Master Theorem

Recurrence Relations

Recurrence Relations: we've seen several different forms of recurrence relation

$T(n) = 2T(n/2) + O(n)$ (MergeSort)

- $T(n) = O(n \log n)$

$T(n) = T(n/2) + O(1)$ (Binary Search)

- $T(n) = O(\log n)$

$T(n) = T(an) + O(n)$, $a<1$

- $T(n) = O(n)$

Can we generalize?

Master Theorem

For recurrence relations of the form $T(n) = aT(n/b) + f(n)$
(we'll ignore floor/ceiling functions, which don't affect asymptotic behavior)

3 special cases: (assume $a \geq 1$, and $b > 1$)

(1) If $f(n) = O(n^{\log a / \log b - \varepsilon})$ for some constant $\varepsilon > 0$

- $T(n) = \Theta(n^{\log a / \log b})$

(2) If $f(n) = \Theta(n^{\log a / \log b})$

- $T(n) = \Theta(n^{\log a / \log b} \log n)$

(3) If $f(n) = \Omega(n^{\log a / \log b + \varepsilon})$ for some constant $\varepsilon > 0$, and if $af(n/b) \leq cf(n)$ for some constant $c < 1$ and sufficiently large n

- $T(n) = \Theta(f(n))$

Note that $\log a / \log b = \log_b a$