Consistency of the MLE for Poisson distribution

Objective: The primary purpose is to explore the consistency of the maximum likelihood estimator ($\hat{\lambda}_n$) for the Poisson distribution as the sample size (n) increases.

Motivation: A data scientist has conducted research which suggests the number of calls in an answering service approaches a Poisson distribution and the telephone operator, on the average, handles six calls every two minutes $\left(\lambda_0^* = 6\,\frac{calls}{two\ minutes}\right)$.

Task: Follow the subsequent steps to examine the consistency of $\hat{\lambda}_n$.

1. Assume that the number of calls addressed by the telephone operator indeed resembles a Poisson distribution.
2. Compute the average number of calls (λ_0) answered in a minute from the given information, use the three-simple rule to obtain λ_0.
3. Open the Shiny app given in the URL: https://fhernanb.shinyapps.io/Cons-MLEs/
4. Using the Shiny app, select the Poisson distribution, the real rate $\left(\lambda_0 = 3\ \frac{calls}{minute}\right)$ and $\delta = 0.07$.
5. Fix the sample size at $n = 50, 175, 560, 994, 1284, 1673, 2000$ and fill the gaps in Table 1 by using the results from the Shiny app.

Table 1. Number of samples and proportions when the maximum likelihood estimate is within or outside the interval ($\lambda_0 \pm \delta$).

Sample size n	Number of samples on which $\lambda_0 - \delta < \hat{\lambda}_n < \lambda_0 + \delta$	Number of samples on which $\hat{\lambda}_n \le \lambda_0 - \delta$ or $\hat{\lambda}_n \ge \lambda_0 + \delta$	Estimated $P(\lvert\hat{\lambda}_n - \lambda_0\rvert < \delta)$	Estimated $P(\lvert\hat{\lambda}_n - \lambda_0\rvert \ge \delta)$
50	2301		0.23	
175				
560		3307		0.331
994				
1284				
1673				
2000				

6. According to the data gathered:

   - What can one infer with regard to the pattern observed?

   - Can one affirm that $\hat{\lambda}_n$ is close to $\lambda_0 = 3$ with high probability (when n is large)?

7. Replicate numerals 4 and 5 with higher values for δ. What can one conclude about the convergence quickness of $\hat{\lambda}_n$ as δ rises?

Consistency of the MLE for Geometric distribution

Objective: The primary purpose is to explore the consistency of the maximum likelihood estimator ($\hat{\pi}_n$) for the Geometric distribution as the sample size (n) increases.

Motivation: From grandparents' expertise, when a loaded dice is thrown repeatedly on a board game, the Geometric distribution might be used for modeling the number of failures until the first time a "6" appears with success probability equals 0.3.

Task: Follow the subsequent steps to examine the consistency of $\hat{\pi}_n$.

1. Assume that the number of failures until the first time a "6" appears truly resembles a Geometric distribution.
2. Open the Shiny app given in the URL: https://fhernanb.shinyapps.io/Cons-MLEs/
3. Using the Shiny app, select the Geometric distribution, the real probability ($\pi_0 = 0.3$) and $\delta = 0.07$.
4. Fix the sample size at $n = 50, 175, 560, 994, 1284, 1673, 2000$ and fill the gaps in Table 2 by using the results from the Shiny app.

Table 2. Number of samples and proportions when the maximum likelihood estimate is within or outside the interval ($\pi_0 \pm \delta$).

Sample size n	Number of samples on which $\pi_0 - \delta < \hat{\pi}_n < \pi_0 + \delta$	Number of samples on which $\hat{\pi}_n \leq \pi_0 - \delta$ or $\hat{\pi}_n \geq \pi_0 + \delta$	Estimated $P(\lvert\hat{\pi}_n - \pi_0\rvert < \delta)$	Estimated $P(\lvert\hat{\pi}_n - \pi_0\rvert \geq \delta)$
50	9437		0.944	
175				
560		0		0
994				
1284				
1673				
2000				

5. According to the data gathered:

    - What can one infer with regard to the pattern observed?

    - Can one affirm that $\hat{\pi}_n$ is close to $\pi_0 = 0.3$ with high probability (when n is large)?

6. Replicate numerals 3 and 4 with higher values for δ. What can one conclude about the convergence quickness of $\hat{\pi}_n$ as δ rises?

Consistency of the MLE for Exponential distribution

Objective: The primary purpose is to explore the consistency of the maximum likelihood estimator ($\hat{\phi}_n$) for the Exponential distribution as the sample size (n) increases.

Motivation: An experimental physicist claims that the lapse of time between detections of an unusual particle by a Geiger counter has an Exponential distribution with mean time equals 0.25 hours.

Task: Follow the subsequent steps to examine the consistency of $\hat{\phi}_n$.

1. Assume that the amount of time between detections of an unusual particle, in effect, has an Exponential distribution.
2. Compute the rate (ϕ_0) from the given information.
3. Open the Shiny app given in the URL: https://fhernanb.shinyapps.io/Cons-MLEs/
4. Using the Shiny app, select the Exponential distribution, the real rate $\left(\phi_0 = \frac{1}{0.25} = 4\,\frac{detections}{hour}\right)$ and $\delta = 0.07$.
5. Fix the sample size at $n = 50, 175, 560, 994, 1284, 1673, 2000$ and fill the gaps in Table 3 by using the results from the Shiny app.

Table 3. Number of samples and proportions when the maximum likelihood estimate is within or outside the interval ($\phi_0 \pm \delta$).

Sample size n	Number of samples on which $\phi_0 - \delta < \hat{\phi}_n < \phi_0 + \delta$	Number of samples on which $\hat{\phi}_n \leq \phi_0 - \delta$ or $\hat{\phi}_n \geq \phi_0 + \delta$	Estimated $P(\lvert\hat{\phi}_n - \phi_0\rvert < \delta)$	Estimated $P(\lvert\hat{\phi}_n - \phi_0\rvert \geq \delta)$
50	948		0.095	
175				
560		6817		0.682
994				
1284				
1673				
2000				

6. According to the data gathered:

   - What can one infer with regard to the pattern observed?

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     ____
     ____
     ____
     ____

   - Can one affirm that $\hat{\phi}_n$ is close to $\phi_0 = 4$ with high probability (when n is large)?

     ____
     ____
     ____
     ____
     ____

7. Replicate numerals 4 and 5 with higher values for δ. What can one conclude about the convergence quickness of $\hat{\phi}_n$ as δ rises?

   ____
   ____
   ____
   ____
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To conclude, infer curves which illustrate the convergence swiftness of the MLEs of the parameters regarded throughout this questionnaire from acquired information in Tables 1, 2 and 3 and color them according to the imposed colors for the distributions.

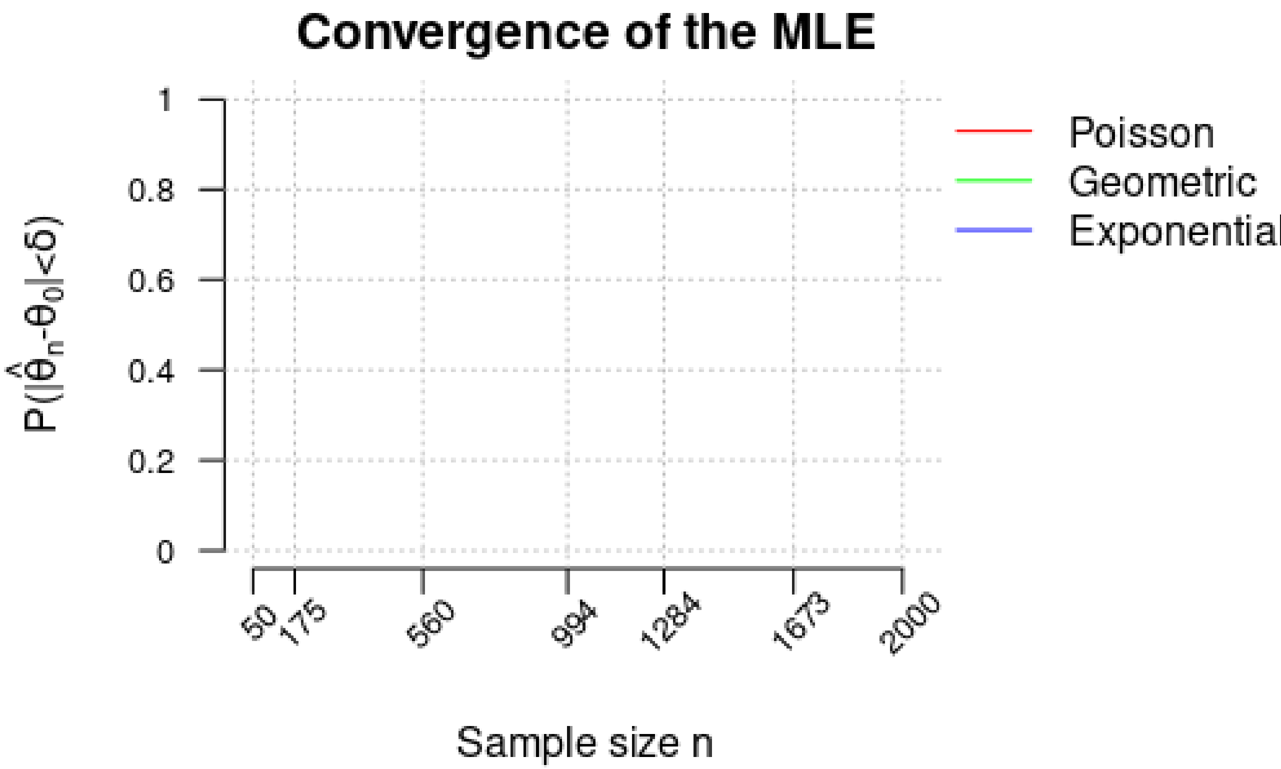

Figure 1. Template to illustrate the convergence swiftness of the MLEs.