POLITECNICO
MILANO 1863

Spacecraft Attitude Dynamics and Control

Lab 4 – Gravity Gradient disturbances and control

Tasks for the Lab.

Task 1: Include the gravity gradient disturbance in your SIMULINK model.

Task 2: Use the nonlinear dynamics to assess the results of the Linear stability analysis.

Task 3: Design a Quaternion Feedback Control.

Task 1:Include the gravity gradient disturbance in SIMULINK

$$I_x\dot{\omega}_x + \left(I_z - I_y\right)\omega_z\omega_y = \frac{3Gm_t}{R^3}\left(I_z - I_y\right)c_3c_2 + u_1$$

$$I_y\dot{\omega}_y + \left(I_x - I_z\right)\omega_x\omega_z = \frac{3Gm_t}{R^3}\left(I_x - I_z\right)c_1c_3 + u_2$$

$$I_z\dot{\omega}_z + \left(I_y - I_x\right)\omega_y\omega_x = \frac{3Gm_t}{R^3}\left(I_y - I_x\right)c_2c_1 + u_3$$

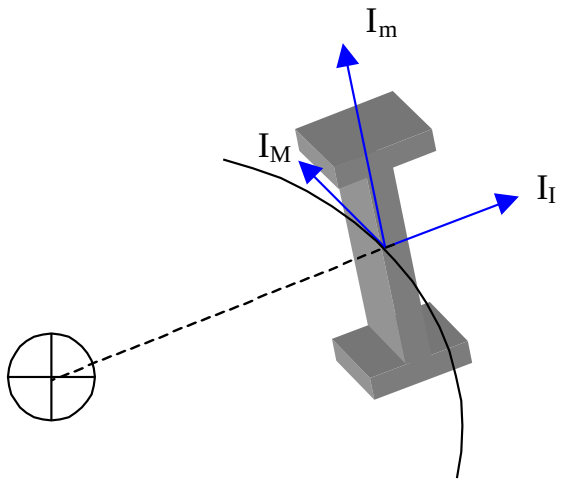
$$R = \frac{a(1-e^2)}{1+e\cos\theta}$$

$$\dot{\theta} = \frac{n(1+e\cos\theta)^2}{(1-e^2)^{3/2}}$$

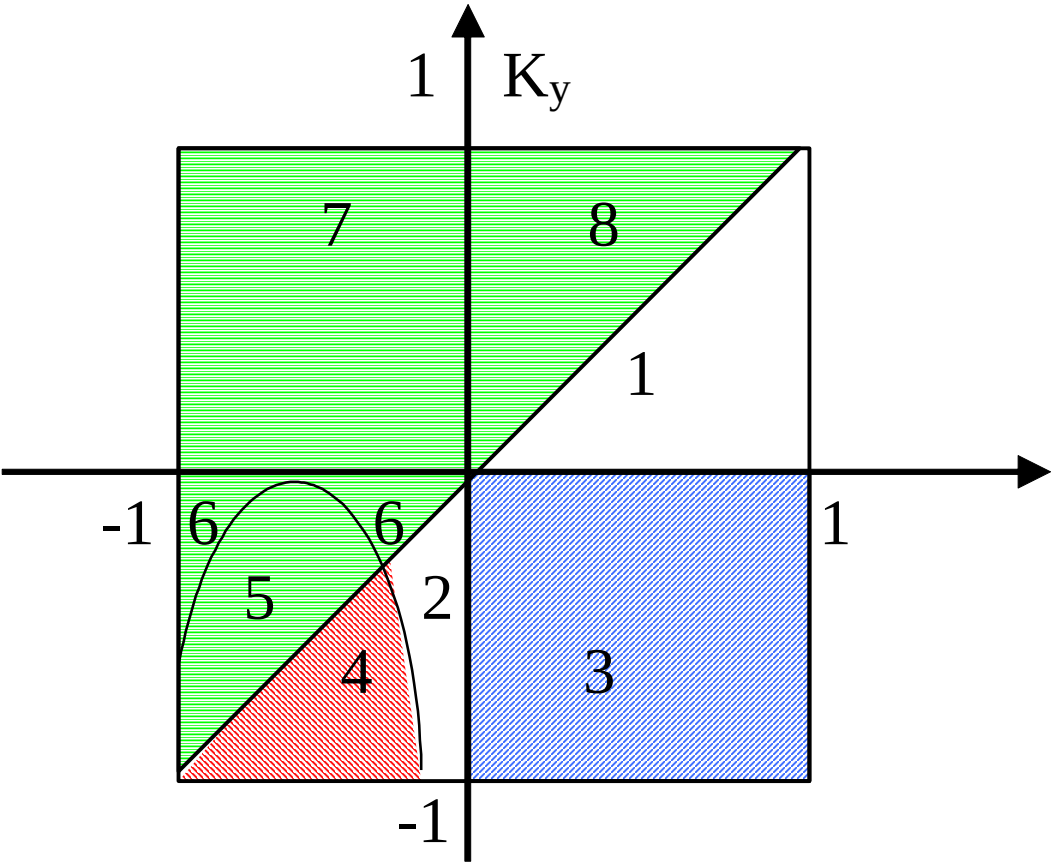
$$\dot{A}_{B/N} = -[\omega_{B/N}]^{\wedge} A_{B/N}$$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = A_{B/L}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$A_{B/L} = A_{B/N}A_{L/N}^T$$



Task 2: Use the nonlinear dynamics to assess the results of the Linear stability analysis



$$K_x = \frac{I_z - I_y}{I_x}$$

$$K_y = \frac{I_z - I_x}{I_y}$$

1. $I_z > I_y > I_x$
2. $I_y > I_x > I_z$
3. $1 + 3K_x + K_y K_x > 4\sqrt{K_y K_x}$

Task 3: Design a Quaternion Feedback Control

$$I_x\dot{\omega}_x+\left(I_z-I_y\right)\omega_z\omega_y=\frac{3Gm_t}{R^3}\left(I_z-I_y\right)c_3c_2+u_1$$

$$I_y\dot{\omega}_y+\left(I_x-I_z\right)\omega_x\omega_z=\frac{3Gm_t}{R^3}\left(I_x-I_z\right)c_1c_3+u_2$$

$$I_z\dot{\omega}_z+\left(I_y-I_x\right)\omega_y\omega_x=\frac{3Gm_t}{R^3}\left(I_y-I_x\right)c_2c_1+u_3$$

$$\frac{d}{dt}\begin{bmatrix}q_1\\q_2\\q_3\\q_4\end{bmatrix}=\frac{1}{2}\begin{bmatrix}0&\omega_3&-\omega_2&\omega_1\\-\omega_3&0&\omega_1&\omega_2\\\omega_2&-\omega_1&0&\omega_3\\-\omega_1&-\omega_2&-\omega_3&0\end{bmatrix}\begin{bmatrix}q_1\\q_2\\q_3\\q_4\end{bmatrix}$$

$$\begin{bmatrix}q_{1e}\\q_{2e}\\q_{3e}\\q_{4e}\end{bmatrix}=\begin{bmatrix}q_{4c}&q_{3c}&-q_{2c}&-q_{1c}\\-q_{3c}&q_{4c}&q_{1c}&-q_{2c}\\q_{2c}&-q_{1c}&q_{4c}&-q_{3c}\\q_{1c}&q_{2c}&q_{3c}&q_{4c}\end{bmatrix}\begin{bmatrix}q_1\\q_2\\q_3\\q_4\end{bmatrix}$$

$$\begin{bmatrix}q_1(0)\\q_2(0)\\q_3(0)\\q_4(0)\end{bmatrix}=\begin{bmatrix}0\\1/\sqrt{3}\\1/\sqrt{3}\\1/\sqrt{3}\end{bmatrix},\begin{bmatrix}q_{1c}\\q_{2c}\\q_{3c}\\q_{4c}\end{bmatrix}=\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

Perform a slew motion

$$\underline{u}=-K\underline{q}_e-C\underline{\omega}_e$$

Perform tracking of the LVLH frame

$$\underline{u}=-K\underline{q}_e-C\underline{\omega}_{B/L}$$

$$\underline{\omega}_{B/L}=\underline{\omega}_{B/N}-A_{B/L}\underline{\omega}_{L/N}$$

Learning outcomes

- Understand how to model the gravity gradient disturbance in SIMULINK.
- Understand how to implement a quaternion feedback control to perform a slew motion.
- Understand how to implement a quaternion feedback control for tracking.