elaciones de recurrencia: PAL07

$$\frac{\partial g_m}{\partial t} = \frac{(2m)!}{2^m m!}(-)(m+\tfrac{1}{2})(1-2xt+t^2)^{-(m+1/2)-1} \times (-2x+2t) = \sum_{n=0}^{\infty} P_{n+m}^m(x)\, n t^{n-1}$$

$$= (2m+1)(x-t)(1-2xt+t^2)^{-1} g_m = \sum_{n=0}^{\infty} P_{n+m}^m(x)\, n t^{n-1}$$

$$= \frac{(2m+1)(x-t)}{(1-2xt+t^2)} \sum_{n=0}^{\infty} P_{n+m}^m t^n = \sum_{n=0}^{\infty} P_{n+m}^m\, n t^{n-1}$$

$$\Rightarrow (2m+1)\sum_{n=0}^{\infty} x P_{n+m}^m t^n - (2m+1)\sum_{n=0}^{\infty} P_{n+m}^m t^{n+1} = \sum_{n=0}^{\infty} P_{n+m}^m\, n t^{n-1} - 2\sum_{n=0}^{\infty} x P_{n+m}^m\, n t^n + \sum_{n=0}^{\infty} P_{n+m}^m\, n t^{n+1}$$

$$(2m+1)\sum_{n=0}^{\infty} x P_{n+m}^m t^n - (2m+1)\sum_{n=1}^{\infty} P_{n+m-1}^m t^n = \sum_{n=0}^{\infty} P_{n+m+1}^m (n+1) t^n - 2\sum_{n=1}^{\infty} x P_{n+m}^m\, n t^n + \sum_{n=1}^{\infty} P_{n+m-1}^m (n-1) t^n$$

$$\Rightarrow (2m+1)\, x P_m^m = P_{m+1}^m$$

$$y \quad (2m+1)\, x P_{n+m}^m - (2m+1) P_{n+m-1}^m = P_{n+m+1}^m (n+1) - 2x P_{n+m}^m\, n + P_{n+m-1}^m (n-1)$$

$$\Rightarrow (2m+2n+1)x P_{n+m}^m - (2m+n) P_{n+m-1}^m - (n+1) P_{n+m+1}^m = 0 \qquad n = 1, 2, \ldots$$

para $n=0 \Rightarrow (2m+1)\, x P_m^m - 2m\, \cancel{P_{m-1}^m} - P_{m+1}^m = 0$ ✓

$$\therefore \boxed{(n+1) P_{n+m+1}^m - (2m+2n+1)\, x P_{n+m}^m + (2m+n) P_{n+m-1}^m = 0} \qquad n = 0, 1, 2, \ldots$$