TRENDS IN INTERNATIONAL MATHEMATICS AND SCIENCE STUDY

TIMSS

TIMSS Advanced 2008 User Guide

for the International Database

Released Items

Advanced Mathematics

TIMSS & PIRLS
International Study Center
Lynch School of Education, Boston College

TIMSS Advanced 2008 User Guide for the International Database

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TIMSS & PIRLS
International Study Center
Lynch School of Education, Boston College

Item ID	Subject	Block	Block Seq	Content Domain	Cognitive Domain	Maximum Points	Key
MA13001	Advanced Mathematics	M1	01	Algebra	Knowing	1	D
MA13002	Advanced Mathematics	M1	02	Algebra	Reasoning	1	A
MA13003	Advanced Mathematics	M1	03	Algebra	Reasoning	1	E
MA13004	Advanced Mathematics	M1	04	Calculus	Knowing	1	B
MA13006	Advanced Mathematics	M1	06	Calculus	Knowing	1	D
MA13007	Advanced Mathematics	M1	07	Geometry	Applying	1	A
MA13008	Advanced Mathematics	M1	08	Geometry	Reasoning	1	D
MA13009	Advanced Mathematics	M1	09	Algebra	Applying	1	C
MA13021	Advanced Mathematics	M3	01	Geometry	Knowing	1	A
MA13024	Advanced Mathematics	M3	04	Calculus	Knowing	1	B
MA13025A	Advanced Mathematics	M3	05	Calculus	Knowing	1	See scoring guide
MA13025B	Advanced Mathematics	M3	05	Calculus	Knowing	1	See scoring guide
MA13026A	Advanced Mathematics	M3	06	Geometry	Applying	1	See scoring guide
MA13026B	Advanced Mathematics	M3	06	Geometry	Applying	1	See scoring guide
MA13027	Advanced Mathematics	M3	07	Algebra	Reasoning	2	See scoring guide
MA13028	Advanced Mathematics	M3	08	Algebra	Knowing	1	See scoring guide
MA13029	Advanced Mathematics	M3	09	Geometry	Reasoning	2	See scoring guide
MA23069	Advanced Mathematics	M6	01	Algebra	Applying	1	D
MA23135	Advanced Mathematics	M6	02	Algebra	Applying	1	See scoring guide
MA23208	Advanced Mathematics	M6	03	Algebra	Reasoning	1	A
MA23165	Advanced Mathematics	M6	04	Calculus	Applying	1	See scoring guide
MA23039	Advanced Mathematics	M6	05	Calculus	Knowing	1	D
MA23159	Advanced Mathematics	M6	06	Calculus	Knowing	1	See scoring guide
MA23198	Advanced Mathematics	M6	07	Calculus	Reasoning	1	See scoring guide
MA23042	Advanced Mathematics	M6	08	Calculus	Knowing	1	B
MA23055	Advanced Mathematics	M6	09	Geometry	Knowing	1	D

Item ID	Subject	Block	Block Seq	Content Domain	Cognitive Domain	Maximum Points	Key
MA23080	Advanced Mathematics	M6	10	Geometry	Reasoning	1	A
MA23021	Advanced Mathematics	M6	11	Geometry	Applying	1	B
MA23004	Advanced Mathematics	M7	01	Algebra	Reasoning	1	B
MA23063	Advanced Mathematics	M7	02	Algebra	Applying	1	B
MA23141	Advanced Mathematics	M7	03	Algebra	Knowing	1	See scoring guide
MA23133	Advanced Mathematics	M7	04	Algebra	Knowing	1	D
MA23158	Advanced Mathematics	M7	05	Calculus	Applying	1	D
MA23151	Advanced Mathematics	M7	06	Calculus	Reasoning	1	C
MA23035A	Advanced Mathematics	M7	07	Calculus	Applying	1	See scoring guide
MA23035B	Advanced Mathematics	M7	07	Calculus	Applying	1	See scoring guide
MA23050	Advanced Mathematics	M7	08	Calculus	Knowing	1	B
MA23041	Advanced Mathematics	M7	09	Calculus	Knowing	1	A
MA23182	Advanced Mathematics	M7	10	Geometry	Applying	1	D
MA23170	Advanced Mathematics	M7	11	Geometry	Applying	1	See scoring guide

Item ID **MA13001**	Advanced Mathematics	Block_Sequence **M1_01**

The functions f and g are defined by $f(x)=x-1$ and $g(x)=(x+3)^2$.

$g(f(x))$ is equal to

(A) $(x-1)(x+3)^2$

(B) $(x+3)^2-1$

(C) $(2x-2)^2$

(D) $(x+2)^2$

(E) x^2+8

MA13001

TIMSS Advanced 2008

Content Domain

Algebra

Cognitive Domain

Knowing

Maximum Points

1

Key

D

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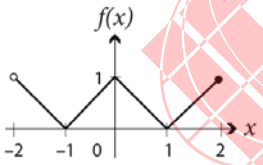
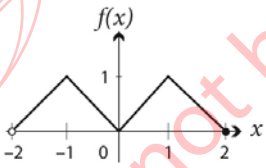
Item ID **MA13002**	Advanced Mathematics	Block_Sequence **M1_02**

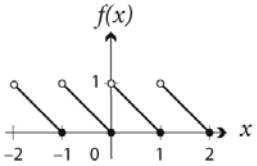
A function f is defined by:

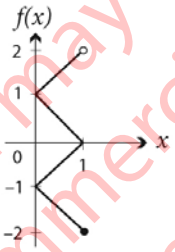
$$f(x) = -x - 1 \quad \text{if} \quad -2 < x \leq -1$$
$$f(x) = x + 1 \quad \text{if} \quad -1 < x \leq 0$$
$$f(x) = -x + 1 \quad \text{if} \quad 0 < x \leq 1$$
$$f(x) = x - 1 \quad \text{if} \quad 1 < x \leq 2$$

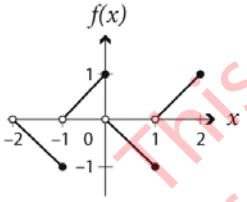
Which is the graph of f?

Ⓐ 

Ⓑ 

Ⓒ 

Ⓓ 

Ⓔ 

MA13002

TIMSS Advanced 2008

Content Domain

Algebra

Cognitive Domain

Reasoning

Maximum Points

1

Key

A

Item ID **MA13003**	Advanced Mathematics	Block_Sequence **M1_03**

Two mathematical models are proposed to predict the return y, in dollars, from the sale of x thousand units of an article (where $0 < x < 5$). Each of these models, P and Q, is based on different marketing methods.

model P: $y = 6x - x^2$
model Q: $y = 2x$

For what values of x does model Q predict a greater return than model P?

(A) $0 < x < 4$
(B) $0 < x < 5$
(C) $3 < x < 5$
(D) $3 < x < 4$
(E) $4 < x < 5$

MA13003

TIMSS Advanced 2008

Content Domain
Algebra

Cognitive Domain
Reasoning

Maximum Points
1

Key
E

Item ID **MA13004**	Advanced Mathematics	Block_Sequence **M1_04**

$\lim_{x \to +\infty} \frac{(2x+1)(x+1)}{3x^2-2}$ is equal to

(A) $-\frac{1}{2}$

(B) $\frac{2}{3}$

(C) 1

(D) 6

(E) ∞

MA13004

TIMSS Advanced 2008

Content Domain

Calculus

Cognitive Domain

Knowing

Maximum Points

1

Key

B

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Item ID **MA13006**	Advanced Mathematics	Block_Sequence **M1_06**

The derivative with respect to x of $\frac{4}{\sqrt{3x-4}}$ is

Ⓐ $12\sqrt{3x-4}$

Ⓑ $\frac{4}{\sqrt{3}}$

Ⓒ $\frac{-2}{(3x-4)^{\frac{3}{2}}}$

Ⓓ $\frac{-6}{(3x-4)^{\frac{3}{2}}}$

Ⓔ $6\sqrt{3x-4}$

MA13006

TIMSS Advanced 2008

Content Domain

Calculus

Cognitive Domain

Knowing

Maximum Points

1

Key

D

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Item ID **MA13007**	Advanced Mathematics	Block_Sequence **M1_07**

MA13007

One side of an equilateral triangle lies along the x-axis. The sum of the slopes of the three sides is

(A) 0
(B) -1
(C) 1
(D) $2\sqrt{3}$
(E) $1+2\sqrt{3}$

TIMSS Advanced 2008

Content Domain
Geometry

Cognitive Domain
Applying

Maximum Points
1

Key
A

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Item ID **MA13008**	Advanced Mathematics	Block_Sequence **M1_08**

MA13008

Triangle PQR is an isosceles right triangle with a right angle at P. If PT is a median of the triangle, then PT has the same length as

(A) PR
(B) PQ
(C) QR
(D) QT

TIMSS Advanced 2008

Content Domain
Geometry

Cognitive Domain
Reasoning

Maximum Points
1

Key
D

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Item ID **MA13009**	Advanced Mathematics	Block_Sequence **M1_09**

MA13009

How many points with integer coordinates are there on the graph of the function $y = \frac{12}{x}, x > 0$?

(A) 2
(B) 4
(C) 6
(D) infinitely many

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Content Domain
Algebra

Cognitive Domain
Applying

Maximum Points
1

Key
C

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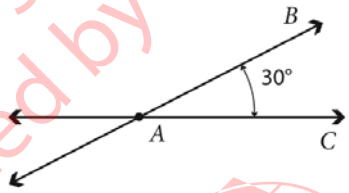
Item ID **MA13021**	Advanced Mathematics	Block_Sequence **M3_01**

As line AB rotates about line AC in space, keeping an angle of 30°, what figure is traced out by the line AB?



(A) a cone
(B) a cylinder
(C) a spiral
(D) a circle
(E) a sphere

MA13021

TIMSS Advanced 2008

Content Domain
Geometry

Cognitive Domain
Knowing

Maximum Points
1

Key
A

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Item ID **MA13024**	Advanced Mathematics	Block_Sequence **M3_04**

$\int_1^2 \left(x - \frac{1}{x^2} \right) dx$ is equal to

(A) $-3\frac{1}{8}$

(B) 1

(C) $2\frac{5}{8}$

(D) 4

(E) $4\frac{1}{2}$

MA13024

TIMSS Advanced 2008

Content Domain

Calculus

Cognitive Domain

Knowing

Maximum Points

1

Key

B

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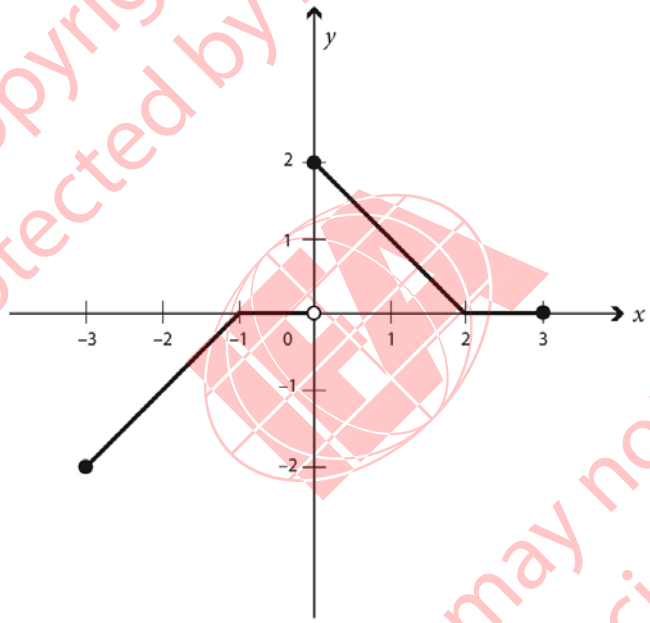
Item ID **MA13025A**	Advanced Mathematics	Block_Sequence **M3_05**

The function $y = f(x), -3 \le x \le 3$, is defined in the following graph



A. For what value(s) of x in the interval $-3 < x < 3$ is the function f NOT continuous?

B. For what value(s) of x in the interval $-3 < x < 3$ is the function f NOT differentiable?

MA13025

TIMSS Advanced 2008

Content Domain

Calculus

Cognitive Domain

Knowing

Maximum Points

1

Key

See scoring guide

TIMSS & PIRLS International Study Center
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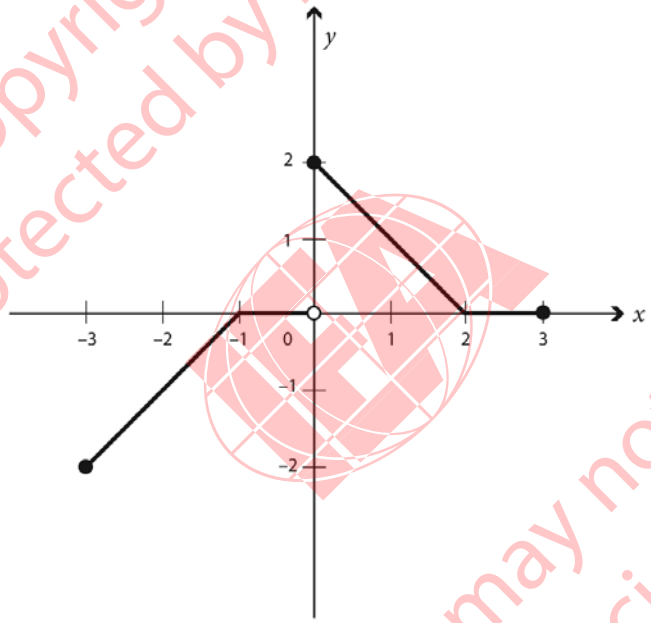
Item ID **MA13025A**	Advanced Mathematics	Block_Sequence **M3_05**

Code	Response	Item: MA13025A
	Correct Response	
10	Correct answer: Only for $x = 0$. Accept answers such as "in points (0, 0) and (0, 2)" as correct.	
	Incorrect Response	
79	Incorrect (including crossed out, erased, stray marks, illegible, or off task) *Examples:* *1. Any of the following answers:* *For $x = -1$ OR For $x = 2$ OR For $x = -1$ and $x = 2$* *2. For all values of x in the interval* $0 \le x \le 2$ ***Note:** Use this code for all answers "interval with endpoints 0 and 2" no matter whether the interval is open or closed.* *3. For <u>no</u> values of x, that is, the function is continuous for all x in the interval* $-3 < x < 3$ *("Continuous" may be confused with "defined.")* *4. x in* $(-3, -1) \cup (0, 2)$ OR x in $(-1, 0) \cup (2, 3)$	
	Nonresponse	
99	Blank	

Code	Response	Item: MA13025B
	Correct Response	
10	Correct answer: for $x = -1$ and for $x = 0$ and for $x = 2$ **Note:** Accept answers that also include $x = -3$ and/or $x = 3$. Accept answers shown as points in the plane instead of values of x (e.g., language such as "point (−1, 0)" instead of $x = -1$).	
	Incorrect Response	
70	For $x = 0$	
71	Any of the following answers: For $x = -1$ OR For $x = 2$ OR For $x = -1$ and for $x = 2$	
72	For all values of x in the interval $-1 \le x \le 0$ and $2 \le x \le 3$ (Misconception: "where f is flat, it has no derivative.")	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task) *Examples:* *1. For <u>NO</u> values of x (i.e., the function is differentiable for all x in the interval $-3 < x < 3$)* *2. For ALL values of x in the interval $-3 < x < 3$ (i.e., the function is nondifferentiable for every value of x).* *3. For ALL values of x in the interval* $-1 \le x \le 0$ or in the interval $2 \le x \le 3$ *4. The question cannot be answered because we do not know the function f' or any similar expression.*	
	Nonresponse	
99	Blank	

Item ID **MA13025B**	Advanced Mathematics	Block_Sequence **M3_05**

The function $y = f(x), -3 \le x \le 3$, is defined in the following graph



A. For what value(s) of x in the interval $-3 < x < 3$ is the function f NOT continuous?

B. For what value(s) of x in the interval $-3 < x < 3$ is the function f NOT differentiable?

MA13025

TIMSS Advanced 2008

Content Domain
Calculus

Cognitive Domain
Knowing

Maximum Points
1

Key
See scoring guide

TIMSS & PIRLS International Study Center
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Item ID **MA13025B**	Advanced Mathematics	Block_Sequence **M3_05**

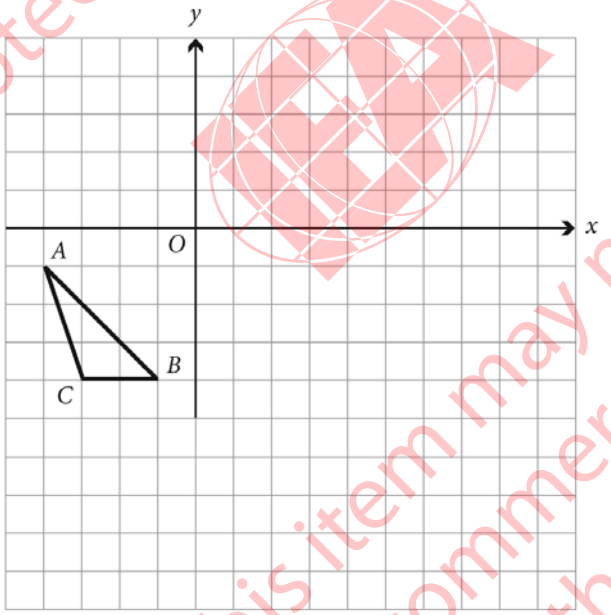
Code	Response	Item: MA13025A
	Correct Response	
10	Correct answer: Only for $x = 0$. Accept answers such as "in points (0, 0) and (0, 2)" as correct.	
	Incorrect Response	
79	Incorrect (including crossed out, erased, stray marks, illegible, or off task) *Examples:* *1. Any of the following answers:* *For $x = -1$ OR For $x = 2$ OR For $x = -1$ and $x = 2$* *2. For all values of x in the interval* $0 \le x \le 2$ ***Note:** Use this code for all answers "interval with endpoints 0 and 2" no matter whether the interval is open or closed.* *3. For <u>no</u> values of x, that is, the function is continuous for all x in the interval* $-3 < x < 3$ *("Continuous" may be confused with "defined.")* *4. x* in $(-3, -1) \cup (0, 2)$ OR *x* in $(-1, 0) \cup (2, 3)$	
	Nonresponse	
99	Blank	

Code	Response	Item: MA13025B
	Correct Response	
10	Correct answer: for $x = -1$ and for $x = 0$ and for $x = 2$ **Note:** Accept answers that also include $x = -3$ and/or $x = 3$. Accept answers shown as points in the plane instead of values of x (e.g., language such as "point $(-1, 0)$" instead of $x = -1$).	
	Incorrect Response	
70	For $x = 0$	
71	Any of the following answers: For $x = -1$ OR For $x = 2$ OR For $x = -1$ and for $x = 2$	
72	For all values of x in the interval $-1 \le x \le 0$ and $2 \le x \le 3$ (Misconception: "where f is flat, it has no derivative.")	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task) *Examples:* *1. For <u>NO</u> values of x (i.e., the function is differentiable for all x in the interval* $-3 < x < 3$*)* *2. For ALL values of x in the interval* $-3 < x < 3$ *(i.e., the function is nondifferentiable for every value of x).* *3. For ALL values of x in the interval* $-1 \le x \le 0$ or in the interval $2 \le x \le 3$ *4. The question cannot be answered because we do not know the function* f' *or any similar expression.*	
	Nonresponse	
99	Blank	

Item ID **MA13026A**	Advanced Mathematics	Block_Sequence **M3_06**

A. Triangle ABC is reflected in the y-axis. On the diagram, draw and label triangle $A'B'C'$, the image of triangle ABC under this reflection.

B. Triangle ABC is rotated through 90° anti-clockwise, centre O. On the diagram, draw and label triangle $A''B''C''$, the image of triangle ABC under this rotation.



MA13026

TIMSS Advanced 2008

Content Domain
Geometry

Cognitive Domain
Applying

Maximum Points
1

Key
See scoring guide

TIMSS & PIRLS International Study Center
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Item ID **MA13026A**	Advanced Mathematics	Block_Sequence **M3_06**

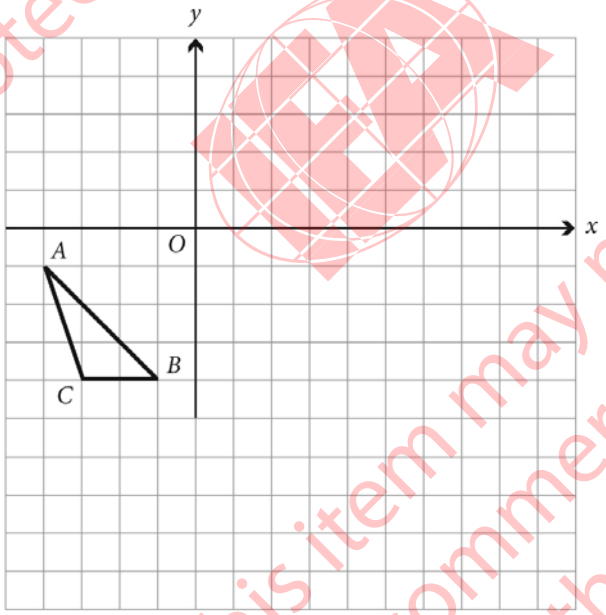
Code	Response	Item: MA13026A
	Correct Response	
10	Correct answer: $(x, y) \rightarrow (-x, y)\frac{1}{2}$ $A'(4, -1)$ $B'(1, -4)$ $C'(3, -4)$	
	Incorrect Response	
70	$A'B'C'$ is the image of triangle ABC under reflection in the x-axis. $A'(-4, 1)$ $B'(-1, 4)$ $C'(-3, 4)$	
71	Image $A'B'C'$ has correct shape and position but is incorrectly labeled	
79	Other incorrect (including sides of a triangle not drawn or vertices not labeled, crossed out, erased, stray marks, illegible, or off task) *Example:* *$A'B'C'$ is the image of triangle ABC under the central symmetry with center O, the origin.* $A'(4, 1)$ $B'(1, 4)$ $C'(3, 4)$	
	Nonresponse	
99	Blank	

Code	Response	Item: MA13026B
	Correct Response	
10	Correct answer: $(x, y) \rightarrow (-y, x)$ $A''(1, -4)$ $B''(4, -1)$ $C''(4, -3)$	
	Incorrect Response	
70	$A''B''C''$ is the correct image of triangle $A'B'C'$ (NOT ABC), as shown in response to Part A, under the rotation 90° counterclockwise with center O.	
71	$A''B''C''$ is the image of triangle ABC under a clockwise 90° rotation with center O. $A''(-1, 4)$ $B''(-4, 1)$ $C''(-4, 3)$	
72	Image $A''B''C''$ has correct shape and position but is incorrectly labeled	
79	Other incorrect (including sides of a triangle not drawn or vertices not labeled, crossed out, erased, stray marks, illegible, or off task) *Examples:* *1.* $A''B''C''$ is the image of triangle ABC under reflection in the x-axis. $A''(-4, 1)$ $B''(-1, 4)$ $C''(-3, 4)$ 2. $A''B''C''$ is the image of triangle ABC under the central symmetry with center O; OR, equivalently, under the rotation of 180° around O. $A''(4, 1)$ $B''(1, 4)$ $C''(3, 4)$	
	Nonresponse	
99	Blank	

Item ID **MA13026B**	Advanced Mathematics	Block_Sequence **M3_06**

A. Triangle ABC is reflected in the y-axis. On the diagram, draw and label triangle $A'B'C'$, the image of triangle ABC under this reflection.

B. Triangle ABC is rotated through 90° anti-clockwise, centre O. On the diagram, draw and label triangle $A''B''C''$, the image of triangle ABC under this rotation.



MA13026

TIMSS Advanced
2008

Content Domain
Geometry

Cognitive Domain
Applying

Maximum Points
1

Key
See scoring guide

TIMSS & PIRLS International Study Center
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Item ID **MA13026B**	Advanced Mathematics	Block_Sequence **M3_06**

Code	Response	Item: MA13026A
	Correct Response	
10	Correct answer: $(x, y) \rightarrow (-x, y)\frac{1}{2}$ $A'(4, -1)$ $B'(1, -4)$ $C'(3, -4)$	
	Incorrect Response	
70	$A'B'C'$ is the image of triangle ABC under reflection in the x-axis. $A'(-4, 1)$ $B'(-1, 4)$ $C'(-3, 4)$	
71	Image $A'B'C'$ has correct shape and position but is incorrectly labeled	
79	Other incorrect (including sides of a triangle not drawn or vertices not labeled, crossed out, erased, stray marks, illegible, or off task) *Example:* *$A'B'C'$ is the image of triangle ABC under the central symmetry with center O, the origin.* $A'(4, 1)$ $B'(1, 4)$ $C'(3, 4)$	
	Nonresponse	
99	Blank	

Code	Response	Item: MA13026B
	Correct Response	
10	Correct answer: $(x, y) \rightarrow (-y, x)$ $A''(1, -4)$ $B''(4, -1)$ $C''(4, -3)$	
	Incorrect Response	
70	$A''B''C''$ is the correct image of triangle $A'B'C'$ (NOT ABC), as shown in response to Part A, under the rotation 90° counterclockwise with center O.	
71	$A''B''C''$ is the image of triangle ABC under a clockwise 90° rotation with center O. $A''(-1, 4)$ $B''(-4, 1)$ $C''(-4, 3)$	
72	Image $A''B''C''$ has correct shape and position but is incorrectly labeled	
79	Other incorrect (including sides of a triangle not drawn or vertices not labeled, crossed out, erased, stray marks, illegible, or off task) *Examples:* 1. $A''B''C''$ is the image of triangle ABC under reflection in the x-axis. $A''(-4, 1)$ $B''(-1, 4)$ $C''(-3, 4)$ 2. $A''B''C''$ is the image of triangle ABC under the central symmetry with center O; OR, equivalently, under the rotation of 180° around O. $A''(4, 1)$ $B''(1, 4)$ $C''(3, 4)$	
	Nonresponse	
99	Blank	

Item ID **MA13027**	Advanced Mathematics	Block_Sequence **M3_07**

A regular polygon of n sides is inscribed in a circle of radius 1.

What is the value of the limit of the perimeter of the polygon as the number of sides n increases to infinity?

MA13027

TIMSS Advanced 2008

Content Domain
Algebra

Cognitive Domain
Reasoning

Maximum Points
2

Key
See scoring guide

TIMSS & PIRLS International Study Center
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Item ID **MA13027**	Advanced Mathematics	Block_Sequence **M3_07**

Code	Response	Item: MA13027
	Correct Response	
20	Any of 2 pi, 2π , 6.28, 6.3, or 2π = 6.28	
	Partially Correct Response	
10	$\lim_{n\to\infty} 2n\sin\frac{\pi}{n}$ **Note:** Accept also $\lim_{n\to\infty} 2n\sin\frac{180}{n}$	
11	2 pi *r* or $2\pi r$ or makes a statement such as "The value of the limit is equal to the circumference of the circle."	
	Incorrect Response	
70	π or pi or 3.14	
71	∞ or "infinity" or "the limit does not exist" or equivalent statement	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task) *Examples:* *1.* $\lim_{n\to\infty} 2n\sin\frac{2\pi}{n}$ *or* $\lim_{n\to\infty} 2n\cos\frac{\pi}{n}$ *or similar formula containing error* *2. 1* *3. "Almost a circle"or similar answers in words, not numerical values, stating that the shape of the polygon will become very close to that of a circle.*	
	Nonresponse	
99	Blank	

Item ID **MA13028**	Advanced Mathematics	Block_Sequence **M3_08**

For every natural number n, $1^2+3^2+\ldots+(2n-1)^2=\frac{n(4n^2-1)}{3}$

To prove this by MATHEMATICAL INDUCTION, what are the essential steps that will need to be carried out? (Do not do the actual proof.)

MA13028

TIMSS Advanced 2008

Content Domain
Algebra

Cognitive Domain
Knowing

Maximum Points
1

Key
See scoring guide

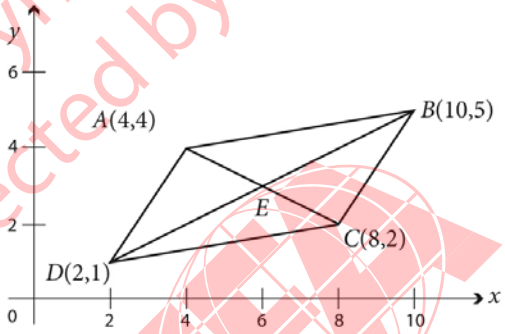
TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA13028**	Advanced Mathematics	Block_Sequence **M3_08**

Code	Response	Item: MA13028
	Correct Response	
10	Correct description of the two steps involved in the proof (i.e., verbal or symbolic statements) equivalent to: Step 1: Prove that the statement is true for $n = 1$. Step 2: Prove if the statement is true for any natural number $n = k$, then it also is true for $n = k + 1$.	
	Incorrect Response	
70	Describes Step 2 correctly but omits Step 1 or describes it incorrectly (e.g., "prove for $n = 0$", or "we must prove it for some small number")	
71	Gives correct proof of statement by induction with or without general statement of the induction method, or performs Step 2 correctly but omits Step 1, or describes Step 1 incorrrectly	
72	Describes Step 2: Prove that if the statement is true for any natural number $n = k$, where k is greater than 1, then it also is true for $n = k - 1$ with an appropriate Step 1.	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task)	
	Nonresponse	
99	Blank	

Item ID **MA13029**	Advanced Mathematics	Block_Sequence **M3_09**

In the quadrilateral $ABCD$ below, diagonals AC and BD intersect at point E. PROVE that E is the midpoint of AC and BD. Show all your work.



MA13029

TIMSS Advanced 2008

Content Domain
Geometry

Cognitive Domain
Reasoning

Maximum Points
2

Key
See scoring guide

Item ID **MA13029**	Advanced Mathematics	Block_Sequence **M3_09**

Code	Response	Item: MA13029
	Correct Response	
20	Any completely correct proof (e.g., showing diagonals have the same midpoint; proving that *ABCD* is a parallelogram and hence diagonals have the same midpoint; proving that *ABCD* is a parallelogram and hence diagonals bisect each other).	
	Partially Correct Response	
10	Method that is partially completed (e.g., shows that point *E* (6, 3) is midpoint of only *AC* or *BD*; or correct proof with step missing or one or two reasons incorrect or missing)	
	Incorrect Response	
79	Incorrect (including crossed out, erased, stray marks, illegible, or off task) *Example:* *States that "From the diagram, it is obvious that ABCD is a parallelogram, and hence its diagonals must bisect each other" or an equivalent statement*	
	Nonresponse	
99	Blank	

Item ID **MA23069**	Advanced Mathematics	Block_Sequence **M6_01**

An infinite geometric series has the first term 3 and the third term $\frac{1}{3}$. All the terms of the series are positive. What is the sum of the series?

Ⓐ $\frac{27}{8}$

Ⓑ $\frac{10}{3}$

Ⓒ $\frac{9}{4}$

Ⓓ $\frac{9}{2}$

MA23069

TIMSS Advanced 2008

Content Domain

Algebra

Cognitive Domain

Applying

Maximum Points

1

Key

D

TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA23135**	Advanced Mathematics	Block_Sequence **M6_02**

$$\frac{x+1}{x-2} > 1$$

For which values of x is the inequality shown above satisfied?

Answer: _____

MA23135

TIMSS Advanced 2008

Content Domain

Algebra

Cognitive Domain

Applying

Maximum Points

1

Key

See scoring guide

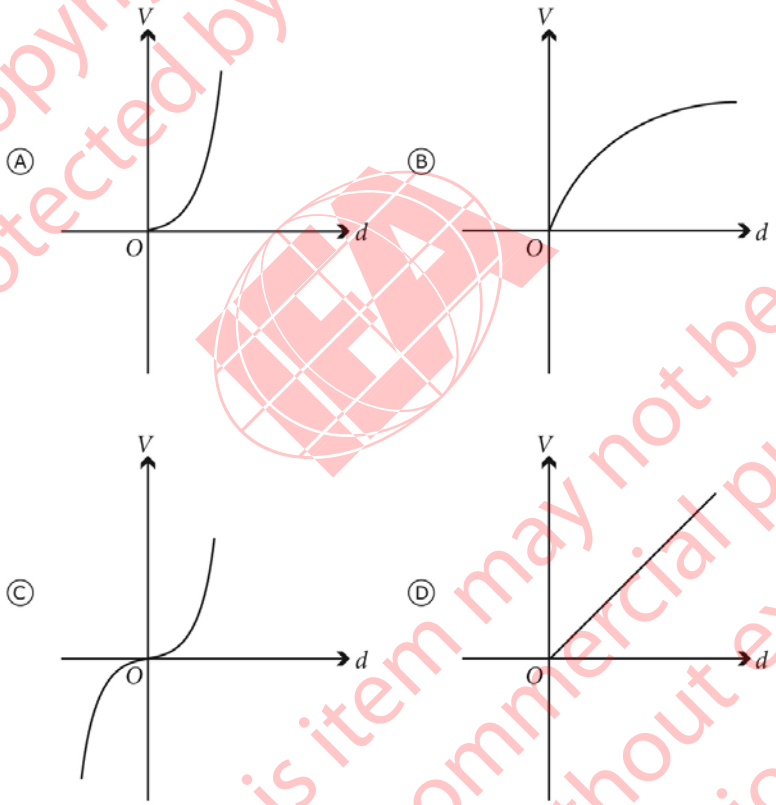
TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA23135**	Advanced Mathematics	Block_Sequence **M6_02**

Code	Response	Item: MA23135
	Correct Response	
10	$x > 2$	
	Incorrect Response	
79	Incorrect (including crossed out, erased, stray marks, illegible, or off task)	
	Nonresponse	
99	Blank	

Item ID **MA23208**	Advanced Mathematics	Block_Sequence **M6_03**

A spherical balloon is blown up. Which graph shows the volume V as a function of the diameter d?



MA23208

TIMSS Advanced 2008

Content Domain

Algebra

Cognitive Domain

Reasoning

Maximum Points

1

Key

A

TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA23165**	Advanced Mathematics	Block_Sequence **M6_04**

Determine $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - 1}$.

Show your work.

MA23165

TIMSS Advanced 2008

Content Domain

Calculus

Cognitive Domain

Applying

Maximum Points

1

Key

See scoring guide

TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA23165**	Advanced Mathematics	Block_Sequence **M6_04**

Code	Response	Item: MA23165
	Correct Response	
10	$\frac{3}{2}$ or equivalent; by algebraic manipulations *Examples:* *1)* $\lim_{x\to 1}\frac{x^2+x-2}{x^2-1}=\lim_{x\to 1}\frac{(x+2)(x-1)}{(x+1)(x-1)}=\lim_{x\to 1}\frac{x+2}{x+1}=\frac{3}{2}$ *2) Let x = h +1, then* $\lim_{h\to 0}\frac{(h+3)}{(h+2)}=\frac{3}{2}$	
11	$\frac{3}{2}$ or equivalent; numerical approximation; substitution of value of *x* close to 1 *Example:* *Let x = 1.001* $\frac{x^2+x-2}{x^2-1}=\frac{1.00201+1.001-2}{1.00201-1}=\frac{0.003}{0.002}$ *Limit is 3/2*	
12	1.5 using a graphing or symbolic calculator	
	Incorrect Response	
70	Calculator used—answer incorrect or explanation inadequate	
71	$\frac{3}{2}$ or equivalent; no method or wrong method given	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task)	
	Nonresponse	
99	Blank	

Item ID **MA23039**	Advanced Mathematics	Block_Sequence **M6_05**

MA23039

$f(x) = e^{\cos x}$

What is $f'(x)$?

(A) $e^{\cos x}$

(B) $e^{-\sin x}$

(C) $e^{\cos x} \cdot \sin x$

(D) $-e^{\cos x} \cdot \sin x$

TIMSS Advanced 2008

Content Domain

Calculus

Cognitive Domain

Knowing

Maximum Points

1

Key

D

TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA23159**	Advanced Mathematics	Block_Sequence **M6_06**

Find $f'(x)$, when $f(x) = \frac{3x+2}{x-1}$.

Show your work.

MA23159

TIMSS Advanced 2008

Content Domain

Calculus

Cognitive Domain

Knowing

Maximum Points

1

Key

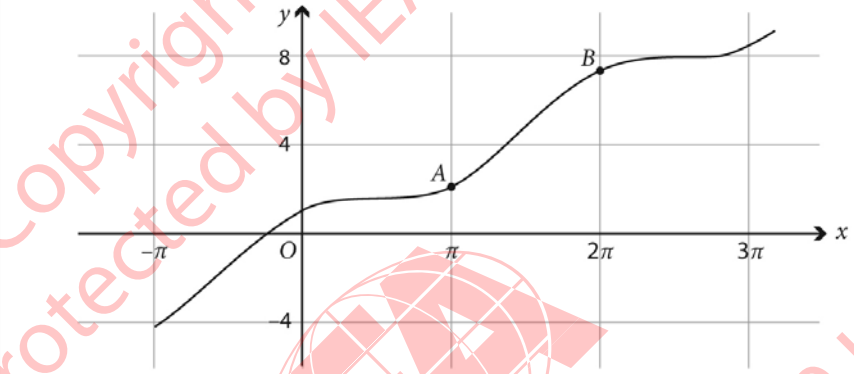
See scoring guide

TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA23159**	Advanced Mathematics	Block_Sequence **M6_06**

Code	Response	Item: MA23159
	Correct Response	
10	Using quotient rule $\left(\frac{u}{v}\right)' = \frac{(u'v - uv')}{v^2}$ or, product rule $(uv)' = u'v + uv'$, obtains $f'(x) = \frac{-5}{(x-1)^2}$	
11	Correct expression using calculator	
	Incorrect Response	
70	Calculator used—answer incorrect or explanation inadequate	
71	Correct answer but no method shown	
72	Using quotient rule but not completing with correct answer	
73	Using product rule but not completing with correct answer	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task)	
	Nonresponse	
99	Blank	

Item ID **MA23198**	Advanced Mathematics	Block_Sequence **M6_07**



Sophia is studying the graph of the function $y = x + \cos x$ shown above. She says that the slope at point A is the same as the slope at point B. Explain why she is correct.

MA23198

TIMSS Advanced 2008

Content Domain

Calculus

Cognitive Domain

Reasoning

Maximum Points

1

Key

See scoring guide

TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA23198**	Advanced Mathematics	Block_Sequence **M6_07**

Code	Response	Item: MA23198
	Correct Response	
10	Explanation involving differentiating and showing the gradient is the same at $x = \pi$ and $x = 2\pi$; or using the nature of the cosine function to establish that the gradient is the same at $x = \pi$ and $x = 2\pi$	
11	Correct answer using calculator with adequate explanation	
	Incorrect Response	
70	Calculator used—answer incorrect or explanation inadequate	
71	Differentiates correctly but does not give adequate explanation of why slopes are equal	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task)	
	Nonresponse	
99	Blank	

Item ID **MA23042**	Advanced Mathematics	Block_Sequence **M6_08**

What is $\int \frac{x^2+2}{x} dx$? $(x>0)$

(A) $\frac{1}{2}x^2 - \frac{2}{x^2} + C$

(B) $\frac{1}{2}x^2 + 2\ln x + C$

(C) $\frac{1}{2}x^2 + \ln 2x + C$

(D) $\frac{4}{3}x^3 + 4x^3 + C$

MA23042

TIMSS Advanced 2008

Content Domain

Calculus

Cognitive Domain

Knowing

Maximum Points

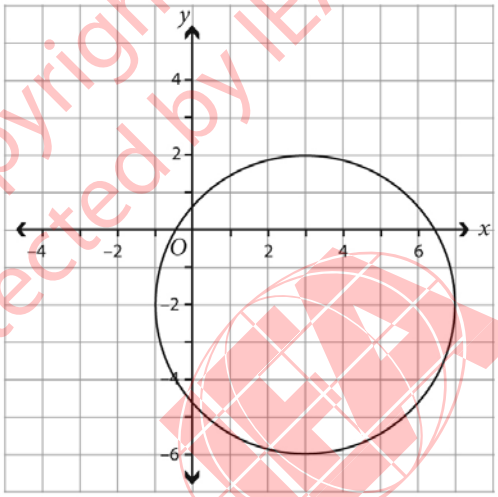
1

Key

B

TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA23055**	Advanced Mathematics	Block_Sequence **M6_09**



What is the equation of the circle shown above?

Ⓐ $x^2 + y^2 - 6x + 4y - 9 = 0$

Ⓑ $x^2 + y^2 + 6x - 4y + 9 = 0$

Ⓒ $x^2 + y^2 + 6x - 4y - 3 = 0$

Ⓓ $x^2 + y^2 - 6x + 4y - 3 = 0$

MA23055

TIMSS Advanced 2008

Content Domain

Geometry

Cognitive Domain

Knowing

Maximum Points

1

Key

D

TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA23080**	Advanced Mathematics	Block_Sequence **M6_10**

How many solutions does the equation $\sin x + \cos x = 2$ have in the interval 0 to 8π?

(A) 0
(B) 2
(C) 4
(D) 8

MA23080

TIMSS Advanced 2008

Content Domain
Geometry

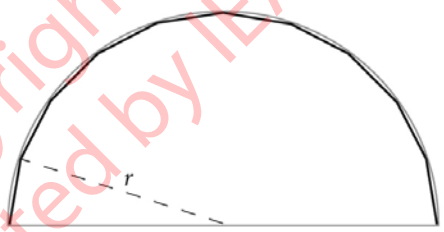
Cognitive Domain
Reasoning

Maximum Points
1

Key
A

TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA23021**	Advanced Mathematics	Block_Sequence **M6_11**



The figure shows a semicircular room seen from above. An architect is placing 10 flat windows in the room as shown. If the radius of the circle is r, which of the following equations would allow the architect to determine the width of each window?

Ⓐ $w = r \sin 9°$

Ⓑ $w = 2r \sin 9°$

Ⓒ $w = r \cos 18°$

Ⓓ $w = 2r \sin 18°$

MA23021

TIMSS Advanced 2008

Content Domain

Geometry

Cognitive Domain

Applying

Maximum Points

1

Key

B

TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA23004**	Advanced Mathematics	Block_Sequence **M7_01**

MA23004

A sheet of paper 0.01 cm thick is cut in two, and one piece is placed on top of the other. The two sheets of paper are then cut in two and made into a pile of 4 sheets. If this process could be repeated 8 more times, how thick would the pile of papers be?

(A) 0.2 cm
(B) 10.24 cm
(C) 20.48 cm
(D) 32.0 cm

TIMSS Advanced 2008

Content Domain
Algebra

Cognitive Domain
Reasoning

Maximum Points
1

Key
B

TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA23063**	Advanced Mathematics	Block_Sequence **M7_02**

MA23063

If $x = -1 + \frac{1}{2}i$, which of the following is equal to $\frac{5}{x}$?

(A) $-5 + i$

(B) $-4 - 2i$

(C) $-4 + 2i$

(D) $4 + 2i$

TIMSS Advanced 2008

Content Domain

Algebra

Cognitive Domain

Applying

Maximum Points

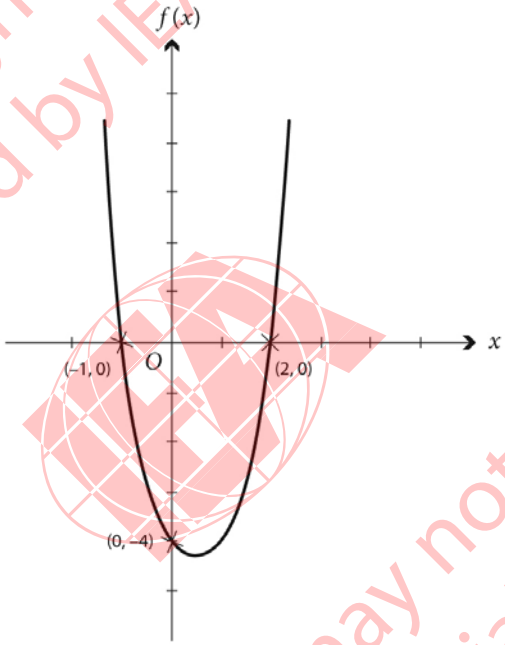
1

Key

B

TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA23141**	Advanced Mathematics	Block_Sequence **M7_03**



The graph of the function f is shown above. The equation of the function f is given by $f(x) = ax^2 + bx + c$. Find the values of a, b, and c.

Show your work.

MA23141

TIMSS Advanced 2008

Content Domain

Algebra

Cognitive Domain

Knowing

Maximum Points

1

Key

See scoring guide

TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA23141**	Advanced Mathematics	Block_Sequence **M7_03**

Code	Response	Item: MA23141
	Correct Response	
10	All values correct: $a = 2$, $b = -2$, $c = -4$, or equivalently giving the full function Method used: factorization	
11	All values correct: $a = 2$, $b = -2$, $c = -4$, or equivalently giving the full function Method used: solving three simultaneous equations	
12	All values correct: $a = 2$, $b = -2$, $c = -4$, or equivalently giving the full function Method used: solving three simultaneous equations by calculator	
13	All values correct: $a = 2$, $b = -2$, $c = -4$, or equivalently giving the full function Method used: quadratic regression by calculator	
19	All values correct: $a = 2$, $b = -2$, $c = -4$, or equivalently giving the full function Other correct method used	
	Incorrect Response	
70	Calculator used—answer incorrect or explanation inadequate (e.g., trial or error method)	
71	All values correct: $a= 2$, $b = -2$, $c = -4$, or equivalently giving the full function. No correct method shown.	
72	$c = -4$ with values of a and b either missing or incorrect.	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task)	
	Nonresponse	
99	Blank	

Item ID **MA23133**	Advanced Mathematics	Block_Sequence **M7_04**

The function f is given by $f(x) = x^2 + 4$. Another function g is given by $g(u) = \sqrt{2u - 1}$. Determine the minimum value of $g(f(x))$.

(A) 0

(B) $\sqrt{3}$

(C) $\sqrt{\frac{7}{2}}$

(D) $\sqrt{7}$

MA23133

TIMSS Advanced 2008

Content Domain

Algebra

Cognitive Domain

Knowing

Maximum Points

1

Key

D

Item ID **MA23158**	Advanced Mathematics	Block_Sequence **M7_05**

MA23158

A car starts braking as it approaches a road junction. After braking for t seconds, the car has traveled a distance of $s(t)$ meters, where $s(t) = -t^2 + 20t$. How far does the car travel from the time the brakes are applied until it stops?

(A) −20 m
(B) 10 m
(C) 50 m
(D) 100 m

TIMSS Advanced 2008

Content Domain
Calculus

Cognitive Domain
Applying

Maximum Points
1

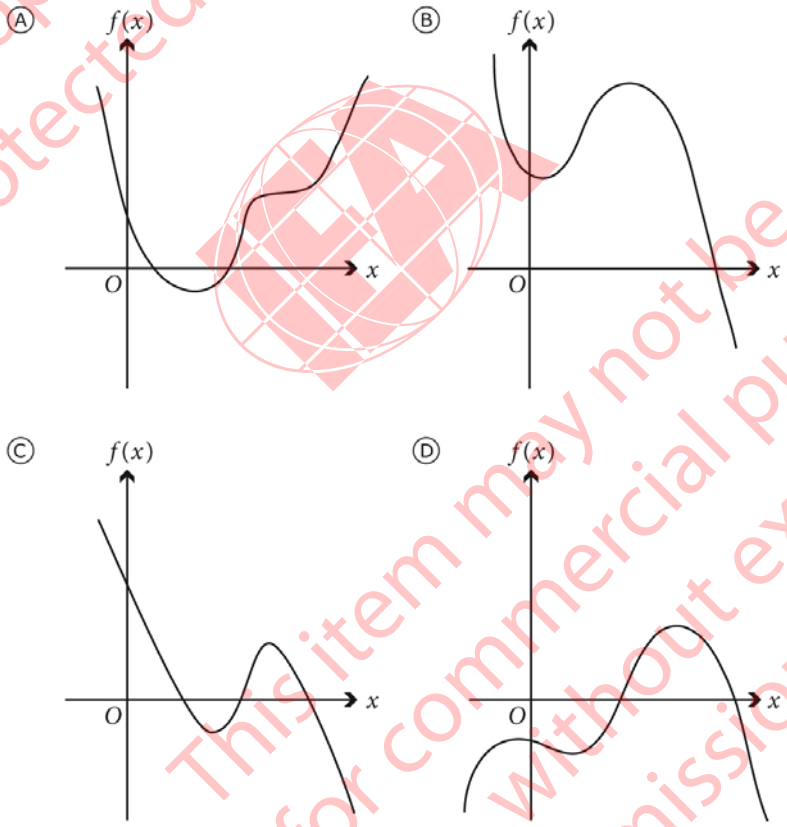
Key
D

TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA23151**	Advanced Mathematics	Block_Sequence **M7_06**

Which one of the graphs below has all of the following properties?

$f(-1)>0, f(3)<0, f'(5)=0, f''(5)<0$



MA23151

TIMSS Advanced 2008

Content Domain

Calculus

Cognitive Domain

Reasoning

Maximum Points

1

Key

C

TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA23035A**	Advanced Mathematics	Block_Sequence **M7_07**

$$f(x) = x^4 - 2x^2$$

A. What are the values of x at the points of intersection of the graph of $f(x)$ with the x-axis?

$x =$ _____

B. What are the maximum and minimum points of the graph of $f(x)$?

Maximum point(s): _____

Minimum point(s): _____

MA23035

TIMSS Advanced 2008

Content Domain

Calculus

Cognitive Domain

Applying

Maximum Points

1

Key

See scoring guide

TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA23035A**	Advanced Mathematics	Block_Sequence **M7_07**

Code	Response	Item: MA23035A
	Correct Response	
10	All three of $-\sqrt{2},\ 0,$ and $\sqrt{2}$. Accept $\left(-\sqrt{2},0\right),\ (0,0),\ \left(\sqrt{2},0\right)$. $\sqrt{2}$ may be given as 1.41, 1.42, or a value between these.	
	Incorrect Response	
70	Any two of $-\sqrt{2},\ 0,$ and $\sqrt{2}$, or $\left(-\sqrt{2},0\right),\ (0,0),\ \left(\sqrt{2},0\right)$. $\sqrt{2}$ may be given as 1.41, 1.42, or a value between these.	
79	Incorrect (including crossed out, erased, stray marks, illegible, or off task)	
	Nonresponse	
99	Blank	

Code	Response	Item : MA23035B
	Correct Response	
10	Maximum $(0,0)$, Minimum $(-1,-1)$ and $(1,-1)$	
	Incorrect Response	
70	Any two of the above correctly identified as maximum or minimum	
71	*x* values only given (i.e., maximum 1, minimum −1 and 1)	
79	Incorrect (including crossed out, erased, stray marks, illegible, or off task)	
	Nonresponse	
99	Blank	

Item ID **MA23035B**	Advanced Mathematics	Block_Sequence **M7_07**

$$f(x) = x^4 - 2x^2$$

A. What are the values of x at the points of intersection of the graph of $f(x)$ with the x-axis?

$x =$ _____

B. What are the maximum and minimum points of the graph of $f(x)$?

Maximum point(s): _____

Minimum point(s): _____

MA23035

TIMSS Advanced 2008

Content Domain

Calculus

Cognitive Domain

Applying

Maximum Points

1

Key

See scoring guide

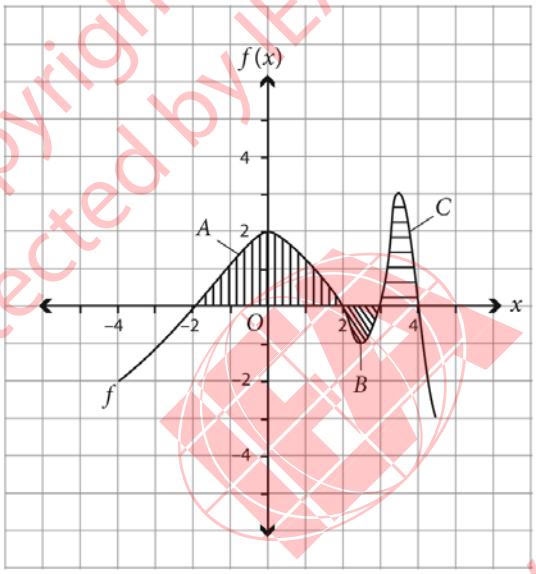
TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA23035B**	Advanced Mathematics	Block_Sequence **M7_07**

Code	Response	Item: MA23035A
	Correct Response	
10	All three of $-\sqrt{2}$, 0, and $\sqrt{2}$. Accept $\left(-\sqrt{2},0\right)$, $(0,0)$, $\left(\sqrt{2},0\right)$. $\sqrt{2}$ may be given as 1.41, 1.42, or a value between these.	
	Incorrect Response	
70	Any two of $-\sqrt{2}$, 0, and $\sqrt{2}$, or $\left(-\sqrt{2},0\right)$, $(0,0)$, $\left(\sqrt{2},0\right)$. $\sqrt{2}$ may be given as 1.41, 1.42, or a value between these.	
79	Incorrect (including crossed out, erased, stray marks, illegible, or off task)	
	Nonresponse	
99	Blank	

Code	Response	Item : MA23035B
	Correct Response	
10	Maximum $(0,0)$, Minimum $(-1,-1)$ and $(1,-1)$	
	Incorrect Response	
70	Any two of the above correctly identified as maximum or minimum	
71	*x* values only given (i.e., maximum 1, minimum −1 and 1)	
79	Incorrect (including crossed out, erased, stray marks, illegible, or off task)	
	Nonresponse	
99	Blank	

Item ID **MA23050**	Advanced Mathematics	Block_Sequence **M7_08**



For the areas between the graph of $f(x)$ and the x-axis shown above, area $A = 4.8$ units, area $B = 0.8$ units, and area $C = 2$ units.

What is the value of the definite integral $\int_{-2}^{4} f(x)dx$?

Ⓐ 5.6
Ⓑ 6.0
Ⓒ 6.8
Ⓓ 7.6

MA23050

TIMSS Advanced 2008

Content Domain
Calculus

Cognitive Domain
Knowing

Maximum Points
1

Key
B

TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA23041**	Advanced Mathematics	Block_Sequence **M7_09**

What is $\int e^{1+4x}\,dx$?

(A) $\frac{1}{4}e^{1+4x}+C$

(B) $e^{1+4x}+C$

(C) $4e^{1+4x}+C$

(D) $e^{x+2x^2}+C$

MA23041

TIMSS Advanced 2008

Content Domain

Calculus

Cognitive Domain

Knowing

Maximum Points

1

Key

A

Item ID **MA23182**	Advanced Mathematics	Block_Sequence **M7_10**

MA23182

$$\sin 2x = \frac{1}{2}$$

What are the possible values for x between 0° and 360°?

Ⓐ 30°, 150°
Ⓑ 195°, 345°
Ⓒ 30°, 150°, 210°, 330°
Ⓓ 15°, 75°, 195°, 255°

TIMSS Advanced 2008

Content Domain
Geometry

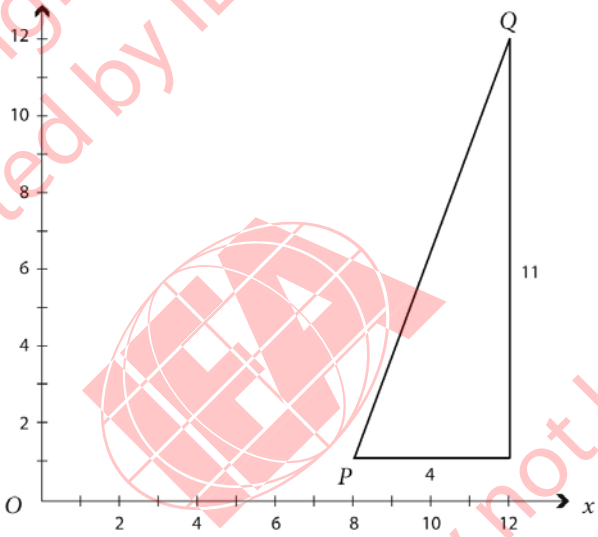
Cognitive Domain
Applying

Maximum Points
1

Key
D

TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA23170**	Advanced Mathematics	Block_Sequence **M7_11**



A straight line l passes through the points A (1,–2) and B (3, 4).
Is the line l parallel with PQ?

Give a reason to support your answer.

MA23170

TIMSS Advanced 2008

Content Domain
Geometry

Cognitive Domain
Applying

Maximum Points
1

Key
See scoring guide

TIMSS & PIRLS International Study Center
Lynch School of Education, Boston College

Item ID **MA23170**	Advanced Mathematics	Block_Sequence **M7_11**

Code	Response	Item: MA23170
	Correct Response	
10	No, with correct work showing gradients are different, and leading to conclusion that l and PQ are not parallel.	
11	No, with correct work leading to conclusion that l and PQ are not parallel using method other than showing gradients are different. For example, shows angle between the lines is not 0^o	
	Incorrect Response	
70	No, with no correct reason	
71	Yes, with or without reason	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task)	
	Nonresponse	
99	Blank	

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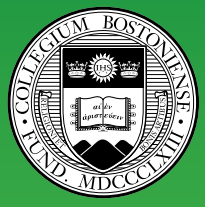

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