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Application of neural networks based SANARX model for identification and control liquid level tank system

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***Abstract*—This paper is devoted to application of artificial Neural Network based Simplified Additive Autoregressive eXogenous model for identification and control of a liquid level tank system consisting of three water reservoirs. A specific restricted connectivity structure of the neural network is trained on input-output data set to identify a nonlinear dynamic single-input single-output model of the liquid level tank system. Parameters of the identified neural network based model can be used to design a dynamic controller for the system. The designed neural network based controller is verified on mathematical model in MATLAB/Simulink environment and applied to the real-time control of the plant. The goal of the control algorithm is to track the desired level of liquid in the upper tank. Experimental result have shown a very good performance of the proposed technique. The designed nonlinear controller is capable of tracking the desired water level for all set points with high degree of accuracy, maximally fast and without significant overshoot.**

I. Introduction

Present paper explores the abilities of the so-called Neural Networks based Simplified Additive Auto Regressive eXogenous (NN-based SANARX) models to be used in closed-loop control of water tank type systems. NN-based SANARX stands for Neural Networks based Simplified Additive Auto Regressive eXogenous model class, see [1]. Being a subclass of a more general ANARX model class [2], SANARX inherits all the advantages of its parent class. Namely, models of this type are always linearizable by the dynamic output feedback. In other words, for a model employing SANARX structure one may always write down equations of the linearizing feedback. The latter means that once coefficients of the model are identified, one just needs to substitute their values into equations describing controller. The main idea of feedback linearization technique consists in modifying the system structure by appropriate feedback, so that the i/o equation of the closed-loop system becomes linear. After that it is possible to apply all the standard control methods for linear systems to meet the required goals.

While the problem of liquid level control in a tank is not new, it does not lost its actuality in time. Level regulators are used in industry to maintain a constant fluid pressure, or a constant fluid supply to a process, or in waste storage [3]. The common examples of possible industrial applications include chemical industry and food processing [4] as well as different irrigation systems like dams, etc. Through the years various techniques have been used to tackle the problem. Recent analytic solutions employ tensor product based methods [5] and decoupling control [6]. In many cases the problem is approached by means of PI [7], PID [4], and fractional-order PID [8] controllers. Recently, methods based on computational intelligence have started to gain popularity and applied either solely or in combination with some classical techniques [9]. While PID controllers is a popular choice in many industrial applications, they do not guarantee that system would work with the same level of accuracy in the entire operating range. Furthermore, adaptation and overregulation are the common problems which one may spot in many other applications, and have to be taken into account during design stage.

Though pure analytic or numeric techniques have known limitations, their combinations with advanced methods of computational intelligence may lead to generic solutions with a broader application range. The present contribution may be seen in application of the classical control technique (linearization via dynamic output feedback) and neural networks based modeling to control a water level in a tank system. In the paper we describe all design steps: starting from collecting the input-output (i/o) data of a process and finishing with implementation and test of the synthesized controller on a real plant. One of the most complicated parts of the overall design procedure was solved using feedback linearization technique. Like any other analytic method the linearization by dynamic output feedback provides one with equations of a controller, whose coefficients are taken directly from the identified model. In other words, selecting NN-SANARX model, one merges together numeric parts of the modeling and control synthesis. To the best knowledge of the authors, there is no any similar research made on the application of NN-SANARX models to control of real plants. Therefore, the research, presented in the paper, can be seen as a preliminary step towards real industrial application.

The paper is organized as follows. In Section II we give a brief overview of the existing theoretical background on ANARX type structures and the main idea of linearization via dynamic output feedback. The next section is devoted to the mathematical model of the process under consideration. Model identification and control of the liquid level tank system are presented in Section III. Concluding remarks and possible

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further developments are drawn in the last two sections.

II. Theoretical background

Hereinafter, we use the notation ξ for a variable $\xi(t)$, $\xi^{[k]}$ for the kth-step forward time shift $\xi(t+k)$ and $\xi^{[-l]}$ for the lth-step backward time shift $\xi(t-l)$ with $k,l \in \mathbb{Z}^+$. Moreover, to simplify exposition of the material, in this paper we restrict our attention to the case of single-input single-output (SISO) systems. The nonlinear control systems are typically represented either by the higher order input-output difference equation

$$y^{[n]} = \varphi\left(y, y^{[1]}, \ldots, y^{[n-1]}, u, u^{[1]}, \ldots, u^{[s]}\right), \tag{1}$$

or by the state equations

$$\begin{aligned} x^{[1]} &= f(x,u) \\ y &= h(x), \end{aligned} \tag{2}$$

where $x : \mathbb{Z} \to \mathcal{X} \subset \mathbb{R}^n$ is the state variable, $u : \mathbb{Z} \to \mathcal{U} \subset \mathbb{R}$ is the input, $y : \mathbb{Z} \to \mathcal{Y} \subset \mathbb{R}$ is the output, $\varphi : \mathcal{Y}^n \times \mathcal{U}^{s+1} \to \mathbb{R}$, $f : \mathcal{X} \times \mathcal{U} \to \mathcal{X}$ and $h : \mathcal{X} \to \mathcal{Y}$ are the real analytic functions. Moreover, we assume that $s < n$ are non-negative integers.

The system, represented by equation (1), is known in the literature as a discrete-time Nonlinear AutoRegressive eXogenous (NARX) model. On the one hand, such structure is capable of identifying a wide class of complex processes with a high degree of accuracy, see [10]. On the other hand, from the control point of view, it has several drawbacks. One of the most important for our studies is linearizability via dynamic output feedback. In case of (1) this property does not always hold, see [11] for details. Thus, to overcome this obstacle, so-called Additive NARX (ANARX) structure was proposed, which is a modification/subclass of the NARX model having separated time instances [12]

$$y^{[n]} = f_1(y^{[n-1]}, u^{[n-1]}) + \cdots + f_n(y, u). \tag{3}$$

Note that model (3) can always be linearized by the dynamic output feedback, see [11]. Another important advantage of ANARX model is that it can always be rewritten in the state-space form as follows

$$\begin{aligned} x_1^{[1]} &= x_2 + f_1(x_1, u) \\ &\vdots \\ x_{n-1}^{[1]} &= x_n + f_{n-1}(x_1, u) \\ x_n^{[1]} &= f_n(x_1, u) \\ y &= x_1. \end{aligned}$$

Remark 1: State-space representation is an important property of the control system, which provides a convenient and compact way for its further modeling and analysis. One of the possible applications is shown in [13], where a state-space controller was constructed.

A. Neural Networks based ANARX model

In order to perform analysis and design of the appropriate controller for the process, one is usually interested in mathematical equations. Note that to maintain a sufficient level of accuracy it is necessary to use nonlinear equations. In fact, one can derive the model from the first principles, e.g. modeling a physical process relying on the Newton equations. However, most likely in many cases such an approach will result in a quite complex model. Therefore, it is more common to start from the measured i/o data of a system and try to determine a mathematical relation between variables without going into the details of what is actually happening inside the process. One of the most common approaches consists in employing formalism based on Neural Networks (NN). Thus, the theory for ANARX models, described above, was adopted to the case of neural networks in [2], [14] and the practical application to identification of different types of processes was shown in [15]. To be more specific NN-ANARX model can be represented by using a neural network with restricted connectivity structure as

$$y^{[n]} = \sum_{i=1}^{n} C_i \phi_i \left(W_i \begin{bmatrix} y^{[n-i]} & u^{[n-i]} \end{bmatrix}^{\mathrm{T}}\right), \tag{4}$$

where $\phi_i(\cdot)$ is an activation function of the ith sublayer neurons, C_i and W_i are $1 \times l_i$ and $l_i \times 2$ dimensional matrices of the ith sublayer output and input synaptic weights, respectively. Here l_i is the number of hidden neurons in the ith sublayer. A schematic diagram of the neural network, representing ANARX structure, is depicted in Fig. 1.

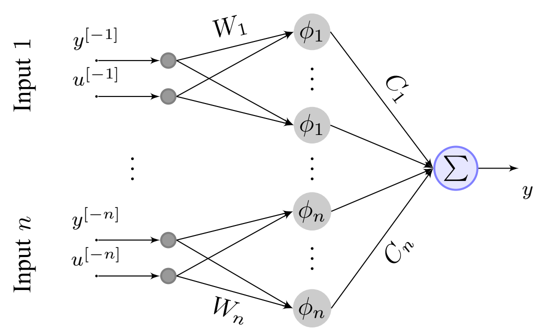

Fig. 1: Representation of NN-ANARX structure

Remark 2: In order to avoid additional notation, further we assume that NN-ANARX model perfectly describes the process meaning that the estimate and the original output coincide, i.e. $\hat{y} = y$, where $\hat{y}$ is the output of the neural network.

Next, the dynamic output feedback can be written by using parameters of the neural network as [16]

$$\eta_1 = C_1 \phi_1 \left(W_1 \begin{bmatrix} y & u \end{bmatrix}^{\mathrm{T}}\right) \tag{5}$$

and

$$\begin{aligned} \eta_1^{[1]} &= \eta_2 - C_2 \phi_2 \left(W_2 \begin{bmatrix} y & u \end{bmatrix}^{\mathrm{T}}\right) \\ &\vdots \\ \eta_{n-2}^{[1]} &= \eta_{n-1} - C_{n-1} \phi_{n-1} \left(W_{n-1} \begin{bmatrix} y & u \end{bmatrix}^{\mathrm{T}}\right) \\ \eta_{n-1}^{[1]} &= v - C_n \phi_n \left(W_n \begin{bmatrix} y & u \end{bmatrix}^{\mathrm{T}}\right). \end{aligned} \tag{6}$$

Here $v : \mathbb{Z} \to \mathcal{V} \subset \mathbb{R}$ is a reference signal (desired output). Note that the application of the dynamic feedback (5) and (6)

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to the model (4) results in the closed-loop system that can be described by the linear model $y^{[n]} = v$.

In order to simplify the calculation of the control signal in (5), we assume like in [1] that $\phi_1(\cdot)$ is a linear function, resulting in a simplified structure of the neural network known as a NN-SANARX model. It means that (5) can be rewritten as follows

$$u = T_2^{-1}(\eta_1 - T_1 y), \tag{7}$$

where T_1 and T_2 are the first and second elements of the vector $C_1 W_1$, respectively. Note that T_2 has to be a nonsingular square matrix. This fact has to be taken into account on the identification stage. The overall structure of the corresponding control system is represented in Fig. 2.

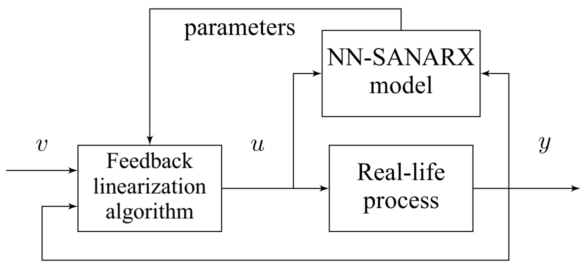

Fig. 2: Control system design procedure

III. Mathematical model

The model of a Multi Tank system is borrowed from the manual, provided by INTECO [17], and depicted in Fig. 3.

The differential equations, describing dynamics of the Tank system, can be derived, assuming the laminar outflow rate of an *ideal fluid* from a tank, by means of mass balance as

$$\begin{aligned}
\dot{x}_1 &= \frac{1}{aw}(u - C_1 x_1^{\alpha_1}) \\
\dot{x}_2 &= \frac{h}{cwh + bwx_2}(C_1 x_1^{\alpha_1} - C_2 x_2^{\alpha_2}) \\
\dot{x}_3 &= \frac{1}{w\sqrt{R^2 - (R - x_3)^2}}(C_2 x_2^{\alpha_2} - C_3 x_3^{\alpha_3}),
\end{aligned} \tag{8}$$

where $u = q$, $x_1 = H_1$, $x_2 = H_2$, $x_3 = H_3$, $h = H_{2\max}$, and the physical meaning of parameters is listed in Table I, where $i \in \{1, 2, 3\}$ is the number of a tank. Furthermore, it is important to mention that x_1, x_2, x_3 and u have natural saturations due to the physical limitations of the system and power of the pump. In addition, control signal has a significant dead zone that has to be taken into account. For more specific details and assumptions made for the model (8), we refer reader to the precise manual available at [17].

From Fig. 3, one may see that the Multi Tank system consists of three serial connected tanks equipped with valves and level sensors. The upper tank has constant cross-sectional area, the middle and lower tanks have variable (conic and spheral) cross sectional areas that determines nonlinearity in the model. In addition, the system is equipped with DC pump providing liquid transportation from the lowest tank to the upper tank. The goal of the DC pump is to adjust the inflow to the upper tank according to the control signals. The DC

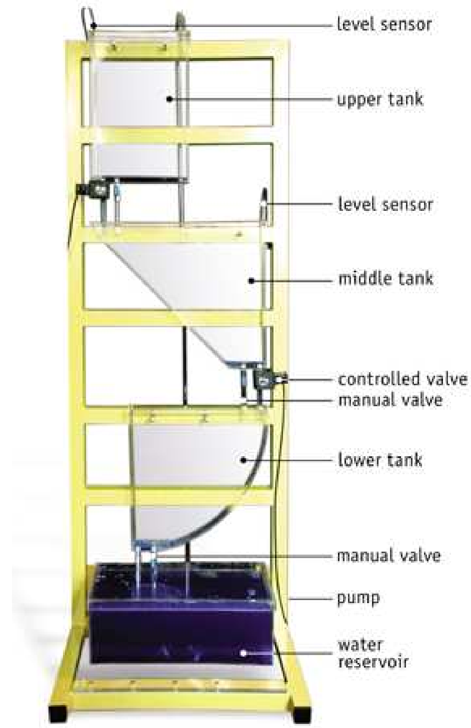

Fig. 3: Model of the Multi Tank system

TABLE I: Nomenclature

Parameter	Physical description
H_i	fluid level in the ith tank
w	width of a tank
a	length of the upper tank
b, c	lengths of the upper and lower part of the middle tank
R	length of the upper part of the lower tank
$H_{2\max}$	height of the middle tank
C_i	resistance of the output orifice of the ith tank
α_i	flow coefficient for the ith tank

pump is supplied from the power interface by an appropriate PWM control signal. The liquid outflows from the tanks due to gravity. The tank valves act as flow resistors. Each valve between tanks can be controlled changing this way the output flow, if necessary, and the number of inputs and outputs of the system. Thus, the system can be reconfigured with respect to requirements. Furthermore, each tank has its own sensor for measuring water level. The numerical values of the parameters from Table I will be provided further in Section IV. The plant is designed to operate with an external PC-based digital controller. The computer communicates with the level sensors, valves and pump by a dedicated I/O board and the power interface. The I/O board is controlled by the real-time software which operates in Simulink using MATLAB Real-Time Windows Target environment. It should be mentioned that in this paper, we focus on the control of the water level in the upper tank.

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IV. Control of liquid level tank system

All the experiments, described in this section, were performed on the equipment available at the laboratory in Department of Computer Control, Tallinn University of Technology, see [18] for more details.

Since we are interested in control of the water level in only one tank, a single-input single-output system can be obtained from equations (8) with inflow and the water level in the first tank being the input u and the output $y := x_1$, respectively. As a results the overall system can be seen as a pump-controlled system. The physical parameters of the plant have the following numerical values $w = 0.035\mathrm{m}$, $a = 0.25\mathrm{m}$, $\alpha_1 = 0.2497$, and the maximal inflow provided by the pump is $1.0284 \cdot 10^{-4}\mathrm{m}^3/\mathrm{s}$. In addition, the resistance of the output orifice of the first tank was determined experimentally $C_1 = 11.08 \cdot 10^{-5}\mathrm{m}^2/\mathrm{s}$, using MATLAB routine provided with the installation package. Next, we describe the identification procedure based on the neural networks approach, recalled in Section II.

The identification data was collected from the real plant with sampling time equals to $0.5\mathrm{sec}$. The input signal was normalized into unit interval $[0, 1]$ for easier training of the neural network. The input-output data was used to train NN-based Simplified ANARX structure by means of gradient descent with adaptive learning rate backpropagation algorithm. The network shown in Fig. 1 was trained with two sublayers, corresponding to the second order ($n = 2$) of the model, and 3 neurons on each sublayer, i.e. $l_1 = l_2 = 3$. The pure linear activation function was chosen on the first and output sublayers as well as hyperbolic tangent sigmoid activation function on the second sublayer. Note that the choice of second order for the identified model is due to the requirements of the control algorithm (5)-(6). In other words the minimal possible order has to be at least 2, see [1]. Moreover, at least one layer of the network has to be nonlinear in order to reflect nonlinearities of the process. The choice of various settings for identification procedure was made off-line. One has to keep in mind that, in principle, this choice is not unique and depends on the particular application. The identified model has the following structure

$$
\begin{aligned}
y = C_1 W_1 \left[y^{[-1]} \quad u^{[-1]} \right]^{\mathrm{T}} + \\
+ C_2 \mathrm{tansig} \left(W_2 \left[y^{[-2]} \quad u^{[-2]} \right]^{\mathrm{T}} \right). \quad (9)
\end{aligned}
$$

In Section II the main advantages of ANARX model were discussed. The main property, we are interested in, is linearization by dynamic output feedback. Since the identified model is of the second order, using (6), (7) and parameters of the identified model (9), we get dynamics of the controller in the following form

$$
\begin{aligned}
u &= T_2^{-1}(\eta_1 - T_1 y) \\
\eta_1^{[1]} &= v - C_2 \mathrm{tansig} \left(W_2 \left[y \quad u \right]^{\mathrm{T}} \right).
\end{aligned} \quad (10)
$$

The reference signal v was chosen as a piecewise constant function defined by Table II.

Note that the function was chosen intentionally this way to illustrate the ability of the proposed method perform well in the whole region of all possible set points. The closed-loop system, consisting of controller (10) and mathematical model (8), was simulated in Simulink environment with reference function v, and the quality of the corresponding control algorithm is depicted in Fig. 4.

TABLE II: Set points

Value [m]	Time interval [sec]
0.20	$0 \leq t < 100$
0.05	$100 \leq t < 160$
0.10	$160 \leq t < 220$
0.15	$220 \leq t < 300$

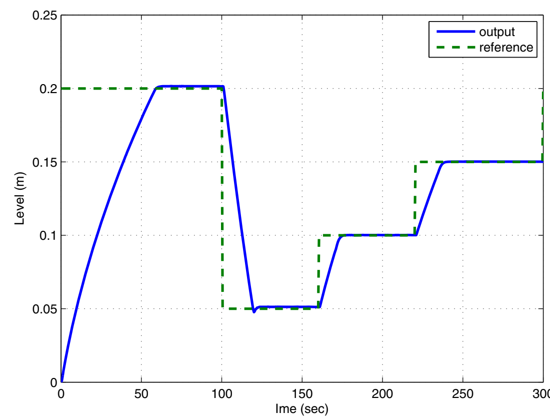

Fig. 4: Verification of the identified model

The small deviation between the reference model and output of the system can be explained by the fact that NN-SANARX model used to design the controller is only approximation of the original process and contains a certain error. Nevertheless, the verification results clearly indicate that the obtained controller can be used to control the real plant. Next, we briefly summarize the mentioned above in the form of the step-by-step procedure.

Algorithm:

- **Step 1.** Collect training data performing real-life experiment.
- **Step 2.** Use *a priori* information of the process to determine important parameters of the neural network such as order of the identified model, number of sublayers, etc.
- **Step 3.** Train neural networks based Simplified ANARX model.
- **Step 4.** According to the pre-specified control requirements, write down either difference or state equations of the controller.
- **Step 5.** Put plant and controller into the closed-loop and verify the designed control system.

After verification of the algorithm we applied it to control the real plant. It should be mentioned that C/C++ builder, provided by Real-Time Windows Target, does not allow functions

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that are not available in the core version of MATLAB. Therefore, activation function, used in (9), has to be implemented by means of standard blocks in the explicit form using formula $\mathrm{tansig}(x) = \frac{2}{1+e^{-2x}} - 1$. The quality of the tracking algorithm is depicted in Fig. 5.

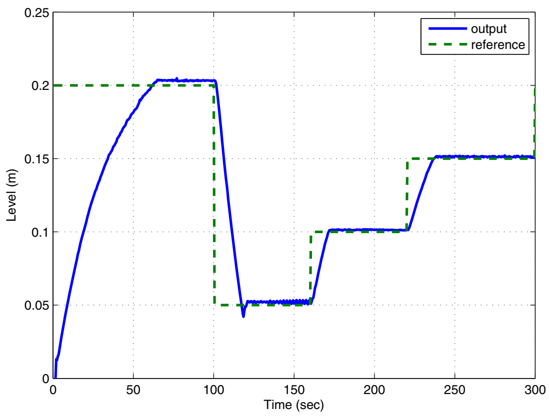

Fig. 5: Level control in the upper tank

It can be seen from Fig. 5 that the control system is capable of tracking the reference signal v and react correctly to the changes in a set point within the $\pm 1\%$ deviation from the final value, when the NN-based SANARX structure and the corresponding control algorithm are used. In addition, it is important to stress that the control system works with the same accuracy on the whole region of set points.

One can see that the result obtained on computer simulation during validation of the designed controller (see Fig. 4) almost coincide with the practical experiment on the real plant presented in Fig. 5. The difference comes from the fact that the identified model is replaced by the real plant in the closed-loop, causing small static error due to the imperfectness of the obtained model. Note that the turbulence near the sensor, caused by the input flow, can be considered as a noise. However, one can easily see that it does not influence the quality of the closed-loop system, and the control algorithm is capable to deal with this noisy measurements.

V. Conclusion and Discussion

In this paper, NN-SANARX model structure and classical technique based on linearization by dynamic output feedback were used to control the liquid level in the real tank system. The choice of NN-SANARX model class has allowed to merge numeric parts of system modeling and control synthesis that reduced computational complexity and in turn simplified implementation. While all the numeric calculations were done in MATLAB, no specialized toolbox functions were used, which one more time demonstrates that the proposed technique is implementation friendly and may be programmed even for specialized embedded devices. We explained all the necessary design steps: starting from collecting the input-output data of a process and finishing with implementation and test of the synthesized controller on a real plant. One may immediately spot that the proposed approach guarantees very low overregulation and stable performance for the entire range of the set points.

Finally, we discuss several important limitations appearing within the proposed approach.

- It is always necessary to assume at least the second-order of the identified model due to the structure control algorithm (5)-(6). In fact, this is natural assumptions that has to be made in majority of the applications.
- From (7) it follows that, in general, one has to care of non-singularity of the matrix T_2^{-1}.
- The model may not necessarily be identifiable with acceptable accuracy by the original SANARX model due to the presence of the high nonlinearity in the data set with respect to the pair $\{y^{[-1]}, u^{[-1]}\}$. However, some other sublayers can be used, see [19] for more details.

Thus, all of the mentioned limitations are not restrictive and can be avoided during one of the pre-implementation steps.

VI. Future work

Another advantage of the proposed technique is that the neural networks based models are naturally suited for adaptation, which is a very important property of intelligent control systems. As a next step, it is planed to apply adaptation to:

- decrease the influence of the modeling error;
- take into account possible disturbances;
- improve performance of the closed-loop.

On the stage presented in the paper, neural network structure (number of layers corresponding to the order of the model and number of neurons) was chosen empirically based on *a priori* knowledge of the process and its complexity. In model (4), presented in Fig. 1, all connections between sublayers as well as between inputs, outputs and the corresponding sublayers were presented. In fact, some of them can be redundant. Combination of neural network learning based modeling with genetic algorithms based approach [20], [21] allows to find an optimal structure of the network (subclass of NN-ANARX model), significantly reduce the number of connections and as a result the number of parameters. This will also makes the subject for our further research.

The control technique proposed in [1] makes possible to apply the proposed algorithm to control of nonlinear multi-input multi-output systems. By using this approach, the result presented in the paper can also be extended to simultaneous control of liquid level in several interconnected tanks which is known to be much more complex task.

If the desired level of liquid in tank or tanks varies in time, a reference model based technique for dynamic output feedback linearization of ANARX model proposed in [16] can be applied to track the varying set point. Moreover, the technique mentioned above makes possible to predefine the desired dynamics of the transient process. However, one has

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to take into account that the required regulation time cannot be less than the one allowed by the physical system. By using this approach it is possible to get rid of any overshoot and guarantee the smooth behavior of the system.

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REFERENCES

[1] E. Petlenkov, “NN-ANARX structure based dynamic output feedback linearization for control of nonlinear MIMO systems,” in *The 15th Mediterranean Conference on Control and Automation*, Athena, Greece, June 2007, pp. 1–6.

[2] Ü. Kotta, F. N. Chowdhury, and S. Nõmm, “On realizability of neural networks-based input-output models in the classical state-space form,” *Automatica*, vol. 42, no. 7, pp. 1211–1216, 2006.

[3] W. C. Dunn, *Fundamentals of Industrial Instrumentation and Process Control*. McGraw-Hill Professional, 2005.

[4] C. Kern and M. Manness, “PID controller tuning for mixed continuous/discrete event processes using dynamic simulation,” in *Dynamic Modeling Control Applications for Industry Workshop*, Vancouver, BC, Canada, May 1997, pp. 37–43.

[5] R. Precup, L.-T. Dioanca, E. M. Petriu, M. Radac, S. Preitl, and C. Dragos, “Tensor product-based real-time control of the liquid levels in a three tank system,” in *IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, Montreal, ON, Canada, October 2010, pp. 768–773.

[6] Q. Wang, L. Xing, and X. Shi, “Decoupling control of three-tank liquid level systems based on feedforward compensation,” in *Chinese Control and Decision Conference*, Guilin, China, June 2009, pp. 5863–5866.

[7] K. Sundaravadivu, V. Jeyakumar, and K. Saravanan, “Design of fractional order PI controller for liquid level control of spherical tank modeled as fractional order system,” in *IEEE International Conference on Control System, Computing and Engineering*, Penang, Malaysia, November 2011, pp. 522–525.

[8] A. Tepljakov, E. Petlenkov, J. Belikov, and M. Halás, “Design and implementation of fractional-order PID controllers for a fluid tank system,” in *The 32nd American Control Conference*, Washington, USA, June 2013, pp. 1780–1785.

[9] L. Liang, “The application of fuzzy PID controller in coupled-tank liquid-level control system,” in *International Conference on Electronics, Communications and Control*, Ningbo, China, September 2008, pp. 2894–2897.

[10] I. J. Leontaritis and S. A. Billings, “Input-output parametric models for non-linear systems Part I: deterministic non-linear systems,” *International Journal of Control*, vol. 41, no. 2, pp. 303–328, 1985.

[11] R. Pothin, Ü. Kotta, and C. H. Moog, “Output feedback linearization of nonlinear discrete time systems,” in *The IFAC Conference on Control Systems Design*, Bratislava, Slovak Republic, 2000, pp. 181–186.

[12] Ü. Kotta, A. S. I. Zinober, and P. Liu, “Transfer equivalence and realization of nonlinear higher order input-output difference equations,” *Automatica*, vol. 37, no. 11, pp. 1771–1778, 2001.

[13] K. Vassiljeva, E. Petlenkov, and J. Belikov, “State-space control of nonlinear systems identified by ANARX and neural network based SANARX models,” in *The World Congress on Computational Intelligence*, Barcelona, Spain, July 2010, pp. 3816–3823.

[14] Ü. Kotta, S. Nõmm, and F. N. Chowdhury, “On a new type of neural network-based input-output model: The ANARMA structure,” in *The 5th IFAC Symposium on Nonlinear Control Systems*, St. Petersburg, Russia, July 2001, pp. 1623–1626.

[15] E. Petlenkov, S. Nõmm, and Ü. Kotta, “Neural networks based ANARX structure for identification and model based control,” in *The 9th International Conference on Control Automation Robotics and Vision*, Singapore, December 2006, pp. 2284–2288.

[16] E. Petlenkov, “Model reference control of nonlinear systems by dynamic output feedback linearization of neural network based ANARX models,” in *The 10th International Conference on Control Automation Robotics and Vision*, Hanoi, Vietnam, December 2008, pp. 1119–1123.

[17] INTECO Sp. z o. o. (2013) INTECO. [Online]. Available: http://www.inteco.com.pl

[18] Tallinn University of Technology. (2013) Alpha Control Lab. [Online]. Available: http://a-lab.ee/

[19] J. Belikov and E. Petlenkov, “NN-ANARX model based control of nonlinear discrete-time systems with input delays,” in *The 3rd IEEE Multi-conference on Systems and Control*, St. Petersburg, Russia, July 2009, pp. 1039–1044.

[20] K. Vassiljeva, E. Petlenkov, and S. Nõmm, “Evolutionary design of the closed loop control on the basis of NN-ANARX model using genetic algorithm,” in *Neural Information Processing*, ser. Lecture Notes in Computer Science. Springer, 2012, pp. 592–599.

[21] K. Vassiljeva, E. Petlenkov, and J. Belikov, “GA based optimization of NN-SANARX model for adaptive control of nonlinear systems,” in *The World Congress on Computational Intelligence*, Brisbane, Australia, June 2012, pp. 1674–1681.

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