

Parallel and Distributed Systems

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A.A. 2021/22

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0.1 Introduction

Prof.: Marco Danelutto

Program Techniques for both parallel (single system, many core) and distributed (clusters of systems) systems. Principles of parallel programming, structured parallel programming, parallel programming lab with standard and advanced (general purpose) **parallel programming frameworks**.

Technical Introduction Each machine has more cores, perhaps multithreaded cores, but also GPUs (maybe with AVX support, which support operations floating point operations, **flops**, in a single instruction).

Between 1950 and 2000 the VLSI technology arised, integrated circuits which nowadays are in the order of 7nm (moving towards 2nm): printed circuits!

In origin, everything happened in a single clock cycle: fetch, decode, execute, write results in registers, with perhaps some memory accesses. Then we had more complex control where in a single clock cycle we do just one of the phases (fetch *or* decode *or* ...), like a **pipeline**. More components are used the higher the frequency but the more power we need to dissipate, and we're coming to a point where the power we need to dissipate is too much and risks to melt the circuit, so we're reaching a **physical limit** in chip miniaturization. But temperature and computing power do not go in tandem: computing power is proportional to the chip dimensions, while temperature is proportional to the area. So it's better to put more processors (**cores**) and let them work together rather than make a bigger single processor. An approach is to have few powerful cores and more less powerful cores (for example, in the Xeon Phi processors). Now, the processors follow this architecture, with the performance of a single core decreasing a bit with every generation but it's leveled by adding more cores.

Up to the 2000, during the single core era, code written years before will run faster on newer machines. Now, code could run slower due to not exploiting more cores and the decreasing in performance of the single core.

With accelerators the situation is even more different: for example GPUs, accelerator for graphics libraries, with their own memory and specialized in certain kinds of operations. This can require the transfer of data between the accelerator's memory and the main memory, so the architecture of the accelerator is impactful on the overall performance.

0.2 General Paradigms of Parallel Programming

Parallelism Execution of different parts of a program on different computing devices at the same time. We can imagine different flows of control (sequences of instruction) that all together are a program and are executed on different computing devices. Note that more flows on a single computing device is **concurrency**, not parallelism.

Concurrency Similar concept: things that *may* happen in parallel respecting the ordering between elements.

Computing Devices

Threads, implying shared memory

Processes, implying separated memories

GPU Cores

Hardware Layouts on a FPGA (Field Programmable Gate Array)

Sequential Task A "program" with its own input data that can be executed by a single computing entity

Overhead Actions required to organize the computation but that are not included in the program. For example: time spent in organizing the result. Basically, time spent orchestrating the parallel computation and not present in the sequential computation.

Speedup Fundamental things that we're looking for, it's the ratio between the sequential time and the parallel time.

$$\text{SpeedUp} = \frac{\text{Sequential time}}{\text{Parallel time}}$$

Assuming the best sequential time.

We have a slightly different measure, too

$$\text{Scalability} = \frac{\text{Parallel time with 1 computing device}}{\text{Parallel time}}$$

Stream of tasks In some cases it's not important considering just one computation but may be useful considering more computations and we want to optimize a set of tasks.

Example: Book Translation With $m = 600$ pages, for example. Let's assume I can translate a page in $t_p = 0.5h$. The sequential task is: take the book and spend time until I can deliver the translated book. The time is circa $m \cdot t_p = 300h$.

In parallel, ideally every page can be translated independently so I can split the book in two pieces of $\frac{m}{2}$ pages each (overhead), giving each half to a person. Both can translate at the same time, so ideally the time required is $\frac{m}{2} \cdot t_p$ for each, producing the translated halves. At this point I get the halves and produce the translated version (overhead). Ideally the time require is more or less $\frac{m}{2} \cdot t_p$, with "more or less" given by the time spent in splitting the book and reuniting the two halves. So the exact time is $T = T_{split} + \frac{m}{2} \cdot t_p + T_{merge}$.

What if the two person have different t_p s? For example $t_1 > t_2$. When a translator finishes, it spends some time synchronizing its work with me. With nw "workers" (translators, in this instance) $T = nw \cdot T_{split} + nw \cdot T_{merge} + \frac{m}{nw} T_{work}$ with $nw \cdot T_{split}$ time spent delivering work to each worker and $nw \cdot T_{merge}$ time in merging each result.

Init is the time where every worker has work to do, and finish is the time where the last worker finished working. So the exact formula is with a single T_{merge} .

So $\frac{m}{nw} T_{work}$ is the time that needs to happen, found in the sequential computation too, whereas the other two factors are **overhead**.

$$\text{SpeedUp} = \frac{\text{Best sequential time}}{\text{Parallel time}}$$

but the parallel time depends on the nw so

$$\text{SpeedUp}(nw) = \frac{\text{Best sequential time}}{\text{Parallel time}(nw)} \simeq \frac{\cancel{m} \cdot \cancel{t_p}}{\frac{\cancel{m}}{nw} \cdot \cancel{t_p}} = nw$$

This not taking into account the overhead. It's a realistic assumption because usually the time splitting the work is very small. But we have to take into account that, in case it's not negligible.

$$\text{SpeedUp}(nw) = \frac{m \cdot t_p}{\frac{m}{nw} \cdot t_p + nw \cdot T_{split} + T_{merge}}$$

Example: Conference Bag $T_{bag} = t_{bag} + t_{pen} + t_{paper} + t_{proc}$ and with m bags we have $T = m \cdot T_{bag}$

We could build a pipeline, a building chain, with 4 people and each person does one task:

One takes the bag and gives to the next

One puts the pen into the bag and passes it

One puts the paper into the bag and passes it

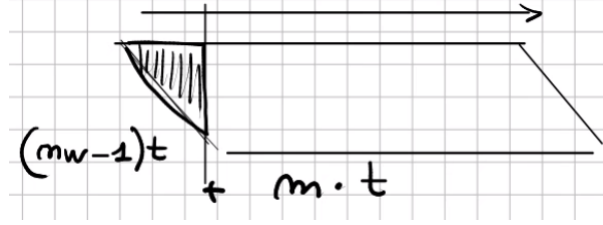
One puts the proceedings into the bag

So $w_b, w_{pen}, w_{paper}, w_{proc}$ workers. When the first worker has passed the bag, it could begin taking the next bag. Same for the others.



So in sequential we have $m \cdot (t_{bag} + t_{pen} + t_{paper} + t_{proc})$, and in parallel per 1 bag we have $t_{bag} + t_{comm} + t_{pen} + t_{comm} + t_{paper} + t_{comm} + t_{proc} + t_{comm}$ with t_{comm} spent passing the bag from one to the other, so total of $m \cdot T_{seq} + m \cdot t_{comm}$. But that's not correct, because we work in parallel: ideally we have a parallelogram of $m \cdot (t_{proc} + t_{comm})$ base, and we require $t_{bag} + t_{pen} + t_{paper} + 3 \cdot t_{comm}$ time to get up to speed and "fill the pipeline". But this required time is negligible, and in the end the overall time is given by the base of the parallelogram.

Pipeline With m tasks and nw stages, with the completion of the stage i required in stage $i + 1$. So the output is $f_{nw}(f_{nw-1}(\dots f_1(x_i) \dots))$. With t time required for each stage.



We spend $(nw - 1)t$ to get the last stage working and $m \cdot t$ time spent by the last stage to complete all the tasks.

$$T_{par}(nw) = (nw - 1) \cdot t + m \cdot t$$

$$\text{SpeedUp}(nw) = \frac{(nw \cdot t) \cdot m}{(nw - 1) \cdot t + m \cdot t}$$

So the higher the m is, the lower is the impact of the time required to get up to speed. So $m \gg nw \Rightarrow T_{par}(nw) \simeq m \cdot t$

Throughput Tasks completed per unit of time.

0.3 Measures

On one side we can have more speed with more resources (computing devices). On the other side we can use more complex applications, with more resources. For example more precise computations, so extra resources not for improving the time but to improve the quality of the computations.

Finally, we could aim at computing results with less energy thanks to parallelism. This is a recent perspective on parallelism.

We've seen the $\text{SpeedUp}(n) = \frac{T_{seq}}{T_{par}(n)}$, where the plot has to lie below the bisection of the cartesian graph.

0.3.1 Base Measurements

Latency L Measure of the wall-clock time between the start and end of the single task.

Service Time T_s It's related to the possibility of executing more tasks. It's the measure of the time between the delivery of two consecutive results, for example between $f(x_i)$ and $f(x_{i+1})$

Even if x_i and x_{i+1} arrive at the same time, f would still be computing $f(x_i)$ so it'll start computing $f(x_{i+1})$ when it has finished.

Completion Time T_c The latency related to a number of tasks. $T_c = L \cdot m$ for x_m, \dots, x_1 inputs to a sequential system.

With a parallel system, instead, we have $T_c \simeq m \cdot T_s$.

Example A 3 stage pipeline, with each node being sequential and with latency L_i for node i .

At t_0 the first stage N_1 gets the first tasks and computes it in L_1 , then N_2 computes in L_2 and N_3 computes in L_3 so a total of $t_0 + L_1 + L_2 + L_3$.

When the pipeline is filled, T_s is dominated by the longest L_i , so $T_s = \max\{L_1, L_2, L_3\}$ and $T_c = \sum L_i + (m - 1)T_s$

If m is large with respect to n = number of stages, the "base of the parallelogram" would be very long, so $m \gg n \Leftrightarrow T_c = m \cdot T_s$

0.3.2 Derived Measurements

SpeedUp

$$\text{SpeedUp}(n) = \frac{T_{seq}}{T_{par}(n)}$$

Could be latencies, service times... depending on what we want to measure the speedup of.

Scalability

$$\text{Scalability}(n) = \frac{T_{par}(1)}{T_{par}(n)}$$

Efficiency

$$\text{Efficiency}(n) = \frac{\text{Ideal parallel time}(n)}{T_{par}(n)} = \frac{\frac{T_{seq}}{n}}{T_{par}(n)} = \frac{T_{seq}}{n \cdot T_{par}(n)} = \frac{\text{SpeedUp}(n)}{n}$$

Measures the tradeoff between what you gain with the speedup and the cost of the speedup.

Throughput

$$\text{Throughput} = \frac{1}{T_s}$$

Amdahl Law Taken the total time of a computation, T_{seq} , it can be divided into something that can and something that cannot be computed in parallel (for example, dividing the book is a sequential activity). So we can say that $T_{seq} = \text{serial fraction} + \text{parallel fraction}$ and the **serial fraction cannot be parallelized**. $f \in [0, 1] \mid f \cdot T_{seq}$ is the serial fraction.

$$T_{seq} = f \cdot T_{seq} + (1 - f) \cdot T_{seq}$$

The parallel fraction can be splitted between the workers, but we would have to compute the serial fraction too. By splitting more and more and more, we have that

$$\lim_{n \rightarrow \infty} T_{par}(n) = f \cdot T_{seq}$$

$$\text{SpeedUp}(n) = \frac{T_{seq}}{f \cdot T_{seq}} = \frac{1}{f}$$

So we have a very low upper bound on the achievable speedup. This is referred to as **strong scaling**: strong meaning using more resources to get the computation faster.

Gustaffson Law

$$\text{SpeedUp}(n) = N - S \cdot (N - 1)$$

With S being the serial fraction. This comes from the fact that we're considering a different perspective: Gustaffson assumes that the computation increases with the parallelism, something that's called **weak scaling**, getting the speedup from using more computational devices, using more data.

Cores In modern computers, we have a main memory (slow), a disk (even slower) and the memory is connected to at least 3 levels of cache. At the bottom we have some cores (4, 8...), each one has its own level 1 cache (usually split in data and instruction cache).

With an activity with a working set that fills the cache, in case of strong scaling splitting the computation across cores we process less data per core because the size of the problem is the same.

With weak scaling, we assume that the data increases so by using more cores we process the same data on all cores but the data grows so we could have extra overhead because of the working set size.

We will have patterns of parallel computation that differentiate in how we process the data.

Application as Graphs The applications can be seen as graphs of sequential nodes with dependencies. The maximum speedup is the work over the span, because in every case I need to go from the first to the goal node. I take the longest one because at least the longest path must be computed, and all the rest can be done in parallel and I assume to have enough resources to compute the rest in the time of the span.

We can use this model

0.4 Technicalities

Examples A simple program that "translates" an ASCII file by transforming lower letters into capital letters. We split the text into $n_{workers}$ parts, we wait for all the threads to finish and then verify the performances. The translator is:

```
1 #include <string>
2
3 char translate_char(char c) {
4     if (islower(c))
5         return(toupper(c));
6     else
7         return(tolower(c));
8 }
```

0.4.1 Threads

We used to write instructions sequentially. At a given point now we **fork** another flow of computation: we get two flows that are executed together **in the same address space**, so the new thread inherits all the memory of the original thread.

Concurrency `todo`

Packaged Threads

Overheads All the time spent that is not involved in the sequential execution: time spent organizing the parallel computation, gathering solutions... so there's a tradeoff between the time spent to setup the parallel activity and the time earned because of the parallel execution.

What can we do to get rid of the setup time? I can create the threads once and reuse them when needed: **threadpools**.

Cache Coherence Protocols Snoopy, or directories of shared data consulted any time I access shared data and propagates the edits.

Coherency works at the cache line level!

In Stencil Reuse cache coherence protocols for each write.

False Sharing Problem We use padding techniques to transform the vector in such way that the original vector has some pad values up to the point where the cache line finishes. This is used in maps.

Disabling Sometimes useful to speed up the computations.

So we have to take into account that we have to ensure locality as much as possible and be careful of the possibilities of fault sharing problems, like similar iterations in short time.

taskset To restrict the cores of a process, `taskset -c 0-3 command` restricts `command` to the cores 0, 1, 2 and 3.

0.5 Patterns

Computations with particular shapes and semantics that can be understood and implemented depending on the situations, not linked to languages and technicalities. Patterns are a useful concepts, allow programmers to reuse experience of other programmers and not reinventing the wheel.

Parallel patterns:

Data parallel

Stream parallel

The same patterns can be referred with different names.

0.5.1 Data Parallel Patterns

Parallelism comes from data: we split the data in pieces, compute a set of results that can be combined into a single final result. The book translation examples is a data parallel pattern. What matters is L .

The general pattern is:

Decomposition

Partial results

Recomposition

Map Pattern Also called applytoall:

\forall item of the collection

Function $f(\text{item})$

$\forall f(\text{item}) \rightarrow$ isomorphic collection

Reduce Pattern Also called fold:

\forall item of the collection

$\oplus(x, y)$

$\oplus(\oplus(a, b), \oplus(c, d))$

Stencil Pattern

In partially overlapping position, e.g. of a matrix or an image

Function $f(\text{item})$

$\forall f(\text{item}) \rightarrow$ isomorphic collection

Different kind of problems: overlapping positions will yield the new value, so we have to account for that.

"Google" mapreduce

\forall items

$f(\text{item}) + \oplus(\text{item}, \text{item})$

Item

What I apply to each item is an f that maps to $\langle \text{key}, \text{value} \rangle$ and \oplus applies the sum to each value.

For example in a document, $f(\text{word}) = \langle w, 1 \rangle$ and $\oplus(\langle w_k, v_1 \rangle, \langle w_k, v_2 \rangle) = \langle w_k, \oplus(v_1, v_2) \rangle$

"The lesson given by the professor", f will output $\langle \text{the}, 1 \rangle, \langle \text{lesson}, 1 \rangle \dots$ and \oplus will for example output $\langle \text{the}, 2 \rangle$.

We can apply the map function over all the data distributed in various databases, for example.

This is **map**(f) and **reduce**(\oplus), but we want something like **map**(**reduce**(\oplus)). Combining elementary patterns to achieve more complex results. Like two nested **for**s.

So I want building blocks, something that guarantees correctness of implementation that can be used to build upon.

For example **map**(**function** $\langle A(B) \rangle$, **collection** $\langle B \rangle$)

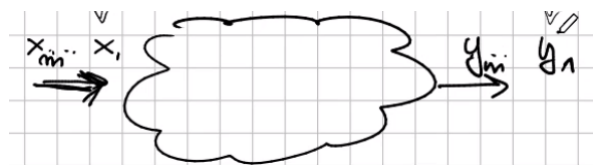
So build a bag of tools that we can combine to undertake common situations with good efficiency, speedup, scalability. . .

0.5.2 Stream Parallel Patterns

Stream of data, flowing in time. In data parallel we process a data collection, while in stream parallel we don't have data appearing all at the same time. So stream as a collection with items appearing at different times. We want to take the single items and try to process in parallel, parallel execution of f over different items of the stream.

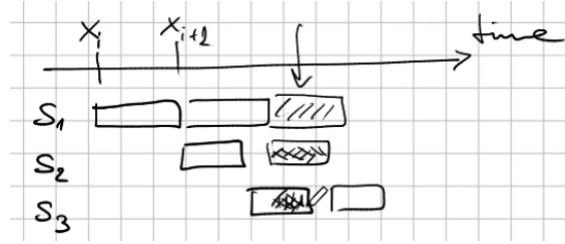
What matters is T_S

Pipeline

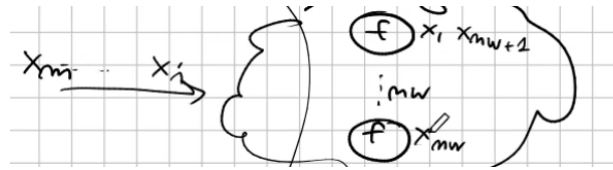


Inside the pipeline we have $f_1 \rightarrow f_2 \rightarrow \dots \rightarrow f_k$ with each f_i corresponding to a phase, with f_i taking input from f_{i-1} .

So $x_i \mapsto f_1(x_i) \mapsto f_2(f_1(x_i)) \mapsto \dots$, and the parallelism is in the computation of different phases of different items (much like what we've seen with the CPU fetch-decode-execute pipeline).



Farm We have a number nw of instances of the same function f each processing one single item.



We have no interference between computations of x_i, x_j with $i \neq j$, no need for synchronization.

$T_C \simeq m \cdot T_S$ and T_S in sequential is $\simeq L$ this means that I can try to decrease the latency by decreasing the stages in the pipeline or the workers in a farm.

Two Tier Model Let's assume a grammar of patterns.

Pat = Seq(f) | DPP | SPP

DPP = Map(Pat) | Reduce(Pat)

SPP = Farm(Pat) | Pipe(Pat, Pat)

This defines parallel computations and we aim at assuring that this can be done, a way of implementing this. So

Map(Pipe(Seq(f), Seq(g)))

can be a data parallel computation where on the single item we compute a 2-stages pipeline of f and g . But each element of the map is given to a single pipeline, or each pipeline receives a single item, so the stream parallelism is useless.

Farm(Map(Seq(f)))

Here we have a stream of items that will be processed by a Farm, each item splitted by Map and processed. This can deliver the result faster and access the next item.

So Data parallel with Stream parallel is not very good, Stream parallel with Data parallel is better: **two tier model**. So we have an initial part of the pattern which is Stream Parallel, the second part is Data Parallel and eventually the last stages (the leafs of the tree) which are sequential.

Parallel Design Patterns Also called **algorithm skeletons**: programming abstraction that model some pattern. The programmer has a framework, libraries, languages and that includes algorithm abstractions.

0.5.3 Composing

These are building blocks, so we can **compose** them. Let's see how that works and what are the expected performances.

Pipeline

We have a number k of stages for m tasks: $\text{Pipeline}(s_1, \dots, s_k)$ meaning that this is a composition yielding $s_k(\dots s_1()) \dots$

$$\text{Input stream} \longrightarrow s_1 \rightarrow \dots \rightarrow s_k \longrightarrow \text{Output stream}$$

With each \rightarrow being a stream $s_i \rightarrow s_{i+1}$ and each s_i taking input from s_{i-1} .

The latency of the pipeline is the sum of the latencies of the stages

$$L(\text{Pipeline}(s_1, \dots, s_k)) = \sum_{i=1}^k L(s_i)$$

We do not consider the time required to pass input to the next stage, t_{comm} , which would be based on size, nature of the computation. . .

The steady state is when all the stage are "filled": the longest of the stages will dominate the service time T_S

$$T_S(\text{Pipeline}(s_1, \dots, s_k)) = \max_{i=1}^k \{T_S(s_i)\} = \max_{i=1}^k \{L(s_i)\}$$

The completion time is T_C

$$T_C(\text{Pipeline}(s_1, \dots, s_k)) = \left(\sum_{i=1}^k L(s_i) \right) + (m-1) \max_{i=1}^k \{L_i\}$$

and when $m \gg k$ we can approximate it with

$$T_C = mT_S$$

because the number of tasks required, the "base of the parallelogram", will dominate the number of tasks, the "height of the parallelogram".

Boundary Conditions We have to take into account the interarrival time T_A , time spent to get another item from the input stream, and the interdeparture time T_D , the time spent to get another item into the output stream.

Let's suppose that $L(s_i) = i$ seconds, so ideally $T_S = k$ seconds: we process 1 item each k seconds. If $T_A > L(s_i)$ we have to wait the second item when I finish processing the first, same thing for the next after the second: the interarrival time looks like an interstage between s_{i-1} and s_i .

T_D behaves at the same way: we have to wait that T_D finishes before giving it out output, behaving like an interstage. So the previous behavior, analyzed before, happens $\Leftrightarrow T_A < T_S$ and $T_D < T_S$ **something that we have always to take into account.**

Farm

Sometimes we denote as $\text{Farm}(s, nw)$, otherwise we omit the number of workers and simply write $\text{Farm}(s)$.

We assume to know L_w and T_w of the workers. We have some scheduler (**emitter** E) that distributes the items from the input stream to the workers, and a gatherer (**collector** C) that gets the results from the workers and delivers them to the output stream. Those can simply be data structures: queues, for example.

$$L(\text{Farm}(s, nw)) = t_E + L_w + t_C$$

This can appear as a pipeline of three stages, where the Emitter produces to the second stage (the workers) which produce to the third stage (the Collector)

$$T_S(\text{Farm}(s, nw)) = \max \left\{ t_E, \frac{T_w}{nw}, t_C \right\}$$

We assume to have m tasks

$$T_C(\text{Farm}(s, nw)) = m \cdot T_S(\text{Farm}(s, nw))$$

With boundary conditions

$$T_S = \max \{T_s(\text{Farm}()), T_A, T_D\}$$

What if we want to achieve a given performance? Compute a nw suitable to achieve a wanted performance by inverting the very same formulas.

With a target $T_S = T_A = 1s$

$$T_S = 1s = \max \left\{ t_E, t_C, \frac{10s}{nw} \right\}$$

But t_E, t_C are negligible so

$$\frac{10s}{nw} = 1s \Rightarrow nw = 10$$

Map

We consider it as made by three phases:

Split: divide the collection into sub collections, a **set of subcollections**

Map: compute the **set of subresults**

Merge: produce the final **collection results** (usually in the same shape as the input collection)

With m dimension of the collection and t_f to compute the function of the map

$$L(\text{Map}) = \frac{m \cdot t_f}{nw} + t_{split} + t_{merge}$$

t_{split} and t_{merge} are non-negligible in distributed architectures, but are negligible in shared-memory architecture.

$$T_S(\text{Map}) = L(\text{Map})$$

No concept of T_C because the map is applied to a single collection.

If I have multiple collections we consider a splitter node as s_1 , map (t_f) as s_2 and merger node as s_3 , giving

$$T_S = \max\{t_{split}, t_{map}, t_{merge}\}$$

$$t_{map} = \frac{m}{nw} t_f$$

Reduce

From a vector we want to output the sum: a scalar from a collection. We split in t_{split} , then each of the nw workers applies the function \oplus to its subcollection in $\frac{m}{nw}t_{\oplus}$ and finally we merge in $nw \cdot t_{\oplus}$ because we have to compute \oplus over all the nw subresults.

$$L = t_{split} + \left(\frac{m}{nw} - 1\right)t_{\oplus} + (nw - 1)t_{\oplus}$$

$$T_S = L$$

With multiple collections, same argument as before

$$T_S = \max\{\dots\}$$

But the computation can be organized in a logarithmic tree, too. We would have $\log_2(m)$ phases each with half the activities of the previous phase.



But we have efficiency 1 only in the first phase, then half, then one fourth...

$$L = t_{split} + \lceil \log_2 m \rceil t_{\oplus}$$

provided that $nw \geq \frac{m}{2}$. If we use threads, the split phase is just telling the threads what they have to do, then we merge with a simple loop.

Stencil

For example computing the average of three neighbor items in a vector, with necessary boundary conditions.

Split phase that produces for example two halves of the vector: when computing the right extreme of the first half, I need to add the first item of the second vector, same with the left extreme of the second half: we have some shared positions. No problem when reading, the problem if we write the shared position. We can use a buffer for the write, using the old values only for reading and swapping the vectors. Or we can use a small buffer to host modified values the neighborhood.

$$L(\text{Stencil}) = t_{split} + \frac{m}{nw}t_{stencil} + t_{merge}$$

Where $t_{stencil}$ includes buffer management with the second solution and t_{merge} includes swapping the buffers in the first solution.

Composition Given a Pipeline(s_1, s_2, s_3) with s_i sequential, s_2 may be data parallel (map), m stream items and each being a vector of k items.

$$L = L_1 + L_2 + L_3$$

$$T_S = \max\{L_1, L_2, L_3\}$$

$$T_C \simeq m \cdot T_S$$

s_2 is sequential but can be turned into a map, takes L_2 so I can imagine $t_f \simeq \frac{L_2}{k}$

$$T_S(\text{Pipeline}) = \max \left\{ L_1, L_3, \frac{L_2}{nw_{map}} \right\}$$

s_2 is the slowest stage, we can use a `Farm(s_2, new_{farm})`, but in this case latency stays the same, while using the `Map` the latency decreases.

Overhead related to memory allocations New objects are created when instantiating inputs and results, for example. This requires allocating memory in the heap, with corresponding `mallocs` and `frees`. So we may need to be smarter, with solutions that work on thread-level memories, small heaps where to allocate objects and releasing when no more needed.

jmalloc library Used in BSD systems, FireFox and Facebook among others.

It manages **chunks of memory** called **arenas**, distributed in a round robin way per thread with each thread using one arena. A metaarena is used a common place.

Arenas A_1, A_2, \dots, A_k are assigned in round robin to th_1, th_2, \dots . When some data comes from an arena, and we free that data we free the original arena not the local one. `jmalloc` has its own API, but other than that it uses the classical API: `malloc` and `free`, same as `stdlib`.

Taking a normal program `a.out`, with `./a.out p1 ...pk` we will go with `malloc` and `free` of the `stdlib`. If we prefix with `LD_PRELOAD=libjmalloc.so ./a.out p1 ...pk` then `malloc` and `free` will be loaded from the `jmalloc` library.

0.6 Load Balancing

Giving the same amount of work to all the cores involved. Even if a single thread takes longer than all the other, we wait that it fishes so we have a lot of empty time in the other threads. The efficiency lowers a lot, and poor speedup too.

One of the reasons could be that the computation per se is unbalanced.

0.6.1 Static Techniques

Related to the usage of different splitting policies.

Chunk policy Take the vector and split into adjacent parts

Cyclic policy First item to first thread, second to second thread...e.g. with two threads: first to th_1 , second to th_2 , third to th_1 , fourth to th_2 ...

Mix A cyclic distribution of blocks. Split into blocks and assign the first to a thread, the second to the next... following the cyclic policy.

0.6.2 Dynamic Techniques

We can do much more, adapting to the situation giving more things to do to the thread that so far have done less.

Autoscheduling Threads are not assigned a block/item or a distribution, but each threads *asks* for something to be computed. Some code like

```
while (more work to do) {
    ask work
    compute
    deliver result
}
```

The threads that gets longer tasks stops asking for more tasks for a while, and more tasks will be executed by the other threads. When tasks are almost finished, it may happen that some thread gets the "last" long task still taking longer than all the other threads. But it's not as impactful as stated before, as it's the worst case.

In general, prefetch $k \simeq 2$ or 3 tasks

Job Stealing Bunch of tasks, cyclic static assignment. With nw threads and m tasks, each thread gets $\frac{m}{nw}$ tasks. With job stealing, the thread that has finished its tasks and perceives that there's more to compute, steals a task from another thread. Problems: "size" of the steal, synchronize accesses, who to target... the solution is a random policy: threads that finishes their own assigned task steal a random number $\in [0, nw]$ and steal that number of tasks.

Autoscheduling + variable size chunks

Template Based Implementation For each pattern (pipe, farm, map, stencil...) we have templates:

for each target architecture (multicore, cloud...)

activity graph

performance modelling, also a way to have an idea of what we can achieve in terms of performance

Let's analyze a template for a pipeline on a shared memory multicore machine (`template(pipeline, SMmulticore)`):

$\longrightarrow \text{thread} \rightarrow \text{thread} \rightarrow \dots \rightarrow \text{thread} \longrightarrow$

With the \rightarrow being communication channels (queues). Modelling the speedup(m) $\simeq m$ with m being the number of stages, so a linear speedup, if and only if $\max\{t_i\} \simeq t_j$ so the times of each stage are more or less the same.

But we can have another template

$x_m \dots x_2 x_1 \longrightarrow \text{queue}$

Where in the queue we put "compute f over x_i " as $\langle f, x_i \rangle$ for each i . Each worker gets $\langle f, x \rangle$ and puts in the queue $\langle g, f(x) \rangle$. For $\langle g, x \rangle$ returns $\langle h, g(x) \rangle$ and for $\langle h, x \rangle$ returns $\langle \text{end}, h(x) \rangle$.

So the queue will have different types of tasks: the input tasks are assigned to f but the intermediate tasks are assigned to g or h : it's a pipeline. Of course, the end tasks means outputting on the output queue.

A problem: the queue is a bottleneck. We can use some techniques: set of queues, local queues... Also the ordering, but i can keep the index i for each stage and use it at the end to keep in a buffer and output in the correct order.

With modeling provided sufficient T_A , the speedup can be proportional to nw and not m .

Another template is a round-robin scheduler that assigns to some workers that deliver results to sorter with respect to the input order. The scheduler gives $\langle i, x_i \rangle$ and the output is $h(g(f(x_i)))$ ordered on i . Pipeline. Uses $nw + 2$ concurrent activities, with a queue for each worker plus a queue for the sorter. With T_A low, we have speedup circa nw .

Changing architecture, from shared memory multicore to a clustered workstation. I can use any template ported to a cluster. For example

$\text{workstation}_1 \rightarrow \text{TCP/IP} \text{ workstation}_2 \dots$

The speedup is circa the number of stages. But we can also have a number of workers in each workstation, with a scheduler and a merger per each. A kind of composed template, the third inside the stages of the first.

$$T_S \simeq \frac{\max\{T_i\}_{nw}}{nw}$$

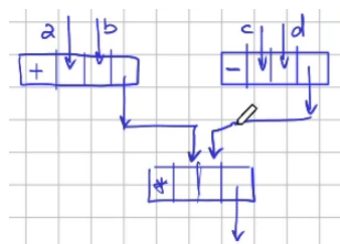
The overhead is in the scheduling and merging parts of each stage. The scheduling is necessary, but in the ordering part we sort data on the input order, but we can do that just at the end of the last stage.

Macro Data Flow Implementation With Data Flow we denote the operations where the order of computations are dictated by the flow of data encountered by the program counter.

So we have nodes that are composed of $\langle \text{operator}, \text{variables}, \text{output} \rangle$. For example,

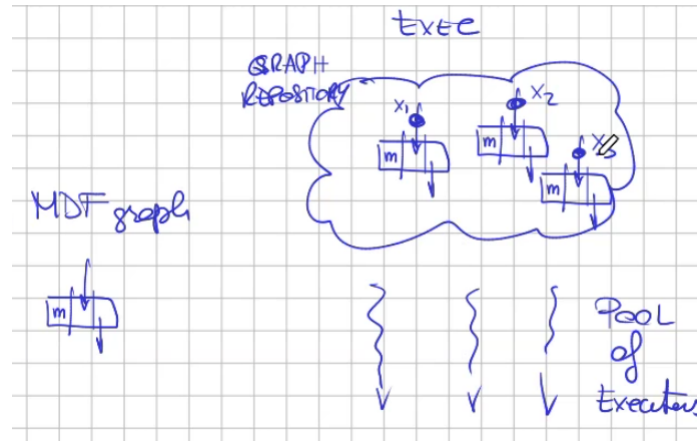
$$(a + b) \cdot (c - d)$$

can be represented as



We have tokens on the inputs, that represent when data are available. When all data is available, the node becomes "fireable". This technique can be used to implement parallel patterns, Macro because instead of considering operations as nodes with functions an input/output nodes we consider macros with full portions of code instead of primitive functions.

Let's consider a program `main.cpp` that uses some pattern internally from a pattern library. First step: from a pattern tree we compile the MDF (Macro Data Flow graph). We have a graph repository and a pool of executors: anytime I have some input data I create a copy of the graph in the repository with the tokens that represent the data.



Each executor executes a loop where they get a fireable instruction, compute and deliver the output tokens.

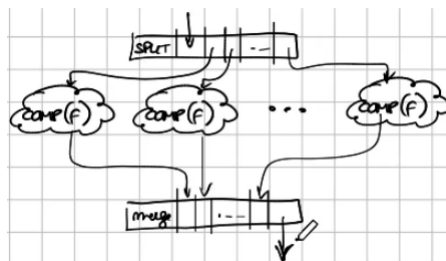
Compiler From the Pattern Tree it outputs the MDF graph.

$\text{Compile}(\text{Pipe}(f, g)) \rightarrow \langle f, \text{in}, A \rangle, \langle g, A, \text{out} \rangle$

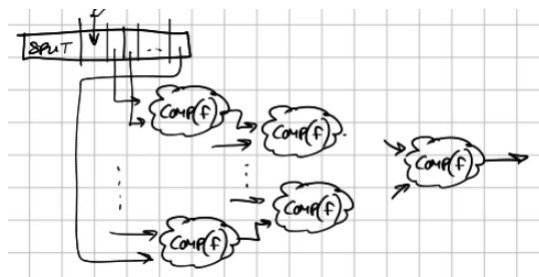
$\text{Compile}(\text{Seq}(f)) \rightarrow \langle f, \text{in}, \text{out} \rangle$

$\text{Compile}(\text{Farm}(f)) \rightarrow \text{Compile}(f)$ for each instance of input

$\text{Compile}(\text{Map}(f)) \rightarrow \text{Compile}(f)$ for each element of the input collection



$\text{Compile}(\text{Reduce}(f)) \rightarrow$



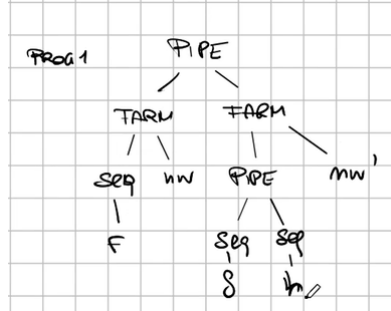
Each one of the graphs has one input token and one output token, mandatory to be able to compose the graph. The Farm parallelism disappears and third

Problems

- Contention over repository
- Token delivery
- Maintain the list of fireable instructions
- Reordering on the out stream

Refactoring Rules The grammar used to write our program is:

Pat = Seq() | Pipe(Pat₁, Pat₂) | Comp(Pat₁, Pat₂) | Farm(Pat, nw) | Map(Pat) | Reduce(Map)
 Prog \Rightarrow Pat : x



Whit **functional semantics** we mean **what is computed**. **Non-functional semantics** refers to **how** the result is computed.

So refactoring rules describe the equivalences in the **functional semantics**. So different performance, parallelism degrees...

Some examples:

$\text{Farm}(x, n) \equiv \text{Farm}(x, m)$ with $n \neq m$

$\text{Pipe}(x, y) \equiv \text{Comp}(x, y)$

$\text{Pipe}(\text{Farm}(x, -), \text{Farm}(y, -)) \equiv \text{Farm}(\text{Pipe}(x, y) -)$

How to figure out the better ones? We will consider a small set of rules:

Pipe introduction/elimination: $\text{Comp}(x, y) \equiv \text{Pipe}(x, y)$

Farm introduction/elimination: $x \equiv \text{Farm}(x)$

Map fusion: $\text{Map}(\text{Comp}(x, y)) \equiv \text{Comp}(\text{Map}(x, y))$

Pardegree change: $\text{Farm}(x, n) \equiv \text{Farm}(x, m)$ with $n \neq m$

The idea is to write a program (so a tree T_0) and apply the rules (via a tool) getting T_1, T_2, \dots all functionally equivalent to T_0 . Usually the goal is a low T_{S_i} . We take the produced trees, mapping each to its T_S and reducing to min to get the minimum T_{S_k} .

But maybe we compute worse trees. I can do another thing: taking T_0 trying to find the path in the possible trees. I apply rules and measure the metric (in this example, T_S). Then I don't go exploring all the subtrees, applying all applicable rules, but I just go one level down and pick the best one each time. We follow a path that always gives a better (or equivalent) performing solution. But this is **not possible**. With a lot of approximations and details ignored, I can figure out the best parallel architecture.

Normal Form Speaking of stream parallel computations. Optimizes the service time T_S (more precisely, the throughput).

We can derive the normal form from every pattern tree, only with stream parallel patterns

1. **Get the frontier**, set of all the sequential leaves in the tree, **left to right**.
2. $\text{Comp}(\text{frontier})$, so sequential composition in the order they appear.
3. $\text{Farm}(\text{Comp}(\text{frontier}))$

This process always aims at optimizing the parallel computation. Unless there are some particular reasons. We take the whole set of things to compute and put them together (1 and 2), meaning creating a chunk of code that do the work: **increase the grain of the computation**.

This optimizes the service time taking also into account resources.

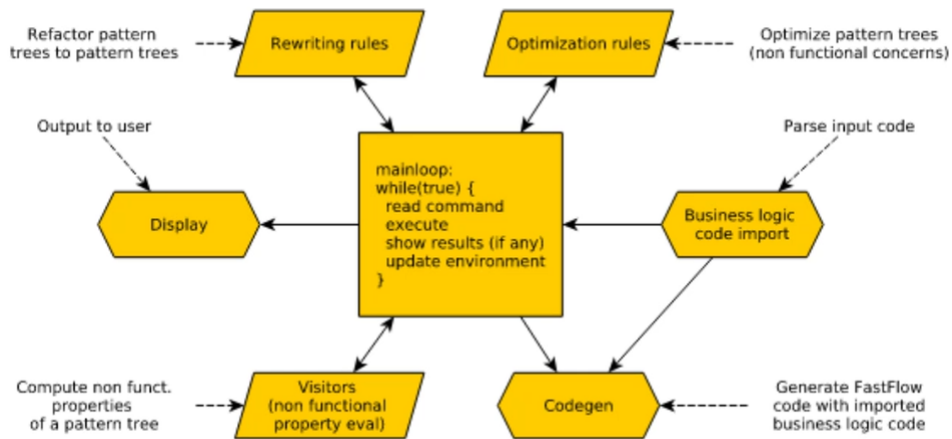
Performance Models Used to evaluate or to optimize.

Evaluate meaning that, having a tree, we evaluate a metric (e.g. T_S) visiting the tree from the leaves. For each leaf we're interested in the sequential time ($T_S = L$ for the sequential nodes). Then we go up one step visiting the parents: we have the parameters of the subtree and can compute T_S of the node given its subtrees. We can repeat until the root.

The visit of a tree is also a pattern, so if I take that pattern (visiting a tree from the leaves upward) passing the tree as a parameter I can get a number of different **visitors**: T_S in this case is a visitor. For example, for Visitor(# resources) we would visit the tree in the same way but using a different formula for computing the contribution of each node.

Optimize the parameters, for example the parallel degrees of the stages. In **rplsh** we used **optimize main with max resources**. How it's implemented? We take the tree, with Pipe in the root, same as before. We turn the tree with the nw figured out, but we have a certain number of resources. For example we want to shrink the number of resources to match the number of cores.

RPLShell



Rewriting Rules are the set of factoring rules.

For example, we have the facts that allow me to rewrite a given tree t as $\text{Farm}(t)$ or $\text{Pipe}(t_1, t_2)$ as $\text{Comp}(t_1, t_2)$.

Optimization Rules the set of rules that for example transform a $\text{Farm}(\text{Farm}(w))$ into $\text{Farm}(w)$ (because two farms linked are bad for performance), known as farmfarmopt , or the farmopt that is used when we have $\text{Farm}(w)$ with a certain T_S of the w and a certain T_S target of the farm in order to set the nw of the farm to $\frac{T_S}{T_{S,w}}$.

Visitors the elements that visit the tree upwards.

Autonomic Management of Non Functional Features In long running computations.

For example the classical variations in network load between night and day. Autonomic because Born from industrial processes management, changing behaviors of the systems. Different architectures of the controller:

MAPE Loop (Monitor, Analyze, Plan, Execute loop)

Execute activities via actuators, and the system has a set of actuator objects that can be invoked to change the behavior of the system (for instance changing nw). The system has sensors, too, that provide data to the monitor part.

We have to devise a strategy to be used. E.g.: if T_A drops, most likely I could just increase nw and viceversa if T_A increases I can drop nw . Being more clever: let's suppose that T_A goes from $T_{A_{\max}}$ to $T_{A_{\min}}$ in a sin fashion, when I'm in the minimum I could take time in taking a decision to increase nw while T_A is already increasing. Same for the maximum, I could spend time making the decision and always arriving late. So our strategy can be update: I can observe the changes and avoid situations of alternating opposite changes (in the example, the adding and taking away workers constantly).

To implement these strategies, I need **sensors** and **actuators**.

We need to observe the interarrival time T_A . Also need a data structure to observe the changes, by doesn't require sensors. These sensors would be in the emitter thread.

We need to increase or decrease nw , so the actuators must be able to achieve that. To add a worker, e.g. a thread, we need to know the queue for that worker, the collector...so the emitter sounds like a good place.

This is implemented in the **control program** of our manager. Very crude `if (condition) do actuator;`.

ECA Rules Set of **Event**, **Condition**, **Action** rules implemented in a rule system (e.g. jboss).

Event: **triggering action**, e.g. T_A change.
Not the event per se to be taken into account.

Condition: **predicates** on monitored values, internal state and events, that state whenever this particular rule has to be fired or not.

Action: **set of actions** on the system or the state that must be **executed after some triggering event when the condition holds true**.

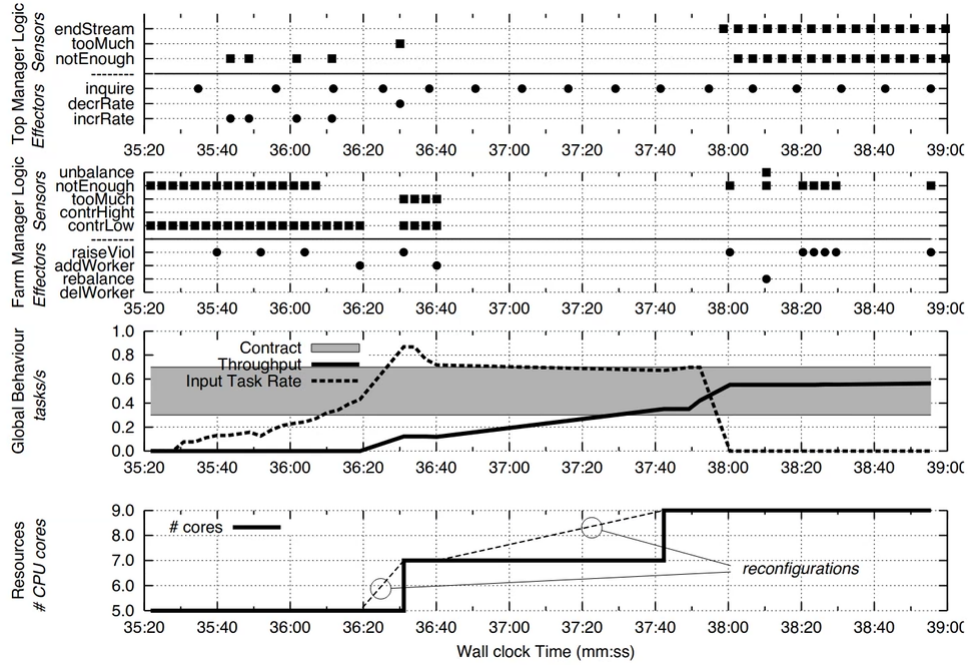
Example

	Event	Condition	Action
R_1	Change T_A	Less than the previous one	Adding a worker
R_2	Change T_A	More than the previous one	Removing a worker
R_3	Change T_A	Less than the previous one AND nw just decreased	nop , update T_A
R_4	Change T_A	More than the previous one AND nw just increased	nop , update T_A

Contracts

1. Propagate the "contract" top-down and try to implement it "best-effort"
2. Manage to keep contracts satisfied

We're taking into accounts performance, but often power consumption is more important. These are both non-functional features. If we take into account both, we could implement a manager for each. But the managers may take decision that contrast with each other. A solution is to build a single manager that takes into account both features, into a single MAPE loop. But this requires that both knowledge are in a single place, bad for the separation of concerns. Another solution is to have two managers, one for each feature, each with its MAPE loop, and the two can communicate: the plan phase inform each other in the hope to have authorization for a change that the execute part needs, if we don't get the authorization we go for a **nop**.



0.7 Vectorization

Very old technique. Given x, y, z as vectors of x_i, y_i, z_i from 0 to $n - 1$ and $\forall i \in [0, n - 1] z_i = f(x_i, y_i)$, implemented e.g. as a `for`. If the computation $f(x_i, y_i)$ is independent of the other loop iterations (that is, x_i is independent from all other $x_j, j \neq i$) then this is easily parallelized. A classical example is matrix multiplication:

$$\forall i \forall j \forall k c_{ij} = a_{ik} \cdot b_{kj}$$

2 steps: we need to have zeroes in the first positions and We have ways to load/store vectors to

Hardware designed specifically, vector processors. Registers that can be seen as vectors of tot bits numbers: 256 bits vector registers seen as 4 64 bits numbers, or 8 32 bits numbers and so on. With special commands, for example `VADD.I32 R1, R2, R3` meaning $R1 = R2 + R3$ with `I32` meaning 32 bit numbers with `I` index.

Conditions In `g++` we can explicitly ask with `-O3 -ftree-vectorize` and with `-fopt-info-vec-[missed/all]` ask what cannot be vectorize.

The conditions are:

- Need to know the number of iterations.

- `for` is ok, `while(c<k)` sometimes cannot be vectorized

- Cannot call external code in the loop body, functions or libraries.

- No conditional code.

- It'd require compiling two paths, so complicates things.

- No overlapping pointers.

`#pragma` Indications to the compiler

`#pragma GCC ivdep` tells the compiler whatever it thinks of the loop, to consider it as independent iterations.

`#pragma GCC unroll n` tells the compiler that the following loop should be unrolled `n` times. Useful for short loops.

So vectorization very useful for mathematical operations, such as differential equations. Requires to take a bit of care in the core to be able to vectorize. If we don't convince the compiler, we can use the `pragma`.

If we vectorize $\text{Farm}(f, nw)$, the speedup with the vectorized f with respect to the non-vectorized f is smaller. Because the vectorized code in general is faster, so $\text{non-vectorized Speedup}(nw) \geq \text{Vectorized Speedup}(nw)$.

But vectorization can radically change the T_S of the stages of a Farm, for instance, and introduce waiting times for workers. You don't change the sequential fraction, just the non-sequential fraction.

Libraries Libraries include operations used often, e.g. **Blas** (basic linear algebra system) or **Lapack** or **NKL**. For instance **blas** provides **gemm** for very optimized matrix multiplications. Use libraries because they are very optimized, exploiting all the possible optimizations.

GRPPI C++ library that implements common parallel patterns, using standard threads or other backends (e.g. **omp**, **tdd** or **fastflow**). The backend is only modeled by the **execution** parameter.

0.8 OpenMP

MPi Directed to clusters and workstations, multicores

CDA/OpenCL For GPU

OpenMP Parallel for, targets shared memory multicores (and GPU/FPGA). Very dated, since 1997 the version 1.0: no GPU, even multicores were not very popular.

At the beginning very simple API that provided launching threads in parallel and something related to the parallel for. Each version improved on that, introducing: tasks, target accelerators, depend clauses on tasks (to tell that a task must be executed only after the completion of another task), memory management, task reduction (among values of different tasks), task affinity...

When we speak about shared memory multicores, we speak of some situation where in the board we have more than one sockets with multicore in each socket.

Concepts

Directives, which OpenMP is based on. Directives, in C/C++ environment, are kind of pragmas which can be understood as portions of compiler

OpenMP is **compiler-based**

As pros:

Few lines of code are required to run parallel code, opposed to writing all the logic by oneself

Keep sequential code, means we can perform functional debugging and then adding parallel pragmas

As main con we need to rewrite the compiler, not entirely (such as the parser), but everything related to the usage of the pragmas must be generated at compile time.

E.g. the map seen in the GRPPI library is written at higher level but in the library, doesn't touch the compiler.

We have something used to **set up parallel activities** and something different used to **organize the parallel computation** using the parallel activities available.

#pragma omp parallel This must be placed before a statement, it tells the compiler that the following block must be executed in *nw* copies (threads) if and only if *nw* is the number of resources/cores of the current machine.

Most directives have clauses used to specify parameters. E.g. **num_threads(nw)** tells to use the variable *nw* instead of the predefined one (the number of cores).

```
1 #include <iostream>
2 #include <omp.h>
3
4 using namespace std;
5 int main(int argc, char** argv) {
6     int nw = 1;
7     if (argc != 1)
8         nw = atoi(argv[1]);
9     #pragma omp parallel num_threads(nw) // clause
10    {
11        auto id = omp_get_thread_num();
12        cout << "Ciao from id " << id << endl;
13    }
14    return(0);
15 }
```

#pragma omp single Specifies that the following statement/block is to be executed by just a single thread.

```
1 #include <iostream>
2 #include <omp.h>
3
4 using namespace std;
5 int main(int argc, char** argv) {
6     int nw = 1;
7     if (argc != 1) {
8         nw = atoi(argv[1]);
9         #pragma omp parallel num_threads(nw)
10        {
11            #pragma single // this will be executed by a single thread, just one enters the
12            // following line, use a block for more lines
13            cout << "Id " << omp_get_thread_num() << " of " << omp_get_num_threads() endl;
14        }
15    } else {
16        #pragma omp parallel
17        {
18            auto id = omp_get_thread_num();
19            cout << "Id " << id << " of " << omp_get_num_threads() endl;
20        }
21    }
22    return(0);
23 }
```

#pragma omp critical To execute the next code in mutual exclusion.

To use OpenMP Compile with flag `-openmp`.

The a.out compiled, when executed, uses the linked library which provides several things including a lock/mutex mechanism. So instead of the `#pragma omp critical` we can use `#pragma omp parallel` and then just the lock/unlock:

```
1 omp_lock_t lock;
2 omp_init_lock(&lock);
3 #pragma omp parallel
4 {
5     // code
6     omp_set_lock(&lock);
7     // mutex code
8     omp_unset_lock(&lock);
9     // code
10 }
```

With the code/unlock declared with respect to the OpenMP library.

Different code for different threads We can do something like

```
1 #pragma omp parallel num_threads(2)
2 {
3     if (omp_get_thread_num() == 0) {
4         // code for thread 1
5     } else {
6         // code for thread 2
7     }
8 }
```

This is **SPMD** code: single program multiple data.

Sections directive

#pragma omp sections: identifies a command which is filled by other **section** directives, each executed possibly in parallel

#pragma omp section

Example

```
1 #pragma omp sections
2 {
3     #pragma omp section
4     {
5         //code 1
6     }
7     #pragma omp section
8     {
9         //code 2
10    }
11 }
```

All the **section** may happen in parallel, how much is defined and can be inherited by other statements. For example I can have a **#pragma omp parallel** before the **#pragma omp sections**, giving the threads explained before (the number of cores as default, or as specified by **num_threads(nw)**).

Variables In most directives you can append clauses that specifies the behavior of variables. For example in parallel pragmas you can append the clauses

private(x) (non initialized local copy local to each thread)

firstprivate(x, y) (get a local copy initialized from previous value)

shared(y, z) (inherit value from the global environment)

lastprivate(x) (this one legal only in sections, not in parallel, local copy then last **section** copied to global environment, take care last one means in syntactic-fashion, the last syntactic section updating the variables copies to global environment)

#pragma omp for Most famous and used.

The default behavior is that the iterations, e.g. from $i = 0$ to $i = n - 1$, are made into nw chunks given each to a thread. Obviously we need independent iterations. There are others scheduling policies, specified with clauses appended to the **#pragma omp for**:

static the default one, with the option of specifying **chunksize(m)** specifying the number of items in each chunk, so $\frac{n-1}{m}$ chunks of m items, given to the threads in a round-robin fashion

dynamic where the threads ask for new chunks when they have finished, chunks are assigned on demand. Same of the static for other things

guided use a number of chunks smaller and smaller: large chunks in the first part of the array, then smaller chunks, then smaller and so on. Intended to be used for unbalanced computations: at the beginning we assign large chunks to each thread, but the threads may finish at different times and get smaller chunks to accomodate for load unbalancing

auto everything decided by the compiler

runtime keyword which inherits the scheduling policy from an environment variable which has to be defined before

You also have other clauses

nowait, removes the implicit barrier to wait all the threads, present in the parallel pragma

reduction([+,*,and,or], x)

#pragma omp task Kind of async, suggestion that if there are unused threads use to compute the task otherwise it's sequential code.

Can define taskgroups to execute in parallel and wait with the implicit barrier at the end.

You can append **tied** or **untied**: with the first anytime you assign/reassign the execution of this task then it would be on the same thread.

The taskloop provides something similar to a map, or a pragma for, but subjecting to the rules of the tasks: could be executed in parallel given enough resources ecc.

Example Sum of a vector with a reduce(+), base case being a vector of length 1 so return the value $v[0]$. Otherwise the recursive case is a vector of n positions: compute the half and split in $v[0, \frac{n}{2}]$ and $v[\frac{n}{2}, n]$ recur. So we need a task for the left half, a task for the right part, checking not to be in the base case.

```

1 int sum (int* restricted v, int start, int end) {
2     auto size = end - start;
3     auto mid = size/2;
4     int x, y; //shared for the subtasks, sum from left and right
5
6     if (size == 1) return v[start]; //base case
7
8     #pragma omp task shared(x) //to share the result with this "main" thread
9     {
10         x = sum(v, start, start+mid);
11     }
12     #pragma omp task shared(y) //same as befor
13     {
14         y = sum(v, end-mid, end);
15     }
16     #pragma omp taskwait //wait for the termination of the tasks, a "barrier"
17     //now both tasks have finished, we have both x and y
18     return x+y;
19 }
20
21 int main(int argc, char** argv) {
22     #pragma omp parallel num_threads(nw) //or from the CLI "export OMP_NUM_THREADS=nw"
23     //...
24     #pragma omp single //single or master
25     sum(v, 0, n-1);
26 }

```

Timing routines Also present.