On k-critical ($claw$, $2K_2$)-free graphs

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Abstract

The coloring of ($claw$, $2K_2$)-free graph is in polynomial.
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1 Introduction

Theorem 1.1 *Let G be a connected claw-free graph with $\alpha(G) \geq 3$. If G contains an odd anti-hole then it contains an odd antihole then it contains a C_5* □

Theorem 1.2 *There is a polytime algorithm for coloring a G with $\alpha(G) = 2$.*

2 Oberservations

In this section, we conclude some observations used to prove that there polytime algorithm for ($claw$, $2K_2$)-free graphs.

Lemma 2.1 *G cannot contain an induced C_ℓ, $\ell \geq 7$.*

Proof. Let G be a C_ℓ, $\ell \geq 7$ with vertices 1, 2, ...,ℓ_{-1}, ℓ. This forms a $2K2$ with (2,3) and ($\ell - 1$, ℓ) but G is ($claw$, $2K_2$)-free. □

Lemma 2.2 *If G contains an C_5 then $\alpha(G) = 2$ or G is disconnnected*

Proof. G contains no k-vertex for $k \in 1, 2, 3$. There can be no edge between an 0-vertex and a 4-vertex or a 5-vertex. Let x be a 0-vertex and y be a 4-vertex, if $xy \in E$ then G contains a $claw(xy, yi, yi_{+2})$. Similar argument applies to 5-vertex. $\alpha(G) = 2$ if G contains no 0-vertex since $\alpha(C_5) = 2$ and all 4-vertex and 5-vertex must have $|E| = v - 2$ otherwise there is a $claw$.

Lemma 2.3 *If G contains an odd anti-hole then $\alpha(G) = 2$ or G is disconnected.*

Proof. By Theorem **??** if connected G contains an odd anti-hole with $\alpha(G) \geq 3$ then G contains an induced C_5 but by Lemma **??** any G containing a C_5 cannot have $\alpha(G)! = 2$ if it is connected. So therefore if connected G contains an odd anti-hole $\alpha(G) = 2$.

Theorem 2.4 *If (claw, $2K_2$)-free then G can be colored in polynomial time*

By Lemmas **??** and **??** any G contains an odd anti-hole has $\alpha(G) = 2$ and using **??** can be colored in polynomial time. If G does not contain an odd-anti hole and by Lemma **??** if is a perfect graph. A perfect graph can be colored in polynomial time.

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References