POLITECNICO
MILANO 1863

# Spacecraft Attitude Dynamics and Control

## Lab 2 - Attitude Kinematics

# Labs – Modelling in Simulink

Lab 1: Analysis of the Euler equations

**Lab 2: Numerically integrating the attitude kinematic equations**

Lab 3: Modelling spacecraft attitude in Earth orbit

Lab 4: Numerical stability analysis in the LVLH frame with gravity gradient

Lab 5: disturbance modelling: air drag, SRP, Earth's magnetic field.

Lab 6: Attitude de-tumbling implementation

Lab 7: Attitude pointing and stabilization implementation

Lab 8: Magnetic attitude control

Lab 9: Control with reaction wheels and thrusters.

Lab 10: Function minimization for attitude determination.

Lab 11: q-method v's TRIAD method for attitude determination

Lab 12: Attitude control with uncertainties

# Tasks for the Lab.

Task 1: Set up the kinematics using DCM representation

Task 2: Test the orthonormality of the solution

Task 3: Plot the pointing vector and look at the behaviour by perturbing from the equilibrium solutions.

Task 4: Integrate the kinematics using Euler angles and map to the DCM.

Task 5: Set up a simple PD controller using Euler angles.

# Task 1: Simulate the dynamics and kinematics using the direct Cosine matrix

Dynamics

$$\dot{\omega}_1 = \frac{I_2 - I_3}{I_1}\omega_2\omega_3$$

$$\dot{\omega}_2 = \frac{I_3 - I_1}{I_2}\omega_1\omega_3$$

$$\dot{\omega}_3 = \frac{I_1 - I_2}{I_3}\omega_2\omega_1$$

Kinematics

$$\frac{dA(t)}{dt} = -[\omega\wedge]A(t)$$

$$[\omega\wedge] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

$$\text{Dynamics} \longrightarrow \text{Kinematics}$$

$$A = \begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{bmatrix}$$

Test if the integration gives an (approximate) orthonormal matrix

# Numerical Integration of the system

Iterative formulas for orthonormalization

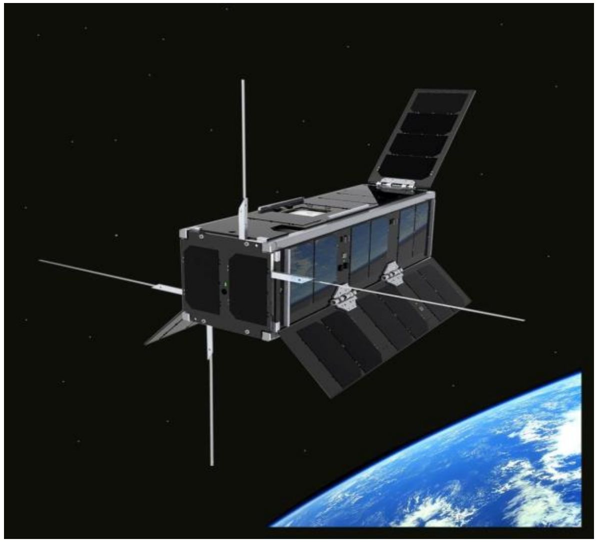
$A_{k+1}(t)= A_k(t)*3/2- A_k(t)*A_k^T(t)*A_k(t)/2$

converges rapidly, with increasing k, to the exact value of A. In a first order approximation it is possible to adopt a single step iteration

$A(t)= A_0(t)*3/2- A_0(t)*A_0^T(t)*A_0(t)/2$

# Task 2: Numerically assess the stability of the equilibrium points by observing the pointing axis

Principal moments of Inertia $I_1 = 0.0109 kgm^2, I_2 = 0.0504 kgm^2, I_3 = 0.055 kgm^2$



$$\omega_1 = 0, \omega_2 = 0, \omega_3 = 0$$
$$\omega_1 = \omega_1(0), \omega_2 = 0, \omega_3 = 0$$
$$\omega_1 = 0, \omega_2 = \omega_2(0), \omega_3 = 0$$
$$\omega_1 = 0, \omega_2 = 0, \omega_3 = \omega_3(0)$$

$$\omega_i(0) = 15 \deg/\sec$$

$$\frac{(I_1 - I_3)(I_1 - I_2)}{I_2 I_3} \omega_1^2(0) > 0$$

$$\frac{dA(t)}{dt} = -[\omega \wedge] A(t)$$

# Task 3: Integrate the system using Euler angles

$$\text{sequence 123}\begin{cases}\dot{\varphi} = \dfrac{\left(\omega_u \cos\psi - \omega_v \sin\psi\right)}{\cos\vartheta} \\ \dot{\vartheta} = \omega_v \cos\psi + \omega_u \sin\psi \\ \dot{\psi} = \omega_w - \left(\omega_u \cos\psi - \omega_v \sin\psi\right)\dfrac{\sin\vartheta}{\cos\vartheta}\end{cases}$$

Map the solution back to the DCM

$$A_{123} = \begin{bmatrix} \cos\psi\cos\vartheta & \cos\psi\sin\vartheta\sin\varphi + \sin\psi\cos\varphi & -\cos\psi\sin\vartheta\cos\varphi + \sin\psi\sin\varphi \\ -\sin\psi\cos\vartheta & -\sin\psi\sin\vartheta\sin\varphi + \cos\psi\cos\varphi & \sin\psi\sin\vartheta\cos\varphi + \cos\psi\sin\varphi \\ \sin\vartheta & -\cos\vartheta\sin\varphi & \cos\vartheta\cos\varphi \end{bmatrix}$$

Test the orthonormaility conditions of the matrix

# Task 4: Implement a simple control law:

Dynamics

Kinematics

$$\dot{\omega}_u = \frac{I_2 - I_3}{I_1}\omega_v\omega_w + u_1$$
$$\dot{\omega}_v = \frac{I_3 - I_1}{I_2}\omega_u\omega_w + u_2$$
$$\dot{\omega}_w = \frac{I_1 - I_2}{I_3}\omega_v\omega_u + u_3$$

$$\text{sequence } 123 \begin{cases} \dot{\varphi} = \dfrac{(\omega_u\cos\psi - \omega_v\sin\psi)}{\cos\vartheta} \\ \dot{\vartheta} = \omega_v\cos\psi + \omega_u\sin\psi \\ \dot{\psi} = \omega_w - (\omega_u\cos\psi - \omega_v\sin\psi)\dfrac{\sin\vartheta}{\cos\vartheta} \end{cases}$$

$$u_1 = -k_{11}\omega_u - k_{12}(\varphi - \varphi_d)$$
$$u_2 = -k_{21}\omega_v - k_{22}(\vartheta - \vartheta_d)$$
$$u_3 = -k_{31}\omega_w - k_{32}(\psi - \psi_d)$$

# Learning objectives

- Numerically integrate the Euler equations along with kinematics in DCM and Euler angle representations.
- Understand pointing stability.
- Set up a simple closed-loop control.