

## \* Top-Down Parsing

Apply By using top-down apply parse tree for the string id+id\*xid

$$E \rightarrow TE'$$

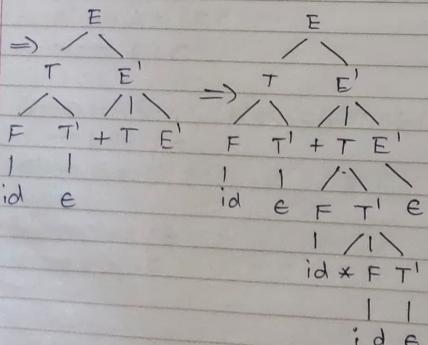
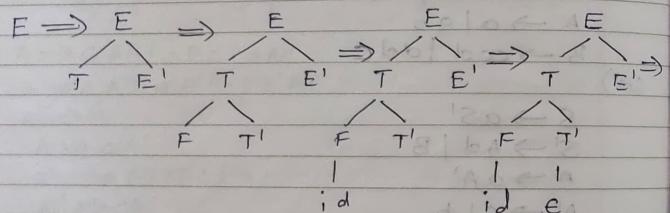
$$E' \rightarrow +TE' \mid \epsilon$$

$$T \rightarrow FT'$$

id+id\*xid

$$T' \rightarrow *FT' \mid \epsilon$$

$$F \rightarrow (CE) \mid id$$



$$\begin{array}{ll} E \xrightarrow{\text{un}} TE' & :: E \rightarrow TE' \\ E \xrightarrow{\text{un}} FT'E' & :: FT \rightarrow FT' \\ E \xrightarrow{\text{un}} id T'E' & :: SF \rightarrow id \\ E \xrightarrow{\text{un}} id +TE' \cancel{\xrightarrow{\text{un}}} & :: E \\ E \xrightarrow{\text{un}} id +FT'E' \cancel{\xrightarrow{\text{un}}} & :: E \\ E \xrightarrow{\text{un}} id +id \cancel{id T'E'} \cancel{\xrightarrow{\text{un}}} & :: E \\ E \xrightarrow{\text{un}} id +id *FT'E' \cancel{\xrightarrow{\text{un}}} & :: E \\ E \xrightarrow{\text{un}} id +id *id T'E' \cancel{\xrightarrow{\text{un}}} & :: E \\ E \xrightarrow{\text{un}} id +id *id E'E' \cancel{\xrightarrow{\text{un}}} & :: E \\ E \xrightarrow{\text{un}} id +id *id \cancel{E'E'} \cancel{\xrightarrow{\text{un}}} & :: E \\ E \xrightarrow{\text{un}} id +id *id \cancel{id} \cancel{\xrightarrow{\text{un}}} & :: E \\ E \xrightarrow{\text{un}} id +id *id \cancel{id} \cancel{\xrightarrow{\text{un}}} & :: E \end{array}$$

## \* Recursive-Descent Parsing

A typical procedure for a non-terminal in a top-down Parser

void AC()

{

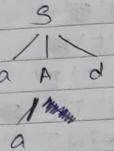
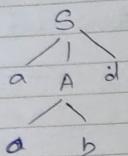
- 1) choose an A-production,  $A \rightarrow x_1 x_2 \dots x_k$ ;
- 2) for  $i=1$  to  $k$  {
- 3) if ( $x_i$  is a non-terminal)
- 4) call procedure  $x_p()$ ;
- 5) else if ( $x_i$  equals the current input symbol  $a$ )
- 6) advance the input to the next symbol;
- 7) else /\* an error has occurred \*/

}

}

g)  $S \rightarrow CAG$   
 $A \rightarrow ab/a$

$w = cad$



### \* FIRST AND FOLLOW functions

Define  $\text{FIRST}(\alpha)$

where  $\alpha$  is any string of grammar symbols,  
 to be the set of terminals that begin  
 strings derived from  $\alpha$ .

Eg:  $A \rightarrow ab/cd/ef$

$$\text{FIRST}(A) = \{a, c, e\}$$

\* if  $\alpha \Rightarrow E$ , then  $E$  can also be in  $\text{FIRST}(\alpha)$

To compute  $\text{FIRST}(X)$  for all grammar  
 Symbols  $X$ , apply the four rules until  
 no more terminals on  $E$  can be added  
 to any first  $\text{FIRST}$  set

① If  $X$  is a terminal, then  
 $\text{FIRST}(X) = \{X\}$

② If  $X \rightarrow E$  is a production then add  $E$  to  $\text{FIRST}(X)$

③ If  $X$  is a non terminal and  $X \rightarrow Y_1 Y_2 \dots Y_k$   
 is a production for some  $k \geq 1$ , then place  
 'a' in  $\text{FIRST}(X)$  if for some  $i$ , 'a' is in  
 $\text{FIRST}(Y_i)$  &  $E$  is in all of  $\text{FIRST}(Y_1) \dots$   
 $\dots \text{FIRST}(Y_{i-1})$  i.e  $Y_1 \dots Y_{i-1} \Rightarrow E$ .

If  $E$  is in  $\text{FIRST}(Y_j)$  for all  $j=1, 2, \dots, k$   
 then add  $E$  to first of  $X$

\* Compute FIRST for the following grammar

does not belong

FOR a production  $X \rightarrow Y_1 Y_2 Y_3$

→ if  $E \notin \text{FIRST}(Y_1)$ , then  $\text{FIRST}(X) = \text{FIRST}(Y_1)$

→ if  $E \in \text{FIRST}(Y_1)$ , then  $\text{FIRST}(X) = \{\text{FIRST}(Y_1) -$   
 $E\} \cup \text{FIRST}(Y_2 Y_3)$

1)  $S \rightarrow aBDh$

$B \rightarrow cC$

$C \rightarrow bC/e$

$D \rightarrow EF$

$E \rightarrow g/g$

$F \rightarrow f/f$

sol)  $\text{FIRST}(S) = \{a\}$

$\text{FIRST}(B) = \{c\}$

$\text{FIRST}(C) = \{b\}$

$\text{FIRST}(D) = \{\text{FIRST}(E) - E\} \cup \text{FIRST}(F)$   
 $= \{g, f, e\}$

$\text{FIRST}(E) = \{g, e\}$

$\text{FIRST}(F) = \{f, e\}$

2)  $S \rightarrow A$   
 $A \rightarrow aB \mid Ad$   
 $B \rightarrow b$   
 $C \rightarrow g$

Sol)  
 $\text{FIRST}(S) \rightarrow \text{Eliminate left recursion}$

$S \rightarrow A$   
 $A \rightarrow aBA' \quad \#$   
 $A' \rightarrow dA' \mid \epsilon$   
 $B \rightarrow b$   
 $C \rightarrow g$

$\text{FIRST}(S) = \{a\}$   
 $\text{FIRST}(A) = \{a\}$   
 $\text{FIRST}(A') = \{d\}$

3)  $S \rightarrow (L) \mid a$   
 $L \rightarrow SL'$   
 $L' \rightarrow SL' \mid \epsilon$

Sol)  
 $\text{FIRST}(S) = \{c, a\}$   
 $\text{FIRST}(L) = \{c, a\}$   
 $\text{FIRST}(L') = \{\epsilon, \epsilon\}$

4)  $S \rightarrow AaAb \mid BbBa$   
 $A \rightarrow \epsilon$   
 $B \rightarrow \epsilon$

Sol)  
 $\text{FIRST}(S) = \{\text{FIRST}(A) - \epsilon\} \cup \text{FIRST}(a) \cup \{\text{FIRST}(B) - \epsilon\} \cup \text{FIRST}(b)$   
 $= \{a, b\}$

$\text{FIRST}(A) = \{\epsilon\}$   
 $\text{FIRST}(B) = \{\epsilon\}$

5)  $E \rightarrow TE'$   
 $E' \rightarrow +TE' \mid G$   
 $T \rightarrow FT'$   
 $T' \rightarrow *FT' \mid \epsilon$   
 $F \rightarrow (E) \mid id$

Sol)  
 $\text{FIRST}(E') = \{+, \epsilon\}$   
 $\text{FIRST}(T') = \{* \mid \epsilon\}$   
 $\text{FIRST}(F) = \{(, id\}$   
 $\text{FIRST}(T) = \text{FIRST}(F) = \{c, id\}$   
 $\text{FIRST}(E) = \text{FIRST}(T) = \{c, id\}$

6)  $S \rightarrow ACB \mid CbB \mid Bg$   
 $A \rightarrow da \mid BC$   
 $B \rightarrow g \mid \epsilon$   
 $C \rightarrow h \mid \epsilon$

Sol)  
 $\text{FIRST}(B) = \{g, \epsilon\}$   
 $\text{FIRST}(C) = \{h, \epsilon\}$   
 $\text{FIRST}(A) = \text{FIRST}(d) \cup \{\text{FIRST}(B) - \epsilon\} \cup \text{FIRST}(C)$   
 $= \{d, g, h, \epsilon\}$   
 $\text{FIRST}(S) = \{\text{FIRST}(A) - \epsilon\} \cup \{\text{FIRST}(C) - \epsilon\} \cup$   
 $\text{FIRST}(B) \cup \text{FIRST}(C) - \epsilon \cup \text{FIRST}(b)$   
 $= \{d, g, h, \epsilon, b, a\}$

## \* Follow Function

Define  $\text{FOLLOW}(A)$  for non-terminal  $A$  to be a set of terminals 'a' that can appear immediately to the right of  $A$  in some sentential form. i.e

$$S \xrightarrow{*} \alpha A \beta$$

To compute  $\text{FOLLOW}(A)$  for all non-terminals, apply the following rules until nothing can be added to any follow set

- 1) place  $\$$  in  $\text{FOLLOW}(S)$  where  $S$  is the start symbol and  $\$$  is input right end marker
- 2) If there is a production  $A \rightarrow \alpha B \beta$ , then everything in  $\text{FIRST}(B)$  except  $\epsilon$  is in  $\text{FOLLOW}(B)$ .
- 3) If there is a production  $A \rightarrow \alpha B$  or  $A \rightarrow \alpha B \beta$  where  $\text{FIRST}(B)$  contains  $\epsilon$  then everything in  $\text{FOLLOW}(A)$  is in  $\text{FOLLOW}(B)$

Compute FOLLOW for the following grammar  
Note: FOLLOW is only for non-terminals

$$\begin{aligned} S &\rightarrow aBDh \\ B &\rightarrow cC \\ C &\rightarrow bc | \epsilon \\ D &\rightarrow EF \\ E &\rightarrow g | \epsilon \\ F &\rightarrow f | \epsilon \end{aligned}$$

Soln)

$$\text{FIRST}(S) = \{a\}$$

$$\text{FIRST}(B) = \{c\}$$

$$\text{FIRST}(C) = \{b, \epsilon\}$$

$$\text{FIRST}(D) = \{g, f, \epsilon\}$$

$$\text{FIRST}(E) = \{g, \epsilon\}$$

$$\text{FIRST}(F) = \{f, \epsilon\}$$

$$\text{FOLLOW}(S) = \{\$\}$$

$$\text{FOLLOW}(D) = \{h\}$$

$$\text{FOLLOW}(B) = \{\text{FIRST}(D) - \epsilon\} \cup \text{FIRST}(h) = \{g, f, h\}$$

$$\text{FOLLOW}(C) = \text{FOLLOW}(B) = \{g, f, h\}$$

$$\text{FOLLOW}(\del{D}) = \text{FOLLOW}(D) = \{h\}$$

$$\begin{aligned} \text{FOLLOW}(E) &= \{\text{FIRST}(F) - \epsilon\} \cup \text{FOLLOW}(D) \\ &= \{f, h\} \end{aligned}$$

$$9) S \rightarrow A$$

$$A \rightarrow aBA'$$

$$A' \rightarrow dA' | \epsilon$$

$$B \rightarrow b$$

$$C \rightarrow g$$

$$\text{Soln)} \quad \text{FIRST}(S) = \{a\} \cancel{, b, d, \epsilon}\}$$

$$\text{FIRST}(A) = \{a\}$$

$$\text{FIRST}(B) = \{b\}$$

$$\text{FIRST}(A') = \{d\}$$

$$\text{FIRST}(C) = \{g\}$$

$$\text{FOLLOW}(S) = \{\$\}$$

$$\text{FOLLOW}(A) = \{\$\}$$

$$\text{FOLLOW}(A') = \{\$\}$$

$$\text{FOLLOW}(B) = \{d, \$\}$$

$$\text{FOLLOW}(C) = \{NA\}$$

Q)  $S \rightarrow (L) | a$

$$L \rightarrow SL^1$$

$$L^1 \rightarrow SL^1 | \epsilon$$

Sol)  $FIRST(S) = \{ (, a \}$

$$FIRST(L) = \{ L, a \}$$

$$FIRST(L^1) = \{ , , \epsilon \}$$

$$FOLLOW(L) = \{ \} \}$$

$$FOLLOW(L^1) = FOLLOW(L) = \{ \} \}$$

$$\begin{aligned} FOLLOW(S) &= \{ FIRST(L^1) - \epsilon \} \cup FOLLOW(L) \\ &\quad \cup \{ FIRST(L^1) - \epsilon \} \cup FOLLOW(L^1) \cup \$ \\ &= \{ , , \$ \} \end{aligned}$$

Q)  $S \rightarrow AaAb | BbBa$

$$A \rightarrow \epsilon$$

$$B \rightarrow \epsilon$$

Sol)  $FOLLOW(S) = \{ \$ \}$

$$FOLLOW(A) = \{ a, b \}$$

$$FOLLOW(B) = \{ b, a \}$$

Q)  $E \rightarrow TE'$

$$E' \rightarrow +TE' | \epsilon$$

$$T \rightarrow FT'$$

$$T' \rightarrow *FT' | \epsilon$$

$$F \rightarrow (E) | id$$

Sol)  $FIRST(F) = \{ (, id \}$

$$FIRST(T') = \{ *, \epsilon \}$$

$$FIRST(T) = \{ (, id, *, \epsilon \}$$

$$FIRST(E') = \{ +, \epsilon \}$$

$$FIRST(E) = \{ (, id, *, +, \epsilon \}$$

$$FOLLOW(E) = \{ \$, ) \}$$

$$FOLLOW(E') = \{ \$, ) \}$$

$$FOLLOW(T) = \{ +, \$, ) \}$$

$$FOLLOW(T') = \{ +, \$, ) \}$$

$$FOLLOW(F) = \{ *, +, \$, ) \}$$

Q)  $S \rightarrow ACB | CbB | Ba$

$$A \rightarrow da | BC$$

$$B \rightarrow g | \epsilon$$

$$C \rightarrow h | \epsilon$$

Sol)  $FIRST(C) = \{ h, \epsilon \}$

$$FIRST(B) = \{ g, \epsilon \}$$

$$FIRST(A) = \{ d, g, h, \epsilon \}$$

$$FIRST(S) = \{ b, d, g, h, a \}$$

$$FOLLOW(A) = \{ g, h, \$ \}$$

$$FOLLOW(B) = \{ \$, a, g, h \}$$

$$FOLLOW(C) = \{ \$, b, g, h \}$$

$$FOLLOW(S) = \{ \$ \}$$

### LL(1) Grammar:

↓  
Producing left most derivation  
Scanning the input from left to right

A grammar G is LL1 if and only if whenever

$A \rightarrow \alpha \mid \beta$  are two disjoint productions of 'G' the following conditions hold

① For no terminal 'a' do both  $\alpha \& \beta$  derived strings begin with 'a'

② Atmost one of  $\alpha \& \beta$  can derive the empty string ③ epsilon( $\epsilon$ )

④ If  $\beta$  derives  $\epsilon$  then  $\alpha$  does not derive any string beginning with terminal in follow of A (FOLLOW(A))

Likewise if  $\alpha$  derives  $\epsilon$  then  $\beta$  does not derive any string beginning with terminal in FOLLOW(A)

### Construction of Predictive Parsing Table

→ INPUT: Grammar G

→ OUTPUT: Parsing table 'M'

→ METHOD: For each production  $A \rightarrow \alpha$  of the grammar do the following

- classmate  
Date \_\_\_\_\_  
Page \_\_\_\_\_
- i) For each terminal 'a' in FIRST( $\alpha$ ) add  $A \rightarrow \alpha$  to M[A,a]
  - ii) If  $\epsilon$  is in FIRST( $\alpha$ ) then for each terminal 'b' in FOLLOW(A) add  $A \rightarrow \alpha$  to M[A,b].
  - iii) If  $\epsilon$  is in FIRST( $\alpha$ ) & \$ is in FOLLOW(A) add  $A \rightarrow \alpha$  to M[A,\$]
  - iv) If after performing the above there is no production at all in M[A,a] then set

10M

• Construct PPT for the following grammar

$$E \rightarrow TE'$$

$$E' \rightarrow +TE' \mid \epsilon$$

$$T \rightarrow FT'$$

$$T' \rightarrow *FT' \mid \epsilon$$

$$F \rightarrow (E) \mid i \mid d$$

Sol:

$$\text{FIRST}(F) = \{ (, ; , id \}$$

$$\text{FIRST}(T) = \{ (, id, *, \epsilon \}$$

$$\text{FIRST}(T') = \{ *, \epsilon \}$$

$$\text{FIRST}(T') = \{ *, \epsilon \}$$

$$\text{FIRST}(E') = \{ +, \epsilon \}$$

## Table Structure:

Non Terminal	Input Symbol					
	id	+	*	(	)	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow FE'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T		$T \rightarrow FT'$		$T \rightarrow FT'$		
T'		$T' \rightarrow E$	$T' \rightarrow FT'$		$T' \rightarrow E$	$T' \rightarrow E$
F	$F \rightarrow id$			$F \rightarrow (E)$		

► Construct predictive parsing table for the following grammar

$$1) S \rightarrow (L) \mid a$$

$$L \rightarrow SL'$$

$$L' \rightarrow , SL' \mid \epsilon$$

Sol)

Non terminal	Input Symbol					
	(	)	a	,	\$	
S	$S \rightarrow (L)$			$S \rightarrow a$		
L		$L \rightarrow SL'$		$L \rightarrow SL'$		
L'			$L' \rightarrow \epsilon$	$L' \rightarrow SL'$		

$$S \rightarrow (L)$$

$$\text{FIRST}((L)) = \{ \}$$

$$S \rightarrow a$$

$$\text{FIRST}(a) = a$$

$$\text{FOLLOW}(L') = \{ \}$$

$$2) S \rightarrow ACB \mid CBB \mid Ba$$

$$A \rightarrow da \mid BC$$

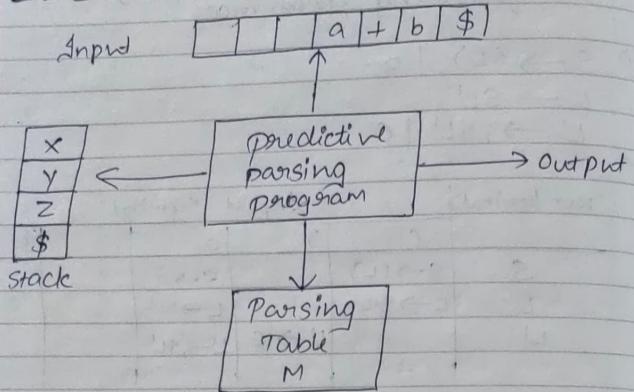
$$B \rightarrow g \mid \epsilon$$

$$C \rightarrow h \mid \epsilon$$

Sol)

Non terminal	Input Symbol					
	a	b	d	g	h	\$
S	$S \rightarrow Ba$	$S \rightarrow CBB$	$S \rightarrow ACB$	$S \rightarrow ACB$	$S \rightarrow Ba$	$S \rightarrow CBB$
A				$A \rightarrow BC$	$A \rightarrow BC$	$A \rightarrow BC$
B	$B \rightarrow \epsilon$			$B \rightarrow g$	$B \rightarrow \epsilon$	$B \rightarrow \epsilon$
C	$C \rightarrow \epsilon$		$C \rightarrow E$	$C \rightarrow h$	$C \rightarrow \epsilon$	$C \rightarrow \epsilon$

## \* Non-Recursive Predictive Parsing



## \* Predictive Parsing Algorithms (Table driven Predictive Parsing)

Input: A string  $w$  & a parsing table  $M$  for a Grammar

Output: If  $w$  is in  $L(G)$ , a leftmost derivation of  $w$  otherwise an error indication

Method: Initially the parser is in a configuration with  $w \$$  in i/p buffer & the start symbol  $S$  of  $G$  is on top of the stack above  $\$$ .

Let ' $a$ ' be the first symbol of  $w$ ; Let ' $x$ ' be the top stack symbol; while ( $x \neq \$$ )

$\text{if } (x = a)$

pop the stack & let ' $a$ ' be the next symbol of  $w$ ;

$\text{else if } (x \text{ is a terminal})$

error();

$\text{else if } (M[x, a] \text{ is an entry})$

error();

$\text{else if } (M[x, a] = x \rightarrow y_1, y_2, \dots, y_k)$

{

    output the production  $x \rightarrow y_1, y_2, \dots, y_k$ ;

    pop the stack;

    push  $y_k, y_{k-1}, \dots, y_1$  onto the stack with  $y_1$  on top;

}

Let  $x$  be the top stack symbol;

}

Matched	Stack	Input	Action
$E \$$	$id + id * id \$$		
$TE' \$$	$id + id * id \$$		Output $E \rightarrow TE'$
$FT'E' \$$	$id + id * id \$$		Output $T \rightarrow FT'$
$id T'E' \$$	$id + id * id \$$		Output $F \rightarrow id$
$id$	$T'E' \$$	$+ id * id \$$	Match id
$id$	$E' \$$	$+ id * id \$$	Output $T \rightarrow E$
$id$	$+ TE' \$$	$+ id * id \$$	Output $E \rightarrow TE'$
$id +$	$TE' \$$	$id * id \$$	Match +
$id +$	$FT'E' \$$	$id * id \$$	Output $T \rightarrow FT'$
$id +$	$id T'E' \$$	$id * id \$$	Output $F \rightarrow id$
$id + id$	$T'E' \$$	$* id \$$	Match id
$id + id$	$* FT'E' \$$	$* id \$$	Output $T \rightarrow FT'$

## Bottom-up Parsing

### UNIT - 3

classmate  
Date \_\_\_\_\_  
Page \_\_\_\_\_

id+id*	FT'E'\$	id\$	OutMatch *
id+id*	idT'E'\$	id\$	Output $F \rightarrow id$
idtid*id	T'E'\$	\$	Match id
idtid*id	E'\$	\$	Output $T' \rightarrow E$
idtid*id	\$	\$	Output $E' \rightarrow E$

#### \* Error Recovery in predictive parsing.

##### 1) Panic mode Error Recovery

→ If MCA,a] is blank, then the input symbol A is skipped

→ If the entry is skipped then the non-terminal on the top of the stack is pop.

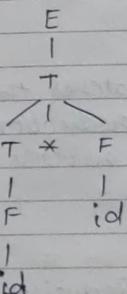
→ If a token on top of the stack does not match the input, then we pop token from stack

#### \* Bottom

$$\begin{aligned} E &\rightarrow \# E + T \mid T \\ T &\rightarrow T * F \mid F \\ F &\rightarrow (E) \mid id \end{aligned}$$

construct id\*id using bottom up approach

$$\begin{array}{ccccccc} \text{Sol}) & id*id & F * id & T * id & T * F & T & T \\ & | & | & | & | & / \backslash & / \backslash \\ & id & F & F & id & T * F & T \\ & & | & & id & | & | \\ & & id & & id & F & id \\ & & & & & | & \\ & & & & & id & \end{array}$$



id\*id, F\*id, T\*id, T\*F, T,E

$$E \xrightarrow{\text{S1}} T \xrightarrow{\text{S2}} T * F \xrightarrow{\text{S3}} T * id \xrightarrow{\text{S4}} F * id \xrightarrow{\text{S5}} id * id$$

→ Bottom up Approach on Bottom up parsing is the reverse of right most derivation

→ Bottom up Parse corresponds to the construct of parse tree for an i/p string beginning

at the leaves & working up towards the root. This process is called Reduction

### \* Reduction

Bottom up parsing is the process of reducing a string  $w$  to the start symbol of the grammar. At each reduction step, a specific substring matching the body of a production is replaced by the non-terminal at the head of that production

Amp

### \* Handle pruning

Handle is a substring that matches the body of a production and whose reduction represents one step along the reverse of a right most derivation

Right Sentential Form	Handle	Reducing production
$id_1 * id_2$	$id_1$	$F \rightarrow id$
$F * id_2$	$F$	$T \rightarrow F$
$T * id_2$	$id_2$	$F \rightarrow id$
$T * F$	$T * F$	$T \not\rightarrow T * F$
$T$	$T$	$E \rightarrow T$

→ The right most derivation in reverse is achieved by handle pruning

### \* Shift-Reduce Parsing

It is a form of bottom up parsing in which a stack holds grammar symbols and input buffer holds the rest of the string to be parsed

#### Initial configuration

Stack	Input
\$	w \$

#### Final configuration

Stack	Input
\$S	\$

Stack	Input	Action
\$ <del>id</del> * <del>id</del> \$	$id_1 * id_2$ \$	shift
\$ <del>id</del> ,	$* id_2$ \$	reduce by $F \rightarrow id$ .
\$ E,	$* id_2$ \$	reduce by $T \rightarrow F$
\$ T	$* id_2$ \$	shift
\$ T *	$id_2$ \$	shift
\$ T * <del>id</del> 2	\$	reduce by $F \rightarrow id$
\$ T * <del>T</del> * F	\$	reduce by $T \rightarrow T * F$
\$ T	\$	reduce by $E \rightarrow T$
\$ E	\$	Accept

Shift, Reduce, Accept, Error

## \* Introduction to L-R Parsing (simple LR)

### LR(k) parsing

- ↓
- left to right scanning of the input.
- constructive right most derivation in reverse.
- No. of input signals of look-ahead.

Why LR Parsers?

## Amp 10m Items & LR(0) automaton

LR(0) automaton = LR(0) grammar + DFA

### Item

States represent sets of items. An LR(0) item of a grammar  $G_1$  is a production of  $G_1$  with a '.' at some position of the body.

Eg:  $A \rightarrow xyz$  production yields the foll' items

- $A \rightarrow .xyz$
- $A \rightarrow x.yz$
- $A \rightarrow xy.z$
- $A \rightarrow xyz.$

One collection of sets of LR(0) items called canonical LR(0) collection provides the basis for constructing a DFA. Such an automaton is called LR(0) automaton.

Amp

To construct the canonical LR(0) collection for a grammar we define an augmented grammar and two functions CLOSURE & GOTO

If  $G_1$  is a grammar with start symbol  $S$  then  $G'_1$  the augmented grammar for  $G_1$  is a  $G_1$  with a new start symbol  $S'$  and production  $S' \rightarrow S$

### CLOSURE of item sets

If  $I$  is a set of item for a grammar  $G_1$  then CLOSURE( $I$ ) is the set of items constructed from  $I$  by the two rules:

- Initially add empty item in  $I$  to CLOSURE( $I$ )
- If  $A \rightarrow x.BB$  is in CLOSURE( $I$ ) &  $B \rightarrow \gamma$  is a production then add the item  $B \rightarrow \gamma$  to CLOSURE( $I$ ) if it is not already there. Apply this rule until no more new items can be added to closure( $I$ )

→ Ex: Compute the closure of the foll' items

$$\begin{aligned} E &\rightarrow E + T \mid T \\ T &\rightarrow T * F \mid F \\ F &\rightarrow (E) \mid id \end{aligned}$$

⇒ augmented  $G'_1 \Rightarrow E' \rightarrow E \quad E \rightarrow E..$

- Compute closure of  $E' \rightarrow E$

$$I \Rightarrow I : E' \rightarrow E$$

CLOSURE(I)

$$\{E' \rightarrow E^2\}$$

$E' \rightarrow E$   
 $E \rightarrow E + T$   
 $E \rightarrow .T$   
 $T \rightarrow .T + F$   
 $T \rightarrow .F$   
 $F \rightarrow .(E)$   
 $F \rightarrow .id$

Q) CLOSURE of  $E \rightarrow .T$

Sol<sup>n</sup> CLOSURE( $E \rightarrow .T$ )

$T \rightarrow .T + F$   
 $T \rightarrow .F$   
 $F \rightarrow .(E)$   
 $F \rightarrow .id$

\* CLOSURE Computation:

Set of Items CLOSURE(I) {

$J = I;$

repeat

for (each item  $A \rightarrow \alpha, BB$  in J)

for (each production  $B \rightarrow \beta$  of G)

if ( $B \rightarrow \beta$  is not in J)

add  $B \rightarrow \beta$  to J;

until no more items are added

on one round;

repeat J;

}

• Compute the closure of the set  $\{ E \rightarrow E + T \}$

$E \rightarrow E + T | T$

$T \rightarrow T * F | F$

$F \rightarrow (E) | id$

Sol<sup>n</sup>

CLOSURE of  $E \rightarrow E + T$  is

$E \rightarrow E + .T$	→ Kernel
$T \rightarrow .T * F$	→ Non-kernel
$T \rightarrow .F$	
$F \rightarrow .(E)$	
$F \rightarrow .id$	

Kernel items: The initial item is  $S \rightarrow .S$  and all items whose dots are not at the left end

Non-kernel items:

All items with their dots at the left end  
except for  $S \rightarrow .S$

Function GOTO

$GOTO(I, x)$  where I is a set of items & x is a grammar symbol is defined to be the closure of the set of all items  $[A \rightarrow \alpha, xB]$  such that  $[A \rightarrow \alpha, xB]$  is in I'

GOTO function is used to define the transitions in LR(0) automata.

GOTO( $I, x$ ) specifies the transition from state  $I$  under input  $x$

- Compute GOTO( $I, +$ )  $\rightarrow I_1$  for the set  $I = \{E\}$

Sol<sup>n</sup>) GOTO( $I, +$ ) is

$$E \rightarrow E + T$$

$$T \rightarrow .T * F$$

$$T \rightarrow .F$$

$$F \rightarrow .(E)$$

$$F \rightarrow .id$$

Compute the GOTO( $I$ )

- Find closure of  $\{E\} \rightarrow E^*$  all terms of  $E^* \rightarrow E$
- $E \rightarrow E + T \mid T$
- $T \rightarrow T * F \mid F$
- $F \rightarrow (E) \mid id$

Sol<sup>n</sup>)  $E^* \rightarrow .E$

$$E \rightarrow E + T$$

$$E \rightarrow .T$$

$$T \rightarrow .T * F$$

$$T \rightarrow .F$$

$$F \rightarrow .(E)$$

$$F \rightarrow .id$$

- Compute GOTO( $I_1, E$ )

Sol<sup>n</sup>)  $E^* \rightarrow E.$

$$E \rightarrow E + T$$

$\downarrow$   
Accept

• Compute GOTO( $I_0, T$ )

Sol<sup>n</sup>)  $E \rightarrow T.$

$$T \rightarrow T * F$$

$\left. \right\} \rightarrow I_2$

• Compute GOTO( $I_0, F$ )

Sol<sup>n</sup>)  $T \rightarrow F. \rightarrow I_3$

• Compute GOTO( $I_0, id$ )

Sol<sup>n</sup>)  $F \rightarrow id. \rightarrow I_4 \quad IS$

• Compute GOTO( $I_0, ()$ )

Sol<sup>n</sup>)  $F \rightarrow .(E)$

$$E \rightarrow .E + T$$

$$E \rightarrow .T$$

$$T \rightarrow .T * F$$

$$T \rightarrow .F$$

$$F \rightarrow .(E)$$

$$F \rightarrow .id$$

GOTO( $I_1, +$ )

$$E \rightarrow E + T$$

$$T \rightarrow .T * F$$

$$T \rightarrow .F$$

$$F \rightarrow .(E)$$

$$F \rightarrow .id$$

GOTO( $I_2, *$ )

$$T \rightarrow T * F$$

$$F \rightarrow .(E)$$

$$F \rightarrow .id$$

$\text{GOTO}(I_5, E)$   
 $F \rightarrow (E,)$     }  
 $E \rightarrow E + T$     }  $\rightarrow I_8$

$\text{GOTO}(I_5, T) \rightarrow I_2$

$\text{GOTO}(I_5, F) \rightarrow I_3$

$\text{GOTO}(I_5, ()) \rightarrow I_5$

$\text{GOTO}(I_5, id) \rightarrow I_4$

$\text{GOTO}(I_8, ))$   
 $F \rightarrow (E) \rightarrow I_9$

$\text{GOTO}(I_8, +) \rightarrow I_6$

$\text{GOTO}(I_6, T)$   
 $E \rightarrow E + T.$     }  
 $T \rightarrow T * F$     }  $\rightarrow I_{10}$

$\text{GOTO}(I_6, F) \rightarrow I_3$

$\text{GOTO}(I_6, ()) \rightarrow I_5$

$\text{GOTO}(I_6, id) \rightarrow I_4$

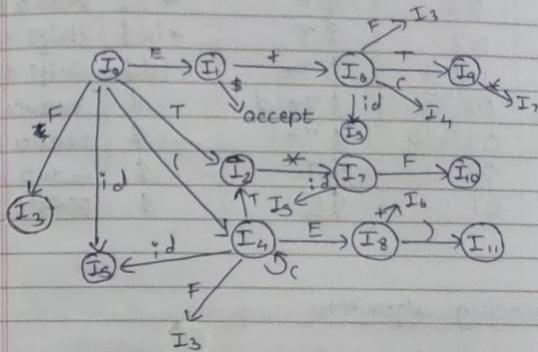
$\text{GOTO}(I_{10}, *) \rightarrow I_7$

$\text{GOTO}(I_7, F)$   
 $T \rightarrow T * F.$   $\rightarrow I_{11}$

$\text{GOTO}(I_7, ()) \rightarrow I_5$

$\text{GOTO}(I_7, id) \rightarrow I_4$

$I_0$  is the start state



↓  
step

Algorithm

Computation of canonical collection of LR(0) items

void items( $G'$ ) {

$C = \{\text{CLOSURE}(\{[S] \rightarrow S\})\};$

repeat

for each set of items  $I$  in  $C$

for each grammar symbol  $x$

if ( $\text{GOTO}(I, x)$  is not empty and not in  $C$ )

add  $\text{GOTO}(I, x)$  to  $C$ ;

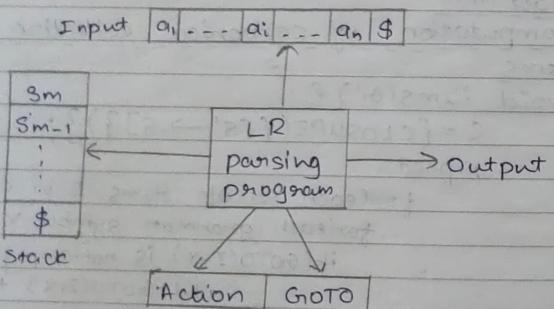
until no new sets of items are added to  $C$  on a round.

## \* Shift-Reduce parser on input id\*id

LINE	Stack	Symbols	Input	Action
1	0	\$	id*ids\$	shift to S
2	05	\$ id	*ids\$	reduce by F id
3	03	\$ F	*id\$	reduce by T F
4	02	\$ T	*id\$	shift to T
5	027	\$ T* x	id\$	shift to S
6	0275	\$ T*xid	\$	reduce by F id
7	02710	\$ T*x F	\$	reduce by T F*
8	02	\$ T	\$	reduce by E T
9	01	\$ E	\$	Final Accept

## \* L-R parsing algorithm

Model of an L-R parser



The Parsing table consists of 2 parts,  
Action & GOTO

The ACTION Function takes as arguments a state 'i' & a terminal 'a' or '\$'. The value of action  $ACTION[i, a]$  can have one of 4 forms

- i) shift j, where j is a state
- ii) Reduce  $A \rightarrow B$
- iii) Accept
- iv) Error

we extend the

The GOTO Function to states. If  $GOTO[i, A] = Ij$ , then GOTO also maps a state 'i' and a non-terminal 'A' to state 'j'

Algorithm

Input: A string  $w$  & L-R parsing table with functions action & goto for a grammar  $G$

Output: If  $w$  is  $L(G)$  i.e (language of that grammar), the reduction steps of a bottom-up parse of  $w$ , otherwise an error indication

Method: Initially, the parser has  $S_0$  on its stack, where  $S_0$  is the initial state &  $w\$$  in the input buffer. The parser then executes the following program.

Let ' $a$ ' be the first symbol of  $w\$$ ;  
while(1){

    Let 's' be the state on top of the stack;  
    if action of  $ACTION[s, a] = \text{shift } t$

        PUSH 't' onto the stack;

Let 'a' be the next input symbol;

else if ( $\text{ACTION}[S, a] = \text{Reduce } A \rightarrow B$ )

POP  $|B|$  symbols of the stack;

Let state 't' now be on top of stack;  
PUSH  $\text{GOTO}[t, A]$  onto the stack;

OUTPUT the production  $A \rightarrow B$ ;

else if ( $\text{ACTION}[S, a] = \text{accept}$ )

break;

else

call error recovery routine;

$E \rightarrow E +$

The codes for the actions are

- 1)  $S_i \rightarrow \text{shift}$  for stack state  $i$
- 2)  $g_j \rightarrow \text{reduce by the production number } j$
- 3)  $\text{acc} \rightarrow \text{accept}$
- 4) blank means error

### \* Constructing SLR parsing table

Input: An augmented grammar  $G'$

Output: The SLR parsing table function  $\text{ACTION}$  &  $\text{GOTO}$  for  $G'$

Method:

- 1) Construct  $C = \{ I_0, I_1, \dots, I_n \}$   
The collection of sets of LR(0) items for  $G'$
- 2) State ' $i$ ' is constructed from ' $I_i$ '. The parsing actions for state ' $i$ ' are determined as follows
  - i) If  $[A \rightarrow x.AB]$  is in ' $I_i$ ' and  $\text{GOTO}[I_i, A] = j$ '  
then set  $\text{ACTION}[i, a]$  to 'shift  $j$ '
  - ii) If  $[A \rightarrow x.]$  is in ' $I_i$ ', then set  $\text{ACTION}[i, a]$  to reduce  $A \rightarrow x$  for all ' $a$ ' in  $\text{FOLLOW}(A)$   
Here ' $A$ ' may not be ' $S$ '
  - iii) If  $[S' \rightarrow S.]$  is in  $I_i$ , then set  $\text{ACTION}[i, \$]$ .
- 3) The  $\text{GOTO}$  transitions for state ' $i$ ' are constructed for all non-terminals ' $A$ ' using the rule if  $\text{GOTO}(I_i, A) = I_j$   
then  $\text{GOTO}[i, A] = j$

### \* Simple LR

#### Constructing SLR Parsing table

State	id	ACTION				GOTO			
		+	*	(	)	\$	E	T	F
0	s5			s4					
1		s6					1	2	3
2			g2	s7					
3			g4	s4					
4	s5			s4					
5			g6	g6			8	2	3
6	s5				g6	g6			
7	s5			s4				9	3
8		s6							10
9			g1	s7			g1	g1	
10			g3	g3			g3	g3	
11			g5	g5			g5	g5	

4) All entries not defined by rules 2 & 3 are made error.

5) The initial state of the parser is the one constructed from the set of items containing  $S' \rightarrow .S$

• Construct LR(0) Automata [SLR Parser], SLR parsing table for the foll grammar

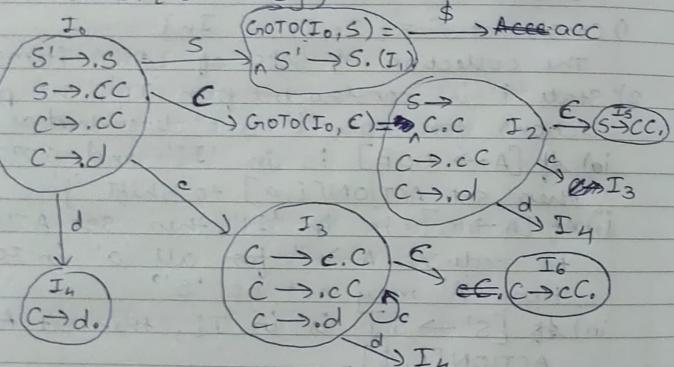
1)  $S \rightarrow CC$

2)  $C \rightarrow cC$

3)  $C \rightarrow d$

So 1)  $S' \rightarrow S$  (First Augment)

CLOSURE({ $S' \rightarrow S$ })



STATE	ACTION			GOTO	
	c	d	\$	S	C
0	$s_3$	$s_4$			
1			acc	1	2
2	$s_3$	$s_4$			5
3	$s_3$	$s_4$			6
4	$g_3$	$g_3$	$g_3$		
5			$g_1$		
6	$g_2$	$g_2$	$g_2$		

→ Parse the string ccdd for the above grammar

LINE	STACK	SYMBOLS	INPUT	ACTION
1	0	\$	ccdd\$	shift to 3
2	03	\$c	ccdd\$	shift to 3
3	033	\$cc	dd\$	shift to 4
4	0334	\$ccd	d\$	Reduce by $C \rightarrow d$
5	0336	\$ccc	,	Reduce by $C \rightarrow CC$
6	036	\$CC	d\$	Reduce by $C \rightarrow CC$
7	02	\$C	d\$	Shift to 4
8	024	\$cd	\$	Reduce by $C \rightarrow d$
9	025	\$CC	.	Reduce by $S \rightarrow CC$
10	01	\$S	\$	accept

## \* Viable prefix

The prefixes of right sentential forms that can appear on the stack of a shift-reduce parser are called viable prefix

$$E \xrightarrow{*_{\text{sim}}} F * id \xrightarrow{*_{\text{sim}}} (E) * id \xrightarrow{*_{\text{sim}}} \dots$$

## \* More powerful LR parsers

① "Canonical LR" or LR

② "Lookahead LR" or LALR

③ Canonical LR(1) items

The general form of LR(1) item is  $[A \rightarrow \alpha, B, a]$  where  $A \rightarrow \alpha B$  is the production and 'a' is a terminal or '\$'

'1' refers to length of the second component called lookahead of the item.

• Constructing LR(1) sets of items

► CLOSURE (Algorithm)

Set of Items CLOSURE(I) {

repeat

for each item  $[A \rightarrow \alpha, B, a]$  in I

for each production  $B \rightarrow \gamma$  in G'

for each terminal b in FIRST(B $\gamma$ )

add  $[B \rightarrow \gamma, b]$  to set I;

repeat until no more items are added to I;  
return I;

3

• Find the closure of the items

$$1) S \rightarrow CC$$

$$2) C \rightarrow cC$$

$$3) C \rightarrow d$$

$$\text{sol: } S' \rightarrow S$$

$$\text{CLOSURE}(f S' \rightarrow .S, \$)$$

$$A \rightarrow \alpha, B\beta, a$$

$$A = S$$

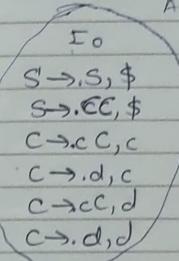
$$B = S$$

$$a = \$$$

$$\text{FIRST}(Ba) = \text{FIRST}(\epsilon\$) \$$$

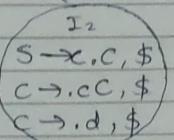
$$\text{FIRST}(Ba) = \text{FIRST}(\$)$$

$$= \{c, d\}$$



• Find the CLOSURE of  $S' \rightarrow S., \$$   
 $\Rightarrow S' = S' \rightarrow S., \$ \rightarrow I_1$

• Find the CLOSURE of  $S \rightarrow C.C, \$$



► GOTO (Algorithm)

set of Items  $\rightarrow \text{GOTO}(I, x) \{$   
 Initialize  $J$  to the empty set;  
 for(each item  $[A \rightarrow \alpha \cdot x \beta, a]$  in  $I$ )  
     add item  $[A \rightarrow \alpha x \cdot \beta, a]$  to set  $J$ ;  
 return  $\text{CLOSURE}(J)$ ;

}

- Find  $\text{GOTO}(I_0, S)$  from previous sum  $\rightarrow$

$S \rightarrow E \cdot C, \$$

$C \rightarrow .CC, \$$

$C \rightarrow .d, \$$

$\text{GOTO}(I_0, C)$

$C \rightarrow c \cdot C, c/d$  —  $I_3$   
 $C \rightarrow .CC, c/d$   
 $C \rightarrow .d, c/d$

$\text{GOTO}(I_2, C)$

$S \rightarrow CC \cdot, \$$  —  $I_5$

$\text{GOTO}(I_2, C)$

$C \rightarrow c \cdot C, \$$  —  $I_6$   
 $C \rightarrow .CC, c/d$   
 $C \rightarrow .d, c/d$