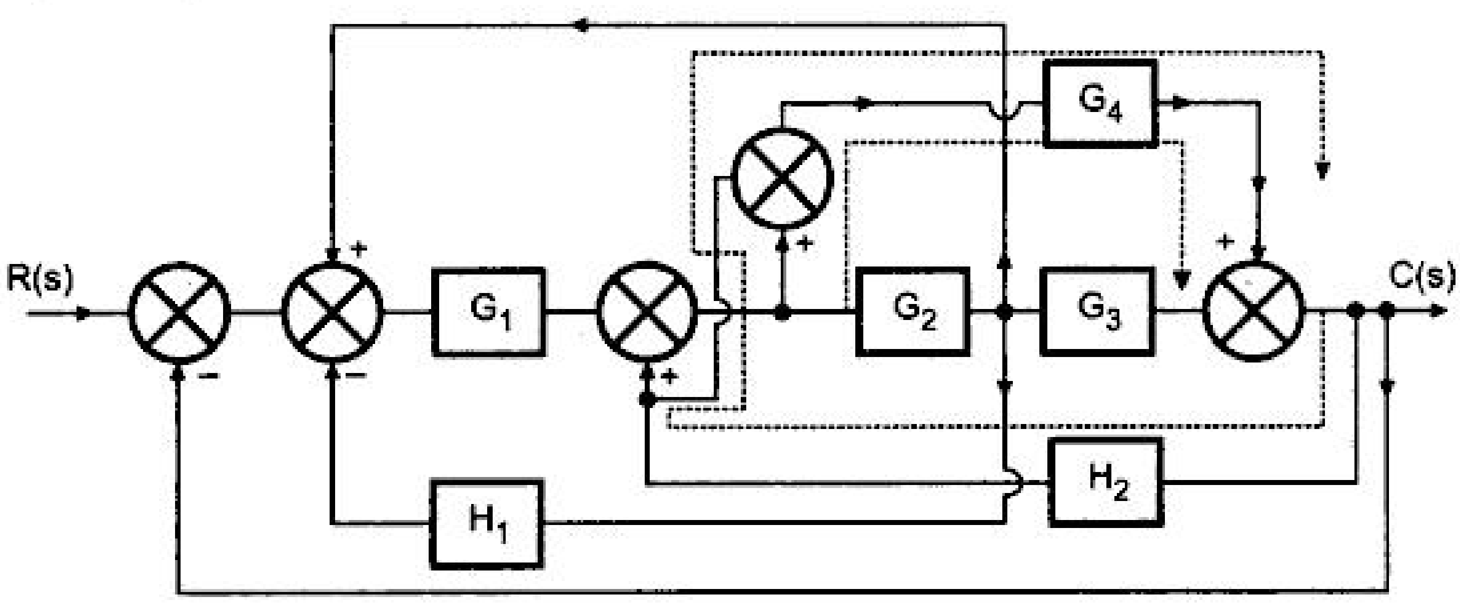
$$\therefore \quad \frac{C}{R} = \frac{G_1(G_2 G_3+G_4)}{1+G_1 G_2 H_1+G_2 G_3 H_2+G_4 H_2+G_1 G_2 G_3+G_1 G_4}$$

Ex. 3.32 *For the block diagram shown, Obtain C(s)/R (s) by using reduction rules.*

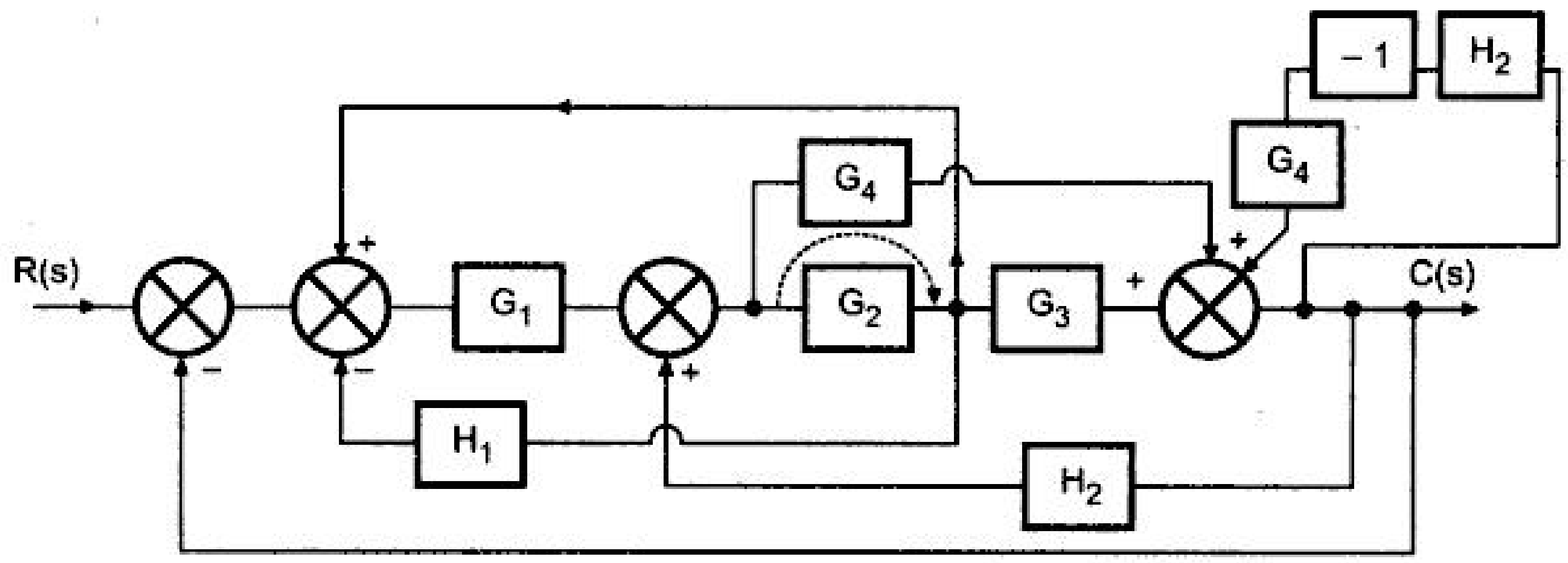
(Mumbai University Jan. 92)



Sol. : Shift the take off point, to the right of summing point. This is the "Critical rule" discussed earlier as rule 10 and 11. In this problem it is necessary to use this rule, which is generally not used to solve simple problems.

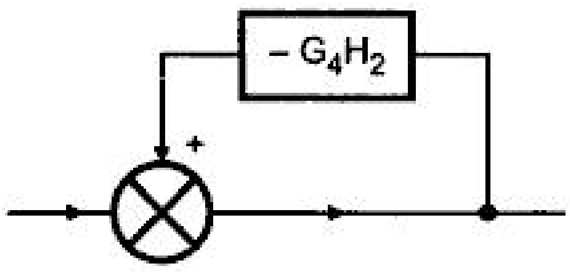


New summing point gets added due to use of critical Rule . This summing point can be eliminated by separating the two paths which are linked by that summing point. The paths are shown dotted. So block diagram reduces as



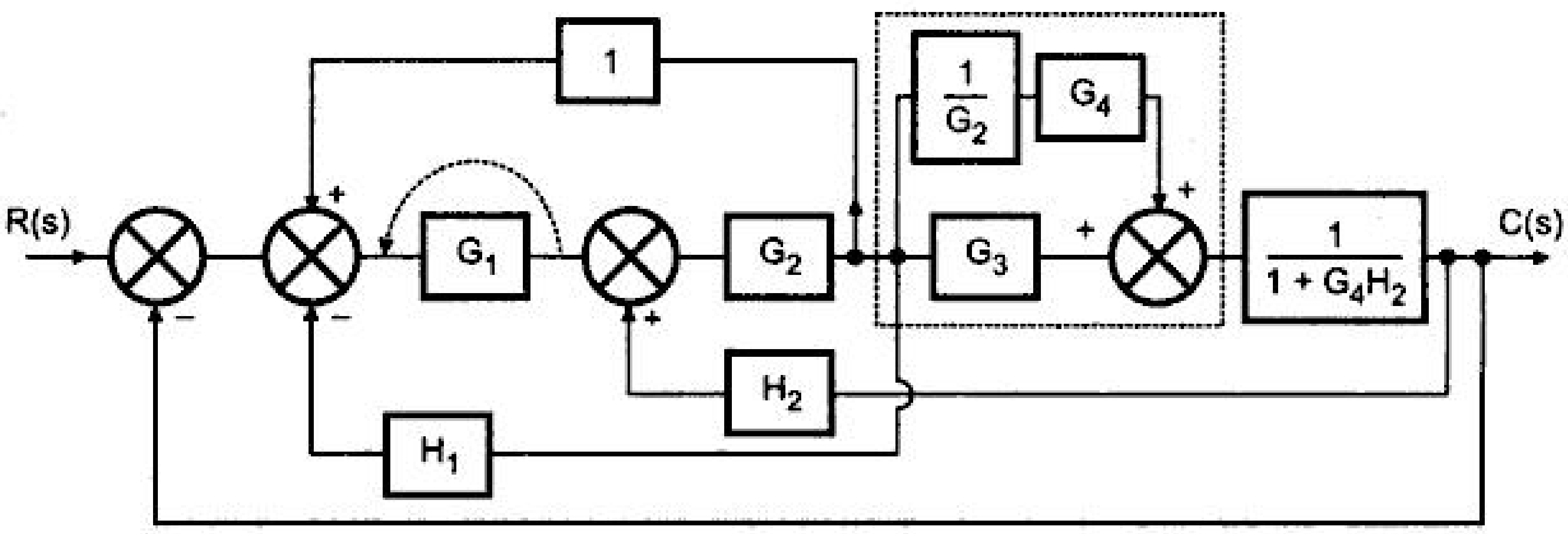
While removing summing point, as sign of one of the signal is negative, the block of transfer function '–1' is connected series with that signal.

So now there exists a minor feedback loop

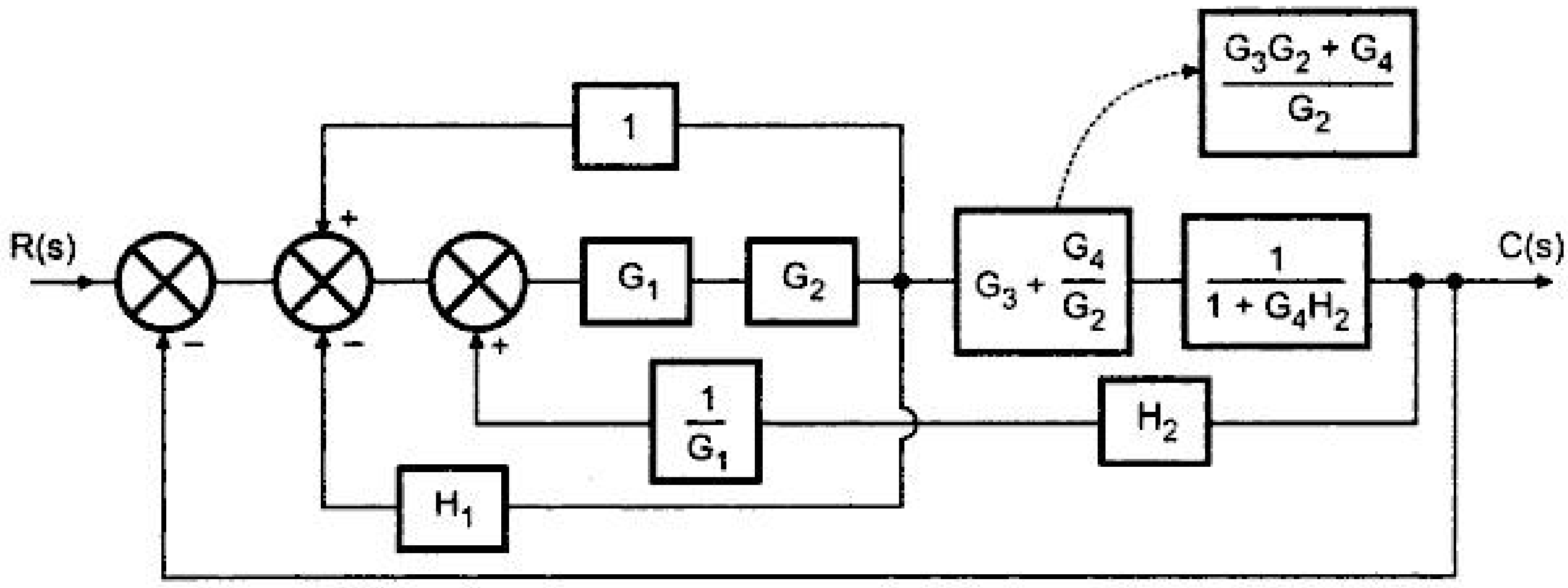


$$\approx \quad \frac{1}{1-(-G_4H_2)} = \frac{1}{1+G_4H_2}$$

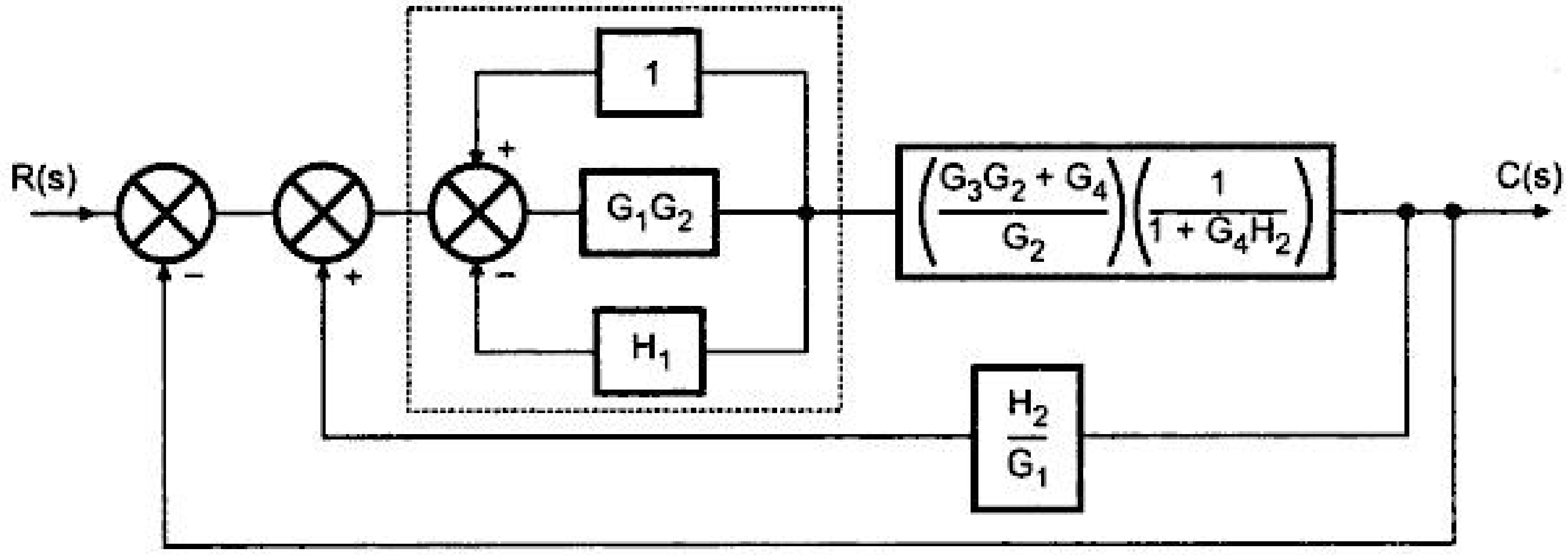
Shifting take off to the right of G_2



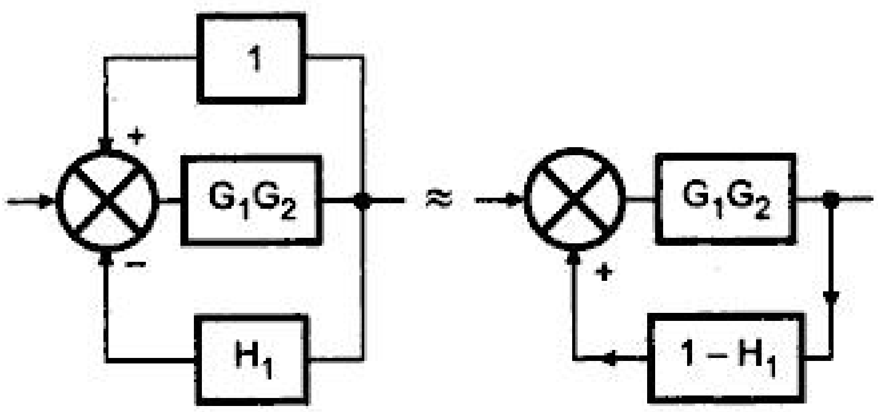
Combining blocks in parallel and shifting summing point to the left of 'G_1'.



Interchanging the summing points using associative law.

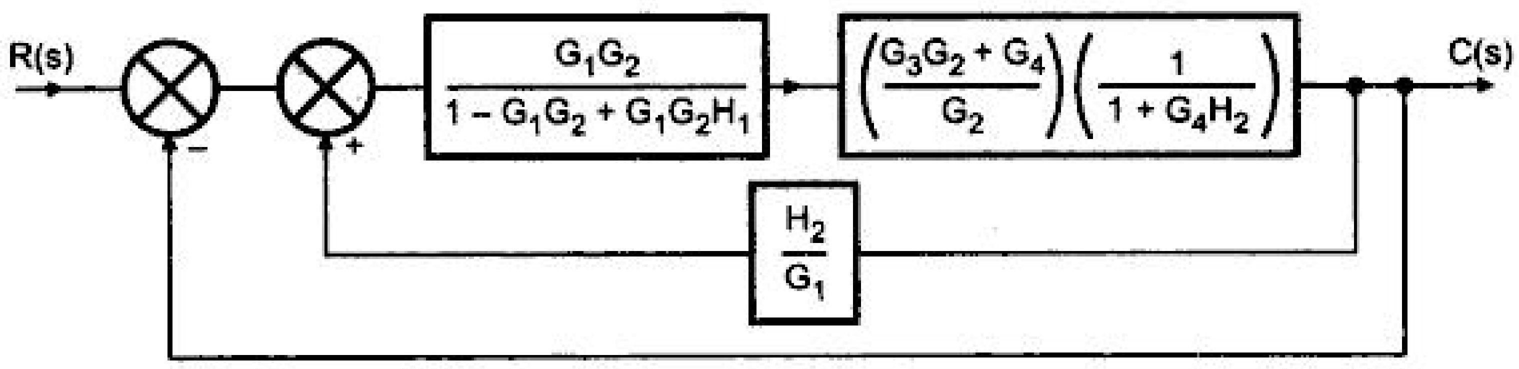


Solving minor feedback loop. The block of '1' and 'H_1' are in parallel so loop becomes

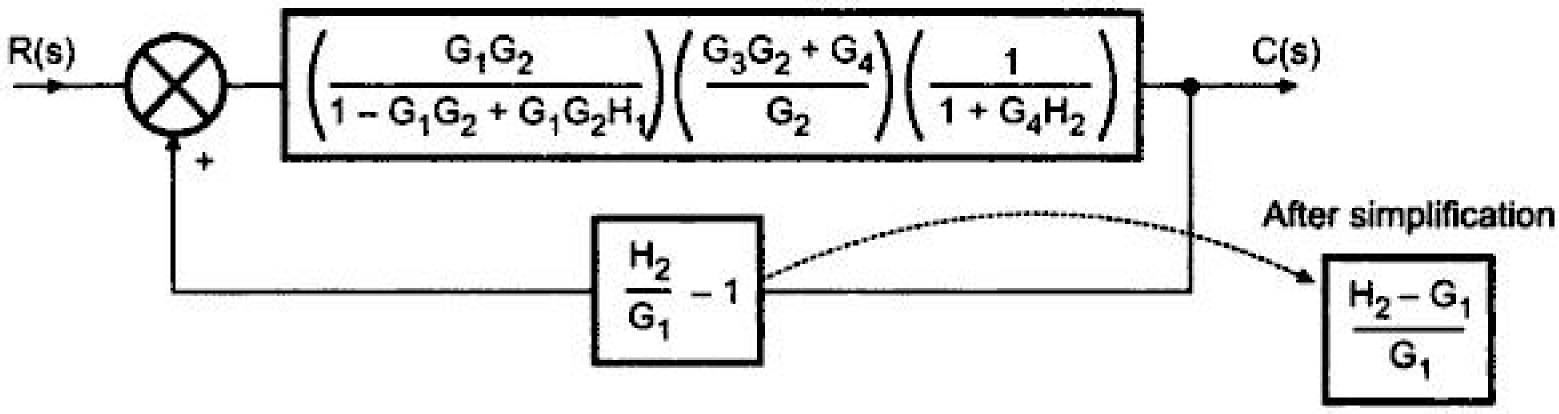


$$\approx \frac{G_1G_2}{1-(1-H_1)G_1G_2} = \frac{G_1G_2}{1-G_1G_2+G_1G_2H_1}$$

$$= \frac{G_1 G_2}{1-(1-H_1)G_1 G_2} = \frac{G_1 G_2}{1-G_1 G_2+G_1 G_2 H_1}$$



Combining the feedback blocks which are in parallel



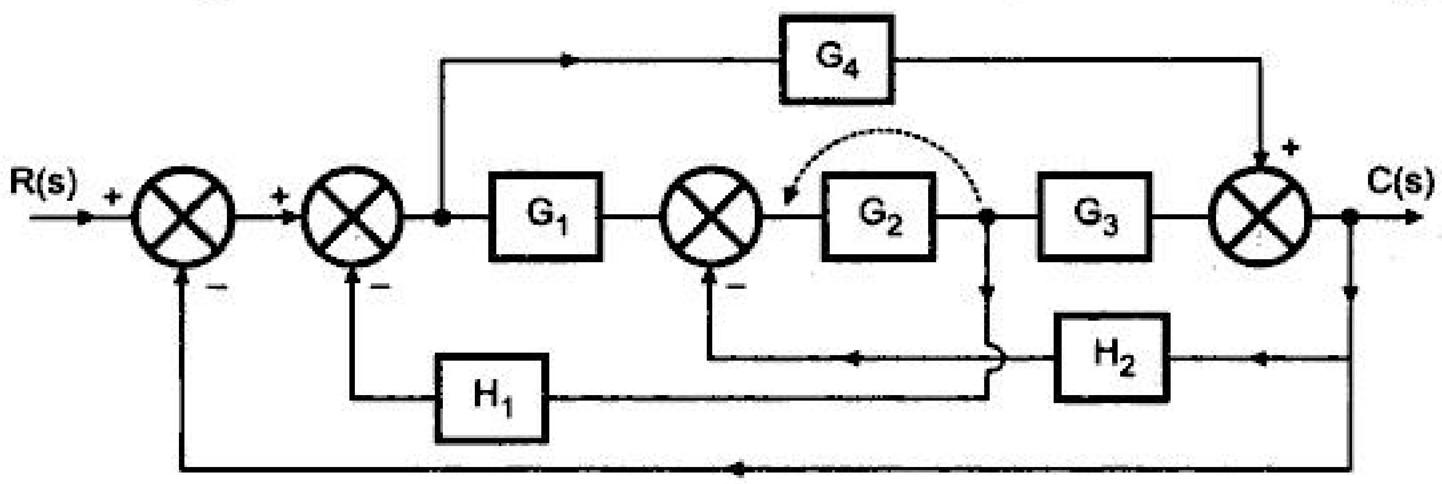
$$\therefore \quad \frac{C(s)}{R(s)} = \frac{\left(\dfrac{G_1 G_2}{1-G_1 G_2+G_1 G_2 H_1}\right)\left(\dfrac{G_3 G_2+G_4}{G_2}\right)\left(\dfrac{1}{1+G_4 H_2}\right)}{1-\left(\dfrac{G_1 G_2}{1-G_1 G_2+G_1 G_2 H_1}\right)\left(\dfrac{G_3 G_2+G_4}{G_2}\right)\left(\dfrac{1}{1+G_4 H_2}\right)\left(\dfrac{H_2-G_1}{G_1}\right)}$$

$$\frac{C(s)}{R(s)} = \frac{G_1(G_3 G_2+G_4)}{(1-G_1 G_2+G_1 G_2 H_1)(1+G_4 H_2)-(G_3 G_2+G_4)(H_2-G_1)}$$

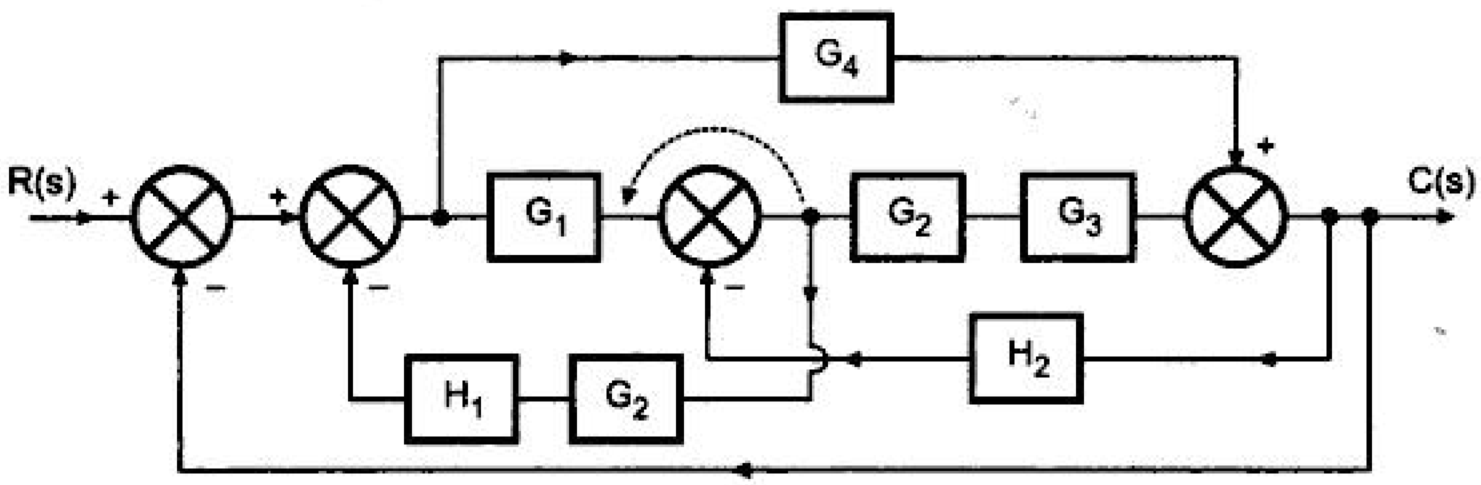
$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3+G_1G_4}{1-G_1G_2+G_1G_2H_1+G_4H_2-G_1G_2G_4H_2+G_1G_2G_4H_1H_2+G_1G_2G_3+G_1G_4-G_2G_3H_2-G_4H_2}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3+G_1G_4}{1-G_1G_2+G_1G_2H_1-G_1G_2G_4H_2+G_1G_2G_4H_1H_2+G_1G_2G_3+G_1G_4-G_2G_3H_2}$$

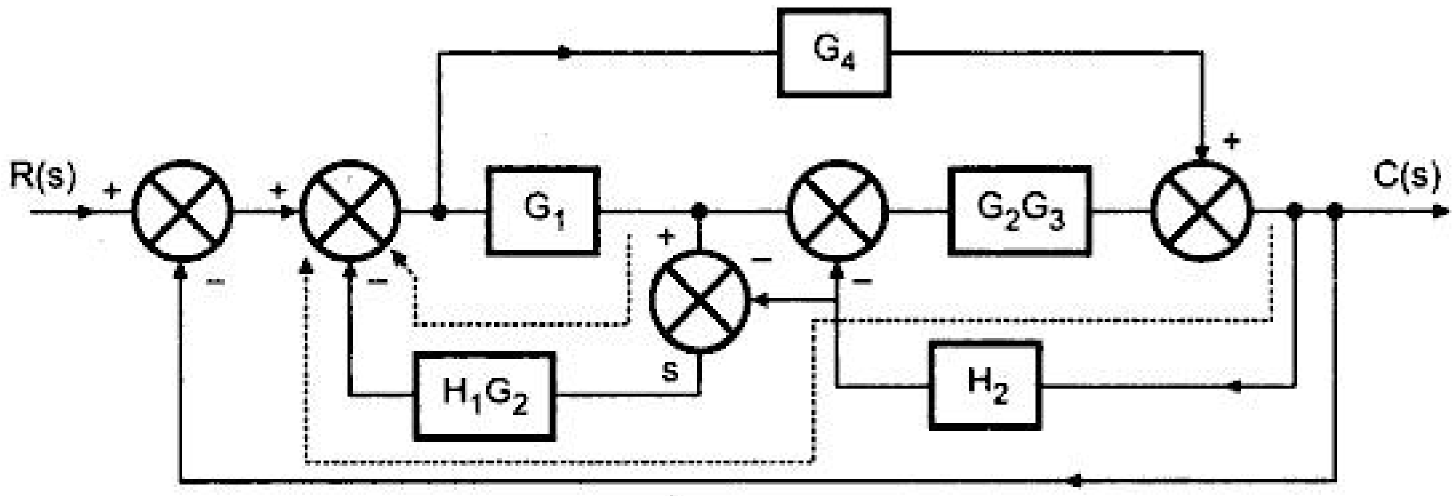
Ex. 3.33 *Use block diagram reduction rules to obtain the transfer function of the block diagram shown below.* **(Mumbai University, July-91)**



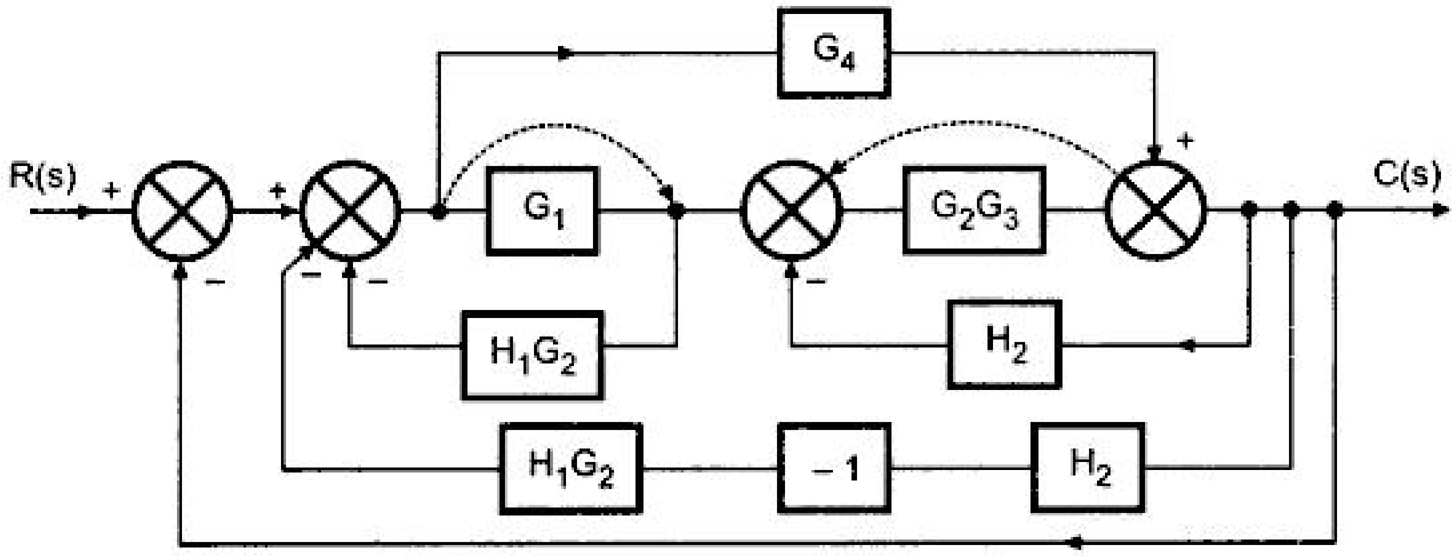
Sol. : Shifting take off point to the left of 'G_2'.



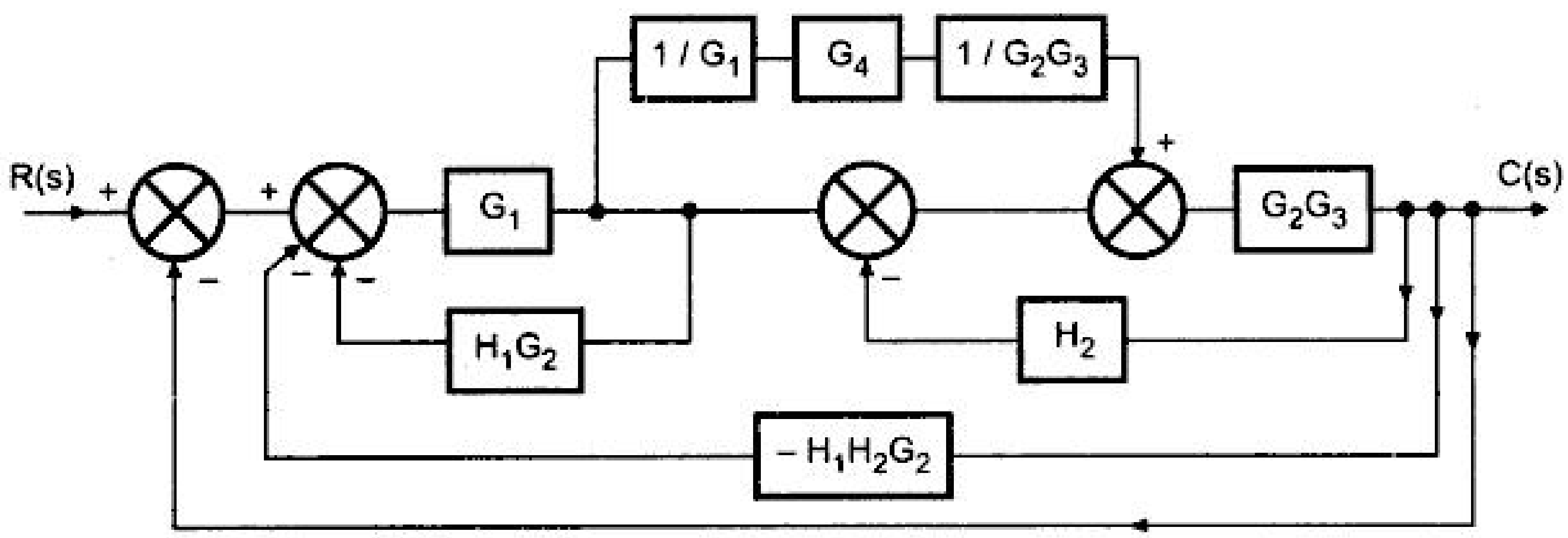
Shifting take off point before the summing point, this is critical rule, (rule 10 and 11), due to this new summing point 'S' gets added in the diagram as shown below.



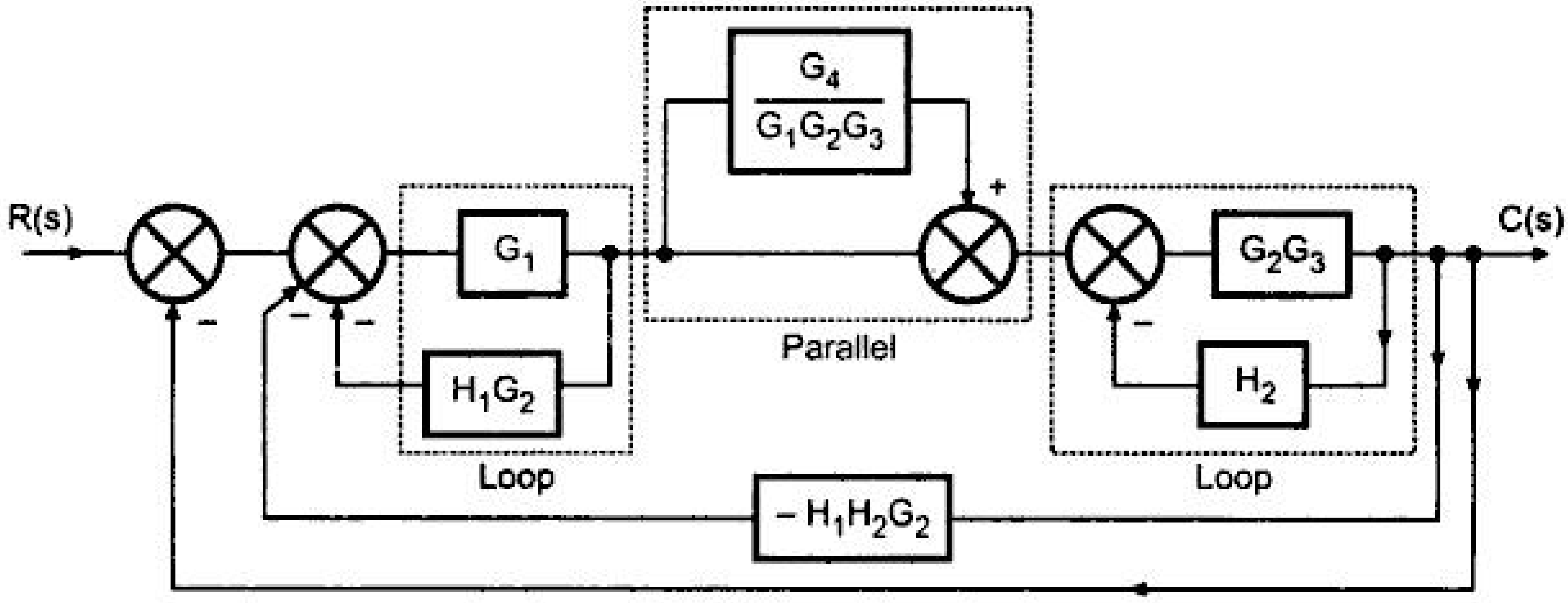
New summing point 's'



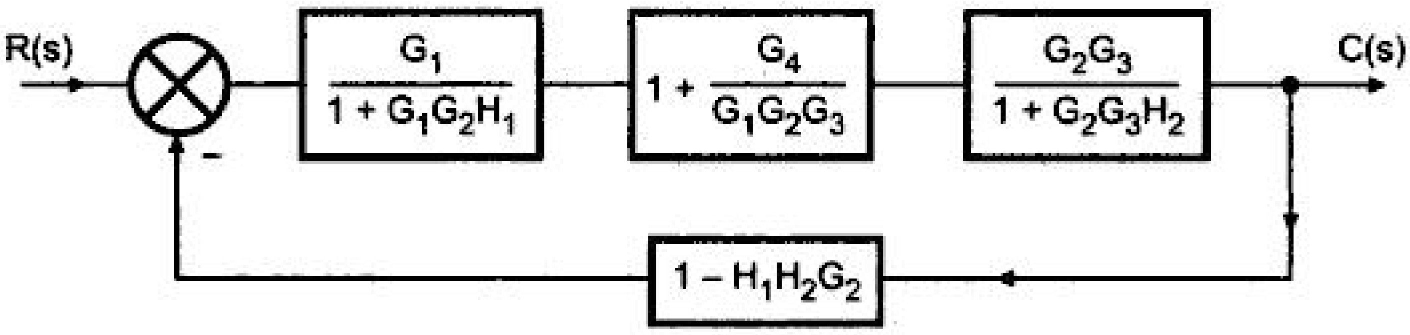
As 'S' is removed, the negative of signal at that point is considered by adding a block of '–1' in series with that signal. Shifting take off after 'G_1' and summing before '$G_2\ G_3$' simultaneously.



Interchanging the summing points we get two minor feedback loops and one combination of parallel blocks.



Combining two parallel feedback paths namely '1' and $-H_1 H_2 G_2$



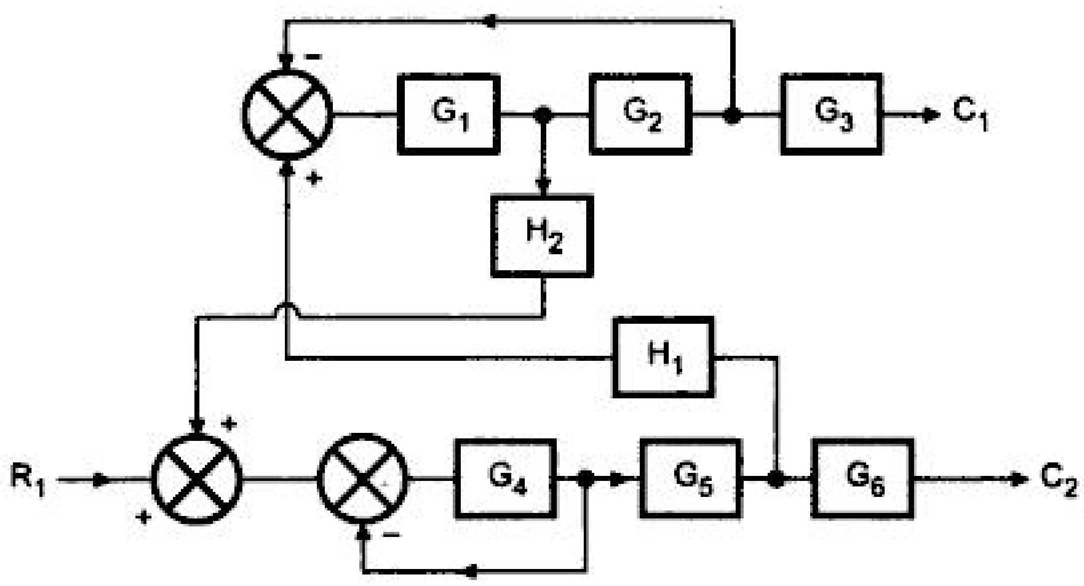
$$\therefore \quad \frac{C(s)}{R(s)} = \frac{\left(\dfrac{G_1}{1+G_1 G_2 H_1}\right)\left(\dfrac{G_1 G_2 G_3 + G_4}{G_1 G_2 G_3}\right)\left(\dfrac{G_2 G_3}{1+G_2 G_3 H_2}\right)}{1+\left(\dfrac{G_1}{1+G_1 G_2 H_1}\right)\left(\dfrac{G_1 G_2 G_3 + G_4}{G_1 G_2 G_3}\right)\left(\dfrac{G_2 G_3}{1+G_2 G_3 H_2}\,(1-H_1 H_2 G_2)\right)}$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 + G_4}{1+G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2^2 G_3 H_1 H_2 + G_1 G_2 G_3 + G_4 - G_1 G_2^2 G_3 H_1 H_2 - H_1 H_2 G_2 G_4}$$

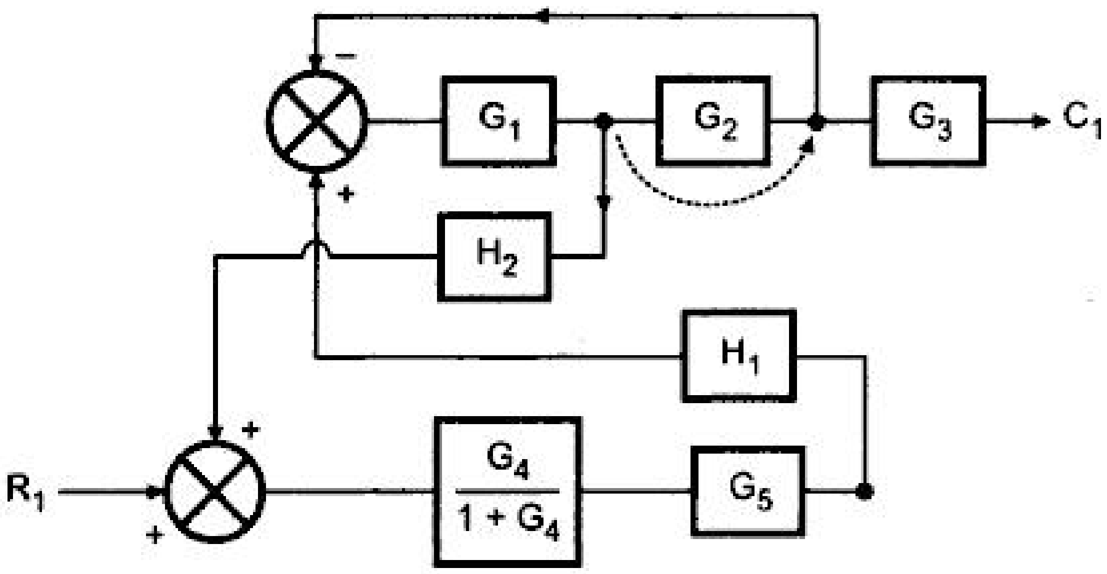
$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 + G_4}{1+G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 + G_4 - H_1 H_2 G_2 G_4}$$

Ex. 3.34 *In the given block diagram, obtain the transfer function of the system* C_1/R_1

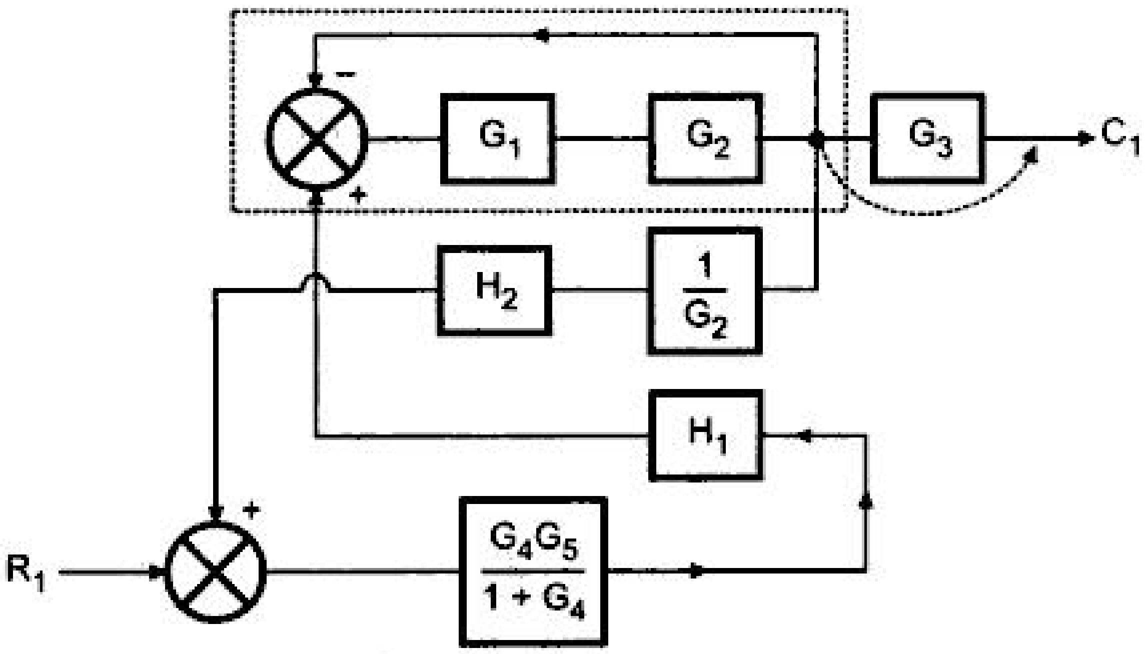
(Mumbai University, Nov.-96)



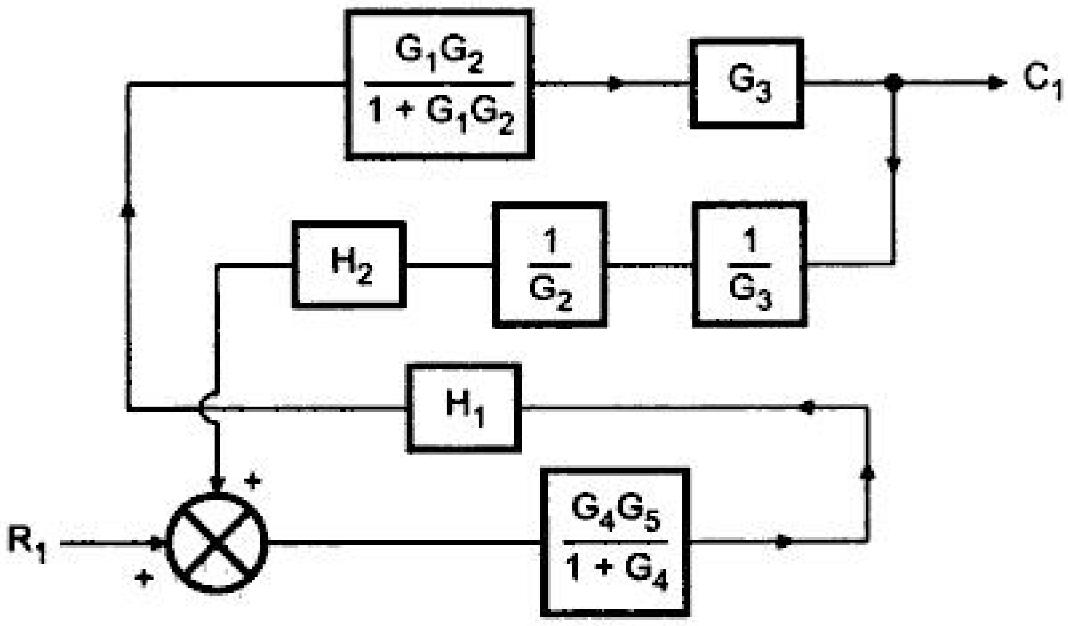
Sol. : Solving the minor feedback loop of 'G_4' and '1'.



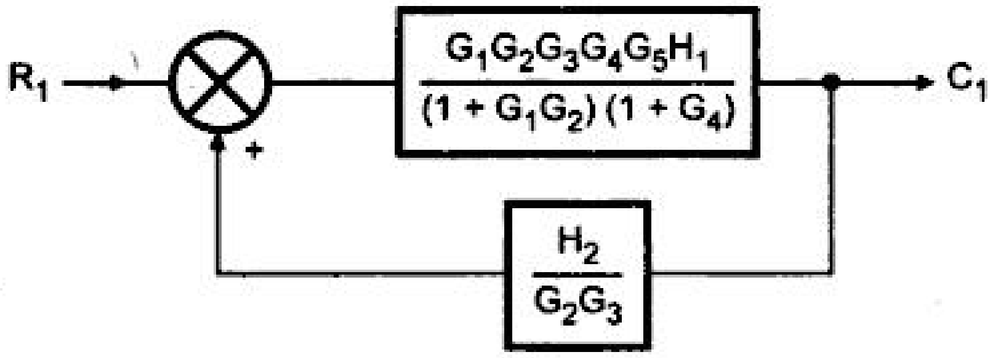
As C_2 is not the focus of interest and hence G_6 becomes meaningless in the block diagram shifting take off point to the right.



Solving minor feedback loop and shifting take off point.



Combining all the blocks in series.



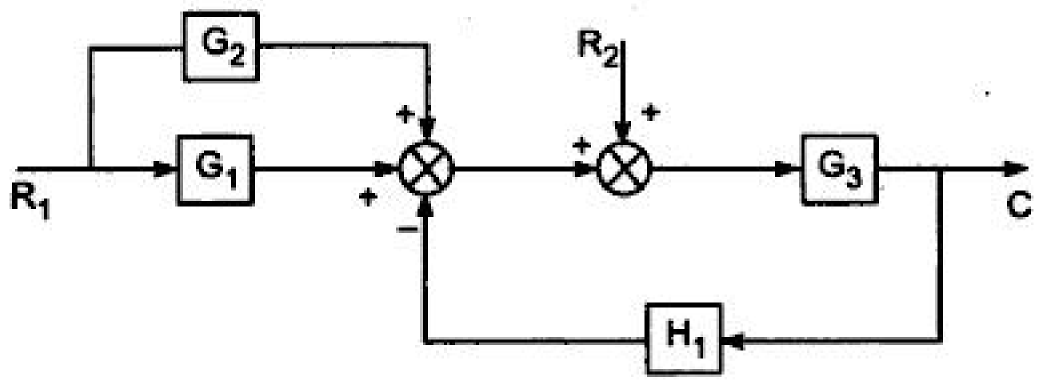
$$\therefore \quad \frac{C_1}{R_1} = \frac{\dfrac{G_1 G_2 G_3 G_4 G_5 H_1}{(1+G_1 G_2)(1+G_4)}}{1 - \dfrac{G_1 G_2 G_3 G_4 G_5 H_1}{(1+G_1 G_2)\ (1+G_4)} \cdot \dfrac{H_2}{G_2 G_3}}$$

$$\therefore \quad \frac{C_1}{R_1} = \frac{G_1 G_2 G_3 G_4 G_5 H_1}{(1+G_1 G_2)(1+G_4) - G_1 G_4 G_5 H_1 H_2}$$

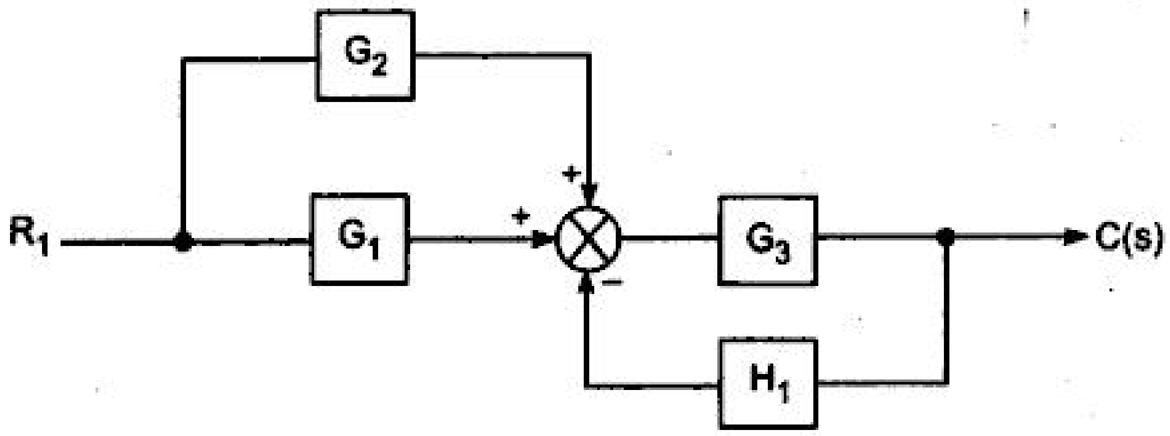
$$\therefore \quad \frac{C_1}{R_1} = \frac{G_1 G_2 G_3 G_4 G_5 H_1}{1+G_1 G_2 + G_4 + G_1 G_2 G_4 - G_1 G_4 G_5 H_1 H_2}$$

Ex. 3.35 *Determine the transfer functions C/R_1 and C/R_2 from the block diagram shown in figure using block diagram reduction techniques.*

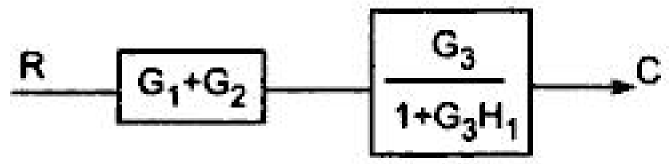
(Mumbai University, May-99)



Ans : For finding C/R_1, assume $R_2 = 0$ hence the block diagram reduces to,

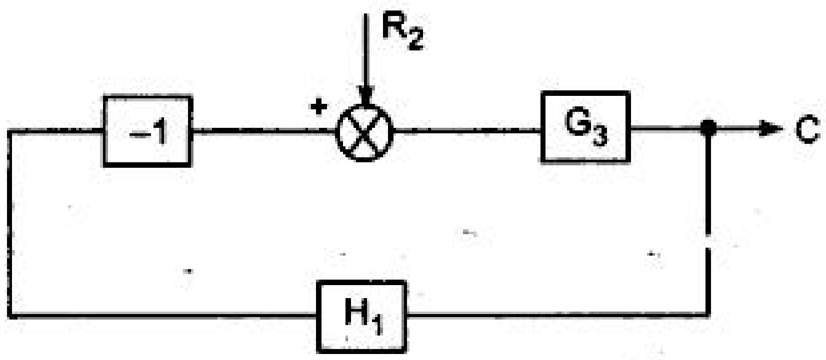


G_1 and G_2 are in parallel while G_3, H, form a minor loop, hence we get

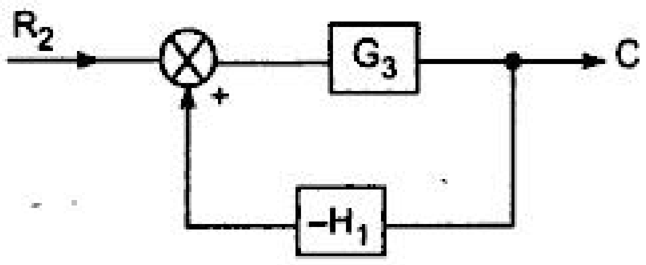


$$\therefore \qquad \frac{C}{R_1} = \frac{(G_1+G_2)G_3}{1+G_3 H_1}$$

To find C/R_2, assume $R_1 = 0$ hence G_1, G_2 will vanish but negative sign of H_1 will continue. So we get,



Redrawing the block diagram,

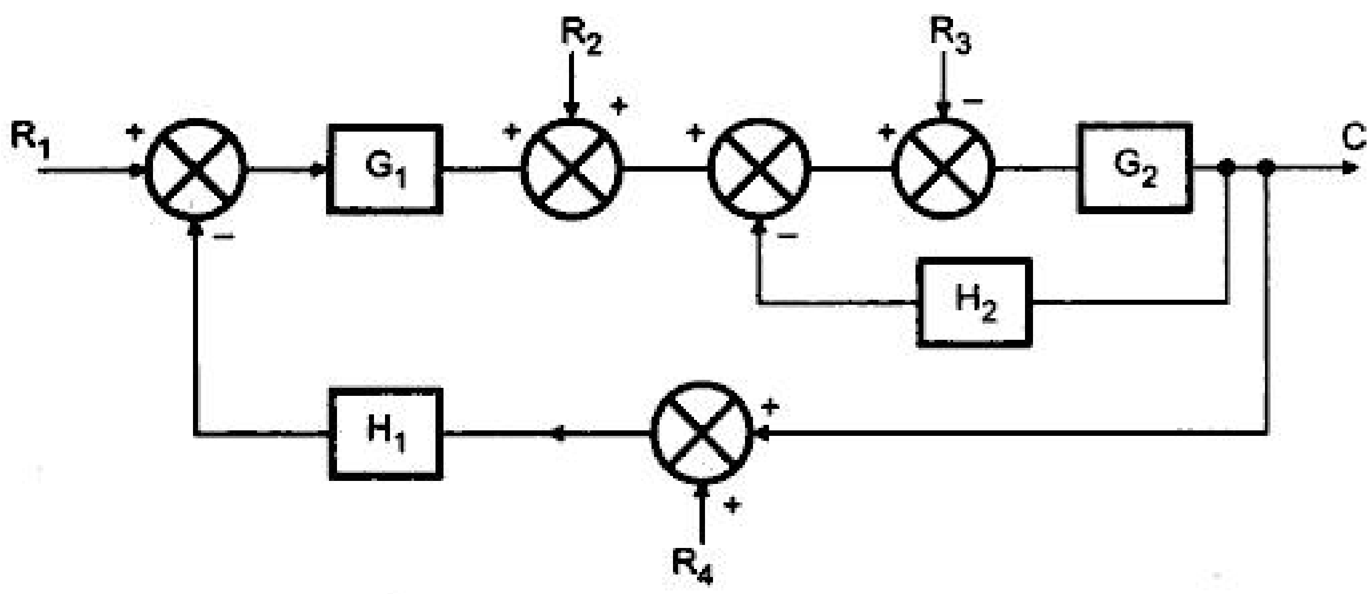


$$\therefore \qquad \frac{C}{R_2} = \frac{G_3}{1-G_3(-H_1)}$$

$$\therefore \qquad \frac{C}{R_2} = \frac{G_3}{1+G_3 H_1}$$

Ex. 3.36 *Find C using - Block diagram reduction techniques*

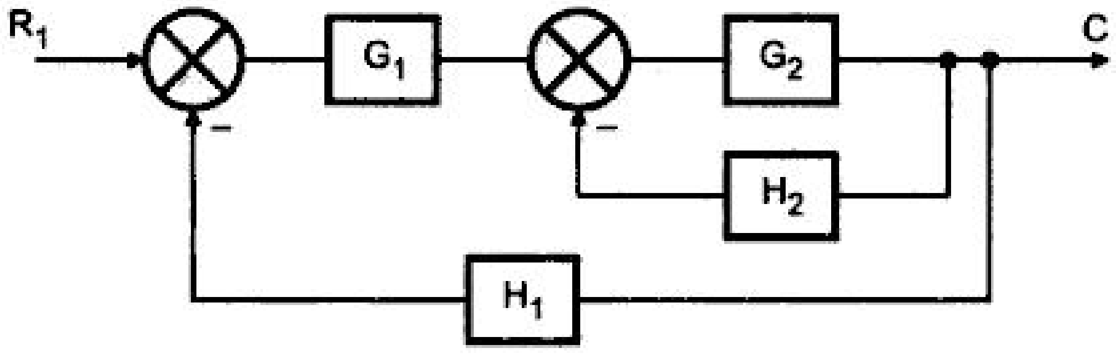
(Mumbai University, May-98)

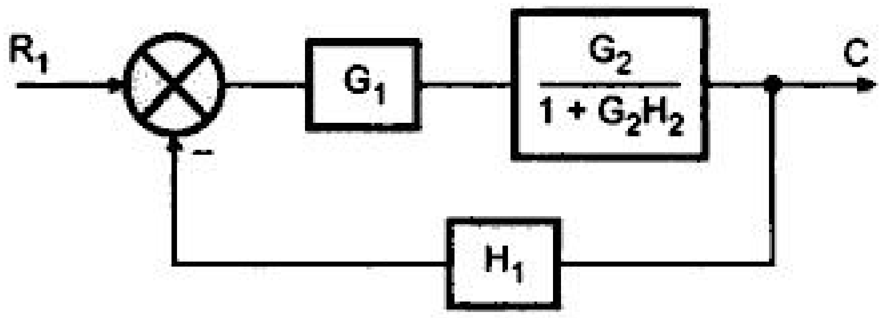


Ans. : By block diagram reduction.

Consider R_1 alone R_2, R_3, R_4 are zero.

Note : Whenever R_1 is zero, as the sign of feedback at R_1 is negative, while removing summing point at R_1, do not forget to insert a block of '−1' to consider effect of negative sign.

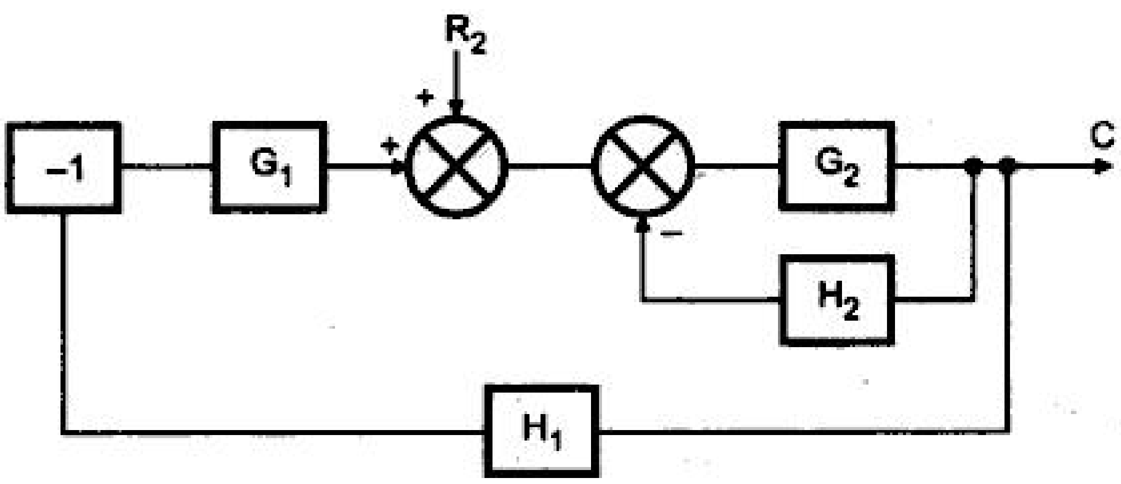




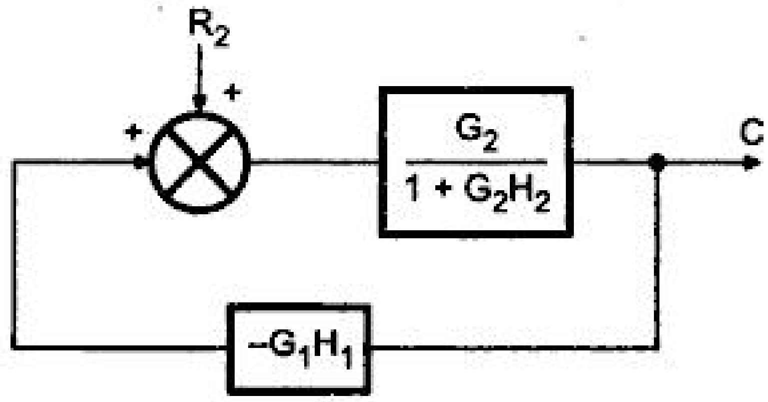
$$\therefore \quad \frac{C}{R_1} = \frac{\dfrac{G_1G_2}{1+G_2H_2}}{1+\dfrac{G_1G_2H_1}{1+G_2H_2}} = \frac{G_1G_2}{1+G_2H_2+G_1G_2H_1}$$

$$\therefore \quad C = \frac{G_1G_2R_1}{1+G_2H_2+G_1G_2H_1} \quad \text{... due to } R_1$$

Consider R_2 alone, with $R_3 = R_1 = R_4 = 0$



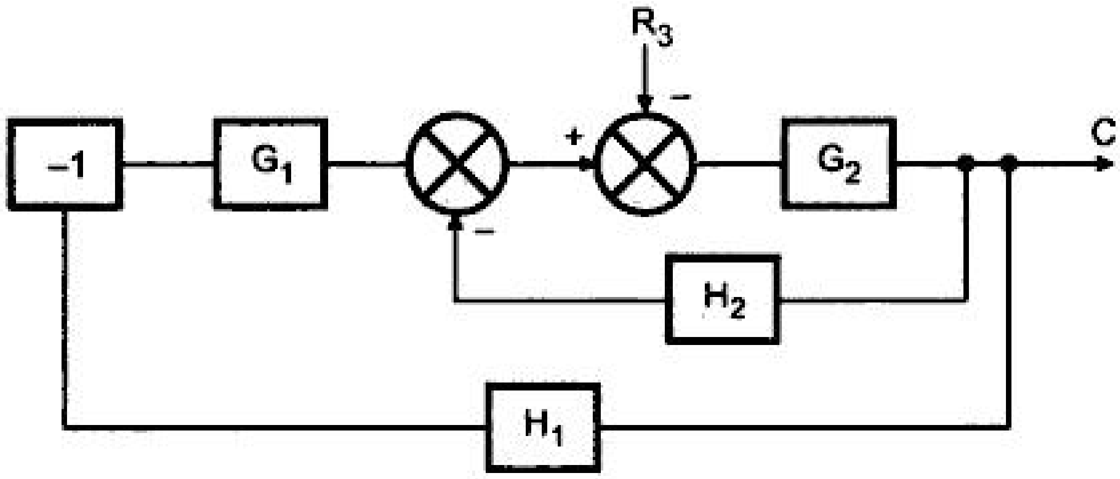
$$\therefore \quad \frac{C}{R_2} = \frac{\dfrac{G_2}{1+G_2H_2}}{1-\left(\dfrac{G_2}{1+G_2H_2}\right)(-G_1H_1)}$$



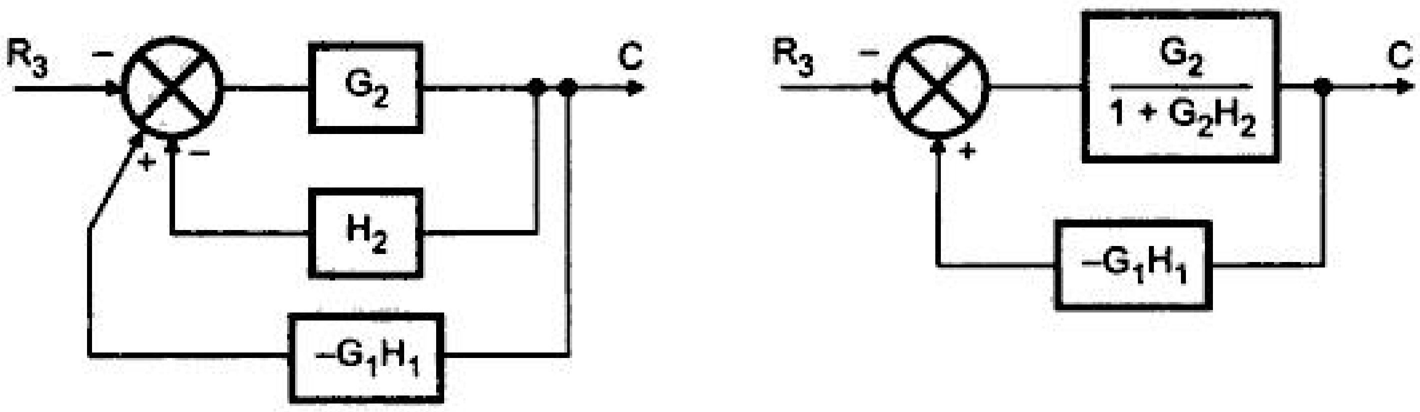
$$= \frac{G_2}{1+G_2H_2+G_1G_2H_1}$$

$$\therefore \qquad C = \frac{G_2R_2}{1+G_2H_2+G_1G_2H_1}$$

Consider R_3 alone, $R_1 = R_2 = R_4 = 0$



Combining two summing points we get,

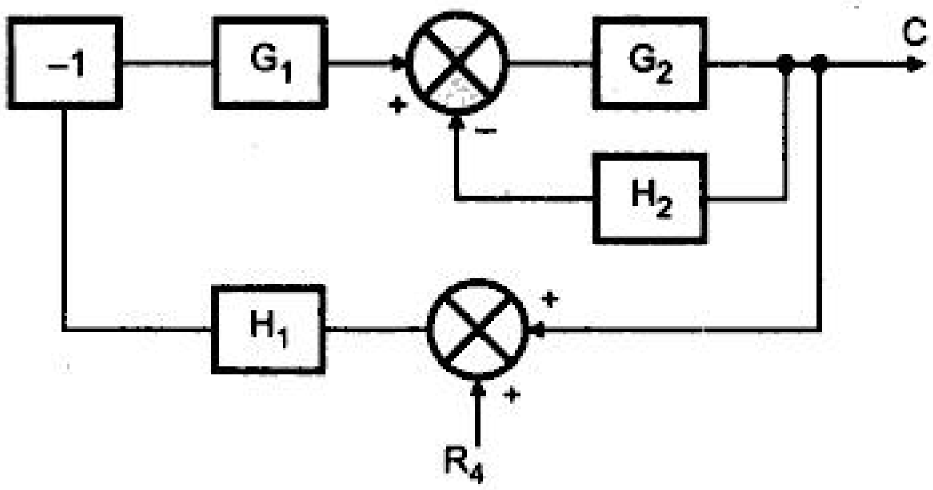


$$\therefore \qquad \frac{C}{-R_3} = \frac{\dfrac{G_2}{1+G_2H_2}}{1-\left(\dfrac{G_2}{1+G_2H_2}\right)(-G_1H_1)}$$

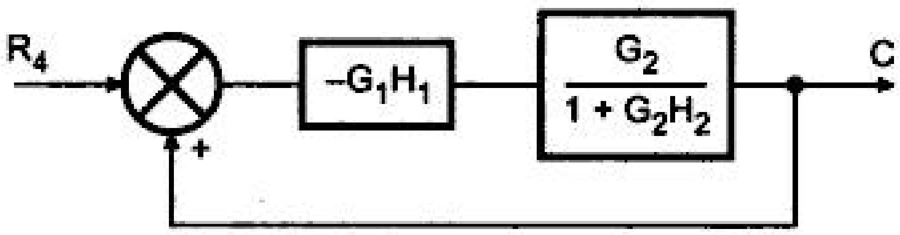
$$= \frac{G_2}{1+G_2H_2+G_1G_2H_1}$$

$$\therefore \qquad C = \frac{-R_3G_2}{1+G_2H_2+G_1G_2H_1}$$

Consider R_1 alone, with $R_1 = R_2 = R_3 = 0$



Solving minor feedback loop and rearranging



$$\therefore \quad \frac{C}{R_4} = \frac{\dfrac{-G_1G_2H_1}{1+G_2H_2}}{1-(-G_1H_1)\left(\dfrac{G_2}{1+G_2H_2}\right)}$$

$$= \frac{-G_1G_2H_1}{1+G_2H_2+G_1G_2H_1}$$

$$\therefore \quad C = \frac{-G_1G_2H_1R_4}{1+G_2H_2+G_1G_2H_1}$$

Combining all the values of C, we get

$$C = \frac{G_1G_2R_1+G_2(R_2-R_3)-G_1G_2H_1R_4}{1+G_2H_2+G_1G_2H_1}$$

Summary

Block diagram is a pictorial representation of the complicated control system.

Block diagram can be easily reduced by using reduction rules.

A simple or canonical form of a block diagram consists of one block in the forward path, one block in the feedback path, one summing point and one take off point.

The rules must be used in the following sequence

i) Reduce series blocks

ii) Reduce parallel blocks

iii) Reduce minor feedback loops.

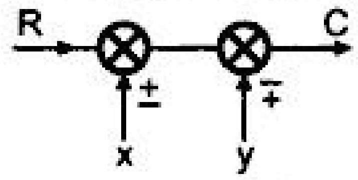
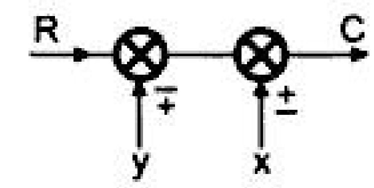
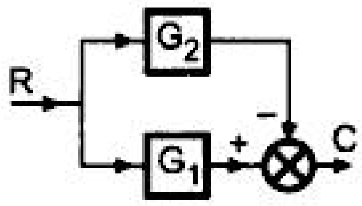
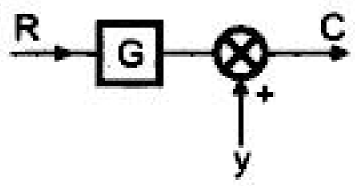
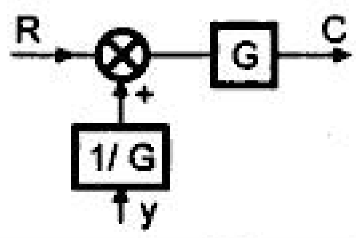
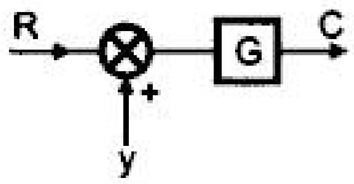
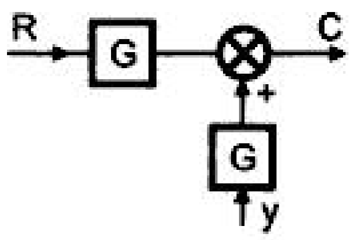
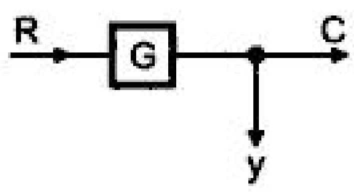
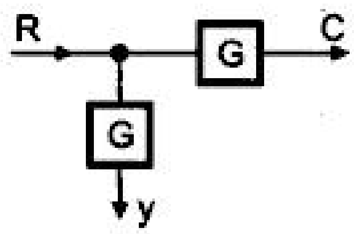
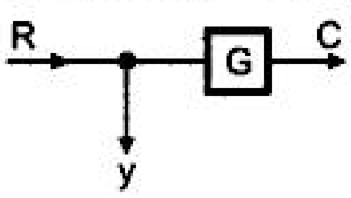
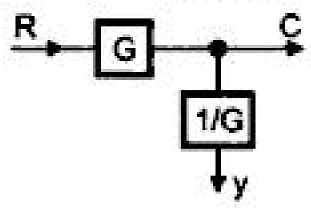
iv) Shift summing point to the left and take off point to the right, as far as possible.

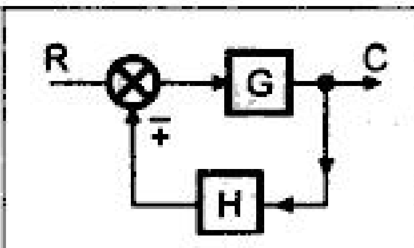
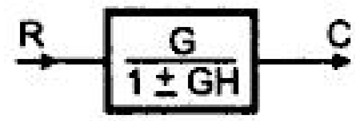
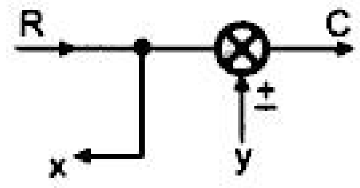
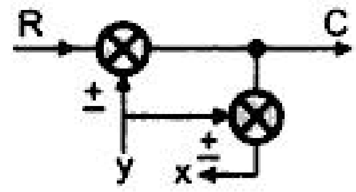
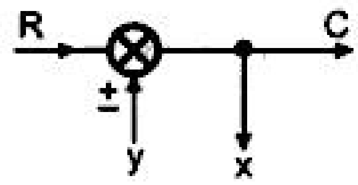
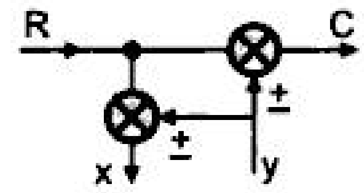
v) Repeat steps (i) to (iv) till canonical form is obtained.

Do not use the critical rules, shifting of takeoff point before and after a summing point, as far as possible.

While identifying series blocks make sure that there is no takeoff or summing point in between. While identifying parallel blocks make sure that there is no takeoff point in between and directions of signals through all the parallel blocks is same. For multiple input multiple output systems use superposition principle. Do not forget to carry forward a negative sign of the signal at the summing point, which is to be removed by adding a block of –1 in series with it.

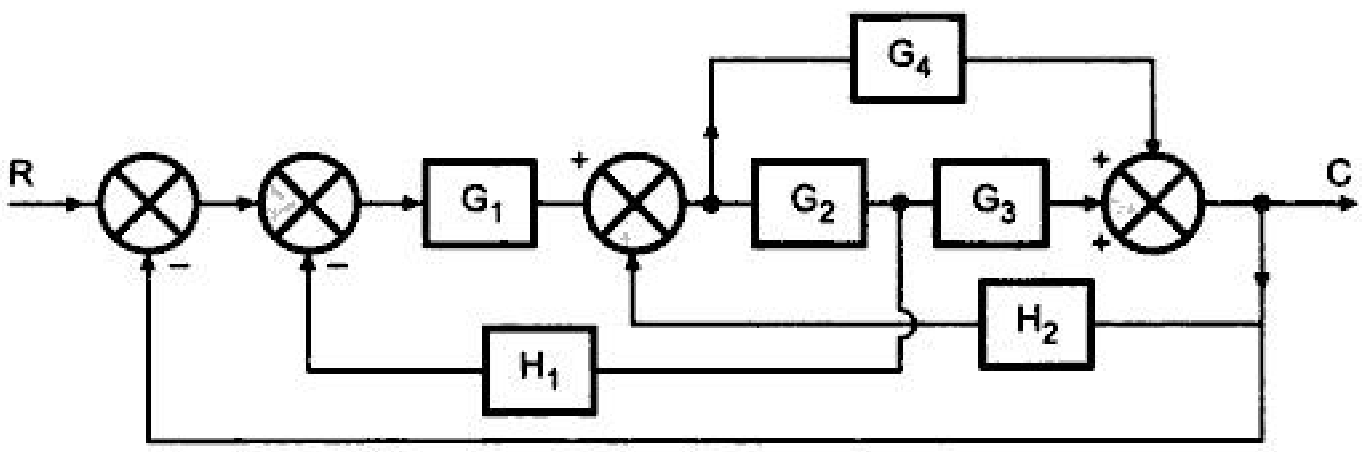
Table of Block Diagram Reduction Rules

1.	Associative Law	The two or more summing points directly connected can be interchanged.		
2.	Blocks in series	Transfer functions of such blocks get multiplied		
3.	Blocks in parallel	Transfer functions of such blocks get added algebraically		
4.	Shifing summing point behind the block	Add a block of T.F. equal to reciprocal of block behind which summing point is to be shifted in series with all signals at that summing point		
5.	Shifting summing point beyond the block	Add a block of T.F. same as the block beyond which summing point is to be shifted, in series with all the signals at that summing point		
6.	Shifting a takeoff point behind the block	Add a block of T.F. equal to the block behind which take off point is to be shifted, in series with all the signals at that takeoff point		
7.	Shifting a takeoff point beyond the block	Add a block of T.F. equal to the reciprocal of the block beyond which takeoff point is to be shifted, in series with all the signals at that takeoff point		

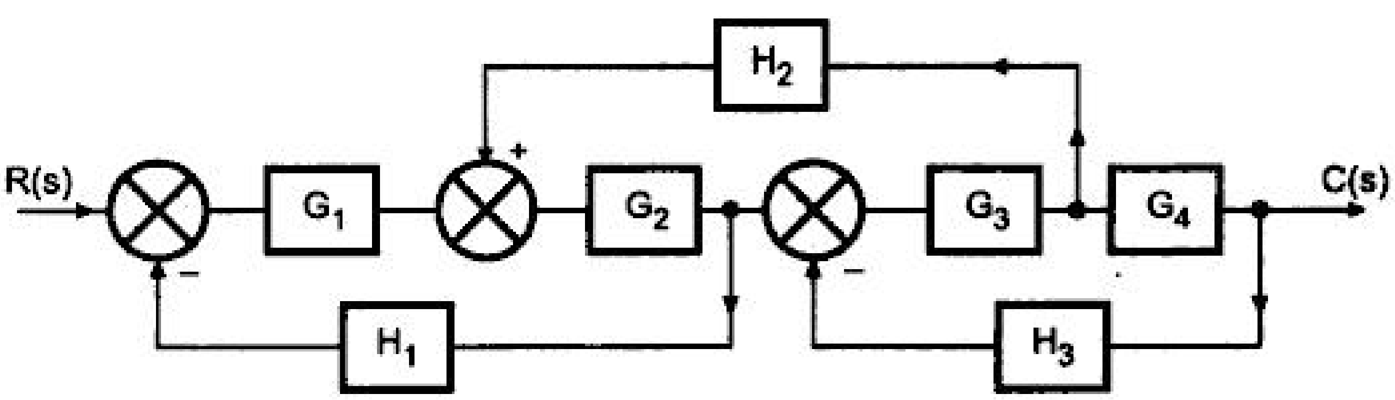
8.	Removing minor feedback loop	Use the standard T.F. of a simple closed loop system		
9.	Shifting takeoff point after summing point	Add a new summing point to compensate for the shift as shown in example		
10.	Shifting a takeoff point before a summing point	Add a summing point to compensate for a shift of takeoff point		

Review Questions

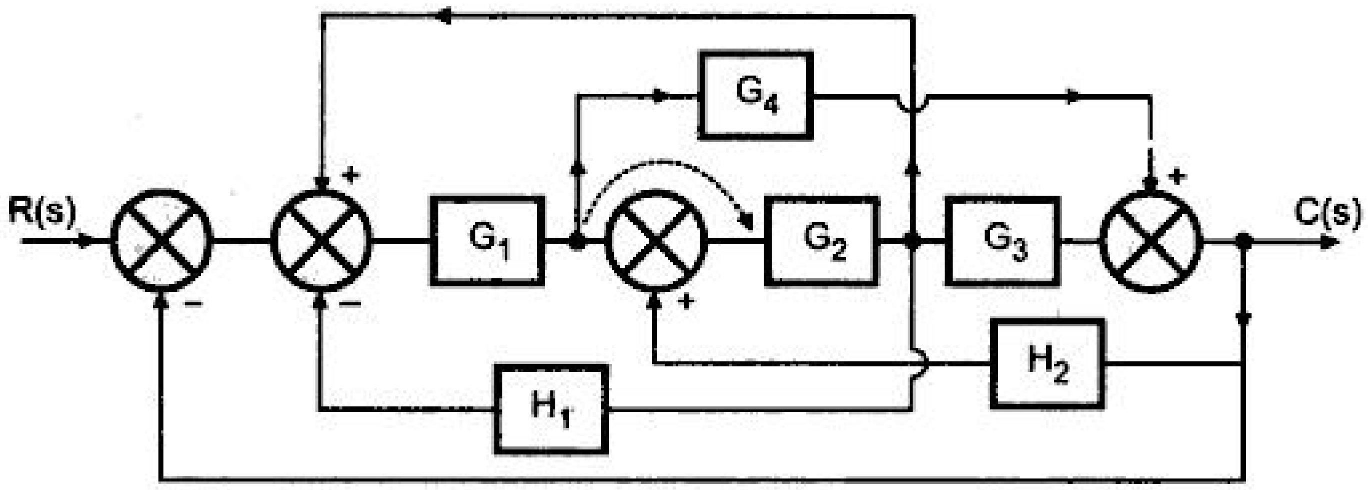
1. *What is block diagram representation ? Explain with suitable example.*
2. *State advantages and disadvantages of the block diagram reduction technique.*
3. *Explain the block diagram reduction rules*
4. *Determine C/R ratio for the system shown below.*



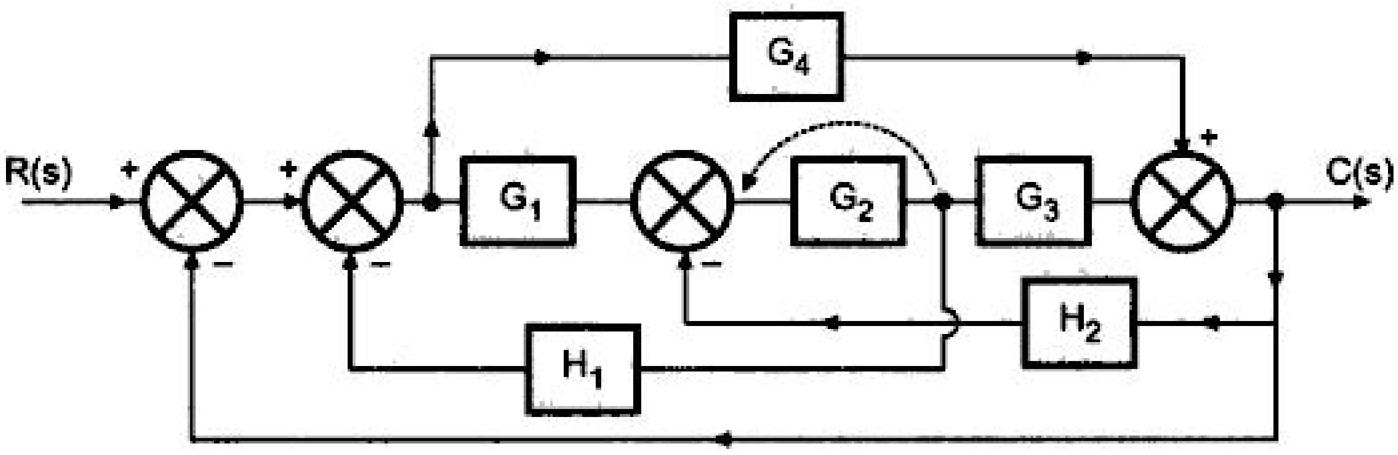
5. *Determine the transfer function of the system using block diagram reduction of the system shown.*



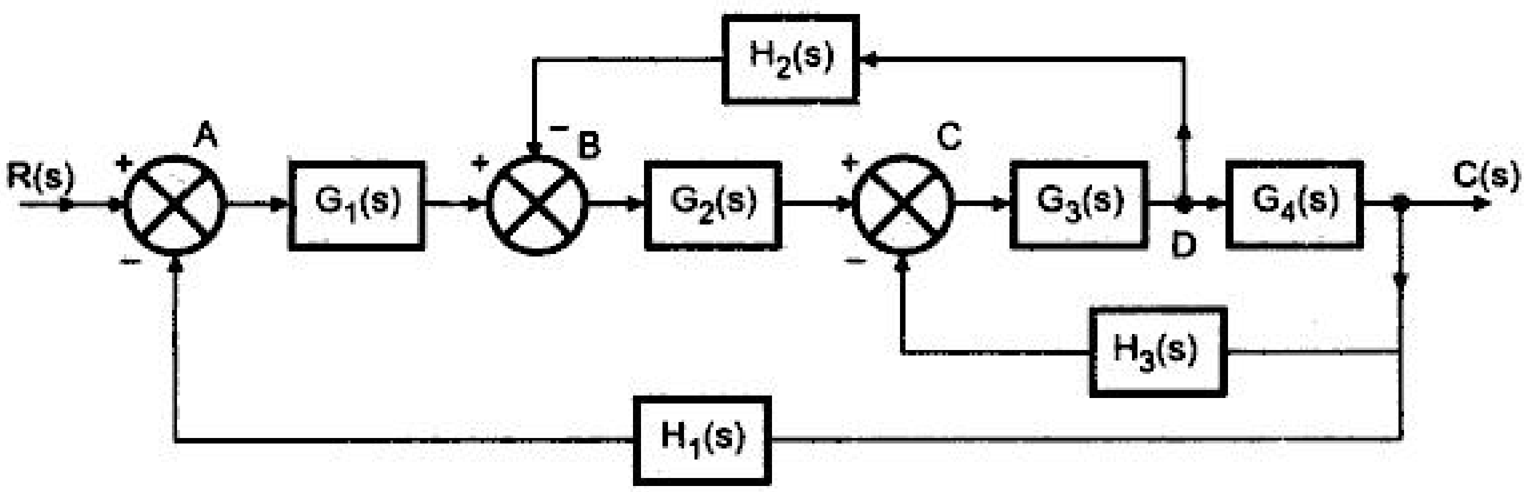
6. *For the block diagram shown, Obtain C(s)/R (s) by using reduction rules*



7. *Use block diagram reduction rules to obtain the transfer function of the block diagram shown below.*

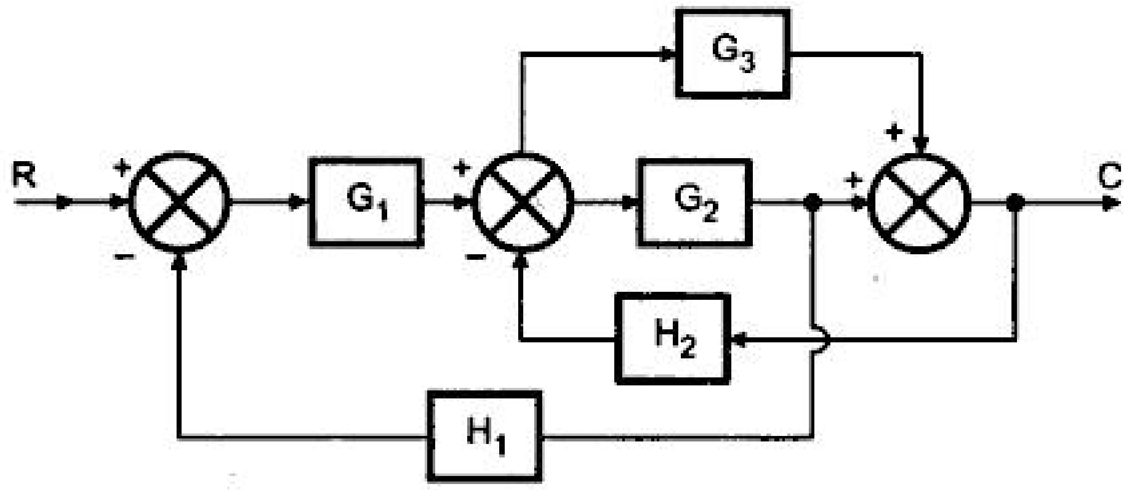


8. *Reduce the block diagram of the multiloop system shown in Fig. below.*



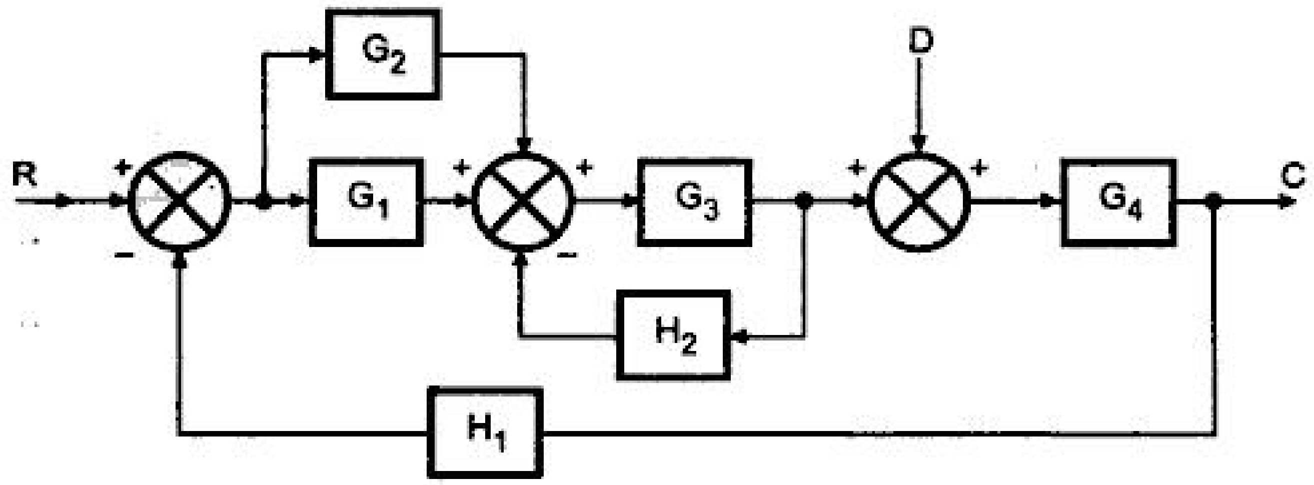
Ans. : $$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{1 + G_3 G_4 H_3 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_1}$$

9. *Determine the overall transfer function relating C and R for the system whose block diagram is shown in Fig.*



$$\text{Ans.}: \frac{C}{R} = \frac{G_1 G_2 + G_1 G_3}{1 + G_2 H_2 + G_1 G_2 H_1 - G_1 G_2 G_3 H_1 H_2}$$

10. *Determine the ratio* $\frac{C}{R}, \frac{C}{D}$ *and the total output for the system whose block diagram is shown in following Fig.*



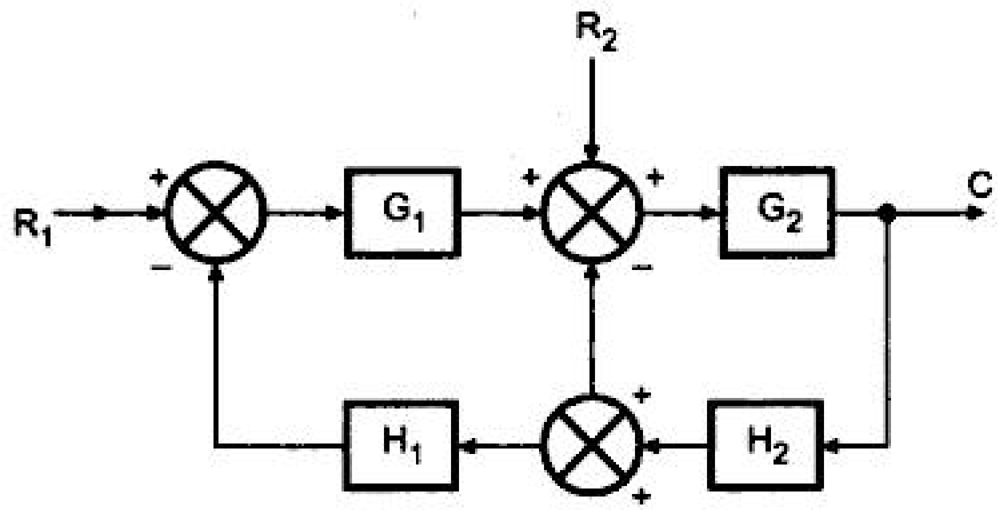
$$\text{Ans.}: \frac{C}{R} = \frac{G_1 G_3 G_4 + G_2 G_3 G_4}{1 + G_3 H_2 + G_1 G_3 G_4 H_1 + G_2 G_3 G_4 H_1}$$

$$\frac{C}{D} = \frac{G_4 (1 + G_3 H_2)}{1 + G_3 H_2 + G_1 G_3 G_4 H_1 + G_2 G_3 G_4 H_1}$$

$$\text{Total output} = \frac{G_1 G_3 G_4 + G_2 G_3 G_4}{1 + G_3 H_2 + G_1 G_3 G_4 H_1 + G_2 G_3 G_4 H_1} R$$

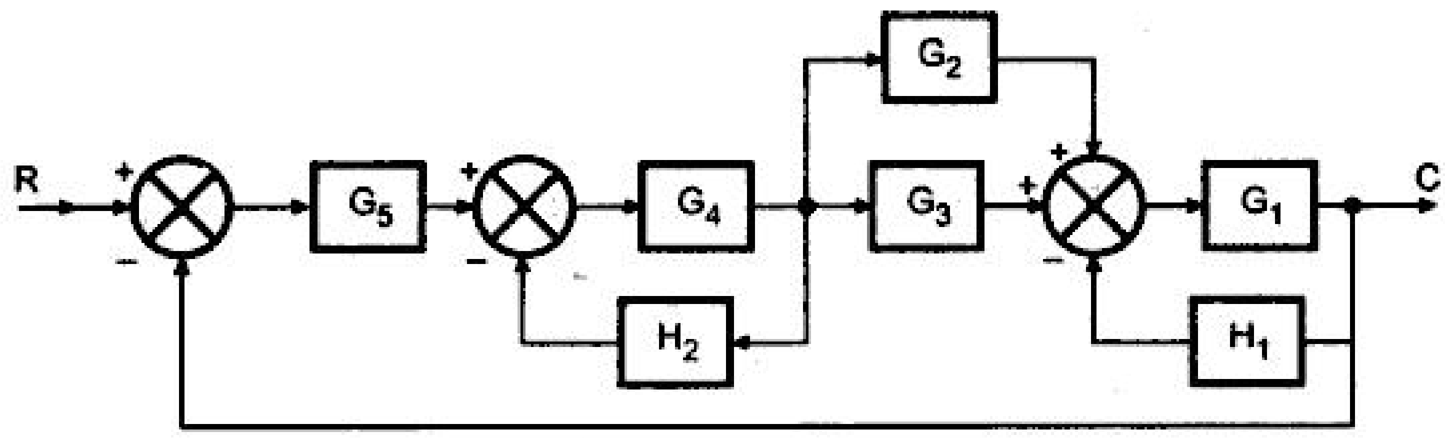
$$+ \frac{G_4 (1 + G_3 H_2)}{1 + G_3 H_2 + G_1 G_3 G_4 H_1 + G_2 G_3 G_4 H_1} D$$

11. Derive an expression for the total output for the system represented by the block diagram in following Fig.



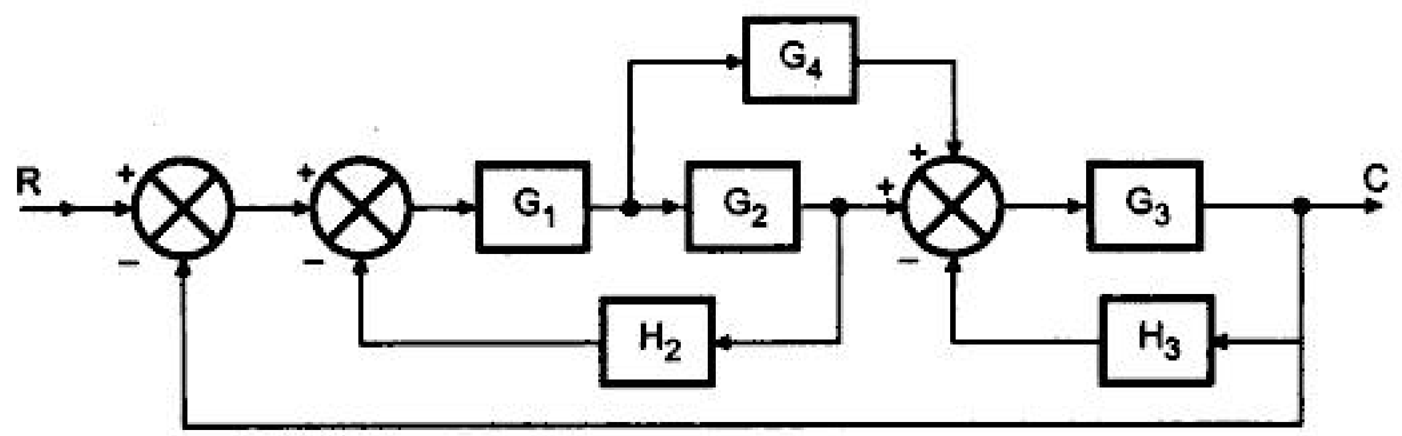
$$\text{Ans.: } C = \frac{G_1\, G_2\, R_1 + G_2\, R_2}{1 + G_1\, G_2\, H_1\, H_2 + G_2 H_2}$$

12. Use block diagram reduction methods to obtain the equivalent transfer function from R to C.



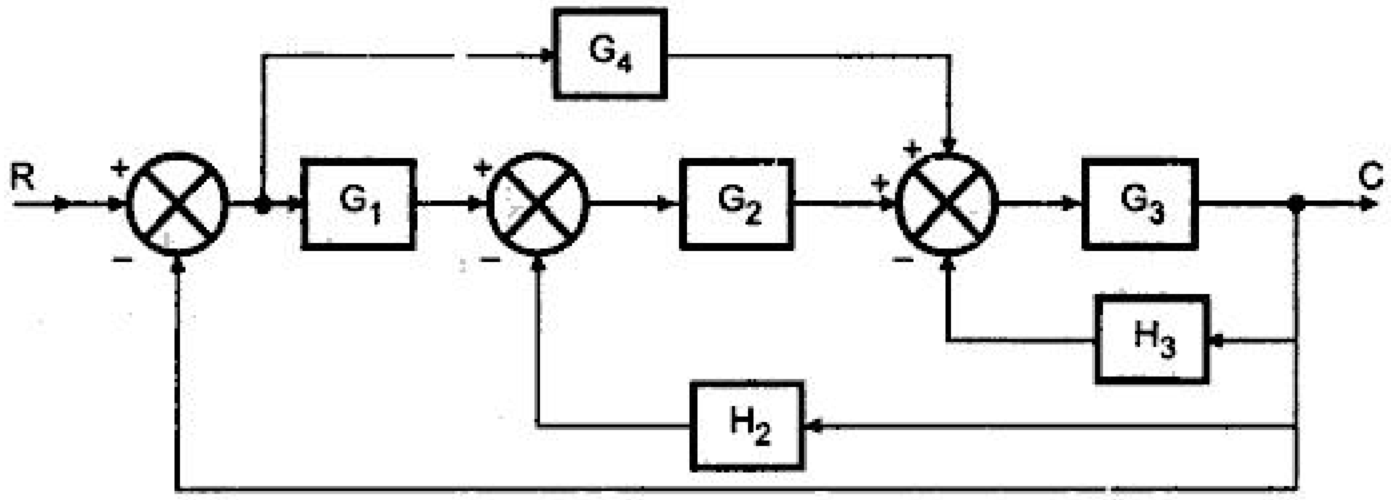
$$\text{Ans.: } \frac{C}{R} = \frac{G_5\, G_4\, (G_2 + G_3)\,(G_1)}{(1 + G_4\, H_2)\,(1 + G_1\, H_1) + G_5\, G_4\, (G_2 + G_3)\, G_1}$$

13. Use block diagram reduction methods to obtain the equivalent transfer function from R to C .



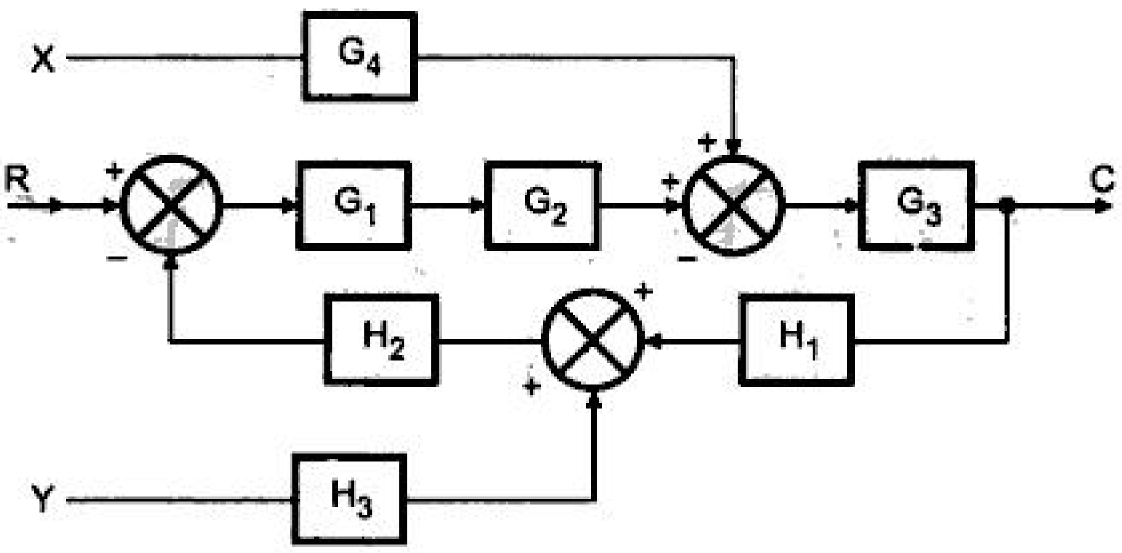
$$\text{Ans.: } \frac{C}{R} = \frac{G_1\, G_3\, (G_2 + G_4)}{(1 + G_1\, G_2\, H_2)\,(1 + G_3\, H_3\,) + G_1 G_3\, (G_2 + G_4)}$$

14. *Find the equivalent transfer function for the Fig. shown below.*



$$\text{Ans.}: \frac{C}{R} = \frac{G_3\,G_4 + G_1\,G_2\,G_3}{1 + G_3\,H_3 + G_2\,H_2\,G_3 + G_3\,G_4 + G_1\,G_2\,G_3}$$

15. *Using block diagram reduction , find the transfer function from each input to the output C.*



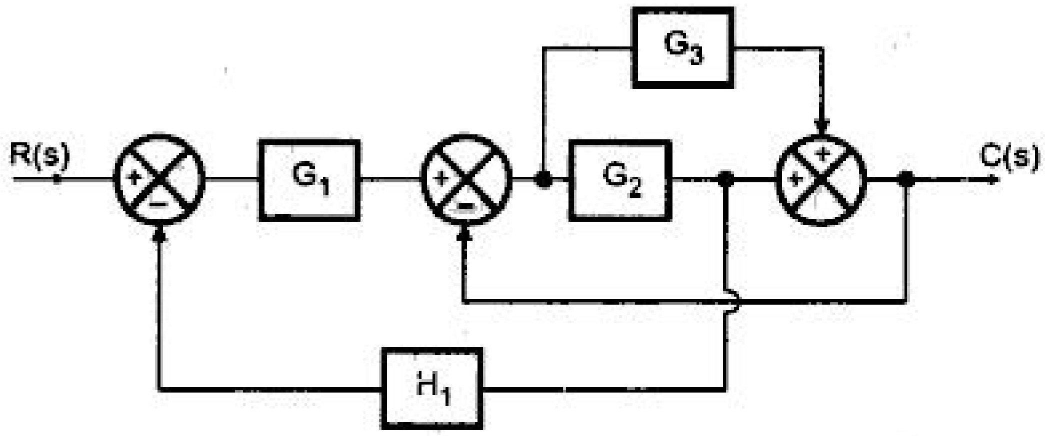
$$\text{Ans.}: \frac{C}{R} = \frac{G_1\,G_2\,G_3}{1 + G_1\,G_2\,G_3\,H_1\,H_2}, \quad \frac{C}{X} = \frac{G_4\,G_3}{1 + G_3\,G_1\,G_2\,H_1\,H_2},$$

$$\frac{C}{Y} = \frac{-G_1\,G_2\,G_3\,H_2\,H_3}{1 + G_1\,G_2\,G_3\,H_1\,H_2}$$

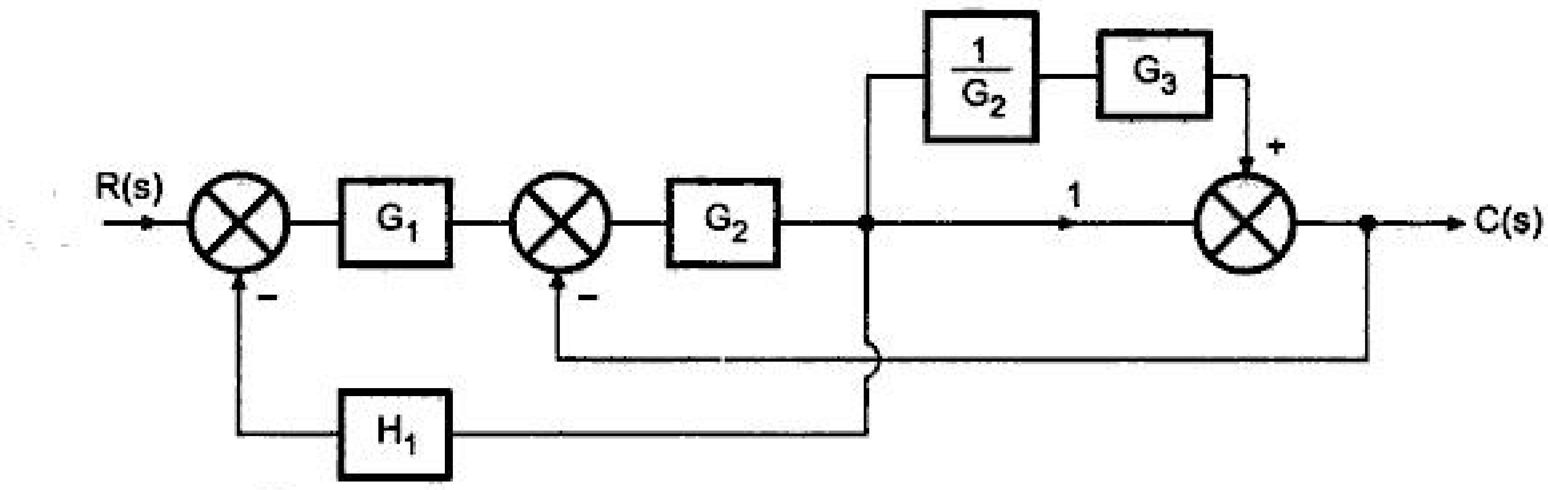
University Questions (New Syllabus)

May-2003

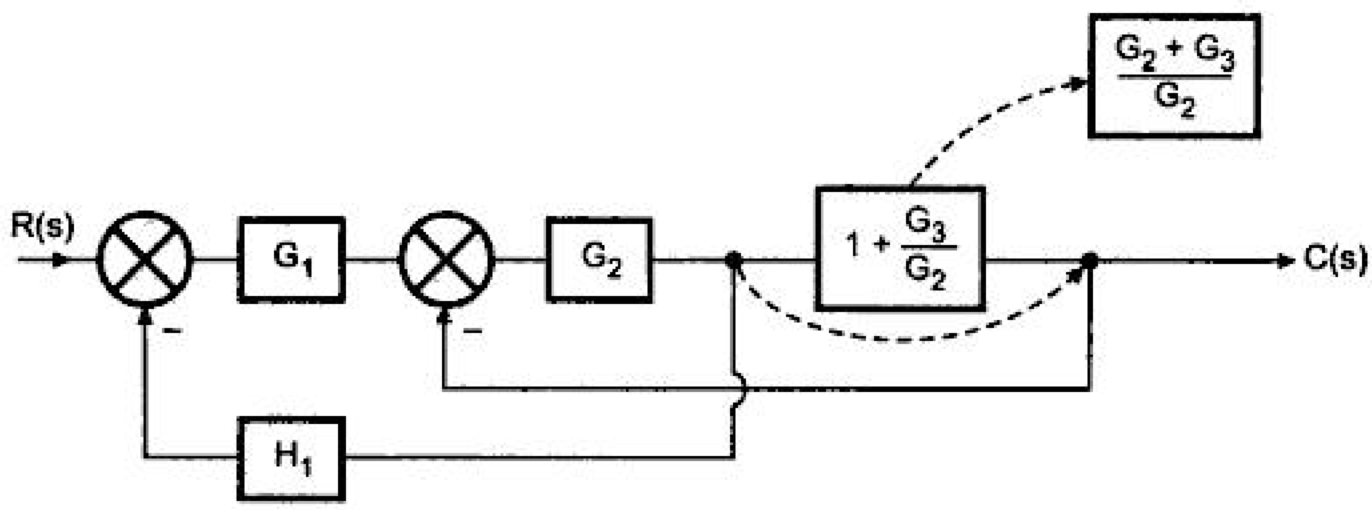
1. *Determine the transfer function* $\frac{C(s)}{R(s)}$ *using block diagram reduction technique for the block diagram shown -* **(8 marks)**



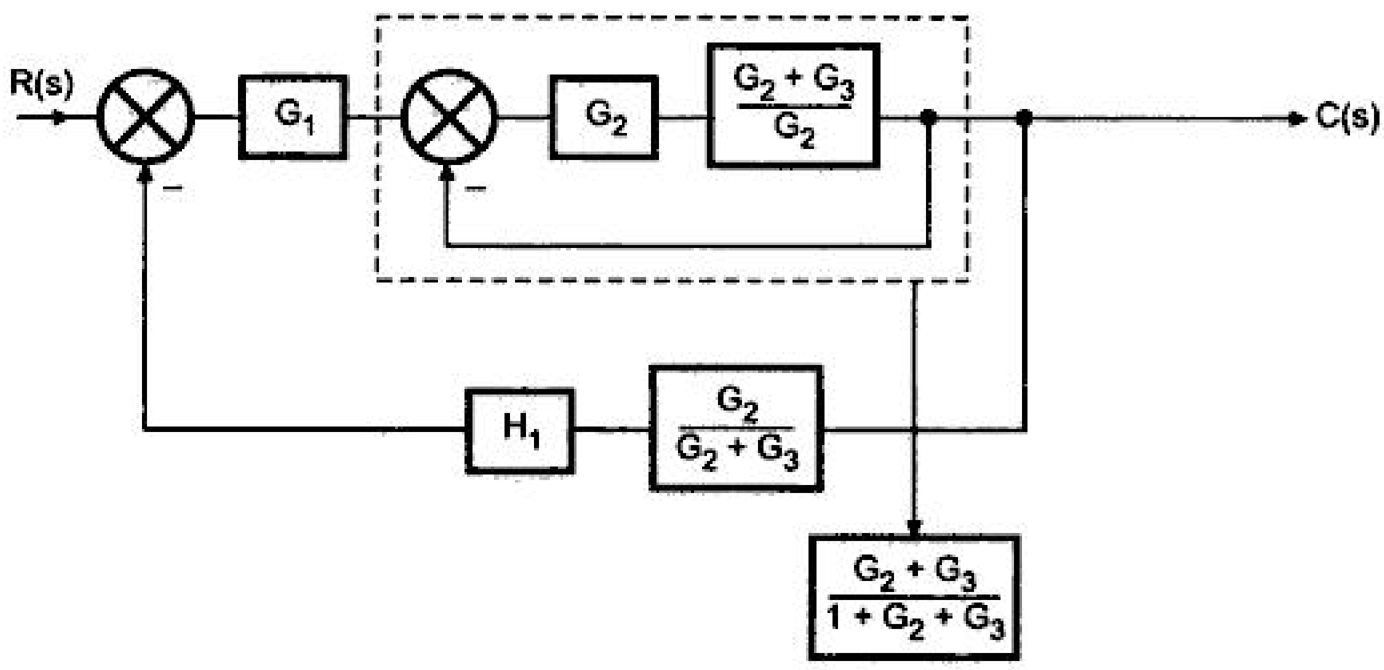
Sol. : Shifting takeoff point of G_3 to the right of G_2 we get,

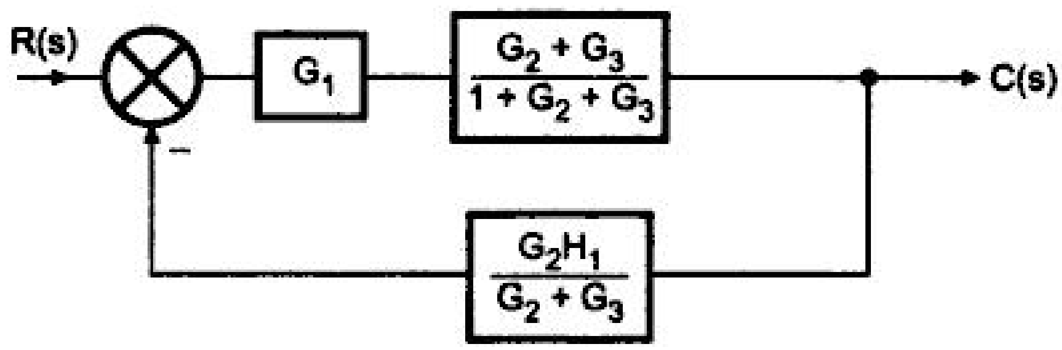


Combining $\frac{G_3}{G_2}$ and 1 in parallel we get,



Shifting takeoff point towards C(s) and interchanging takeoff points,





$$\therefore \frac{C(s)}{R(s)} = \frac{\dfrac{G_1(G_2+G_3)}{1+G_2+G_3}}{1+\dfrac{G_1(G_2+G_3)}{(1+G_1+G_3)}\times\dfrac{G_2H_1}{(G_2+G_3)}} = \frac{G_1(G_2+G_3)}{1+G_2+G_3+G_1G_2H_1}$$

❑❑❑

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

function or branch gain joining I and V. The direction of arrow is from I to V. This is shown in the Fig. 4.2.

So all the branches represent the cause and effect relationship existing between the various variables. The branch transfer function is also called as **branch gain** or **branch transmittance** in signal flow graph terminology.

4.2 Properties of Signal Flow Graph :

1) The signal flow graph is applicable only to linear time invariant systems.

2) The signal in the system flows along the branches and along the arrowheads associated with the branches.

3) The signal gets multiplied by the branch gain or branch transmittance when it travels along it.

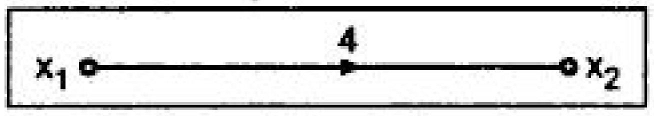


Fig. 4.3

e.g. Consider signal flow graph shown in the Fig. 4.3

The signal from x_1 gets multiplied by 4 when it travels along the branch joining x_1 to x_2. So we can say value of x_2 is 4 times the value of x_1.

4) The value of variable represented by any node is an algebraic sum of all the signals entering at the node

e.g. Consider the variable x_2. At that node, 3 signal are entering from x_1 , x_3 and x_4, so value of x_2 depends on the variables x_1, x_3 and x_4. The branch gains indicate the exact contribution of each variable in generating x_2. So value of x_2 is algebraic sum of all such signals entering. So we can write,

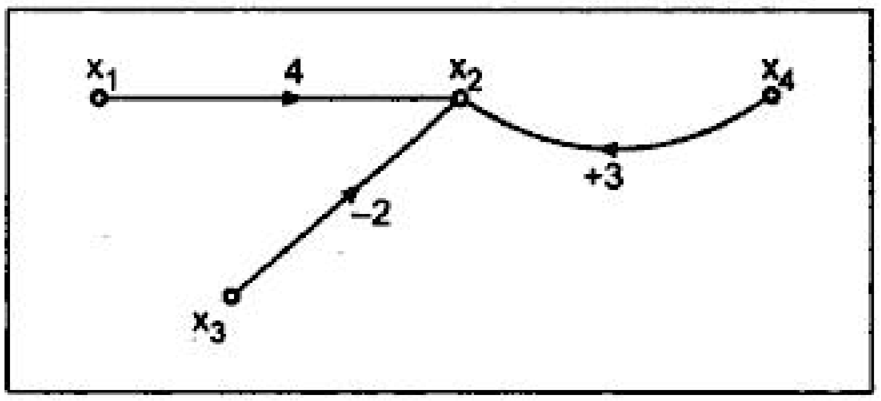


Fig. 4.4

$$x_2 = 4x_1 - 2x_3 + 3x_4$$

5) The value of the variable represented by any node is available to all the branches leaving that node. The number of branches leaving a node does not affect the value of variable represented by that node.

e.g. Consider signal flow graph represented in Fig. 4.5. The value of x_2 can be obtained from signals entering at x_2.

i.e. $x_2 = 5x_1 - 2x_3$

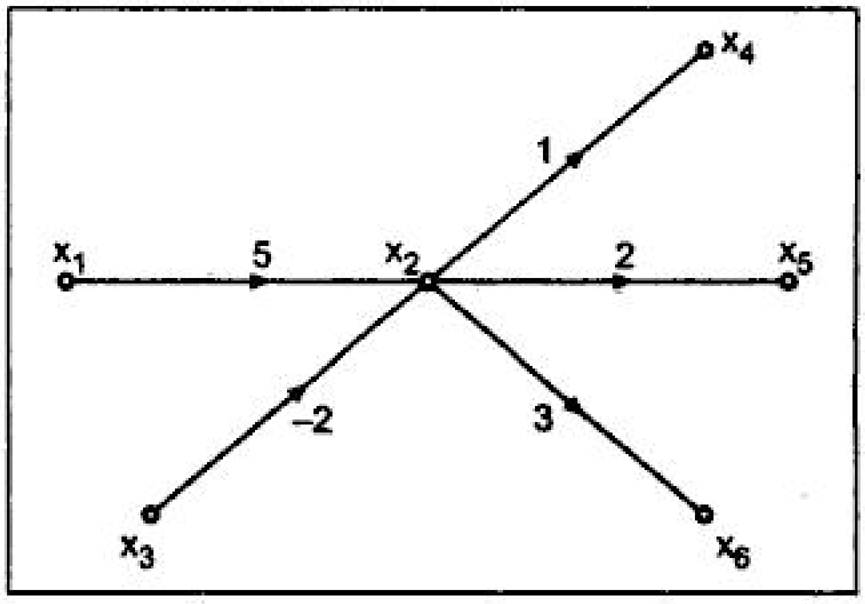


Fig. 4.5

Now there are three branches leaving x_2 , joining to x_4, x_5 and x_6 that means x_4, x_5 and x_6 variables depend on $2x_2$.

So we can write $x_4 = x_2$, $x_5 = 2x_2$, $x_6 = 3x_2$

i.e. for all branches leaving from x_2 the value of x_2 available is same and number of such branches do not affect the value of x_2.

So value of a variable represented by node depends on signals entering and this value is available to all the branches leaving from that node.

6) For a given system signal flow graph is not unique. Many other graphs can be drawn by writing system equations in different manner.

4.3 Terminology used in Signal Flow Graph :

Consider a Signal flow graph shown in the Fig. 4.6

i) **Source Node :** The node having only outgoing branches is known as source or input node. e.g. x_0 is source node.

ii) **Sink Node :** The node having only incoming branches is known as sink or output node. e.g. x_5 is sink node.

iii) **Chain Node :** A node having incoming and outgoing branches is known as chain node. e.g. x_1 , x_2 , x_3 and x_4 .

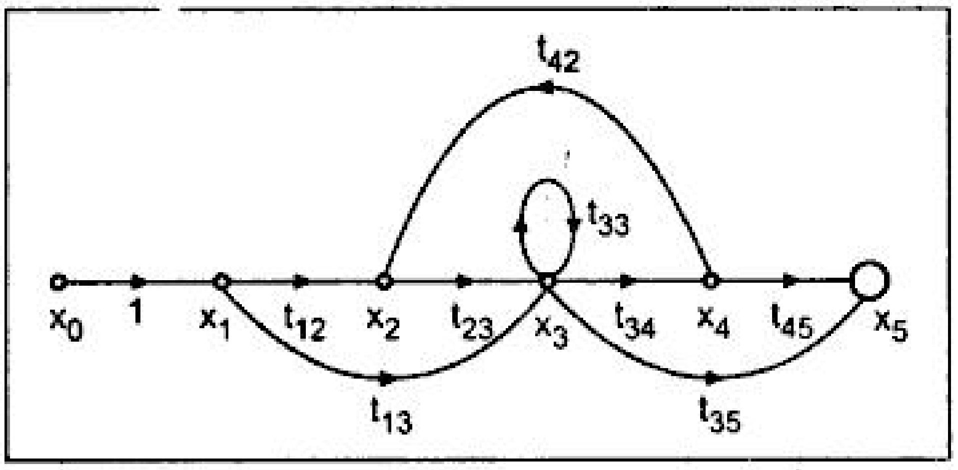


Fig. 4.6

iv) **Forward Path :** A path from the input to output node is defined as forward path.

e.g. $x_0 - x_1 - x_2 - x_3 - x_4 - x_5$ First forward path

$x_0 - x_1 - x_3 - x_4 - x_5$ Second forward path.

$x_0 - x_1 - x_3 - x_5$ Third forward path.

$x_0 - x_1 - x_2 - x_3 - x_5$ Fourth forward path.

No node is to be traced twice.

v) **Feedback Loop :** A loop which originates and terminates at the same node is known as feedback path i.e. $x_2 - x_3 - x_4 - x_2$. No node is to be traced twice.

vi) **Self Loop :** A feedback loop consisting of only one node is called as self loop. i.e. t_{33} at x_3 is self loop. A self loop can not appear while defining a forward path or feedback loop as node containing it gets traced twice which is not allowed.

vii) **Path Gain :** The product of branch gains while going through a forward path is known as path gain. i.e. path gain for path $x_0 - x_1 - x_2 - x_3 - x_4 - x_5$ is ,

$1 \times t_{12} \times t_{23} \times t_{34} \times t_{45}$. This can be also called forward path gain.

viii) **Dummy Node :** If there exists incoming and outgoing branches both at first and last node representing input and output variables, then as per definition these can not be called as source and sink nodes. In such a case a separate input and output nodes can be created by adding branches with gain 1. Such nodes are called as dummy nodes.

e.g.

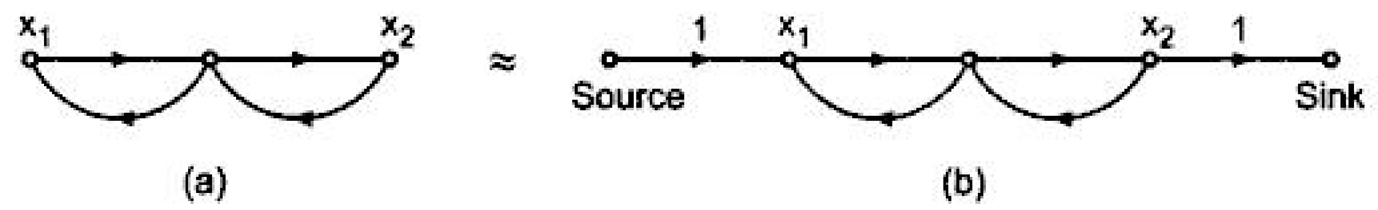


Fig. 4.7

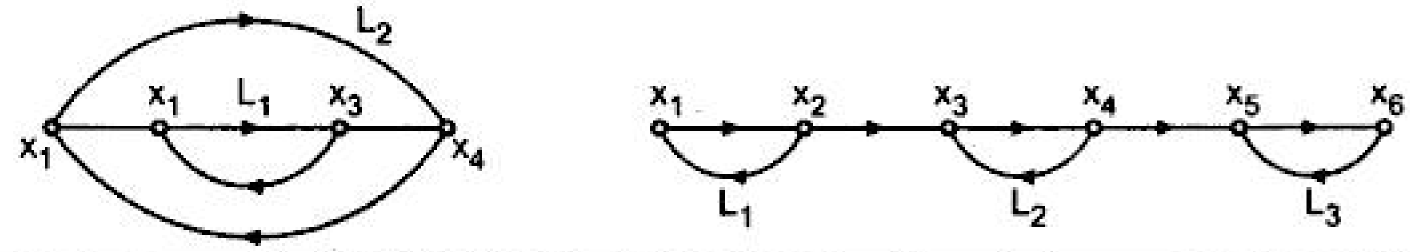
In the signal flow graph x_1 and x_2 are input and output variables but as per definition not input and output nodes. Such independent nodes can be generated by adding branches of gain 1 as shown in the Fig. 4.7 (b).

Note : Such creation of dummy nodes is not necessary. Without this also signal flow graph can be analysed to get the overall transfer function.

Addition of branches of gain 1 is possible only before starting node and after the last node. In between the chain nodes such branches of gain 1 cannot be added.

ix) **Non-touching loops :** If there is no node common in between the two or more loops, such loops are said to be non-touching loops.

Fig. 4.8 (a)&(b) show a combination of non-touching loops of two and three loops.

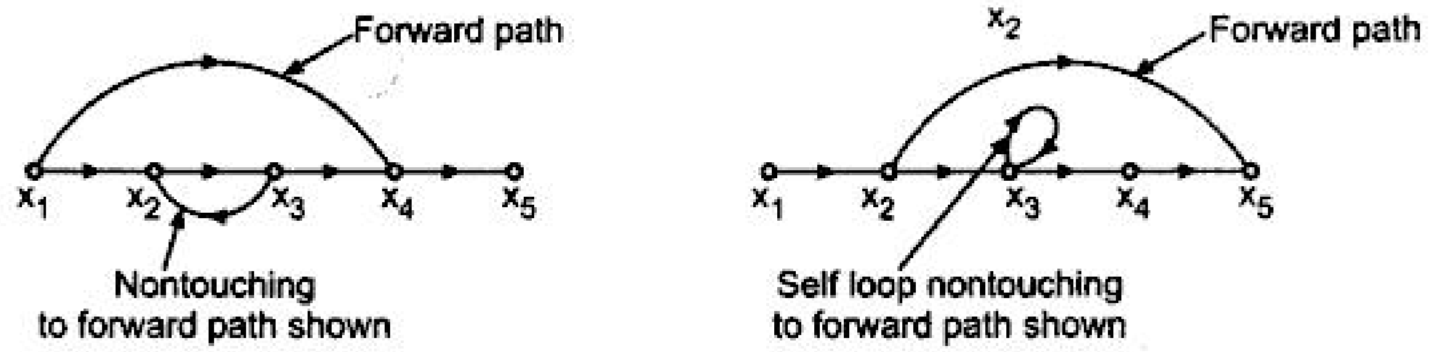


(a) Two non-touching loops **(b) Three non-touching loops**

Fig. 4.8

Similarly if there is no node common in between a forward path and a feedback loop, a loop is said to be non-touching to that forward path.

Fig. 4.9 (a) and (b) shows such a loop which is non-touching to a forward path.



(a) **(b)**

Fig. 4.9

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

4.4.2 From given block diagram :

Steps :

i) Name all the summing points and take-off points in the block diagram.

ii) Represent each summing and take-off point by a separate node in signal flow graph.

iii) Connect them by the branches instead of blocks, indicating block transfer functions as the gains of the corresponding branches.

iv) Show the input and output nodes separately if required, to complete signal flow graph.

Example :

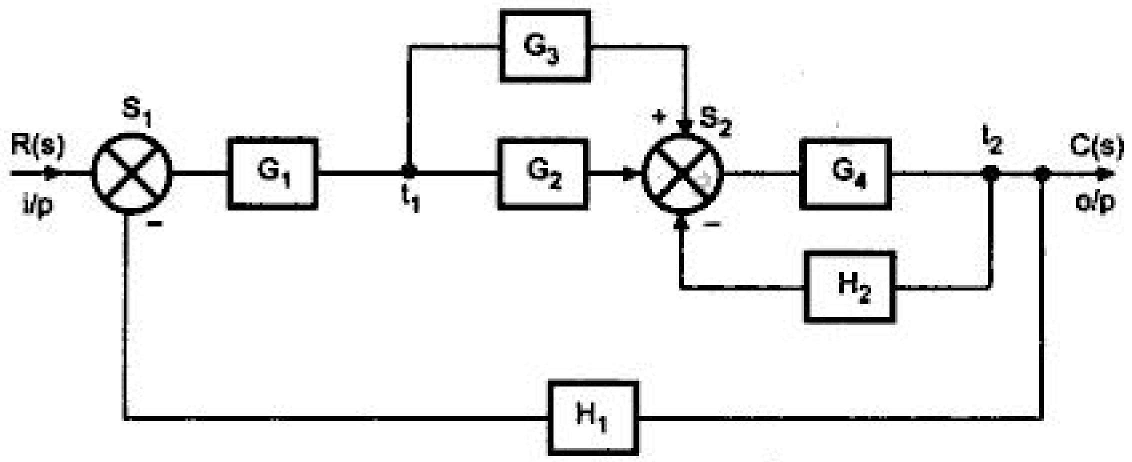


Fig. 4.12

Naming summing and take-off points as shown in Fig. 4.13.

Note : Make sure that if summing and take-off points are near each other in a given block diagram, they are to be represented by separate nodes in the corresponding signal flow graph.

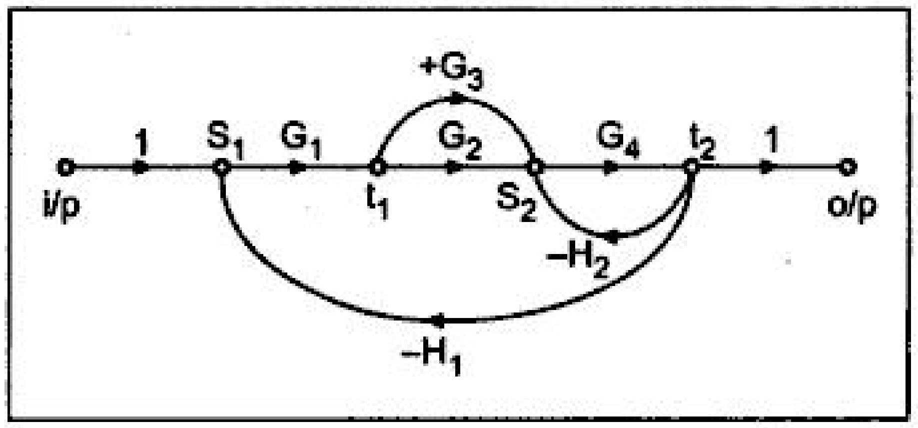


Fig. 4.13

4.5 Mason's Gain Formula :

It is seen earlier that in block diagram representation, we have to apply reduction rules, one after the other to obtain simple form of the sysem and hence overall transfer function. We have to draw the reduced block diagram after every step. This is time consuming. In signal flow graph approach, once S.F.G is obtained direct use of one formula leads to the overall system transfer function $\frac{C(s)}{R(s)}$. This formula is stated by Mason and hence referred as Mason's Gain formula. The formula can be stated as :

Hence T.F. can be obtained by substituting these values in

$$\text{T.F.} = \frac{T_1\Delta_1 + T_2\,\Delta_2 + T_3\,\Delta_3}{\Delta}$$

Ex. 4.1 *Find the overall T.F. by using Mason's gain formula for the signal flow graph given in Fig. 4.14*

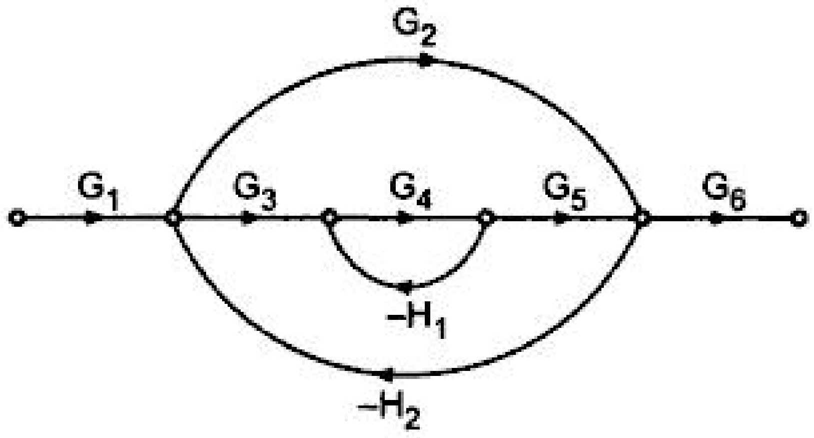


Fig. 4.14

Sol. : Two forward paths, K = 2,

$$T_1 = G_1\,G_3\,G_4\,G_5\,G_6$$

$$T_2 = G_1\,G_2\,G_6$$

Loops are,

$$L_1 = -G_4\,H_1$$

$$L_2 = -G_3\,G_4\,G_5 H_2$$

$$L_3 = -G_2\,H_2$$

Out of these, L_1 and L_3 is combination of 2 non-touching loops

$$\Delta = 1 - [L_1 + L_2 + L_3] + [L_1\,L_3]$$

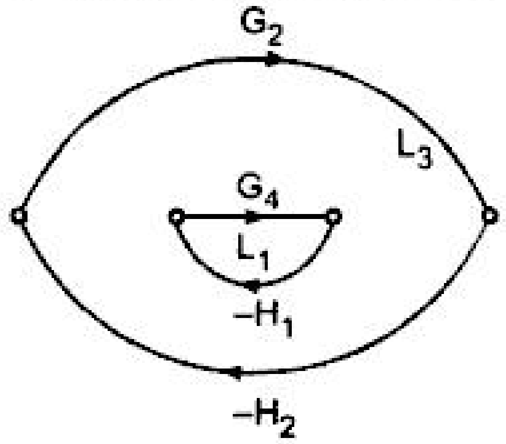


Fig. 4.15 Non touching loops

Δ_1 = Eliminate L_1, L_2, L_3 as all are touching to T_1 from Δ

$\therefore$ $\Delta_1 = 1$

Δ_2 = Eliminate L_2 and L_3, as they are touching to T_2, from Δ. But L_1 is non-touching hence keep it as it is in Δ.

$\therefore$ $\Delta_2 = 1 - [L_1]$

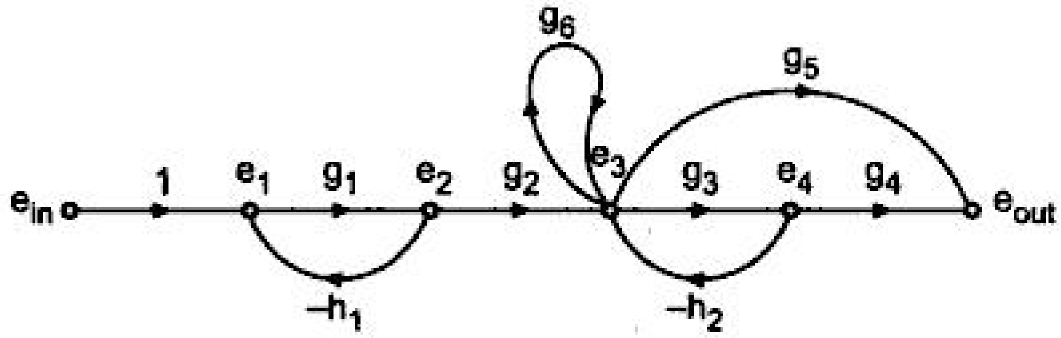
You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

4.7 Application of the General Gain Formula Between Output Nodes and Non Input nodes :

It was derived earlier that Mason's gain formula is used to get a relation between output node and input node called transfer function.

But often, it is required to calculate the relation between output node variable and a non input node variable.

Example :



Now it is required to calculate $\frac{e_{out}}{e_2}$ i.e. dependence of e_{out} on e_2 and e_2 is not input variable.

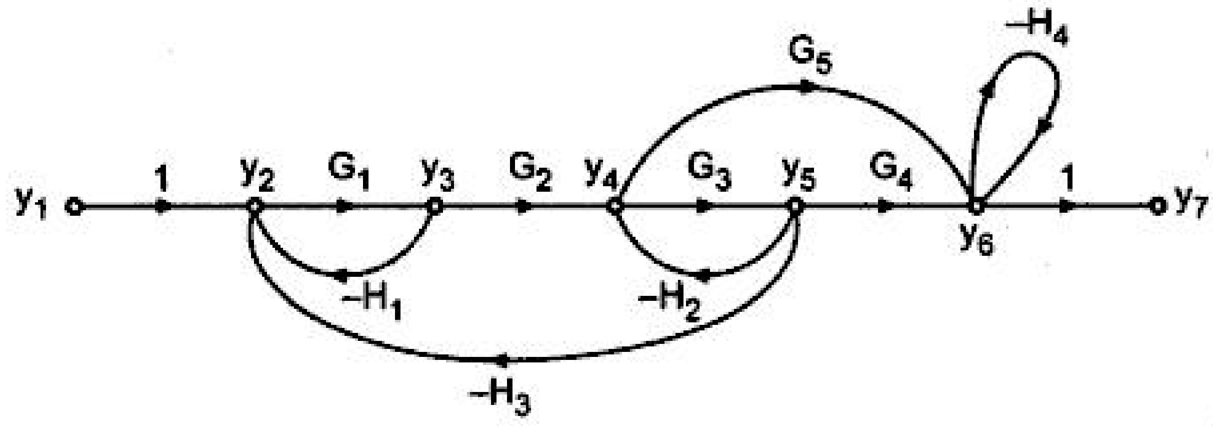
So $\frac{e_{out}}{e_2}$ can be expressed as

$$\frac{e_{out}}{e_2} = \frac{\frac{e_{out}}{e_{in}}}{\frac{e_2}{e_{in}}} = \frac{\left.\frac{\Sigma T_K \Delta_K}{\Delta}\right|_{\text{from } e_{in} \text{ to } e_2}}{\left.\frac{\Sigma T_K \Delta_K}{\Delta}\right|_{\text{from } e_{in} \text{ to } e_{out}}}$$

Since Δ is independent of inputs and outputs,

$$\frac{e_{out}}{e_2} = \frac{\Sigma T_K \Delta_K|_{\text{from } e_{in} \text{ to } e_{out}}}{\Sigma T_K \Delta_K|_{\text{from } e_{in} \text{ to } e_2}}$$

Ex. 4.2 *Calculate $\frac{y_7}{y_2}$ of the system, whose signal flow graph is given below*



You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

$$L_3 = a_{22}$$

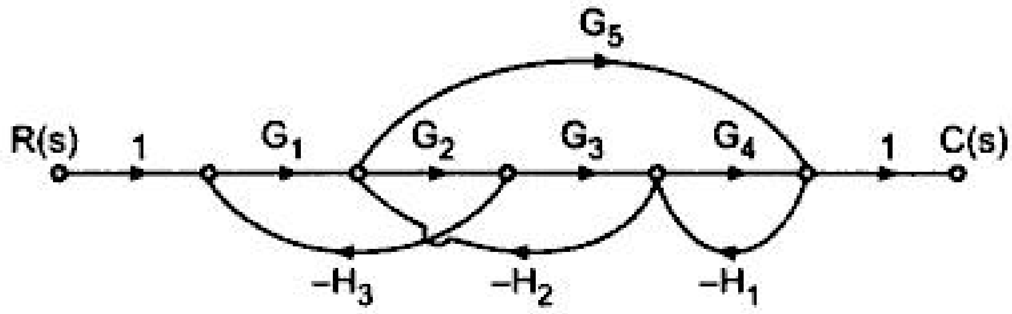
$$\therefore \quad \Delta = 1 - [L_1 + L_2 + L_3] + [L_2 L_3]$$

$$= 1 - a_{21} a_{12} - a_{11} - a_{22} - a_{11} a_{22}$$

$\Delta_1 = 1 - a_{11}$ as a_{11} loop is nontouching to the forward path.

$$\frac{Y_2}{U_2} = \frac{T_1 \Delta_1}{\Delta} = \frac{b_2 (1 - a_{11})}{1 - a_{21} a_{12} - a_{11} - a_{22} + a_{11} a_{22}}$$

Ex. 4.4 *Find* $\frac{C(s)}{R(s)}$ *for S.F.G. shown in following figure.*



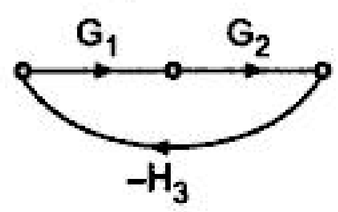
Sol. : Number of forward paths = K = 2

$$\therefore \quad \text{T.F.} = \frac{\sum_{K+1}^{2} T_K \Delta_K}{\Delta} \text{ using Mason's Gain Formula}$$

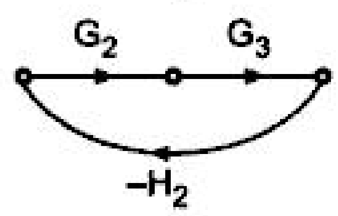
$$T_1 = G_1 G_2 G_3 G_4$$

$$T_2 = G_1 G_5$$

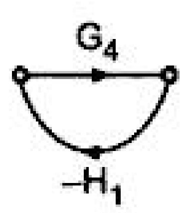
Individual feedback loops



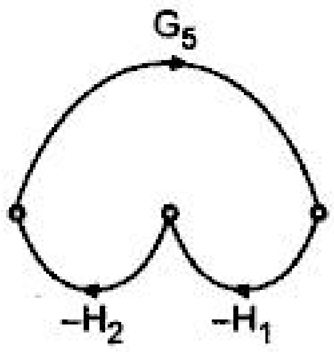
$L_1 = -G_1 G_2 H_3$



$L_2 = -G_2 G_3 H_2$



$L_3 = -G_4 H_1$

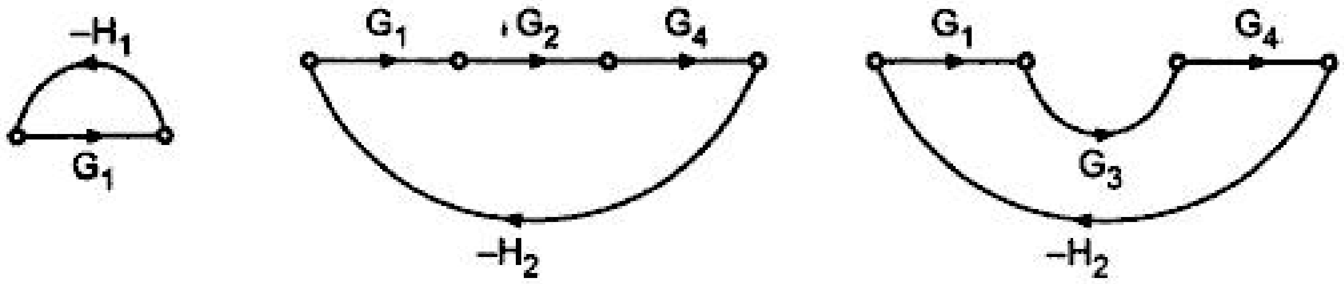


$L_3 = +G_5 H_1 H_2$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Individual feedback loops :



$$L_1 = -G_1H_1 \qquad L_2 = -G_1G_2G_4H_2 \qquad L_3 = G_1G_3G_4H_2$$

$\therefore$ $\Delta = 1-[L_1+L_2+L_3]$ All loops are touching

$\therefore$ $\Delta = 1+G_1H_1+G_1G_2G_4H_2+G_1G_3G_4H_2$

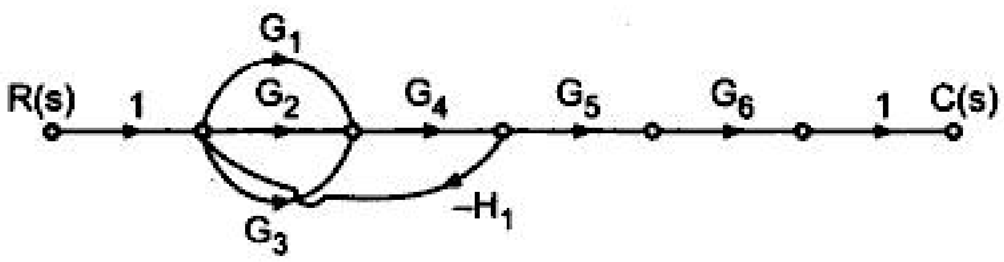
Consider $T_1 = G_1G_2G_4$ All loops are touching, $\therefore \Delta_1 = 1$

$T_2 = G_1G_3G_4$ All loops are touching, $\therefore \Delta_2 = 1$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{T_1\Delta_1+T_2\Delta_2}{\Delta} = \frac{G_1G_2G_4\cdot 1+G_1G_3G_4\cdot 1}{\Delta}$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G_1G_2G_4+G_1G_3G_4}{1+G_1H_1+G_1G_2G_4H_2+G_1G_3G_4H_2}$$

Ex. 4.7 *Find* $\dfrac{C(s)}{R(s)}$



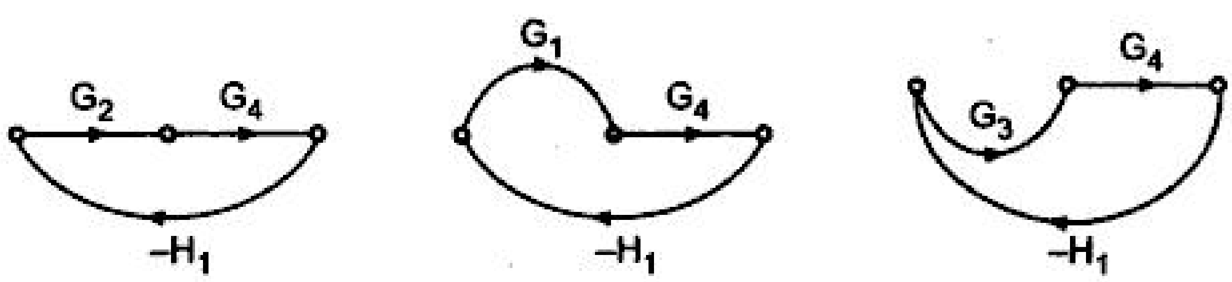
Sol. : Number of forward paths K = 3

$\therefore$ By Mason's gain formula,

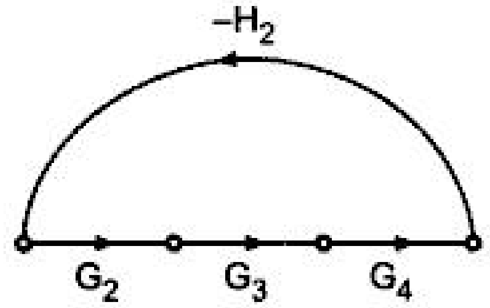
$$\text{T.F.} = \frac{\sum_{K=1}^{3} T_K\Delta_K}{\Delta} = \frac{T_1\Delta_1+T_2\Delta_2+T_3\Delta_3}{\Delta}$$

$$T_1 = G_2G_4G_5G_6 \qquad T_2 = G_1G_4G_5G_6 \qquad T_3 = G_3G_4G_5G_6$$

Individual feedback loops



$$L_1 = -G_2G_4H_1 \qquad L_2 = -G_1G_4H_1 \qquad L_3 = -G_3G_4H_1$$



$L_4 = -G_2 G_3 G_4 H_2$ $\qquad$ $L_5 = G_5$

Combinations of two nontouching loops

i) L_1 and L_2 $\qquad$ ii) L_1 and L_3 $\qquad$ iii) L_1 and L_5 $\qquad$ iv) L_2 and L_5

Combination of three nontouching loops.

i) L_1 , L_2 and L_5

$$\therefore \quad \Delta = 1-[L_1+L_2+L_3+L_4+L_5]+[L_1 L_2+L_1 L_3+L_1 L_5+L_2 L_5]-[L_1 L_2 L_5]$$

$$\Delta = 1+G_1 H_1+G_3 H_4+G_4 H_3+G_2 G_3 G_4 H_2-G_5+G_1 G_3 H_1 H_4$$
$$+G_1 G_4 H_1 H_3-G_1 H_1 G_5-G_3 H_4 G_5-G_1 G_3 G_5 H_1 H_4$$

Consider $\quad T_1 = G_1 G_2 G_3 G_4$

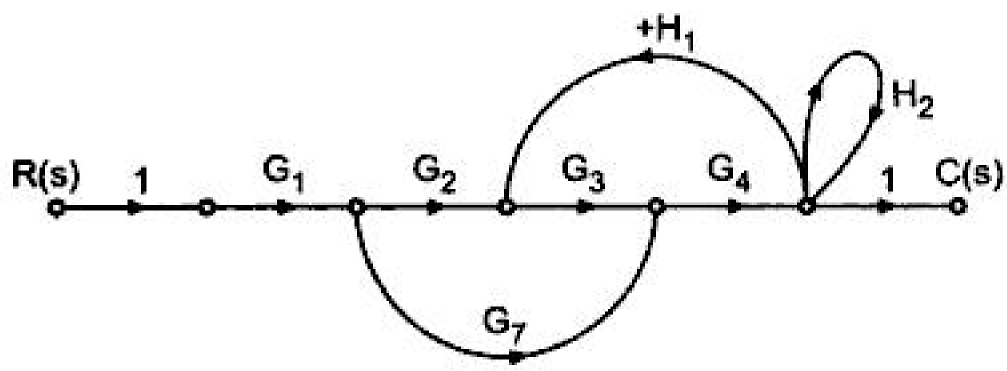
All loops are touching to above forward path.

$$\therefore \quad \Delta_1 = 1$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{T_1 \Delta_1}{\Delta} = \frac{G_1 G_2 G_3 G_4 \cdot 1}{\Delta}$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{1+G_1 H_1+G_3 H_4+G_4 H_3+G_2 G_3 G_4 H_2+G_5+G_1 G_3 H_1 H_4}$$
$$+G_1 G_4 H_1 H_3-G_1 H_1 G_5-G_3 G_5 H_4-G_1 G_3 G_5 H_1 H_4$$

Ex. 4.12 *Find* $\frac{C(s)}{R(s)}$



Sol. : Number of forward paths $= K = 2$

$$\text{T.F.} = \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta} \qquad \text{... By Mason's gain formula}$$

$T_1 = G_1 G_2 G_3 G_4$ and $T_2 = G_1 G_7 G_4$

Combinations of two non-touching loops are

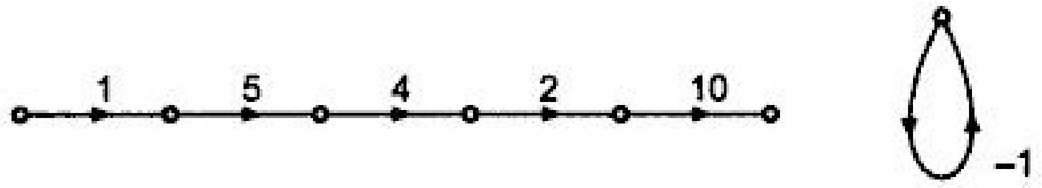
i) L_1 and L_3 and ii) L_2 and L_3

No combination of three non-touching loops :

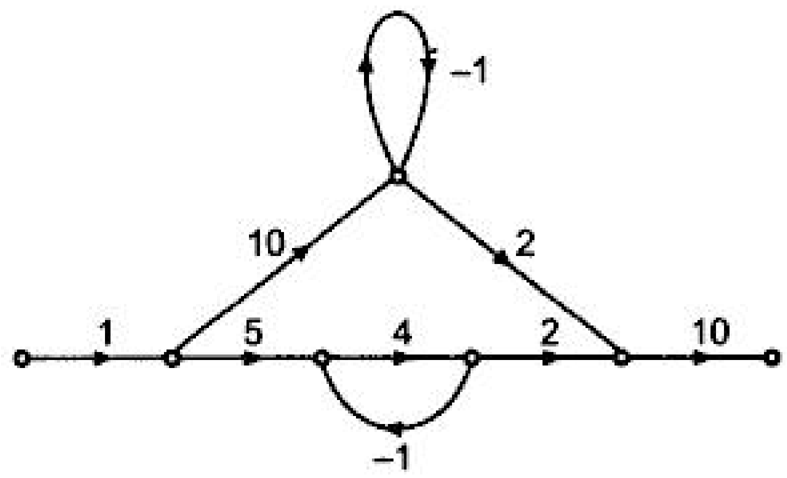
$$\therefore \quad \Delta = 1-[L_1+L_2+L_3]+[L_1 L_3+L_2 L_3]$$
$$= 1-[-4-4-1]+[4+4] = 1+9+8 = 18$$

Consider T_1, L_3 loop is nontouching

$$\therefore \quad \Delta_1 = 1 - L_3 = 1 - [-1] = 2$$



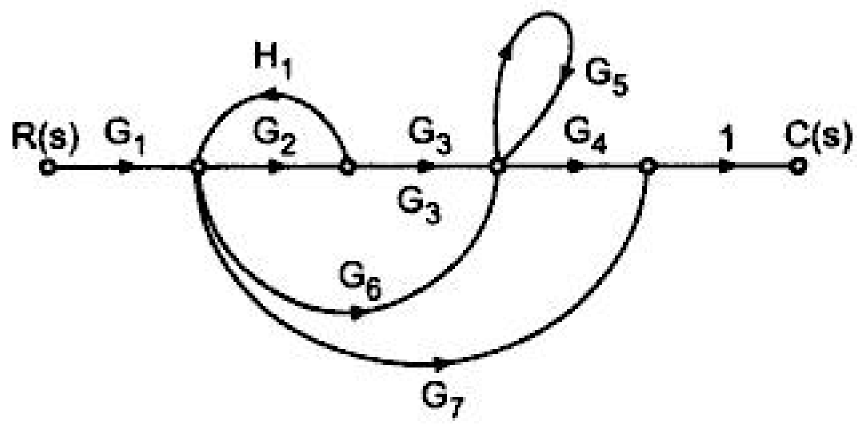
Consider T_2, L_1 is nontouching



$$\Delta_2 = 1 - L_1 = 1 - [-4] = 5$$

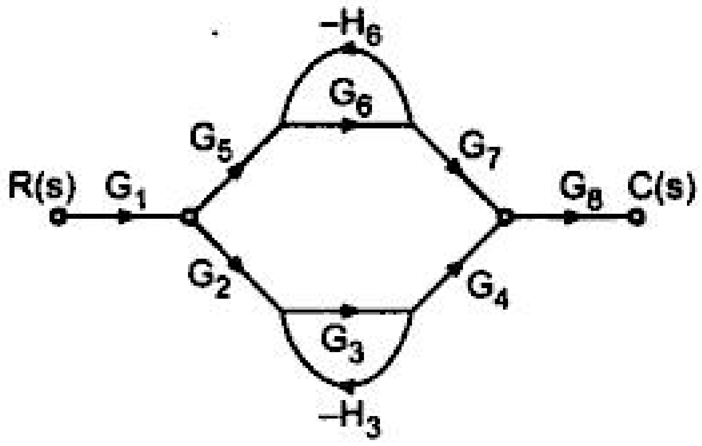
$$\therefore \quad \frac{C(s)}{R(s)} = \frac{T_1\,\Delta_1 + T_2\,\Delta_2}{\Delta} = \frac{400\times 2 + 200\times 5}{18} = \frac{800+1000}{18} = 100$$

Ex. 4.15 *Find* $\frac{C(s)}{R(s)}$



Sol. : Number of forward paths = K = 3

Ex. 4.17 *Find $\frac{C(s)}{R(s)}$ by Mason's gain formula.*

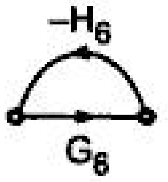


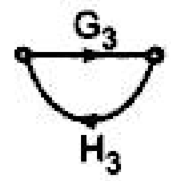
Sol. : Number of forward paths = K = 2

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{\sum_{K=1}^{2} T_K \Delta_K}{\Delta} = \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta} \quad \text{... Mason's gain formula}$$

$\therefore \quad T_1 = G_1 G_5 G_6 G_7 G_8 \qquad T_2 = G_1 G_2 G_3 G_4 G_8$

Individual loops are,



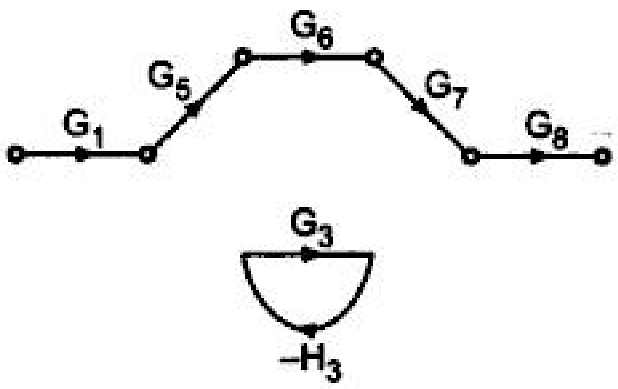


$$L_1 = -G_6 H_6 \qquad L_2 = -G_3 H_3$$

Both L_1 and L_2 are non-touching to each other

$$\therefore \quad \Delta = 1 - [L_1 + L_2] + [L_1 L_2]$$
$$= 1 + G_6 H_6 + G_3 H_3 + G_3 G_6 H_3 H_6$$

Consider T_1, L_2 is non-touching



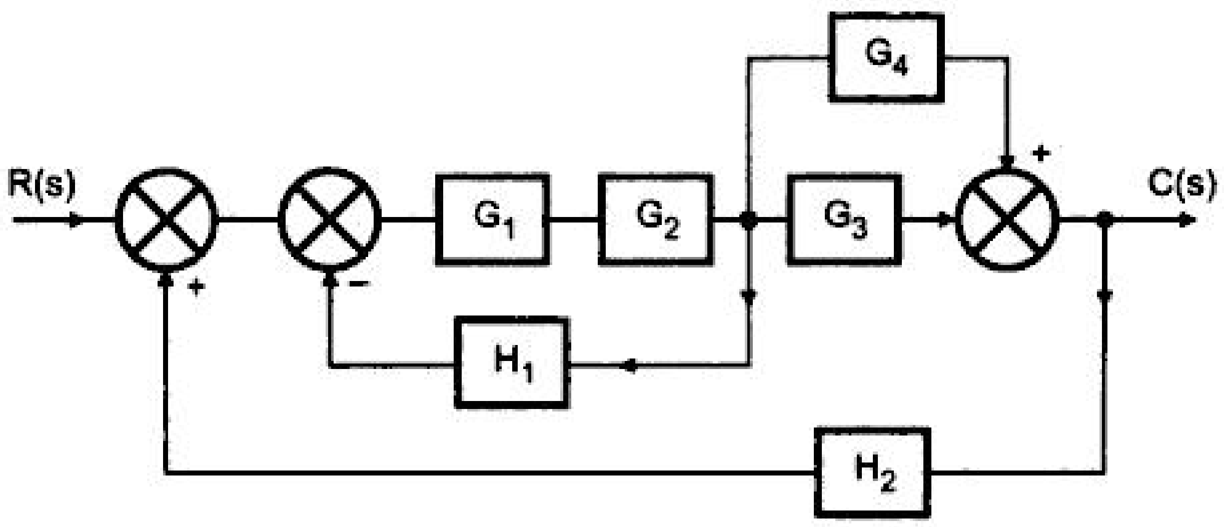
$$\therefore \quad \Delta_1 = 1 - L_2 = 1 + G_3 H_3$$

Consider T_2, L_1 is non-touching

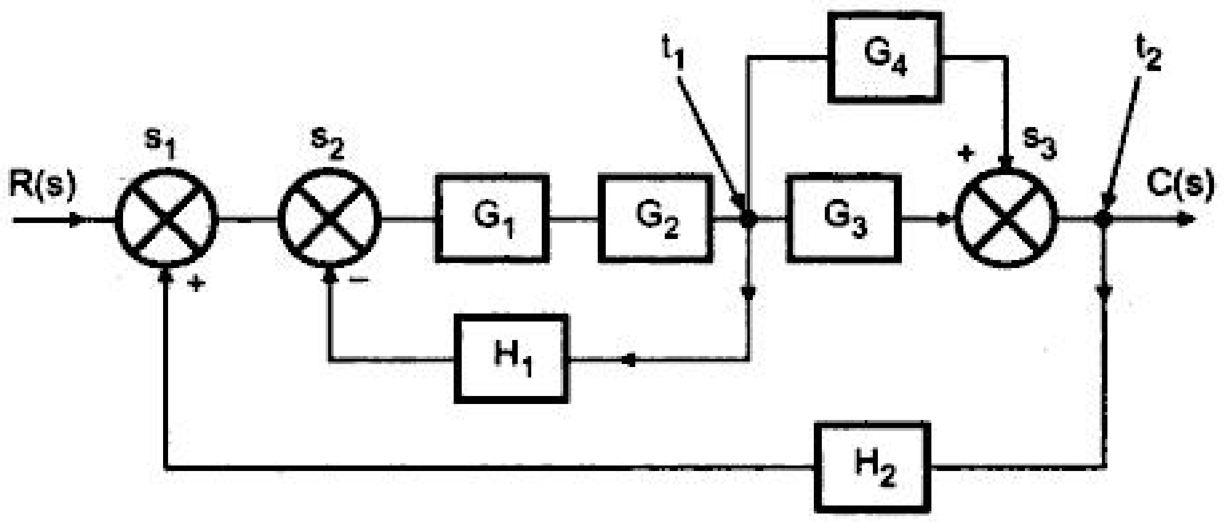
$$\therefore \quad \Delta_2 = 1 - L_1 = 1 + G_6 H_6$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

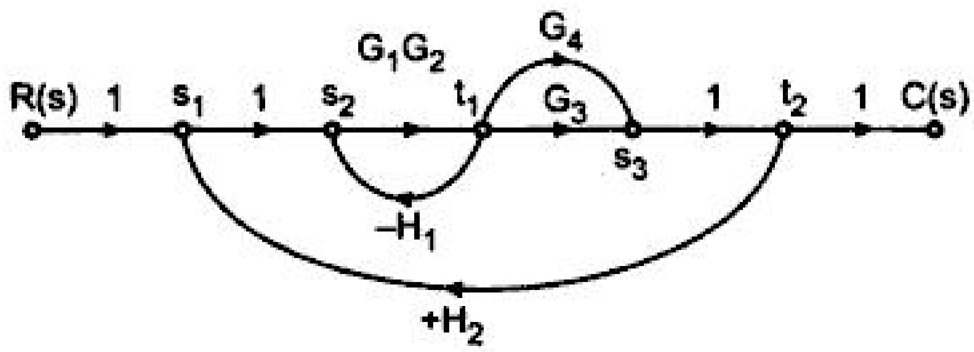
Ex. 4.19 *Draw the corresponding signal flow graph of given block diagram.and find* $\frac{C(s)}{R(s)}$.



Sol. : To draw signal flow graph from given block diagram use the method discussed earlier. Name all the summing points and take-off points of the given block diagram as shown.



The complete S.F.G. for given block diagram, is shown below.

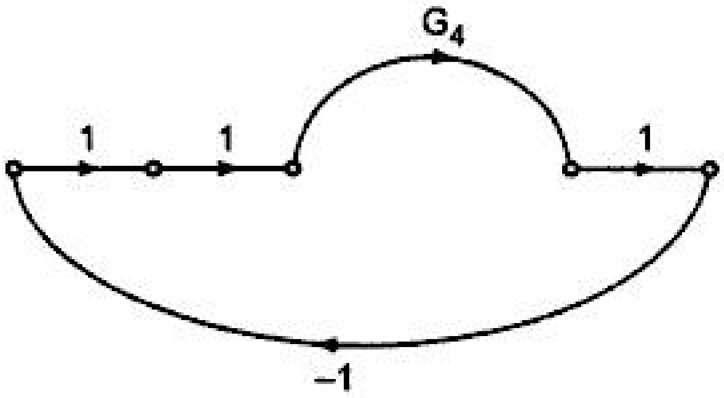


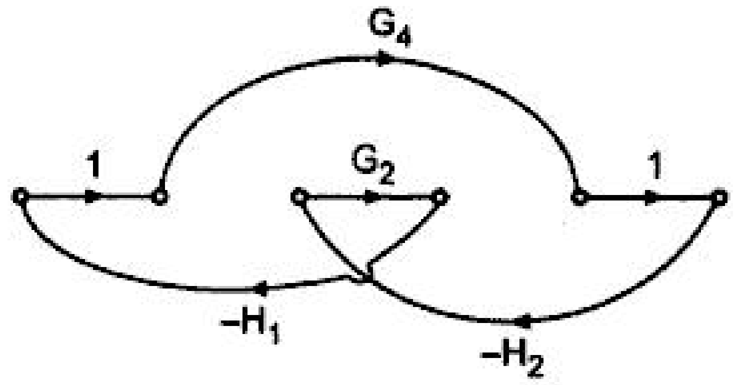
To find $\frac{C(s)}{R(s)}$, use Mason's gain formula.

Number of forward paths = K = 2

$$T_1 = G_1 G_2 G_3 \text{ and } T_2 = G_1 G_2 G_4$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.





$L_4 = -G_4$ $\qquad\qquad$ $L_5 = +G_2 G_4 H_1 H_2$

No combination of non-touching loops.

$$\therefore \quad \Delta = 1-[L_1+L_2+L_3+L_4+L_5]$$

$$= 1+G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 + G_4 - G_2 G_4 H_1 H_2$$

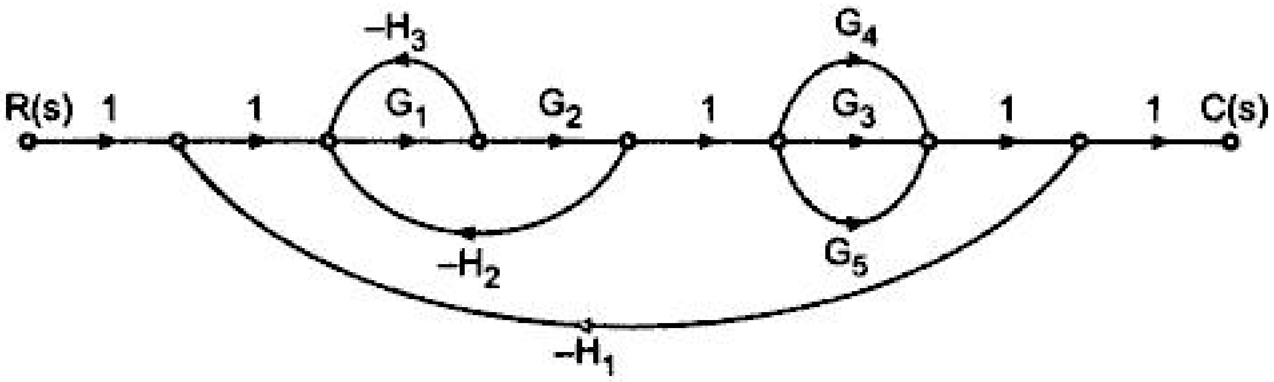
Consider T_1, all loops are touching $\qquad \therefore \Delta_1 = 1$

For T_2, all loops are touching $\qquad \therefore \Delta_2 = 1$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta} = \frac{G_1 G_2 G_3 \cdot 1 + G_4 \cdot 1}{\Delta}$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 + G_4}{1+G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 + G_4 - G_2 G_4 H_1 H_2}$$

Ex. 4.22 *Find* $\frac{C(s)}{R(s)}$

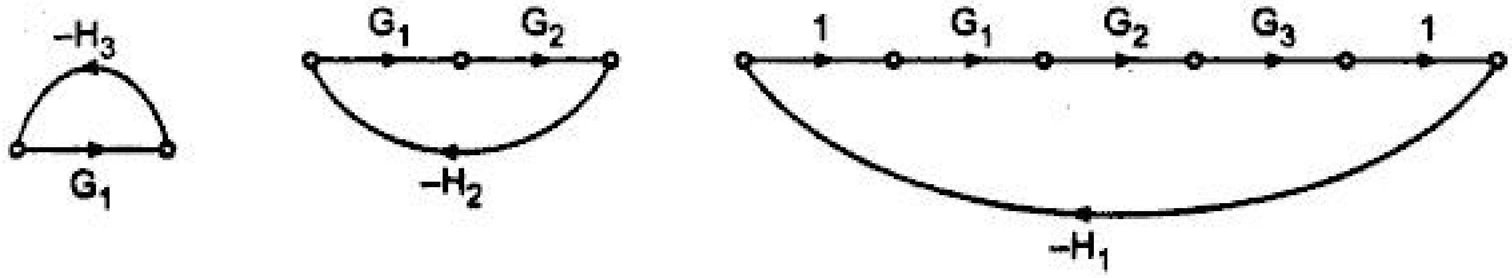


Sol. : Number of forward paths = K = 3

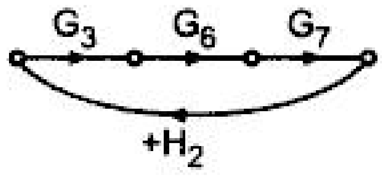
$$\therefore \quad \frac{C(s)}{R(s)} = \frac{T_1 \Delta_1 + T_2 \Delta_2 + T_3 \Delta_3}{\Delta} \qquad \text{.... Mason's gain formula}$$

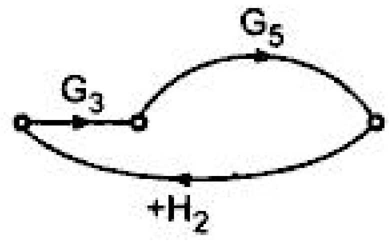
$T_1 = G_1 G_2 G_3$, $T_2 = G_1 G_2 G_4$ $\quad T_3 = G_1 G_2 G_5$

Individual loops are,



You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.





$$L_5 = G_3 G_6 G_7 H_2 \qquad L_6 = G_3 G_5 H_2$$

No combination of nontouching loops.

$$\therefore \quad \Delta = 1 - [L_1 + L_2 + L_3 + L_4 + L_5 + L_6]$$

All loops are touching to all the forward paths from T_1 to T_4

$$\therefore \quad \Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1$$

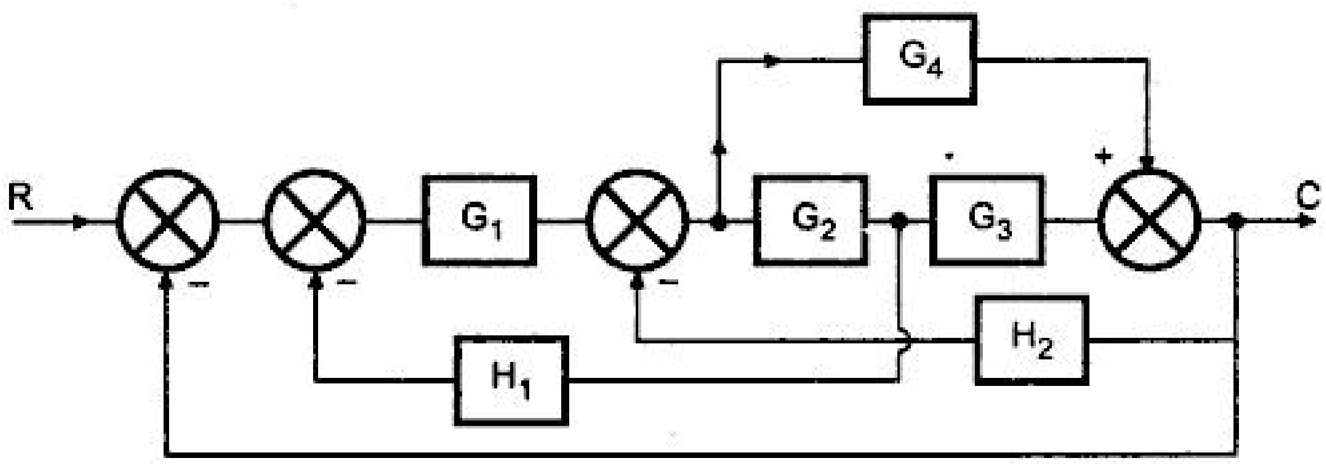
$\therefore$ Using Mason's Gain Formula

$$\frac{X_2}{X_1} = \frac{\Sigma T_K \Delta_K}{\Delta}$$

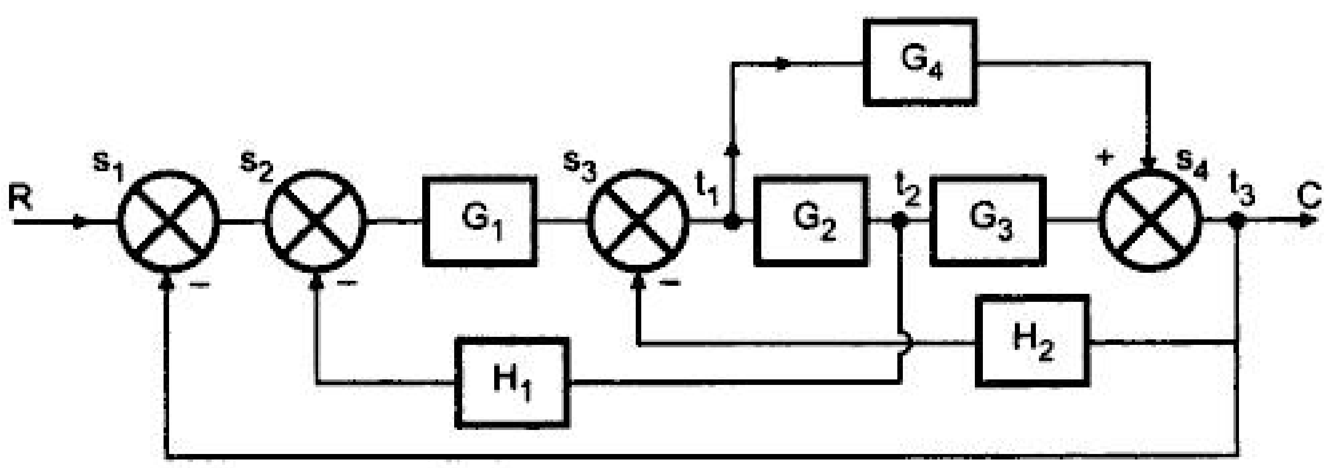
$$= \frac{T_1 \Delta_1 + T_2 \Delta_2 + T_3 \Delta_3 + T_4 \Delta_4}{\Delta}$$

$$\frac{X_2}{X_1} = \frac{G_1 G_2 G_3 G_6 G_7 G_8 + G_1 G_4 G_6 G_7 G_8 + G_1 G_2 G_3 C_5 G_8 + G_1 G_4 G_5 G_8}{1 - G_2 G_3 G_6 G_7 G_8 H_1 - G_4 G_6 G_7 G_8 H_1 - G_2 G_3 G_5 G_8 H_1 - G_4 G_5 G_8 H_1 - G_3 G_6 G_7 H_2 - G_3 G_5 H_2}$$

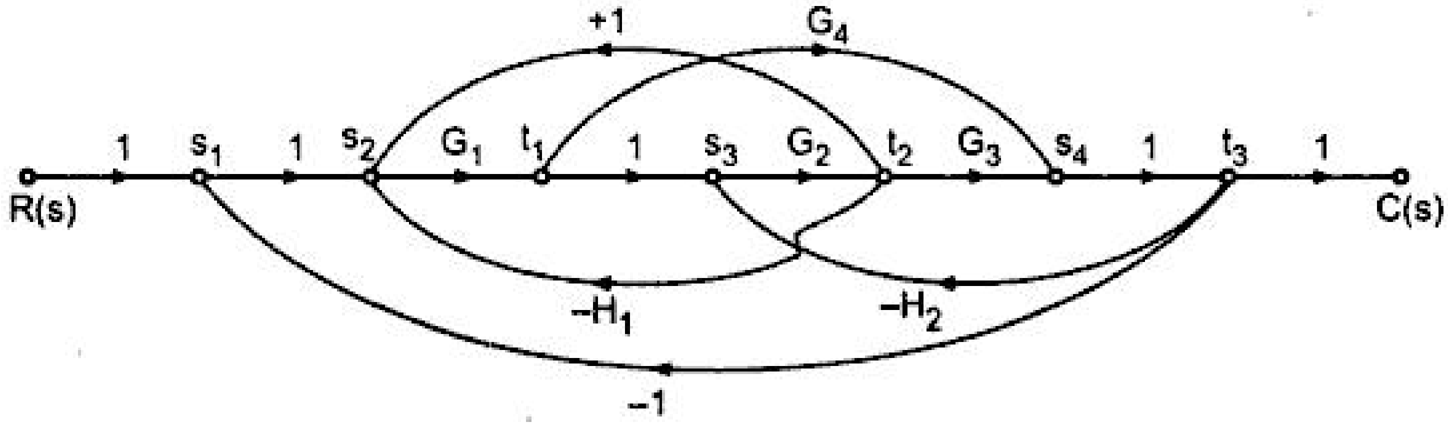
Ex. 4.31 *Use Mason's gain formula to calculate C/R of the system shown below.*



Sol. : Name all the summing and take off points as below.



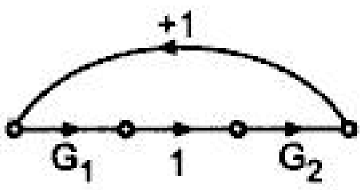
Represent each summing and take off point by separate node and join them to get a signal flow graph.

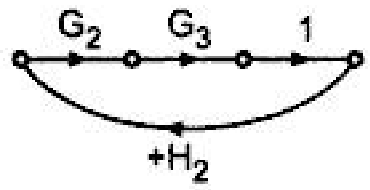


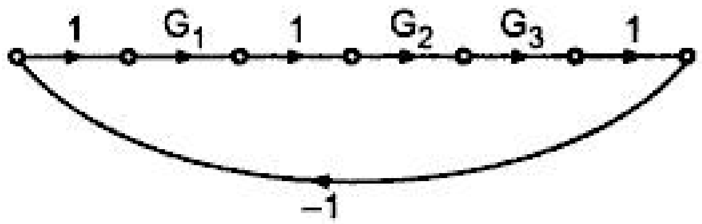
Number of forward paths K = 2

Forward path gains $T_1 = G_1 G_2 G_3 \quad T_2 = G_4 G_1$

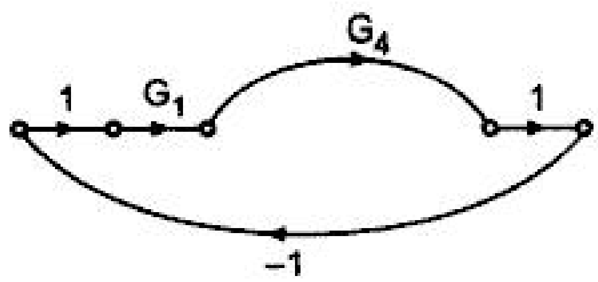
Individual feedback loops are

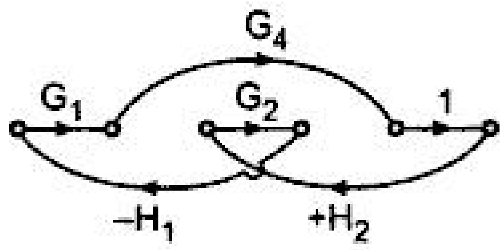


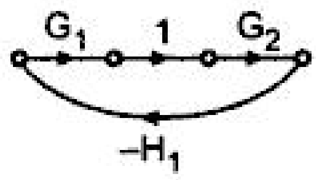




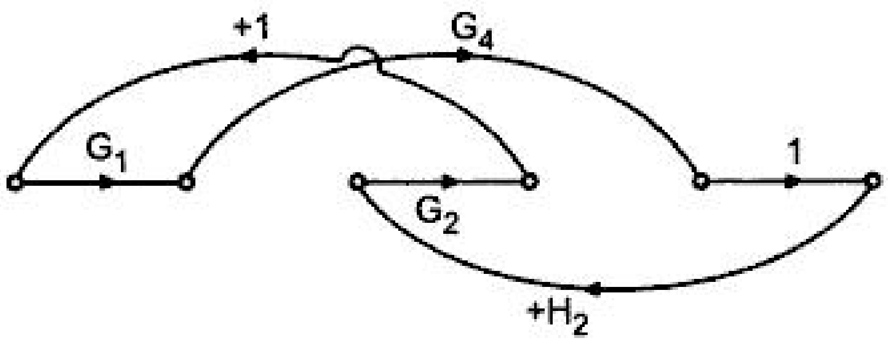
$L_1 = + G_1 G_2 \qquad L_2 = G_2 G_3 H_2 \qquad L_3 = - G_1 G_2 G_3$







$L_4 = - G_1 G_4 \qquad L_5 = - G_1 G_2 G_4 H_1 H_2 \qquad L_6 = - G_1 G_2 H_1$



$$L_7 = G_1 G_2 G_4 H_2$$

There is no combination of nontouching loops.

$\therefore \qquad \Delta = 1 - [L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7]$

All loops are touching to both the forward paths

$\therefore$ Eliminating all loop gains from Δ we get,

$$\Delta_1 = \Delta_2 = 1$$

$\therefore$ Using Mason's Gain formula,

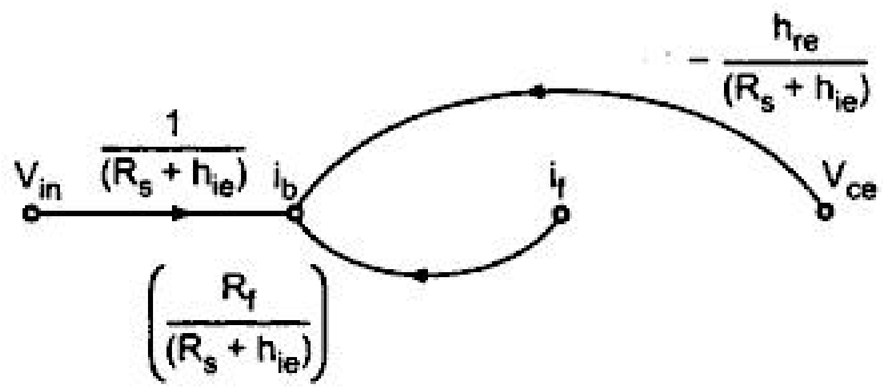
You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

From (3) we can write equation for i_b as

$$i_b = \frac{V_{in} + i_f R_f - h_{re} V_{ce}}{(R_s + h_{ie})}$$

i.e. $$i_b = \frac{1}{(R_s + h_{ie})} V_{in} + \frac{R_f}{(R_s + h_{ie})} i_f - \frac{h_{re}}{(R_s + h_{ie})} V_{ce} \quad \text{... (10)}$$

This can be simulated by signal flow graph as

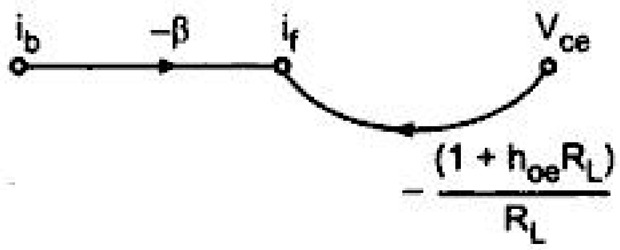


From (8) we can write equation for i_f as,

$$i_f = \frac{- h_{oe} R_L V_{ce} - \beta R_L i_b - V_{ce}}{R_L}$$

i.e. $$i_f = \frac{-(1 + h_{oe} R_L)}{R_L} V_{ce} - \beta i_b \quad \text{... (11)}$$

This can be simulated as

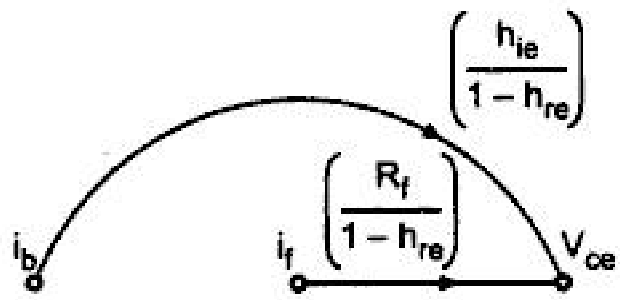


Equation (9) is directly the output equation

$$V_{ce} = i_f R_f + i_b h_{ie} + h_{re} V_{ce}$$

i.e. $$V_{ce} = \left(\frac{R_f}{1 - h_{re}}\right) i_f + \left(\frac{h_{ie}}{1 - h_{re}}\right) i_b \quad \text{... (12)}$$

This can be simulated as,

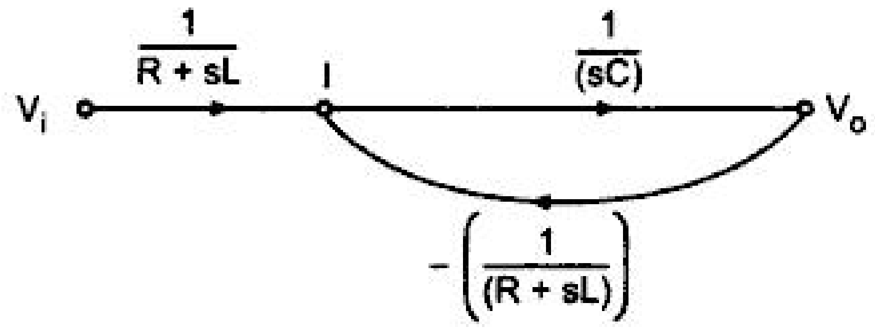


You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Combining above two graphs total signal flow graph is,



Use Mason's Gain Formula.

Number of forward paths = K = 1

$$T_1 = \frac{1}{sC(R+sL)}$$

Individual loops $L_1 = -\frac{1}{sC(R+sL)}$

$$\therefore \quad \Delta = 1 - [L_1] = 1 + \frac{1}{sC(R+sL)}$$

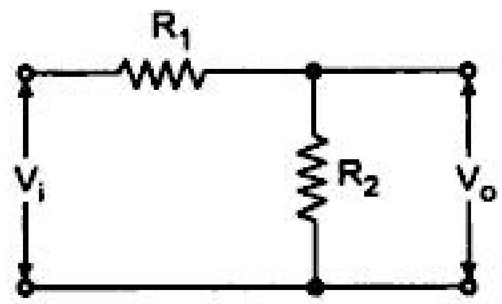
$$= \frac{1+sRC+s^2LC}{sC(R+sL)}$$

For T_1, L_1 is touching $\therefore \Delta_1 = 1$

$$\therefore \quad \frac{V_o(s)}{V_i(s)} = \frac{T_1\Delta_1}{\Delta} = \frac{\frac{1}{sC(R+sL)\cdot 1}}{\Delta} = \frac{\frac{1}{sC(R+sL)}}{\frac{1+sRC+s^2LC}{sC(R+sL)}}$$

$$\therefore \quad \frac{V_o(s)}{V_i(s)} = \frac{1}{s^2LC+sRC+1}$$

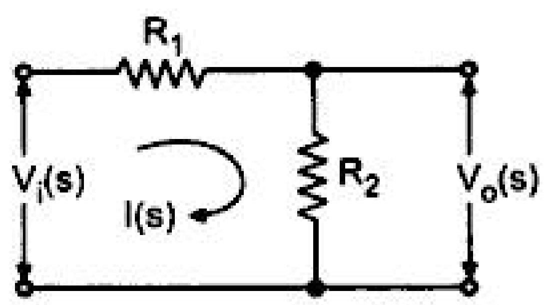
Ex. 4.37 *Find the transfer function for the given network.*



Sol. : Laplace transform of given network.

$$I(s) = \frac{V_i - V_o}{R_1} \quad \dots (1)$$

$$V_o(s) = I(s)\, R_2 \quad \dots (2)$$



You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Out of three, L_1 and L_3 are nontouching

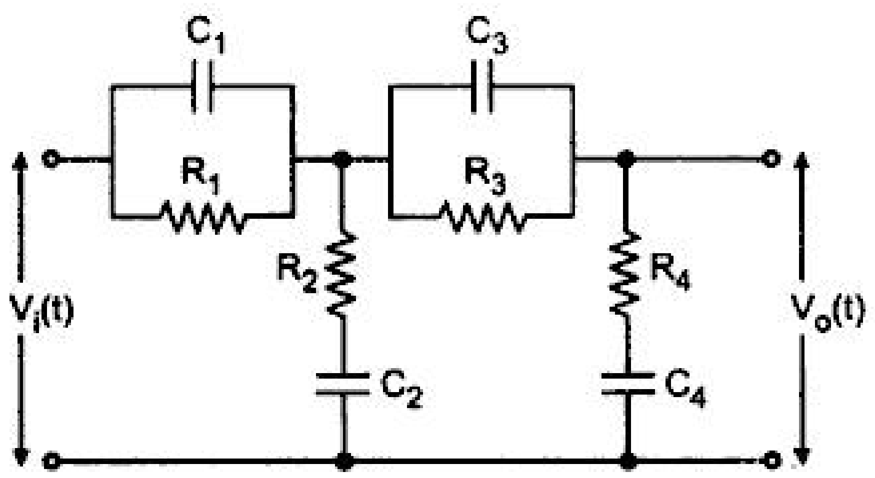
$$\begin{aligned}\Delta &= 1-[L_1+L_2+L_3]+[L_1 L_3]\\ &= 1-[-sR_1 C_1 - sR_1 C_2 - sR_2 C_2]+\left[s^2 R_1 C_1 R_2 C_2\right]\\ &= 1+s\,[R_1 C_1 + R_1 C_2 + R_2 C_2] + s^2\;[R_1 C_1 R_2 C_2]\end{aligned}$$

$$\therefore \qquad \frac{V_o(s)}{V_i(s)} = \frac{T_1\,\Delta_1}{\Delta}$$

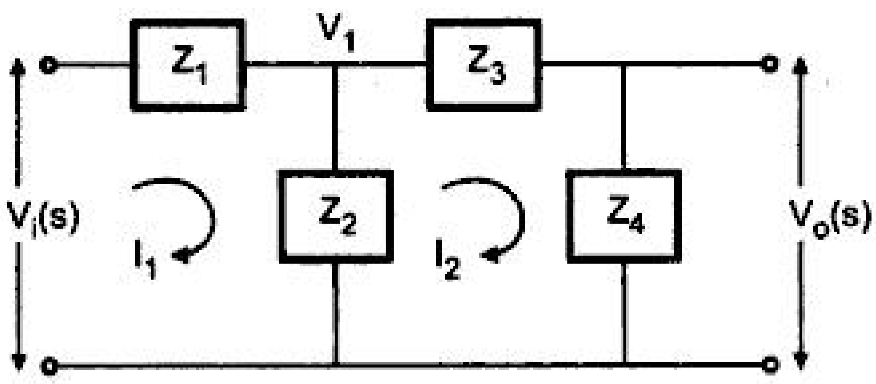
All loops are touching to T_1, $\therefore\ \Delta_1 = 1$

$$\therefore \qquad \frac{V_o(s)}{V_i(s)} = \frac{s^2 R_1 C_1 R_2 C_2}{1+s\,[R_1 C_1 + R_1 C_2 + R_2 C_2]+s^2\;[R_1 C_1 R_2 C_2]}$$

Ex. 4.42 *Find out the transfer function* $\frac{V_o(s)}{V_i(s)}$ *of the given electrical network.*



Sol. : Converting given network to its laplace form and assuming different loop currents and voltages as shown we get.



$$Z_1 = R_1 \,||\, \frac{1}{sC_1} = \frac{R_1 \times \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{R_1}{1+sR_1 C_1}$$

$$Z_3 = R_3 \,||\, \frac{1}{sC_3} = \frac{R_3}{1+sR_3 C_3}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

$$\frac{V_o}{V_i} = \frac{\Sigma T_K \Delta_K}{\Delta}\ ; \text{ No. of forward paths} = 1$$

$$\therefore \quad \frac{V_o}{V_i} = \frac{T_1 \Delta_1}{\Delta}$$

$$T_1 = \frac{L}{R_1 R_2 C}$$

Individual feedback loops are

$$L_1 = -\frac{1}{sR_1 C}, \qquad L_2 = -\frac{1}{sR_2 C}, \qquad L_3 = -\frac{sL}{R_2}$$

L_1 and L_3 are nontouching

$$\therefore \quad \Delta = 1 - [L_1 + L_2 + L_3] + [L_1 L_3]$$

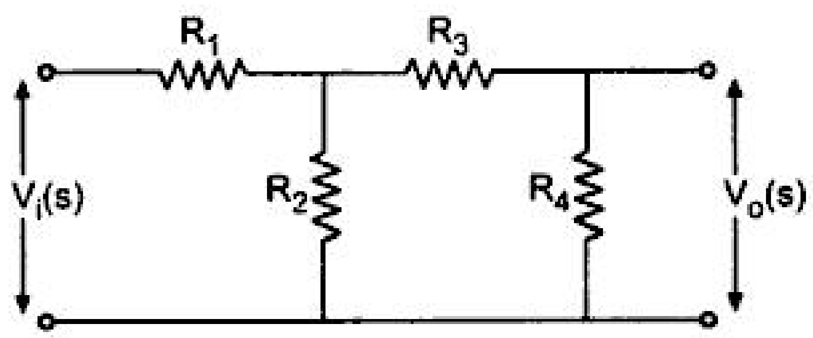
$$\Delta = 1 + \frac{1}{sR_1 C} + \frac{1}{sR_2 C} + \frac{sL}{R_2} + \frac{L}{R_1 R_2 C}$$

As all loops are touching to T_1, $\Delta_1 = 1$

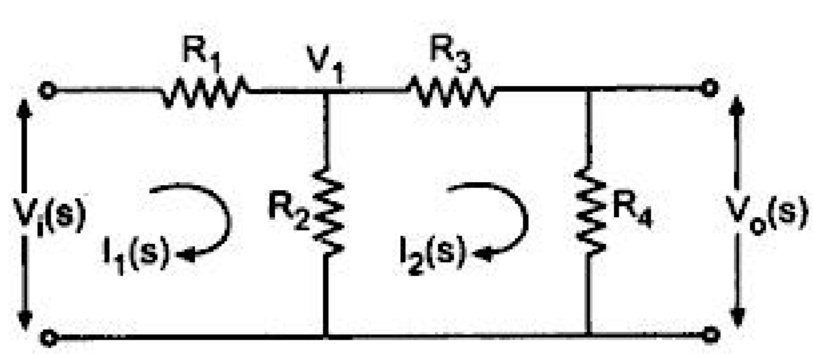
$$\therefore \quad \frac{V_o}{V_i} = \frac{\dfrac{L}{R_1 R_2 C}}{1 + \dfrac{1}{sR_1 C} + \dfrac{1}{sR_2 C} + \dfrac{sL}{R_2} + \dfrac{L}{R_1 R_2 C}}$$

$$\therefore \quad \frac{V_o}{V_i} = \frac{sL}{sR_1 R_2 C + R_2 + R_1 + s^2 L R_1 C + sL}$$

Ex. 4.44 *Find the T.F. of the given network*

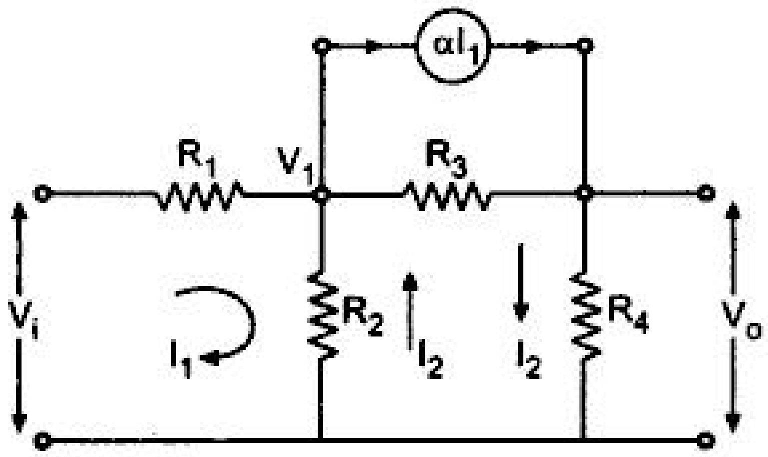


Sol. : Laplace Transform of the given network is,



You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Ex. 4.46 *Draw signal flow graph for the given electrical network if the different branch currents are as shown.* **(Mumbai University Dec. 98)**

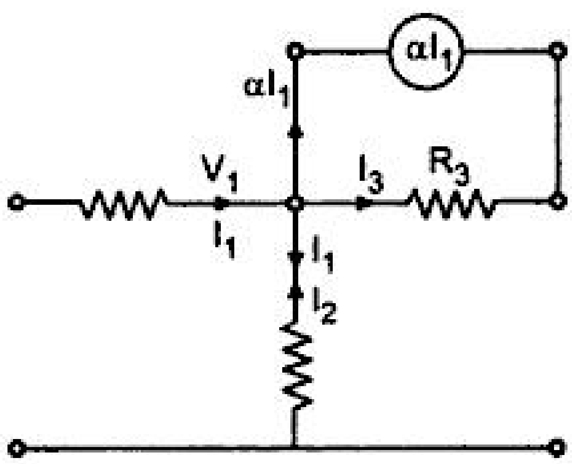


Sol. : Equations for different currents and voltages are as follows

$$I_1 = \frac{(V_i - V_1)}{R_1} \qquad \text{... (I)}$$

$$V_1 = (I_1 - I_2) R_2 \qquad \text{... (II)}$$

Now I_2 is current flowing through `$R_4$' and `R_2' but current flowing through R_3 is not I_2. Consider the network.



At node V_1, currents entering = current leaving.

Now let I_3 be the current flowing through R_3.

$$\therefore \quad I_1 + I_2 = \alpha I_1 + I_3 + I_1$$

$$\therefore \quad I_3 = I_2 + I_1(1-\alpha) - I_1 = I_2 - \alpha I_1 = \frac{(V_1 - V_o)}{R_3}$$

$$\therefore \quad I_2 - \alpha I_1 = \frac{V_1 - V_o}{R_3}$$

$$\therefore \quad I_2 = \frac{(V_1 - V_o)}{R_3} + \alpha I_1 \qquad \text{... (III)}$$

and $$V_o = I_2 R_4 \qquad \text{... (IV)}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

$$C = \frac{G_1G_2R_1}{1+G_2H_2+G_1G_2H_1}$$

For R_2 acting alone,

$$T_1 = G_2 \qquad L_1 = -G_2H_2 \qquad L_2 = -G_1G_2H_1$$

No nontouching combination.

$$\Delta = 1-[L_1+L_2]$$
$$= 1+G_2H_2+G_1G_2H_1$$

and $\quad \Delta_1 = 1$

$$\therefore \quad \frac{C}{R_2} = \frac{T_1\Delta_1}{\Delta}$$
$$= \frac{G_2}{1+G_2H_2+G_1G_2H_1}$$
$$C = \frac{G_2R_2}{1+G_2H_2+G_1G_2H_1}$$

For R_3 acting alone,

$$T_1 = -G_2 \qquad L_1 = -G_2H_2 \qquad L_2 = -G_1G_2H_1$$

No nontouching combination.

$$\Delta = 1-[L_1+L_2] = 1+G_2H_2+G_1G_2H_1$$

and $\quad \Delta_1 = 1$

$$\therefore \quad \frac{C}{R_3} = \frac{T_1\Delta_1}{\Delta} = \frac{-G_2}{1+G_2H_2+G_1G_2H_1}$$
$$C = \frac{-G_2R_3}{1+G_2H_2+G_1G_2H_1}$$

For R_4 acting alone,

$$T_1 = -G_1G_2H_1 \qquad L_1 = -G_2H_2 \qquad L_2 = -G_1G_2H_1$$

No nontouching combination.

$$\Delta = 1-[L_1+L_2]$$
$$= 1+G_2H_2+G_1G_2H_1$$

and $\quad \Delta_1 = 1$

$$\therefore \quad \frac{C}{R_4} = \frac{T_1\Delta_1}{\Delta} = \frac{-G_1G_2H_1}{1+G_2H_2+G_1G_2H_1}$$

$$\therefore \quad C = \frac{-G_1G_2H_1R_4}{1+G_2H_2+G_1G_2H_1}$$

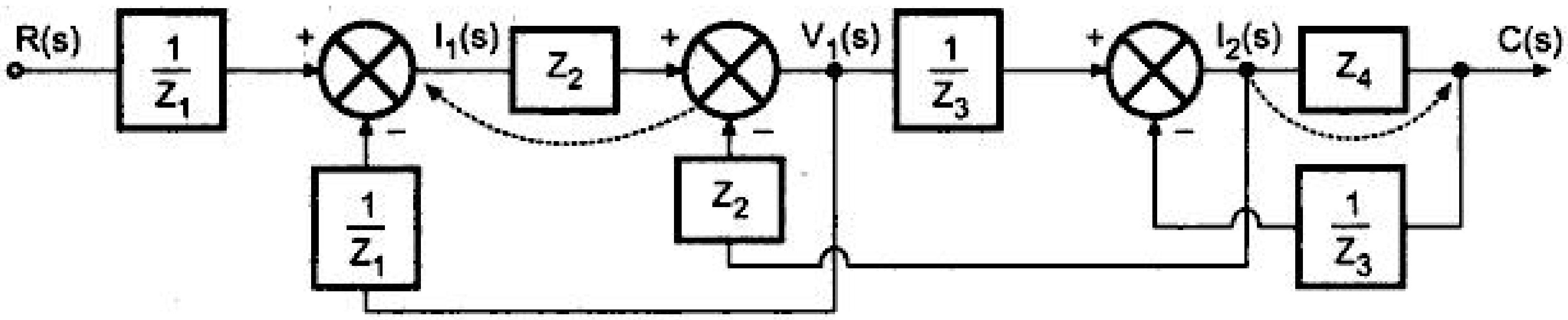
$$\therefore \quad C = \frac{G_1G_2R_1+G_2(R_2-R_3)-G_1G_2H_1R_4}{1+G_2H_2+G_1G_2H_1}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

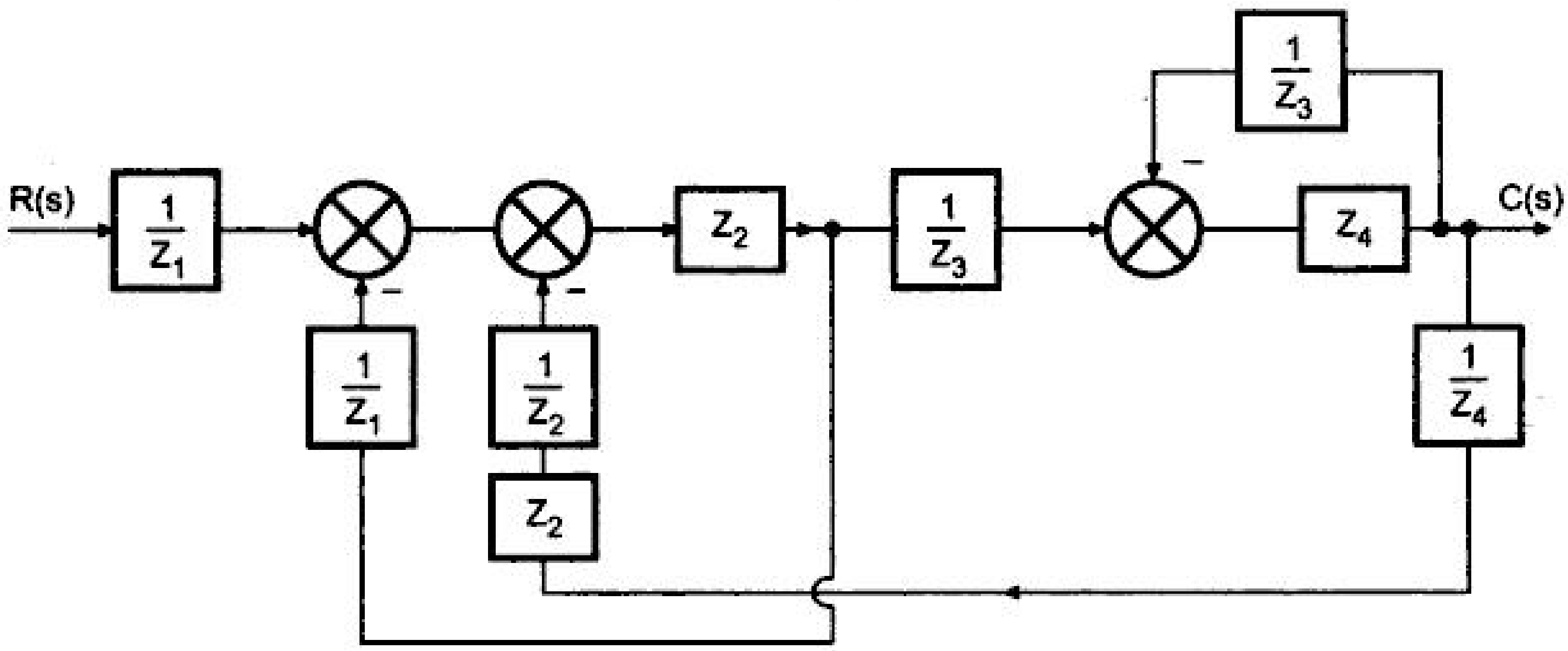
$$\therefore \qquad I_2(s) = \frac{1}{Z_3} V_1(s) - \frac{1}{Z_3} C(s) \qquad \text{... (3)}$$

$$\text{and} \qquad C(s) = Z_4\, I_2(s) \qquad \text{... (4)}$$

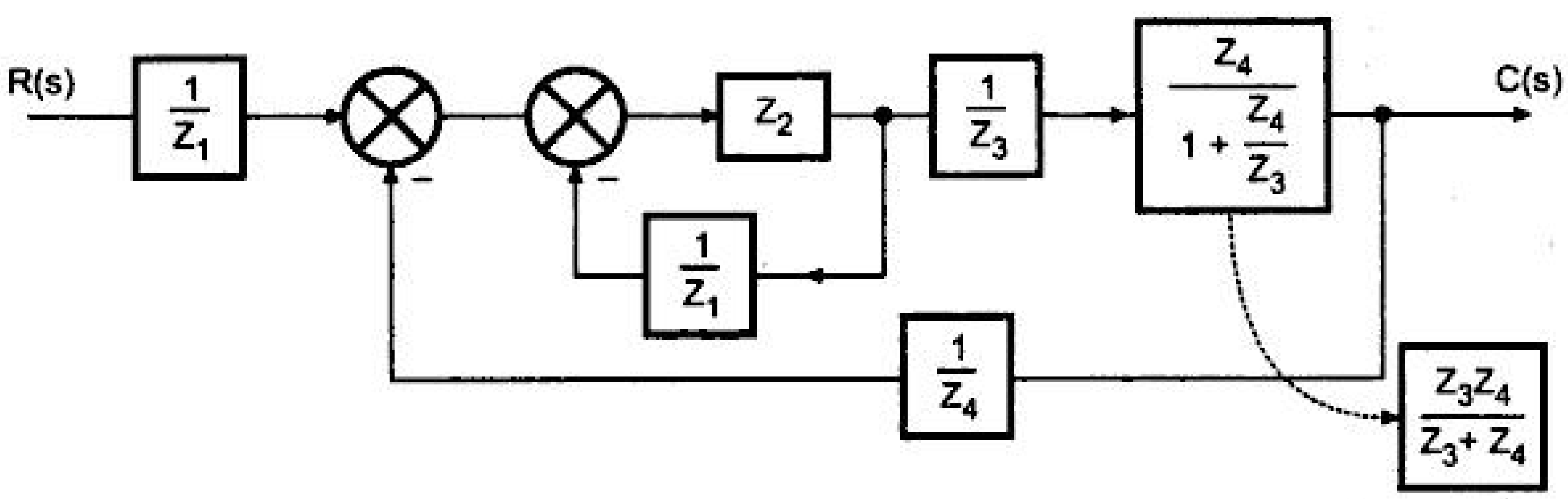
The corresponding block diagram can be obtained by combining individual block diagram simulation of each equation as,



Shifting summing point to the left and take off to the right we get,



Interchanging summing points using associative law and solving minor feedback loop of $\frac{1}{Z_3}$ and Z_4 we get,



You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Sol. : The forward path gains are

$$T_1 = G_1G_2G_3 \qquad T_2 = G_4$$

The various feedback loop gains are,

$$L_1 = -G_1G_2H_1 \qquad L_2 = -G_1G_2G_3 \qquad L_3 = -G_2G_3H_2$$

$$L_4 = -G_4 \qquad L_5 = G_4(-H_2)(G_2)(-H_1) = G_2G_4H_1H_2$$

All are touching to each other

$$\therefore \quad \Delta = 1-[L_1+L_2+L_3+L_4+L_5]$$

Now $\quad \Delta_1 = 1$ and $\Delta_2 = 1$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{T_1\Delta_1+T_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2G_3+G_4}{1+G_1G_2H_1+G_1G_2G_3+G_2G_3H_2+G_4-G_2G_4H_1H_2}$$

Summary

Signal flow graph is another technique of representating complicated control systems. In this method, variables of the system are represented by separate nodes.

The value of the variable represented by any node is an algebraic sum of all the signals entering at that node.

The signal gets multiplied by the branch gain when it travels along the branch.

The number of outgoing branches from a node does not affect the value of the variable represented by that node.

The signal flow graph is mainly useful when a set of linear equations is available for a system.

The signal flow graph is not the unique property of the system. Number of signal flow graphs can be constructed by writing system equations in different manner.

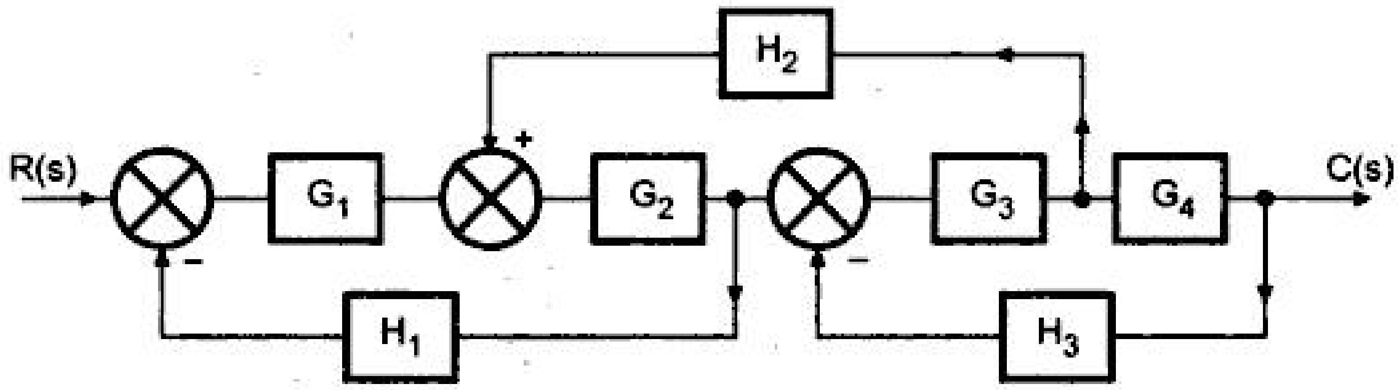
While obtaining signal flow graph from the block diagram, each summing and takeoff point must be represented by a separate node. In signal flow graph gain of the forward path or feedback loop is just the product of all the branch gains which are involved in the formation of corresponding forward path or feedback loop. Self loop is loop originating and terminating at the same node. While identifying various forward paths and feedback loops, no node should be traced twice. Each node should be traced only once. The loops having no node common in between are called as non-touching loops.

The resultant transfer function of the system can be obtained from signal flow graph by using Mason's Gain Formula.

The Mason's Gain Formula is

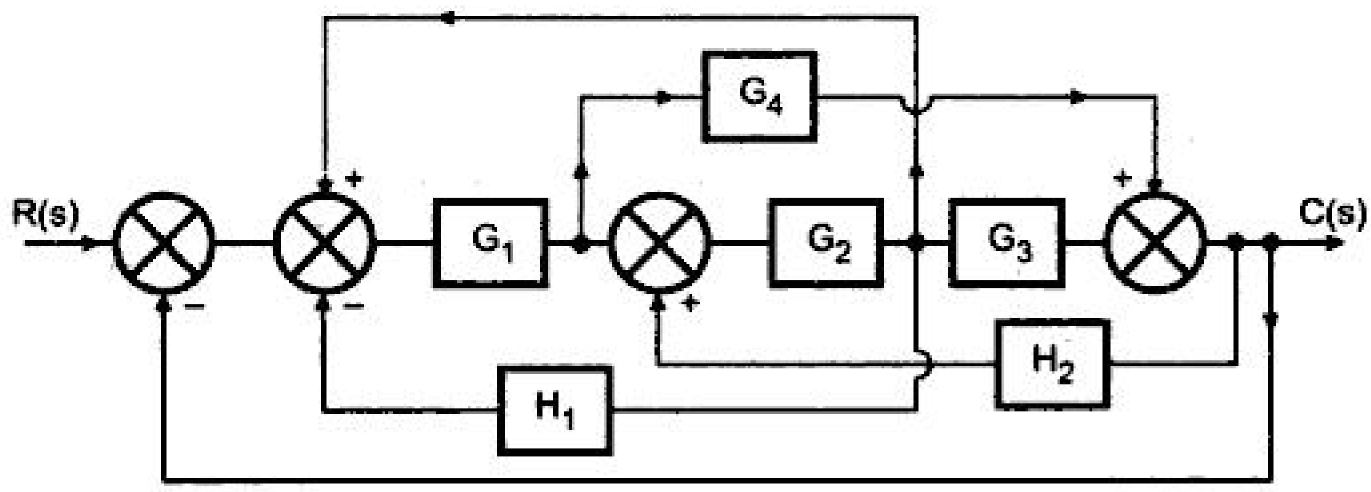
You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

15. Draw the signal flow graph and obtain the transfer function of the system shown below.



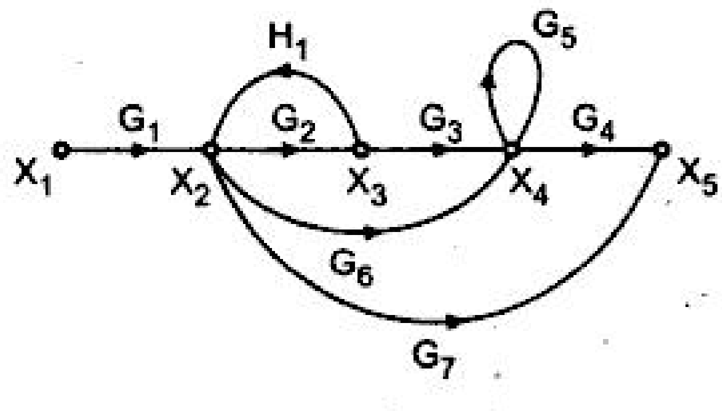
$$\textbf{Ans. :}\ \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{1 + G_1 G_2 H_1 - G_2 G_3 H_2 + G_3 G_4 H_3 + G_1 G_2 G_3 G_4 H_1 H_3}$$

16. Use Mason's Gain Formula to obtain C(s)/R(s) of the system shown below.



$$\textbf{Ans. :}\ \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 + G_1 G_4}{1 - G_1 G_2 - G_2 G_3 H_2 + G_1 G_2 G_3 + G_1 G_4 + G_1 G_2 G_4 H_1 H_2 + G_1 G_2 H_1 - G_1 G_2 G_4 H_2}$$

17. For the signal flow graph shown in following Fig. determine the ratio x_5 / x_1. Use Mason's gain formula for signal flow graphs.



$$\textbf{Ans. :}\ \frac{x_5}{x_1} = \frac{G_1 G_2 G_3 G_4 + G_1 G_4 G_6 + G_1 G_7 (1 - G_5)}{1 - G_1 H_1 + G_2 G_5 H_1 - G_5}$$

18. Using Mason's gain formula determine the system output for input R and disturbances D_1 and D_2 *for the system described in the block diagram of Fig.*

5
Mathematical Modeling of Systems

5.1 Introduction

To study and examine a physical system it is always necessary to have some types of equivalent representation which will describe the various components and their relations in the system. In last chapter, we have seen block diagram and signal flow graph representation of the system.

But to obtain the block diagram and hence the transfer function, it is necessary to analyse the given system. And to obtain block diagram, it is necessary to obtain the set of mathematical equations describing the dynamic behaviour of the system. This is called **mathematical modeling** of the system. The various operations in the system are represented by the mathematical equations, in such model. Generally it is very difficult to represent the exact mathematical relations between the system parameters. Hence first, a simple physical model is obtained with some approximations whose behaviour is almost same as that of system under consideration . When appropriate physical laws are used on such physical model, a mathematical model is obtained.

Most control systems contain mechanical or electrical or both types of elements and components. To analyse such systems, it is necessary to convert such systems into mathematical models based on transfer function approach. From mathematical angle of view, models of mechanical and electrical components are exactly analogous to each other. Not only this but we can show that for given mechanical system there is always an analogous electrical network exists and vice versa. The mathematical equations describing both the systems are exactly same in nature.

As we are well familiar with the behaviour of electrical networks and methods of writing equations for it, it will be better if we can draw equivalent electrical networks for given mechanical systems. This will help us in writing system equations in simplified manner and with more detailed understanding.

In this chapter we will learn,

i) What are analogous networks.

ii) How to derive electrical - mechanical analogous networks.

iii) How to write equilibrium equations for analogous networks.

iv) How to solve mechanical systems of different types i.e. rotational, containing gear train etc. and get analogous electrical networks.

v) Derivations of the transfer functions of various elements and systems which are very commonly used in control system analysis.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Similarly if friction is between two moving surfaces, it is shown as below.

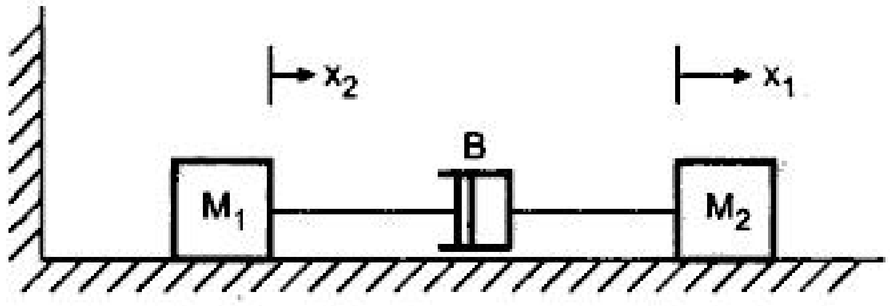


Fig. 5.8

In such a case, opposing force is given by,

$$F_{frictional} = B\left[\frac{d\,x_1(t)}{dt} - \frac{d\,x_2(t)}{dt}\right]$$

Taking laplace,

$$F_{frictional}(s) = Bs\,[X_1(s) - X_2(s)]$$

Thus if the force applied to mass M_2 is f(t) then due to friction between the masses M_1 and M_2, the force getting transmitted to M_1 is always less than F(t). Hence the displacement of mass M_1 is different than the displacement of mass M_2. **The friction between two moving points, causes a change in displacement from one point to other.** Frictional force also behaves exactly in same manner, in rotational systems, only linear frictional constant becomes torsional frictional constant but denoted by same symbol 'B' only.

5.3 Rotational Motion

This is the motion about a fixed axis. In such systems, the force gets replaced by a moment about the fixed axis i.e. (force × distance from fixed axis) which is called **Torque.**

So extension of Newton's law states that the sum of the torques applied to a rigid body or a system must be equal to sum of the torques consumed by the different elements of the system in order to produce angular displacement (θ), angular velocity (ω) and angular acceleration (α) in them. As previously stated, spring and friction behaves in same manner in rotational systems. The property of system which stores kinetic energy in rotational system is called **Inertia** and denoted by 'J' i.e. moment of inertia. Opposing torque due to inertia 'J' is proportional to the angular acceleration (α) of that inertia.

$$\therefore \qquad T_{\text{due to inertia}} = J\frac{d^2\theta(t)}{dt^2} \qquad \text{where } \alpha = \frac{d^2\theta}{dt^2}$$

Taking Laplace,

$$T_{\text{due to inertia}}(s) = J\,s^2\,\theta(s)$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

5.6.1. Gear Train With Inertia and Friction

In practice, gears do have inertia and friction which can not be neglected. Consider such practical gear arrangement connected to the load, shown below.

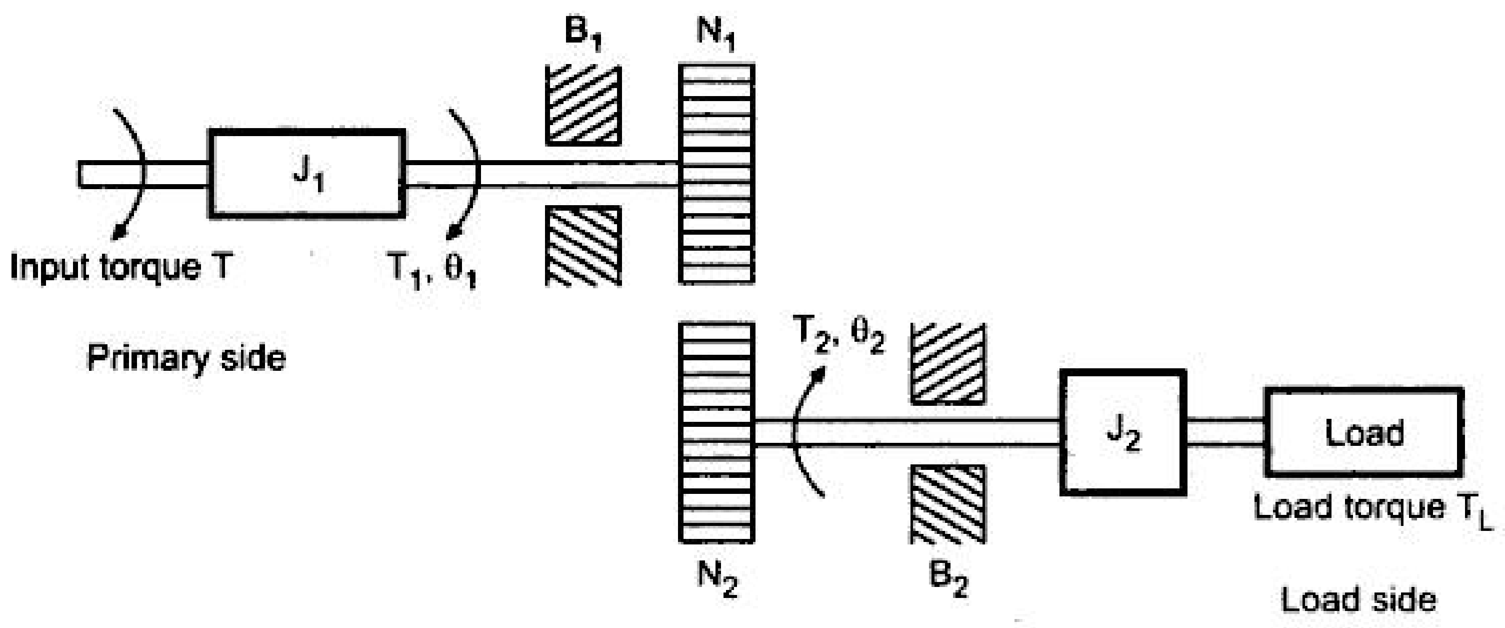


Fig. 5.15

T = applied torque

T_1, T_2 = torque transmitted to gears

N_1, N_2 = number of teeth

θ_1, θ_2 = angular displacements

J_1, J_2 = inertia of gears.

B_1, B_2 = friction coefficients

Torque equation of side 1 is,

$$T = J_1 \frac{d^2\theta_1(t)}{dt^2} + B_1 \frac{d\theta_1(t)}{dt} + T_1(t) \quad \text{... (1)}$$

Torque equation of side 2 is,

$$T_2 = J_2 \frac{d^2\theta_2(t)}{dt^2} + B_2 \frac{d\theta_2(t)}{dt} + T_L(t) \quad \text{... (2)}$$

Now $$\frac{T_1}{T_2} = \frac{N_1}{N_2} = \frac{\theta_2}{\theta_1}$$

$\therefore$ $$T_2 = \frac{N_2}{N_1} T_1$$

Substituting in equation (2)

$\therefore$ $$\frac{N_2}{N_1} T_1 = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_L$$

$\therefore$ $$T_1 = \frac{N_1}{N_2} J_2 \frac{d^2\theta_2}{dt^2} + \frac{N_1}{N_2} B_2 \frac{d\theta_2}{dt} + \frac{N_1}{N_2} T_L \quad \text{... (3)}$$

Substituting value of T_1 in (1)

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

According to Newton's law of motion, applied force will cause displacement x(t) in spring, acceleration to mass M against frictional force having constant B.

$$\therefore \qquad F(t) = Ma + Bv + K\,x(t)$$

Where, a = acceleration, v = velocity

$$\therefore \qquad F(t) = M\frac{d^2\,x(t)}{dt^2} + B\frac{d\,x(t)}{dt} + K\,x(t)$$

Taking Laplace, $F(s) = Ms^2\,X(s) + Bs\,X(s) + K\,X(s)$

This is equilibrium equation for the given system.

Now we will try to derive analogous electrical network.

5.8.2 Force Voltage Analogy (Loop Analysis)

In this method, to the force in mechanical system, voltage is assumed to be analogous one. Accordingly we will try to derive other analogous terms. Consider electric network as shown in the Fig. 5.24.

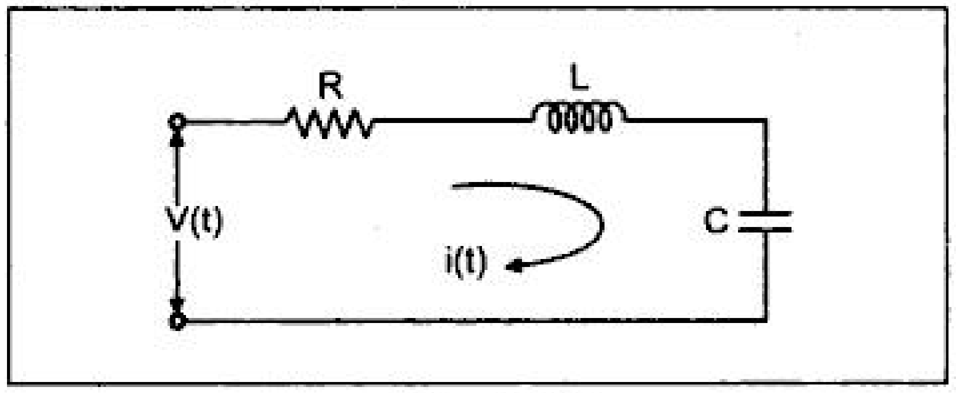


Fig. 5.24

The equation according to Kirchhoff's law can be written as,

$$V(t) = i(t)\,R + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt$$

Taking Lapalce,

$$V(s) = I(s)\,R + Ls\,I(s) + \frac{I(s)}{sC}$$

But we cannot compare F(s) and V(s) unless we bring them into same form.

For this we will use current as rate of flow of charge.

$$\therefore \qquad i(t) = \frac{dq}{dt}$$

i.e. $\qquad I(s) = s\,q(s) \quad \text{or} \quad q(s) = \frac{I(s)}{s}$

Replacing in above equation,

$$V(s) = L\,s^2 q(s) + Rs\,q(s) + \frac{1}{C}q(s)$$

Comparing equations for F(s) and V(s) it is clear that,

i) Inductance 'L' is analogous to mass M

ii) Resistance 'R' is analogous to friction B.

iii) Reciprocal of capacitor i.e. $1/C$ is analogous to spring of constant K.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

i) Force -Voltage Method :

$$F(s) = V(s)$$

$$X(s) = q(s), \text{ charge}$$

$$\therefore \qquad V(s) = q(s)\left[s^2 L + \frac{1}{C} + sR\right]$$

Replacing $\qquad q(s) = \dfrac{I(s)}{s}$

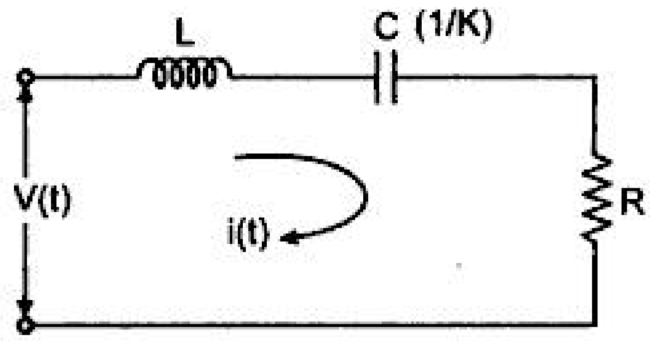
$$V(s) = I(s)\left[sL + \frac{1}{sC} + R\right]$$

$$s = \frac{d}{dt}, \quad \frac{1}{s} = \int dt$$

i.e. $\qquad V(t) = L\dfrac{di}{dt} + \dfrac{1}{C}\int i\,dt + iR$

Simulating using loop method :

Analogous to `K' is capacitor `C' only but its value is proportional to $\frac{1}{K}$. To indicate this, the general method is to write (1/K) in bracket near to `C' which is indicating value of the capacitor in terms of spring constant `K'.



ii) Force - Current Method :

$$F(s) = I(s) \text{ and } \phi(s) = X(s)$$

Using analogous quantities, equation (1) becomes,

$$I(s) = \phi(s)\cdot\left[s^2 C + \frac{1}{R}s + \frac{1}{L}\right]$$

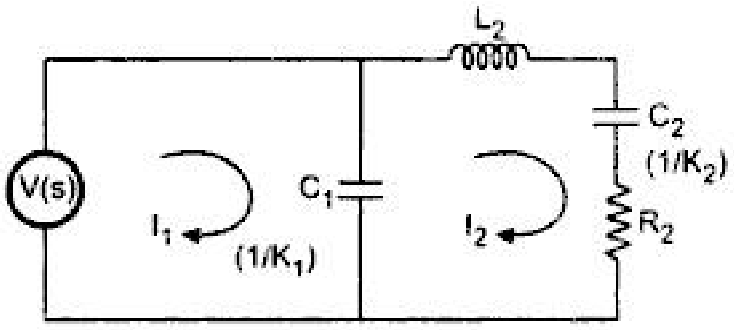
Replacing $\qquad \phi(s) = \dfrac{V(s)}{s}$

$$\therefore \qquad I(s) = V(s)\left[sC + \frac{1}{R} + \frac{1}{sL}\right]$$

i.e. $\qquad i(t) = C\dfrac{dV(t)}{dt} + \dfrac{V(t)}{R} + \dfrac{1}{L}\int V(t)\,dt$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

$$0 = \frac{1}{C_1}(q_2 - q_1) + L_2 s^2 q_2 + \frac{1}{C_2} q_2 + R_2 s q_2 \qquad \text{... (2) Loop (2)}$$



Same displacement same current

Equations in terms of I_1 and I_2 can be written by using $i(t) = \frac{dq}{dt}$ i.e. $I(s) = s\,q(s)$ as explained earlier.

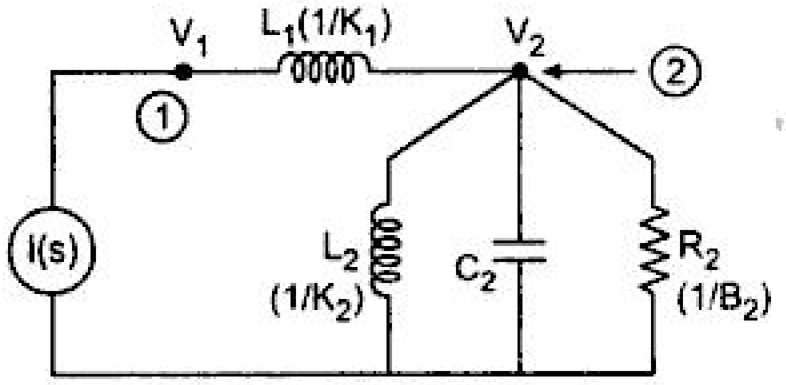
(ii) F – I analogy : $M \rightarrow C$, $B \rightarrow 1/R$, $K \rightarrow 1/L$

$$\therefore \qquad I(s) = \frac{1}{L_1}(\phi_1 - \phi_2) \qquad \text{... (1) Node (1)}$$

$$0 = \frac{1}{L_1}(\phi_2 - \phi_1) + C_2 s^2 \phi_2 + \frac{1}{R_2} s\phi_2 + \frac{1}{L_2}\phi_2 \qquad \text{... (2) Node (1)}$$

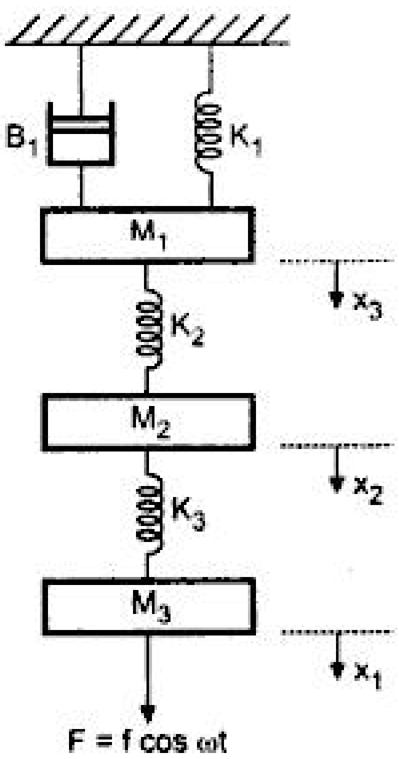
Equations in terms of $V_1(s)$ and $V_2(s)$ can be obtained by using the relation,

$$V(t) = \frac{d\phi}{dt} \qquad \text{i.e.} \quad V(s) = s\,\phi(s) \quad \text{as explained earlier}$$



Same displacement same voltage

Ex. 5.4



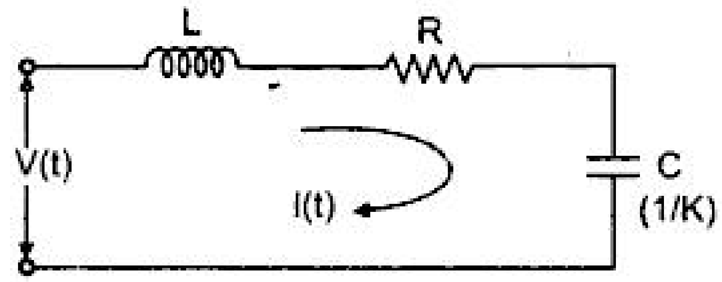
You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

i) Force -Voltage Analogy :

$$V(s) = q(s)\left[Ls^2 + Rs + \frac{1}{C}\right] \quad \text{... replace } q(s) = sI(s)$$

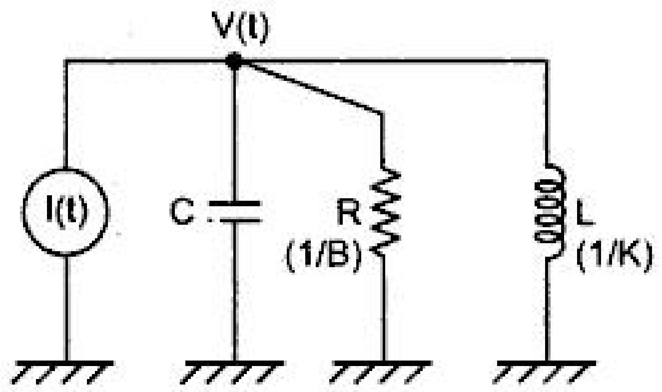
i.e. $$V(s) = Ls\,I(s) + R\,I(s) + \frac{1}{sC}I(s) \quad \text{... Loop equation.}$$



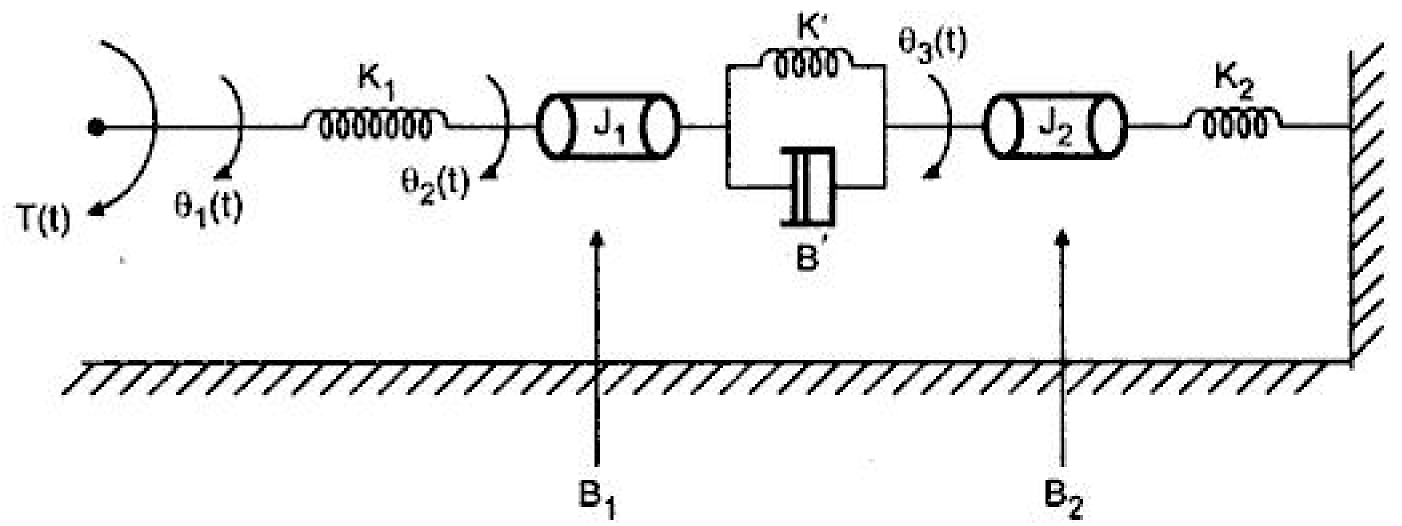
ii) Force - Current Analogy :

$$I(s) = \phi(s)\left[Cs^2 + \frac{1}{R}s + \frac{1}{L}\right] \quad \text{... replace } \phi(s) = \frac{V(s)}{s}$$

i.e. $$I(s) = Cs\,V(s) + \frac{V(s)}{R} + \frac{1}{sL}V(s) \quad \text{... Node equation}$$



Ex. 5.7 *For a given rotational system, obtain the electrical analogous systems based on inverse and direct analogous methods.*



Sol. : Rotational System :

As torque is directly applied to spring K_1, two different displacements at two ends of the spring. So no element is under $\theta_1(t)$, K_1 between θ_1 and θ_2. Then J_1 and B_1 under $\theta_2(t)$. Now because of K' and B' together, change from $\theta_2(t)$ to $\theta_3(t)$. So K' and B' are in parallel between θ_2 and θ_3 and J_2, K_2 and B_2 are under $\theta_3(t)$.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Given

θ = *angular displacement of the motor shaft*

K_f = *gain f error detector = 7.64 V/rad*

R_a = $2\,\Omega$, L_a = *negligible*

K_b = 5.5×10^{-2} V / rad / sec

K_T = 6×10^{-5} *Nm/A*

J_m = 1×10^{-5} Kg - m^2

f_m = *negligible*

J_L = 4.4×10^{-3} Kg - m^2

f_L = 4×10^{-2} *Nm/ (rad/sec)*

N_1 = *1*

N_2 = *10*

Ans.: $\dfrac{\theta_c(s)}{\theta_r(s)} = \dfrac{42.3}{s^2 + 7.7\,s + 42.3}$

10. *Determine the transfer function of the armature controlled d.c. servomotor.* **(May-97, 98)**
11. *How does the a.c. servomotor differ from the conventional a.c. motor ? Derive the transfer function of an a.c. servomotor.* **(Dec.-97)**
12. *Derive the transfer function of a field controlled d.c. servomotor.* **(May-99)**

❑❑❑

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

t	$V_o(t)$
0	0
RC	0.632
2RC	0.860
3RC	0.950
4RC	0.982
∞	1

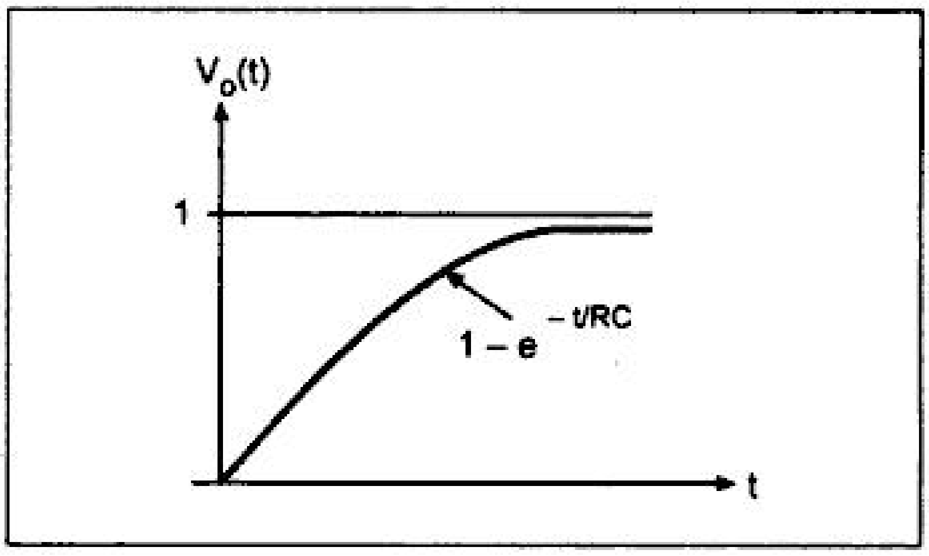


Fig. 6.18 (b)

The response is purely exponential.

Now suppose input is changed to step of `A' units.

Then $$V_i(s) = \frac{A}{s}$$

$\therefore$ $$V_o(s) = \frac{A}{s(1+sRC)} = \frac{A'}{s} + \frac{B'}{1+sRC}$$

$\therefore$ $$A = A'(1+sRC) + sB'$$

$\therefore$ $$A'(RC+B') = 0 \quad \text{and} \quad A' = A$$

$\therefore$ $$B' = -ARC$$

$\therefore$ $$V_o(s) = \frac{A}{s} - \frac{ARC}{1+sRC} = \frac{A}{s} - \frac{A}{s+1/RC}$$

$\therefore$ $$V_o(t) = A\left[1-e^{-t/RC}\right]$$

So rate of decay is not changed but the steady state value has changed. The corresponding response can be shown as in Fig. 6.19.

But if values of R and C are changed i.e. location of pole s = – 1/RC is changed, the transient output will behave in a different way as rate of decay will get affected without change in its steady state.

The transient term is vanishing as it contains exponential term of negative index which is only because pole of the system is having real negative part.

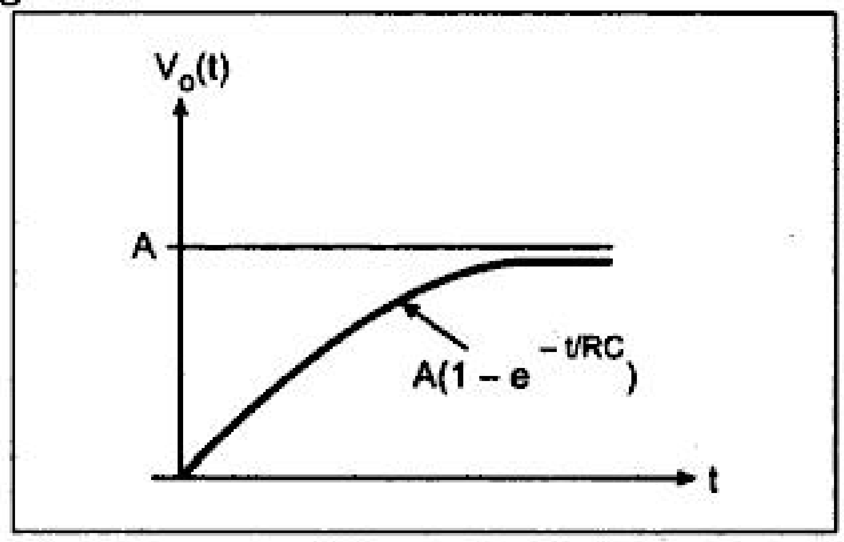


Fig. 6.19

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

i) $1 < \xi < \infty$, the roots are

$$s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

i.e. real, unequal and negative, say $-K_1$ and $-K_2$

$$\therefore \quad C(s) = \frac{\omega_n^2}{s(s+K_1)(s+K_2)}$$

$$= \frac{A}{s} + \frac{B}{s+K_1} + \frac{C}{s+K_2}$$

Taking Laplace inverse

C(t) will take the following form,

$$C(t) = C_{ss} + Be^{-K_1 t} + Ce^{-K_2 t},$$

Where $\quad C_{ss}$ = Steady state output = A

The output is purely exponential. This means damping is so high that there are no oscillations in the output and is purely exponential. Hence such systems are called as '**Overdamped**'.

Hence nature of response will be as shown in Fig. 6.22.

As ξ increases, output will take more time to reach its steady state and hence become sluggish and slow.

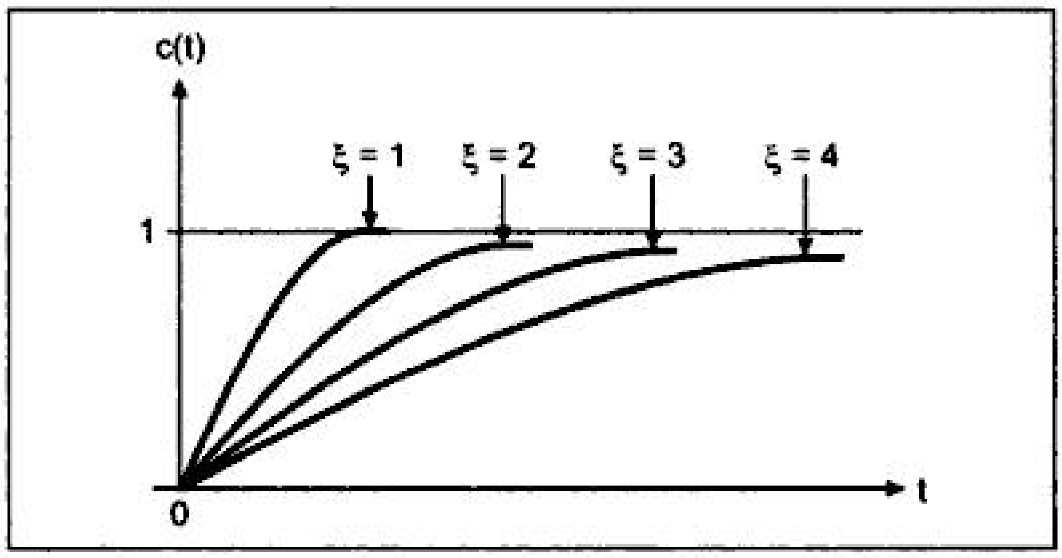


Fig. 6.22

ii) When $\xi = 1$, the roots are,

$$s_{1,2} = -\omega_n, -\omega_n$$

i.e. real equal and negative.

$$\therefore \quad C(s) = \frac{\omega_n^2}{s(s+\omega_n)(s+\omega_n)} = \frac{\omega_n^2}{s(s+\omega_n)^2} = \frac{A}{s} + \frac{B}{(s+\omega_n)^2} + \frac{C}{(s+\omega_n)}$$

Taking Laplace Inverse, C(t) will take the following form ....(Refer appendix A for Laplace Inverse)

$$C(t) = C_{ss} + Bte^{-\omega_n t} + Ce^{-\omega_n t}$$

Where $\quad C_{ss}$ = Steady state output value = A

This is purely exponential. But in comparison with overdamped case, settling time required for this case is less and because of repetitive occurrence of roots the system is called as '*Critically Damped*'. This is critical value of damping ratio because if it is decreased further roots will become complex conjugates and this is the least value of damping ratio for which roots are real and negative.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

$$C(s) = \frac{a_1}{s} + \frac{a_2 s + a_3}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\therefore \quad \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)} = \frac{a_1(s^2 + 2\xi\omega_n s + \omega_n^2) + s(a_2 s + a_3)}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

equating numerators on both sides,

$$\omega_n^2 = s^2(a_1 + a_2) + s(a_1 2\xi\omega_n + a_3) + a_1\omega_n^2$$

$\therefore \quad a_1\omega_n^2 = \omega_n^2$ equating constant

$a_1 + a_2 = 0$ equating coefficients of s^2

$a_1 2\xi\omega_n + a_3 = 0$ equating coefficients of s

$\therefore \quad a_1 = 1 \quad a_2 = -1 \quad a_3 = -2\xi\omega_n$

As $\xi\omega_n = \alpha$ assumed earlier for ease of computations.

$\therefore \quad a_1 = 1 \qquad a_2 = -1 \qquad a_3 = -2\alpha$

$$\therefore \quad C(s) = \frac{1}{s} + \frac{-s - 2\alpha}{s^2 + 2\alpha s + \omega_n^2}$$

$$\therefore \quad C(s) = \frac{1}{s} - \left\{\frac{s + 2\alpha}{s^2 + 2\alpha s + \omega_n^2}\right\}$$

Taking negative sign outside.

Now consider $s^2 + 2\alpha s + \omega_n^2$,

For completing square,

$$\text{Last term} = \frac{(\text{middle term})^2}{4 \times \text{first term}} = \frac{4\alpha^2}{4 \times 1} = \alpha^2$$

So adjusting denominator as $s^2 + 2\alpha s + \alpha^2 + \omega_n^2 - \alpha^2$

i.e. denominator $= (s + \alpha)^2 + \omega_n^2 - \alpha^2$

but $\alpha = \xi\omega_n \quad \therefore \quad \alpha^2 = \xi^2\omega_n^2$

Substituting in above we get, $(s + \alpha)^2 + \omega_n^2 - \xi^2\omega_n^2$

i.e. denominator $= (s + \alpha)^2 + \omega_n^2(1 - \xi^2)$

Now $\omega_d = \omega_n\sqrt{1 - \xi^2}$

$\therefore \quad \omega_d^2 = \omega_n^2(1 - \xi^2)$

Substituting this in the expression of C(s) we get,

Where $\theta = \tan^{-1} \dfrac{\sqrt{1-\xi^2}}{\xi}$

As at $t = T_p$, C(t) will achieve its maxima, according to Maxima theorem.

$$\left.\frac{dC(t)}{dt}\right|_{t=T_p} = 0$$

So differentiating C(t) w.r.t. `t' we can write

$$\text{i.e.} -\frac{e^{-\xi\omega_n t}(-\xi\omega_n)\sin(\omega_d t+\theta)}{\sqrt{1-\xi^2}} - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\omega_d \cos(\omega_d t+\theta) = 0$$

Substituting $\omega_d = \omega_n \sqrt{1-\xi^2}$

$$\frac{\xi\omega_n e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t+\theta) - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\omega_n \sqrt{1-\xi^2}\cos(\omega_d t+\theta) = 0$$

$$\therefore \quad \xi\sin(\omega_d t+\theta) - \sqrt{1-\xi^2}\cos(\omega_d t+\theta) = 0$$

$$\therefore \quad \tan(\omega_d t+\theta) = \frac{\sqrt{1-\xi^2}}{\xi}$$

Now $\theta = \tan^{-1}\dfrac{\sqrt{1-\xi^2}}{\xi}$

$$\therefore \quad \frac{\sqrt{1-\xi^2}}{\xi} = \tan\theta$$

$$\therefore \quad \tan(\omega_d t+\theta) = \tan\theta$$

From trignometric formula,

$$\tan(n\pi+\theta) = \tan\theta$$

$\therefore \quad \omega_d t = n\pi \qquad$ where n = 1, 2,3

But T_p , time required for first peak overshoot. $\therefore$ n = 1

$$\omega_d T_p = \pi$$

$$\therefore \quad T_p = \frac{\pi}{\omega_d}$$

$$= \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$C(t) \text{ at } (t = T_s) = 0.95$$

$$\therefore \quad 0.95 = 1 - e^{-\xi\omega_n T_s}$$

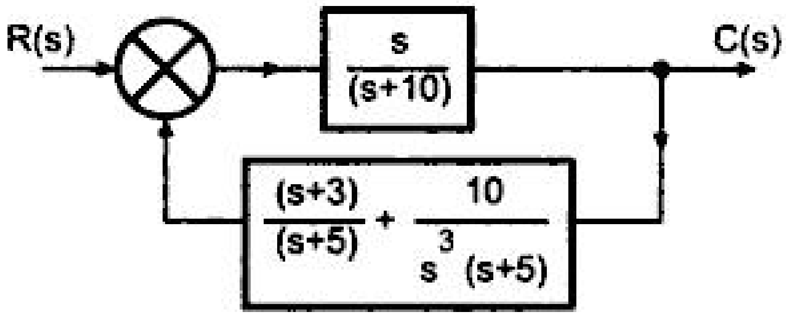
$$\therefore \quad T_s = \frac{2.995}{\xi\omega_n} \approx \frac{3}{\xi\omega_n}$$

Transient Response Specifications
$$C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d + \theta)$$ for underdamped system, unit step input Where $\omega_d = \omega_n \sqrt{1-\xi^2}$ and $\theta = \tan^{-1}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right]$ radians.
For a step of A units, $$C(t) = A\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)\right]$$
$$T_d = \frac{1 + 0.7\,\xi}{\omega_n} \text{ sec}$$
$$T_p = \frac{\pi}{\omega_d} \text{ sec}$$
$$T_r = \frac{\pi - \theta}{\omega_d} \text{ sec}$$
$$\%\, M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$$
$$T_s = \frac{4}{\xi\,\omega_n} \text{ sec},$$ for ± 2 % tolerance band.

Table 6.5

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Sol. : Given system can be reduced by combining two blocks connected in parallel in feedback path.



$$\therefore \quad G(s) = \frac{s}{s+10},$$

$$H(s) = \frac{(s+3)}{(s+5)} + \frac{10}{s^3\,(s+5)} = \frac{s^3\,(s+3)+10}{s^3\,(s+5)}$$

$$G(s)\,H(s) = \frac{s}{(s+10)} \cdot \frac{(s^4+3s^3+10)}{s^3\,(s+5)} = \frac{(s^4+3s^3+10)}{s^2\,(s+5)\,(s+10)}$$

$$K_p = \text{positional error coefficient} = \lim_{s\to 0} G(s)\,H(s)$$

$$= \lim_{s\to 0} \frac{s^4+3s^3+10}{s^2\,(s+5)\,(s+10)}$$

$$= \infty$$

$$K_v = \text{Velocity error coefficient}$$

$$= \lim_{s\to 0} sG(s)\,H(s) = \lim_{s\to 0} \frac{s^4+3s^3+10}{s(s+5)\,(s+10)}$$

$$= \infty$$

$$K_a = \text{Acceleration error coefficient}$$

$$= \lim_{s\to 0} s^2G(s)\,H(s) = \lim_{s\to 0} \frac{s^4+3s^3+10}{(s+5)\,(s+10)} = \frac{10}{5.10}$$

$$= 0.2$$

Ex. 6.4 *For unity feedback system having* $G(s) = \dfrac{10\,(s+1)}{s^2\,(s+2)\,(s+10)}$

Determine :

(i) Type of system

(ii) Error coefficients and

(iii) Steady state error for input as $1 + 4t + \dfrac{t^2}{2}$.

$$\therefore \qquad e_{ss} = \frac{A}{1+K_p} = \frac{10}{1+\frac{25}{4}} = \frac{40}{29} = 1.379$$

ii) $r(t) = 5t, \quad \therefore \quad R(s) = \frac{5}{s^2},$ so $A = 5$ ramp

$$\therefore \qquad e_{ss} = \frac{A}{K_v} = \frac{5}{0} = \infty$$

iii) $r(t) = 10 + 5t + \frac{6}{2}t^2$

i.e. $A_1 = 10$ step, $A_2 = 5$ ramp, $A_3 = 6$ parabolic

$$\therefore \qquad e_{ss} = \frac{A_1}{1+K_p} + \frac{A_2}{K_v} + \frac{A_3}{K_a} = \frac{10}{1+\frac{25}{4}} + \frac{5}{0} + \frac{6}{0}$$

$$= \infty$$

Thus as system is type 0 system, error is finite only for step input.

Ex. 6.6 *Determine steady state error for a system having* $G(s)\,H(s) = \frac{100}{s^2\,(1+0.5s)\,(s+2)}$ *and input as* $r(t) = t^2$

Sol. : e_{ss} = Steady state error $= \lim_{s \to 0} \frac{sR(s)}{1+G(s)\,H(s)}$

$$r(t) = t^2,$$

$$R(s) = \frac{2!}{s^3} = \frac{2}{s^3}$$

$$\therefore \qquad e_{ss} = \lim_{s \to 0} \frac{s \cdot \frac{2}{s^3}}{1+\frac{100}{s^2\,(1+0.5s)\,(s+2)}} = \lim_{s \to 0} \frac{2}{s^2 + \frac{s^2 \cdot 100}{s^2\,(1+0.5s)\,(s+2)}}$$

$$e_{ss} = \frac{2}{0 + \lim_{s \to 0} \frac{100}{(1+05\,s)\,(s+2)}}$$

$$\therefore \qquad e_{ss} = \frac{2}{0+\frac{100}{2}} = 0.04$$

Thus error is 4%.

Note : The result obtained is just by mechanical application of formula derived without investigating the stability of the system. In fact this system is unstable and for unstable systems, K_p, K_v, K_a are undefined and hence e_{ss} is also undefined.

$$= 0 + \frac{6}{K}, \quad K = 10 \text{ given}$$

$$\therefore \quad e_{ss} = \frac{6}{10} = 0.6$$

Ex. 6.11 *For a unity feedback system* $G(s) = \dfrac{200}{s\,(s+8)}$ *and* $r(t) = 2t$ *determine steady state error. If it is desired to reduce this existing error by 5% find new value of gain of the system.*

Sol. : The input is 2t i.e. ramp of magnitude 2, so K_v will control the error.

$$\therefore \quad K_v = \lim_{s \to 0} sG(s)\,H(s) = \lim_{s \to 0} s \cdot \frac{200}{s(s+8)} \cdot 1 = 25$$

$$\therefore \quad e_{ss} = \frac{A}{K_v} = \frac{2}{25} = 0.08$$

This error is to be reduced by 5% of existing value, with new gain of G(s) as K_2 instead of 200.

$$\therefore \quad e_{ss1} = e_{ss} - \left(\frac{5}{100} \times e_{ss}\right) = 0.08 - \frac{5 \times 0.08}{100} = 0.076$$

New error is 0.076.

$$\text{New} \quad G(s) = \frac{K_2}{s(s+8)} \text{ and } \quad H(s) = 1 \quad \text{with same input}$$

$$\therefore \quad K_v = \lim_{s \to 0} sG(s)\,H(s) = \frac{s \cdot K_2}{s(s+8)} = \frac{K_2}{8}$$

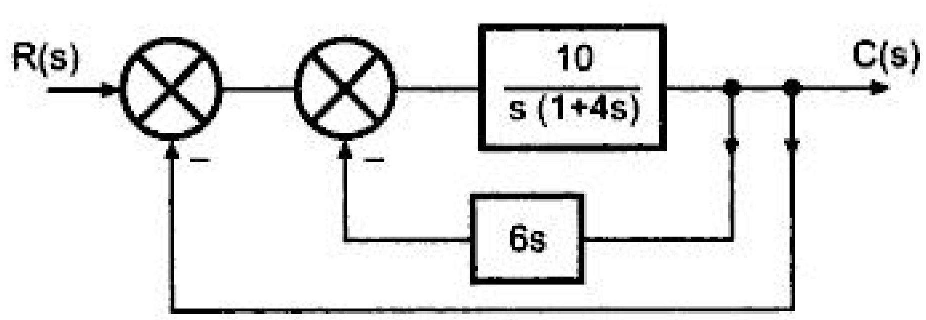
$$\therefore \quad e_{ss1} = \frac{A}{K_v} = \frac{2}{\left(\frac{K_2}{8}\right)} = \frac{16}{K_2}$$

$$\therefore \quad 0.076 = \frac{16}{K_2}$$

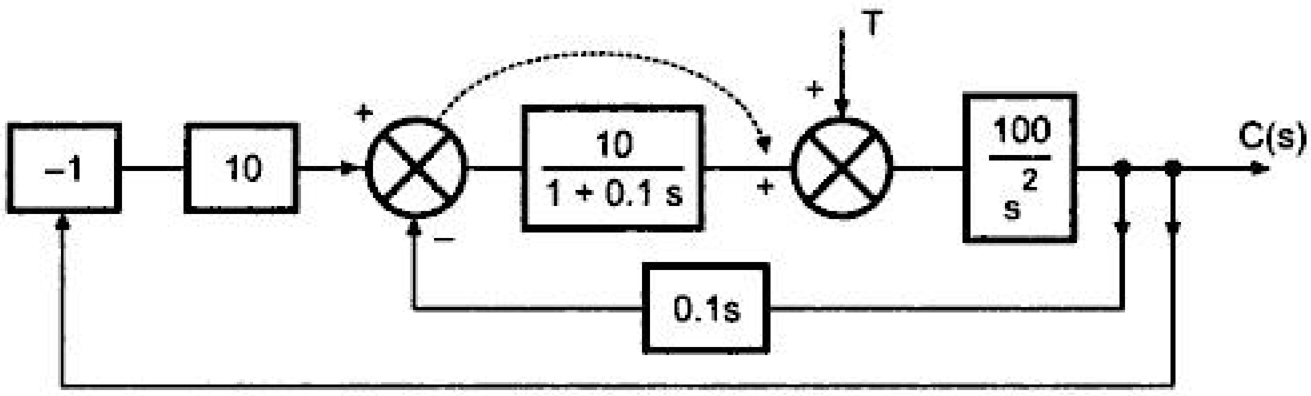
$$\therefore \quad K_2 = 210.52$$

So new gain is 210.52

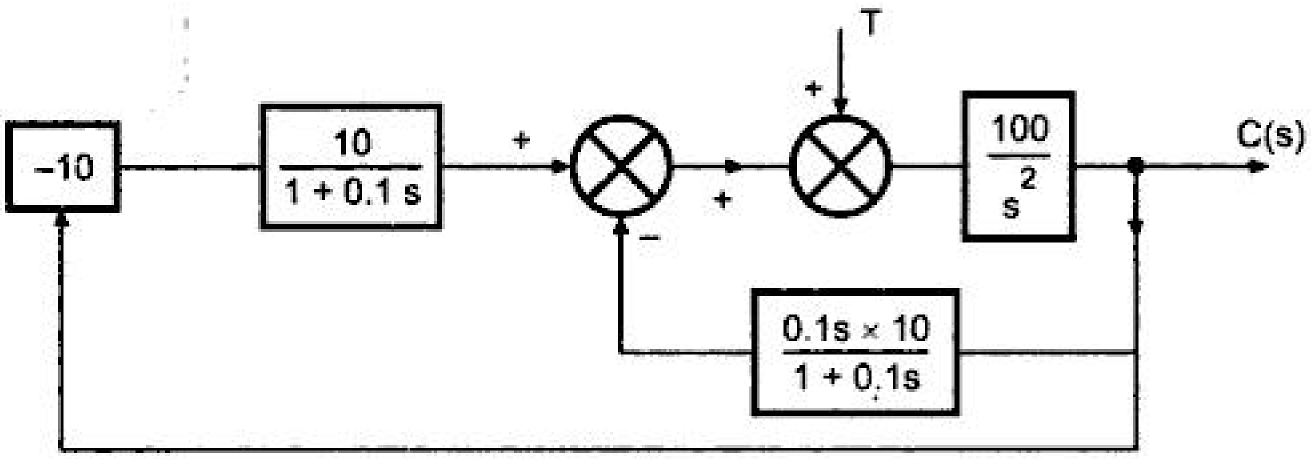
Ex. 6.12 *For a given system find error coefficients and type of the system.*



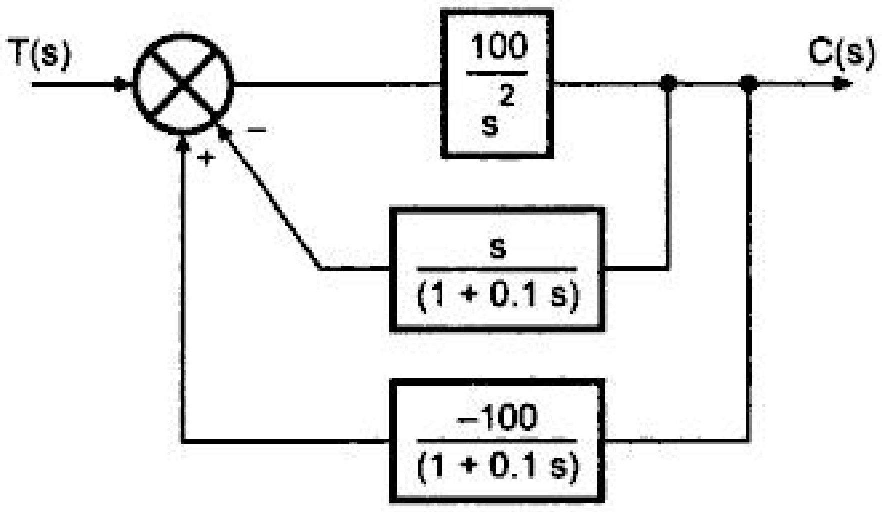
You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.



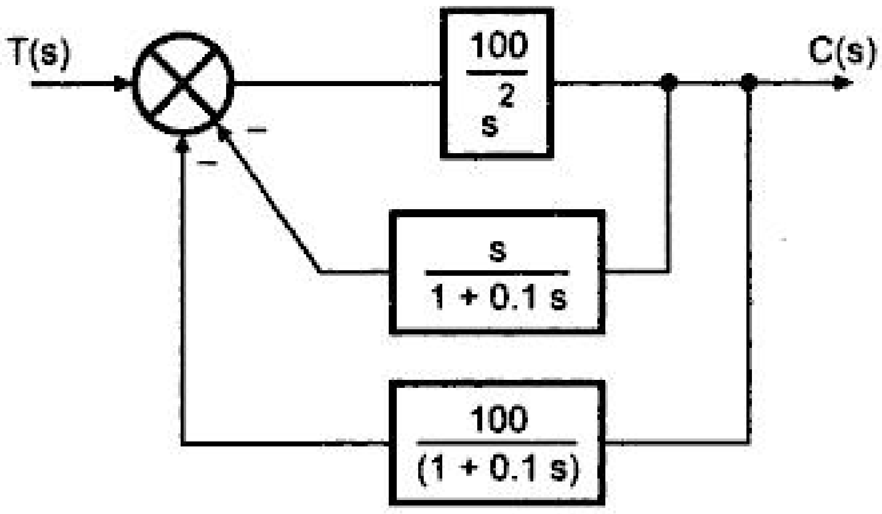
Shifting summing point to right.

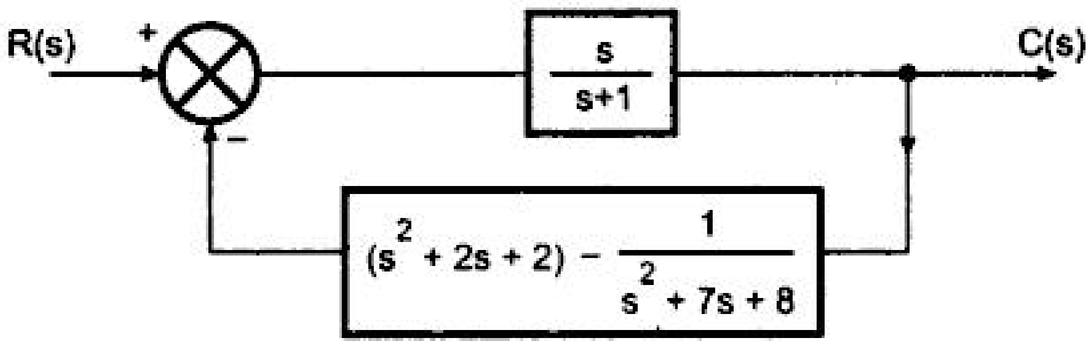


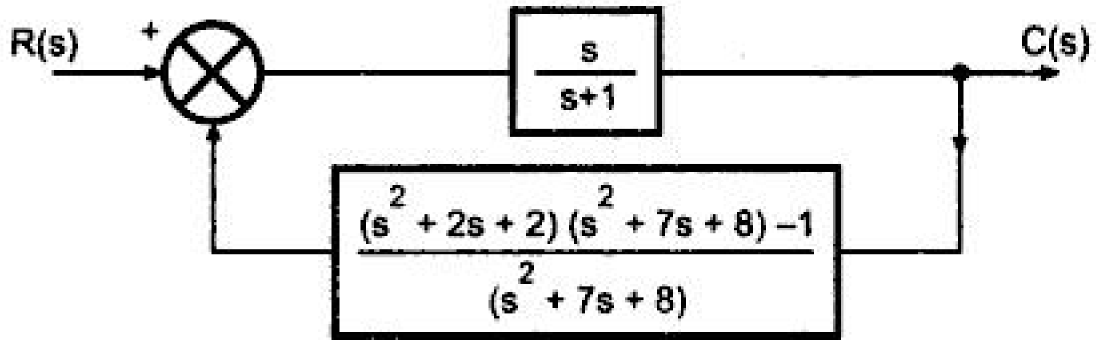
Combining the two summing points and redrawing the diagram.



Negative sign of $\left(\frac{-100}{1+0.1s}\right)$ can be taken out to change sign of the signal at the summing point from positive to negative.







$$\therefore \qquad G(s)\,H(s) = \frac{s}{(s+1)} \times \frac{(s^2+2s+2)\ (s^2+7s+8)-1}{(s^2+7s+8)}$$

$$K_p = \lim_{s \to 0} G(s)\,H(s) = 0$$

$$K_v = \lim_{s \to 0} sG(s)\,H(s) = 0$$

$$K_a = \lim_{s \to 0} s^2G(s)\,H(s) = 0$$

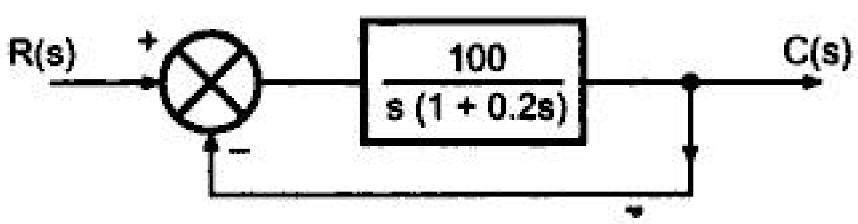
a) Unit step

$$e_{ss} = \frac{1}{1+K_p} = 1$$

b) Unit ramp input

$$e_{ss} = \frac{1}{K_v} = \infty$$

Ex. 6.19 *Evaluate the steady state error of the system when input applied is* $r(t) = 3 + 4t$

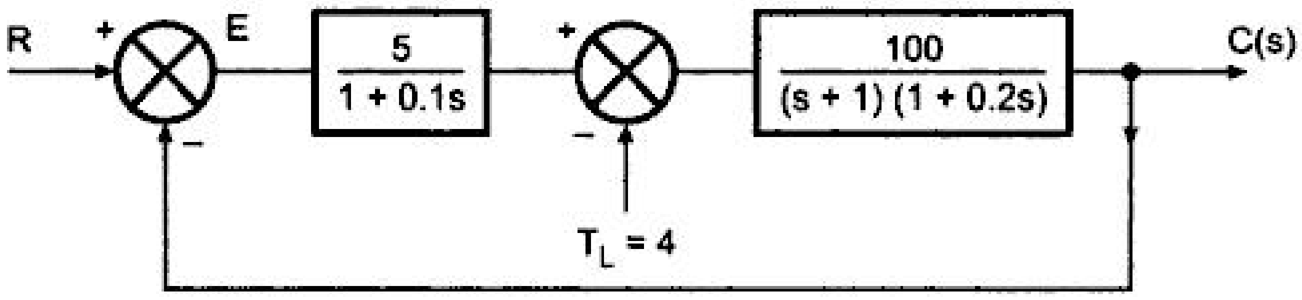


(Mumbai University, May 93)

Sol. : For the system,

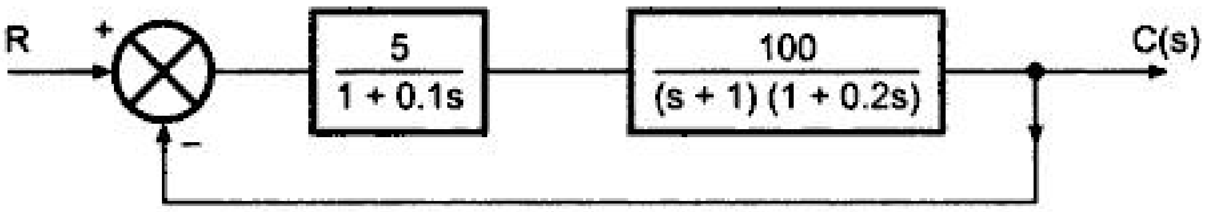
$$G(s)\,H(s) = \frac{100}{s(1+0.2s)}$$

Ex. 6.23 *In the system given, the command input is R = 10 and disturbance signal is T_L = 4, what is the steady state error.* **(Mumbai University, Nov. 94)**



Sol. : Using superposition principle, consider inputs separately.

a) R acting, $T_L = 0$

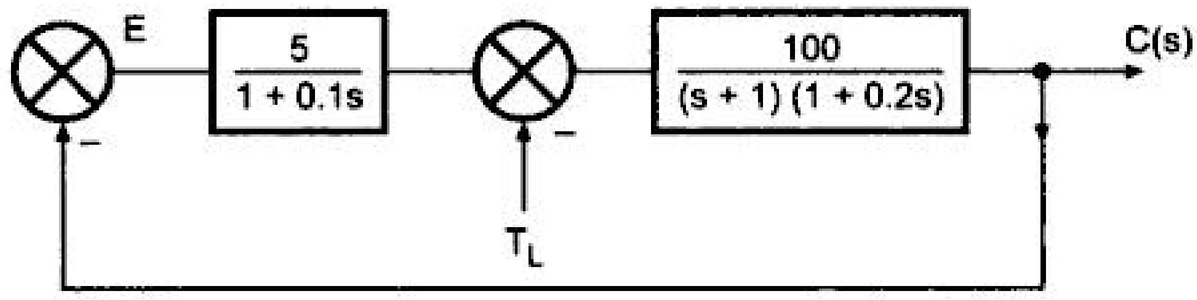


$$\therefore \qquad G(s)\,H(s) = \frac{500}{(1+0.1s)\;(s+1)\;(1+0.2s)}$$

For step input

$$K_p = \lim_{s\to 0} G(s)\,H(s) = 500$$

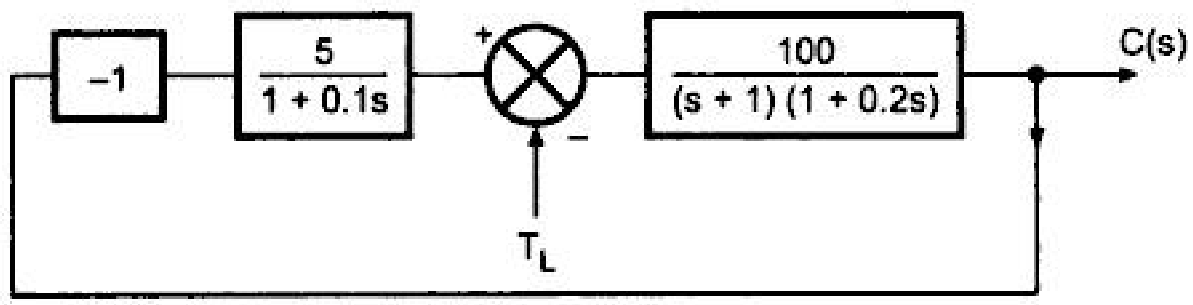
$$\therefore \qquad e_{ss1} = \frac{A}{1+K_p} \text{ where A = magnitude of step} = \frac{10}{1+500} = \frac{10}{501}$$



b) T_L acting, R = 0

$$E(s) = -\,C(s)$$

As system is not in standard form, error coefficient method cannot be used.



Comparing the two expressions.

$\omega_n^2 = 16$ hence $\omega_n = 4$ rad/sec i.e. natural frequency of oscillations then

$2\xi\,\omega_n = 8$ $\quad\therefore\ \xi\omega_n = 4$ $\quad\therefore\ \xi = 1$ i.e. damping ratio.

i.e. system is critically damped. Now damped frequency of oscillations is given by,

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 0 \text{ rad/sec}$$

As system is critically damped there are no oscillations and hence no damped frequency of oscillations.

$$\omega_n = 4 \text{ rad/sec},\quad \xi = 1,\quad \omega_d = 0 \text{ rad/sec}$$

Ex. 6.28 *For a system having T.F.* $= \dfrac{64}{s^2+5s+64}$ *, for unity step input determine*

i) ω_n *ii)* ξ *iii)* ω_d *, iv) Time for peak overshoot* T_p*.*

Sol. : From, T.F. it is clear that system is second order hence comparing given,

T.f. with standard T.F. of second order system $\dfrac{\omega_n^2}{s^2+2\xi\,\omega_n\,s+\omega_n^2}$.

$$\omega_n^2 = 64 \qquad \therefore\ \omega_n = 8 \text{ rad/sec}$$

$$2\xi\,\omega_n = 5 \qquad \therefore\ \xi\,\omega_n = 2.5 \qquad \therefore\ \xi = 0.3125$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 8\sqrt{1-(0.3125)^2}$$

$$\omega_d = 7.599 \text{ rad/sec}$$

Time for peak over shoot,

$$T_p = \frac{\pi}{\omega_d}\text{ sec.} = \frac{\pi}{7.599} = 0.4134 \text{ sec}$$

$$\omega_n = 8 \text{ rad/sec},\quad \xi = 0.3125,\quad \omega_d = 7.599 \text{ rad/sec},$$

$$T_p = 0.4134 \text{ sec}$$

Ex. 6.29 *A second order system is given by* $\dfrac{C(s)}{R(s)} = \dfrac{25}{s^2+6s+25}$. *Find its rise time, peak time, peak overshoot and settling time if subjected to unit step input. Also calculate expression for its output response.*

Sol. : Comparing the T.F. with the standard form $\dfrac{\omega_n^2}{s^2+2\xi\,\omega_n\,s+\omega_n^2}$

$$\omega_n^2 = 25 \quad\text{and}\quad 2\xi\,\omega_n = 6$$

$$\omega_n = 5 \qquad \therefore\ \xi = 0.6$$

$$\theta = \tan^{-1}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right] = 0.9272 \text{ radians}$$

Comparing this with standard T.F. of second order system $\dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$$\therefore \quad \omega_n^2 = 8$$

$$\therefore \quad \omega_n = 2.83 \text{ rad/sec}$$

$$2\xi\omega_n = 4 \qquad \therefore \ \xi = 0.7067$$

$$\therefore \quad \omega_d = \omega_n\sqrt{1-\xi^2} = 2.83\sqrt{1-(0.7067)^2} = 2.002 \text{ rad/sec}$$

$$\therefore \quad T_p = \text{time for peak overshoot}$$

$$= \frac{\pi}{\omega_d} = \frac{\pi}{2.002} = 1.57\text{sec}$$

$$\% \ M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = e^{-\pi\times 0.706/\sqrt{1-(0.706)^2}\times 100}$$

$$= 4.33\%$$

$$T_s = \text{settling time} = \frac{4}{\xi\omega_n} = \frac{4}{0.7067\times 2.83} = 2 \text{ sec}$$

$$C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

Where $\theta = \tan^{-1}\left(\dfrac{\sqrt{1-\xi^2}}{\xi}\right) = 45° = \dfrac{\pi}{4} \text{ rad}$

$$\therefore \quad C(t) = 1 - \frac{e^{-0.7067\times 2.87\, t}}{\sqrt{1-7067^2}} \sin\left(2t + \frac{\pi}{4}\right)$$

$$C(t) = 1 - 1.41\, e^{-2t} \sin\left(2t + \frac{\pi}{4}\right)$$

$$\omega_n = 2.83 \text{ rad/sec} \qquad T_p = 1.57 \text{ sec}$$

$$\omega_d = 42.002 \text{ rad/sec} \qquad \% \ M_p = 4.33\%$$

$$T_s = 2 \text{ sec} \qquad \xi = 0.7067$$

Ex. 6.33 *Open loop T.F. of unity feedback system is* $G(s) = \dfrac{K}{(1+Ts)\,s}$ *where K and T are constants. Determine factor by which gain `K' should be multiplied so that overshoot of unit step response be reduced from 75% to 25%.*

Sol. : Closed loop T.F. $= \dfrac{G(s)}{1+G(s)}$, $\quad H(s) = 1$

T.F. is $$\frac{Y(s)}{X(s)} = \frac{12}{s^2+7s+12}$$

Comparing denominator with standard form

$\omega_n = \sqrt{12}$, $\quad 2\xi\omega_n = 7 \quad \therefore \xi = 1.010363$

As $\xi > 1$, system is overdamped, hence the output will not contain any oscillations. Hence standard expression for C(t) cannot be used.

Now input is unit step, so X(s) = 1/s Substituting in T.F.

$$Y(s) = \frac{12}{s(s^2+7s+12)}$$ Use partial fraction method

$$= \frac{12}{s(s+3)(s+4)} = \frac{A}{s}+\frac{B}{s+3}+\frac{C}{s+4}$$

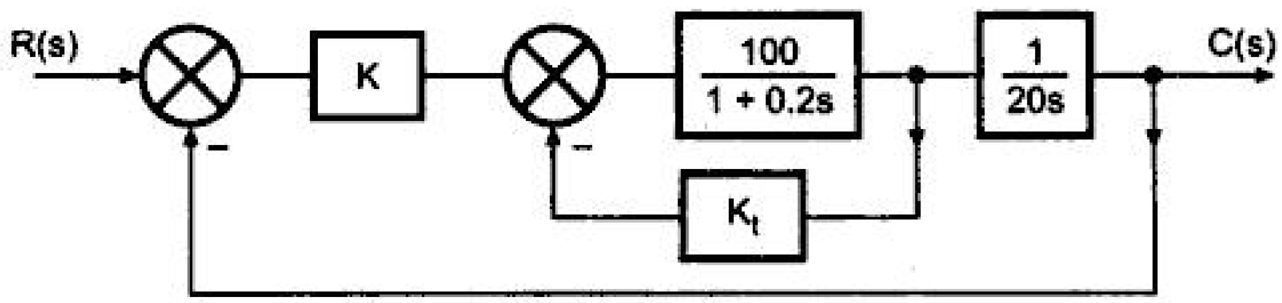
where, A = 1, B = − 4, C = 3

$$Y(s) = \frac{1}{s}-\frac{4}{s+3}+\frac{3}{s+4}$$

Taking Laplace inverse,

$$Y(t) = 1-4e^{-3t}+3e^{-4t}$$

Ex. 6.36 *For a control system shown in figure, find the values of K and K_t so that the damping ratio of system is 0.6 and settling time is 0.1 sec. Use $T_s = \frac{3.2}{\xi\omega_n}$. Assume unit step input.*



Sol. : Using block diagram reduction rule, reduction of inner loop is,

$$\frac{\dfrac{100}{1+0.2s}}{1+\dfrac{100}{1+0.2s}\cdot K_t} = \frac{100}{1+0.2s+100\,K_t}$$

$$\therefore \quad \text{overall } G(s) = K\cdot\frac{100}{1+0.2s+100\,K_t}\cdot\frac{1}{20s} = \frac{5K}{s\,[1+100\,K_t+0.2s]}$$

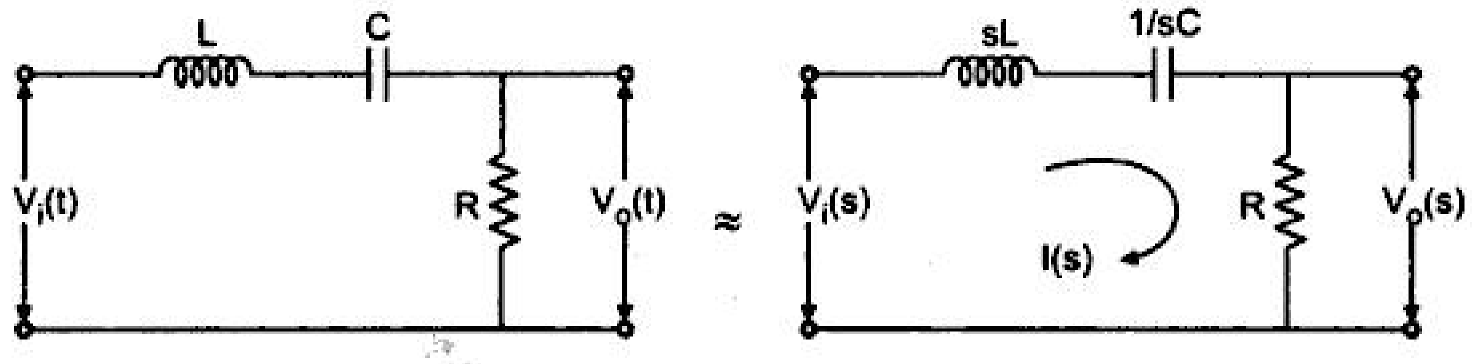
$$H(s) = 1$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Ex. 6.38 *Obtain the step response of series R-L-C circuit, with $R = 1\Omega$, $L = 1H$, $C = 1F$ and output is taken across `R'. Assume input as a step of 10V.*

(Pune University, May 94)

Sol. : The circuit is as shown in following figure.



To find $\frac{V_o(s)}{V_i(s)}$, we can write the equations as,

$$V_i(s) = s\,L\,I(s) + \frac{1}{sC} I(s) + RI(s)$$

while $$V_o(s) = I(s)\,R$$

Substituting $I(s) = \frac{V_o(s)}{R}$ in first equation

$$\therefore \quad V_i(s) = \frac{V_o(s)}{R}\left[sL + \frac{1}{sC} + R\right]$$

$$\therefore \quad V_i(s) = \frac{V_o(s)}{sCR}\left[s^2 LC + 1 + sRC\right]$$

$$\therefore \quad \frac{V_o(s)}{V_{i(s)}} = \frac{sCR}{s^2 LC + sRC + 1.} = \frac{s \cdot R/L}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

Substituting R = L = C = 1

$$\frac{V_o(s)}{V_i(s)} = \frac{s}{s^2 + s + 1}$$

Comparing the denominator

$$\omega_n^2 = 1 \qquad \therefore \omega_n = 1$$

$$2\xi\omega_n = 1 \qquad \therefore \xi = 0.5$$

Note : The standard expression for C(t) is derived for T.F. in the form $\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$ and not for the T.F. having `s' terms in its numerator. Hence standard expression for C(t) cannot be used and output must be calculated by direct partial fraction approach.

Sol. : $$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{1}{s(Js-K)}}{1+\frac{1}{s(Js-K)}(s+m)}$$

$$= \frac{1}{Js^2 - Ks + s + m}$$

$$\frac{C(s)}{R(s)} = \frac{I/J}{s^2 + \left[\frac{-K+1}{J}\right]s + \frac{m}{J}}$$

Comparing denominator with $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$\therefore$ $$\omega_n^2 = \frac{m}{J} \quad \therefore \omega_n = \sqrt{\frac{m}{J}}$$

$$2\xi\omega_n = \frac{[-K+1]}{J}$$

$\therefore$ $$\xi = \frac{[-K+1]}{2J}\sqrt{\frac{J}{m}} = \frac{[-K+1]}{2\sqrt{Jm}}$$

For $M_p = 25\%$, we can calculate ξ

$$25 = 100 \times e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$\therefore$ $$ln\ 0.25 = \frac{-\pi\xi}{\sqrt{1-\xi^2}}$$

$\therefore$ $$1.9218 = \frac{\pi^2\xi^2}{1-\xi^2}$$

$\therefore$ $$\xi^2 = 0.1629$$

$$\xi = 0.4037$$

Now $$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$\therefore$ $$2 = \frac{\pi}{\omega_n\sqrt{1-(0.4037)^2}}$$

$\therefore$ $$\omega_n = 1.7169 \text{ rad/sec}$$

Now steady state of system is 5 units.

Now steady state is defined as,

$$C_{ss} = \lim_{t\to\infty} C(t)$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

$$\omega_n^2 = 100 \quad \therefore \quad \omega_n = 10$$

and $$2\xi\omega_n = 2 \quad \therefore \quad \xi = 0.1$$

$\therefore$ $$\omega_d = \omega_n\sqrt{1-\xi^2} = 9.9498$$

$\therefore$ i) Response of system is

$$C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \theta)$$

where $$\theta = \tan^{-1}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right] \text{ in radians}$$

$$\theta = 1.47062 \text{ radians}$$

$\therefore$ $$C(t) = 1 - \frac{e^{-0.1\times 10t}}{\sqrt{1-(0.1)^2}}\sin(9.9498t + 1.47062)$$

$\therefore$ $$C(t) = 1 - 1.005\, e^{-t}\sin(9.9498t + 1.4706)$$

$$\lim_{t\to\infty} C(t) = C_{ss} = 1$$

$\therefore$ Steady state error $e_{ss} = 0$

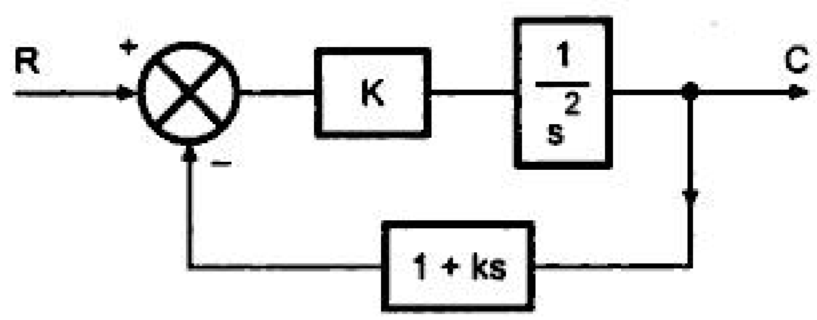
ii) For step input,

$$K_p = \lim_{s\to 0} G(s)\, H(s) = \lim_{s\to 0} \frac{100}{s(s+2)} = \infty$$

$\therefore$ $$e_{ss} = \frac{1}{1+K_p} = 0.$$

Ex. 6.47 *For the servomechanism shown below, determine the values of K and k, so that maximum overshoot for unit step input is 25% and peak time is 2 seconds.*

Sol. :



$$G(s) = \frac{K}{s^2}$$

$$H(s) = 1 + ks$$

The characteristic equation is $1 + G(s)\, H(s) = 0$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

$\therefore$ Frequency of oscillation $= \dfrac{\omega_d}{2\pi}$ (as $\omega = 2\pi f$)

$$= 0.9549 \text{ cycles/sec.}$$

$$\text{Time for 1 cycle} = 1/0.9549 = 1.0471 \text{ sec/cycle}$$

$$T_s = 0.375 \text{ sec}$$

$\therefore$ Number of cycles before it settles are

$$= \frac{0.375}{1.0471}$$

$$= 0.358$$

As output not even completes one cycle before attaining steady state hence practically second undershoot will not exist. Hence mathematically we got its value as 5×10^{-8} i.e. almost zero.

Ex. 6.49 *For a unity feedback system* $G(s) = \dfrac{36}{s(s+0.72)}$*. Determine characteristic equation and hence calculate damping ratio, peak time, settling time, peak overshoot and number of cycles completed before output settles for a unit step input.*

(Mumbai University, May 93)

Sol. : Characteristic equation is 1 + G(s) H(s) = 0

$$\therefore \quad 1 + \frac{36}{s(s+0.72)} = 0$$

i.e. $$s^2 + 0.72s + 36 = 0$$

Comparing with

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\therefore \quad \omega_n^2 = 36, \quad \therefore \quad \omega_n = 6 \text{ rad/sec}$$

and $$2\xi\omega_n = 0.72, \quad \therefore \quad \xi = 0.06$$

$$\omega_d = \omega_n \sqrt{1-\xi^2}$$

$$\therefore \quad \omega_d = 5.9891 \text{ rad/sec.}$$

$\therefore$ Peak time $$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{5.9891} = 0.5245 \text{ sec.}$$

Settling time $$T_s = \frac{4}{\xi\omega_n} = \frac{4}{0.06 \times 6} = 11.11 \text{ sec.}$$

Peak overshoot $$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

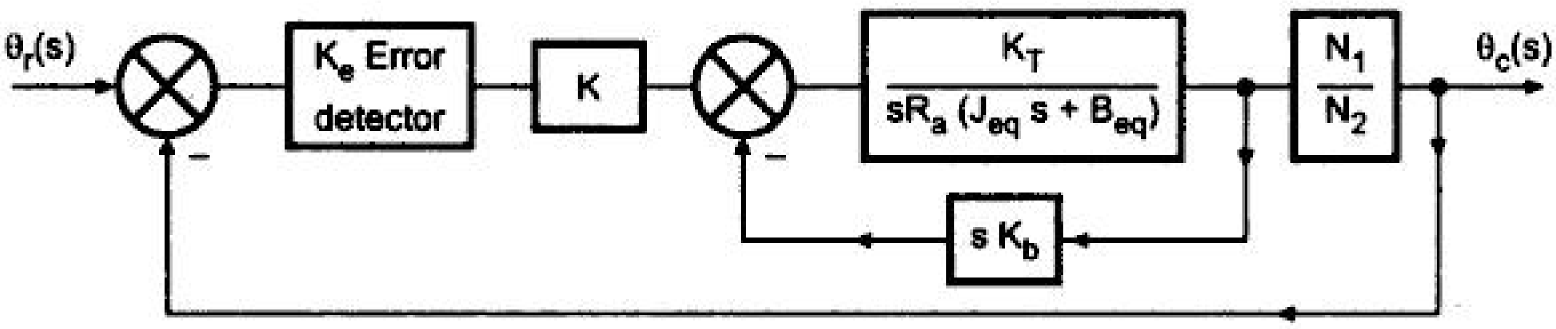
$$= 0.8279$$

i.e. $$\% M_p = 82.79\%$$

Now $$\omega_d = 5.9891 \text{ rad/sec.}$$

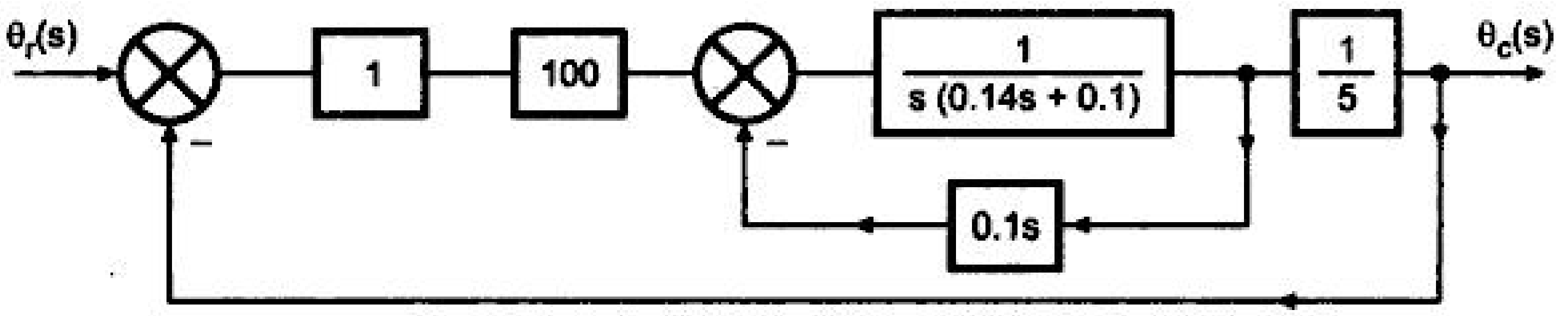
You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

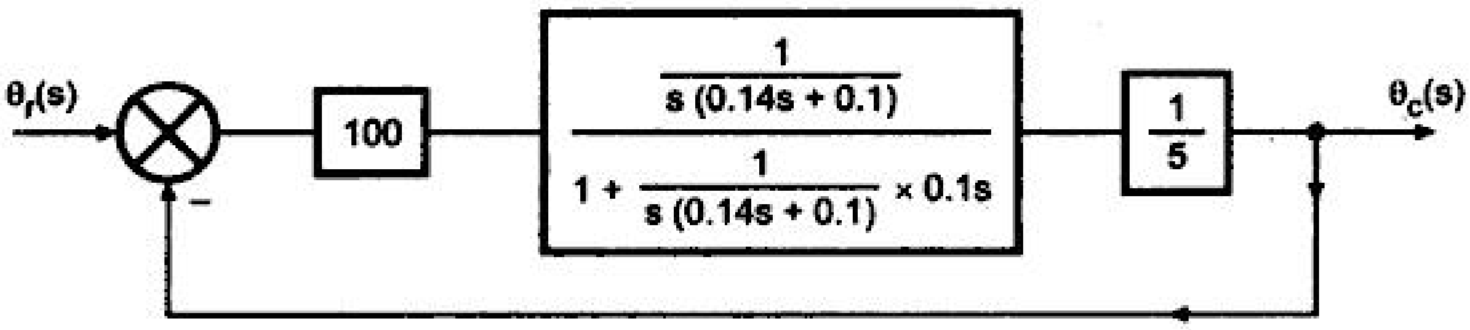
The block diagram is

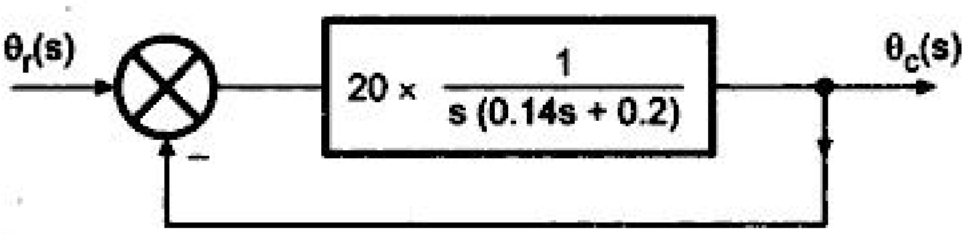


$$* \; K_b = 0.1 \text{ V/(rad/sec)}$$
$$* \; K = 100$$
$$* \; K_e = 1 \text{ V/rad}$$
$$* \; R_a = 1 \, \Omega$$

* Assume torque constant K_T of the motor as 1.







$$\frac{\theta_C(s)}{\theta r(s)} = \frac{\dfrac{20}{s(0.14s+0.2)}}{1+\dfrac{20}{s(0.14s+0.2)}}$$

$$\frac{\theta_C(s)}{\theta r(s)} = \frac{20}{0.14s^2+0.2s+20} = \frac{142.857}{s^2+1.4285s+142.857}$$

The characteristic equation is

$$s^2 + 1.4285s + 142.857 = 0$$

Comparing with

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

$$\theta = \tan^{-1}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right] \text{ rad} = 0.4636 \text{ radians}$$

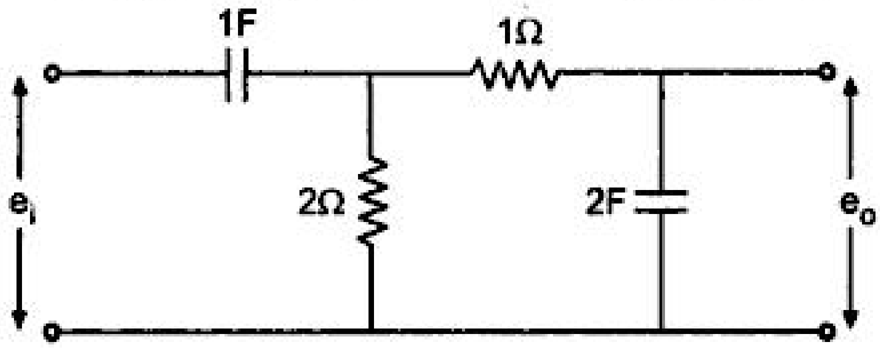
$$T_p = \frac{\pi}{\omega_d} = 3.1415 \text{ sec}$$

$$T_s = \frac{4}{\xi\omega_n} = 2 \text{ sec}$$

$$T_r = \frac{\pi-\theta}{\omega_d} = 2.6779 \text{ sec}$$

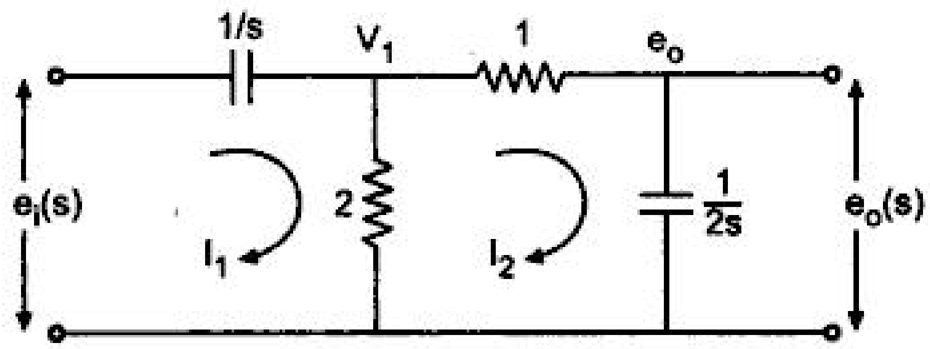
$$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = 0.1867\%$$

Ex. 6.56 *Find the impulse response of electrical circuit given below :*



(Mumbai University, May 95)

Sol. : Taking laplace of the network



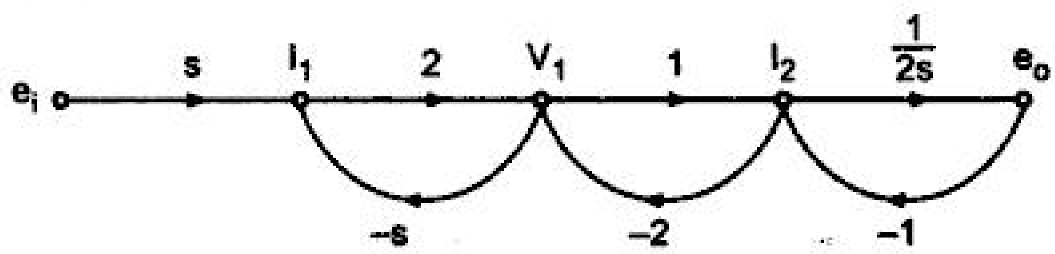
$$I_1 = \frac{e_i - v_1}{\frac{1}{s}} = s(e_i - v_1) \qquad \text{..... (1)}$$

$$v_1 = 2(I_1 - I_2) \qquad \text{.... (2)}$$

$$I_2 = \frac{v_1 - e_0}{1} \qquad \text{..... (3)}$$

$$e_0 = I_2 \times \frac{1}{2s} \qquad \text{..... (4)}$$

∴Signal flow graph is



You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

$$= \frac{G(s)}{\left[\dfrac{G(s)}{1+G(s)H(s)}\right]} \cdot \frac{1}{[1+G(s)H(s)]^2}$$

$$\therefore \qquad S_G^T = \frac{1}{1+G(s)H(s)} \qquad \text{... (6)}$$

Comparing the two equations (5) and (6), it can be observed that due to the feedback the sensitivity function gets reduced by the factor $1/[1+G(s)H(s)]$ compared to an open loop system. And less the value of sensitivity function, less sensitive is the system to the variations in the forward path transfer function G (s).

Sensitivity of T (s) with respect to H (s)

Let us calculate the sensitivity function which indicates the sensitivity of the overall transfer function T (s) with respect to the feedback path transfer function H (s). Such a function can be expressed as,

$$S_H^T = \frac{H(s)}{T(s)} \cdot \frac{\partial T(s)}{\partial H(s)}$$

For a cloased loop system,

$$T(s) = \frac{G(s)}{1+G(s)H(s)}$$

$$\therefore \qquad \frac{\partial T(s)}{\partial H(s)} = \frac{[1+G(s)H(s)][0]-[G(s)][G(s)]}{[1+G(s)H(s)]^2}$$

$$= \frac{-[G(s)]^2}{[1+G(s)H(s)]^2}$$

$$\therefore \qquad S_H^T = \frac{H(s)}{T(s)} \cdot \frac{-[G(s)]^2}{[1+G(s)H(s)]^2}$$

$$= \frac{H(s)}{\left[\dfrac{G(s)}{1+G(s)H(s)}\right]} \cdot \frac{-[G(s)]^2}{[1+G(s)H(s)]^2}$$

$$= \frac{-G(s)H(s)}{1+G(s)H(s)} \qquad \text{... (7)}$$

It can be observed from the equations (6) and (7) that the closed loop system is more sensitive to variations in the feedback path parameters than the variations in the forward path parameters. Thus, the specifications of the feedback elements must be observed strictly as compared to the specifications of the forward path elements.

6.18.4 Effect of Feedback on Time Constant of a Control System

Consider an open loop system with overall transfer function as,

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

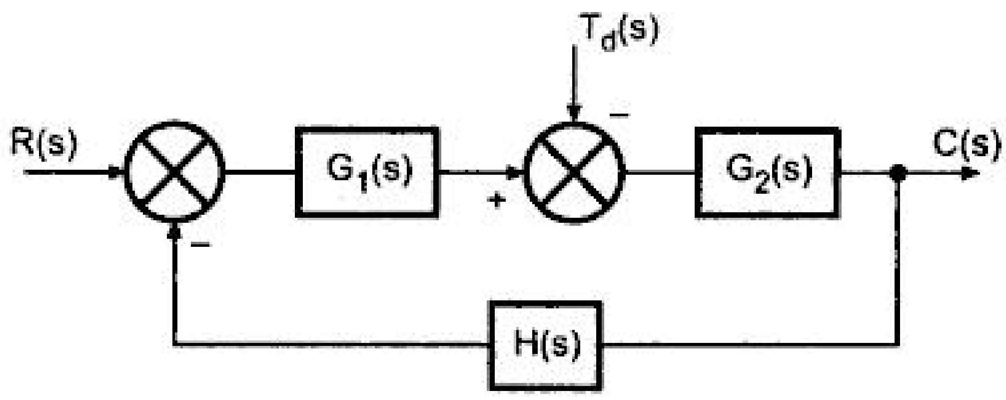


Fig. 6.34

Assuming R (s) to be zero, let us obtain the ratio C (s) / T_d (s) to study the effect of disturbance on output. With R (s) = 0, system becomes.

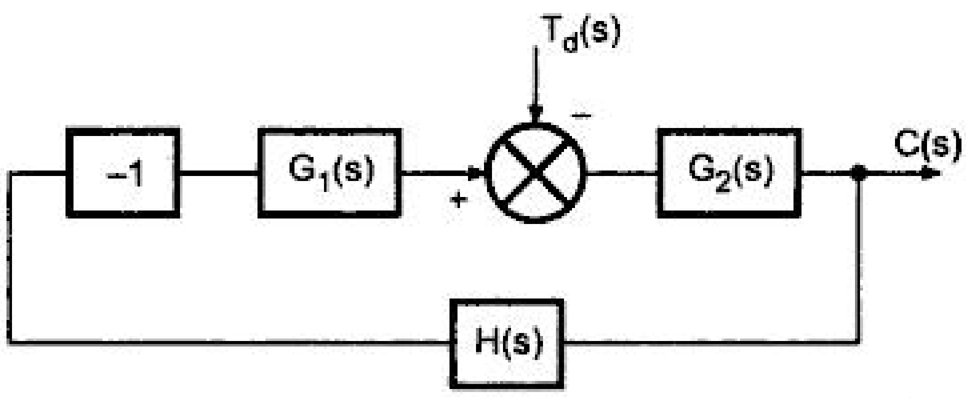


Fig. 6.35

The resultant elements are,

$$G(s) = G_2(s)$$

$$H'(s) = -G_1(s)\,H(s)$$

Positive feedback

Negative input

$$\therefore \quad \frac{C(s)}{-T_d(s)} = \frac{G_2(s)}{1-[G_2(s)(-G_1(s)\,H(s))]}$$

$$\therefore \quad \frac{C(s)}{T_d(s)} = \frac{-G_2(s)}{1+G_1\,G_2\,H(s)}$$

$$\therefore \quad C(s) = \frac{-T_d(s)\,G_2}{1+G_1\,G_2\,H}$$

In the denominator assume that $1 << G_1\,G_2\,H$ hence we get,

$$C(s) = \frac{-T_d(s)}{G_1\,H(s)}$$

Thus to make the effect of disturbance on the output as small as possible, the G_1 (s) must be selected as large as possible.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Now $$S_K^T = \frac{K}{T(s)} \cdot \frac{\partial T(s)}{\partial K}$$

$$T(s) = \frac{C(s)}{R(s)} = \frac{\dfrac{K}{s(s+a)}}{1+\dfrac{K}{s(s+a)}}$$

$$= \frac{K}{s^2 + as + K}$$

$\therefore$ $$\frac{\partial T(s)}{\partial K} = \frac{\left(s^2+as+K\right)(1) - K(1)}{\left(s^2+as+K\right)^2} = \frac{s^2+as}{\left(s^2+as+K\right)^2}$$

$\therefore$ $$S_K^T = \frac{K}{\left(\dfrac{K}{s^2+as+K}\right)} \times \frac{\left(s^2+as\right)}{\left(s^2+as+K\right)^2}$$

$$= \frac{s(s+a)}{s^2+as+K}$$

$\therefore$ $$S_K^T = \frac{s(s+4)}{\left(s^2+4s+20\right)}$$

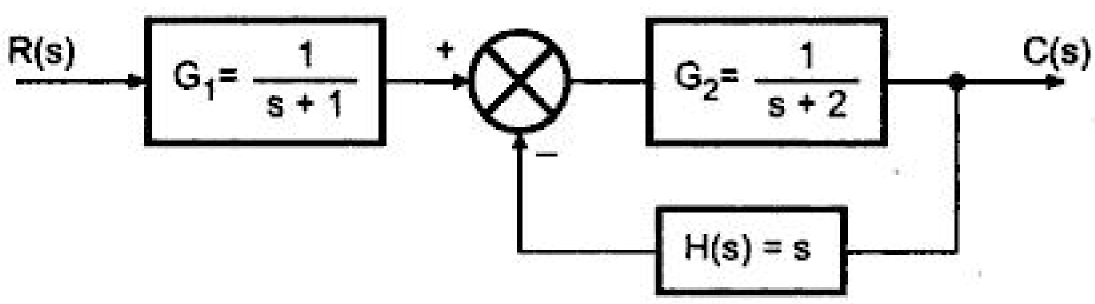
Now $$S_a^T = \frac{a}{T(s)} \cdot \frac{\partial T(s)}{\partial a}$$

$$\frac{\partial T(s)}{\partial a} = \frac{\left(s^2+as+K\right)(0) - K(s)}{\left(s^2+as+K\right)^2} = \frac{-K}{\left(s^2+as+K\right)^2}$$

$\therefore$ $$S_a^T = \frac{a}{\left(\dfrac{K}{s^2+as+K}\right)} \times \frac{-Ks}{\left(s^2+as+K\right)^2} = \frac{-as}{s^2+as+K}$$

$\therefore$ $$S_a^T = \frac{-4s}{s^2+4s+20}$$

Ex. 6.62 *In the system shown in the figure, find the system sensitivities i) $S_{G_2}^T$ ii) $S_{G_2}^T$.*



You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Ex. 6.64 *A thermometer requires one minute to indicate 98% of the response. Assuming first order system. Find its time constant.If thermometer is placed in a bath, the temperature of which varies linearly at the rate of* 10^0 *per minute, how much error does the thermometer show ?*

(Mumbai University, May-96, Dec.-96)

Sol. : The response of first order system to the step input is given as

$$c_{(t)} = A\,(t - e^{-t/T})$$

where

$$A = \text{Final response value}$$

$$T = \text{Time constant}$$

Now

$$c_{(t)} = 0.98\text{ A for } t = 60 \text{ sec}$$

$$\therefore \quad 0.98\text{ A} = A\,(1 - e^{-60/T})$$

$$\therefore \quad 0.98 = 1 - e^{-60t}$$

$$\therefore \quad e^{-60/T} = 0.02$$

$$\therefore \quad -\frac{60}{T} = -3.912$$

$$\therefore \quad T = 15.33 \text{ sec}$$

Now,

$$\theta_i = \text{input temperature}$$

$$\theta_0 = \text{output temperature}$$

For first order system,

$$\frac{\theta_0(s)}{\theta_i(s)} = \frac{1}{1+Ts} = \frac{1}{1+15.33\,s}$$

$$\therefore \quad \theta_0 = \theta_i\left[\frac{1}{1+15.33\,s}\right]$$

$$\text{Now error} = \theta_i(s) - \theta_0(s)$$

$$\therefore \quad E_{(s)} = \theta_i(s) - \theta_i\left[\frac{1}{1+15.33\,s}\right] = \theta_i\left[1 - \frac{1}{1+15.33\,s}\right]$$

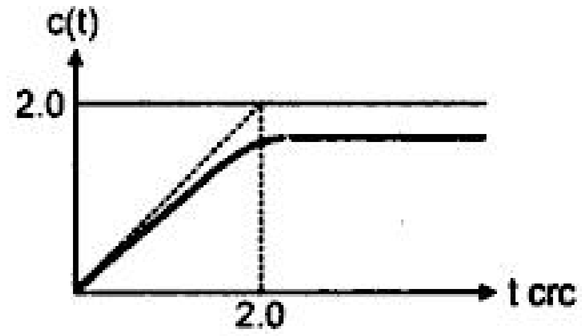
$$= \frac{15.33\,s\,\theta_i}{(1+15.33\,s)}$$

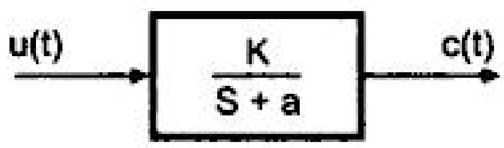
But input temperature is varying as a rate of 10^0 C/min i.e. $\frac{1}{6}$ ^{0}C/sec

$$\therefore \quad \theta_i(t) = \frac{1}{6} \times t$$

$$\therefore \quad \theta_i(s) = \frac{1}{6\,s^2}$$

Ex. 6.67 *A first order system and its response to a unit step input are shown in figure below. Determine `a' and `k'.* **(Mumbai University, Dec.97)**





Sol. :

$$R(s) = \frac{1}{s}$$

$$T(s) = \frac{C(s)}{R(s)} = \frac{K}{s+a}$$

$$\therefore \quad C(s) = \frac{K}{s+a}$$

Finding partial fractions, we get

$$C(s) = \frac{A_1}{s} + \frac{A_2}{s+a}$$

where $A_1 = \frac{K}{a}$ and $A_2 = -\frac{K}{a}$

$$\therefore \quad C(s) = \frac{\left(\frac{K}{a}\right)}{s} - \frac{\left(\frac{K}{a}\right)}{s+a}$$

Taking inverse laplace transform,

$$\therefore \quad C(t) = \frac{K}{a} - \frac{K}{a} e^{-at}$$

The steady state of C(t) is 2

$$\therefore \quad \lim_{t \to \infty} C(t) = 2$$

$$\therefore \quad \frac{K}{a} = 2$$

While slope at $t = 0$ is $\frac{2.0}{2.0} = 1$

$$\text{i.e.} \quad \left.\frac{dC(t)}{dt}\right|_{t=0} = 1$$

$$\therefore \quad -\frac{K}{a} \times (-a)\, e^{-at}\Big|_{t=0} = 1$$

$$\therefore \quad K = 1$$

$$\therefore \quad a = 1/2$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

6.19.1 Mean Square Error (Ems)

This is defined as
$$J = \text{Ems} = \lim_{\tau \to \infty}\left[\frac{1}{2\tau}\int_{-\tau}^{\tau} e^2(t)dt\right]$$

This index is mathematically convenient to deal with and is found to be useful for designing lower order system.

Advantages :

a) Easily handle mathematically.

b) Applicable to statistical inputs i.e. step.

c) Can be used with TYPE zero systems where $e_{ss} \to 0$ rapidly as $t \to \infty$

Disadvantages :

a) Minimization of Ems leads to lightly damped higher order systems with higher values of settling time and % overshoot.

b) It is relatively insensitive to parameter variations.

6.19.2 Integral Square Error Criterion (ISE)

This is defined as

$$J = \int_0^{\infty} e^2(t)\,dt$$

This is closely related to Ems and therefore has precisely the same advantages and disadvantages. As before it has got poor sensitivity to parameter variations. This index is extensively used for statistical inputs because of ease of computing the integral. Due to minimization of this index, there is minimization of power consumption for some systems.

For linear second order system I.S.E. is worked out to be

$$J = \frac{1+4\xi^2}{4\xi\omega n}$$

Which has a minimum value when $\xi = 0.5$.

6.19.3 Integral of time multiplied square error criterion (ITSE)

This is defined as

$$J = \int_0^{\infty} t\,e^2(t)\,dt$$

This is generally desirable to step response patterns when used as a basis for optimum design. This has a characteristics that in unit step response of the system a large initial error is weighted lightly, while errors occurring late in the transient response are penalized heavily.

$\therefore$ Magnitude of ramp input = A = $\frac{\pi}{30}$

Now
$$K_V = \lim_{s \to 0} s\,G(s)\,H(s)$$
$$= \lim_{s \to 0} s \cdot \frac{K_T}{s\,(s\,J_L + B_L)}$$
$$= \frac{K_T}{B_L} = \frac{2 \times 10^4}{3 \times 10^3}$$
$$= 6.67$$

$\therefore$
$$e_{ss} = \frac{A}{K_V} = \frac{\left(\frac{\pi}{30}\right)}{6.67}$$
$$= 0.0157 \text{ rad} = 0.9^\circ$$

Ex. 6.70 *A servo mechanism controlling the angular position of the shaft has the following specifications.*

amplifier gain = 300 A per radian error

motor gain = 0.8 Nm per ampere

load inertia = 2 kg – m^2

viscous friction constant = 17 N-m/(rad/sec)

Find the expression for the output response, if the input is a step change of 90°.

Sol. : The given values are,

$$K_a = 300 \text{ A/rad of error}$$
$$K_m = 0.8 \text{ N-m/A}$$
$$J = 2\,\text{kg} - \text{m}^2$$
$$B = 17 \text{ N-m/(rad/sec)}$$
$$K = K_p\, K_a\, K_m$$
$$= 300 \times 0.8 \text{ with } K_p \text{ assumed one}$$
$$= 240$$

$\therefore$
$$G(s) = \frac{K}{s(s\,J + B)}, \quad H(s) = 1$$

$\therefore$
$$\frac{C(s)}{R(s)} = \frac{\frac{K}{s\,(s\,J + B)}}{1 + \frac{K}{s\,(s\,J + B)}}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Now
$$K_V = \lim_{s\to 0} s\,G(s)\,H(s)$$
$$= \lim_{s\to 0} s\cdot\frac{K}{s(1+sT)}$$
$$= K$$
$$e_{ss} = \frac{A}{K_V} \quad \text{where } A = \text{magnitude of ramp input}$$
$\therefore$
$$e_{ss} = \frac{(1/12)}{K}$$
and
$$e_{ss} = 0.1^\circ \text{ allowed}$$
$\therefore$
$$0.1 = \frac{1}{12\,K}$$
$\therefore$
$$K = \frac{1}{1.2} \;/\text{ degree}$$

As $K_V = K$, the require velocity error coefficient is $\frac{1}{1.2}$ / degree.

Ex. 6.73 *The open loop transfer function of a unity feedback system is given by,* $G(s) = e^{-2s}$. *Sketch the output of the feedback system for a unit step input. Assume that the system is initially relaxed.*

(Gate - 94)

Sol. : As system has unity feedback,

$\therefore$
$$H(s) = 1$$
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\,H(s)} = \frac{G(s)}{1+G(s)}$$
$$= \frac{e^{-2s}}{1+e^{-2s}}$$

Dividing both numerator and denominator by e^{-2s},

$\therefore$
$$\frac{C(s)}{R(s)} = \frac{1}{1+e^{2s}}$$

The system is excited by unit step input

$\therefore$
$$r(t) = 1 \quad \text{for } t \geq 0$$
$\therefore$
$$R(s) = \frac{1}{s}$$

Substituting in C (s)

$\therefore$
$$C(s) = \frac{1}{s}\cdot\frac{1}{1+e^{2s}}$$
$\therefore$
$$C(s)\left[1+e^{2s}\right] = \frac{1}{s}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{5}{s(1+0.5s)}}{1+\frac{5}{s(1+0.5s)}} = \frac{5}{0.5s^2+s+5}$$

Make coefficient of s^2 as 1 to bring it in standard form,

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{10}{s^2+2s+10}$$

Comparing denominator with standard form,

$$\omega_n^2 = 10$$

$$\therefore \quad \omega_n = \sqrt{10}$$

and $\quad 2\xi\omega_n = 2$

$$\therefore \quad \xi = 0.3162$$

This is the required damping ratio.

For error calculation :

$$G(s)H(s) = \frac{5}{s(1+0.5s)}$$

For ramp input magnitude is unity i.e. A=1

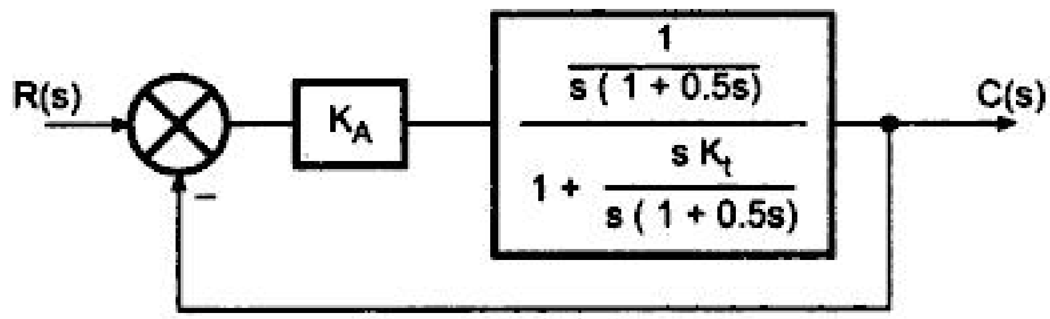
$$\therefore \quad K_V = \lim_{s\to 0} sG(s)H(s) = \lim_{s\to 0} \frac{s.5}{s(1+0.5s)} = 5$$

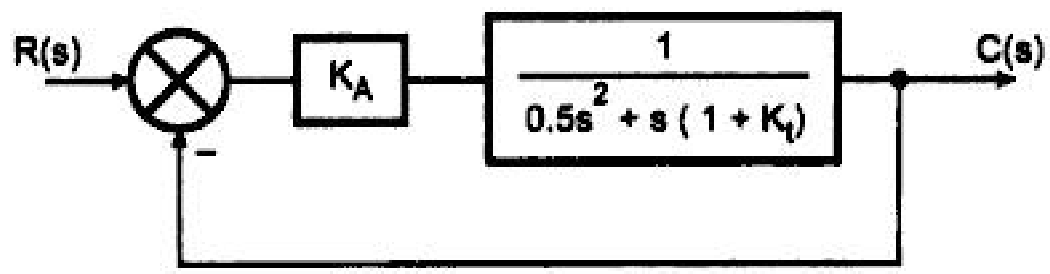
$$\therefore \quad e_{ss} = \frac{A}{K_V} = \frac{1}{5} = 0.2$$

Case b) The derivative feedback is introduced in the system.

The system becomes,

$$\therefore \quad G(s) = \frac{K_A}{s[0.5s+1+K_t]} \quad \text{and} \quad H(s)=1$$





You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

$$\therefore \quad \frac{C(s)}{T_L(s)} = -\left[\frac{\dfrac{1}{(0.15\,s^2 + 0.9s)}}{1 - \dfrac{1}{(0.15\,s^2 + 0.9\,s)}.(-6)}\right]$$

$$= -\left[\frac{1}{0.15\,s^2 + 0.9\,s + 6}\right]$$

$$T_L(s) = \frac{1}{s}$$

$$\therefore \quad C(s) = \frac{-1}{s\left[0.15\,s^2 + 0.9\,s + 6\right]}$$

$$C_{ss} = \lim_{s \to 0} sC(s)$$

$$= \lim_{s \to 0} s.\frac{-1}{s[0.15s^2 + 0.9s + 6]}$$

$$= -\frac{1}{6}$$

$$= -0.166$$

Summary

A time response of any system can be divided into two parts (i) Steady state response and (ii) Transient response. Transient response is the output before it reaches and stabilises to its final value.

Steady state is the output which remains when the transient output completely vanishes from the system output.

$$C(t) = C_t(t) + C_{ss}$$

The standard inputs used for control system analysis are (i) Step (ii) Ramp (iii)Parabolic and (iv) Impulse. The steady state error e_{ss} is the difference between the acutal output and the reference input.

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s)$$

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)\,H(s)}$$

The static error coefficients are

$$K_p = \lim_{s \to 0} G(s)\,H(s) \text{ to be used for Step inputs}$$

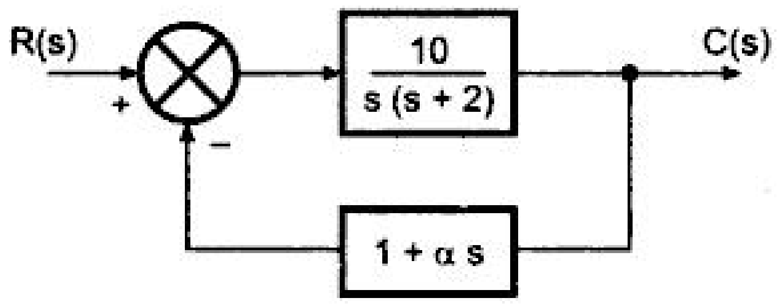
$$K_v = \lim_{s \to 0} sG(s)\,H(s) \text{ to be used for Ramp inputs}$$

18. *The open loop transfer function of a unity feedback control system is given by*

$$G(s) = \frac{K}{s(1 + sT)}$$

The input to the system is 1 R.P.M. and the steady state error being 0.25. Calculate the natural frequency of oscillations if the system is critically damped. **[Ans. :** 48 rad/s**]**

19. *The block diagram of a position control system with velocity feedback is shown in fig. Determine the value of α so that the step response has maximum overshoot of 10 percent. What is the steady state error?*



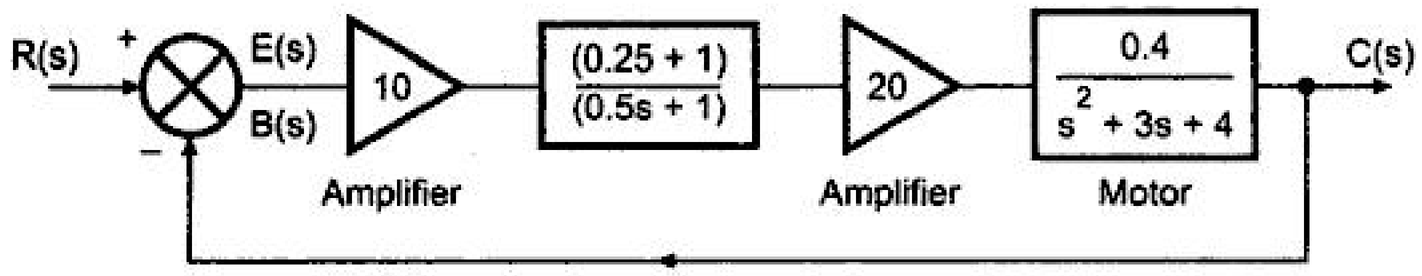
[Ans. : $\alpha = 0.1739$, $e_{ss} = 0$**]**

20. *The closed loop transfer function of a second order system with proportional plus error-rate feedback is given by* $\frac{C(s)}{R(s)} = \frac{K(s+z)}{s^2 + 4s + 8}$ *where parameters K and z are adjustable*

a) *If $r(t) = t$; determine the values of K and z such that the steady state error is zero.*

b) *For these values of K and z, determine the steady state error to a unit parabolic input; i.e. $r(t) = -1/2t^2$.*
[Ans. : (a) K = 4, z = 2 **(b)** 0.25**]**

21. *For the system shown in fig. determine the steady state error for*

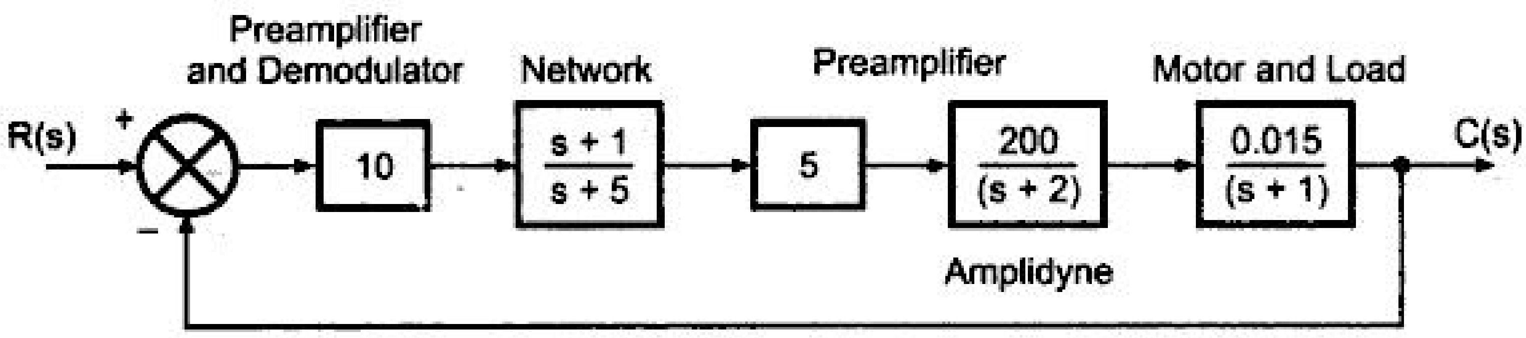


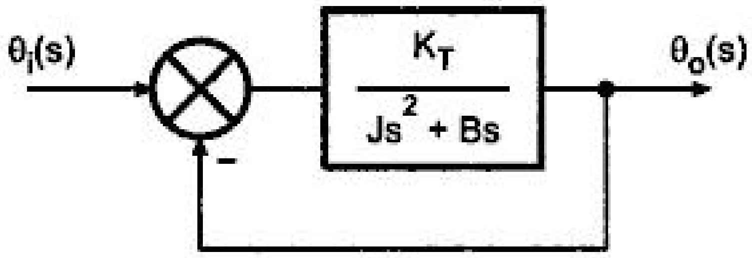
i) A unit step input

ii) A unit step velocity input

iii) A unit step acceleration input **[Ans. : (i)** 0.0475 **(ii)** ∞ **(iii)** ∞ **]**

22. *The block diagram of a fire control system with unity feedback is described in fig. Using generalised error series determine the steady state error of the system when the system input is*





Find :

a) The step response if input is step type of one radian. Also determine rise time, peak time and peak overshoot.

b) The S.S. error if input is constant angular velocity of 1 r.p.m.

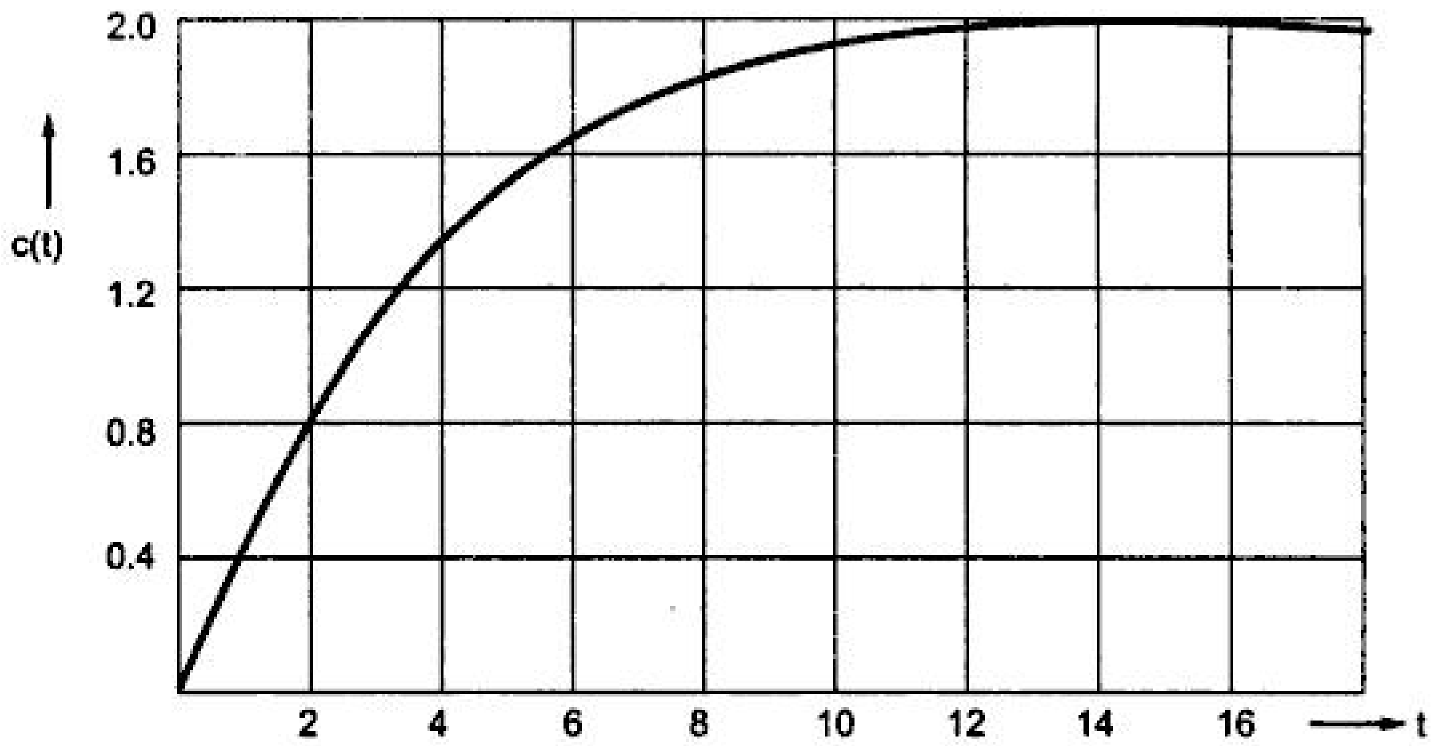
c) The S.S. error which exists when a torque of 1200 Nm is applied to the load shaft.

(Ans. : $1 - 1.75\, e^{-13.4\,t} \sin(9.35\,t + 0.61)$, 0.6°, 1.72°**)**

University Questions (New Syllabus)

May-2003

1. *The plot shows the unit step response of a first order system. What is the transfer function of the system ?* **(8 marks)**



Sol. : Let the transfer function of first order system be,

$$\frac{C(s)}{R(s)} = \frac{K}{s+a} \quad \text{and} \quad R(s) = \frac{1}{s} \text{ as unit step}$$

$$\therefore \qquad C(s) = \frac{K}{s(s+a)} = \frac{K/a}{s} - \frac{K/a}{s+a}$$

$$\therefore \qquad c(t) = \frac{K}{a} - \frac{K}{a} e^{-at}$$

From graph, $\lim_{t\to\infty} c(t) = 2$

$$\therefore \qquad 2 = \frac{K}{a} - \frac{K}{a} e^{-\infty} = \frac{K}{a}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

7
Stability of the System

7.1 Introduction :

As we have seen earlier that every system for small amount of time has to pass through a transient period. Now whether system will reach to its intended steady state after passing through transients or not? The answer to this question means to define whether system is stable or unstable. This is stability analysis.

For example, consider that we want to go from one station to other. The station where we want to reach is our final steady state. The travelling period is the transient period. Now anything may happen during the travelling period. And due to conditions like bad weather, road accident etc. there is a chance that we may not reach to next station in time. The analysis of, whether the given system can reach steady state; passing through the transients successfully is called as **Stability Analysis** of the system.

In this chapter we will study

i) The stability and the factors on which system stability depends.

ii) Stability analysis and locations of closed loop poles.

iii) Stability analysis using Hurwitz method.

iv) Stability analysis using Routh-Hurwitz method.

v) Special cases of Routh's array and

vi) Applications of Routh-Hurwitz's method.

In this chapter we will define stability, we will see the factors on which it depends and how to analyze the stability of given system by Routh-Hurwitz's method.

7.2 Concept of Stability :

Consider a system i.e. a deep container with an object placed inside it as shown in Fig. 7.1.

Now if we apply a force to take out the object, as the depth of container is more, it will oscillate and will settle down again at original position.

If we assume that the force required to take out the object tends to infinity i.e. always object will oscillate when force is applied and will settle down but will not come out, such a system is called as ***absolutely stable*** system. No change in parameters, disturbances, changes the output. As against this, consider a container which is pointed one, on which we try to keep a circular object.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Ex. 8.49 *Plot the root locus for a unity feedback system whose forward transfer function is given as*

$$G(s) = \frac{10(s+1)}{s(s-3)}$$

Sol. :
$$G(s)\,H(s) = \frac{10(s+1)}{s(s-3)}$$

Step 1 : P = 2, Z = 1. For root locus assume 10 as K.

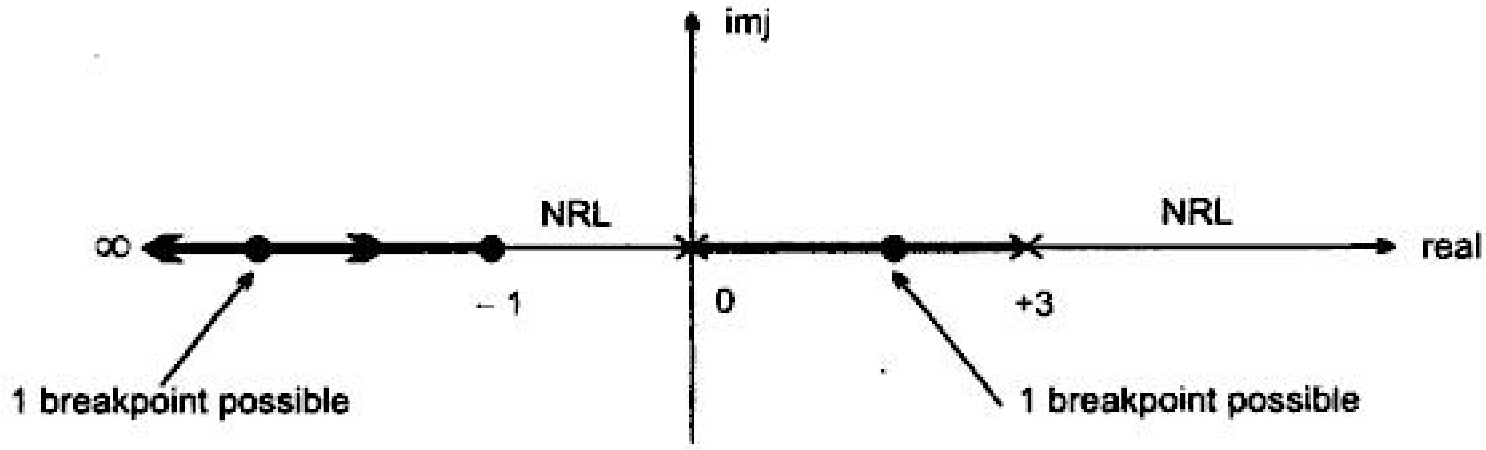
$\therefore$ Number of branches = N = 2

One branch will approach to ∞.

Starting points = 0, +3

Terminating point = –1, ∞

Step 2 : Pole zero Plot



Step 3 : Angles of asymptote

$$\theta = \frac{(2q+1)180°}{P-Z} = \frac{180°}{1}$$
$$= 180°$$

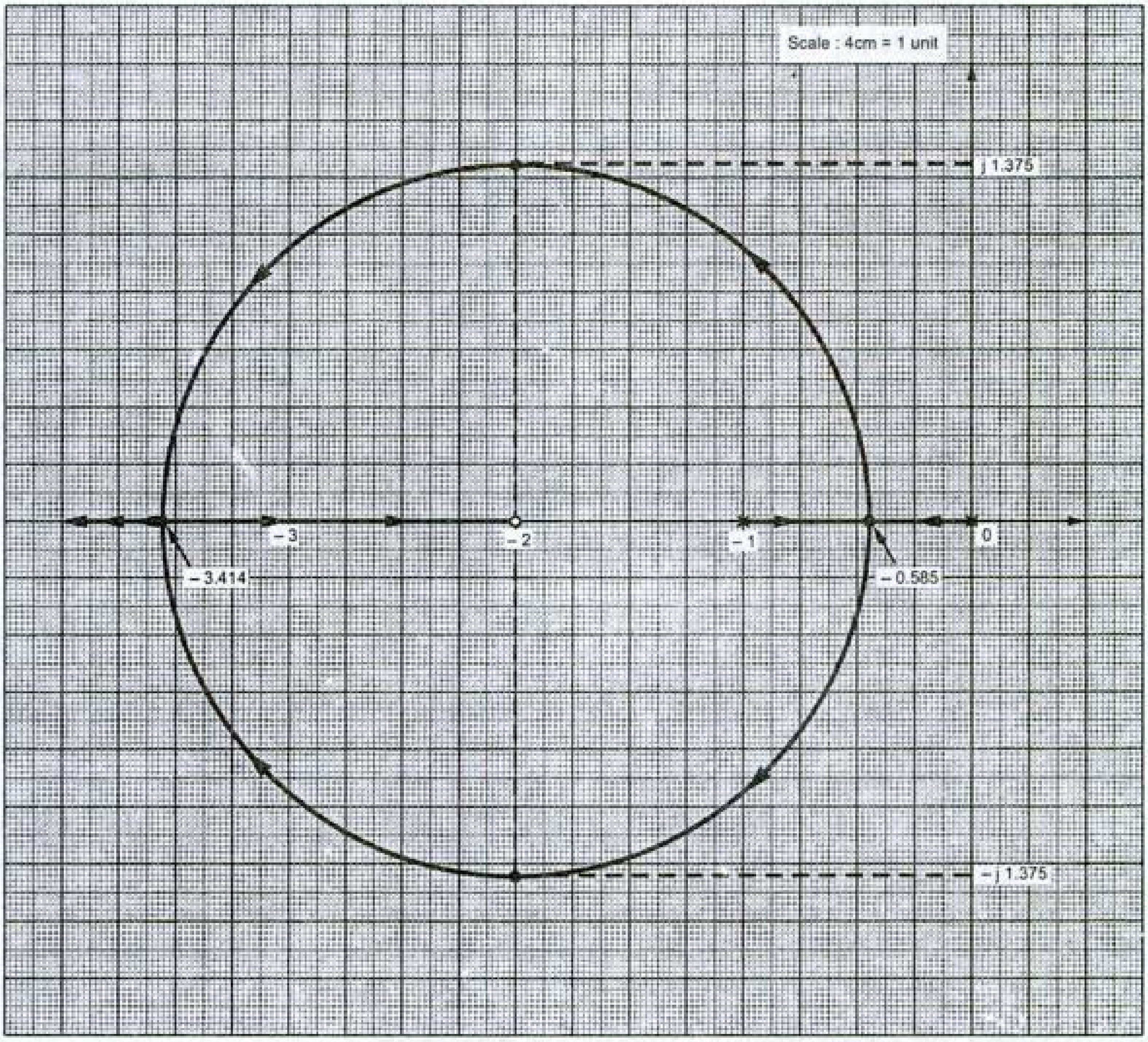
One asymptote which is negative real axis.

Step 4 : As negative real axis is asymptote, centroid is not necessary.

Step 5 : Breakaway point

$$1 + G(s)\,H(s) = 0$$
$$1 + \frac{K(s+1)}{s(s-3)} = 0$$
$$\therefore \quad s(s-3) + K(s+1) = 0$$
$$\therefore \quad K = \frac{-s^2+3s}{(s+1)}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.


Scale : 4cm = 1 unit
j 1.375
− 3
− 2
− 1
0
− 3.414
− 0.585
− j 1.375

❑❑❑

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

9.5 Apparatus Required for Frequency Response :

Obviously we need a variable frequency oscillator (O), an amplitude measuring device (M) and a phase measuring device (P) to obtain the ratio of output amplitude, to the input amplitude and the phase angle between the input and the output amplitude.

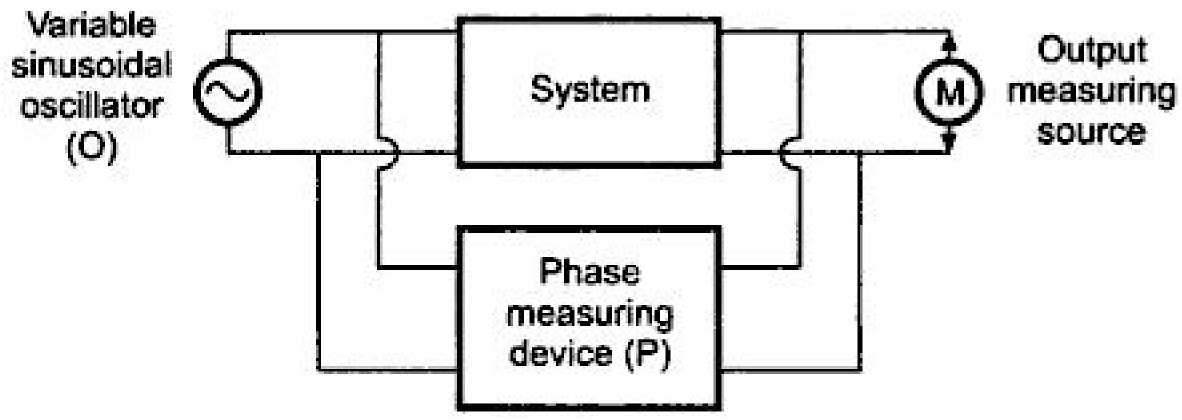


Fig. 9.3 Transfer function analyser

For example for an electronic circuit the sinusoidal oscillator may have frequency range from 10 Hz to 100 KHz and voltage may be measured using CRO (or electronic voltmeter), the phase may be measured using CRO (or phase meter).

However for servomechanisms, the useful frequency range may be very low say 0.001 Hz to 10 Hz. A special low frequency oscillator, a special CRO having long persistance screen and a low frequency phase measuring device is necessary. This system is supplied by many manufacturers and is generally known as a 'Transfer Function Analyser'.

For non electronic systems such as pneumatic systems the transfer function analyser should supply air pressure whose variation is sinusoidal. Such systems have also been built and used.

At each input frequency one must wait for the transients to die out and the steady state sinusoidal output to result. Then using the amplitude measuring device (such as electronic voltmeter), the input and output are measured and 'Gain' determined. Similarly using the phase measuring device, the 'Phase' is determined. This procedure is repeated at other frequencies. The results are tabulated giving at each frequency, the gain and the phase. By this procedure we generate the data for frequency response.

9.6 Relation Between Transfer Function and Frequency Response :

Consider that the transfer function of a system is T(s).

then $$T(s) = \frac{\tau(s)}{R(s)}$$

The frequency response function or 'system function' is then obtained by simply

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

9.11 Determination of ω_{gc} and P.M. for Standard Second Order System:

Consider $$G(s) = \frac{\omega_n^2}{s(s+2\xi\,\omega_n)}, \quad H(s) = 1$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\,H(s)} = \frac{\omega_n^2}{s^2+2\xi\,\omega_n\,s+\omega_n^2}$$

But to find ω_{gc} and P.M. we have to consider the transfer function G(s) H(s).

$$G(s)\,H(s) = \frac{\omega_n^2}{s(s+2\xi\,\omega_n)} = \frac{\omega_n^2}{s^2+2\xi\,\omega_n s}$$

$$\therefore \quad G(j\omega)\,H(j\omega) = \frac{\omega_n^2}{(j\omega)^2+2\xi\,\omega_n\,j\omega} = \frac{\omega_n^2}{-\omega^2+2\xi\,\omega_n\,j\omega}$$

ω_{gc} is the gain crossover frequency for which magnitude of $G(j\omega)\,H(j\omega)$ is 0 dB i.e. so log $|G(j\omega)\,H(j\omega)| = 0$ dB

$$\therefore \quad |G(j\omega)\,H(j\omega)| = 1 \quad \text{for } \omega = \omega_{gc}$$

For ω_{gc} $|G(j\omega)\,H(j\omega)| = 0$ dB i.e. $|G(j\omega)\,H(j\omega)| = 1$

$$\text{Now} \quad |G(j\omega)\,H(j\omega)| = \frac{\omega_n^2}{\sqrt{(-\omega^2)^2+(2\xi\,\omega\omega_n)^2}} = \frac{\omega_n^2}{\sqrt{\omega^4+4\xi^2\,\omega_2\,\omega_n^2}} = 1$$

Squaring both sides.

$$\frac{\omega_n^4}{\omega^4+4\,\xi^2\,\omega^2\,\omega_n^2} = 1$$

$$\therefore \quad \omega_n^4 = \omega^4+4\,\xi^2\,\omega^2\,\omega_n^2$$

$$\therefore \quad \omega^4+4\,\xi^2\,\omega^2\,\omega_n^2-\omega_n^4 = 0$$

Now ω_n is constant, we want to determine 'ω' which is ω_{gc} as $|G(j\omega)\,H(j\omega)|$ is equated to 1. It is quadratic in ω^2. i.e. ω_{gc}^2

$$\therefore \quad \omega_{gc}^2 = \frac{-4\,\xi^2\,\omega_n^2 \pm \sqrt{(4\,\xi^2\,\omega_n^2)^2-4\times 1\times(-\omega_n^4)}}{2\times 1}$$

$$= \frac{-4\,\xi^2\,\omega_n^2 \pm \sqrt{16\,\xi^4\,\omega_n^4+4\,\omega_n^4}}{2}$$

$$= -2\xi^2\,\omega_n^2 \pm \frac{\omega_n^2\sqrt{16\xi^4+4}}{2}$$

$$\omega_{gc}^2 = -2\xi^2\,\omega_n^2 \pm \omega_n^2\,\sqrt{4\,\xi^4+1}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Ex. 9.11 *A system has 30% overshoot and settling time of 5 seconds, for a step input. Determine the transfer function. Sketch frequency response and determine the bandwith. The steady state error is 2%.*

(Mumbai University Dec. 95)

Sol. :

$$M_p = 30\%$$

$$0.3 = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$\therefore \quad ln\,(0.3) = \frac{-\pi\xi}{\sqrt{1-\xi^2}}$$

$$\therefore \quad -1.2039 = \frac{-\pi\xi}{\sqrt{1-\xi^2}}$$

Solving,

$$\therefore \quad \xi^2 = 0.128$$

$$\therefore \quad \xi = 0.3578$$

$$T_s = \frac{4}{\xi\omega_n} = 5$$

$$\therefore \quad \omega_n = 2.2355 \text{ rad/sec}$$

$$\therefore \quad \text{T.F.} = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$= \frac{5}{s^2 + 1.6s + 5}$$

For bandwidth,

$$20 \log \frac{1}{\sqrt{(1-x^2)^2 + 4\xi^2 x^2}} = -3 \text{ dB} \qquad \text{where } x = \frac{\omega_b}{\omega_n}$$

$$\therefore \quad -20 \log \sqrt{(1-x^2)^2 + 4\xi^2 x^2} = -3$$

$$\therefore \quad \sqrt{(1-x^2)^2 + 4\xi^2 x^2} = 1.4125$$

$$\therefore \quad x^4 - 1.488x^2 - 0.9951 = 0 \qquad \text{for } \xi = 0.3578$$

$$\therefore \quad x^2 = 1.9884, \qquad x = 1.4101$$

i.e.

$$\frac{\omega_b}{\omega_n} = 1.4101$$

$$\therefore \quad \omega_b = \text{Bandwidth} = 3.152$$

Sol. : The transfer function is,

$$\frac{C(s)}{R(s)} = \frac{\frac{K}{s(s+a)}}{1+\frac{K}{s(s+a)}} = \frac{K}{s^2+as+K}$$

$$\therefore \quad \omega_n^2 = K \quad \text{i.e.} \quad \omega_n = \sqrt{K}$$

and $$2\xi\omega_n = a \text{ i.e. } \xi = \frac{a}{2\sqrt{K}}$$

Now $$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = 1.04$$

$$\therefore \quad 1 = 4.3264\,\xi^2\,(1-\xi^2)$$

$$\therefore \quad 4.3264\,\xi^4 - 4.3264\,\xi^2 + 1 = 0$$

$$\therefore \quad \xi^2 = \frac{4.3264 \pm \sqrt{(4.3264)^2 - 4\times 4.3264}}{2\times 4.3264} = 0.637,\ 0.362$$

$$\therefore \quad \xi = 0.7983,\ 0.6016$$

For M_r, ξ must be less than 0.707 hence $\xi = 0.6016$.

$$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

$$\therefore \quad 11.55 = \omega_n\sqrt{1-2\times(0.6016)^2}$$

$$\therefore \quad \omega_n = 21.9788 \text{ rad/sec}$$

$$\therefore \quad K = \omega_n^2 = 483.071$$

$$\therefore \quad a = 26.4445$$

ii) $$T_s = \frac{4}{\xi\omega_n} = 0.3025 \text{ sec}$$

$$\text{B.W.} = \omega_n\sqrt{1-2\xi^2+\sqrt{2-4\xi^2+4\xi^4}}$$

$$= 25.1902$$

2. *Compare time and frequency domain specifications.* **(8 marks)**

❑❑❑

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

K = Resultant system gain $\quad$ P = Type of the system

$T_1, T_2, T_a, T_b, \ldots\ldots$ = Time constants of different poles and zeros.

Each of the factor involved in G(s)H(s) above will contribute to magnitude and phase angle variations of $G(j\omega)\,H(j\omega)$ in frequency domain. Frequency domain transfer function can be obtained by substituting. $s = j\omega$ in above expression

$$G(j\omega)\,H(j\omega) = \frac{K(1+T_1\,j\omega)\,(1+T_2\,j\omega)\ldots\ldots}{(j\omega)^P\,(1+T_a\,j\omega)\,(1+T_b\,j\omega)\ldots\ldots}$$

Now basic factors which very frequently occur in the above form can be identified and studied separately.

List of such basic factors is ,

1) Resultant system gain K, constant factor. (When $G(j\omega)\,H(j\omega)$ is expressed in time constant form).

2) Poles or zeros at the origin. (Integral and Derivative factors) i.e. $(j\omega)^{\pm P}$

   Either poles or zeros at origin will be present.

3) Simple poles and zeros also called as first order factors of the form $(1+j\omega T)^{\pm 1}$

4) Quadratic factors which cannot be factoriesed into real factors, of the form

$$\left(1+\frac{2\xi}{\omega_n}s+\frac{s^2}{\omega_n^2}\right) \approx 1+2\xi\,j\left(\frac{\omega}{\omega_n}\right)+\left(\frac{j\omega}{\omega_n}\right)^2$$

Once the behaviour of such factors is clear in frequency domain then by adding logarithmic plots of such factors, the resultant logarithmic plot for any $G(j\omega)\,H(j\omega)$ can be obtained. The process of obtaining logarithmic plots for such factors can be simplified by using asymptotic approximations for each factor. But by adding corrections to such plot if necessary accurate plot may be obtained.

10.4 Bode Plots of Standard Factors of G(jω) H(jω) :

For each factor procedure to obtain its Bode Plot can be divided into following steps.

Step 1 : Replace 's' by 'jω' to convert it to frequency domain.

Step 2 : Find its magnitude as a function of 'ω'.

Step 3 : Express the magnitude in 'dB' by $20\log_{10}|G(j\omega)\,H(j\omega)|$

Step 4 : Find phase angle by using $\tan^{-1}\left[\dfrac{\text{imaginary part}}{\text{real part}}\right] = \phi$ in degres.

Step 5 : With required approximations, plot magnitude in dB and phase angle in degrees against Log ω by varying ω from 0 to ∞.

Let us start with the basic factors one by one.

In general for P number of zeros at the origin

$$G(s)\,H(s) = s^P$$

$$\therefore \quad G(j\omega)\,H(j\omega) = j\omega \cdot j\omega \cdot j\omega \text{......P time}$$

$$\therefore \quad |G(j\omega)\,H(j\omega)| = \omega^P$$

$$\therefore \quad \text{Magnitude in dB} = 20 \text{ xP Log } \omega$$

i.e. $$\text{slope} = +\,20 \text{ xP dB/decade}$$

So it gives family of lines with slopes as +20, + 40 ..... + 20 xP dB/decade passing through intersection point of $\omega = 1$ with 0 dB line as shown in the Fig. 10.8.

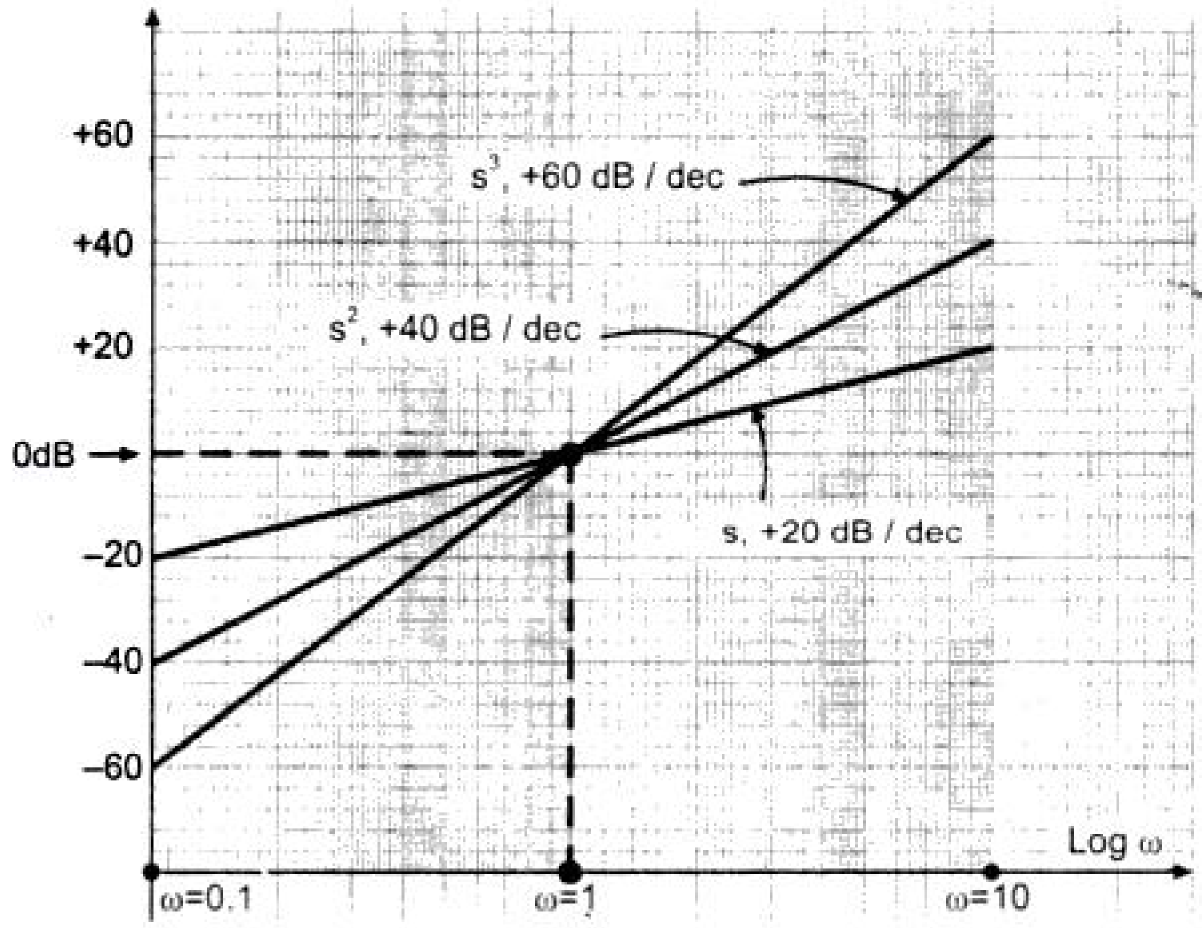


Fig. 10.8 Contribution by zeros at origin

A zero at the origin increases the magnitude at a rate of +20 dB/dec.

Phase Angle Plot : Consider 1 pole at the origin

$$G(s)\,H(s) = \frac{1}{s} \quad G(j\omega)\,H(j\omega) = \frac{1}{j\omega}$$

$$\therefore \quad \angle G(j\omega)\,H(j\omega) = \angle\frac{1}{j\omega}\angle\frac{1}{j\omega} = \frac{0^\circ}{90^\circ \cdot 90^\circ} = -180^\circ$$

This is independent of `ω'. So phase angle plot of `pole at origin is line parallel to x-axis contributing – 90° to phase angle.

For 2 pole at origin, $G(s)\,H(s) = \dfrac{1}{s^2}$

This can be shown as below.

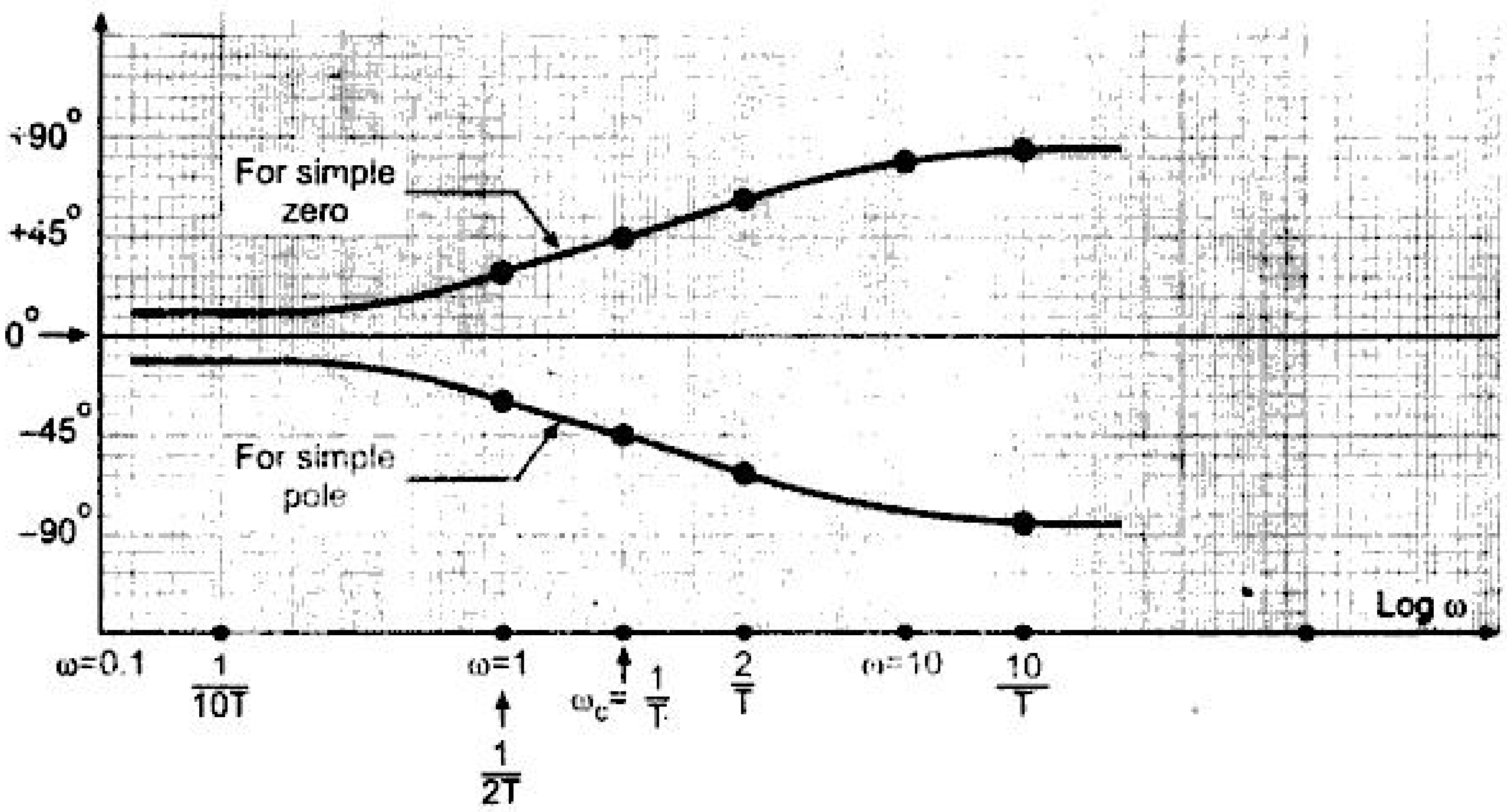


Fig. 10.14

The shapes will remain same, for various values of 'T' time constants.

It is important to note that phase angle is ± 45° for a zero or pole at $\omega = \omega_c = 1/T$

Ex. 10.1 *Sketch the Bode Plot for the system having*

$$G(s)\,H(s) = \frac{20}{s(1+0.1\,s)}$$

Sol. : First see that given G(s) H(s) is in the proper time constant form or not. If not arrange it in the time constant form. Now identify the factors.

i) K = 20 ∴ Its magnitude = 20 Log 20 = +26 dB

ii) 1 pole at origin. Its magnitude plot is straight line passing through intersection point of ω = 1 and 0 dB with slope − 20 dB/decade.

iii) Simple pole → $\frac{1}{1+0.1\,s}$, comparing with $\frac{1}{1+Ts}$

$$\therefore \quad T = 0.1$$

$$\therefore \quad \omega_c = \frac{1}{T} = \frac{1}{0.1} = 10$$

i.e. Asymptotic magnitude plot is 0 dB up to $\omega = \omega_c = 10$ and then straight line of slope − 20 dB/decade. Procedure to Plot resultant

i) Draw 20 Log K line.

ii) Draw line for 1 pole at origin

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Hence it is necessary to modify magnitude plot for 2nd order quadratic pole as shown above at its corner frequency for various values of `ξ'.

Students can use table 10.1 to decide correction for given ξ or find the corection using the formula,

$$\text{Correction} = -20 \text{ Log } 2\xi \text{ dB at } \omega = \omega_n \text{ of pole}$$

Positive correction upwards and negative correction down wards.

The magnitude plot for a quadratic zero can be obtained by reversing the sign of the slope of basic asymptote and then by reversing the signs of the corrections at corner frequency for various values of `ξ'. Hence it looks like as shown in fig. 10.19

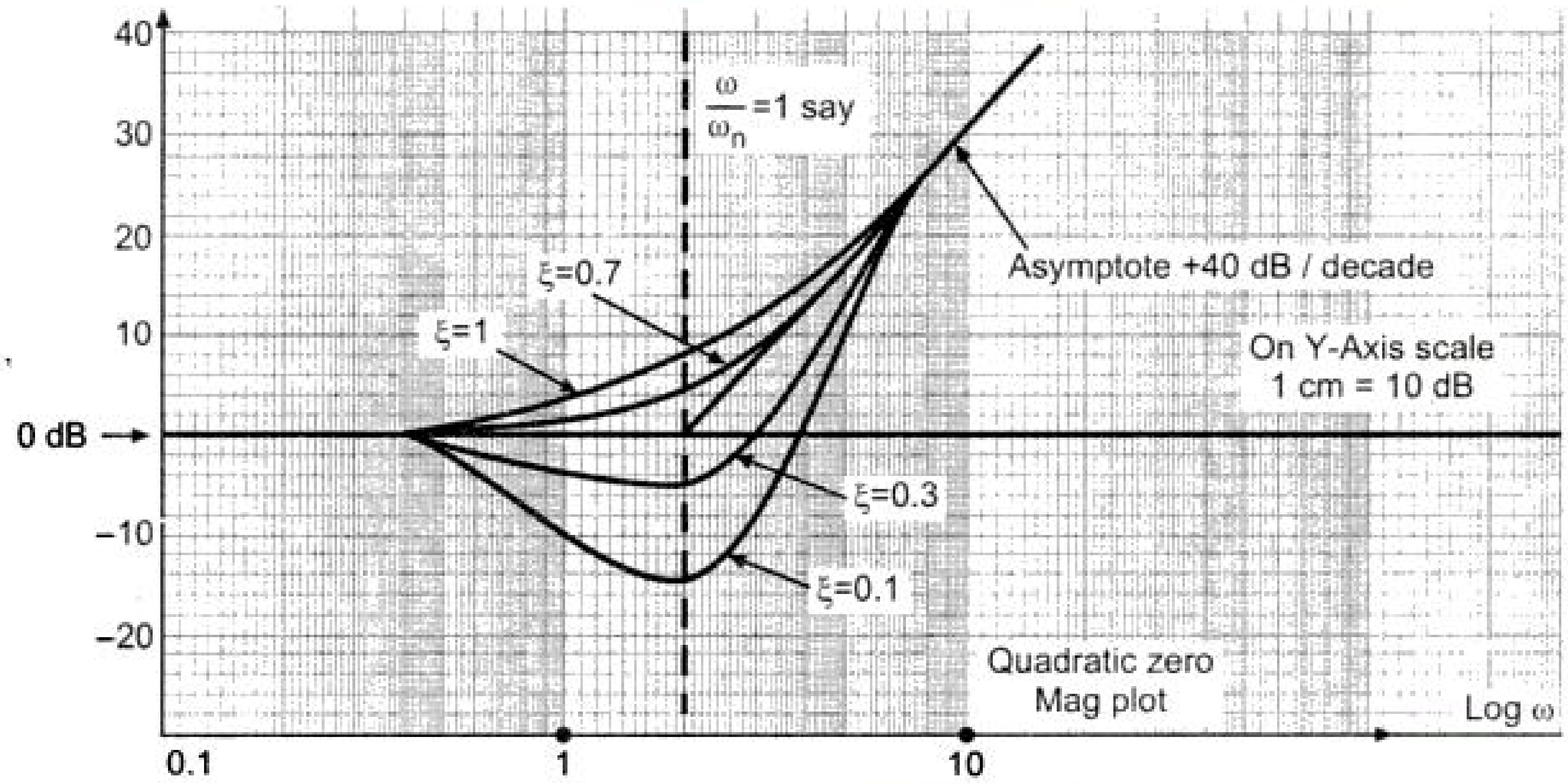


Fig. 10.19 Quadratic zero

Let us see phase angle table :

$$G(j\omega)\, H(j\omega) = \frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2 + j\, 2\xi\left(\frac{\omega}{\omega_n}\right)} \quad \text{for a quadratic pole}$$

$$\therefore \quad \angle G(j\omega)\, H(j\omega) = \frac{0°}{\tan^{-1}\left\{\frac{2\xi\,(\omega/\omega_n)}{1-(\omega/\omega_n)^2}\right\}}$$

$$\angle G(j\omega)\, H(j\omega) = \tan^{-1}\left\{\frac{2\xi\,(\omega/\omega_n)}{1-(\omega/\omega_n)^2}\right\}$$

The table for $\xi = 0.3$ is shown below

$$\phi = \tan^{-1}\left\{\frac{2\times 0.3\times \omega/\omega_n}{1-(\omega/\omega_n)^2}\right\}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Ex. 10.5 *For a unity feedback system* $G(s) = \dfrac{800\,(s+2)}{s^2\,(s+10)\,(s+40)}$

Sketch the Bode plot, asymptotic in nature. Comment on stability.

Sol. : *Step 1* : Arrange G(s) H(s) in time constant form.

$$G(s)\,H(s) = \frac{800 \times 2 \times \left(\dfrac{s}{2}+1\right)}{s^2 \times 10\left(1+\dfrac{s}{10}\right) \times 40 \times \left(1+\dfrac{s}{40}\right)} = \frac{4\left(1+\dfrac{s}{2}\right)}{s^2\left(1+\dfrac{s}{10}\right)\left(1+\dfrac{s}{40}\right)}$$

Step 2 : Factors are

i) Constant K = 4,

ii) 2 poles at the origin, $1/s^2$

iii) Simple zero, $1+\frac{s}{2}$, $T_1 = 1/2$, $\omega_{c1} = \dfrac{1}{T_1} = 2$ rad/sec.

iv) Simple pole, $\dfrac{1}{1+\dfrac{s}{10}}$, $T_2 = 1/10$, $\omega_{c2} = \dfrac{1}{T_2} = 10$ rad/sec.

v) Simple pole, $\dfrac{1}{1+\dfrac{s}{40}}$, $T_3 = 1/40$, $\omega_{c3} = \dfrac{1}{T_3} = 40$ rad/sec.

Step 3 : Magnitude plot analysis

i) For K = 4, 20 Log K = 20 Log 4 = 12dB.

ii) 2 poles at the origin i.e. $1/s^2$. It contributes a straight line of slope – 40 dB/decade passing through intersection point of $\omega = 1$ and 0 dB. So starting slope becomes – 40 dB/decade.

iii) Shift intersection point of $\omega = 1$ and 0 dB on 20 Log K line and draw parallel line to – 40 dB/decade. This represents addition of K and $1/s^2$. This resultant will continue till first corner frequency $\omega_{c1} = 2$.

iv) At $\omega_{c1} = 2$, simple zero occurs which contributes + 20 dB/dec individually and hence resultant slope from '2' onwards becomes – 40 +20 = – 20 dB/dec. This continues till $\omega_{c1} = 10$.

v) At $\omega_{c2} = 10$, simple pole occurs which contributes – 20 dB/dec individually and hence resultant slope from 10 onwards becomes – 20 – 20 = – 40 dB/dec again. This continues till $\omega_{c3} = 40$.

vi) At $\omega_{c3} = 40$, simple pole occurs which contributes – 20 dB/dec individually and hence resultant slope from 40 onwards becomes – 40 – 20 = – 60 dB/dec. This continues upto $\omega \to \infty$ as there is no other factor present in G(s) H(s).

Step 4 : Phase Angle Plot

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Ex. 10.7 *For a certain feedback system* $G(s)\,H(s) = \dfrac{3\,(s+1)\,(s+6)}{s^2\,(s^2+18\,s+400)}$

Sketch the Bode plot and comment on G.M. P.M. and stability

Sol.: *Step 1* : Arrange G(s) H(s) in time constant form.

$$G(s)\,H(s) = \frac{3\,(s+1)\,(6)\,(1+s/6)}{s^2\,(400)\left(1+\frac{18}{400}s+\frac{s^2}{400}\right)} = \frac{0.045\,(1+s)\left(1+\frac{s}{6}\right)}{s^2\left(1+0.045\,s+\frac{s^2}{400}\right)}$$

Step 2 : Factors : i) Constant K = 0.045, ii) $\frac{1}{s^2}$, 2 poles at the origin

iii) Simple zero , (1 + s) , $T_1 = 1$, $\omega_{c1} = 1$

iv) Simple zero , $(1+\frac{s}{6})$, $T_2 = \frac{1}{6}$, $\omega_{c2} = 6$

v) Quadratic pole, $\dfrac{1}{(1+0.045\,s+\frac{s^2}{400})}$, $\omega_{c3} = \omega_n = 20$

Now compare $s^2+18\,s+400$ *with* $s^2+2\xi\,\omega_n\,s+\omega_n^2$

$\therefore\ \omega_n^2 = 400,\quad \omega_n = 20,\quad$ and $2\xi\,\omega_n = 18,\quad \therefore \xi = 0.45$

Its corner frequency is 20 while as $\xi = 0.45$, magnitude plot will exhibit + 2 dB overshoot at $\omega_{c3} = 20$ (Referring to correction table given in discussion of quadratic pole).

Step 3 :

i) K = 0.045 $\therefore$ its contribution is 20 Log K = 20 Log 0.045 = – 27 dB.

ii) $\frac{1}{s^2}$, 2 poles at the origin so magnitude plot is straight line of slope – 40 dB/dec passing through intersection point of ω = 1 and 0dB. This is starting slope of magnitude plot.

iii) Shift intersection point of ω = 1 and 0 dB line on 20 Log K line i.e. – 27dB downwards (as K < 1) and from that point draw parallel to – 40 dB/dec line. This will represent addition of K/s^2.This will continue till first factor becomes dominant having least corner frequency i.e ω_{c1}=1

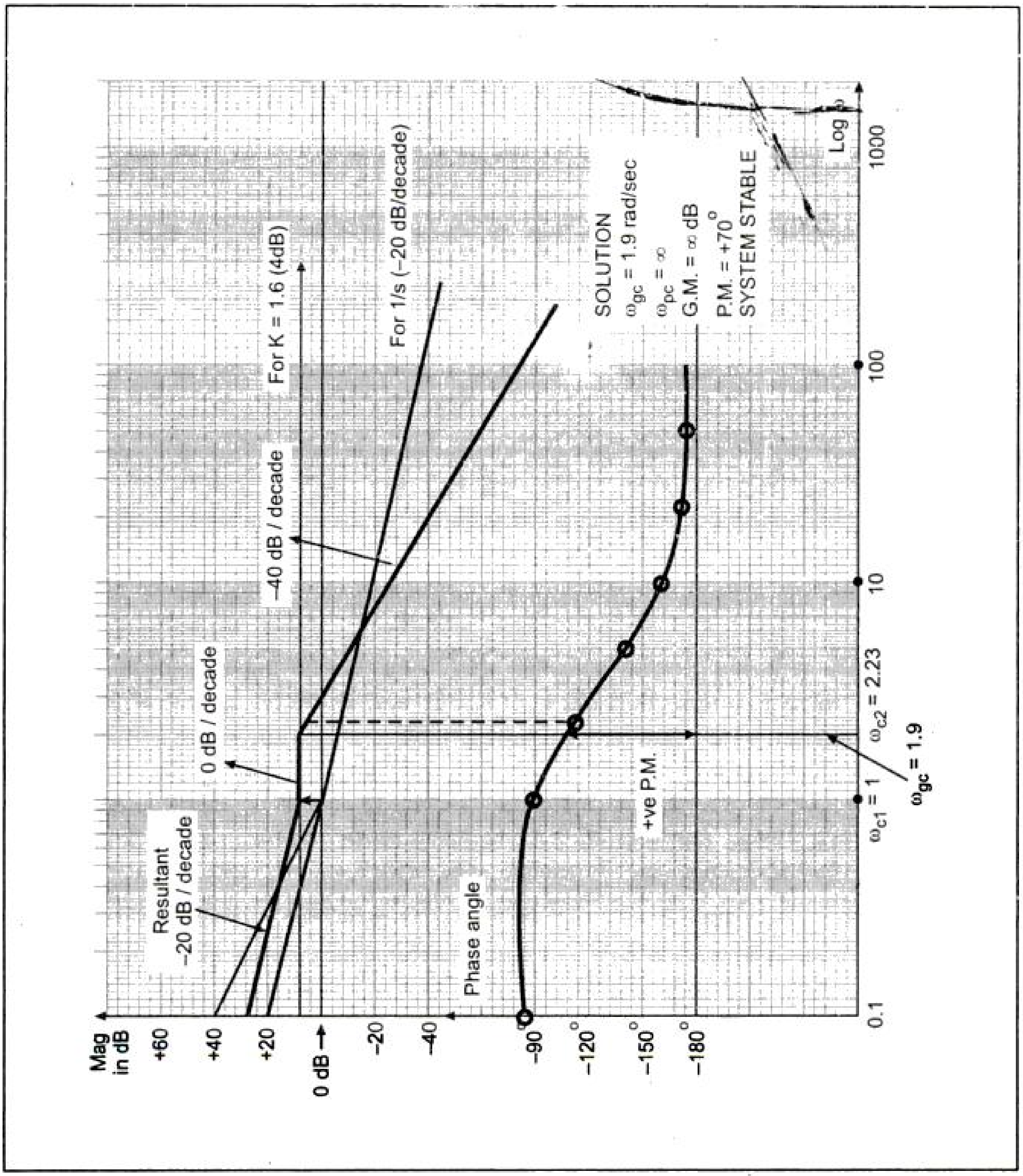
iv) At ω_{c1}=1 , simple zero occurs, contributing + 20 dB/decade individually hence resultant will have slope – 40 + 20 = – 20 dB/dec. Hence '1' onwards slope of resultant will be – 20 dB/dec contributing up to next corner frequency ω_{c2} = 6.

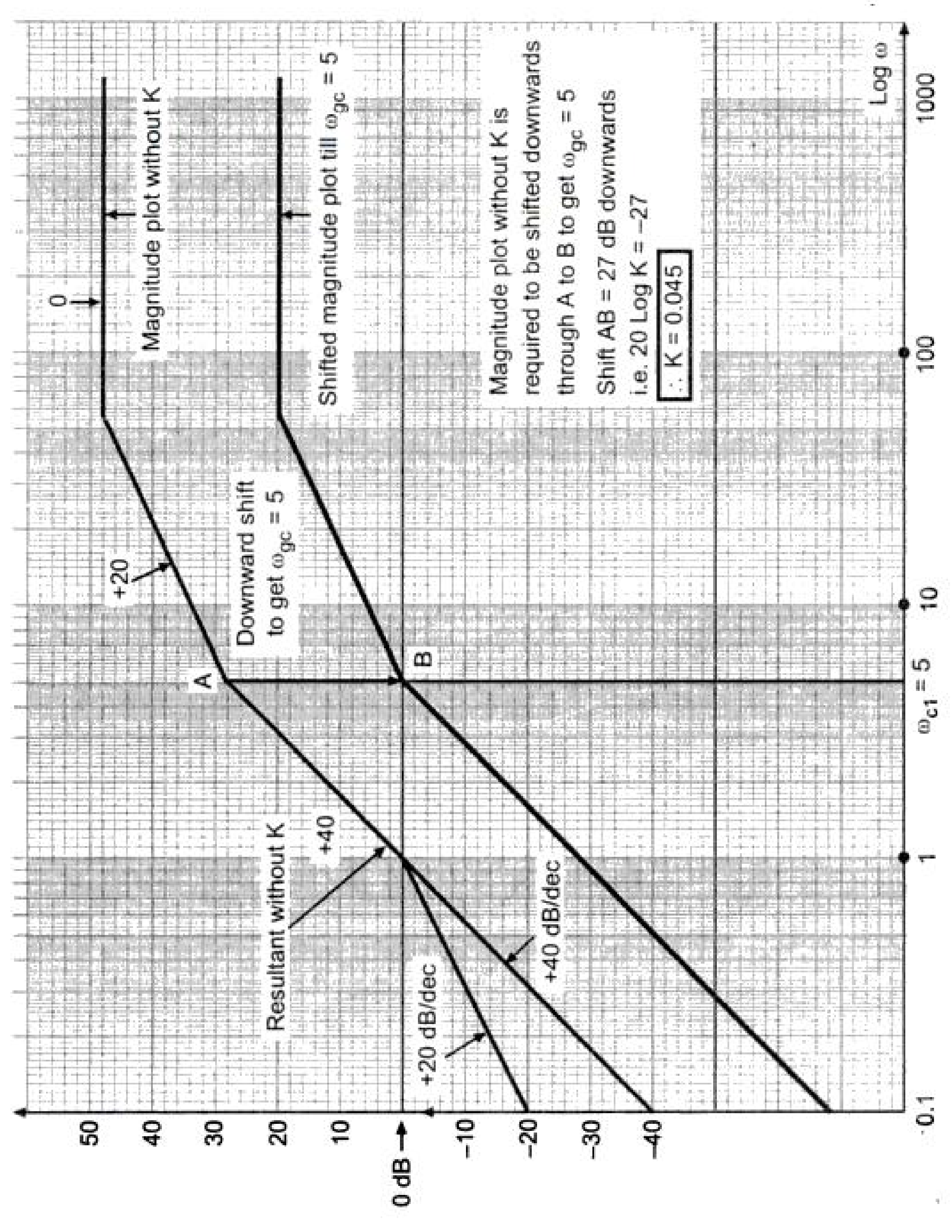
v) At ω_{c2} = 6 , another simple zero occurs contributing + 20 dB/dec individually making the slope of the resultant will become – 20 + 20 = 0 dB/dec from 6 onwards i.e. line parallel to x-axis till next corner frequency ω_{c3} = 20.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Step 5 : Sketch the Bode plot and obtain the solution.




Magnitude plot without K
0
Shifted magnitude plot till $\omega_{gc} = 5$
Magnitude plot without K is required to be shifted downwards through A to B to get $\omega_{gc} = 5$
Shift AB = 27 dB downwards
i.e. 20 Log K = –27
∴ K = 0.045
Downward shift to get $\omega_{gc} = 5$
+20
A
B
Resultant without K
+40
+40 dB/dec
+20 dB/dec
50
40
30
20
10
0 dB
–10
–20
–30
–40
0.1
1
$\omega_{c1} = 5$
10
100
1000
Log ω

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

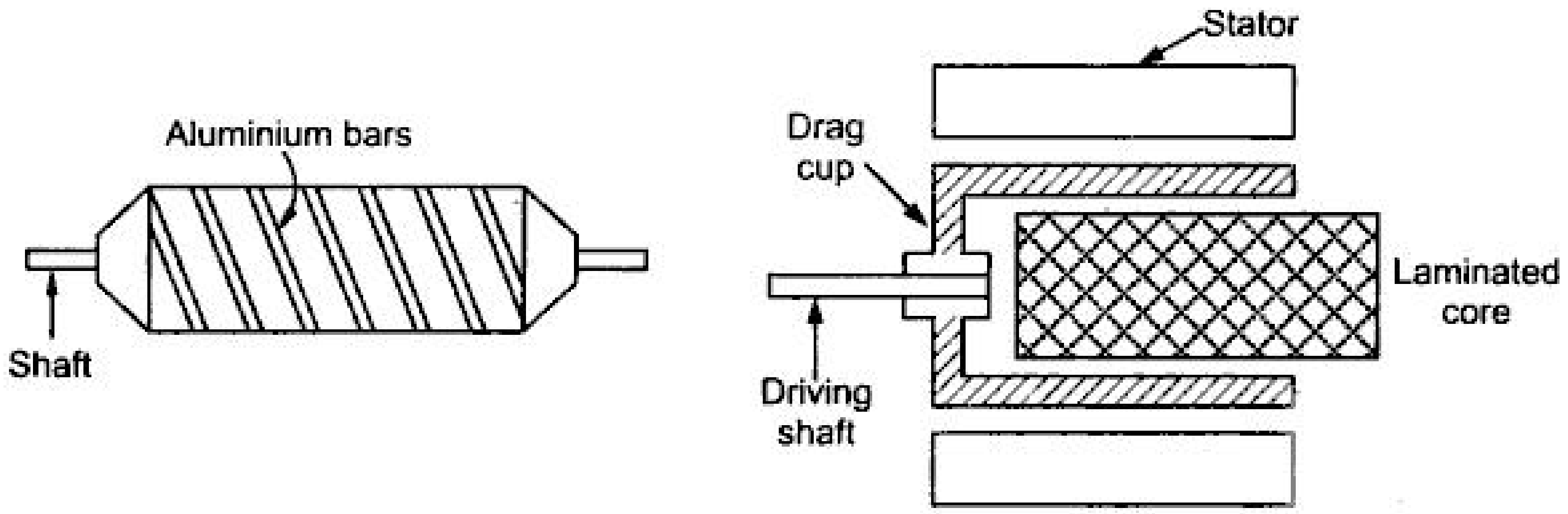
You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.



(a) Squirrel cage rotor (b) Drag cup type rotor

Fig. 13.2

13.3.3 Torque-speed Characteristics :

The torque-speed characteristics of a two phase induction motor, mainly depends on the ratio of reactance to resistance. For small X to R ratio i.e. high resistance low reactance motor the characteristics is much more linear while it is nonlinear for large X to R ratio as shown in the Fig. 13.3.

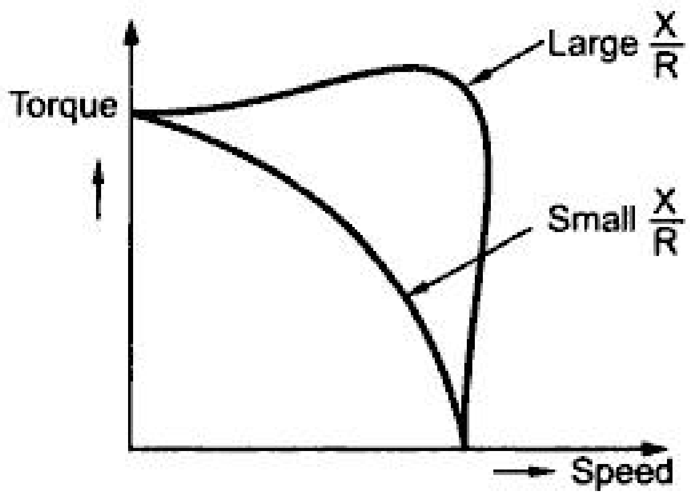


Fig. 13.3

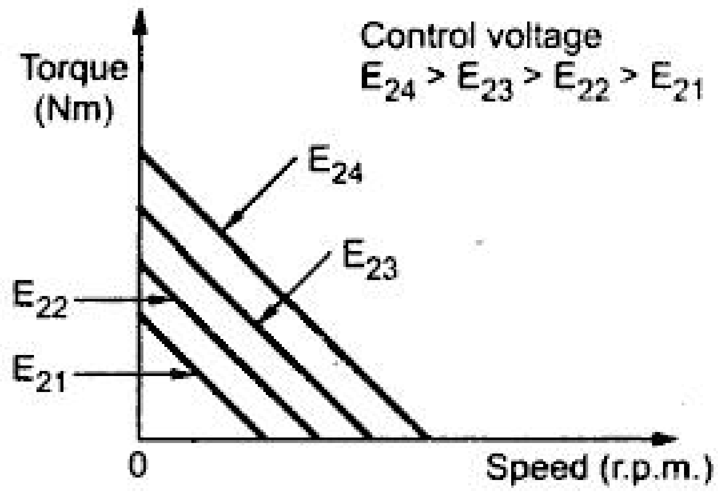


Fig. 13.4

In practice, design of the motor is so as to get almost linear torque-speed characteristics. Fig. 13.4 shows the torque-speed characteristics for various control voltages. The torque varies almost linearly with speed. All the characteristics are equally spaced for equal increments of control voltage. It is generally operated with low speeds.

13.3.4 Features of A. C. Servomotor :

The a.c. servomotor has following features :

i) Light in weight ii) Robust construction iii) Reliable and stable operation. iv) Smooth and noise free operation. v) Large torque to weight ratio vi) Large R to X ratio i.e. small X to R ratio. vii) No brushes or slip rings hence maintenance free

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

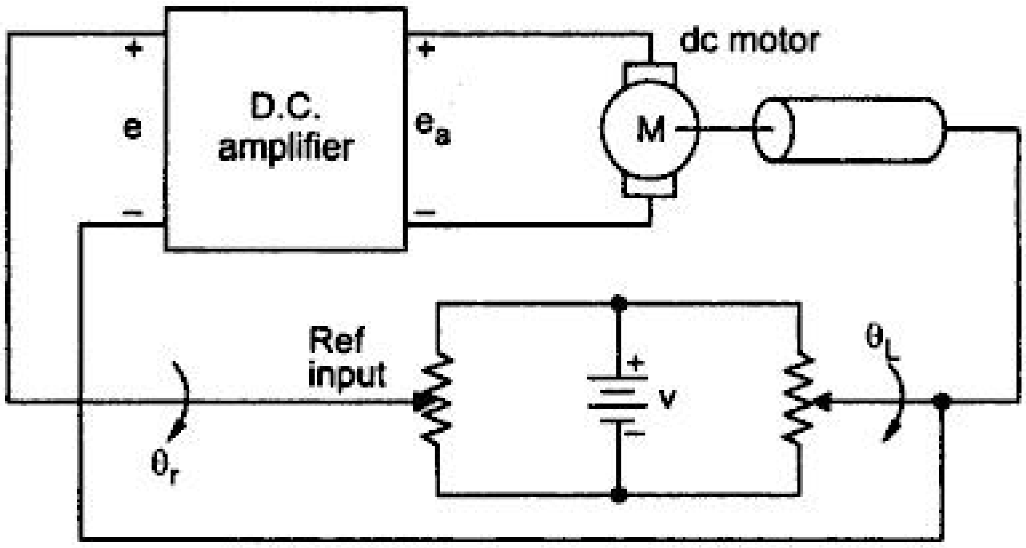


Fig. 13.27

13.15.1 Potentiometer as an Error Detector :

The potentiometer is used to multiply any voltage by a constant α, where 0<α<1 can be accomplished by the potentiometer. The output $e_0 = \frac{R_0}{R_i\ e_i}$ this type of potentiometer is commonly used.

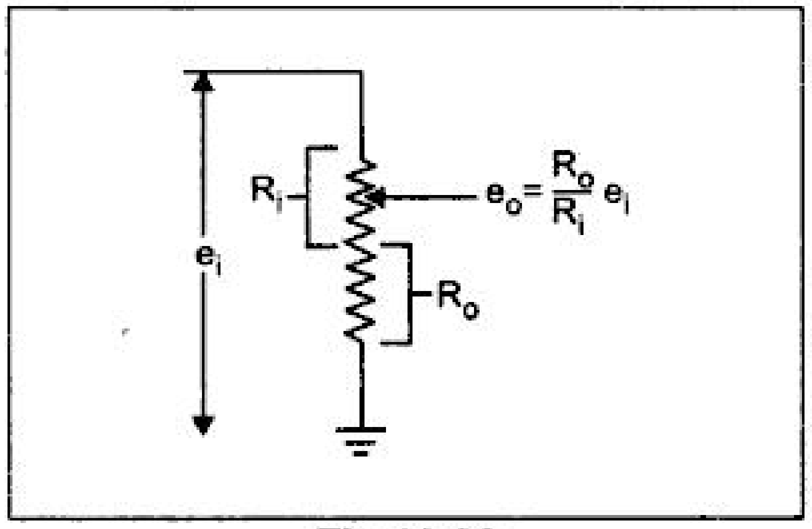


Fig.13.28

To study the use of potentiometer as error detector consider the automatic tank level control system as shown below.

This control system can maintain the liquid level h of the tank within accurate tolerance of the desired liquid level even though the o/p flow rate through the value V_1 is varied.

The liquid level is sensed by float, which positions the slider B on a potentiometer. The slider arm A of another potentiometer is positioned corresponding to desired liquid level H. When liquid level rises or falls, the potentiometers gives error voltage proportional to the change in liquid level.

The error voltage actuates the motor through a power amplifier which in tern condition the point in order to restore the desired liquid level.

Thus the control system automatically attempts to correct any deviation between the actual and desired levels in the tank. The arrangement is shown in the Fig. 13.29.

13.15.2 Types of Potentiometers :

i) Single turn and multiturn potentiometer.

ii) Linear motion potentiometer

iii) Wire wound or carbon film potentiometer.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

14.7.1 Step Response of Integral Mode

The step response of the integral control mode is shown in the Fig. 14.9.

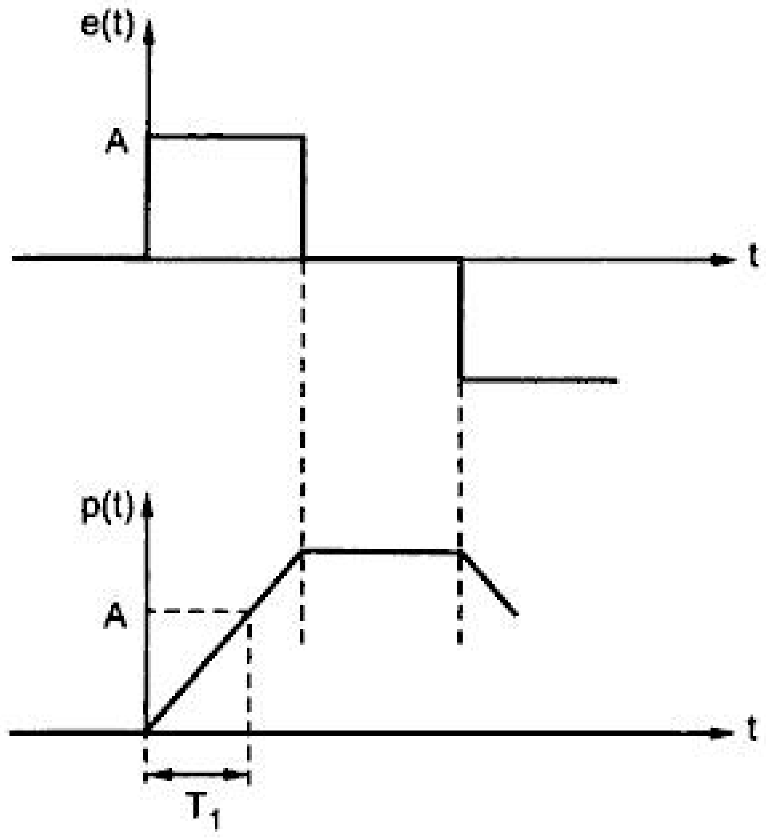


Fig. 14.9 Step response

The integration time constant is the time taken for the output to change by an amount equal to the input error step. This is shown in the Fig. 14.9.

It can be seen that when error is positive, the output p(t) ramps up. For zero error, there is no change in the output. And when error is negative, the output p(t) ramps down.

14.7.2 Characteristics of Integral Mode

The integrating controller is relatively slow controller. It changes its output at a rate which is dependent on the integrating time constant, until the error signal is cancelled. Compared to the proportional control, the integral control requires time to build up an appreciable output. However it continues to act till the error signal disappears. This corrects the problem of the offset error in the proportional controller.

For example, let us assume that the integral controller is used to control the armature current of a d.c. motor and to keep its value constant at 500 A. As long as the armature current is less than 500 A, the armature voltage, controlled by the controller, will increase. Thus the output of the controller will increase and will continue to do so till the error becomes zero i.e. armature current becomes 500 A. Then the controller output will remain at that value reached. This is possible because the output of the controller can remain at any value within its range, if the input is zero. The controller must not be overdriven as it will not then be effective.

Thus for an integral mode,

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

$$\frac{C(s)}{R(s)} = \frac{\dfrac{(s+30)\,3.33}{s(s+12)}}{1+\dfrac{(s+30)\,3.33}{s(s+12)}} = \frac{3.33(s+30)}{s^2+12s+3.33s+100}$$

$$= \frac{3.33\,(s+30)}{s^2+15.33s+100}$$

$\therefore \quad \omega_n^2 = 100 \qquad \therefore \omega_n = 10$

$2\xi\,\omega_n = 15.33$

$\therefore \quad \xi = \dfrac{15.33}{2\times 10} = 0.7665$

$\therefore\ \xi$ is improved, $\omega_d = \omega_n\sqrt{1-\xi^2} = 10\sqrt{1-(0.7665)^2}$

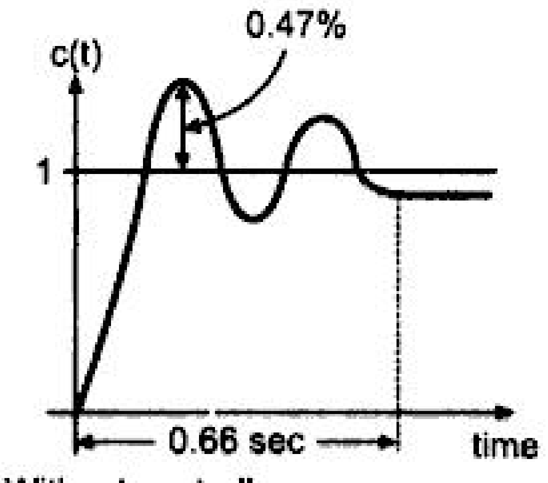
$= 6.4224$ rad/sec

$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = 2.353\ \%$

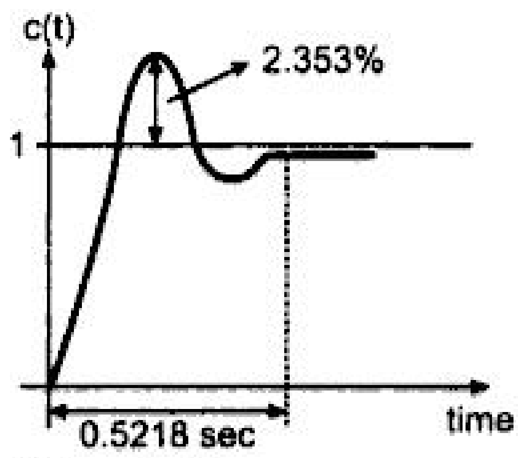
Overshoot decreased to 2.3% from 9.47%.

$T_s = \dfrac{4}{\xi\,\omega_n} = 0.5218$ sec

Comparison : Following Figure shows comparison between system with controller and system without controller.

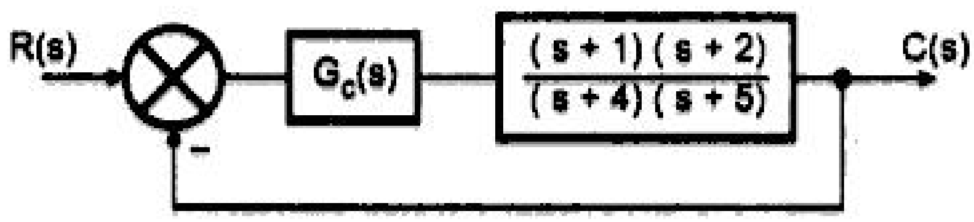


Without controller



With controller

Ex. 14.8 *Consider the system shown in the Fig. where $G_c(s)$ is the transfer function of a compensator. Find the steady state error of the system if the compensator is i) P type and ii) PI type. Assume unit step input.* **(Gate-88)**



15
Miscellaneous Questions

1) *Write a short note on compensating networks and their use in compensation.*

Ans. : For satisfactory performance of the system, the gain is set first. But while gain is increased, improving steady–state behaviour, it may result in poor stability. In order to alter the overall behaviour so that the system will behave as desired, an additional device inserted into the system which is called as `compensator'.

If a compensator is placed in series with transfer function G(s) then it is called as series compensator.

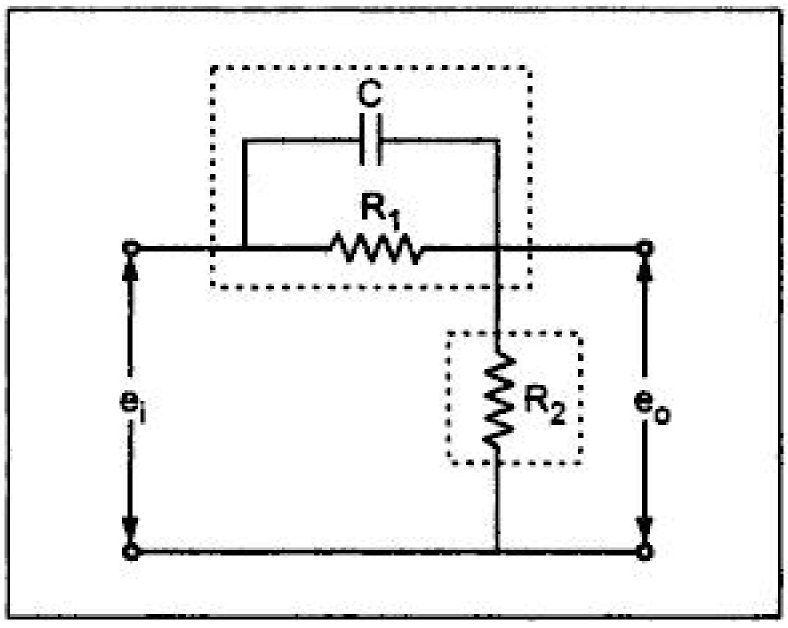
Widely used series compensators are lead compensators, lag compensators and lag–lead compensators.

Lead network :

Transfer function of lead network

$$\frac{E_o(s)}{E_i(s)} = \frac{\alpha\,(1+T\,s)}{(1+\alpha\,T\,s)}$$

where
$$\alpha = \frac{R_2}{R_1+R_2}$$
$$T = R_1\,C$$



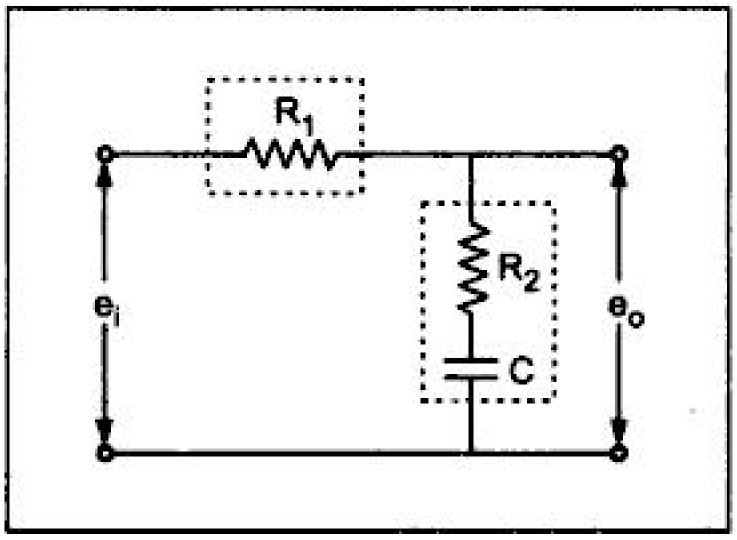
Lead compensation essentially yields an appreciable improvement in transient response and a small improvement in steady state accuracy.

Lag Network :

Transfer function of lag network

$$\frac{E_o(s)}{E_i(s)} = \frac{1+Ts}{1+\beta\,Ts}$$

where
$$\beta = \frac{R_1+R_2}{R_2},$$
$$T = R_2\,C$$

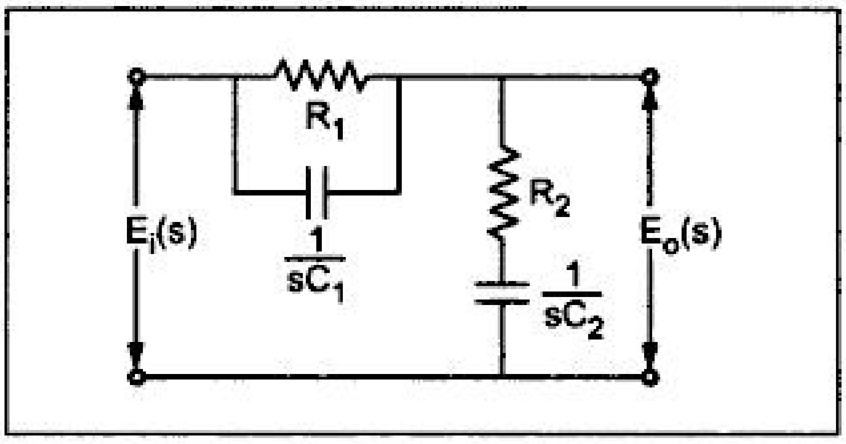


Lag compensation yields an appreciable improvement in steady–state accuracy at the expense of increasing the transient–response time.

Both lead and lag compensators raises the order of system by one.

$$Z = \frac{R_1 \cdot \dfrac{1}{sC_1}}{R_1 + \dfrac{1}{sC_1}}$$

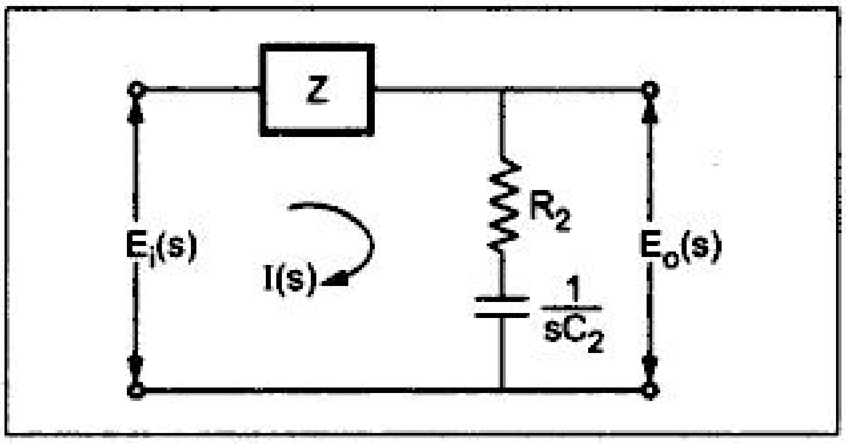
$$= \frac{R_1}{1 + sC_1 R_1}$$



Replacing parallel combination by impedance Z,

Apply KVL,

$$E_i(s) = ZI(s) + R_2 I(s) + \frac{I}{sC_2} I(s)$$



$$E_o(s) = I(s)\left[R_2 + \frac{1}{sC_2}\right]$$

$$\therefore \quad I(s) = \frac{sC_2\, E_o(s)}{1 + sR_2\, C_2}$$

$$\therefore \quad E_i(s) = \frac{sC_2\, E_o(s)}{(1 + sR_2\, C_2)}\left[Z + R_2 + \frac{1}{sC_2}\right]$$

$$\therefore \quad E_i(s) = \frac{sC_2\, E_o(s)}{(1 + sR_2\, C_2)}\left[\frac{ZsC_2 + 1 + sC_2\, R_2}{sC_2}\right]$$

$$\therefore \quad \frac{E_o(s)}{E_i(s)} = \frac{(1 + sR_2\, C_2)}{(1 + sC_2\, R_2 + ZsC_2)}$$

Substituting `Z'

$$\frac{E_o(s)}{E_i(s)} = \frac{(1 + sR_2\, C_2)}{\left[1 + sC_2\, R_2 + \dfrac{R_1\, sC_2}{(1 + sC_1\, R_1)}\right]}$$

$$\therefore \quad \frac{E_o(s)}{E_i(s)} = \frac{(1 + sC_1\, R_1)\,(1 + sR_2\, C_2)}{(1 + sC_2\, R_2)\,(1 + sC_1\, R_1) + sR_1\, C_2}$$

This is the required transfer function.

6) *Draw the polar plots for lag, lead and lag–lead network. Prove that it is semicircle in first quadrant. Also prove that the frequency ω_m, maximising the lead angle ϕ is*

$$\omega_m = \frac{1}{T\sqrt{\alpha}}$$

Where $\alpha = \dfrac{R_2}{R_1 + R_2}$ *and* $T = R_1 C$

(Mumbai University Dec. 96)

Ans. : From the given transfer function, the frequency domain transfer function can be obtained by replacing s by jω

13) *Discuss the advantages of the Bode plot technique.*

Ans. : Advantages of the Bode plot technique are as follows :

1) It shows both low frequency and high frequency characteristic of transfer function in one diagram.
2) They can be constructed with easy.
3) Gain margin and phase margin can be obtained with minimum computational efforts from Bode plots.
4) Bode plots indicate clearly relative stability of a system.
5) Data for constructing polar plots of complex transfer function can be easily obtained from Bode plots.
6) Without the knowledge of the transfer function, the bode plot of stable open loop system can be obtained experimentally.
7) These methods are easy to use for design of control systems and for finding absolute as well as relative stability of the system. Calculations are simple and methods of design are well tested.
8) The apparatus required for obtaining Bode plot practically is simple and inexpensive, and easy to use.

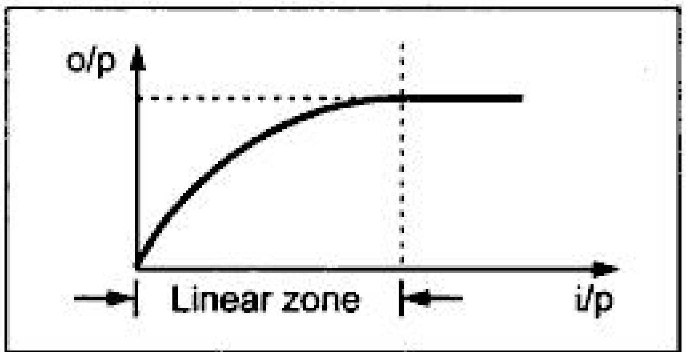
14) *Which are the different types of nonlinearities? Why most of the systems are nonlinear?*

Ans. : The various types of nonlinearities can be classified as

a) Inherent (unintentional) nonlinearities and b) Intentional nonlinearities.

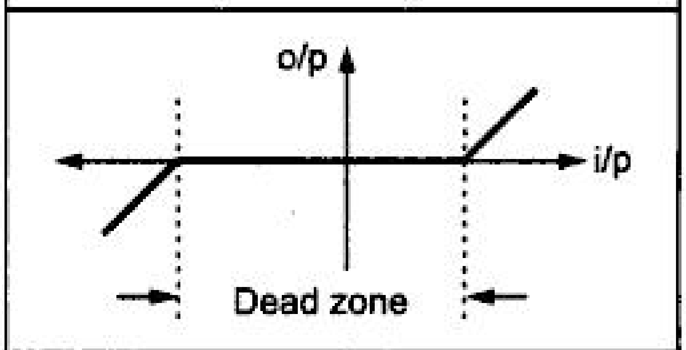
a) Inherent nonlinearities are :

i) **Saturation** : $K = \frac{\text{Output}}{\text{Input}}$, as the value of K goes on decreasing the linear zone goes on decreasing. e.g. amplifiers, motors, generators, ironcore devices.



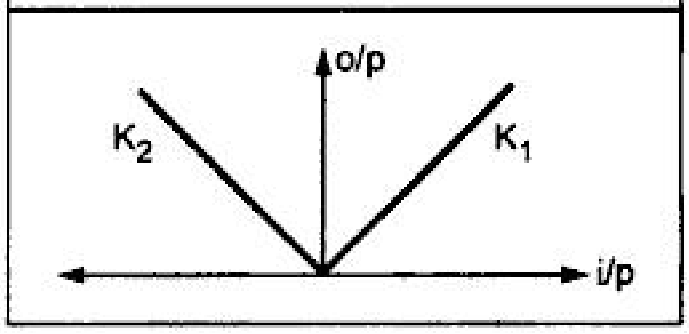
ii) **Dead space / Dead zone** : This type of nonlinearity occurs, because of the inability of certain components to react to sufficient small amplitude levels of input variables.

e.g. relays, contactor devices, hydraulic values.



iii) **Absolute value nonlinearity** : Though direction of input is changed, output remains in any one particular direction.

e.g. Moving iron type instruments, Electromagnets, Turbine flow transducers.



You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Contents

Edition - 2007

Rs. 300/-

ISBN 81-89411-59-4

9788189411596

Technical Publications Pune®

1, Amit Residency, 412 Shaniwar Peth, Pune - 411030

☎(020) 24495496/97, Email : technical@vtubooks.com

Visit us at : www.vtubooks.com