On polynomial coloring for ($claw$, $co-diamond$)-free graphs

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Abstract

The coloring of ($claw$, $co-diamond$)-free graph is in polynomial.
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1 Introduction

Theorem 1.1 *Let G be a connected claw-free graph with $\alpha(G) \geq 3$. If G contains an odd anti-hole then it contains an odd antihole then it contains a C_5* □

Theorem 1.2 *There is a polytime algorithm for coloring a G with $\alpha(G) = 2$.*

2 Oberservations

In this section, we conclude some observations used to prove that there is a polytime algorithm for ($claw$, $co-diamond$)-free graphs.

Lemma 2.1 *G cannot contain an induced C_ℓ, $\ell \geq 7$.*

Proof. Let G be a C_ℓ, $\ell \geq 7$ with vertices 1, 2, ...,ℓ_{-1}, ℓ. This forms a $co-diamond$ with (1,2) and 4, ℓ but G is ($claw$, $co-diamond$)-free. □

For the following observations let $N_{i,i+1}$ be the set of 2-vertex on a C_5, $N_{i,i+1,i+2}$ be the set of 3-vertex on a C_5, $N_{i,i+1,i+2,i+3}$ be the set of 4-vertex on a C_5 in which G contains an C_5. Let N_i be the set of all i-vertex.

Lemma 2.2 $|N_{i,i+1}| \leq 1$

Proof. Let x be a vertex from $N_{i,i+1}$ and y be a vertex from $N_{i,i+1}$ such that $x \neq y$. If $xy \notin E$ then there is a $claw(i_{+1}i_{+2}, i_{+1}x, i_{+1}y)$. If $xy \in E$ then there is a $co-diamond(xy, i_{-1}, i_{+2}$.

Lemma 2.3 *N_2 forms a clique K_n, $n = |N_2|$*

Proof. Let x be a vertex from $N_{i,i+1}$ and y be a vertex from $N_{j,j+1}$. Without loss of generality suppose $xy \notin E$ if $j = i+1$ then there is a $co-diamond$ (y, xi, i_{+3}), but if $j = i+2$ then there is a $co-diamond$ $(y, xi, j+1)$ so therefore $xy \in E$.

Lemma 2.4 *The set $N_{i,i+1,i+2}$ forms a clique K_n, $n = |N_{i,i+1,i+2}|$*

Proof. Let x be a vertex from $N_{i,i+1,i+2}$ and y be a vertex from $N_{i,i+1,i+2}$ such that $x \neq y$. If $x \notin E$ there is a $claw(xi, yi, ii_{-1})$.

Lemma 2.5 *The set $N_{i,i+1,i+2,i+3}$ forms a clqiue K_n, $n = |N_{i,i+1,i+2,i+3}|$*

Proof. Let x be a vertex from $N_{i,i+1,i+2,i+3}$ and y be a vertex from $N_{i,i+1,i+2,i+3}$ such that $x \neq y$. If $x \notin E$ there is a $claw(xi, yi, ii_{-1})$.

Lemma 2.6 $\alpha(N_5) \leq 2$

Proof. Let x, y, z be vertices from N_5 such that $x \neq y, x \neq z, y \neq z$ and $xy \notin E, xz \notin E, yz \notin E$. There is a claw (xi, yi, zi). Therefore $\alpha(N_5) \leq 2$.

Lemma 2.7 $\alpha(N_4) \leq 2$

Proof. Let x, y, z be vertices from N_4 such that $x \neq y, x \neq z, y \neq z$ and $xy \notin E, xz \notin E, yz \notin E$. There is a claw (xi, yi, zi). Therefore $\alpha(N_4) \leq 2$.

Lemma 2.8 $\alpha(N_3) \leq 2$

Proof. Let x, y, z be vertices from N_3 such that $x \neq y, x \neq z, y \neq z$ and $xy \notin E, xz \notin E, yz \notin E$. If $x \in N_{i,i+1},\ y \in N_{i,i+1},\ , and z \in N_{i,i+1},$ then by Lemma **??** forms a clique otherwise there exists a vertex i such that $xi \in E$ but $yi \notin E and zi \notin E$ which means there is $co-diamond(xi, y, z)$. Therefore $\alpha(N_3) \leq 2$.

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References