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BLG 354E Signals & Systems Homework II

Q1) Before finding the values for every graph, there is a pattern. ~~~~

- Since $N=8$ for all tables, $C_k = \frac{1}{8}\sum_{n=0}^{7} x[n]\, e^{-j\frac{k\pi}{4}\cdot n}$ for all x functions, $\Omega = \frac{2\pi}{8} = \frac{\pi}{4}$
- Since $e^{jx} = \cos(x) + i\cdot\sin(x)$, $C_k = \frac{1}{8}\sum_{n=0}^{7} x[n]\left(\cos\left(-\frac{k\pi n}{4}\right) + i\sin\left(-\frac{k\pi n}{4}\right)\right)$
- Since $\cos(x)$ is even and $\sin(x)$ is odd; $\boxed{C_k = \frac{1}{8}\sum_{n=0}^{7} x[n]\left(\cos\left(\frac{k\pi n}{4}\right) - i\sin\left(\frac{k\pi n}{4}\right)\right)}$
- I have calculated the values of $\left(\cos\left(\frac{k\pi n}{4}\right) - i\sin\left(\frac{k\pi n}{4}\right)\right)$ for $k: 0 \dots 7$, $n: 0 \dots 7$ for all functions given in the question, using a python script,
- Table of values:

	n=0	n=1	n=2	n=3	n=4	n=5	n=6	n=7
k=0	+1	+1	+1	+1	+1	+1	+1	+1
k=1	+1	$\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$	$-i$	$-\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$	-1	$-\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$	$+i$	$\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$
k=2	+1	$-i$	-1	$+i$	$+1$	$-i$	-1	$+i$
k=3	+1	$-\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$	$+i$	$\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$	-1	$\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$	$-i$	$-\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$
k=4	+1	-1	1	-1	1	-1	1	-1
k=5	+1	$-\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$	$-i$	$\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$	-1	$\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$	$+i$	$-\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$
k=6	+1	$+i$	-1	$-i$	$+1$	$+i$	-1	$-i$
k=7	+1	$\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$	$+i$	$-\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$	-1	$-\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$	$-i$	$\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$

- With using this table and multiplying $x[n]$s with adequate values, C_ks for x functions can be found easily.

- The program used to calculate these values is provided in the screenshot below,

Q1 a)

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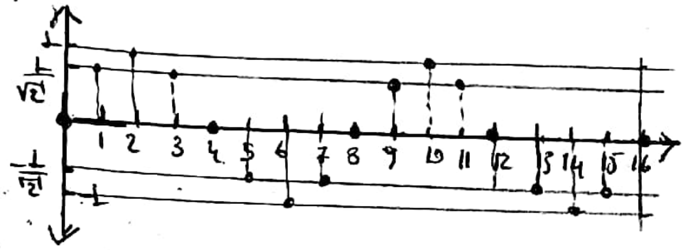

$N=8,\ \Omega = \frac{2\pi}{8} = \frac{\pi}{4}$

$x[0]=0 \qquad x[4]=0$

$x[1]=\frac{1}{\sqrt{2}} \qquad x[5]=\frac{-1}{\sqrt{2}}$

$x[2]=1 \qquad x[6]=-1$

$x[3]=\frac{1}{\sqrt{2}} \qquad x[7]=\frac{-1}{\sqrt{2}}$

I will use the table to calculate c_ks, $\boxed{c_k = \frac{1}{8}\sum_{n=0}^{7} x[n]\left(\cos\left(\frac{nk\pi}{4}\right) - i\sin\left(\frac{nk\pi}{4}\right)\right)}$

	n=0	n=1	n=2	n=3	n=4	n=5	n=6	n=7	Sum
k=0	1	1	1	1	1	1	1	1	—
c_0	0	$\cancel{\frac{1}{\sqrt{2}}}$	$\cancel{1}$	$\cancel{\frac{1}{\sqrt{2}}}$	0	$\cancel{-\frac{1}{\sqrt{2}}}$	$\cancel{-1}$	$\cancel{-\frac{1}{\sqrt{2}}}$	0
k=1	1	$\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$	$-i$	$\frac{-1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$	-1	$\frac{-1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$	i	$\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$	—
c_1	0	$\cancel{\frac{1}{2}} - \frac{i}{2}$	$-i$	$\cancel{\frac{-1}{2}} - \frac{i}{2}$	0	$\cancel{\frac{1}{2}} - \frac{i}{2}$	$-i$	$\cancel{\frac{-1}{2}} - \frac{i}{2}$	$-4i$
k=2	1	$-i$	-1	i	$+1$	$-i$	-1	i	—
c_2	0	$\cancel{\frac{-i}{\sqrt{2}}}$	$\cancel{-1}$	$\cancel{\frac{i}{\sqrt{2}}}$	0	$\cancel{\frac{i}{\sqrt{2}}}$	$\cancel{1}$	$\cancel{\frac{-i}{\sqrt{2}}}$	0
k=3	1	$\frac{-1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$	i	$\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$	-1	$\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$	$-i$	$\frac{-1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$	—
c_3	0	$\cancel{\frac{-1}{2}} \cancel{-\frac{i}{2}}$	$\cancel{i}$	$\cancel{\frac{1}{2}} \cancel{-\frac{i}{2}}$	0	$\cancel{\frac{-1}{2}} \cancel{-\frac{i}{2}}$	$\cancel{i}$	$\cancel{\frac{1}{2}} \cancel{-\frac{i}{2}}$	0
k=4	1	-1	1	-1	1	-1	1	-1	[illegible]
c_4	0	$\cancel{\frac{-1}{\sqrt{2}}}$	$\cancel{1}$	$\cancel{\frac{-1}{\sqrt{2}}}$	0	$\cancel{\frac{1}{\sqrt{2}}}$	$\cancel{-1}$	$\cancel{\frac{1}{\sqrt{2}}}$	0
k=5	1	$\frac{-1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$	$-i$	$\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$	-1	$\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$	i	$\frac{-1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$	—
c_5	0	$\cancel{\frac{-1}{2}} + \cancel{\frac{i}{2}}$	$\cancel{-i}$	$\cancel{\frac{1}{2}} + \cancel{\frac{i}{2}}$	0	$\cancel{\frac{-1}{2}} + \cancel{\frac{i}{2}}$	$\cancel{-i}$	$\cancel{\frac{1}{2}} + \cancel{\frac{i}{2}}$	0
k=6	1	$+i$	-1	$-i$	$+1$	$+i$	-1	$-i$	—
c_6	0	$\cancel{\frac{i}{\sqrt{2}}}$	$\cancel{-1}$	$\cancel{\frac{-i}{\sqrt{2}}}$	0	$\cancel{\frac{-i}{\sqrt{2}}}$	$\cancel{1}$	$\cancel{\frac{i}{\sqrt{2}}}$	0
k=7	1	$\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$	$+i$	$\frac{-1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$	-1	$\frac{-1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$	$-i$	$\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$	—
c_7	0	$\cancel{\frac{1}{2}} + \frac{i}{2}$	i	$\cancel{\frac{-1}{2}} + \frac{i}{2}$	0	$\cancel{\frac{1}{2}} + \frac{i}{2}$	i	$\cancel{\frac{-1}{2}} + \frac{i}{2}$	$4i$
x_n	[illegible]	$\frac{1}{\sqrt{2}}$	1	$\frac{1}{\sqrt{2}}$	0	$\frac{-1}{\sqrt{2}}$	-1	$\frac{-1}{\sqrt{2}}$	—

$c_0 = 0 \qquad c_4 = 0$

$c_1 = \frac{4i}{8} = \frac{i}{2} \qquad c_5 = 0$

$c_2 = 0 \qquad c_6 = 0$

$c_3 = 0 \qquad c_7 = \frac{4i}{8} = \frac{i}{2}$

$$x[n] = \sum_{k=0}^{7} c_k e^{jk\Omega n} = \sum_{k=0}^{7} c_k \left(\cos\left(k.\frac{\pi}{4}.n\right) + i\sin\left(k.\frac{\pi}{4}.n\right)\right)$$

$$= \frac{i}{2}\left(\cos\left(\frac{\pi n}{4}\right) + i\sin\left(\frac{\pi n}{4}\right) + \cos\left(\frac{7\pi n}{4}\right) + i\sin\left(7\frac{\pi n}{4}\right)\right)$$

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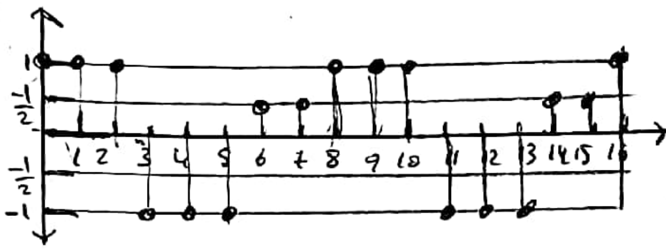
Q1b)



$N=8,\ \Omega = \frac{2\pi}{8} = \frac{\pi}{4}$

$x[0]=1 \quad x[4]=-1$
$x[1]=1 \quad x[5]=-1$
$x[2]=1 \quad x[6]=1/2$
$x[3]=-1 \quad x[7]=1/2$

I will use the table as in the previous example. $C_k = \frac{1}{8}\sum_{n=0}^{7} x[n]\, e^{-j\Omega n k}$

x_n	1	1	1	-1	-1	-1	$1/2$	$1/2$	—
n	0	1	2	3	4	5	6	7	SUM
$k=0$	1	1	1	1	1	1	1	1	—
C_0	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	$\cancel{-1}$	$\cancel{-1}$	$\cancel{-1}$	$1/2$	$1/2$	$\frac{1}{8}$
$k=1$	1	$\frac{1}{\sqrt2}-\frac{i}{\sqrt2}$	$-i$	$\frac{-1}{\sqrt2}-\frac{i}{\sqrt2}$	-1	$\frac{-1}{\sqrt2}+\frac{i}{\sqrt2}$	$+i$	$\frac{+1}{\sqrt2}+\frac{i}{\sqrt2}$	—
C_1	1	$\frac{1}{\sqrt2}-\cancel{\frac{i}{\sqrt2}}$	$-i$	$\frac{+1}{\sqrt2}+\cancel{\frac{i}{\sqrt2}}$	1	$\frac{1}{\sqrt2}-\frac{i}{\sqrt2}$	$+i/2$	$\frac{1}{2\sqrt2}+\frac{i}{2\sqrt2}$	$0{,}559$ $-0{,}107i$
$k=2$	1	$-i$	-1	i	1	$-i$	-1	i	—
C_2	$\cancel{1}$	$-i$	$\cancel{-1}$	$\cancel{-1}$	-1	$\cancel{+i}$	$-\frac{1}{2}$	$\frac{+1}{2}$	$\frac{-3}{16}+\frac{i}{16}$
$k=3$	1	$\frac{-1}{\sqrt2}\ \frac{-i}{\sqrt2}$	$+i$	$\frac{1}{\sqrt2}-\frac{i}{\sqrt2}$	-1	$\frac{1}{\sqrt2}+\frac{1}{\sqrt2}$	$-i$	$\frac{-1}{\sqrt2}+\frac{1}{\sqrt2}$	—
C_3	1	$\frac{-1}{\sqrt2}-\cancel{\frac{i}{\sqrt2}}$	$+i$	$\frac{-1}{\sqrt2}+\cancel{\frac{i}{\sqrt2}}$	1	$\frac{-1}{\sqrt2}-\frac{i}{\sqrt2}$	$\frac{-i}{2}$	$\frac{-1}{2\sqrt2}+\frac{i}{2\sqrt2}$	$-0{,}059$ $+0{,}018i$
$k=4$	1	-1	1	-1	1	-1	1	-1	—
C_4	$\cancel{1}$	$\cancel{-1}$	1	1	$\cancel{-1}$	$\cancel{1}$	$\cancel{1/2}$	$\cancel{-1/2}$	$\frac{1}{4}$
$k=5$	1	$\frac{-1}{\sqrt2}+\frac{i}{\sqrt2}$	$-i$	$\frac{1}{\sqrt2}+\frac{i}{\sqrt2}$	-1	$\frac{1}{\sqrt2}-\frac{i}{\sqrt2}$	$+i$	$-\frac{1}{\sqrt2}-\frac{i}{\sqrt2}$	—
C_5	1	$\frac{-1}{\sqrt2}+\cancel{\frac{i}{\sqrt2}}$	$-i$	$\frac{+1}{\sqrt2}-\cancel{\frac{i}{\sqrt2}}$	1	$\frac{-1}{\sqrt2}+\frac{i}{\sqrt2}$	$i/2$	$\frac{-1}{2\sqrt2}-\frac{i}{2\sqrt2}$	$-0{,}059$ $-0{,}018i$
$k=6$	1	$+i$	-1	$-i$	$+1$	$+i$	-1	$-i$	=
C_6	$\cancel{1}$	1	-1	$\cancel{i}$	$\cancel{-1}$	$\cancel{-i}$	$-1/2$	$-i/2$	$\frac{-3}{16}+\frac{i}{16}$
$k=7$	1	$\frac{1}{\sqrt2}+\frac{i}{\sqrt2}$	$+i$	$\frac{-1}{\sqrt2}+\frac{i}{\sqrt2}$	-1	$\frac{-1}{\sqrt2}-\frac{i}{\sqrt2}$	$-i$	$\frac{1}{\sqrt2}-\frac{i}{\sqrt2}$	—
C_7	1	$\frac{1}{\sqrt2}+\frac{i}{\sqrt2}$	i	$\frac{1}{\sqrt2}-\cancel{\frac{i}{\sqrt2}}$	1	$\frac{1}{\sqrt2}+\cancel{\frac{i}{\sqrt2}}$	$-i/2$	$\frac{1}{2\sqrt2}-\frac{i}{2\sqrt2}$	$0{,}559$ $+0{,}107i$

$C_0 = \frac{1}{8}$

$C_4 = \frac{2}{8} = \frac{1}{4}$

$C_2 = \dfrac{-\frac{3}{2}+\frac{i}{2}}{8} = \frac{-3}{16}+\frac{i}{16}$

$C_6 = \dfrac{-\frac{3}{2}+\frac{i}{2}}{8} = \frac{-3}{16}+\frac{i}{16}$

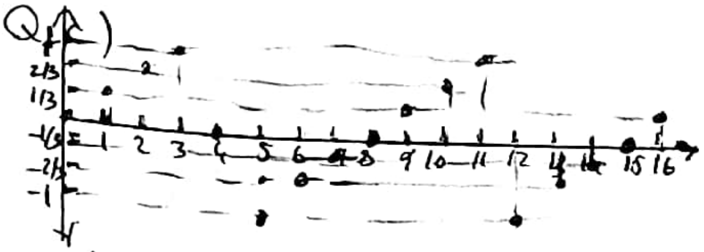
$C_1 = \dfrac{2+\frac{3}{\sqrt2}+\frac{1}{2\sqrt2}-i-\frac{i}{\sqrt2}+\frac{i}{2}+\frac{i}{2\sqrt2}}{8} = \dfrac{\frac{8+7\sqrt2}{4}+\frac{-2-\sqrt2}{4}i}{8} = \begin{matrix}0{,}559\\-0{,}107i\end{matrix}$

$C_3 = \dfrac{2-\frac{3}{\sqrt2}-\frac{1}{2\sqrt2}+i-\frac{i}{2}+\frac{i}{\sqrt2}+\frac{i}{2\sqrt2}}{8} = \dfrac{\frac{8-7\sqrt2}{4}+\frac{2-\sqrt2}{4}i}{8} = \begin{matrix}-0{,}059\\+0{,}018i\end{matrix}$

$C_5 = \dfrac{2+\frac{-3}{\sqrt2}-\frac{1}{2\sqrt2}-i+\frac{i}{\sqrt2}+\frac{i}{2}-\frac{i}{2\sqrt2}}{8} = \dfrac{\frac{8-7\sqrt2}{4}+\frac{-2+\sqrt2}{4}i}{8} = \begin{matrix}-0{,}059\\-0{,}018i\end{matrix}$

$C_7 = \dfrac{2+\frac{3}{\sqrt2}+\frac{1}{2\sqrt2}+i+\frac{i}{\sqrt2}-\frac{i}{2}-\frac{i}{2\sqrt2}}{8} = \dfrac{\frac{8+7\sqrt2}{4}+\frac{2+\sqrt2}{4}i}{8} = \begin{matrix}0{,}559\\+0{,}107i\end{matrix}$

Q1)



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$N=8,\ \Omega=\frac{2\pi}{8}=\pi/4$

$x[0]=0$
$x[1]=1/3$
$x[2]=2/3$
$x[3]=1$
$x[4]=0$
$x[5]=-1$
$x[6]=-2/3$
$x[7]=-1/3$

n	0	1	2	3	4	5	6	7	—
x[n]	0	1/3	2/3	1	0	-1	-2/3	-1/3	Sum
k=0	1	1	1	1	1	1	1	1	—
C_0	0	1/3	2/3	1	0	-1	-2/3	-1/3	0
k=1	1	$\frac{1}{\sqrt2}-\frac{i}{\sqrt2}$	$-i$	$\frac{-1}{\sqrt2}-\frac{i}{\sqrt2}$	-1	$\frac{-1}{\sqrt2}+\frac{i}{\sqrt2}$	$+i$	$+\frac{1}{\sqrt2}+\frac{i}{\sqrt2}$	—
C_1	0	$\cancel{\frac{1}{3\sqrt2}}-\frac{i}{3\sqrt2}$	$\frac{-2i}{3}$	$\cancel{\frac{-1}{\sqrt2}}-\frac{i}{\sqrt2}$	0	$\cancel{\frac{1}{\sqrt2}}-\frac{i}{\sqrt2}$	$\frac{-2i}{3}$	$\cancel{\frac{-1}{3\sqrt2}}-\frac{i}{3\sqrt2}$	$-0,402i$
k=2	1	$-i$	-1	i	1	$-i$	-1	i	—
C_2	0	$-i/3$	$\cancel{-2/3}$	i	0	i	$\cancel{2/3}$	$-i/3$	$0,166i$
k=3	1	$\frac{-1}{\sqrt2}-\frac{i}{\sqrt2}$	$+i$	$\frac{1}{\sqrt2}-\frac{i}{\sqrt2}$	-1	$\frac{1}{\sqrt2}+\frac{i}{\sqrt2}$	$\mp i$	$\frac{-1}{\sqrt2}+\frac{i}{\sqrt2}$	—
C_3	0	$\cancel{\frac{-1}{3\sqrt2}}-\frac{i}{3\sqrt2}$	$\frac{2i}{3}$	$\cancel{\frac{1}{\sqrt2}}-\frac{i}{\sqrt2}$	0	$\cancel{\frac{-1}{\sqrt2}}-\frac{i}{\sqrt2}$	$\frac{2i}{3}$	$\cancel{\frac{1}{3\sqrt2}}-\frac{i}{3\sqrt2}$	$-0,069i$
k=4	1	-1	1	-1	1	-1	1	-1	—
C_4	0	$-1/3$	2/3	-1	0	1	$-2/3$	1/3	0
k=5	1	$\frac{-1}{\sqrt2}+\frac{i}{\sqrt2}$	$-i$	$\frac{1}{\sqrt2}+\frac{i}{\sqrt2}$	-1	$\frac{1}{\sqrt2}-\frac{i}{\sqrt2}$	$+i$	$\frac{-1}{\sqrt2}-\frac{i}{\sqrt2}$	—
C_5	0	$\cancel{\frac{-1}{3\sqrt2}}+\frac{i}{3\sqrt2}$	$-2i/3$	$\cancel{\frac{1}{\sqrt2}}+\frac{i}{\sqrt2}$	0	$\cancel{\frac{-1}{\sqrt2}}+\frac{i}{\sqrt2}$	$\cancel{-2i/3}$	$\cancel{\frac{+1}{3\sqrt2}}+\frac{i}{3\sqrt2}$	$0,069i$
k=6	1	$+i$	-1	$-i$	$+1$	i	-1	$-i$	—
C_6	0	$+1/3$	$\cancel{-2/3}$	$-i$	0	$-i$	$+2/3$	$+1/3$	$-0,166i$
k=7	1	$\frac{1}{\sqrt2}+\frac{i}{\sqrt2}$	$+i$	$\frac{-1}{\sqrt2}+\frac{i}{\sqrt2}$	-1	$\frac{-1}{\sqrt2}-\frac{i}{\sqrt2}$	$-i$	$\frac{1}{\sqrt2}-\frac{i}{\sqrt2}$	—
C_7	0	$\cancel{\frac{1}{3\sqrt2}}+\frac{i}{3\sqrt2}$	$\frac{2i}{3}$	$\cancel{\frac{-1}{\sqrt2}}+\frac{i}{\sqrt2}$	0	$\cancel{\frac{1}{\sqrt2}}+\frac{i}{\sqrt2}$	$\frac{2i}{3}$	$\cancel{\frac{-1}{3\sqrt2}}+\frac{i}{3\sqrt2}$	$+0,402i$

$C_0=C_4=0$

$$C_2=\frac{\frac{-i}{3}+i+i-\frac{i}{3}}{8}=\frac{\frac{4i}{3}}{8}=\frac{i}{6}=0,166i$$

$$C_6=\frac{\frac{i}{3}-i-i+\frac{i}{3}}{8}=\frac{\frac{-4i}{3}}{8}=\frac{-i}{6}=-0,166i$$

$$C_5=\left(\frac{i}{3\sqrt2}-\frac{2i}{3}+\frac{i}{\sqrt2}\right)\cdot\cancel{x}=\frac{(\sqrt2-4+3\sqrt2)i}{24}=\frac{(\sqrt2-1)i}{6}$$

$$C_3=\left(\frac{-i}{3\sqrt2}+\frac{2i}{3}-\frac{i}{\sqrt2}\right)\cdot\cancel{x}=\frac{(-\sqrt2+4-3\sqrt2)i}{24}=\frac{(-\sqrt2+1)i}{6}$$

$$C_1=\frac{\left(\frac{-i}{3\sqrt2}-\frac{2i}{3}-\frac{i}{\sqrt2}\right)\cdot\cancel{2}}{\cancel{8}\,4}=\frac{(-\sqrt2-4-3\sqrt2)i}{24}$$

$$C_7=\frac{\left(\frac{i}{3\sqrt2}+\frac{2i}{3}+\frac{i}{\sqrt2}\right)\cdot2}{8}=\frac{(-\sqrt2-1)i}{6}=\frac{(\sqrt2+1)i}{6}$$

Q1d)

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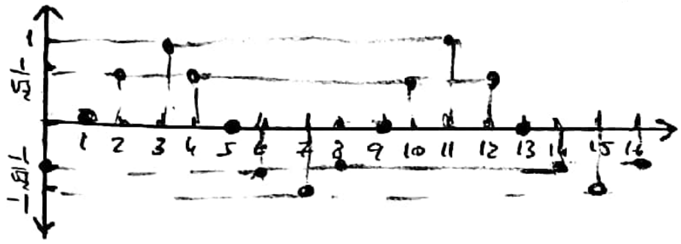


$N=8 \quad \Omega = \frac{2\pi}{8} = \frac{\pi}{4}$

$x[0] = \frac{-1}{\sqrt2}$ $\quad x_4 = \frac{1}{\sqrt2}$

$x[1] = 0$ $\quad x_5 = 0$

$x[2] = \frac{1}{\sqrt2}$ $\quad x_6 = \frac{-1}{\sqrt2}$

$x[3] = 1$ $\quad x_7 = -1$

n	0	1	2	3	4	5	6	7	~~8~~
x_n	$\frac{-1}{\sqrt2}$	0	$\frac{1}{\sqrt2}$	1	$\frac{1}{\sqrt2}$	0	$\frac{-1}{\sqrt2}$	-1	sum
k=0	1	1	1	1	+1	1	1	1	—
c_0	$\cancel{\frac{-1}{\sqrt2}}$	0	$\cancel{+\frac{1}{\sqrt2}}$	$\cancel{+1}$	$\cancel{+\frac{1}{\sqrt2}}$	0	$\cancel{-\frac{1}{\sqrt2}}$	$\cancel{-1}$	0
k=1	1	$\frac{1}{\sqrt2} - \frac{i}{\sqrt2}$	$-i$	$\frac{-1}{\sqrt2} - \frac{i}{\sqrt2}$	-1	$\frac{-1}{\sqrt2} + \frac{i}{\sqrt2}$	$+i$	$\frac{1}{\sqrt2} + \frac{i}{\sqrt2}$	—
c_1	$\frac{-1}{\sqrt2}$	0	$\frac{-i}{\sqrt2}$	$\frac{-1}{\sqrt2} - \frac{i}{\sqrt2}$	$\frac{-1}{\sqrt2}$	0	$\frac{-i}{\sqrt2}$	$-\frac{1}{\sqrt2} - \frac{i}{\sqrt2}$	$-0{,}353$ $-0{,}353i$
k=2	1	$-i$	-1	$+i$	$+1$	$-i$	-1	$+i$	—
c_2	$\cancel{\frac{-1}{\sqrt2}}$	0	$\cancel{\frac{-1}{\sqrt2}}$	$\cancel{+i}$	$\cancel{+\frac{1}{\sqrt2}}$	0	$\cancel{+\frac{1}{\sqrt2}}$	$\cancel{-i}$	0
k=3	1	$\frac{-1}{\sqrt2} - \frac{i}{\sqrt2}$	$+i$	$\frac{1}{\sqrt2} - \frac{i}{\sqrt2}$	-1	$\frac{1}{\sqrt2} + \frac{i}{\sqrt2}$	$-i$	$\frac{-1}{\sqrt2} + \frac{i}{\sqrt2}$	—
c_3	$\cancel{\frac{-1}{\sqrt2}}$	0	$\cancel{\frac{i}{\sqrt2}}$	$\cancel{\frac{1}{\sqrt2}}\ \cancel{\frac{i}{\sqrt2}}$	$\cancel{\frac{-1}{\sqrt2}}$	0	$\cancel{\frac{+i}{\sqrt2}}$	$\cancel{\frac{+1}{\sqrt2}} - \cancel{\frac{i}{\sqrt2}}$	0
k=4	1	-1	1	-1	+1	-1	1	-1	—
c_4	$\cancel{\frac{-1}{\sqrt2}}$	0	$\cancel{\frac{1}{\sqrt2}}$	$\cancel{-1}$	$\cancel{\frac{1}{\sqrt2}}$	0	$\cancel{\frac{1}{\sqrt2}}$	$\cancel{1}$	0
k=5	1	$\frac{-1}{\sqrt2} + \frac{i}{\sqrt2}$	$-i$	$\frac{1}{\sqrt2} + \frac{i}{\sqrt2}$	-1	$\frac{+1}{\sqrt2} - \frac{i}{\sqrt2}$	$+i$	$\frac{-1}{\sqrt2} - \frac{i}{\sqrt2}$	—
c_5	$\cancel{\frac{-1}{\sqrt2}}$	0	$\cancel{\frac{-i}{\sqrt2}}$	$\cancel{\frac{+1}{\sqrt2}} + \cancel{\frac{i}{\sqrt2}}$	$\cancel{\frac{-1}{\sqrt2}}$	0	$\cancel{\frac{-i}{\sqrt2}}$	$\cancel{\frac{+1}{\sqrt2}} + \cancel{\frac{i}{\sqrt2}}$	0
k=6	1	$+i$	-1	$-i$	+1	$+i$	-1	$-i$	—
c_6	$\cancel{\frac{-1}{\sqrt2}}$	0	$\cancel{\frac{-1}{\sqrt2}}$	$\cancel{-i}$	$\cancel{+\frac{1}{\sqrt2}}$	0	$\cancel{+\frac{1}{\sqrt2}}$	$\cancel{+i}$	0
k=7	1	$\frac{1}{\sqrt2} + \frac{i}{\sqrt2}$	$+i$	$\frac{-1}{\sqrt2} + \frac{i}{\sqrt2}$	-1	$\frac{-1}{\sqrt2} - \frac{i}{\sqrt2}$	$-i$	$\frac{1}{\sqrt2} - \frac{i}{\sqrt2}$	—
c_7	$\frac{-1}{\sqrt2}$	0	$\frac{+i}{\sqrt2}$	$\frac{-1}{\sqrt2} + \frac{i}{\sqrt2}$	$-\frac{1}{\sqrt2}$	0	$\frac{+i}{\sqrt2}$	$\frac{-1}{\sqrt2} + \frac{i}{\sqrt2}$	$-0{,}353$ $+0{,}353i$

$C_0 = C_2 = C_3 = C_4 = C_5 = C_6 = 0$

$$C_1 = \frac{\frac{-\cancel{4}}{\sqrt2} - \frac{\cancel{4}i}{\sqrt2}}{\cancel{8}\,2} = \frac{-1-i}{2\sqrt2} = \frac{-\sqrt2}{4} - \frac{\sqrt2 i}{4} = -0{,}353 - 0{,}353i$$

$$C_7 = \frac{\frac{-\cancel{4}}{\sqrt2} + \frac{\cancel{4}i}{\sqrt2}}{\cancel{8}\,2} = \frac{-1+i}{2\sqrt2} = \frac{-\sqrt2}{4} + \frac{\sqrt2 i}{4} = -0{,}353 + 0{,}353i$$

hw1.py

hw1.py > ...

```python
# Fatih Baskın
# 150210710

import numpy as np


PI = np.pi
OMEGA = PI / 4


def calc_k_n_exp(k: int, n: int) -> np.complex128:
    return np.exp(-1j * k * OMEGA * n)


def table_n_exp(k: int, n: int) -> list:
    return [calc_k_n_exp(k, i) for i in range(n)]


def table_k_exp(k: int, n: int) -> list:
    return [table_n_exp(i, n) for i in range(k)]


if __name__ == "__main__":
    n = 8
    k = 8
    table_ = table_k_exp(k, n)

    for i in range(k):
        print(f"Table for k = {i}", end=": ")
        for j in range(n):
            print(f"{table_[i][j]:.2f}", end=" ")
        print()
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PORTS

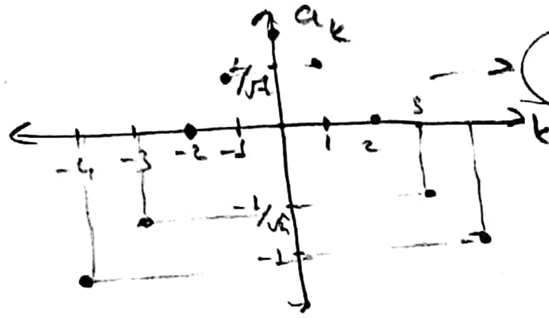
```
Table for k = 3: 1.00+0.00j -0.71-0.71j -0.00+1.00j 0.71-0.71j -1.00-0.00j 0.71+0.71j 0.00-1.00j -0.71+0.71j
Table for k = 4: 1.00+0.00j -1.00-0.00j 1.00+0.00j -1.00-0.00j 1.00+0.00j -1.00-0.00j 1.00+0.00j -1.00-0.00j
Table for k = 5: 1.00+0.00j -0.71+0.71j 0.00-1.00j 0.71+0.71j -1.00-0.00j 0.71-0.71j -0.00+1.00j -0.71-0.71j
Table for k = 6: 1.00+0.00j -0.00+1.00j -1.00-0.00j 0.00-1.00j 1.00+0.00j -0.00+1.00j -1.00-0.00j -0.00-1.00j
Table for k = 7: 1.00+0.00j 0.71+0.71j -0.00+1.00j -0.71+0.71j -1.00-0.00j -0.71-0.71j -0.00-1.00j 0.71-0.71j
PS C:\Users\Fatih\Desktop\signals_hw2> & c:/Users/Fatih/Desktop/signals_hw2/.venv/Scripts/python.exe c:/Users/Fatih/Desktop/signals_hw2/hw1.py
Table for k = 0: 1.00+0.00j 1.00+0.00j 1.00+0.00j 1.00+0.00j 1.00+0.00j 1.00+0.00j 1.00+0.00j 1.00+0.00j
Table for k = 1: 1.00+0.00j 0.71-0.71j 0.00-1.00j -0.71-0.71j -1.00-0.00j -0.71+0.71j -0.00+1.00j 0.71+0.71j
Table for k = 2: 1.00+0.00j 0.00-1.00j -1.00-0.00j -0.00+1.00j 1.00+0.00j 0.00-1.00j -1.00-0.00j -0.00+1.00j
Table for k = 3: 1.00+0.00j -0.71-0.71j -0.00+1.00j 0.71-0.71j -1.00-0.00j 0.71+0.71j 0.00-1.00j -0.71+0.71j
Table for k = 4: 1.00+0.00j -1.00-0.00j 1.00+0.00j -1.00-0.00j 1.00+0.00j -1.00-0.00j 1.00+0.00j -1.00-0.00j
Table for k = 5: 1.00+0.00j -0.71+0.71j 0.00-1.00j 0.71+0.71j -1.00-0.00j 0.71-0.71j -0.00+1.00j -0.71-0.71j
Table for k = 6: 1.00+0.00j -0.00+1.00j -1.00-0.00j 0.00-1.00j 1.00+0.00j -0.00+1.00j -1.00-0.00j -0.00-1.00j
Table for k = 7: 1.00+0.00j 0.71+0.71j -0.00+1.00j -0.71+0.71j -1.00-0.00j -0.71-0.71j -0.00-1.00j 0.71-0.71j
PS C:\Users\Fatih\Desktop\signals_hw2>
```

Q2)

Fatih Besker

150210710

a)

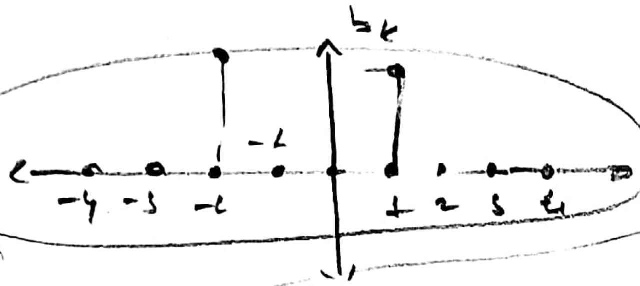


$\cos\left(\frac{\pi}{4} \cdot k\right)$

$k=0 \rightarrow \cos 0 = 1$

$k=1 \rightarrow \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$k=2 \rightarrow \cos \frac{\pi}{2} = 0$



$b_k = \delta[k-1] + \delta[k+2]$

$\Omega = \frac{2\pi}{8} = \frac{\pi}{4}$

$$\cos \frac{\pi k}{4} = \frac{e^{\frac{j2\pi k}{8}} + e^{\frac{-j2\pi k}{8}}}{2} = \frac{1}{8} \sum_n x[n] e^{-j\frac{2\pi}{8}kn}$$

$n = 1, n = -1$

$x[n] = 4$

$$x[n] = 4\left(\delta[n-1] + \delta[n+1]\right)$$

b)

$$x[n] = \sum_k b_k \cdot e^{j\frac{2\pi}{8}kn} = e^{j\frac{2\pi}{8}n} + e^{-j\frac{2\pi}{8}2n}$$

$$= \cos \frac{\pi}{4} n + i \sin \frac{\pi}{4} n + \underbrace{\cos\left(\frac{-\pi}{2} n\right)}_{\text{even}} + \underbrace{i \sin\left(\frac{-\pi}{2} n\right)}_{\text{odd}}$$

$$= \cos\left(\frac{\pi}{4} n\right) + \cos\left(\frac{\pi}{2} n\right) + i \sin\left(\frac{\pi}{4} n\right) - i \sin\left(\frac{\pi}{2} n\right)$$

$$x[n] = \cos\left(\frac{\pi}{2} n\right) + \cos\left(\frac{\pi}{4} n\right) + \left(\sin\left(\frac{\pi}{4} n\right) - \sin\left(\frac{\pi}{2} n\right)\right) \cdot i$$

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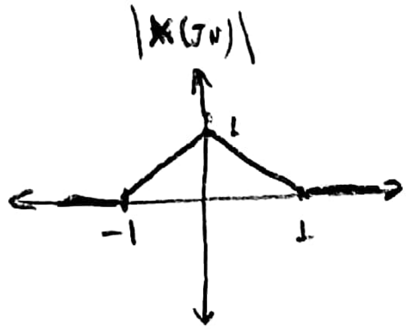
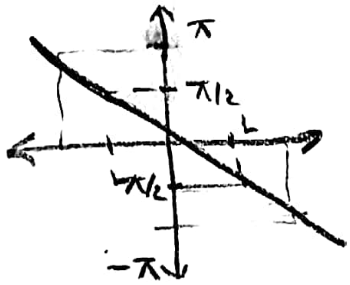
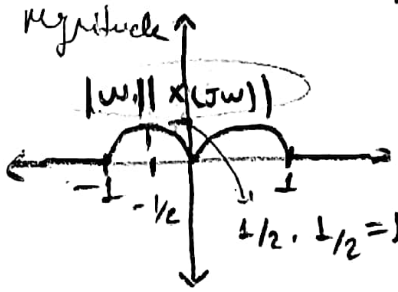
Q3)





phase: $\tan^{-1}(w)$

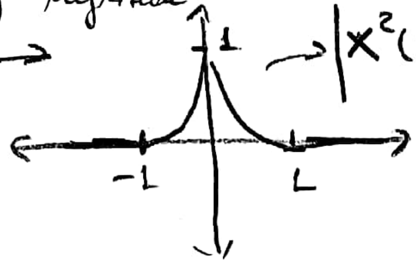
$\frac{dx(t)}{dt} \rightarrow jw.X(w) \rightarrow$



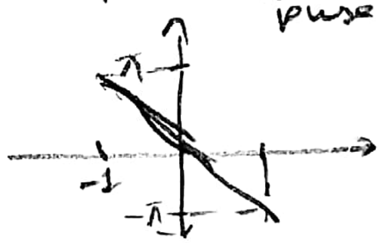
Magnitude: M5
Phase: ?

$x * x(t) \rightarrow X(w).X(w)$

Magnitude: M3
Phase: A2



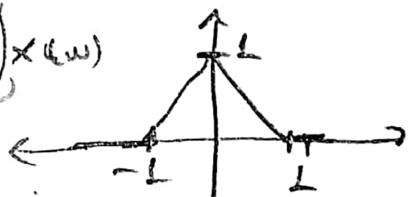
Phase: 2x current phase



$x\left(t-\frac{\pi}{2}\right) \rightarrow e^{-jw\frac{\pi}{2}} X(w)$

Magnitude: M1 $= \left(\cos\left(-\frac{\pi}{2}w\right) + i.\sin\left(-\frac{\pi}{2}w\right)\right) X(w)$

Phase A2 $= \left(\cos\left(\frac{\pi}{2}w\right) - i.\sin\left(\frac{\pi}{2}w\right)\right) X(w)$



Magnitude using $|X(jw) X(-jw)|$

$$\left(\cos\left(\frac{\pi}{2}jw\right) - i\sin\left(\frac{\pi}{2}jw\right)\right)\left(\cos\left(-\frac{\pi}{2}jw\right) - i\sin\left(\frac{\pi}{2}jw\right)\right)$$

$$\left(\cos\left(\frac{\pi}{2}jw\right) - i\sin\left(\frac{\pi}{2}jw\right)\right)\left(\cos\left(\frac{\pi}{2}jw\right) + i\sin\left(\frac{\pi}{2}jw\right)\right)$$

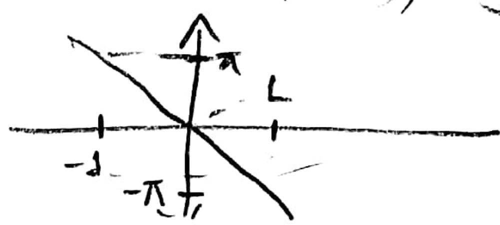
$$= \cos^2\left(\frac{\pi}{2}jw\right) - i\sin\left(\frac{\pi}{2}jw\right)$$

$$= \left(\cos^2 + \sin^2\right) \quad 1$$

1

phase: $\tan^{-1}\left(\frac{-\sin\frac{\pi}{2}w}{\cos\frac{\pi}{2}w}\right) = \tan^{-1}\left(-\tan^{-1}\left(\frac{\pi}{2}w\right)\right) = -\frac{\pi}{2}w$

→ added to current



Q3 continued)

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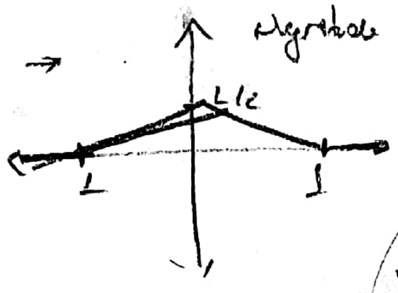
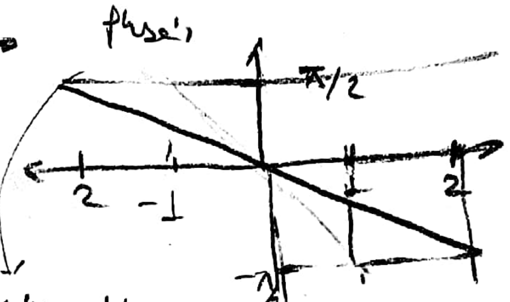
- $x(2t) \rightarrow \frac{1}{|2|} X\left(\frac{w}{2}\right) \rightarrow$



Magnitude: M4
Phase: A3

Phase:



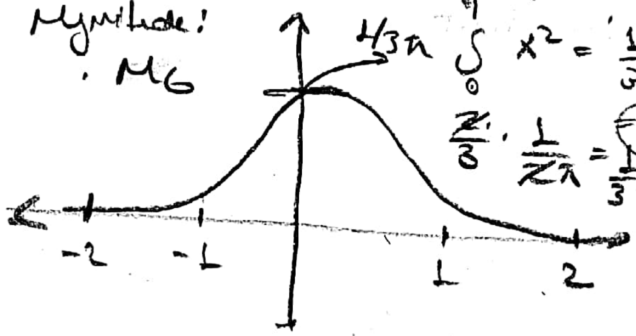
makes the graph longer,

- $x^2(t) \rightarrow \frac{1}{2\pi} X * X(w) \rightarrow$ Convolution of two triangle functions!

Magnitude: M6

Phase: Convolution frequency domain does not affect phase response, A_1

Magnitude:
. M6



$4/3\pi \quad \int_0^1 x^2 = \frac{1}{3} \cdot x^3 \Big|_0^1$

$\frac{2}{3} \cdot \frac{1}{2\pi} = \frac{1}{3\pi}$ $\frac{1}{3}$

Table 1

Signal	Magnitude	Phase
$\frac{d\,x(t)}{dt}$	M5	
$x * x(t)$	M3	A2
$x(t - \frac{\pi}{2})$	M1	A2
$x(2t)$	M4	A2
$x^2(t)$	M6	A1

Couldn't find