Función generadora para fnes asociadas de Legendre

Sabemos que $g(x,t) = (1-2xt+t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)\,t^n$

$$\Rightarrow \frac{d^m}{dx^m} g(x,t) = \sum_{n=m}^{\infty} \frac{d^m P_n}{dx^m} t^n \Rightarrow (1-x^2)^{m/2} \frac{d^m}{dx^m} g(x,t) = \sum_{n=m}^{\infty} P_n^m(x)\,t^n$$

$$\frac{d}{dx} g = t\cdot(1-2xt+t^2)^{-3/2} \; ; \; \frac{d^2}{dx^2} g = t^2\cdot 1\cdot 3\cdot(1-2xt+t^2)^{-5/2}$$

$$\frac{d^3}{dx^3} g = t^3\cdot 1\cdot 3\cdot 5\,(1-2xt+t^2)^{-7/2} = t^3 \cdot \frac{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6}{2\cdot 4\cdot 6}(1-2xt+t^2)^{-7/2} = t^3\frac{6!}{2^3\cdot 3!}(\;)^{-7/2}$$

etc $$\frac{d^m}{dx^m} g = t^m \frac{(2m)!}{2^m (m!)} (1-2xt+t^2)^{-(2m+1)/2}$$

$$\frac{(2m)!}{2^m\cdot m!} t^m (1-x^2)^{m/2}\cdot(1-2xt+t^2)^{-(2m+1)/2} = \sum_{n=m}^{\infty} P_n^m(x)\,t^n \qquad P_n^m = 0 \text{ si } m>n$$

ó $$\boxed{\frac{(2m)!}{2^m m!}(1-x^2)^{m/2}(1-2xt+t^2)^{-(m+1/2)}} = \sum_{n=m}^{\infty} P_n^m(x)\,t^{n-m} = \sum_{n=0}^{\infty} P_{n+m}^m\, t^n$$

Es útil definir los polinomios $\mathcal{P}_n^m(x) := (1-x^2)^{-m/2} P_n^m(x) = \frac{d^m}{dx^m} P_n(x)$

$$\boxed{g_m(x,t) := \frac{(2m)!}{2^m m!}(1-2xt+t^2)^{-(m+1/2)} = \sum_{n=0}^{\infty} \mathcal{P}_{n+m}^m(x)\,t^n}$$