Unit I

Numerical Analysis

i) Transcidental / Algebraic
ii) Ordinary differential equation

Algebraic to transcedental

An algebraic eq^n of degree n is

$$p(x) = a_0 x^n + a_1 x^{n-1} + \dots a_n = 0$$

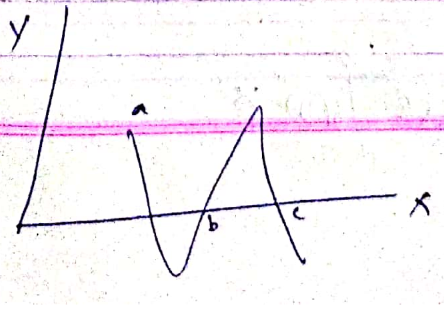
$a_0 \neq 0$, here $n \geq 1$.

Transcedental eq^n are non algebraic equation involving trans. functions such as exponential, logarithmic, trignometric functions

A general form of an algebraic / trans equation is $f(x) = 0$
where $f(x)$ is defined & continuous on an interval $a < x < b$.

Root :-
A value 'a' for which $f(a) = 0$ is known as root or solution of the equation $f(x) = 0$ or & a is called zero of the function $\{f(x)\}$.

Geometrically root of $f(x) = 0$



Note

*) App. solution of an algebraic / transcedental equation are to be found by new method consisting of

i) Isolating the roots and
ii) Then improving the value of approx. roots.

Graphical method

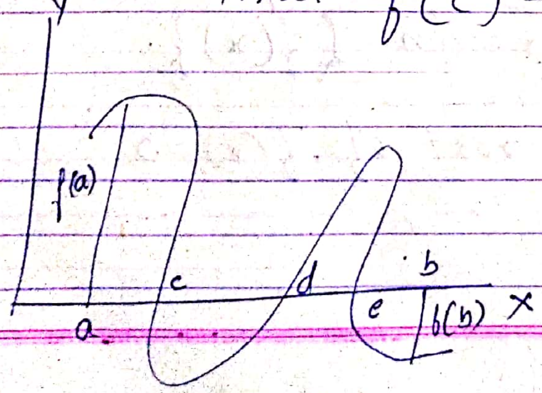
→ are cumbersome.
∴ They should Theorem.

Theorem :-

If a continuous function $f(x)$ assumes values of opposite signs at the end points of an interval $[a, b]$ i.e., $f(a) \cdot f(b) < 0$, then the interval will contain atleast one root of the equation $f(x) = 0$

That is,
there exists c belongs to (a, b) such that $f(c) = 0$.

$f(a) \cdot f(b) < 0$



Descarte's rule of sign

The no. of +ve roots of $f(x) = 0$ cannot exceed the no. of changes of sign in $f(x)$

Also, the no. of -ve roots of $f(x)$ cannot exceed the no. of changes of sign in $f(-x)$.

(*) $x^5 + 2x^4 + x^3 + 2x^2 + 5x$ } Change rule of sign.

$+ \quad + \quad - \quad - \quad +$

Regula-falsi method or Method of interpolation or method of false position.

This is a geometrical method to find an approx. root of an eq^n

This method is equivalent to replacing the curve $y = f(x)$ by a chord that passes through the points.

$A(a, f(a))$ and $B(b, f(b))$

Formula =>

$$\boxed{x = \frac{af(b) - bf(a)}{f(b) - f(a)}}$$

Eg. Find the root of the equation $\cos x = xe^x$ using regula-falsi meth (5 decimal places)

→ $\left(\text{Let } f(x) = \cos x - xe^x = 0\right)$

$f(0) = \cos 0 - 0e^0 = 1$

$f(1) = \cos 1 - 1e^1 = -2.1779$

the root lies between 0 & 1.
(radian mode)

$$x_{1} = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$= a = 0 \quad b = 1 \quad f(a) = 1$
$f(b) = -2.17798$

$$x_1 = \frac{0 f(1) - 1 f(0)}{f(1) - f(0)} = 0.31467$$

$f(0.31467) = 0.51987$

$\therefore$ Root lies b two (0.31467) and 1

$\therefore$ Using formula

$$= \frac{0 \times -2.17798 - 1 \times 1}{-2.17798 - 1}$$

$= 0.31467$

$f(x) = \cos(0.31467) - 0.31467 e^{0.31467}$

$= 0.51986 \simeq 0.51987$

One $f()$ is +ve & one $f()$ is -ve
$f(1)$ is -ve.

Root lies b/w 0.31467 to 1

Since $f(0.31467) > 0$ & $f(1) < 0$
$a = 0.31467 \qquad b = 1$

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$$x_2 = \frac{0.31467\, f(1) - 1\, f(0.31467)}{f(1) - f(0.31467)}$$

$$= 0.44673$$

$f(0.44673) = 0.20356.$

$f(x) = \cos x - xe^{x} = 0.20356.$

$\therefore$ The root lies b/w 0.44673 & 1

$f(1)$ is $-ve$ $\quad f(x_2)$ is $+ve$.

$x_2 =$

-2.17798×0.20355

$$x_3 = \frac{0.44673\, f(1) - 1\, f(0.44673)}{f(b) - f(0.44673)}$$

$f(a)$

$$= 0.49407$$

Continuing in this way

x_4

x_5

x_6

x_7

x_8

x_9

$\therefore$ Req. root is 0.5177 to 4 decimal places.

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How to solve.

$$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$\cos x = x e^{x}$

$f(x) = \cos x - x e^{x} = 0$

$f(0) = \cos 0 - 0e^{0} = 1$

$f(1) = \cos 1 - 1e^{1} = -2.17798$

Root 0 & 1

$a = 0$ $b = 1$

$f(a) = 1$ $f(b) = -2.17798$

$x_1 = 0.31467$

$f(x_1) = \cos(0.31467) - 0.31467 e^{0.31467}$

$= 0.51987$

Root 0.31467 and 1

$a = 0.31467$ $b = 1$

$f(0.31467) = 0.51986$

$f(a) = \cos(0.31467) - 0.31467 e^{0.31467}$

$= 0.51986$

$f(b) = -2.17798$

$= 0.44673.$

Root lies b/w 0.44673 and 1

$a = 0.44673$ $b = 1$

$f(a) = 0.2035$ $f(b) = -2.17798$

after formulating

$= 0.49407$

Examples.

(1) Use R.F method to find real root of the equation $\cos x = 3x - 1$ correct to three decimal places.

→ Let $f(x) = \cos x + 1 - 3x$

for $x = 0$; $f(0) = 2$
i.e $\cos(0) + 1 - 3 \times 0 = 2$.

for $x = 1$, $f(1) = -1.46$
$\cos(1) + 1 - 3 \times 1 = -1.46$

$\because f(0) \cdot f(1) < 0$, root lies in $(0, 1)$

Also $f(0.6) = 0.0253$
$f(0.7) = -0.3352$

$\because f(0.6) \cdot f(0.7) < 0$
root lies in $(\underset{a}{0.6}, \underset{b}{0.7})$.

Now,

$a = 0.6$
$b = 0.7$
$f(a) = 0.0253$
$f(b) = -0.3352$

By R.F method we have,

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

→

$$= \frac{0.6(-0.3352) - 0.7(0.0253)}{-0.3352 - 0.0253}$$

$$x_1 = 0.607$$

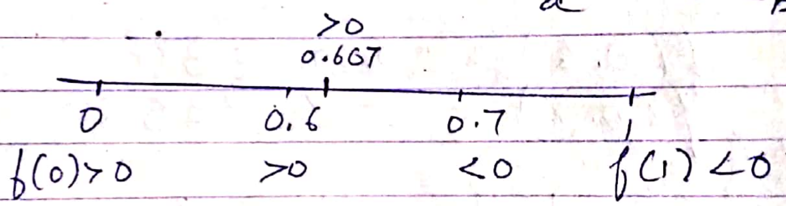
$$f(x_1) = f(0.607)$$

$$= \cos(0.607) + 1 - 3(0.607)$$

$$= 0.00036 > 0$$

$$f(0.607) \cdot f(0.7) < 0$$

Root lies in $(\underset{a}{0.607})$ & $(\underset{b}{0.7})$



$$x_2 = \frac{(0.607)(-0.3352) - (0.7)(0.025)}{-0.3352 - 0.0253}$$

$$x_2 = 0.607$$

$\because$ x_1 and x_2 are identical

$\therefore$ $x_2 = 0.607$ is the approx root of the given equation

(2) $\cos x = xe^{x}$

→ Let $f(x) = \cos x - xe^{x} = 0$

Root lies in the interval $(\underset{a}{0.5}, \underset{b}{0.6})$

$\because f(0.5) \cdot f(0.6) < 0$

i.e $f(0.5) = \overset{f(a)}{0.0532}$

$f(0.6) = -0.267 \quad f(b)$

Now

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$= \frac{0.5(-0.267) - 0.6(0.0532)}{-0.267 - 0.0532}$$

$$= 0.516614$$

$f(x_1) = f(0.516614) = 0.00347 > 0$

$\because f(0.516614) \cdot f(0.6) < 0$

Root lies in $(\underset{a}{0.516614}), (\underset{b}{0.6})$

Now,

$x_2 = 0.51768 \simeq 0.5177$

$f(x_2) = f(0.51768) = 0.00023 > 0$

Root lies in $(\underset{a}{0.51768}, \underset{b}{0.6})$

$x_3 = 0.51775$

$\therefore$ Since x_2 & x_3 are identical

$x_3 = 0.51775$ is the approx root

(3) Use the method of the false position to find the 4^{th} root of 32 correct to 3 decimal places.

$\rightarrow$ Let $x = (32)^{1/4} \Rightarrow x^4 - 32 = 0$

$$f(x) = x^4 - 32$$

Root lies b/w $(\underset{a}{2.3}, \underset{b}{2.4})$

$f(2.3) = -4.015$

$f(2.4) = 1.1776$

$f(2.3) \cdot f(2.4) < 0$

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)} \qquad \frac{AD - BC}{D - C}$$

$$= \frac{2.3(1.1776) - 2.4(-4.015)}{1.1776 + 4.015}$$

$$x_1 = 2.37732$$

$f(x_1) = f(2.37732) = -0.0588$

$f(2.37732) \cdot f(2.4) < 0$

$f(a) = -\ldots$

$f(b) = 1\ldots$

Root lies in $(\underset{a}{2.37732}, \underset{b}{2.4})$

$x_2 = 2.37839$ / 2.385

$f(x_2) = f(2.37839) = -0.0013$

$f(2.37839) \cdot f(2.4) < 0$

Root lies in $(\underset{a}{2.37839}, \underset{b}{2.4})$

$x_3 = 2.37841$ //

x_2 & x_3 are identical,
$x_3 = 2.37841$ is root
correct to three decimal places.

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Newton-Raphson method

or

Method of Tangents

Geometrically N-R method is equivalent to replacing a small arc of curve $y = f(x)$ by a tangent line drawn at a point of the curve

In general,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$n = 0, 1, 2, 3 \ldots$

Formula,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$0.4343\,[x \ln(x)] - 1.2$

(1) Find the root of $x \log_{10} x = 1.2$

$\rightarrow$ $f(x) = x \log_{10} x - 1.2$ — (1)

$f(x) = \dfrac{x \log_e x}{\log_e 10} - 1.2$

$= x \log_{10} e \log_e x - 1.2$

$= 0.4343\, x \log_e x - 1.2$

$f'(x) = 0.4343 \left[x \cdot \frac{1}{x} + \log x \right]$

$= 0.4343\,(1 + \log x)$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$f(2) = -0.59794 < 0$

$f(3) = 0.23136 > 0$

root lies b/w 2 & 3

x_0 $\because |f(3)| < |f(2)|$

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 3 - \dfrac{f(3)}{f'(3)} = 2.7461$

$f(2.7) = -0.035$

$f(2.8) = 0.052$

Root lies b/w 2.7 & 2.8

$$x_2 = 2.7461 - \frac{f(2.7461)}{f'(2.7461)}$$

$$= 2.74062$$

$$x_3 = 2.7406 - \frac{f(2.7406)}{f'(2.7406)}$$

$$= 2.7406$$

Examples.

(2). $x \sin x + \cos x = 0$

$$f(x) = x \sin x + \cos x$$

(diff) $$f'(x) = x \cos x + \sin x - \sin x$$

$$f'(x) = x \cos x$$

Let $x_0 = \pi$

By N-R method we have

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= \pi - \frac{\sin(\pi) + \cos(\pi)}{\pi \cos \pi}$$

$$= 2.8232$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.8232 - \frac{\sin(2.8232) + \cos(2.8232)}{2.8232 \cos 2.8232}$$

$$x_2 = 2.7985$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.7983$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 2.7983$$

x_3 & x_4 are identical

$x_4 = 2.7983$ is approx. root

~~$x_1 . x_2 . x_3 \ldots x_n$ if it converges to α is taken as the root of the equation $f(x) = 0$~~

~~Note~~:-

Fixed point iteration method.

Let $f(x) = 0$ be the given equation

let us write this equation

$$x = \phi(x) \quad \text{--- (1)}$$

Let x_0 be the initial approx. value to the actual root 'α' & subs. $x = x_0$ in RHS of (1) we get,

$$x_1 = \phi(x_0) \quad \text{--- (2)}$$

Again put $x = x_1$ in (2)

$$x_2 = \phi(x_1)$$

$$\cdots \quad \cdots \quad \cdots$$

$$x_n = \phi(x_{n-1})$$

The sequence of approx. roots $x_1, x_2, x_3 \ldots x_n$ if it converges to α is taken as the root of the equation $f(x) = 0$.

Note:— 1) The smaller the values of $\phi'(x)$, the more rapid will be the convergence.

2). The sufficient condition for the convergence is $\boxed{|\phi'(x)| < 1}$ for all x in the interval I containing the root $x = \alpha$

(*). $f(x) = 0$ can be algebraically expressed as $x = \phi(x)$

[A point say α is fixed point if it satisfied
$x = \phi(x)$] //

Egs.

(1) . Find the root of the equation $x^2+x-1=0$.
Using fixed root iteration

→ Let $f(x) = x^2+x-1=0$

$x^2+x=1$

$x(1+x)=1$

$x = \frac{1}{1+x} = \phi(x)$

Now, $\phi'(x) = -\frac{1}{(1+x)^2}$

Consider the function $f(x) = x^2+x-1=0$

Root lies in the interval $(0.6, 0.7)$

$f(0.6) = -0.04$

$f(0.7) = 0.19$

Let $x_0 = 0.6$

Also, $|\phi'(0.6)| = 0.3906 < 1$;

$|\phi'(0.7)| = 0.346 < 1$

$x_0 = 0.6$ be the initial approx.

$x_1 = \phi(x_0) = \frac{1}{1+x_0} = \frac{1}{1+0.6} = 0.625$

$x_2 = \phi(x_1) = \frac{1}{1+0.625} = 0.61538$

$x_3 = \phi(x_2) = \frac{1}{1+0.61538} = 0.61904$

$x_4 = 0.61764$

$x_5 = 0.61818$

$x_6 = 0.61797$

$x_7 = 0.61805$

$x_8 = 0.61802$

$$\boxed{x_8 = 0.61802}$$

is the required root //

(2) $x = \frac{1}{2} + \sin x$

Let $f(x) = \frac{1}{2} + \sin x - x = 0$

Root lies in $(1.4, 1.5)$

$f(1.4) = 0.0854$ $= 0.0025$

$f(1.5) = -2.163$ (-2.505)

Consider $x = \frac{1}{2} + \sin x = \phi(x)$

Also, $\phi'(x) = \cos x$

$|\phi'(x)| = |\phi'(1.4)| = \left.\begin{matrix} \\ |\cos(1.4)| = \end{matrix}\right\}$ $0.1699 < 1$ 0.0707

Let $x_0 = 1.4$ / 1.5

$x_1 = \phi(x_0) = \frac{1}{2} + \sin(x_0)$

$= \frac{1}{2} + \sin(1.4)$ (1.5)

$= 1.48544$ 1.4974

$x_2 = \phi(x_1) = \frac{1}{2} + \sin(1.48544)$

$= 1.49635$ 1.4973

$$x_3 = \phi(x_2)$$

$$= \frac{1}{2} + \sin(1.49635)$$

$$= 1.49723$$

$$x_4 = \phi(x_3)$$

$$= \frac{1}{2} + \sin(1.49723)$$

$$= 1.497295$$

$$x_5 = 1.497296$$

$x_5 = 1.49729$ is the required root//.

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A 1^{st} order ODE $\frac{dy}{dx} = f(x,y)$ with an initial con[illegible]
$y(x_0) = y_0$ can be solved using i) Taylor's series
ii) Modified Euler's method iii) Rungekutta method

(*) Taylor's series method

$$y(x) = y(x_0) + y'(x_0)(x-x_0) + \frac{y'(x_0)}{2!}(x-x_0)^2 + \frac{y''(x_0)}{3!}(x-x_0)^3 + \cdots$$

Numerical soln to 1^{st} order of ODE

(1). y at $x = 0.2$ for $\frac{dy}{dx} = 2y + 3e^x$ $y(0$[illegible]

$y' = 2y + 3e^x$

$$y(x) = y(x_0) + (x-x_0)\,y'(x_0) + \frac{(x-x_0)^2}{2!}y''(\cdot$$
$$+ \frac{(x-x_0)^3}{3!}y'''(x_0)$$

Put $x_0 = 0$ $x = 0.2$ we get,

$$y(x) = y(0) + (0.2)\,y'(0) + \frac{(0.2)^2}{2!}y''(0) + \frac{(0.2)^3}{3!}y'''(0) + \frac{(0.2)^4}{4!}y^{IV}(0) \quad \text{--- (1)}$$

Consider,

$y' = 2y + 3e^x$; $y'(0) = 2y_0 + 3e^{x_0} = 2(1) + 3e^0 =$

$y'' = 2y' + 3e^x$; $y''(0) = 2y_0' + 3e^{x_0} = 2(5) + 3e^0$

$y''' = 2y'' + 3e^x$; $y'''(0) =$ $= 2(13) + 3e^0 =$

~~$y^{IV} = 2y''' + 3e^x$; $y^{IV}(0) =$ $= 2(29) + 3e^0$~~

$= 61$

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Subs in (1) we get,

$$y(0.2) = 1 + 0.2(5) + \frac{(0.2)^2}{2!}(13) + \frac{(0.2)^3}{3!}(29) + \frac{(0.2)^4}{4!}(61)$$

$$y(0.2) = 2.30276$$ //

2. $y' = x - y^2$

Euler's method.

$$\frac{dy}{dx} = f(x,y)$$

cond^n $y(x_0) = x_0$

Formula :-

$$y_{n+1} = y_n + hf(x_n, y_n)$$

Ex.

Q. Use Euler's mthd. to find approx value of $\frac{dy}{dx} = x+y$ & $y = 1$ when

$f(x,y) = x+y$, $x_0 = 0$ $h = 0.1$

$y_0 = 1$ $n = 10$

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Again. $y_{n+1} = y_n + hf(x_n, y_n)$

(1)

x	y	$f(x,y) = x + y$
0	1	1.1
0.1	1.1	1.22
0.2	1.22	1.362
0.3	1.36	
0.4	1.52	
0.5	1.71	
0.6	1.93	
0.7	2.18	
0.8	2.46	
0.9	2.78	
1.0	3.148	3.562

$y(1) = 3.18$

(check once)

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2. $\frac{dy}{dx} = \frac{y-x}{y+x}$

$y = 1$ at $x = 0$ y for $x = 0.1$

$n = 5$ $h = 0.02$

$f(x+y) = \frac{y-x}{y+x}$

$x_0 = 0$

$y_0 = 1$

Step = 0.02

x	y
0	1 1.02
0.02	1.039
0.04	1.057
0.08	1.074
0.08	1.091
0.1	1.09~~1~~

Formula = $y + 0.02\left(\frac{y-x}{y+x}\right)$

Again :—

$\frac{dy}{dx} = \frac{y-x}{y+x}$

$f(x+y) = \frac{y-x}{y+x}$

x	y
0	1
0.02	1.02
0.04	1.039
0.06	1.057
0.08	1.074
0.1	1.091

Modified Euler's method

i) $\longrightarrow y_1^0 = y_0 + hf(x_0, y_0)$

ii)

$$y_1^1 = y_0 + \frac{h}{2}\left[f(x_0 + y_0) + f(x_1, y_1^0)\right]$$

iii)

$$y_1^2 = y_0 + \frac{h}{2}\left[f(x_0 + y_0) + f(x_1, y_1^1)\right]$$

and so on . . . . . .

This is a predictor - corrector method
We first predict the method of y using Euler's method.

① $y(20.2)$

$$\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right) \text{ with } y(20) = 5$$

$f(x) = \log_{10}\left(\frac{x}{y}\right)$ $\quad x_0 = 20 \quad h = 0.2$

$y_0 = 5$

By data, $x_0 = 20 \quad y_0 = 5 \quad h = 0.2$

$x_1 = x_0 + h = 20 + 0.2 = 20.2$

$f(x, y) = \log_{10}\left(\frac{x}{y}\right)$

$f(x_0 y_0) = f(20, 5) = \log\left(\frac{20}{5}\right)$

$= 0.6021$

$E.F = 5 + 0.2[0.6021]$

$= 5.1204$ //

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Modified EF

$$y_0' = y_0 + \frac{h}{2}\left[f(x_0 y_0) + f(x_1, y_1^{(0)})\right]$$

$$= 5 + \frac{0.2}{2}\left[0.6021 + \log\left(\frac{x_1}{y_1^0}\right)\right]$$

$$= 5 + \frac{0.2}{2}\left[0.6021 + \log\left(\frac{20.2)}{5.1204}\right)\right]$$

$$= 5.1198$$

$$\overline{5.1198}$$

$$= 5.1198$$

$$y(0.2) = 5.1198$$

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(2) $y = 0.1$ given $\frac{dy}{dx} = x^2 + y$.

$y(0) = 1$ taking $h = 0.05$

considering the accuracy upto 2 app. in each step

$f(x,y) = x^2 + y$

$x_0 = 0$ $\quad h = 0.05$

$y_0 = 1$

$x_1 = 0.05$ $\quad f(0,1)$

$f(x_0 y_0) = 1$

$E.F = y_0 + hf(x_0 y_0)$

$1 + 0.05(1)$

$= 1.05$

Modified

$1 + \frac{0.05}{2}[1 + x_1^2 + y_1^0]$

$= 1.0513$

$y_1^2 = \frac{1 + 0.05}{2}[1 + x_1^2 + y^1]$

$= 1 + \frac{0.05}{2}(1 + \frac{(0.05)}{2})$

$[1 + (0.05)^2 + 1.0513]$

$= 1.0513$

$y(0.05) = 1.0513$.

11) Let $x_0 = 0.05$

$y_0 = 1.0513$, $h = 0.05$

Euler's formula $= y_1^0 = y_0 + hf[x_0\, y_0]$

$= 1.0512 + 0.05\,[1.0538]$

$= 1.0538$

MEM $\rightarrow$ $y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0\, y_0) + f(x_1, y_1^{(0)})\right]$

$= 1.0538 + \frac{0.05}{2}\left[1.0538 + (0.1)^2 + 1.104\right]$

$= 1.1055$

$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0\, y_0) + f(x_1, y_1^{1})\right]$

$= 1.0538 + \frac{0.05}{2}\left[1.0538 + (0.1)^2 + 1.1055\right]$

$= 1.1055$

$y(0.1) = 1.1055$

(*) $x + |\sqrt{y}|$ with initial cond $y = 1$ at $x = 0$ for the range $0 \le x \le 0.4$ in steps of 0.2.

$$E.F = 1 + 0.2(1)$$
$$= 1.2$$

$x_1 = 0.2$

$y(0.2) = 1.22 \approx 1.2309$.

Do this!

→ Rungekutta method

– Formula →

$$y_1 = y_0 + \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

where,

$$k_1 = hf(x_0, y_0)$$

$$k_2 = hf\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_1\right)$$

$$k_3 = hf\left(x_0 + \frac{1}{2}h, y + \frac{1}{2}k_2\right)$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

1) Use RK method to find approx value of y when $x = 0.2$ given than $\frac{dy}{dx} = x + y^2$ & $y = 1$ where $x = 0$ & $h = 0.1$

→ $f(x, y) = x + y^2$ $\quad x_0 = 0$, $y_0 = 1$

$$k_1 = hf(x_0 y_0) = 0.1 f(0,1) = 0.100$$

$$k_2 = hf\left(x_0 + \frac{1}{2}h + y_0 + \frac{1}{2}k_1\right) = 0.11$$

$$k_3 = hf\left(x_0 + \frac{1}{2}h, y + \frac{1}{2}k_2\right) =$$

$(x_1 = x_0 + h)$ $\quad 1 + \frac{0.1}{2} = 1 + 0.5$

$x_{\phi} = (x_0 + h)$

(*) $f(x,y) = x + y^2$

$x_0 = 0$ $h = 0.1$

$y_0 = 1$

$f(x,y) = x + y^2$

$0 + 1^2$

$0 + 1^2 = 1$.

$K_1 = h.[f(x_0, y_0)]$ $= 1$

$= 0.1 [0 \times 1]$

$= 0.1 \times 1$

$K_1 = 0.100$

$K_2 = hf\left(x_0 + \frac{h}{2}\right) \times \left(y_0 + \frac{K_1}{2}\right)$

$= 0.1 \left(0 + \frac{0.1}{2}\right) * \left(1 + \frac{0.100}{2}\right)$

$= 0.1\, f(0.05) \times (1.05)^2$

$= 0.1 (0.05) + (1.1)^2$

$0.05 + (1.1)^2$

$= 0.126$.

$0.05 + 1.1^2$

$= 0.1 + f(0.05) \times (1.1)$

$0.05 + (1.1)^2$

0.11525

$\frac{dy}{dx} = x+y^2$ $y(0) = 1$, find $y(0.2)$ when $h=0.1$

$\rightarrow f(x,y) = x+y^2$

$x_0 = 0$ $y_0 = 1$
$x_1 = 0.1$ $y_1 = 1.1164$
$x_2 = 0.2$ $y_2 =$
$x_3 = 0.3$

$K_1 = hf(x_0 y_0)$
$h(x_0 + y_0^2)$
$= 0.1(0+1) = 0.100$

$K_2 = hf(x_0 + h/2, y_0 + K_1/2)$
$h[(x_0 + h/2) + (y_0 + K_1/2)^2]$
$= 0.1\left[\left[0 + \frac{0.1}{2}\right] + \left(1 + \frac{0.1}{2}\right)^2\right]$
$= 0.11525$

$K_3 = 0.1\left[\left[0 + \frac{0.1}{2}\right] + \left[1 + \frac{K_2 (0.11525)}{2}\right]^2\right]$
$= 0.11685$

$K_4 = \cancel{0.11702} \; hf((x_0 + h) + (y_0 + K_3)^2)$
$= 0.1[(0.1+0) + (1+0.11685)^2]$
$= 0.1347$

$K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$=$

$= 0.11648$

$y_1 = y_0 + K$
$= 1 + 0.11648$
$= 1.1164$

Same as before but here $y_1 = 1.1164$

2.

II $K_1 = 0.1[0.1 + (1.1164)^2]$
$= 0.1346$

$K_4 = 0.1[(0.1+0) + (1.1164 + 0.1576)]$
$= 0.1823$

$K = 0.1572$

$y(0.2) = y_2 = y_1 + K$
$1.1164 + 0.1572$
$= 1.2737$

(2). Find value of y when $x = 0.2$ given that $\frac{dy}{dx} = x+y$

$y = 1$ & $x = 0$

$\rightarrow x_0 = 0 \quad y_0 = 1 \quad h = 0.2 \quad f(x, y) = x+y$

$K_1 = hf(x_0 y_0)$
$= 0.2 f(0, 1) = 0.2$

$K_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2})$
$= 0.2 f(0.1, 1.1) = 0.24$

$K_3 = kf(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2})$
$= 0.2 f(0.2, 1.\cancel{22}) = 0.244$

$K_4 = kf(x_0 + h, y_0 + K_3)$
$= 0.2 f(0.2, 1.244) = 0.2888$

$y(x_0 + h) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + K_4)$
$\Rightarrow y(0.2) = 1.2428.$

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③ Use R-K method to find an approx. value of y when $x = 1.1$ given that $y = 1.2$ when $x = 1$ & $\frac{dy}{dx} = 3x + y^2$

$y(1.1) = 1.7278$