

## Chapter 1:

1. State Euler's Theorem for homogeneous function of two variables. If  $u = \tan^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$  then show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$ .
2. Obtain the maximum value of  $xyz$  such that  $x + y + z = 24$ .  
If  $u = \log \frac{x^2 + y^2}{x + y}$ , then show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 1$
2. Find the minimum value of  $x^2 + y^2 + z^2$  when  $x + y + z = 3a$ .
  1. State Euler's Theorem for a homogeneous function of two independent variables and verify it for the function  $u = x^n \tan^{-1}\left(\frac{y}{x}\right)$
  2. Find the extreme value of  $x^2 + y^2 + z^2$  connected by the relation  $ax + by + cz = p$ .
  2. Find the minimum value of the function  $x^2 + xy + y^2 + 3z^2$  under the condition  $x + 2y + 4z = 60$ .
13. If  $u = \operatorname{cosec}^{-1}\left(\frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}\right)$ , show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -\frac{1}{6} \tan u$ .
14. Find the extreme values of  $x^2 + y^2 + z^2$  subjected to the condition  $x + y + z - 1 = 0$  and  $xyz + 1 = 0$ .
- ✓ 1. State Euler's theorem for homogeneous function of two variables. If  $u = \cos^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right)$ , then prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -\frac{1}{2} \cot u$ .
2. Find the minimum value of  $x^2 + xy + y^2 + 3z^2$  under the condition  $x + 2y + 4z = 60$ .

## Chapter 2:

Yes bata yeti matra gara aaru na gara....

3. Evaluate:  $\int_0^a \int_{\sqrt{x}}^a \frac{y^2}{\sqrt{y^4 - a^2 x^2}} dy dx$  by changing order of integration.
4. Evaluate  $\int_0^a \int_y^a \frac{x dx dy}{x^2 + y^2}$  by changing order of integration.
3. Evaluate  $\iint xy(x + y) dx dy$  over the area between  $y = x^2$  and  $y = x$ .
4. Evaluate the integral by changing to polar coordinates  $\int_0^a \int_0^{\sqrt{a^2 - x^2}} y^2 \sqrt{x^2 + y^2} dy dx$
3. Evaluate:  $\int_0^\infty \int_0^\infty e^{-(x^2 + y^2)} dy dx$  by changing to polar coordinates.
4. Evaluate:  $\iiint_V x dv$  where  $V$  is bounded by the coordinate planes and the plane  $x + y + z = 1$
5. Evaluate the integral by changing to polar coordinates  $\int_0^a \int_0^{\sqrt{a^2 - x^2}} \sqrt{x^2 + y^2} dy dx$ .
3. Evaluate:  $\iint_R xy dx dy$  where  $R$  is the region over the area of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  in the first quadrant.
4. Find by double integration, the volume bounded by the plane  $z = 0$ , surface  $z = x^2 + y^2 + 2$  and the cylinder  $x^2 + y^2 = 4$ .

## Chapter 3:

5. Show that  $\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2}$  and  $3x-2y+z+5=0=2x+3y+4z-4$  are coplanar lines and find the point of intersection.
5. Obtain the equation of the plane passing through the line of intersection of two planes through the line of intersection of two planes  $7x-4y+7z+16=0$  and  $4x-3y-2z+13=0$  and perpendicular to plane  $x-y-2z+5=0$
5. Find the equation of the plane through the line  $2x+3y-5z=4$  and  $3x-4y+5z=6$  and parallel to the coordinate axes.
5. Show that the lines  $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$  and  $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$  are coplanar. Find their common point.
6. Show that the shortest distance between the lines  $x+a=2y=-12z$  and  $x=y+2a=6z-6a$  is  $2a$ .
7. Obtain the equation of tangent plane to sphere  $x^2+y^2+z^2+6x-2z+1=0$  which passes through the line  $3(16-x)=3z=2y+30$
6. Find the length and equation of the shortest distance between the line  $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ ,  $2x-3y+27=0$ ;  $2y-z+20=0$
7. Find the equation of the sphere having the circle  $x^2+y^2+z^2+7y-2z+2=0$ ,  $2x+3y+4z-8=0$  as a great circle.
6. Find the length and equation of shortest distance between the lines  $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$  and  $2x-3y+27=0$ ,  $2y-z+20=0$ .
7. Obtain the centre and radius of the circle  $x^2+y^2+z^2+x+y+z=4$ ,  $x+y+z=0$ .
6. Find the S.D between the lines  $\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1}$  and  $\frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$ . Find also the equation of shortest distance.
7. Find the equation of spheres passing through the circle  $x^2+y^2+z^2-6x-2z+5=0$ ,  $y=0$  and touching the plane  $3y+4z+5=0$ .
5. Find the distance of the point  $(1, -2, 3)$  from the plane  $x-y+z=5$  measured parallel to the line  $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$
6. Show that the lines  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  and  $4x-3y+1=0=5x-3z+2$  are coplanar and find their point of intersection.
7. Obtain the equation of the tangent planes to the sphere  $x^2+y^2+z^2+6x-2z+1=0$  which passes through the line  $x+z-16=0, 2y-3z+30=0$

Cone and cylinder ko question na gara.....

## Chapter 4:

9. Solve the initial value problem:  $y''-4y'+3y=10e^{-2x}$ ,  $y(0)=1$ ,  $y'(0)=3$
10. Solve the differential equation by power series method:  $y''-y=0$



11. Solve in series, the Legendre's equation  $(1-x^2)y''-2xy'+n(n+1)y=0$
9. Solve by power series method the differential equation  $y''+xy'+y=0$
10. Express the following in terms of Legendre's Polynomials  $f(x)=5x^3+x$
9. Solve by power series method the differential equation  $y''-4xy'+(4x^2-2)y=0$
10. Express  $f(x)=x^3-5x^2+x+2$  in terms of Legendre's polynomial.
9. Solve by Power series method  $y''-y=x$ .
10. Express in terms of Legendre's polynomials  $f(x)=x^3-5x^2+6x+1$ .
10. Prove that the Legendre's function  $x^5 = \frac{8}{63} \left[ P_5(x) + \frac{7}{2} P_3(x) + \frac{27}{8} P_1(x) \right]$
9. Solve by power series method the differential equation  $y''+xy'+y=0$
10. Find the general solution of the Legendre's differential equation.

Bessell's Functions sajilo lauxa bhane matra gara natra nagara 5 marks matrai ho.....

11. Show that  $J_{\left(\frac{5}{2}\right)}(x) = \sqrt{\frac{2}{\pi x}} \left( \frac{3}{x} \sin x + \frac{3-x^2}{x^2} \cos x \right)$

11. Prove the Bessel's Function

$$J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \left( \frac{\sin x}{x} - \cos x \right)$$

11. Show that  $4J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$ .

11. Prove Bessel's Function  $\frac{d[x^{-n}J_n(x)]}{dx} = -x^{-n}J_{n+1}$

10. Show that:

$$J_{\frac{5}{2}}(x) = \frac{\sqrt{2}}{\pi x} \left( \frac{3-x^2}{x^2} \sin x - \frac{3}{x} \cos x \right)$$

10. Prove the following Bessel's function:  $J_{n+3}(x) + J_{n+5}(x) = \frac{2}{x}(n+4)J_{n+4}(x)$ .

## Chapter 5:

Yo chapter bata yeti gare aaru herna pardaina ....

12. Prove that 
$$\begin{bmatrix} \vec{a} \times \vec{b} & \vec{c} \times \vec{d} & \vec{e} \times \vec{f} \end{bmatrix} = \begin{bmatrix} \vec{a} & \vec{b} & \vec{d} \end{bmatrix} \begin{bmatrix} \vec{c} & \vec{e} & \vec{f} \end{bmatrix} - \begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix} \begin{bmatrix} \vec{d} & \vec{e} & \vec{f} \end{bmatrix}$$

13. Prove that the necessary and sufficient conditions for the vector function  $\vec{a}$  of scalar variable  $t$  to have constant direction is  $\vec{a} \times \frac{d\vec{a}}{dt} = 0$

12. Find the set of reciprocal system to the set of vectors  $2\vec{i} + 3\vec{j} - \vec{k}$ ,  $-\vec{i} + 2\vec{j} - 3\vec{k}$  and  $3\vec{i} - 4\vec{j} + 2\vec{k}$

13. Prove that the necessary and sufficient condition for the vector function of scalar variable 't' have constant magnitude is  $\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$

14. Find the directional derivative of  $\phi(x, y, z) = xy^2 + yz^3$  at the point  $(2, -1, 1)$  in the direction of vector  $\vec{i} + 2\vec{j} + 2\vec{k}$

12. Prove that 
$$\begin{bmatrix} \vec{b} \times \vec{c} & \vec{c} \times \vec{a} & \vec{a} \times \vec{b} \end{bmatrix} = \begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix}^2$$

14. Find the directional derivative of  $\phi(x, y, z) = xy^2 + yz^3$  at the point  $(2, -1, 1)$  in the direction of vector  $\vec{i} + 2\vec{j} + 2\vec{k}$ .

12. If  $\vec{a}, \vec{b}, \vec{c}$  and  $\vec{a}', \vec{b}', \vec{c}'$  are the reciprocal system of vectors then prove that

$$\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \frac{\vec{a} + \vec{b} + \vec{c}}{[\vec{a} \ \vec{b} \ \vec{c}]}, \begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix} \neq 0$$

9. Show that  $\begin{bmatrix} \vec{a} + \vec{b} & \vec{b} + \vec{c} & \vec{c} + \vec{a} \end{bmatrix} = 2[\vec{a} \ \vec{b} \ \vec{c}]$

10. If  $\vec{a}, \vec{b}, \vec{c}$  and  $\vec{a}', \vec{b}', \vec{c}'$  are reciprocal system of vectors, then show that  $\vec{a} \cdot \vec{a}' + \vec{b} \cdot \vec{b}' + \vec{c} \cdot \vec{c}' = 3$ .

or

14. Find the angle between the normal to the surfaces  $x \log z = y^2 - 1$  and  $x^2y + z = 2$  at the point  $(1, 1, 1)$ .

or

14. Find the angle between the surface  $x^2 + y^2 + z^2 = 9$  and  $z = x^2 + y^2 - 3$  at the point  $(2, -1, 2)$

## Chapter 6:

15. Test the convergence of the series:

$$x + \frac{3}{5}x^2 + \frac{8}{10}x^3 + \frac{15}{17}x^4 + \dots + \frac{n^2 - 1}{n^2 + 1}x^n + \dots, x > 0.$$

16. Find the interval and radius of convergence of power series:

$$\frac{1}{1.2}(x-2) + \frac{1}{2.3}(x-2)^2 + \frac{1}{3.4}(x-2)^3 + \dots + \frac{1}{n(n+1)}(x-2)^n + \dots$$

15. Test convergent or divergent of the series  $1 + \frac{x}{2} = \frac{2!}{3^2}x^2 + \frac{3!}{4^3}x^3 + \dots \infty$

16. Find the interval and radius of convergence of the power series  $\sum_{n=1}^{\infty} \frac{2^n (x-3)^n}{n+3}$



15. Test the convergence of the series

$$\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \dots, x > 0$$

16. Find the interval and radius of convergence of the power series  $\sum_{n=1}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$ .

15. Test the series for convergence or divergence

$$2x + \frac{3x^2}{8} + \frac{4x^3}{27} + \dots + \frac{(x+1)x^n}{n^3} + \dots \quad (x > 0)$$

16. Find the interval of convergence and the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{2^n (x-3)^n}{n+3}$$

16. Find the radius and interval of convergence of the power series  $\sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{n+1}$

15. Determine whether the following series is convergent or divergent:

$$1 + \frac{1^2}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \frac{4^4}{5^5} + \dots$$

15. Determine whether the series  $\sum \frac{n}{1+n\sqrt{n+1}}$  is convergent or divergent.

16. Find the radius and interval of convergence of the power series  $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n \cdot 2^n}$