

$$(3) \Rightarrow C_m^m = \oint P_m(\cos\theta') Y_m^{*m}(\theta, \varphi) d\Omega$$

$$(4) \Rightarrow = \sum_{m'=-m}^m (a_m^{mm'})^* \oint P_m(\cos\theta') Y_m^{*m'}(\theta', \varphi') d\Omega'$$

ya que $d\Omega' = d\Omega$
($ds' = ds$ bajo
 $r' = r$ rotacion)

$$= \sum_{m'=-m}^m (a_m^{mm'})^* \cdot \sqrt{\frac{4\pi}{2m+1}} \oint Y_m^0(\theta', \varphi') Y_m^{*m'}(\theta', \varphi') d\Omega'$$

$$= \sqrt{\frac{4\pi}{2m+1}} \sum_{m'=-m}^m (a_m^{mm'})^* \delta_{m',0}$$

$$= \sqrt{\frac{4\pi}{2m+1}} (a_m^{m0})^*$$

$$\text{Si } \hat{r}_2 = \hat{n} \Rightarrow \theta = \theta_2 ; \varphi = \varphi_2 , \text{ y } \gamma = \theta' = 0$$

$$\text{Entonces } (4) \Rightarrow Y_m^m(\theta_2, \varphi_2) = \sum_{m'=-m}^m a_m^{mm'}(\theta_2, \varphi_2) Y_m^{*m'}(0, \varphi')$$

$$\text{Además } Y_m^{*m'}(\theta, \varphi') = (-1)^{m'} \sqrt{\frac{2m+1}{4\pi}} \sqrt{\frac{(m-m')!}{(m+m')!}} P_{\ell}^{m'}(1) e^{im'\varphi'} \quad \text{y} \quad P_{\ell}^{m'}(1) = \begin{cases} 0 & , m' \neq 0 \\ 1 & , m' = 0 \end{cases}$$

$$\Rightarrow Y_m^m(\theta_2, \varphi_2) = \sqrt{\frac{2m+1}{4\pi}} a_m^{m0} \Rightarrow C_m^m(\theta_2, \varphi_2) = \frac{4\pi}{2m+1} Y_m^{*m}(\theta_2, \varphi_2)$$

$$\Rightarrow P_m(\cos\gamma) = \frac{4\pi}{2m+1} \sum_{m=-m}^m Y_m^{*m}(\theta_2, \varphi_2) Y_m^m(\theta, \varphi)$$