Ortogonalidad: Podemos demostrar que $\int_{-1}^{1} P_n^m P_p^m \, dx = 0$ si $n \neq p$

de manera similar al caso de Pol. $P_n(x)$

$$\frac{d}{dx}\left[(1-x^2)\frac{dP_n^m}{dx}\right] + \left[n(n+1) - \frac{m^2}{1-x^2}\right] P_n^m = 0 \qquad / \cdot P_p^m$$

$$\rightarrow P_p^m \frac{d}{dx}\left[(1-x^2)\frac{dP_n^m}{dx}\right] + \left[n(n+1) - \frac{m^2}{1-x^2}\right] P_n^m P_p^m = 0$$

$$\frac{d}{dx}\left[P_p^m (1-x^2)\frac{dP_n^m}{dx}\right] - \frac{dP_p^m}{dx}(1-x^2)\frac{dP_n^m}{dx} + \left[n(n+1) - \frac{m^2}{1-x^2}\right] P_n^m P_p^m = 0 \qquad / \int_{-1}^{1}$$

$$\underbrace{P_p^m (1-x^2)\frac{dP_n^m}{dx}\Big|_{-1}^{1}}_{=0} - \int_{-1}^{1} \frac{dP_p^m}{dx}(1-x^2)\frac{dP_n^m}{dx} + \int_{-1}^{1}\left[n(n+1) - \frac{m^2}{1-x^2}\right] P_n^m P_p^m \, dx = 0$$

Vemos de $P_n^m(x) = (1-x^2)^{m/2} \frac{d^m}{dx^m} P_n(x)$ que

$$\frac{dP_n^m}{dx} = -m(1-x^2)^{\frac{m}{2}-1} \frac{d^m}{dx^m} P_n(x) + (1-x^2)^{m/2} \frac{d^{m+1}}{dx^{m+1}} P_n(x)$$

$$(1-x^2)\frac{dP_n^m}{dx} = -m(1-x^2)^{\frac{m}{2}} \frac{d^m}{dx^m} P_n(x) + (1-x^2)^{\frac{m}{2}+1} \frac{d^{m+1}}{dx^{m+1}} P_n(x)$$

$$\rightarrow (1-x^2)^{1/2} \frac{dP_n^m}{dx}\Big|_{-1}^{1} = 0 \qquad \text{y} \qquad P_p^m(\pm 1) = 0$$

$$\Rightarrow \int_{-1}^{1} [n(n+1) - p(p+1)] P_n^m P_p^m \, dx = 0 \quad \Rightarrow \int_{-1}^{1} P_n^m P_p^m \, dx = 0 \qquad n \neq m$$

$= 1, 2, \ldots$