Parallel and Distributed Systems

Federico Matteoni

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0.1 Introduction

Prof.: Marco Danelutto

Program Techniques for both parallel (single system, many core) and distributed (clusters of systems) systems. Principles of parallel programming, structured parallel programming, parallel programming lab with standard and advanced (general purpose) **parallel programming frameworks**.

Technical Introduction Each machine has more cores, perhaps multithreaded cores, but also GPUs (maybe with AVX support, which support operations floating point operations, **flops**, in a single instruction).
Between 1950 and 2000 the VLSI technology arised, integrated circuits which nowadays are in the order of 7nm (moving towards 2nm): printed circuits!
In origin, everything happened in a single clock cycle: fetch, decode, execute, write results in registers, with perhaps some memory accesses. The we had more complex control where in a single clock cycle we do just one of the phases (fetch *or* decode *or*...), like a **pipeline**. More components are used the higher the frequency but the more power we need to dissipate, and we're coming to a point were the power we need to dissipate is too much and risks to melt the circuit, so we're reaching a **physical limit** in chip miniaturization. But temperature and computing power do not go in tandem: computing power is proportional to the chip dimensions, while temperature is proportional to the area. So it's better to put more processors (**cores**) and let them work together rather than make a bigger single processor. An approach is to have few powerful cores and more less powerful cores (for example, in the Xeon Phi processors). Now, the processors follow this architecture, with the performance of a single core decreasing a bit with every generation but it's leveled by adding more cores.

Up to the 2000, during the single core era, code written years before will run faster on newer machines. Now, code could run slower due to not exploiting more cores and the decreasing in performance of the single core.
With accelerators the situation is even more different: for example GPUs, accelerator for graphics libraries, with their own memory and specialized in certain kinds of operations. This can require the transfer of data between the accelerator's memory and the main memory, so the architecture of the accelerator is impactful on the overall performance.

0.2 General Paradigms of Parallel Programming

Parallelism Execution of different parts of a program on different computing devices at the same time. We can imagine different flows of control (sequences of instruction) that all together are a program and are executed on different computing devices. Note that more flows on a singe computing device is **concurrency**, not parallelism.

Concurrency Similar concept: things that *may* happen in parallel respecting the ordering between elements.

Computing Devices

Threads, implying shared memory

Processes, implying separated memories

GPU Cores

Hardware Layouts on a FPGA (Field Programmable Gate Array)

Sequential Task A "program" with its own input data that can be executed by a single computing entity

Overhead Actions required to organize the computation but that are note included in the program. For example: time spent in organizing the result. Basically, time spent orchestrating the parallel computation and not present in the sequential computation.

Speedup Fundamental things that we're looking for, it's the ratio between the sequential time and the parallel time.

$$\text{SpeedUp} = \frac{\text{Sequential time}}{\text{Parallel time}}$$

Assuming the best sequential time.
We have a slightly different measure, too

$$\text{Scalability} = \frac{\text{Parallel time with 1 computing device}}{\text{Parallel time}}$$

Stream of tasks In some cases it's not important considering just one computation but may be useful considering more computations and we want to optimize a set of tasks.

Example: Book Translation With $m = 600$ pages, for example. Let's assume I can translate a page in $t_p = 0.5$h. The sequential task is: take the book and spend time until I can deliver the translated book. The time is circa $m \cdot t_p = 300$h.
In parallel, ideally every page can be translated independently so I can split the book in two pieces of $\frac{m}{2}$ pages each (overhead), giving each half to a person. Both can translate at the same time, so ideally the time required is $\frac{m}{2} \cdot t_p$ for each, producing the translated halves. At this point I get the halves and produce the translated version (overhead). Ideally the time require is more or less $\frac{m}{2} \cdot t_p$, with "more or less" given by the time spent in splitting the book and reuniting the two halves. So the exact time is $T = T_{split} + \frac{m}{2} \cdot t_p + T_{merge}$.
What if the two person have different t_ps? For example $t_1 > t_2$. When a translator finishes, it spends some time synchronizing its work with me. With nw "workers" (translators, in this instance) $T = nw \cdot T_{split} + nw \cdot T_{merge} + \frac{m}{nw} T_{work}$ with $nw \cdot T_{split}$ time spent delivering work to each worker and $nw \cdot T_{merge}$ time in merging each result.

Init is the time where every worker has work to do, and finish is the time where the last worker finished working. So the exact formula is with a single T_{merge}.
So $\frac{m}{nw} T_{work}$ is the time that needs to happen, found in the sequential computation too, whereas the other two factors are **overhead**.

$$\text{SpeedUp} = \frac{\text{Best sequential time}}{\text{Parallel time}}$$

but the parallel time depends on the nw so

$$\text{SpeedUp}(nw) = \frac{\text{Best sequential time}}{\text{Parallel time}(nw)} \simeq \frac{\cancel{m} \cdot \cancel{t_p}}{\frac{\cancel{m}}{nw} \cdot \cancel{t_p}} = nw$$

This not taking into account the overhead. It's a realistic assumption because usually the time splitting the work is very small. But we have to take into account that, in case it's not negligible.

$$\text{SpeedUp}(nw) = \frac{m \cdot t_p}{\frac{m}{nw} \cdot t_p + \underline{nw \cdot T_{split} + T_{merge}}}$$

Example: Conference Bag $T_{bag} = t_{bag} + t_{pen} + t_{paper} + t_{proc}$ and with m bags we have $T = m \cdot T_{bag}$
We could build a pipeline, a building chain, with 4 people and each person does one task:

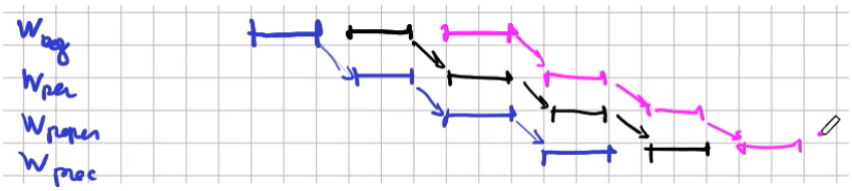
One takes the bag and gives to the next

One puts the pen into the bag and passes it

One puts the paper into the bag and passes it

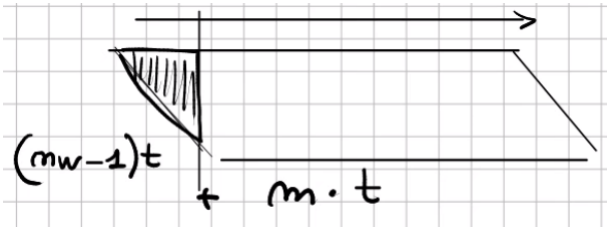
One puts the proceedings into the bag

So $w_b, w_{pen}, w_{paper}, w_{proc}$ workers. When the first worker has passed the bag, it could begin taking the next bag. Same for the others.



So in sequential we have $m \cdot (t_{bag} + t_{pen} + t_{paper} + t_{proc})$, and in parallel per 1 bag we have $t_{bag} + t_{comm} + t_{pen} + t_{comm} + t_{paper} + t_{comm} + t_{proc} + t_{comm}$ with t_{comm} spent passing the bag from one to the other, so total of $m \cdot T_{seq} + m \cdot t_{comm}$. But that's not correct, because we work in parallel: ideally we have a parallelogram of $m \cdot (t_{proc} + t_{comm}$ base, and we require $t_{bag} + t_{pen} + t_{paper} + 3 \cdot t_{comm}$ time to get up to speed and "fill the pipeline". But this required time is negligible, and in the end the overall time is given by the base of the parallelogram.

Pipeline With m tasks and nw stages, with the completion of the stage i required in stage $i+1$. So the output is $f_{nw}(f_{nw-1}(\dots f_1(x_i)\dots))$. With t time required for each stage.



We spend $(nw-1)t$ to get the last stage working and $m \cdot t$ time spent by the last stage to complete all the tasks.

$$T_{par}(nw) = (nw-1) \cdot t + m \cdot t$$

$$\text{SpeedUp}(nw) = \frac{(nw \cdot t) \cdot m}{(nw-1) \cdot t + m \cdot t}$$

So the higher the m is, the lower is the impact of the time required to get up to speed. So $m >> nw \Rightarrow T_{par}(nw) \simeq m \cdot t$

Throughput Tasks completed per unit of time.

0.3 Measures

On one side we can have more speed with more resources (computing devices). On the other side we can use more complex applications, with more resources. For example more precise computations, so extra resources not for improving the time but to improve the quality of the computations.
Finally, we could aim at computing results with less energy thanks to parallelism. This is a recent perspective on parallelism.

We've seen the $\text{SpeedUp}(n) = \frac{T_{seq}}{T_{par}(n)}$, where the plot has to lie below the bisection of the cartesian graph.

0.3.1 Base Measurements

Latency L Measure of the wall-clock time between the start and end of the single task.

Service Time T_s It's related to the possibility of executing more tasks. It's the measure of the time between the delivery of two consecutive results, for example between $f(x_i)$ and $f(x_{i+1})$
Even if x_i and x_{i+1} arrive at the same time, f would still be computing $f(x_i)$ so it'll start computing $f(x_{i+1})$ when it has finished.

Completion Time T_c The latency related to a number of tasks. $T_c = L \cdot m$ for $x_m, \dots, x_1$ inputs to a sequential system.
With a parallel system, instead, we have $T_c \simeq m \cdot T_s$.

Example A 3 stage pipeline, with each node being sequential and with latency L_i for node i.
At t_0 the first stage N_1 gets the first tasks and computes it in L_1, then N_2 computes in L_2 and N_3 computes in L_3 so a total of $t_0 + L_1 + L_2 + L_3$.
When the pipeline is filled, T_s is dominated by the longest L_i, so $T_s = \max\{L_1, L_2, L_3\}$ and $T_c = \sum L_i + (m-1)T_s$
If m is large with respect to n = number of stages, the "base of the parallelogram" would be very long, so $m >> n \Leftrightarrow T_c = m \cdot T_s$

0.3.2 Derived Measurements

SpeedUp

$$\text{SpeedUp}(n) = \frac{T_{seq}}{T_{par}(n)}$$

Could be latencies, service times... depending on what we want to measure the speedup of.

Scalability

$$\text{Scalability}(n) = \frac{T_{par}(1)}{T_{par}(n)}$$

Efficiency

$$\text{Efficiency}(n) = \frac{\text{Ideal parallel time}(n)}{T_{par}(n)} = \frac{\frac{T_{seq}}{n}}{T_{par}(n)} = \frac{T_{seq}}{n \cdot T_{par}(n)} = \frac{\text{SpeedUp}(n)}{n}$$

Measures the tradeoff between what you gain with the speedup and the cost of the speedup.

Throughput

$$\text{Throughput} = \frac{1}{T_s}$$

Amdahl Law Taken the total time of a computation, T_{seq}, it can be divided into something that can and something that cannot be computed in parallel (for example, dividing the book is a sequential activity). So we can say that T_{seq} = serial fraction + parallel fraction and the **serial fraction cannot be parallelized**. $f \in [0,1] \mid f \cdot T_{seq}$ is the serial fraction.

$$T_{seq} = f \cdot T_{seq} + (1-f) \cdot T_{seq}$$

The parallel fraction can be splitted between the workers, but we would have to compute the serial fraction too. By splitting more and more and more, we have that

$$\lim_{n \to \infty} T_{par}(n) = f \cdot T_{seq}$$

$$\text{SpeedUp}(n) = \frac{T_{seq}}{f \cdot T_{seq}} = \frac{1}{f}$$

So we have a very low upper bound on the achievable speedup. This is referred to as **strong scaling**: strong meaning using more resources to get the computation faster.

Gustaffson Law

$$\text{SpeedUp}(n) = N - S \cdot (N-1)$$

With S being the serial fraction. This comes from the fact that we're considering a different perspective: Gustaffson assumes that the computation increases with the parallelism, something that's called **weak scaling**, getting the speedup from using more computational devices, using more data.

Cores In modern computers, we have a main memory (slow), a disk (even slower) and the memory is connected to at least 3 levels of cache. At the bottom we have some cores (4, 8...), each one has its own level 1 cache (usually split in data and instruction cache).
With an activity with a working set that fills the cache, in case of strong scaling splitting the computation across cores we process less data per core because the size of the problem is the same.
With weak scaling, we assume that the data increases so by using more cores we process the same data on all cores but the data grows so we could have extra overhead because of the working set size.
We will have patterns of parallel computation that differentiate in how we process the data.

Application as Graphs The applications can be seen as graphs of sequential nodes with dependencies. The maximum speedup is the work over the span, because in every case I need to go from the first to the goal node. I take the longest one because at least the longest path must be computed, and all the rest can be done in parallel and I assume to have enough resources to compute the rest in the time of the span.
We can use this model

0.4 Technicalities

Examples A simple program that "translates" an ASCII file by transforming lower letters into capital letters. We split the text into $n_{workers}$ parts, we wait for all the threads to finish and then verify the performances.
The translator is:

```
#include <string>

char translate_char(char c) {
  if (islower(c))
    return(toupper(c));
  else
    return(tolower(c));
}
```

0.4.1 Threads

We used to write instructions sequentially. At a given point now we `fork` another flow of computation: we get two flows that are executed together **in the same address space**, so the new thread inherits all the memory of the original thread.

Concurrence `todo`

0.5 Patterns

Computations with particular shapes and semantics that can be understood and implemented depending on the situations, not linked to languages and technicalities. Patterns are a useful concepts, allow programmers to reuse experience of other programmers an not reinventing the wheel.
Parallel patterns:

Data parallel

Stream parallel

The same patterns can be referred with different names.

0.5.1 Data Parallel Patterns

Parallelism comes from data: we split the data in pieces, compute a set of results that can be combined into a single final result. The book translation examples is a data parallel pattern. What matters is L.
The general pattern is:

Decomposition

Partial results

Recomposition

Map Pattern Also called applytoall:

$\forall$ item of the collection

Function f(item)

$\forall$ f(item) $\rightarrow$ isomorphic collection

Reduce Pattern Also called fold:

$\forall$ item of the collection

$\oplus(x, y)$

$\oplus(\oplus(a, b), \oplus(c, d))$

Stencil Pattern

In partially overlapping position, e.g. of a matrix or an image

Function f(item)

$\forall$ f(item) $\rightarrow$ isomorphic collection

Different kind of problems: overlapping positions will yield the new value, so we have to account for that.

"Google" mapreduce

$\forall$ items

f(item) + $\oplus$(item,item)

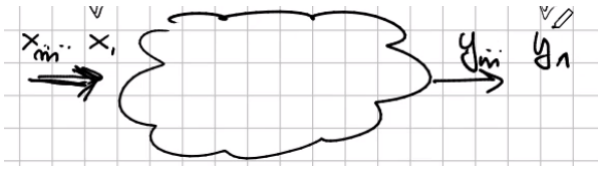
Item

What I apply to each item is an f that maps to $\langle$key, value$\rangle$ and $\oplus$ applies the sum to each value.
For example in a document, $f(\text{word}) = \langle w, 1\rangle$ and $\oplus(\langle w_k, v_1\rangle, \langle w_k, v_2\rangle) = \langle w_k, \oplus(v_1, v_2)\rangle$
"The lesson given by the professor", f will output $\langle$the, 1$\rangle, \langle$lesson, 1$\rangle \ldots$ and $\oplus$ will for example output $\langle$the, 2$\rangle$.
We can apply the map function over all the data distributed in various databases, for example.
This is `map(f)` and `reduce(`$\oplus$`)`, but we want something like `map(reduce(`$\oplus$`))`. Combining elementary patterns to achieve more complex results. Like two nested `for`s.
So I want building blocks, something that guarantees correctness of implementation that can be used to build upon.
For example `map(function<A(B)>, collection<B>)`
So build a bag of tools that we can combine to undertake common situations with good efficiency, speedup, scalability...

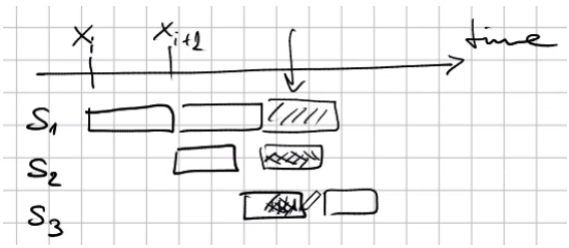
0.5.2 Stream Parallel Patterns

Stream of data, flowing in time. In data parallel we process a data collection, while in stream parallel we don't have data appearing all at the same time. So stream as a collection with items appearing at different times. We want to take the single items and try to process in parallel, parallel execution of f over different items of the stream.
What matters is T_S

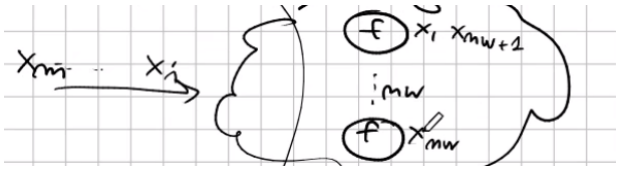
Pipeline



Inside the pipeline we have $f_1 \to f_2 \to \ldots \to f_k$ with each f_i corresponding to a phase, with f_i taking input from f_{i-1}.
So $x_i \mapsto f_1(x_i) \mapsto f_2(f_1(x_i)) \mapsto \ldots$, and the parallelism is in the computation of different phases of different items (much like what we've seen with the CPU fetch-decode-execute pipeline).



Farm We have a number nw of instances of the same function f each processing one single item.



We have no interference between computations of x_i, x_j with $i \neq j$, no need for synchronization.

$T_C \simeq m \cdot T_S$ and T_S in sequential is $\simeq L$ this means that I can try to decrease the latency by decreasing the stages in the pipeline or the workers in a farm.

Two Tier Model Let's assume a grammar of patterns.

Pat = Seq(f) | DPP | SPP

DPP = Map(Pat) | Reduce(Pat)

SPP = Farm(Pat) | Pipe(Pat, Pat)

This defines parallel computations and we aim at assuring that this can be done, a way of implementing this. So

Map(Pipe(Seq(f), Seq(g)))

can be a data parallel computation where on the single item we compute a 2-stages pipeline of f and g. But each element of the map is given to a single pipeline, or each pipeline receives a single item, so the stream parallelism is useless.

Farm(Map(Seq(f)))

Here we have a stream of items that will be processed by a Farm, each item splitted by Map and processed. This can deliver the result faster and access the next item.
So Data parallel with Stream parallel is not very good, Stream parallel with Data parallel is better: **two tier model**. So we have an initial part of the pattern which is Stream Parallel, the second part is Data Parallel and eventually the last stages (the leafs of the tree) which are sequential.

Parallel Design Patterns Also called **algorithm skeletons**: programming abstraction that model some pattern. The programmer has a framework, libraries, languages and that includes algorithm abstractions.

0.5.3 Composing

These are building blocks, so we can **compose** them. Let's see how that works and what are the expected performances.

Pipeline

We have a number k of stages for m tasks: Pipeline$(s_1, \ldots, s_k)$ meaning that this is a composition yielding $s_k(\ldots s_1() \ldots)$

$$\text{Input stream} \longrightarrow s_1 \to \ldots \to s_k \longrightarrow \text{Output stream}$$

With each $\to$ being a stream $s_i \to s_{i+1}$ and each s_i taking input from s_{i-1}.
The latency of the pipeline is the sum of the latencies of the stages

$$L(\text{Pipeline}(s_1, \ldots, s_k)) = \sum_{i=1}^{k} L(s_i)$$

We do not consider the time required to pass input to the next stage, t_{comm}, which would be based on size, nature of the computation...
The steady state is when all the stage are "filled": the longest of the stages will dominate the service time T_S

$$T_S(\text{Pipeline}(s_1, \ldots, s_k)) = \max_{i=1}^{k}\{T_S(s_i)\} = \max_{i=1}^{k}\{L(s_i)\}$$

The completion time is T_C

$$T_C(\text{Pipeline}(s_1, \ldots, s_k)) = \left(\sum_{i=1}^{k} L(s_i)\right) + (m-1)\max_{i=1}^{k}\{L_i\}$$

and when $m >> k$ we can approximate it with

$$T_C = mT_S$$

because the number of tasks required, the "base of the parallelogram", will dominate the number of tasks, the "height of the parallelogram".

Boundary Conditions We have to take into account the interarrival time T_A, time spent to get another item from the input stream, and the interdeparture time T_D, the time spent to get another item into the output stream.
Let's suppose that $L(s_i) = i$ seconds, so ideally $T_S = k$ seconds: we process 1 item each k seconds. If $T_A > L(s_i)$ we have to wait the second item when I finish processing the first, same thing for the next after the second: the interrarival time looks like an interstage between s_{i-1} and s_i.
T_D behaves at the same way: we have to wait that T_D finishes before giving it out output, behaving like an interstage.
So the previous behavior, analyzed before, happens $\Leftrightarrow T_A < T_S$ and $T_D < T_S$ **something that we have always to take into account**.

Farm

Sometimes we denote as $\text{Farm}(s, nw)$, otherwise we omit the number of workers and simply write $\text{Farm}(s)$.
We assume to know L_w and T_w of the workers. We have some scheduler (**emitter** E) that distributes the items from the input stream to the workers, and a gatherer (**collector** C) that gets the results from the workers and delivers them to the output stream. Those can simply be data structures: queues, for example.

$$L(\text{Farm}(s, nw)) = t_E + L_w + t_C$$

This can appear as a pipeline of three stages, where the Emitter produces to the second stage (the workers) which produce to the third stage (the Collector)

$$T_S(\text{Farm}(s, nw)) = \max\left\{t_E, \frac{T_w}{nw}, t_C\right\}$$

We assume to have m tasks

$$T_C(\text{Farm}(s, nw)) = m \cdot T_S(\text{Farm}(s, nw))$$

With boundary conditions

$$T_S = \max\{T_s(\text{Farm}()), T_A, T_D\}$$

What if we want to achieve a given performance? Compute a nw suitable to achieve a wanted performance by inverting the very same formulas.
With a target $T_S = T_A = 1\text{s}$

$$T_S = 1\text{s} = \max\left\{t_E, t_C, \frac{10\text{s}}{nw}\right\}$$

But t_E, t_C are negligible so

$$\frac{10\text{s}}{nw} = 1\text{s} \Rightarrow nw = 10$$