La rel. anterior relaciona fnes P_n^m con igual m, pero n diferente

Podemos derivar una relación entre fnes de $\neq m$

$$g_{m+1} = \frac{(2m+2)!}{2^{2m+1}(m+1)!}(1-2xt+t^2)^{-(m+1+1/2)} = \frac{(2m+2)(2m+1)}{2\,m+1}(1-2xt+t^2)^{-1} \cdot g_m$$

$$= (2m+1)(1-2xt+t^2)\,g_m$$

$$\Rightarrow \quad (1-2xt+t^2)\,g_{m+1} = (2m+1)\,g_m$$

$$\Rightarrow \quad (1-2xt+t^2)\sum_{n=0}^{\infty} P_{n+m+1}^{m+1}\,t^n = (2m+1)\sum_{n=0}^{\infty} P_{n+m}^{m}\,t^n$$

$$\sum_{n=0}^{\infty} P_{n+m+1}^{m+1}\,t^n - 2x\sum_{n=0}^{\infty} P_{n+m+1}^{m+1}\,t^{n+1} + \sum_{n=0}^{\infty} P_{n+m+1}^{m+1}\,t^{n+2} = (2m+1)\sum_{n=0}^{\infty} P_{n+m}^{m}\,t^n$$

$$\underbrace{-2x\sum_{n=1}^{\infty} P_{n+m}^{m+1}\,t^n}_{-2x\sum_{n=0}^{\infty} P_{n+m}^{m+1}\,t^n} \qquad + \underbrace{\sum_{n=2}^{\infty} P_{n+m-1}^{m+1}\,t^n}_{\sum_{n=0}^{\infty} P_{n+m-1}^{m+1}\,t^n}$$

ya que $P_m^{m+1} = 0$ ya que $P_{m-1}^{m+1} = P_m^{m+1} = 0$

$$\Rightarrow \quad \boxed{P_{n+m+1}^{m+1} - 2x\,P_{n+m+1}^{m+1} - P_{n+m-1}^{m+1} = (2m+1)\,P_{n+m}^{m} \qquad n = 1, 2, \ldots}$$