Contents

Homework 4 #2 Introduction

```
% Geneva Porter, 2019.11.07
% Homework 4, Problem 2, Math 693A
% Professor Uduak George, SDSU

% This assignment applies the standard conjugate gradient method to solve
% the linear systems describing the Hilbert matrix in 5, 8, 12, and 20
% dimensions. The residual norms and eigenvalue spreads are plotted.
% Condition factors for CG and steepest descent methods are compared.
```

Establishing variables and parameters

```
clear
clc
format short

Dimension = [5, 8, 12, 20]';
Condition_Number = zeros(4,1);
```

Forming each n-dimensional matrix A

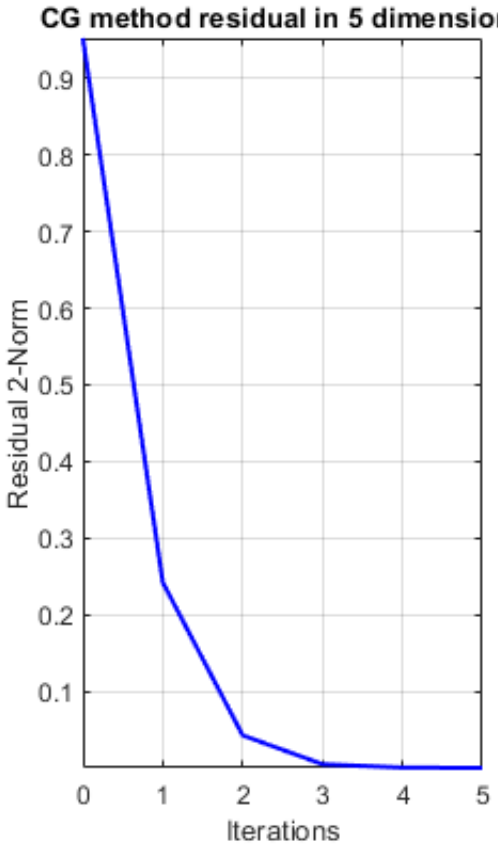
```
for k=1:4

    figure(k)
    clf
    hold on

    A = zeros(Dimension(k));
    b = ones(Dimension(k),1);
    x = zeros(Dimension(k),1);

    for i=1:Dimension(k)
        for j=1:Dimension(k)
            A(i,j) = 1/(i+j-1);
        end
    end

    % Compute condition number
    Condition_Number(k) = cond(A);

    % Plotting the norm of the residual
```

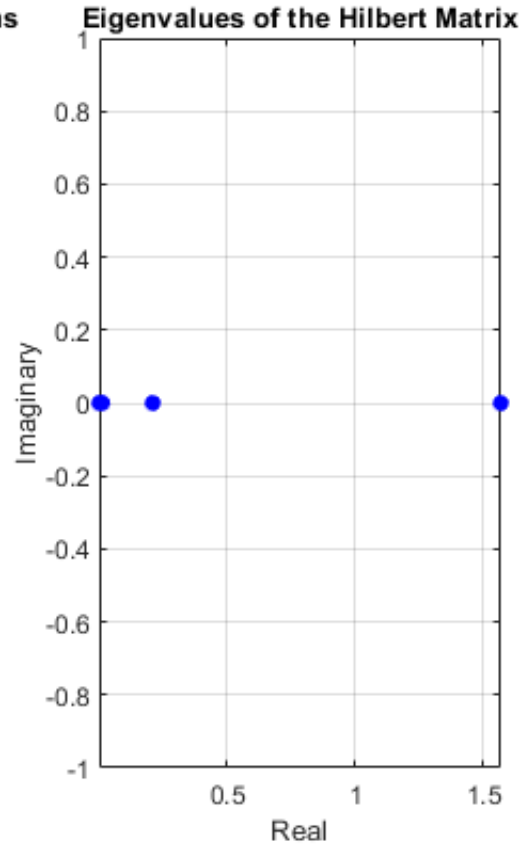
```
    subplot(1,2,1);
    r_norm = cg_standard(A,b,x,"no");
    plot_cg(r_norm, Dimension(k));

    % Plotting the spread of the eigenvalues
    subplot(1,2,2);
    plot_eg(A);


end
```

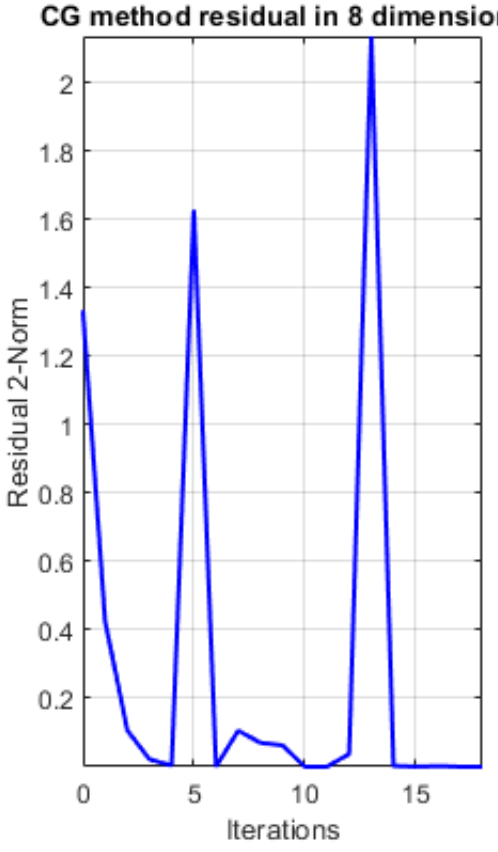
Computing other values

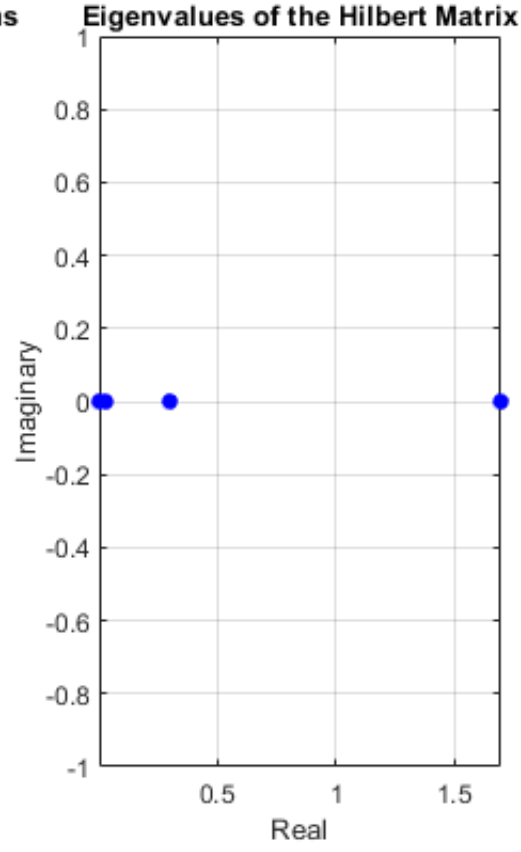
```
% Steepest descent comparison

CG_Condition_Factor = (Condition_Number.^(1/2) - 1) ...
                      ./ (Condition_Number.^(1/2) + 1);
SD_Condition_Factor = (Condition_Number - 1) ...
                      ./ (Condition_Number + 1);

Data = table(Dimension, Condition_Number, ...
       CG_Condition_Factor, SD_Condition_Factor);

disp(Data);
```

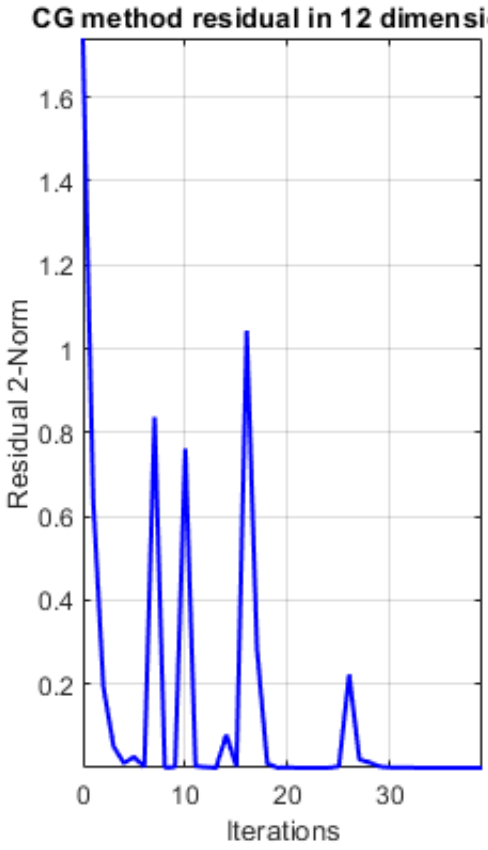
Dimension	Condition_Number	CG_Condition_Factor	SD_Condition_Factor
5	4.7661e+05	0.99711	1
8	1.5258e+10	0.99998	1
12	1.6212e+16	1	1
20	2.1065e+18	1	1

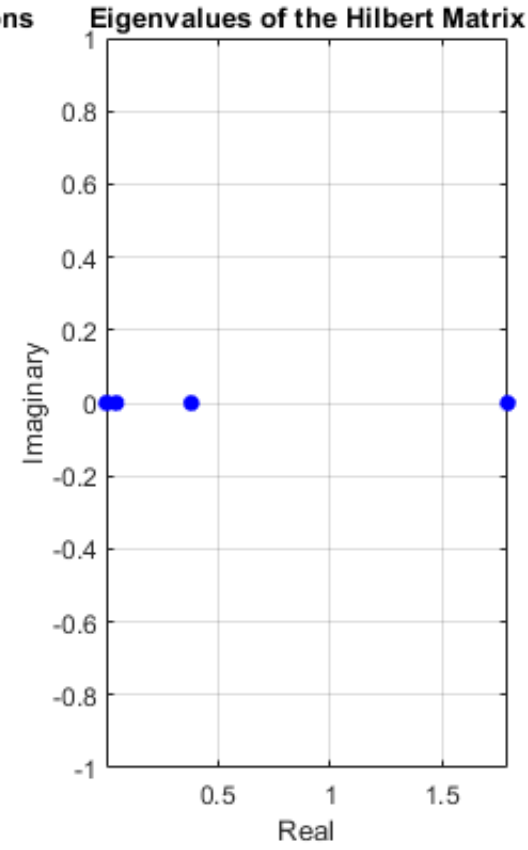

CG method residual in 5 dimensions
Residual 2-Norm
Iterations
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5

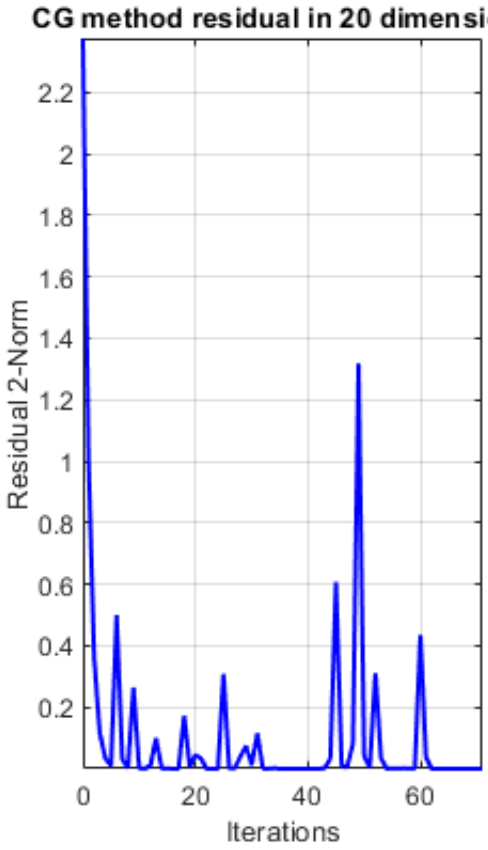

Eigenvalues of the Hilbert Matrix
Imaginary
Real
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0.5
1
1.5

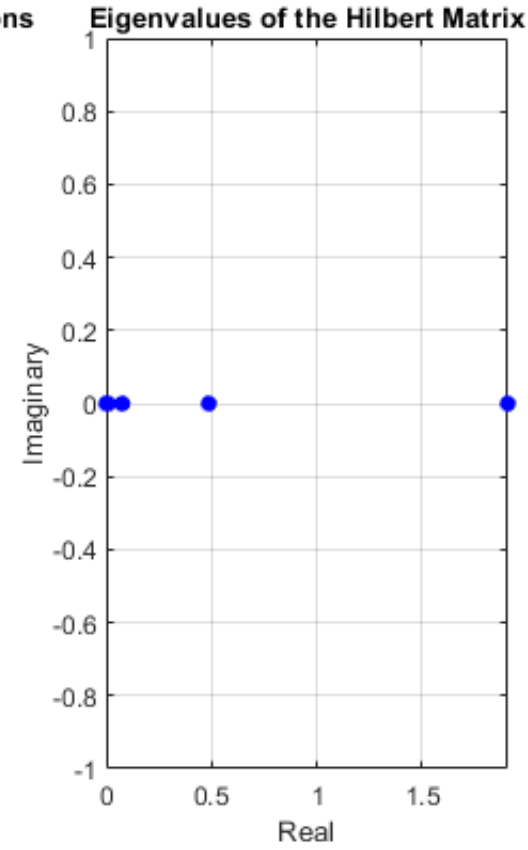

CG method residual in 8 dimensions
Residual 2-Norm
Iterations
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
5
10
15


Eigenvalues of the Hilbert Matrix
Imaginary
Real
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0.5
1
1.5


CG method residual in 12 dimensions
Residual 2-Norm
Iterations
0
10
20
30
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6


Eigenvalues of the Hilbert Matrix
Imaginary
Real
0.5
1
1.5
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1


CG method residual in 20 dimensions
Residual 2-Norm
Iterations
0
20
40
60
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2


Eigenvalues of the Hilbert Matrix
Imaginary
Real
0
0.5
1
1.5
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

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