POLITECNICO
MILANO 1863

Spacecraft Attitude Dynamics and Control

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Lab 1 – Analysis of the Euler Equations

Labs – Modelling in Simulink

Lab 1: Analysis of the Euler equations

Lab 2: Numerically integrating the attitude kinematic equations

Lab 3: Linear Modelling spacecraft attitude in an Earth orbit

Lab 4: Numerical stability analysis in the LVLH frame with gravity gradient

Lab 5: disturbance modelling: air drag, SRP, Earth's magnetic field.

Lab 6: Attitude de-tumbling implementation

Lab 7: Attitude pointing and stabilization implementation

Lab 8: Magnetic attitude control

Lab 9: Control with reaction wheels and thrusters.

Lab 10: Function minimization for attitude determination.

Lab 11: q-method v's TRIAD method for attitude determination

Lab 12: Attitude control with uncertainties

Tasks for the Lab.

Task 1: Set up the Euler equations in Simulink (40-50 mins)

Task 2: Plot the conserved quantities and show that they coincide with the numerical solutions (40 mins)

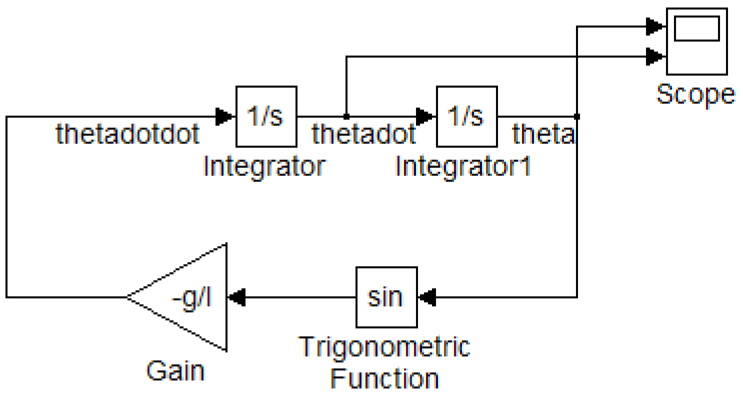
Task 3: analyse the stability of the equilibrium points numerically (30 mins)

Task 4: Verify that the numerical solutions match the analytic solutions (if there is time)

Integrating the nonlinear pendulum equations

$$I\ddot{\theta} + k\dot{\theta} + \frac{g}{L}\sin\theta = 0$$

$$I\ddot{\theta} = -\frac{g}{L}\sin\theta$$



Use Simulink defined blocks

Or we can use an S-function

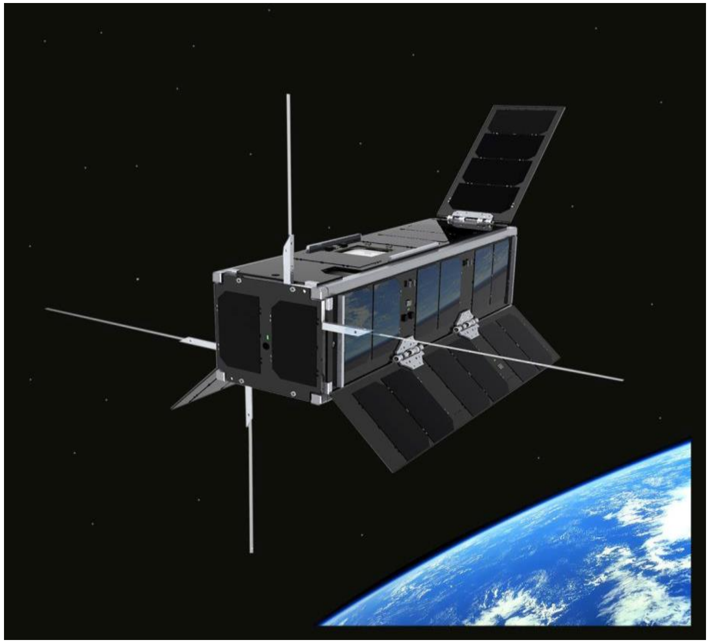
$$\dot{\theta} = \Omega$$

$$\dot{\Omega} = -\frac{k}{I}\dot{\theta} - \frac{g}{LI}\sin\theta$$

Task 1: Numerically integrate the equations of motion in Simulink

Principal moments of Inertia $I_1 = 0.0109 kgm^2, I_2 = 0.0504 kgm^2, I_3 = 0.07 kgm^2$

With initial conditions $\omega_1(0) = 0.45 rad/s, \omega_2(0) = 0.52 rad/s, \omega_2(0) = 0.55 rad/s$



$$\dot{\omega}_1 = \frac{I_2 - I_3}{I_1}\omega_2\omega_3$$
$$\dot{\omega}_2 = \frac{I_3 - I_1}{I_2}\omega_1\omega_3$$
$$\dot{\omega}_3 = \frac{I_1 - I_2}{I_3}\omega_2\omega_1$$

$$I_1 = 0.0109 kgm^2, I_2 = 0.0504 kgm^2, I_3 = 0.0504 kgm^2$$

$$\omega_1(0) = 5 revs/\min, \omega_2(0) = 0.05 rad/s, \omega_2(0) = 0.05 rad/s$$

Task 2: Conserved quantities - Plot the following

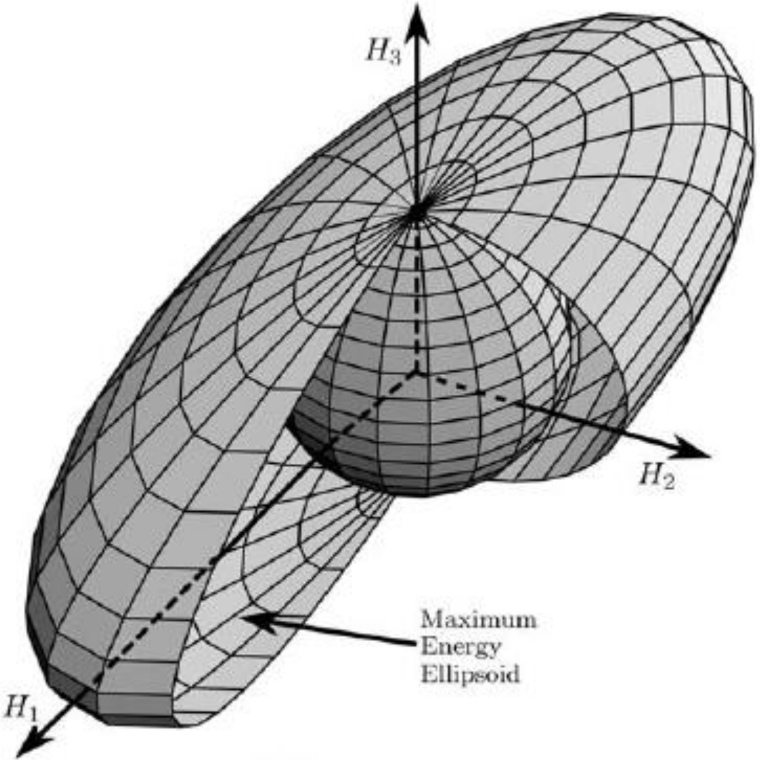
$$H = [I_1\omega_1 \quad I_2\omega_2 \quad I_3\omega_3]^T$$

$$T = \frac{1}{2}(I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2)$$

$$H_1 = I_1\omega_1$$

$$H_2 = I_2\omega_2$$

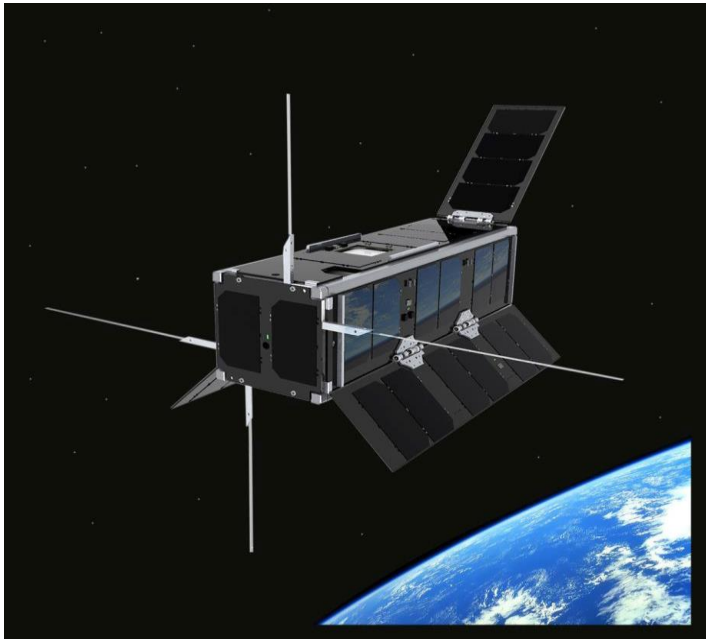
$$H_3 = I_3\omega_3$$



Task 2: Plot the surfaces and numerically integrate the Euler equations

Principal moments of Inertia $I_1 = 0.0109 kgm^2, I_2 = 0.0504 kgm^2, I_3 = 0.07 kgm^2$

With initial conditions $\omega_1(0) = 0.45 rad/s, \omega_2(0) = 0.52 rad/s, \omega_2(0) = 0.55 rad/s$



$$H = [I_1\omega_1 \quad I_2\omega_2 \quad I_3\omega_3]^T$$

$$T = \frac{1}{2}(I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2)$$

PLOT

$$\dot{\omega}_1 = \frac{I_2 - I_3}{I_1}\omega_2\omega_3$$

$$\dot{\omega}_2 = \frac{I_3 - I_1}{I_2}\omega_1\omega_3$$

$$\dot{\omega}_3 = \frac{I_1 - I_2}{I_3}\omega_2\omega_1$$

INTEGRATE

Code for surface plots (only available in Matlab 2017b)

Compute the constants H, T

```
I1 = 0.0109;
I2 = 0.0504;
I3 = 0.07;
w1=0.45;
w2=0.52;
w3=0.55;
H=I1^2*w1^2 + I2^2*w2^2 + I3^2*w3^2
T =(1/2)*(I1*w1^2 + I2*w2^2 + I3*w3^2)
```

Plot the momentum sphere

```
fimplicit3(@(x,y,z) x.^2+y.^2+z.^2 - H, [-0.1 0.1 -0.1 0.1 -0.1 0.1])
```

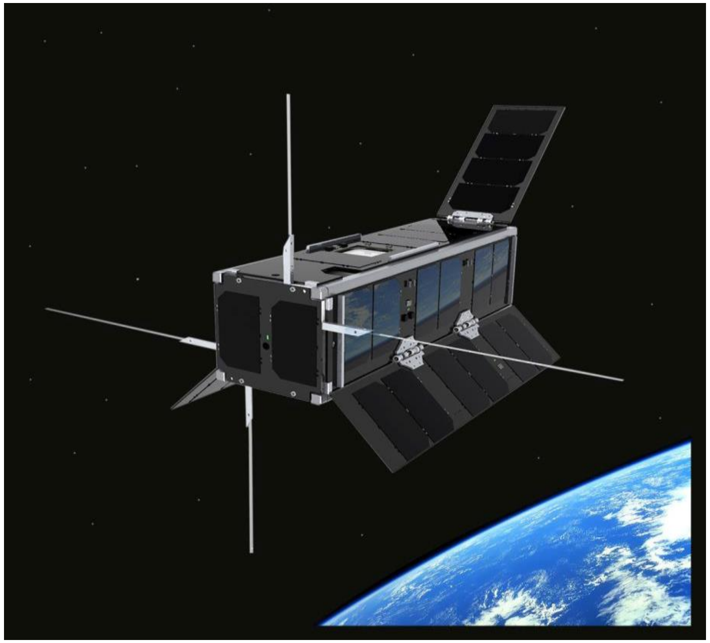
Plot the energy ellipsoid T

```
fimplicit3(@(x,y,z) x.^2/(I1)+y.^2/(I2)+z.^2/(I3) - 2*T, [-0.1 0.1 -0.1 0.1 -0.1 0.1])
```

You can also use the code on BEEP titled “conserved_quantities.m”

Task 3: Numerically assess the stability of the equilibrium points

Principal moments of Inertia $I_1 = 0.0109 kgm^2, I_2 = 0.0504 kgm^2, I_3 = 0.055 kgm^2$



$$\omega_1 = 0, \omega_2 = 0, \omega_3 = 0$$
$$\omega_1 = \omega_1(0), \omega_2 = 0, \omega_3 = 0$$
$$\omega_1 = 0, \omega_2 = \omega_2(0), \omega_3 = 0$$
$$\omega_1 = 0, \omega_2 = 0, \omega_3 = \omega_3(0)$$

$$\omega_i(0) = 15 \deg/\sec$$

$$\dot{\omega}_1 = \frac{I_2 - I_3}{I_1} \omega_2 \omega_3$$

$$\dot{\omega}_2 = \frac{I_3 - I_1}{I_2} \omega_1 \omega_3$$

$$\dot{\omega}_3 = \frac{I_1 - I_2}{I_3} \omega_2 \omega_1$$

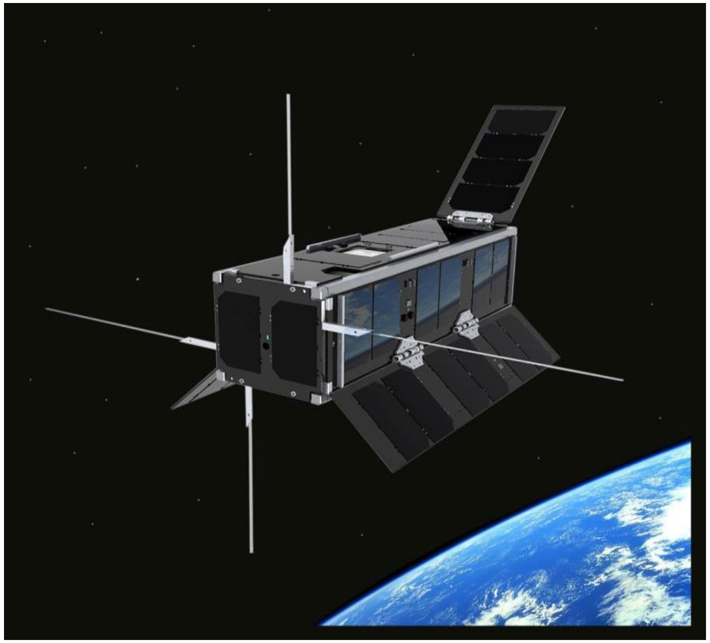
$$\frac{(I_1 - I_3)(I_1 - I_2)}{I_2 I_3} \omega_1^2(0) > 0$$

Task 4: Match the analytical results in Matlab to the numerical results in Simulink

Principal moments of Inertia

$$I_1 = 0.0109kgm^2, I_2 = 0.0504kgm^2, I_3 = 0.0504kgm^2$$

With initial conditions $\omega_1(0) = 0.45rad/s, \omega_2(0) = 0.52rad/s, \omega_2(0) = 0.55rad/s$



```
x0=1;
r=1;
a=1;
b=0.5;
t = linspace(0,2*pi,20);
x = x0*ones(1,length(t));
y = r*sin(a*t+b);
z = r*cos(a*t+b);
plot3(x,y,z)
```

Learning objectives

- Numerically integrate the Euler equations in Simulink
- Evaluate the numerical solution against the geometric solution for verification.
- Numerically assess the stability of equilibrium points.
- Verify the numerical integration of a symmetric spacecraft against the analytic solution.