Consistency of the MLE for Poisson distribution

Objective: The primary purpose is to explore the consistency of the maximum likelihood estimator ($\hat{\lambda}_n$) for the Poisson distribution as the sample size (n) increases.

Motivation: A data scientist has conducted research which suggests the number of calls in an answering service approaches a Poisson distribution and the telephone operator, on the average, handles six calls every two minutes $\left(\lambda^* = 6 \frac{calls}{two\ minutes}\right)$.

Task: Follow the subsequent steps to examine the consistency of $\hat{\lambda}_n$.

1. Assume that the number of calls addressed by the telephone operator indeed resembles a Poisson distribution.
2. Compute the average number of calls (λ) answered in a minute from the given information, use the three-simple rule to obtain λ.
3. Open the Shiny app given in the URL: https://mateo.shinyapps.io/Cons-MLEs/
4. Using the Shiny app, select the Poisson distribution, the real rate $\left(\lambda = 3 \frac{calls}{minute}\right)$ and $\delta = 0.07$.
5. Fix the sample size at $n = 50, 175, 560, 994, 1284, 1673, 2000$ and fill the gaps in Table 1 by using the results from the Shiny app.

Table 1. Number of samples and proportions when the maximum likelihood estimator is within or outside the interval ($\lambda \pm \delta$).

Sample size n	Number of samples on which $\lambda - \delta < \hat{\lambda}_n < \lambda + \delta$	Number of samples on which $\hat{\lambda}_n \leq \lambda - \delta$ or $\hat{\lambda}_n \geq \lambda + \delta$	Estimated $P(\lvert\hat{\lambda}_n - \lambda\rvert < \delta)$	Estimated $P(\lvert\hat{\lambda}_n - \lambda\rvert \geq \delta)$
50	2301		0.23	
175				
560		3307		0.331
994				
1284				
1673				
2000				

6. According to the data gathered:

    - What can one infer with regard to the pattern observed?

    - Can one affirm that $\hat{\lambda}_n$ is close to $\lambda = 3$ with high probability (when n is large)?

7. Replicate numerals 4 and 5 with higher values for δ. What can one conclude about the convergence quickness of $\hat{\lambda}_n$ as δ rises?

Consistency of the MLE for Geometric distribution

Objective: The primary purpose is to explore the consistency of the maximum likelihood estimator ($\hat{\pi}_n$) for the Geometric distribution as the sample size (n) increases.

Motivation: From grandparents' expertise, when a loaded dice is thrown repeatedly on a board game, the Geometric distribution might be used for modeling the number of failures until the first time a "6" appears with success probability equals 0.3.

Task: Follow the subsequent steps to examine the consistency of $\hat{\pi}_n$.

1. Assume that the number of failures until the first time a "6" appears truly resembles a Geometric distribution.
2. Open the Shiny app given in the URL: https://mateo.shinyapps.io/Cons-MLEs/
3. Using the Shiny app, select the Geometric distribution, the real probability ($\pi = 0.3$) and $\delta = 0.07$.
4. Fix the sample size at $n = 50, 175, 560, 994, 1284, 1673, 2000$ and fill the gaps in Table 2 by using the results from the Shiny app.

Table 2. Number of samples and proportions when the maximum likelihood estimator is within or outside the interval ($\pi \pm \delta$).

Sample size n	Number of samples on which $\pi - \delta < \hat{\pi}_n < \pi + \delta$	Number of samples on which $\hat{\pi}_n \leq \pi - \delta$ or $\hat{\pi}_n \geq \pi + \delta$	Estimated $P(\lvert\hat{\pi}_n - \pi\rvert < \delta)$	Estimated $P(\lvert\hat{\pi}_n - \pi\rvert \geq \delta)$
50	9437		0.944	
175				
560		0		0
994				
1284				
1673				
2000				

5. According to the data gathered:

    - What can one infer with regard to the pattern observed?

    - Can one affirm that $\hat{\pi}_n$ is close to $\pi = 0.3$ with high probability (when n is large)?

6. Replicate numerals 3 and 4 with higher values for δ. What can one conclude about the convergence quickness of $\hat{\pi}_n$ as δ rises?

Consistency of the MLE for Exponential distribution

Objective: The primary purpose is to explore the consistency of the maximum likelihood estimator ($\hat{\phi}_n$) for the Exponential distribution as the sample size (n) increases.

Motivation: An experimental physicist claims that the lapse of time between detections of an unusual particle by a Geiger counter has an Exponential distribution with mean time equals 0.25 hours.

Task: Follow the subsequent steps to examine the consistency of $\hat{\phi}_n$.

1. Assume that the amount of time between detections of an unusual particle, in effect, has an Exponential distribution.
2. Compute the rate (ϕ) from the given information.
3. Open the Shiny app given in the URL: https://mateo.shinyapps.io/Cons-MLEs/
4. Using the Shiny app, select the Exponential distribution, the real rate $\left(\phi = \frac{1}{0.25} = 4\frac{detections}{hour}\right)$ and $\delta = 0.07$.
5. Fix the sample size at $n = 50, 175, 560, 994, 1284, 1673, 2000$ and fill the gaps in Table 3 by using the results from the Shiny app.

Table 3. Number of samples and proportions when the maximum likelihood estimator is within or outside the interval $(\phi \pm \delta)$.

Sample size n	Number of samples on which $\phi - \delta < \hat{\phi}_n < \phi + \delta$	Number of samples on which $\hat{\phi}_n \leq \phi - \delta$ or $\hat{\phi}_n \geq \phi + \delta$	Estimated $P(\lvert\hat{\phi}_n - \phi\rvert < \delta)$	Estimated $P(\lvert\hat{\phi}_n - \phi\rvert \geq \delta)$
50	948		0.095	
175				
560		6817		0.682
994				
1284				
1673				
2000				

6. According to the data gathered:

   - What can one infer with regard to the pattern observed?

   - Can one affirm that $\hat{\phi}_n$ is close to $\phi = 4$ with high probability (when n is large)?

7. Replicate numerals 4 and 5 with higher values for δ. What can one conclude about the convergence quickness of $\hat{\phi}_n$ as δ rises?

To conclude, infer curves which illustrate the convergence swiftness of the MLEs of the parameters regarded throughout this questionnaire from acquired information in Tables 1, 2 and 3 and color them according to the imposed colors for the distributions.

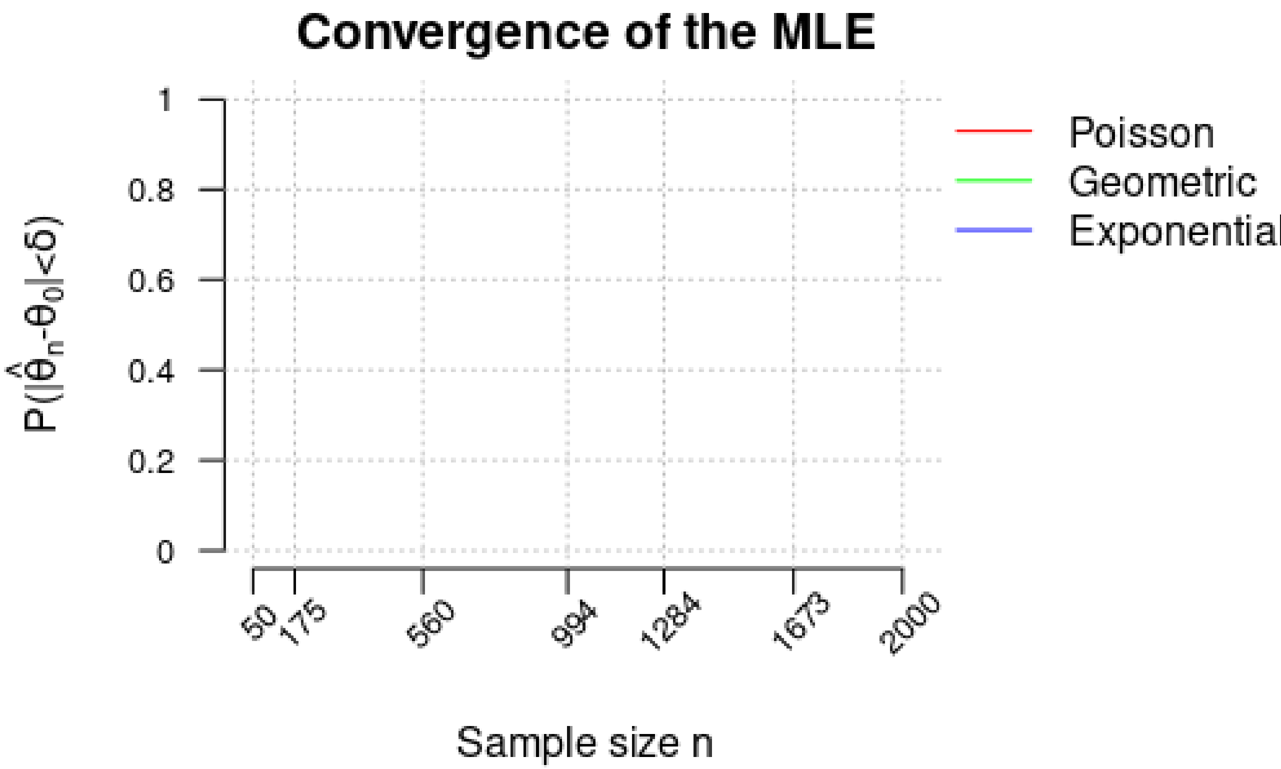

Figure 1. Template to illustrate the convergence swiftness of the MLEs.