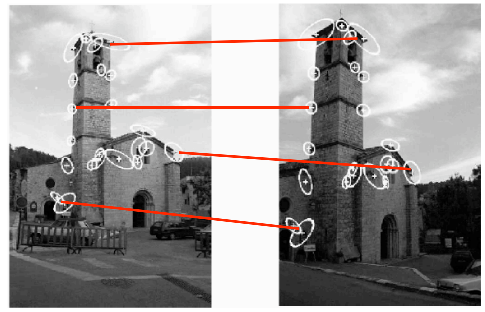
Lecture 06: Harris Corner Detector

Reading: T&V Section 4.3

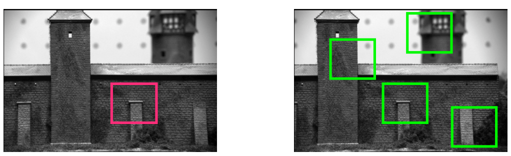
Motivation: Matchng Problem

Vision tasks such as stereo and motion estimation require finding corresponding features across two or more views.

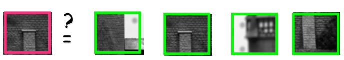


Motivation: Patch Matching

Elements to be matched are image patches of fixed size

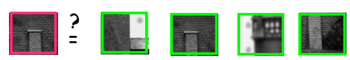


Task: find the best (most similar) patch in a second image



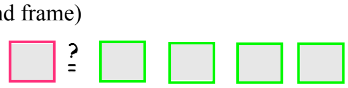
Not all Patches are Created Equal!

Inituition: this would be a good patch for matching, since it is very distinctive (there is only one patch in the second frame that looks similar).

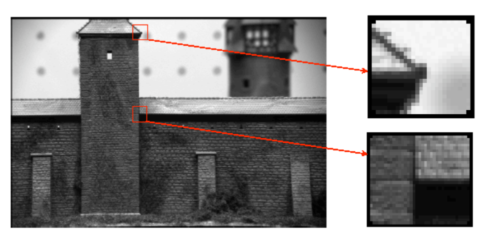


Not all Patches are Created Equal!

Inituition: this would be a BAD patch for matching, since it is not very distinctive (there are many similar patches in the second frame)



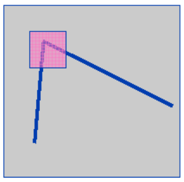
What are Corners?



- Intuitively, junctions of contours.
- Generally more stable features over changes of viewpoint
- Intuitively, large variations in the neighborhood of the point in all directions
- They are good features to match!

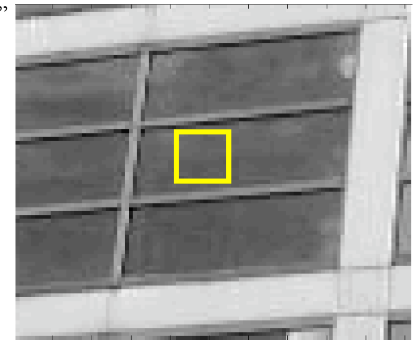
Corner Points: Basic Idea

- We should easily recognize the point by looking at intensity values within a small window
- Shifting the window in *any direction* should yield a *large change* in appearance.

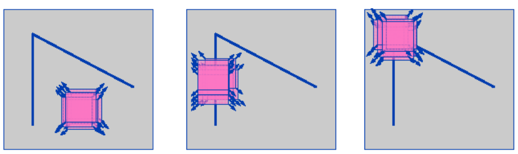


Appearance Change in Neighborhood of a Patch

Interactive "demo"



Harris Corner Detector: Basic Idea



"flat" region: no change in all directions

"edge": no change along the edge direction

"corner": significant change in all directions

Harris corner detector gives a mathematical approach for determining which case holds.

Harris Detector: Mathematics

Change of intensity for the shift $[u,v]$:

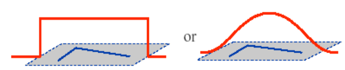
$$E(u,v) = \sum_{x,y} w(x,y) \left[I(x+u, y+v) - I(x,y) \right]^2$$

Window function

Shifted intensity

Intensity

Window function $w(x,y)$ =



or

1 in window, 0 outside

Gaussian

Harris Detector: Intuition

Change of intensity for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y) \left[I(x+u, y+v) - I(x,y) \right]^2$$

Window function

Shifted intensity

Intensity

For nearly constant patches, this will be near 0.
For very distinctive patches, this will be larger.
Hence... we want patches where E(u,v) is LARGE.

Taylor Series for 2D Functions

$$f(x+u, y+v) = f(x,y) + u f_x(x,y) + v f_y(x,y) +$$

First partial derivatives

$$\frac{1}{2!}\left[u^2 f_{xx}(x,y) + uv f_{xy} x,y + v^2 f_{yy}(x,y)\right] +$$

Second partial derivatives

$$\frac{1}{3!}\left[u^3 f_{xxx}(x,y) + u^2 v f_{xxy}(x,y) + uv^2 f_{xyy}(x,y) + v^3 f_{yyy}(x,y)\right]$$

Third partial derivatives

$+ \ldots$ (Higher order terms)

First order approx

$$f(x+u, y+v) \approx f(x,y) + u f_x(x,y) + v f_y(x,y)$$

Robert Collins
CSE486, Penn State

Harris Corner Derivation

$$\sum [I(x+u,y+v) - I(x,y)]^2$$

$$\approx \sum [I(x,y) + uI_x + vI_y - I(x,y)]^2 \quad \text{First order approx}$$

$$= \sum u^2 I_x^2 + 2uvI_xI_y + v^2 I_y^2$$

$$= \sum [\,u \;\; v\,] \begin{bmatrix} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \quad \text{Rewrite as matrix equation}$$

$$= [\,u \;\; v\,] \left(\sum \begin{bmatrix} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{bmatrix} \right) \begin{bmatrix} u \\ v \end{bmatrix}$$

Robert Collins
CSE486, Penn State

Harris Detector: Mathematics

For small shifts $[u,v]$ we have a *bilinear* approximation:

$$E(u,v) \cong [u,v] \quad M \quad \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{bmatrix}$$

Windowing function - computing a weighted sum (simplest case, w=1)

Note: these are just products of components of the gradient, Ix, Iy

Robert Collins
CSE486, Penn State

Intuitive Way to Understand Harris

Treat gradient vectors as a set of (dx,dy) points with a center of mass defined as being at (0,0).

Fit an ellipse to that set of points via scatter matrix

Analyze ellipse parameters for varying cases…

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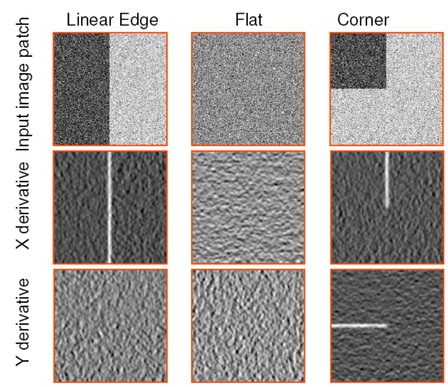
Example: Cases and 2D Derivatives



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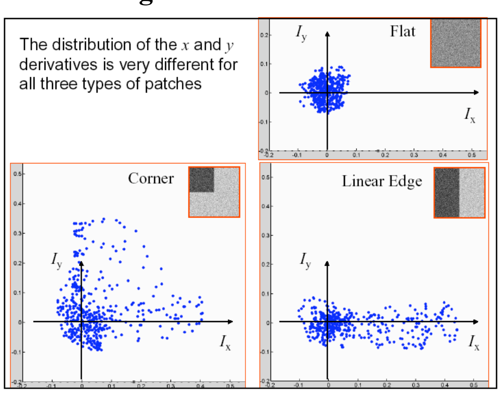
Plotting Derivatives as 2D Points



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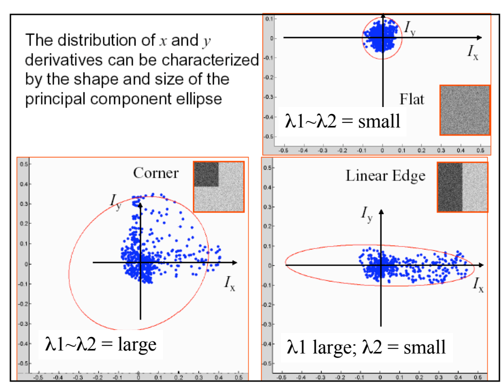
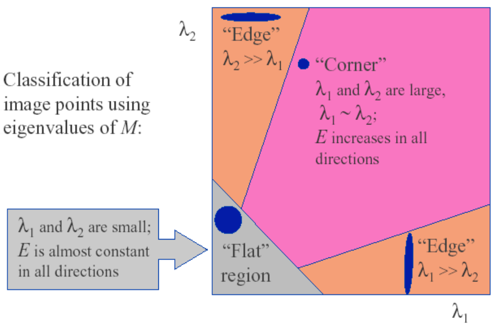
Fitting Ellipse to each Set of Points



Classification via Eigenvalues

Classification of image points using eigenvalues of M:



Corner Response Measure

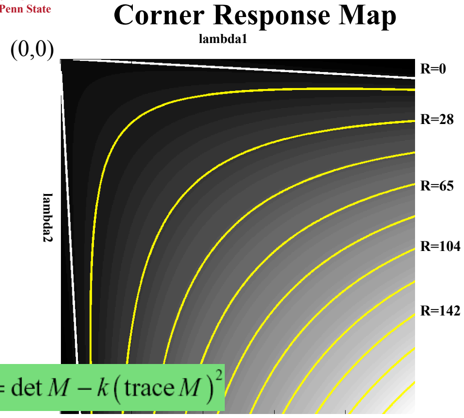
Measure of corner response:

$$R = \det M - k\left(\operatorname{trace} M\right)^2$$

$$\det M = \lambda_1 \lambda_2$$
$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

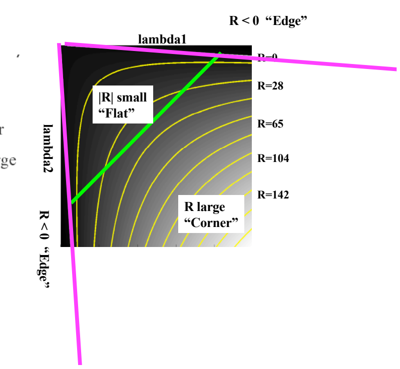
(k is an empirically determined constant; k = 0.04 - 0.06)

Corner Response Map

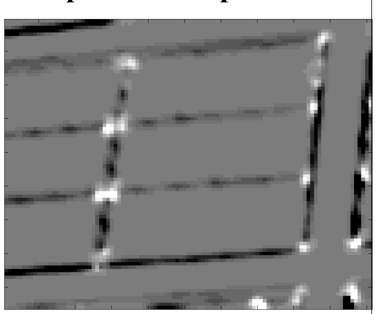


Corner Response Map

- R depends only on eigenvalues of M
- R is large for a corner
- R is negative with large magnitude for an edge
- $|R|$ is small for a flat region



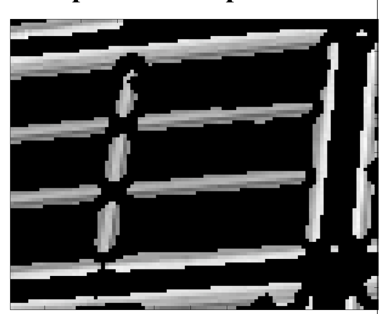
Corner Response Example

Harris R score.

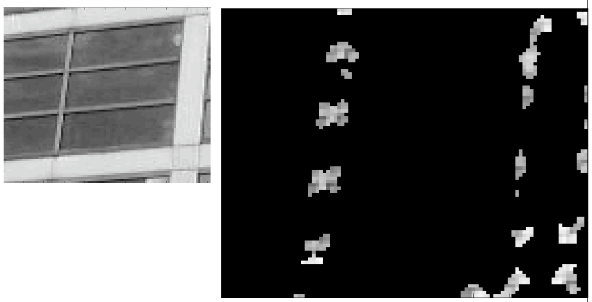
Ix, Iy computed using Sobel operator
Windowing function w = Gaussian, sigma=1

Corner Response Example

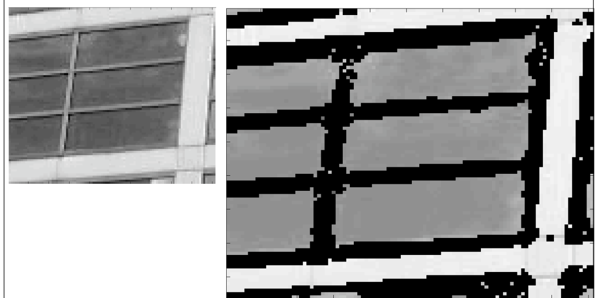
Threshold: R < -10000
(edges)

Corner Response Example



Threshold: > 10000
(corners)

Corner Response Example



Threshold: -10000 < R < 10000
(neither edges nor corners)

Harris Corner Detection Algorithm

1. Compute x and y derivatives of image

$$I_x = G_\sigma^x * I \quad I_y = G_\sigma^y * I$$

2. Compute products of derivatives at every pixel

$$I_{x2} = I_x . I_x \quad I_{y2} = I_y . I_y \quad I_{xy} = I_x . I_y$$

3. Compute the sums of the products of derivatives at each pixel

$$S_{x2} = G_{\sigma'} * I_{x2} \quad S_{y2} = G_{\sigma'} * I_{y2} \quad S_{xy} = G_{\sigma'} * I_{xy}$$

4. Define at each pixel (x, y) the matrix

$$H(x,y) = \begin{bmatrix} S_{x2}(x,y) & S_{xy}(x,y) \\ S_{xy}(x,y) & S_{y2}(x,y) \end{bmatrix}$$

5. Compute the response of the detector at each pixel

$$R = Det(H) - k(Trace(H))^2$$

6. Threshold on value of R. Compute nonmax suppression.