

## 1 IEEE 754 Floating Point

Given a 32-bit single precision IEEE 754 floating point number  $A$ , we have the three parts:

- $A[31:31]$  - Sign bit, 0 for +ve, 1 for -ve;
- $A[30:23]$  - Exponent bits (8 bits)
- $A[22:0]$  - Mantissa bits (24 bits)

From a decimal form, we can calculate the IEEE 754 representation in the following steps (taking number **85.125** as an example):

1. Covert integer part and decimal part into binary
  - (a)  $85 = (1010101)_2$
  - (b)  $.125 = (.001)_2$
2. We have 1010101.001 now. Make it only have one bit in the integer part, hence we do right shift for 6 bits. ( $1010101.001 = 1.010101001 \cdot 2^6$ )
3. We calculate the exponent bit. For single precision, exponent = 127 + power of 2. Hence we have 133 for this example. ( $133 = (10000101)_2$ )
4. Put everything together with 0 fills at the back, we have the binary representation in IEEE 754 format:  
 $(0 \ 1000101 \ 010101001 \ 00000000000000)_2 = 0x42AA4000$
5. Sometimes numbers cannot be represented precisely in binary (e.g. **0.3**), we do the binary conversion until we have the full 23-bit Mantissa.

## 2 Binary conversion

To convert a decimal point to binary, taking **0.125** as an example, we do this:

Decimal Number	Result	Binary
$0.125 \cdot 2$	<b>0.25</b>	0
$0.25 \cdot 2$	<b>0.50</b>	0
$0.50 \cdot 2$	<b>1.00</b>	1
$0.00 \cdot 2$	<b>0.00</b>	0

\* Binary number is read from top to bottom. In this case,  $.125 = (.001)_2$ .

## 3 Negating Numbers

To negate a binary number, we have

- Sign-and-Magnitude: just flip MSB to negate a number ( $0001_2 \rightarrow 1001_2$ )
- 1s Complement: flip every bits to negate a number ( $0001_2 \rightarrow 1110_2$ )

- 2s Complement: add 1 to **LSB** of 1s Complement ( $0001.01_2 \rightarrow 1110.10_2 \rightarrow 1110.11_2$ )

We can detect overflow if either happens:

- +ve add +ve become -ve
- -ve add -ve become +ve

When performing binary addition on **1s-complement** numbers, if there is a carry out of the MSB, add 1 to the result and dispose the MSB. We do **NOT** need to do so for addition on **2s-complement** numbers.

## 4 Boolean algebra laws and theorems

Duality: if the AND/OR operators and identity elements 0/1 in a Boolean equation are interchanged, it remains valid (but brackets are to be added when necessary). (e.g.  $X + X \cdot Y = X \rightarrow X \cdot (X + Y) = X$ )

Laws:

- Identity:  $A + 0 = 0 + A = A$
- Inverse/Complement:  $A + A' = 1$
- Commutative:  $A + B = B + A$
- Associative:  $A + (B + C) = (A + B) + C$
- Distributive:  $A + (B \cdot C) = A \cdot B + A \cdot C$

Theorems:

- Idempotency:  $X + X = X$
- One element/Zero element:  $X + 1 = 1$
- Involution:  $(X')' = X$
- Absorption 1:  $X + X \cdot Y = X$
- Absorption 2:  $X + X' \cdot Y = X + Y$
- DeMorgans':  $(X + Y)' = X' \cdot Y'$
- Consensus:  $X \cdot Y + X' \cdot Z + Y \cdot Z = X \cdot Y + X' \cdot Z$

## 5 Flip-flop reference

Characteristic Tables:

J	K	Q <sup>+</sup>	S	R	Q <sup>+</sup>
0	0	Q	0	0	Q
0	1	0	0	1	0
1	0	1	1	0	1
1	1	Q(t)'	1	1	Unpredictable

  

D	Q <sup>+</sup>	T	Q <sup>+</sup>
0	0	0	Q
1	1	1	Q'

Excitation Tables:

Q	Q <sup>+</sup>	J	K	Q	Q <sup>+</sup>	S	R
0	0	0	X	0	0	0	X
0	1	1	X	0	1	1	0
1	0	X	1	1	0	0	1
1	1	X	0	1	1	X	0

  

Q	Q <sup>+</sup>	D	Q	Q <sup>+</sup>	T
0	0	0	0	0	0
0	1	1	0	1	1
1	0	0	1	0	1
1	1	1	1	1	0

$D = Q^+$                        $T = Q \oplus Q^+$

## 6 Notes about pipeline

Considering instruction A and B. B is just the instruction executed after A.

- A pipeline stage of B will always be behind the same stage for A, even if the resource is not occupied and there is no data dependency / control hazard. (e.g. D stage of instruction B cannot be executed at cycle 3, even D is vacant)

Inst	1	2	3	4	5	6	7	8
A	F			D	E	M	W	
B		F			D	E	M	W

- For  $N$  instructions, ideal 5-stage pipeline takes  $N + 2$  cycles to execute.
- Delays introduced when no forwarding, no early branching and no branch prediction.
  - Data dependency: **2 cycles**

– Control hazard (beq/bne, j): **3 cycles**

- Delays introduced when there is forwarding: **1 cycle** for data dependency after **lw**.
- Data dependency for **beq/bne** instruction can lead to delay **when forwarding and early branching is implemented**. This is because data is required in stage 2 (D), not stage 3 (E), of the branching instruction.
- When branch prediction is wrong, flush everything including the correct ones to be executed. This is especially true when there is **no early branching**. Considering the following example: assuming not taken is used, but actually A branched to C. C and D is executed once again after everything flushed.

Inst	1	2	3	4	5	6	7	8
A (beq)	F	D	E	M	W			
B		F	D	E	X	X		
C			F	D	X	X	X	
D				F	X	X	X	X
C (again)					F	D	E	M

## 7 Cache

Average Access Time formula:

$$\text{Hit rate} \times \text{Hit time} + (1 - \text{Hit rate}) \times \text{Miss penalty}$$

## 8 Quick reference

if (a < b) {...} else {...}

slt \$t0, \$a, \$b
beq \$t0, \$zero, else

if (a >= b) {...} else {...} : flip a and b in the above example.

## 9 Miscellaneous notes

- Zero-extension and Sign-extension is different especially in the case that the number starts with 1 in binary.
- MIPS does not have **not** instruction. We use **nor \$1, \$1, \$zero** to do bitwise not.