Convergence of the maximum of a sample from a Uniform distribution

Objective: The primary purpose is to explore the convergence of $X_{(n)}$ for the Uniform distribution as the sample size (n) increases.

Task: Follow the subsequent steps to examine the convergence of $X_{(n)}$:

1. Open the shiny application given by the URL: https://fhernanb.shinyapps.io/Conv-Prob-Dist/
2. Using the shiny application, select the Uniform distribution, the sample size n, the parameter γ, and ϵ based on the information given by Table 1.
3. Fill the gaps of Table 1 by using the results from the shiny application in order to infer curves on Figure 1 (convergence quickness of $X_{(n)}$) with their respective color, which is associated to each value of ϵ.

			$n = 3$	$n = 44$	$n = 101$	$n = 197$
$\gamma = 8.6$	$\epsilon = 0.1$	$F_{X_{(n)}}(\gamma - \epsilon) = P(\lvert X_{(n)} - \gamma \rvert \geq \epsilon)$	0.96			
	$\epsilon = 0.3$	$F_{X_{(n)}}(\gamma - \epsilon) = P(\lvert X_{(n)} - \gamma \rvert \geq \epsilon)$				0
	$\epsilon = 0.5$	$F_{X_{(n)}}(\gamma - \epsilon) = P(\lvert X_{(n)} - \gamma \rvert \geq \epsilon)$		0.07		

Table 1: Assessment of the empirical $F_{X_{(n)}}(\gamma - \epsilon)$ as n increases.

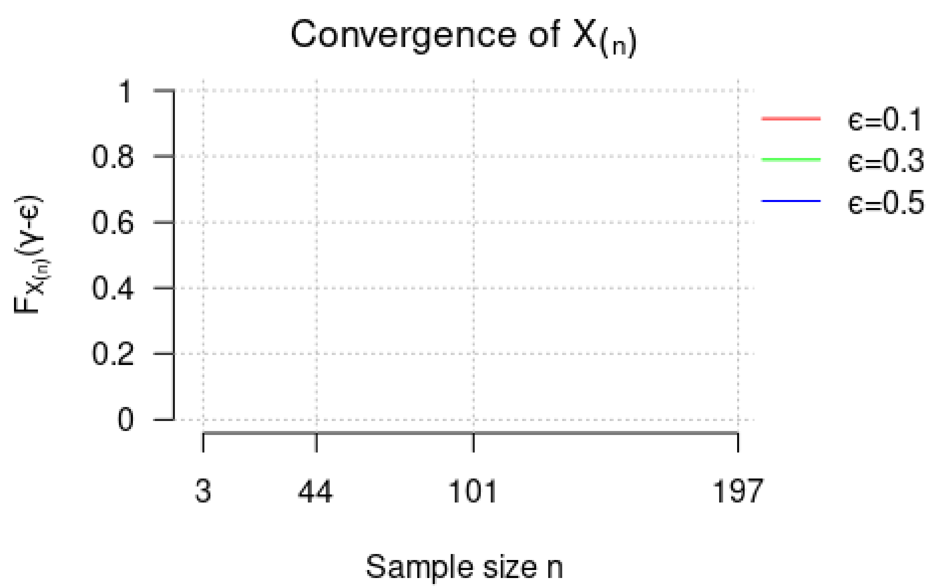

Figure 1: Template to illustrate the convergence quickness of $X_{(n)}$ for each value of ϵ.

4. In accordance with Table 1 and Figure 1:

- What can be inferred with regard to the pattern observed?

- It can be affirmed that $X_{(n)}$ is close to $\gamma = 8.6$ with high probability (when n is large)?

- What can be concluded about the convergence quickness of $X_{(n)}$ as ϵ rises?

Convergence of the minimum of a sample from a Shifted Exponential distribution

Objective: The primary purpose is to explore the convergence of $X_{(1)}$ for the Shifted Exponential distribution as the sample size (n) increases.

Task: Follow the subsequent steps to examine the convergence of $X_{(1)}$:

1. Open the shiny application given by the URL: https://fhernanb.shinyapps.io/Conv-Prob-Dist/
2. Using the shiny application, select the Shifted Exponential distribution, the sample size n, the parameter γ, and ϵ based on the information given by Table 2.
3. Fill the gaps of Table 2 by using the results from the shiny application in order to infer curves on Figure 2 (convergence quickness of $X_{(1)}$) with their respective color, which is associated to each value of ϵ.

			$n = 2$	$n = 25$	$n = 115$	$n = 185$
$\gamma = 4.2$	$\epsilon = 0.1$	$F_{X_{(1)}}(\gamma + \epsilon) = P(\lvert X_{(1)} - \gamma \rvert < \epsilon)$			0.99	
	$\epsilon = 0.3$	$F_{X_{(1)}}(\gamma + \epsilon) = P(\lvert X_{(1)} - \gamma \rvert < \epsilon)$	0.44			
	$\epsilon = 0.5$	$F_{X_{(1)}}(\gamma + \epsilon) = P(\lvert X_{(1)} - \gamma \rvert < \epsilon)$				0.99

Table 2: Assessment of the empirical $F_{X_{(1)}}(\gamma + \epsilon)$ as n increases.

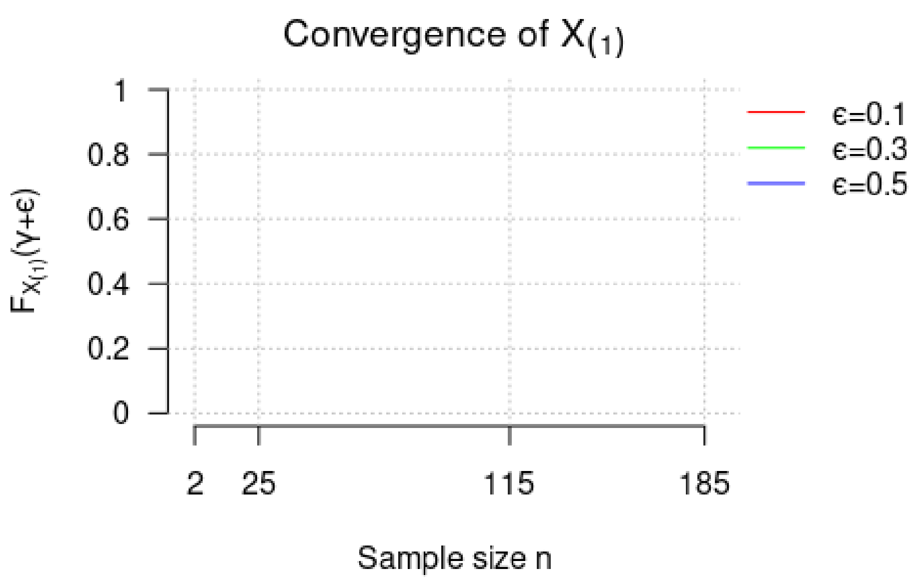

Figure 2: Template to illustrate the convergence quickness of $X_{(1)}$ for each value of ϵ.

4. In accordance with Table 2 and Figure 2:

- What can be inferred with regard to the pattern observed?

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- It can be affirmed that $X_{(1)}$ is close to $\gamma = 4.2$ with high probability (when n is large)?

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- What can be concluded about the convergence quickness of $X_{(1)}$ as ϵ rises?

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