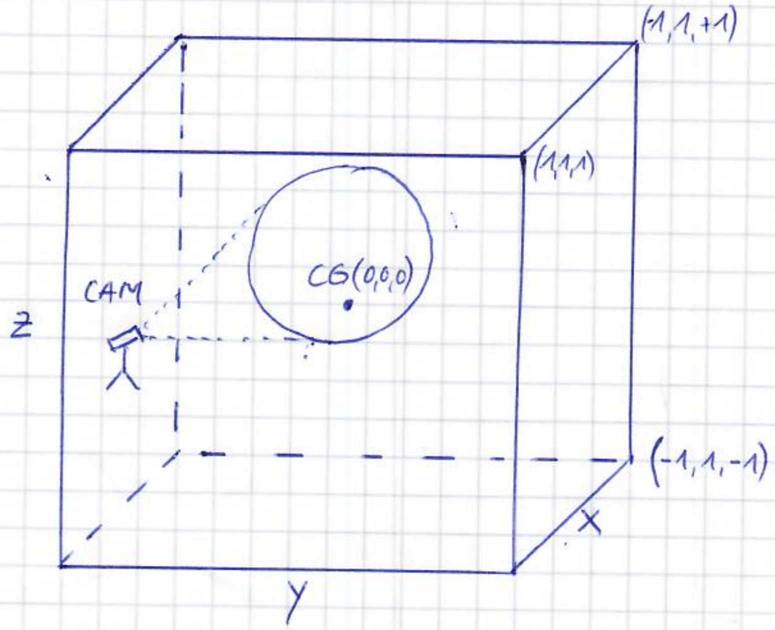
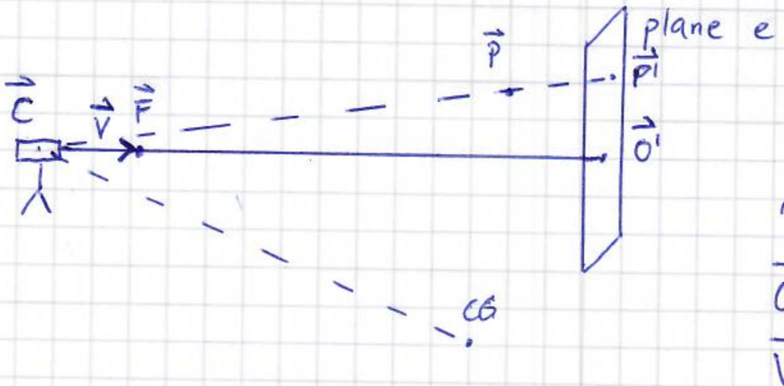
SINGULERSUM

Math behind the project.
Singulersum is the „universe" where all 3D objects are placed within.

$P(p_x, p_y, p_z) \in \mathbb{R}^3, \quad p_x \in [-1; 1], \quad p_y \in [-1; 1], \quad p_z \in [-1; 1]$





$\lambda \frac{\vec{CP}}{\|\vec{CP}\|} = \vec{p'}, \quad \vec{p'} \in e$

λ := Distance $\vec{p'}$ from $\vec{C}$

$\vec{O'}$:= Center view point on plane e

$\vec{V}$:= View vector, where camera is looking at.

$\vec{C}$:= Camera position

plane e is tangential to the circumference sphere of Singulersum.
$\vec{F}$:= Focus Point (where camera looks at)

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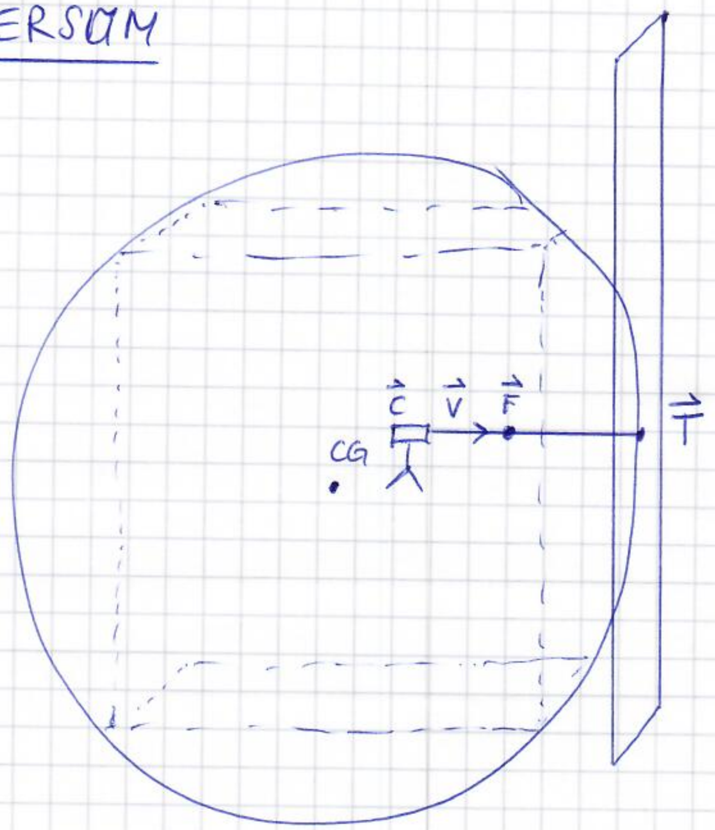

$\vec{V} = \overrightarrow{CF} = \vec{F} - \vec{C}$ # DocRef. 1

$LV = \|\vec{V}\|$ # DocRef. 2

From $\vec{V}$ derive azimuth and altitude.

$$\text{azimut (view)} = \operatorname{atan} \frac{V_y}{V_x}$$

$$\text{altitude} = \operatorname{atan} \frac{V_z}{\sqrt{V_x^2 + V_y^2}}$$

see DocRef. ~~2~~3/4

With this, transform View vector $\vec{V}$ such that $\vec{V}$ „lies" on the X-axis $\left(\vec{V}' = \begin{pmatrix} LV \\ 0 \\ 0 \end{pmatrix}\right)$.

Using Rotation-Matrix calculus (VectorMath.py, DocRef. 5)
This gives the new view vector $\vec{V}'$ (V_prime) and $\vec{C}'$ (C-prime).

Now calculate Tangential Point $\vec{T} = \frac{1}{\|\vec{V}'\|} \cdot \vec{V}'$ # DocRef. 6

Using Tangential point $\vec{T}$ and $\vec{V}'$ as normal vector, calculate View plane e

$$e = \vec{V}' \cdot (\vec{x} - \vec{T}) \quad \text{\# DocRef. 7}$$

if not otherwise stated, DocRef.# references are in Camera.py.
They are all in either Camera.py OR VectorMath.py

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Using $\vec{C'}$ and $\vec{V'}$, create line equation cl (camera line)

$$cl(\lambda) = \vec{C'} + \lambda \vec{V'} \quad \text{\# DocRef. 8}$$

and project Focal point to the plane. Note that Focal point is „encoded" in $\vec{V'}$ since original $\vec{V} = \vec{CF} = \vec{F} - \vec{C}$.

$$\text{solve} \left\{ e = \vec{V'} \cdot \left(cl(\lambda) - \vec{T'} \right) \Rightarrow \vec{V'} \cdot \begin{pmatrix} C'_x + \lambda V'_x - T'_x \\ C'_y + \lambda V'_y - T'_y \\ C'_z + \lambda V'_z - T'_z \end{pmatrix} \right.$$

\# DocRef. 9

Every point $\vec{p}$ of Singulersum can be projected onto view plane e using:

$\vec{p'} = R_M \cdot \vec{p}$, where R_M is the rotation matrix for view azimuth and altitude correction. # DocRef. 5, VectorMath.py

↑

\# DocRef. 10

Displace $\vec{p'}$, so that camera is in center: $\vec{C} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$\vec{p'} = \vec{p'} - \vec{C'} \quad \text{\# DocRef. 11}$$

Project $\vec{p'}$ onto $\vec{e} \Rightarrow \vec{p''}$

$cpl(\lambda) = \vec{C'} \cdot \lambda(\vec{p'})$ # DocRef. 12 cpl: camera-point-line

solve $cpl(\lambda)$ intersecting e. # DocRef. 13, VectorMath.py

since $\vec{V'}$ „lies" on x-axis; the y and z axis are the 2D (x, y) coordinates.

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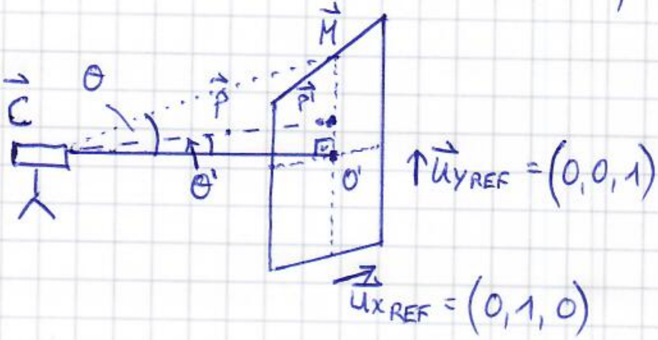
FIELD OF VIEW Calc (DocRef. 14)



θ' can be computed for any point $\vec{p}$.

$\theta = \frac{FOV}{2}$; $\text{a}\cancel{\text{cos}}\,\text{tan}\ \frac{\overline{MO'}}{\overline{CO'}} = \theta$

$$\tan \theta' = \frac{\overline{O'P'}}{\overline{CO'}}$$

$$\tan \theta = \frac{\overline{O'M}}{\overline{CO'}}$$

$$\Rightarrow \tan\theta' + \tan\theta = \frac{\overline{O'P'}}{\overline{CO'}} + \frac{\overline{O'M}}{\overline{CO'}} = \frac{\overline{O'P'} + \overline{O'M}}{\overline{CO'}} \Rightarrow$$

$$\Rightarrow (\tan\theta' + \tan\theta)\,\overline{CO'} = \overline{O'P'} + \overline{O'M} \Rightarrow \underbrace{\overline{O'M}}_{\text{what is needed to know.}} = \underbrace{\overline{CO'}(\tan\theta + \tan\theta') - \overline{O'P'}}_{\text{all therms are known} \Rightarrow \text{possible to calculate}}$$

what is needed to know.

al therms are known
=> possible to calculate