

# Cheatsheet di Analisi Matematica 1

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**Limiti Notevoli** — Sostituire  $x$  con  $f(x)$  per ottenere lo stesso risultato

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\tan(x)}{x} = 0$$

$$\lim_{x \rightarrow 0} \frac{\alpha^x - 1}{x} = \ln(\alpha)$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{\alpha}{x}\right)^{nx} = e^{n\alpha}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x^2} = \frac{1}{2}$$

$$\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$$

$$\lim_{x \rightarrow \infty} \frac{(1+x)^\alpha - 1}{x} = \alpha$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

**Derivate e integrali** di funzioni elementari

Funzione	Derivata	Integrale
$x$	$\frac{d}{dx} x = 1$	$\int x \, dx = \frac{x^2}{2} + c$
$x^n$	$\frac{d}{dx} x^n = n x^{n-1}$	$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$
$\sin(x)$	$\frac{d}{dx} \sin(x) = \cos(x)$	$\int \sin(x) \, dx = -\cos(x) + c$
$\cos(x)$	$\frac{d}{dx} \cos(x) = -\sin(x)$	$\int \cos(x) \, dx = \sin(x) + c$
$\frac{1}{x}$	$\frac{d}{dx} \frac{1}{x} = -\frac{1}{x^2}$	$\int \frac{1}{x} \, dx = \ln(x) + c$
$\ln(x)$	$\frac{d}{dx} \ln(x) = \frac{1}{x}$	$\int \ln(x) \, dx = x(\ln(x) - 1) + c$ (per parti)

$$\frac{d}{dx} \tan(x) = \frac{1}{\cos^2(x)}$$

$$\frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \arccos(x) = -\frac{1}{\sqrt{1-x^2}}$$

**Derivate** – Regole di derivazione

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx} f(x) \cdot g(x) = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

**Integrali** – Regole di integrazione

$$\int f(g(x)) \cdot g'(x) \, dx = F(g(x)) + c$$

*Integrale per parti*

$$\int f(x) \cdot g(x) \, dx = F(x)g(x) - \int F(x) \cdot g'(x) \, dx$$

**Differenziali** – Soluzioni generali

$$\begin{aligned} y' &= a(x) \cdot y + b(x) \\ \Rightarrow y &= e^{A(x)} \cdot \int e^{-A(x)} \cdot b(x) \, dx + c \\ \text{con } A(x) &= \int a(x) \, dx \end{aligned}$$

$$\begin{aligned} y' &= a(x) \cdot f(y) \text{ Variabili separabili} \\ \Rightarrow \int \frac{1}{f(y(x))} \, dy &= \int a(x) \, dx \end{aligned}$$

$$\sin^2(x) + \cos^2(x) = 1 \quad \sin(2x) = 2 \sin(x) \cos(x)$$

$$y'' + ay' + by = 0 \text{ Di secondo grado } \lambda^2 + a\lambda + b = 0 \rightarrow \lambda_1, \lambda_2$$

$$\lambda_1 \neq \lambda_2 \in \mathbb{R} \Rightarrow y(x) = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$$

$$\lambda_1 = \lambda_2 \in \mathbb{R} \Rightarrow y(x) = c_1 e^{\lambda_1 x} + x c_2 e^{\lambda_1 x}$$

$$\begin{aligned} \lambda_1 &= \alpha + i\beta, \lambda_2 = \alpha - i\beta \\ \Rightarrow y(x) &= e^{\alpha x} (c_1 \cos(\beta x) + c_2 \sin(\beta x)) \end{aligned}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) \quad \tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$$

## Studio di funzione

### 1. Insieme di definizione

$$\frac{1}{f(x)} \Rightarrow f(x) \neq 0$$

$$\log_a(f(x)) \Rightarrow f(x) > 0$$

$$\sqrt[n]{f(x)} \Rightarrow f(x) \geq 0 \text{ per } n \text{ pari}$$

$$\text{per } \arcsin(f(x)) \text{ e } \arccos(f(x)) \Rightarrow -1 \leq f(x) \leq 1$$

### 2. Asintoti

#### Orizzontali

$$\lim_{x \rightarrow \pm\infty} f(x) = l \neq \infty$$

#### Verticali

$$\lim_{x \rightarrow x_0} f(x) = \infty$$

$x_0$  punto di discontinuità

#### Obliqui

$y(x) = mx + q$  asintoto obliquo se

$$\lim_{x \rightarrow \pm\infty} \frac{f(x)}{x} = m \neq \infty, 0$$

$$\lim_{x \rightarrow \pm\infty} f(x) - mx = q$$

### 3. Derivate e segno

$$f'(x) = \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$f''(x)$$

$$f'(x_0) \geq 0 \Rightarrow f(x_0) \text{ crescente}$$

$$f''(x_0) > 0 \Rightarrow f(x_0) \text{ convessa}$$

$$f'(x_0) \leq 0 \Rightarrow f(x_0) \text{ decrescente}$$

$$f''(x_0) < 0 \Rightarrow f(x_0) \text{ concava}$$

$$x_0 \text{ angoloso se } f'_+(x_0) \neq f'_-(x_0) \text{ finiti}$$

$$x_0 \text{ flesso se } f''(x_0) = 0$$

$$x_0 \text{ cuspidale se } f'_+(x_0) = \pm\infty \wedge f'_-(x_0) = \mp\infty$$

$$x_0 \text{ minimo se } f(x) \text{ decrescente} \rightarrow x_0 \rightarrow \text{crescente}$$

$$x_0 \text{ massimo se } f(x) \text{ crescente} \rightarrow x_0 \rightarrow \text{decrescente}$$

#### Valori di sin e cos

Angolo	sin	cos
$0 = 2\pi = 360$	0	1
$30 = \frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$45 = \frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$60 = \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$90 = \frac{\pi}{2}$	1	0
$180 = \pi$	0	-1
$270 = \frac{3\pi}{2}$	-1	0

$$\lim_{x \rightarrow +\infty} e^x = +\infty$$

$$\lim_{x \rightarrow -\infty} e^x = 0^+$$

$$\lim_{x \rightarrow 0^+} \ln(x) = -\infty$$

$$\lim_{x \rightarrow +\infty} \ln(x) = +\infty$$