Neutrosophic Numbers
Why, What and Fun

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Acknowledgement

This talk is based on the edited literature on Neutrosophic logic and math by Florentin Smarandache listed in the references in the end slides.

Florentin Smarandache

- December 10, 1954; Bălceşti, Romania
- Romanian-American, Chair and Professor of mathematics and science, University of New Mexico, Gallup
- Was refused an exit visa in 1986 by the Romanian regime to attend the International Congress of Mathematicians at UC, Berkeley
- Fled Romania in 1988, leaving behind his son and pregnant wife
- In 1990, after two years in refugee camps in Turkey, he emigrated to the United States

Neutrosophic Numbers: Why

Neutrosophic Numbers: Why

- World, and so the life, is full of **indeterminacy**
- State of not being measured, counted, clearly known or unknown
- Kind of uncertainty
- More precise imprecision is required

Neutrosophic Numbers: Why

- For example
  - In quantum theory, uncertainty about the energy and the momentum of particles because in the subatomic world they don't have exact positions

Neutrosophic Numbers: Why

- For example
  - Mathematical relationships in connection with Psychology, Sociology, Economics, and Literature

Neutrosophic Numbers: Why

- Example

A manufacturing plant has got done a survey of its annual sales by two independent observers on different samples of the same size.

Both survey findings are close, yet different.

The manager decided to present survey results of Sold Quantity (in thousands) together, taking the [min, max].

Neutrosophic Numbers: Why

Year	Sold Quantity (in thousands)
2016	[4, 6]
2017	[7, 8]
2018	5.5 or 6.0
2019	(8.0, 8.8)
2020	7.5

Neutrosophic Numbers: Why

- Example 2

  Time/day spent by an American
  T=watching TV: between [4,5] hours
  B=reading books: between [1,2] hours
  D=driving: between [1,3] hours
  S=sleeping: between [6,9] hours

Neutrosophic Numbers: Why

- To study information with indeterminacy, alternative framework
- Florentin Smarandache introduced NEUTROSOPHY PHILOSOPHY (1980) and generalized the logics of fuzzy, intuitive, paraconsistent, multi-valent, dialetheism (can be both true and false simultaneously) to the NEUTROSOPHIC LOGIC (1995)

https://vixra.org/pdf/1010.0051v1.pdf

Neutrosophic Numbers: Why

- Neutrosophic logic and framework has useful applications in
  - statistics
  - pattern recognition
  - image processing
  - artificial intelligence
  - neural networks
  - evolutionary programming
  - neutrosophic dynamic systems
  - quantum mechanics

Neutrosophy Philosophy

Neutrosophy Philosophy (1980)

- To study the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra

- Considers a proposition, theory, event, concept, or entity "A" in relation to its opposite, "Anti-A" and that which is not A, "Non-A", and that which is neither "A" nor "Anti-A", denoted by "Neut-A".

Neutrosophic Logic (1995)

- A generalization of fuzzy logic (a degree of truth between 0 and 1) based on Neutrosophy

- A proposition is t true, i indeterminate, and f false, where $t, i,$ and f are real values from the ranges T, I, F, with no restrictions on T, I, F, or the sum $n = t + i + f$

Neutrosophic Logic

- Generalizes:
  - intuitionistic logic (supports incomplete theories)
  - fuzzy logic (a degree of truth between 0 and 1)
  - Boolean logic (values are either true or false)
  - multi-valued logic (more than two possible values)
  - paraconsistent logic (deals with contradictions)
  - Dialetheism (both true and false; true contradictions)

Neutrosophic Numbers

Neutrosophic Numbers

- A classical Neutrosophic Number has the standard form:

$a + bI,$
a and $b(\neq 0) =$ real or complex coefficients
I = indeterminacy, such $0 \cdot I = 0$ and $I^2 = I$

- $I^n = I$ for all positive integer n

Neutrosophic Numbers

- If the coefficients a, b are real, then $a + bI$ is called *Neutrosophic Real Number*

- Examples: $6 + 5I$, $-9 - \frac{5}{3}I$

Neutrosophic Numbers

- If the coefficients a, b or $both$ are complex, then $a + bI$ is called *Neutrosophic Complex Number*

- Examples: $(5 + 2i) + (2 - 8i)I, \quad I + i + 9I - iI$, where $i = \sqrt{-1}$

Neutrosophic Numbers

- A neutrosophic complex number can be re-written as:

$$a + bi + cI + diI$$

$a, b, c,$ and d are real

Degenerated Neutrosophic Numbers

- Any real number can be written as a neutrosophic number; called a degenerated neutrosophic number

- Example: $9 = 9 + 0 \cdot I$

  or $9 = 9 + 0 \cdot i + 0 \cdot I + 0 \cdot i \cdot I$

- A true neutrosophic number contains the indeterminacy I with a *non-zero* coefficient

Time for Fun with NNs

Neutrosophic Numbers

- Let the neutrosophic numbers be:

$$-2-4I, \quad -1+0I, \quad 3+5I, \quad 6+7I$$

- Sum = ?

Neutrosophic Numbers

- Let the neutrosophic numbers be:

$$-2-4I, \quad -1+0I, \quad 3+5I, \quad 6+7I$$

- Sum

$$(-2-4I) + (-1+0I) + (3+5I) + (6+7I)$$

$$=(-2-1+3+6) + (-4+0+5+7)I$$

$$=6+8I$$

Neutrosophic Numbers

Find the Square of

$$(-3.5 - 6I)^2$$

Remember: $I^n = I$

Neutrosophic Numbers

Find $(-3.5 - 6I)^2$

$$= (-3.5)^2 + 2(-3.5)(-6I) + (-6I)^2$$
$$= 12.25 + 42I + 36I^2$$
$$= 12.25 + 42I + 36I$$
$$= 12.25 + 78I$$

Neutrosophic Numbers

In general, Square formula:

$$(a \pm bI)^2 = (a)^2 + (\pm 2ab + b^2)I$$

Neutrosophic Numbers

- Find $(2 + 3I) \div (1 + I)$

- Will division (of these two neutrosophic numbers) be a neutrosophic number? Guess.

Hint: Denote: $(2 + 3I)/(1 + I) = x + yI$

Neutrosophic Numbers

Denote: $(2+3I)/(1+I) = x + yI$

Thus, $(1+I)(x+yI) = x + yI + xI + yI^2$ *(Note $I^2 = I$)*

$= \boldsymbol{x + (x+2y)I \equiv 2 + 3I}$

Identifying the coefficients,

$x = 2$ and $x + 2y = 3$ or $x = 2, y = 0.5$

Answer: $\boldsymbol{(2+3I)/(1+I) = 2 + 0.5I}$ **(neutrosophic number)**

Neutrosophic Numbers

Find $(2 + 3I) \div (8 + 12I)$

Guess whether the division (of these two neutrosophic numbers) will be a neutrosophic number?

Neutrosophic Numbers

Denote: $(2+3I)/(8+12I) = x + yI$

Thus, $(8+12I)(x+yI) = \mathbf{8x + (12x + 20y)I \equiv 2 + 3I}$

Identifying the coefficients,

$8x = 2$ and $12x + 20y = 3$ or $x = 0.25, y = 0$

Answer: $\mathbf{\frac{2+3I}{8+12I} = 0.25}$ **(a real number)**

Neutrosophic Numbers

Find $(2 + 3I) \div (1 - I)$

Guess whether the division (of these two neutrosophic numbers) will be a neutrosophic number?

Neutrosophic Numbers

Denote: $(2+3I)/(1-I) = x + yI$

Thus, $(1-I)(x+yI) = \boldsymbol{x - xI \equiv 2 + 3I}$

Identifying the coefficients, $\boldsymbol{x = 2}$ **and** $\boldsymbol{-x = 3}$ **(impossible)**

Answer: $\boldsymbol{\frac{2+3I}{8+12I} = undefined}$

Neutrosophic Numbers

Find $I \div I$

Do you agree answer is obviously 1?

Neutrosophic Numbers

Denote: $I/I = x + yI$

Thus, $(I)(x + yI) = \boldsymbol{(x + y)I \equiv 1I}$

Hence, $x + y = 1$, where x and y are unknown real numbers

Neutrosophic Numbers

Since, $\boldsymbol{x} \in \boldsymbol{\mathcal{R}}$ and $\boldsymbol{y} = \mathbf{1} - \boldsymbol{x}$,

we get infinitely many solutions of $(x + yI)$

$\mathbf{1}, \boldsymbol{I}, \mathbf{2} - \boldsymbol{I}$, etc.

However, the division result should be unique,

$\frac{\boldsymbol{I}}{\boldsymbol{I}}$ **is undefined**

Neutrosophic Numbers

- Let $(a_1 + b_1 I) \div (a_2 + b_2 I) = x + yI$

- By multiplying

$$a_1 + b_1 I \equiv (x + yI)(a_2 + b_2 I)$$

or

$$\boldsymbol{a_1 + b_1 I \equiv (a_2 x) + (b_2 x + a_2 y + b_2 y) I}$$

and identifying the coefficients

$$a_2 x = a_1 \quad \text{and} \quad b_2 x + (a_2 + b_2) y = b_1$$

Neutrosophic Numbers

- Unique solution exists only when the determinant of second order

$$\begin{vmatrix} a_2 & 0 \\ b_2 & a_2 + b_2 \end{vmatrix} \neq 0$$

or, $$a_2(a_2+b_2) \neq 0$$

- Hence $\boldsymbol{a_2 \neq 0}$ **and** $\boldsymbol{a_2 \neq -b_2}$ are the conditions for the division of $(a_1 + b_1 I) \div (a_2 + b_2 I)$ to exist

Neutrosophic Numbers

- Then

$$x = \frac{a_1}{a_2}$$

$$y = \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)}$$

and

$$\boldsymbol{\frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} I}$$

Neutrosophic Numbers

Since $$\frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2+b_2)} I,$$

(i) $$\frac{a + b\,I}{ka + k\,b\,I} = \frac{1}{k}$$

for non-zero real numbers $k, a \neq 0$ and $a \neq -b$

(ii) Divisions by $I, -I$, and, in general by $k\,I$, are undefined.

$$\frac{a + b\,I}{k\,I} = \text{undefined}$$

for any real numbers k, a and b

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Neutrosophic Numbers

(iii) $\dfrac{a+bI}{c} = \dfrac{a}{c} + \dfrac{b}{c} I$ for $c \neq 0$

(iv) $\dfrac{c}{a+bI} = \dfrac{c}{a} - \dfrac{bc}{a(a+b)} I$ for $a \neq 0, a \neq b$

Neutrosophic Numbers

(square root)

Find $\sqrt{9+7I}$

How many solutions? Guess?

Neutrosophic Numbers

Denote: $\sqrt{9+7I} = x + yI$

Squaring , $\mathbf{9+7}\boldsymbol{I}\ \mathbf{=} (x+yI)^2 \mathbf{=} \boldsymbol{x}^{\mathbf{2}} + (\mathbf{2}\boldsymbol{xy} + \boldsymbol{y}^{\mathbf{2}})\boldsymbol{I}$

and identifying the coefficients,

$x^2 = 9$ or $x = \mp 3$ and $2xy + y^2 = 7$

Neutrosophic Numbers

Find y:

$$x = 3 \qquad\qquad x = -3$$
$$6y + y^2 = 7 \qquad -6y + y^2 = 7$$
$$y^2 + 6y - 7 = 0 \quad y^2 - 6y - 7 = 0$$
$$(y+7)(y-1) = 0 \qquad (y-7)(y+1) = 0$$
$$y = -7, 1 \qquad\qquad y = 7, -1$$

- Thus, $(x, y) = (3, -7), (3,1), (-3,7), (-3, -1)$

and $\sqrt{9 + 7I} = 3 - 7I, 3 + I, -3 + 7I, -3 - I$ **(four solutions)**

Neutrosophic Numbers

Example. Find $\sqrt{I}$

How many solutions?

Neutrosophic Numbers

Denote: $\sqrt{I} = x + yI$

Squaring, $I = (x + yI)^2 = x^2 + (2xy + y^2)I$

Identifying coefficients

$x^2 = 0$ or $x = 0$ and $2xy + y^2 = 1$ or $y = \mp 1$

Thus, $\sqrt{I} = x + yI = 0 \mp I = \mp I$

Neutrosophic Numbers

- Let $\sqrt{a + bI} = x + yI$

- By squaring

$$a + bI = x^2 + (2xy + y^2)I$$

and identifying the coefficients

$$x^2 = a \quad \text{and} \quad (2xy + y^2) = b$$

Neutrosophic Numbers

Thus, $x = \pm\sqrt{a}$ and $y^2 \pm 2\sqrt{a} \cdot y - b = 0$

Solving for y:

$y = (\mp 2\sqrt{a} \pm \sqrt{4a + 4b}\,)/2(1)$ = $\mp\sqrt{a} \pm \sqrt{a + b}$

Solutions: (x,y)

$(\sqrt{a}, -\sqrt{a} + \sqrt{a + b})$ $(\sqrt{a}, -\sqrt{a} - \sqrt{a + b})$

$(-\sqrt{a}, \sqrt{a} + \sqrt{a + b})$ $(-\sqrt{a}, \sqrt{a} - \sqrt{a + b})$

Neutrosophic Numbers

Thus, $\sqrt{a+bI}$

$=\sqrt{a}+(-\sqrt{a}+\sqrt{a+b})I$, or

$=\sqrt{a}-(\sqrt{a}+\sqrt{a+b})I$, or

$=-\sqrt{a}+(\sqrt{a}+\sqrt{a+b})I$, or

$=-\sqrt{a}+(\sqrt{a}-\sqrt{a+b})I$

Neutrosophic Numbers

- Root index $n \geq 2$ of any neutrosophic number $\sqrt[n]{a+bI}$

Let $\sqrt[n]{a+bI} = x + yI$

Thus, $a + bI = (x+yI)^n = x^n + \left(\sum_{k=0}^{n-1} C_k^n x^k y^{n-k}\right) I$

Neutrosophic Numbers

- Identifying the coefficients

$x^n = a$ or $\boldsymbol{x = \sqrt[n]{a}}$ **, if** $\boldsymbol{n}$ **is odd and** $\boldsymbol{x = \pm\sqrt[n]{a}}$ **, if** $\boldsymbol{n}$ **is even**

$\boldsymbol{y}$ **is the solution of** $\boldsymbol{\sum_{k=0}^{n-1} C_k^n x^k y^{n-k} = b}$

- When the x and y solutions are real (complex), we get neutrosophic real (complex) solutions

Neutrosophic Numbers

- Consider $\sqrt[n]{I}$

  Let $\sqrt[n]{I} = x + yI$

  Thus, $I = (x + yI)^n = x^n + \left(\sum_{k=0}^{n-1} C_k^n x^k y^{n-k}\right) I$

Neutrosophic Numbers

- Identifying the coefficients

$x^n = 0$ or $\boldsymbol{x = 0}$

$\sum_{k=0}^{n-1} C_k^n x^k y^{n-k} = 1$ or $y^n = 1$ or $\boldsymbol{y = \sqrt[n]{1}}$

n solutions: a real solution $y = 1$ and $n - 1$ complex solutions if we are interested in neutrosophic complex solutions as roots index n of 1.

Home Work

- Compute the square root of **a neutrosophic complex number**

$$a + bi + cI + diI$$

where *a, b, c, d* are reals.

Hint: Consider $\sqrt{}(a + bi + cI + diI\ ) = x + yi + zI + wiI$

Square both sides and find x, y, z, w.

Neutrosophic Polynomials

- A polynomial whose coefficients (at least one containing I) are neutrosophic numbers is called **Neutrosophic Polynomials**

- **Neutrosophic Real Polynomials** if its coefficients are neutrosophic real numbers, and **Neutrosophic Complex Polynomials** if its coefficients are neutrosophic complex numbers

Neutrosophic Polynomials

- Example of a neutrosophic real polynomial

$$P(x) = x^2 + (2 - I)x - 5 + 3I$$

- Example of a neutrosophic complex polynomial

$$Q(x) = 3x^3 + (1 + 6i)x^2 + 5Ix - 4iI$$

Neutrosophic Polynomial Equations: Solution

- Let the neutrosophic real polynomial equation be

$$6x^2 + (10 - I)x + 3I = 0$$

- Solve it using the quadratic fromula

Neutrosophic Polynomial Equations: Solution

- $6x^2 + (10 - I)x + 3I = 0$

$$x = \frac{-(10-I) \pm \sqrt{(10-I)^2 - 4(6)(3I)}}{2(6)}$$

$$= \frac{-(10-I) \pm \sqrt{100 - 20I + I^2 - 72I}}{12}$$

$$= \frac{-(10-I) \pm \sqrt{100 - 20I + I \ - 72I}}{12}$$

$$= \frac{-10 + I \pm \sqrt{100 - 91I}}{12}$$

Neutrosophic Polynomial Equations: Solution

- Compte $\sqrt{100-91I}$
- Let $\sqrt{100-91I} = a + bI$

or, squaring $100 - 91I = a^2 + (2ab + b^2)I$

Thus, $a^2 = 100$ or $a = \pm 10$

And $a^2 + (2ab + b^2) = -91$

Neutrosophic Polynomial Equations: Solution

- If $a = 10$,

$$b^2 + 20b - 91 = 0$$

- Solving by quadratic formula

$$b = \frac{-20 \pm \sqrt{20^2 - 4(1)(91)}}{2(1)}$$

$$= -7, -13$$

Neutrosophic Polynomial Equations: Solution

- If $a = -10$,

$$b^2 - 20b + 91 = 0$$

- Solving by quadratic formula

$$b = \frac{20 \pm \sqrt{-20^2 - 4(1)(91)}}{2(1)}$$

$$= 13, 7$$

Neutrosophic Polynomial Equations: Solution

- Thus,

$$(a, b) = (10, -7), (10, -13), (-10,13), (-10,7) \text{ and}$$

$$\sqrt{100 - 91I} \quad = 10 - 7I$$

or $$= -10 + 7I$$

or $$= 10 - 13I$$

or $$= -10 + 13I$$

Neutrosophic Polynomial Equations: Solution

- To find $x = \frac{-10+I\pm\sqrt{100-91I}}{12}$
- Since, there is $\pm$ in front of the radical, $10-7I$ and $-10+7I$, we get the same values for x. Similarly, for 10−13I and −10+13I.

$$x_{1,2}= \frac{-10+I\pm(10-7I)}{12} = \frac{-I}{2}, \frac{5}{3}+\frac{2}{3}I$$

$$x_{3,4} = \frac{-10+I\pm(10-13I)}{12} = -I, \frac{-10}{6}+\frac{7}{6}I$$

Neutrosophic Polynomial Equations: Solution

- The neutrosophic real equation of degree 2

$$6x^2 + (10 - I)x + 3I = 0$$

- We have four neutrosophic solutions

$$\{\frac{-I}{2}, \frac{5}{3} + \frac{2}{3}I, -I, \frac{-10}{6} + \frac{7}{6}I\}$$

Neutrosophic Polynomial Equations: Solution

- First neutrosophic factoring of

$$6x^2 + (10 - I)x + 3I$$

- $P(x) = 6x^2 + (10 - I)x + 3I$

$$= 6\left[x - \frac{-I}{2}\right]\left[x - \left(\frac{5}{3} + \frac{2}{3}I\right)\right]$$

Neutrosophic Polynomial Equations: Solution

- Second neutrosophic factoring of
$$6x^2 + (10 - I)x + 3I$$
- $P(x) = 6x^2 + (10 - I)x + 3I$
$$= [x - (-I)\,]\left[x - \left(\frac{-10}{6} + \frac{7}{6}I\right)\right]$$
- Differently from the classical polynomial with real or complex coefficients, **the neutrosophic polynomials do not have a unique factoring.**

References

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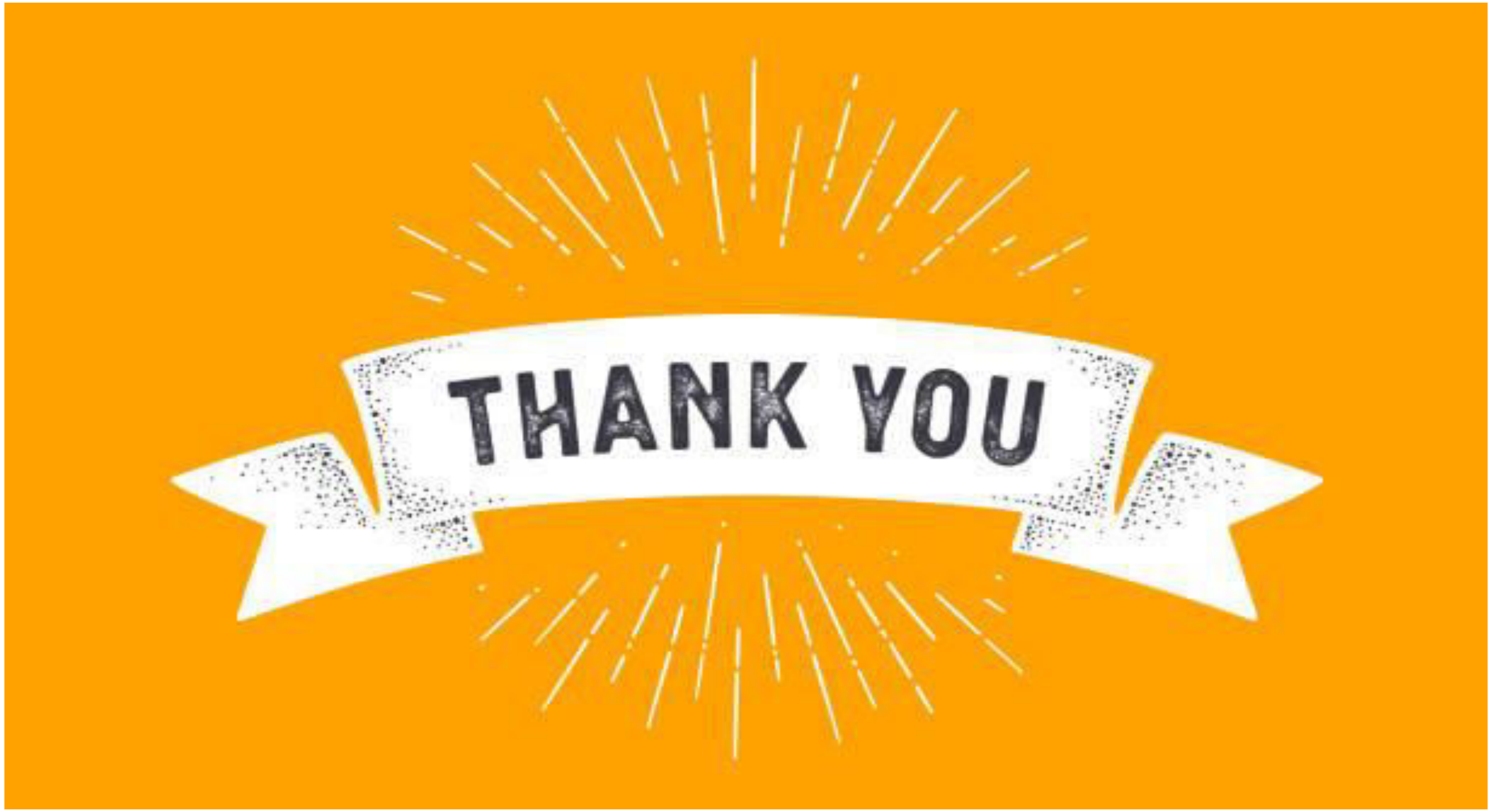
THANK YOU