GLM Cheatsheet

- *Random component*: This specifies the response variable y and its probability distribution. The observations[1] $\boldsymbol{y} = (y_1, \ldots, y_n)^{\mathrm{T}}$ on that distribution are treated as independent.
- *Linear predictor*: For a *parameter vector* $\boldsymbol{\beta} = (\beta_1, \beta_2, \ldots, \beta_p)^{\mathrm{T}}$ and a $n \times p$ *model matrix* $\boldsymbol{X}$ that contains values of p explanatory variables for the n observations, the linear predictor is $\boldsymbol{X\beta}$.
- *Link function*: This is a function g applied to each component of $E(\boldsymbol{y})$ that relates it to the linear predictor,

$$\mu_i = E(y_i)$$

$$g(\mu_i) = \eta_i = \sum_{j=1}^{p} \beta_j x_{ij}, \quad i = 1, \ldots, n.$$

GLM Cheatsheet

Family	pdf or pmf	Mean	Variance	Observaciones
Normal	$\mathcal{P}(y;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(y-\mu)^2}{2\sigma^2}\right\}$	μ	$\phi = \sigma^2$	
Binomial	$\mathcal{P}(y;\mu,m) = \binom{m}{my} \mu^{my}(1-\mu)^{m(1-y)}$	μ	$\mu\,(1-\mu)\,/\,m$	
Poisson	$\mathcal{P}(y;\mu) = \frac{\exp(-\mu)\mu^y}{y!}$	μ	μ	
Negative binomial	$\mathcal{P}(y_i;\mu_i,k) = \frac{\Gamma(y_i+k)}{\Gamma(y_i+1)\Gamma(k)} \left(\frac{\mu_i}{\mu_i+k}\right)^{y_i} \left(1-\frac{\mu_i}{\mu_i+k}\right)^{k}$	μ	$\mu + \mu^2/k$	k es el parámetro de precisión. $\phi = 1/k$ es el parámetro de dispersión. El valor de k se acostumbra a llamar `theta` en R.
Gamma	$\mathcal{P}(y;\mu,\phi) = \left(\frac{y}{\phi\mu}\right)^{1/\phi} \frac{1}{y} \exp\left(-\frac{y}{\phi\mu}\right) \frac{1}{\Gamma(1/\phi)}$	μ	$\phi\,\mu^2$	
Inverse Gaussian	$\mathcal{P}(y;\mu,\phi) = (2\pi y^3\phi)^{-1/2} \exp\left\{-\frac{1}{2\phi}\frac{(y-\mu)^2}{y\mu^2}\right\}$	μ	$\phi\,\mu^3$	

Table 15.2 Canonical Link, Response Range, and Conditional Variance Function for Exponential Families

Family	*Canonical Link*	*Range of Y_i*	$V(Y_i \mid \eta_i)$
Gaussian	Identity	$(-\infty, +\infty)$	ϕ
Binomial	Logit	$\frac{0, 1, ..., n_i}{n_i}$	$\frac{\mu_i(1-\mu_i)}{n_i}$
Poisson	Log	$0,1,2,...$	μ_i
Gamma	Inverse	$(0,\infty)$	$\phi\mu_i^2$
Inverse-Gaussian	Inverse-square	$(0,\infty)$	$\phi\mu_i^3$

NOTE: ϕ is the dispersion parameter, η_i is the linear predictor, and μ_i is the expectation of Y_i (the response). In the binomial family, n_i is the number of trials.

Table 5.1 Common EDMs, showing their variance function $V(\mu)$, cumulant function $\kappa(\theta)$, canonical parameter θ, dispersion parameter ϕ, unit deviance $d(y,\mu)$, support S (the permissible values of y), domain Ω for μ and domain Θ for θ. For the Tweedie distributions, the case $\xi = 2$ is the gamma distribution, and $\xi = 1$ with $\phi = 1$ is the Poisson distribution. $\mathbb{R}$ refers to the real line; $\mathbb{N}$ refers to the natural numbers $1, 2, \ldots$; superscript $+$ means positive values only; superscript $-$ means negative values only; subscript 0 means zero is included in the space (Sect. 5.3.5)

EDM	$V(\mu)$	$\kappa(\theta)$	θ	ϕ	$d(y,\mu)$	S	Ω	Θ	Reference
Normal	1	$\theta^2/2$	μ	σ^2	$(y-\mu)^2$	$\mathbb{R}$	$\mathbb{R}$	$\mathbb{R}$	Chaps. 2 and 3
Binomial	$\mu(1-\mu)$	$\dfrac{\exp\theta}{1+\exp\theta}$	$\log\dfrac{\mu}{1-\mu}$	$\dfrac{1}{m}$	$2\left\{y\log\dfrac{y}{\mu}+(1-y)\log\dfrac{1-y}{1-\mu}\right\}$	$\dfrac{0,1,\ldots m}{m}$	$(0,1)$	$\mathbb{R}$	Chap. 9
Negative binomial	$\mu+\dfrac{\mu^2}{k}$	$-\log(1-\exp\theta)$	$\log\dfrac{\mu}{\mu+k}$	1	$2\left\{y\log\dfrac{y}{\mu}-(y+k)\log\dfrac{y+k}{\mu+k}\right\}$	$\mathbb{N}_0$	$\mathbb{R}^+$	$\mathbb{R}^-$	Chap. 10
Poisson	μ	$\exp\theta$	$\log\mu$	1	$2\left\{y\log\dfrac{y}{\mu}-(y-\mu)\right\}$	$\mathbb{N}_0$	$\mathbb{R}^+$	$\mathbb{R}$	Chap. 10
Gamma	μ^2	$-\log(-\theta)$	$-\dfrac{1}{\mu}$	ϕ	$2\left\{-\log\dfrac{y}{\mu}+\dfrac{y-\mu}{\mu}\right\}$	$\mathbb{R}^+$	$\mathbb{R}^+$	$\mathbb{R}$	Chap. 11
Inverse Gaussian	μ^3	$-\sqrt{-2\theta}$	$-\dfrac{1}{2\mu^2}$	ϕ	$\dfrac{(y-\mu)^2}{\mu^2 y}$	$\mathbb{R}^+$	$\mathbb{R}^+$	$\mathbb{R}_0^-$	Chap. 11
Tweedie ($\xi \le 0$ or $\xi \ge 1$)	μ^ξ	$\dfrac{\{(1-\xi)\theta\}^{(2-\xi)/(1-\xi)}}{2-\xi}$ for $\xi \ne 2$	$\dfrac{\mu^{1-\xi}}{1-\xi}$ for $\xi \ne 1$	ϕ	$2\left\{\dfrac{\max(y,0)^{2-\xi}}{(1-\xi)(2-\xi)}-\dfrac{y\mu^{1-\xi}}{1-\xi}+\dfrac{\mu^{2-\xi}}{2-\xi}\right\}$ for $\xi \ne 1, 2$	$\xi < 0$: $\mathbb{R}$	$\mathbb{R}^+$	$\mathbb{R}_0^+$	Chap. 12
						$1 < \xi < 2$: $\mathbb{R}_0^+$	$\mathbb{R}^+$	$\mathbb{R}^-$	
						$\xi > 2$: $\mathbb{R}^+$	$\mathbb{R}^+$	$\mathbb{R}_0^-$	

Los capítulos de la última columna corresponden a libro “Generalized linear models with examples in R” de Dunn & Smyth (2018).

Table 15.1 Some Common Link Functions and Their Inverses

Link	$\eta_i = g(\mu_i)$	$\mu_i = g^{-1}(\eta_i)$
Identity	μ_i	η_i
Log	$\log_e \mu_i$	e^{η_i}
Inverse	μ_i^{-1}	η_i^{-1}
Inverse-square	μ_i^{-2}	$\eta_i^{-1/2}$
Square-root	$\sqrt{\mu_i}$	η_i^2
Logit	$\log_e \frac{\mu_i}{1-\mu_i}$	$\frac{e^{\eta_i}}{1+e^{\eta_i}}$
Probit	$\Phi^{-1}(\mu_i)$	$\Phi(\eta_i)$
Log-log	$-\log_e[-\log_e(\mu_i)]$	$\exp[-\exp(-\eta_i)]$
Complementary log-log	$\log_e[-\log_e(1-\mu_i)]$	$1-\exp[-\exp(\eta_i)]$

NOTE: μ_i is the expected value of the response, η_i is the linear predictor, and $\Phi(\cdot)$ is the cumulative distribution function of the standard-normal distribution.

Table 6.4 The link functions accepted by different `glm()` families in R are indicated using a tick ✓. The default (and canonical) links used by R are indicated with stars ★ (Sect. 6.9)

Link function	`gaussian`	`binomial` and `quasibinomial`	`poisson` and `quasipoisson`	`Gamma`	`inverse.gaussian`	`quasi`
`identity`	★		✓	✓	✓	★
`log`	✓		★	✓	✓	✓
`inverse`	✓			★	✓	✓
`sqrt`			✓			✓
`1/mu^2`					★	✓
`logit`		★				✓
`probit`		✓				✓
`cauchit`		✓				
`cloglog`		✓				✓
`power`						✓

Table 15.9 Functions $a(\cdot)$, $b(\cdot)$, and $c(\cdot)$ for Constructing the Exponential Families

Family	$a(\phi)$	$b(\theta)$	$c(y, \phi)$
Gaussian	ϕ	$\theta^2/2$	$-\frac{1}{2}[y^2/\phi + \log_e(2\pi\phi)]$
Binomial	$1/n$	$\log_e(1+e^\theta)$	$\log_e\binom{n}{ny}$
Poisson	1	e^θ	$-\log_e y!$
Gamma	ϕ	$-\log_e(-\theta)$	$\phi^{-1}\log_e(y/\phi) - \log_e y - \log_e\Gamma(\phi^{-1})$
Inverse-Gaussian	ϕ	$-\sqrt{-2\theta}$	$-\frac{1}{2}\left[\log_e(\pi\phi y^3) + 1/(\phi y)\right]$

NOTE: In this table, n is the number of binomial trials, and $\Gamma(\cdot)$ is the gamma function.