TA02

Pero $(\hat{L}^2)' P_n(\cos\gamma) = (\hat{L}^2)' P_n(\cos\theta')$ ... (porque $P_n(\cos\theta) \propto Y_n^0$)

$$= n(n+1) P_n(\cos\theta') = n(n+1) \sum_{n',m} C_{n'}^{m} Y_{n'}^{m} \quad \ldots (2)$$

Por otro lado, como $(\hat{L}^2)' = (\hat{L}^2)$ entonces

$$(\hat{L}^2)' P_n(\cos\gamma) = (\hat{L}^2) P_n(\cos\gamma) \overset{(1)}{=} \sum_{n',m} C_{n'}^{m} (\theta_2,\varphi_2)(\hat{L}^2 Y_{n'}^{m})(\theta,\varphi)$$

$$= \sum_{n',m} C_{n'}^{m} (\theta_2,\varphi_2)\, n'(n'+1)\, Y_{n'}^{m}(\theta,\varphi) \quad \ldots (3)$$

Comparando (2) con (3) $\Rightarrow$

$$\sum_{n',m} [n(n+1) - n'(n'+1)]\, C_{n'}^{m}(\theta_2,\varphi_2)\, Y_{n'}^{m}(\theta,\varphi) = 0$$

Y_n^m son l.i $\Rightarrow$ $[n(n+1) - n'(n'+1)]\, C_{n'}^{m} = 0$ $\Rightarrow$ $C_{n'}^{m} = 0$ si $n' \neq n$

$\Rightarrow$ sólo quedan los términos con $n' = n$

$$\Rightarrow P_n(\cos\gamma) = \sum_{m=-n}^{n} C_n^m(\theta_2,\varphi_2) \cdot Y_n^m(\theta,\varphi) \quad \ldots (3)$$

análoga% podemos escribir (la T. de C. depende de θ_2 y φ_2)

$$Y_n^m(\theta,\varphi) = \sum_{m'=-l}^{l} a_n^{mm'}(\theta_2,\varphi_2) \cdot Y_n^{m'}(\theta',\varphi') \quad \ldots (4)$$

[Bajo el cambio de S.C $K \to K'$ $(\theta,\varphi) = f(\theta',\varphi')$ y son fnes finitas $\Rightarrow$ pueden expandirse en serie de $Y_n^m(\theta',\varphi')$

$\Rightarrow Y_n^m(\theta,\varphi) = \sum_{n',m'} a_{nn'}^{mm'} Y_{n'}^{m'}(\theta',\varphi')$ $\Rightarrow$ $(\hat{L}^2) Y_n^m = n(n+1) Y_n^m = (\hat{L}'^2) Y_n^m$

$\Rightarrow \sum_{n',m'} a_{nn'}^{mm'}\, n(n+1) Y_{n'}^{m'} = \sum_{n',m'} n'(n'+1) Y_{n'}^{m'}$ $\Rightarrow$ $a_{nn'}^{mm'} = 0$ si $n \neq n'$

$\Rightarrow$ sólo contribuyen $a_{nn}^{mm'} =: a_n^{mm'}$]