WELL-ROUNDED FACTS ABOUT SPHERES FOR TYPSTS

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This document showcases a layout for handouts inspired by the work of Edward Tufte (Figure 1), and typeset of the tufte-LaTex class. The contents of the handouts were copied from (cite Weissman)

Computing the Volume

The closed n-dimensional ball of radius r, centered at the origin, is defined by:

$$B^n = \{\vec{x} \in \mathbb{R}^n \mid x_1^2 + x_2^2 + \dots + x_n^2 \leq r^2\}.$$

As a bounded closed subset of $\mathbb{R}^n$, the n-ball has a well-defined volume, which we call $V_n(r)$. A table of volumes is given in the margin. Of course, the word "volume" might be a bit misleading in this degree of generality. In dimension 0, the "volume" $V_0(r)$ is the cardinality of the one-point set $\mathbb{R}^0$. In dimension 1, the "volume", $V_1(r) = 2r$, is simply the length of the line segment $[-r, r]$. In dimension 2, the "volume" of a circle is its area, and $V_2(r) = \pi r^2$.

This, in essence, is the definition of π. In dimension Don't confuse a definition with a computation. Of course, one could "compute" the area of a circle with an integral, but such an argument would necessarily be circular, pun intended. Perhaps the only fact that needs to be proven is that the circumference is the derivative of the area, as functions of the radius, which follows from Stokes theorem. 3, the "volume" of a sphere is its volume as the word is used by the English-speaking community at large.

There is one fact about volumes of balls – the functions $V_{n(r)}$ – that can be deduced from the simplest change of variables: a ball of radius r can be obtained by scaling a unit ball by r. It follows that $V_n(r) = V_n(1) r^n$.

Basic slicing

Slicing the n-dimensional ball like an egg is helpful for computing the volume $V_n(r)$:

Layout inconsistencies

- leading between author and date
- margin note font size should be set automatically

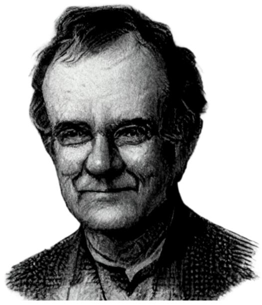

Figure 1: Edward Tufte.

n	$C_n = V_n(1)$	$V_n(r)$
0	1	1
1	2	$2r$
2	π	πr^2
3	$\frac{4}{3}\pi$	$\frac{4}{3}\pi r^3$
4	$\frac{1}{2}\pi^2$	$\frac{1}{2}\pi^2 r^4$
5	$\frac{8}{15}\pi^2$	$\frac{8}{15}\pi^2 r^5$

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Scaling is a particularly simple instance of the technique of change of variables. Scaling a measurable subset of $\mathbb{R}^n$ by r changes its volume by a factor of r^n.

- figure caption should be left aligned
- table fractions should be slanted

Roadmap

- add image
- add references in the margin
- integrate python/R rendering via Quarto

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Conclusion

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