A Bayes filtering exercise on tracking a mobile robot with a fixed camera

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Abstract

This document is the text of a simple exercise about tracking a mobile robot by exploiting the images obtained from a fixed camera observing the plane where the robot moves. The objective of the exercise it to experiment with the writing of the system equations, and on filtering the measurements to obtain the robot pose. Being an exercise, a set of simplifications has been introduced. Examples are: the robot motion takes place in a plane, the camera optical axis is aligned with the Z world axis, the image axes are aligned with the X and Y world axes, the camera has no translation of the principal point, no skew, and a unit aspect ratio. The exercise bases on the observations of the robot by the camera, whose functioning is simulated. In particular, it is supposed the sensor software can detect and recognize 2 points on the robot body whose position is known w.r.t. the robot body reference. Realistic mistakes in the sensor, i.e., camera and software, will be dealt with in the version of the exercise for non - parametric filtering. In this exercise we will work under the hypothesis of Kalman filtering being applicable. A working knowledge of representation of roto-translations, at least in the plane, is required. A similar requirement applies for some proficiency with programming in matlab. Developing the solution requires also knowledge of differential odometry; to obtain this last requirement solve the roto-translation in the plane exercise from the 3^{rd} year Robotics course.

I. Problem definition

A reference frame that is fixed with a mobile object is usually called *body* frame. In this exercise the mobile object is a mobile robot, therefore the body-attached frame will be called the *robot* frame (x_r, y_r, z_r). This is the reference frame w.r.t. which the coordinates of the robot points and/or the equations of the robot body surfaces are known.

The mobile robot moves on an horizontal plane.

The reference frame *w* (for world) is the one into which the robot motion has to be described. It is also the frame where the world points are known. The equation of the plane where the motion takes place is $z_w = 0$. See also Figure 1.

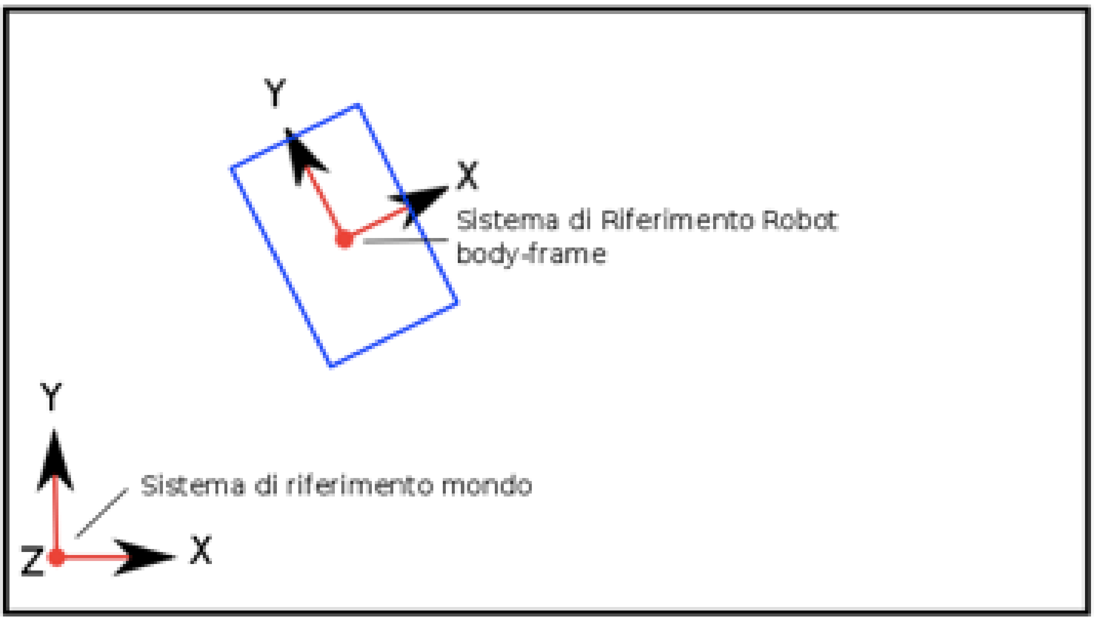

Fig. 1. Sketch of the reference frames involved in the problem, view from above. The red dot means the axis, i.e., Z in this case, is orthogonal to the paper, pointing toward the observer. “Sistema di riferimento mondo” stands for “world reference frame”, while “Sistema di riferimento robot” stands for “robot reference frame”

The *pose* of the robot is the set of values taken by the variables defining how the robot is positioned w.r.t. the world; the robot pose includes both the position of the origin of the robot frame as well as the orientation of the axes of the robot frame.

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System time is discrete: $t_0, t_1, \ldots, t_n$.

The robot is observed by a camera, whose projection centre is at height h (a parameter whose value is known, e.g., $10m$) above the world origin. The camera looks downward, toward the ground plane, and its optical axis is perfectly aligned with the z_w axis. Happily enough, the sensor axes (u, v) are perfectly aligned with the world axes (x_w, y_w).

Let u be the image axis aligned with x_w, v the one aligned with (the opposite of) y_w, f (a parameter whose value is known, e.g., $600pix$) the camera focal length, h the height (along z_w) of the camera, i.e., the height of its projection centre, see also Figure 2.

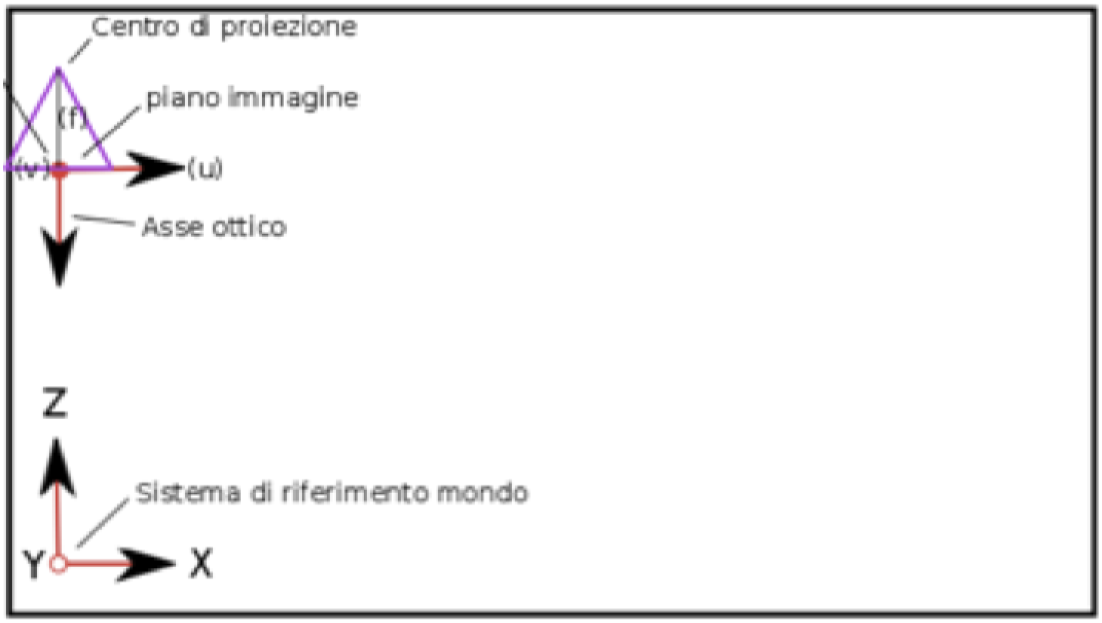

Fig. 2. Sketch of the reference frames, including the camera observing the robot, frontal view. The red dot means the axis, i.e., v in this case, is orthogonal to the paper, pointing toward the observer. The red circle means the axis, i.e., y_w in this case, is orthogonal to the paper, but points away from the observer. "Centro di proiezione" stands for "projection centre", "piano immagine" stands for "image plane", "asse ottico" stands for "optical axis", and "Sistema di riferimento mondo" stands for "world reference frame"

Therefore, the relationship between the coordinates of the world points and the respective image points, disregarding the quantization implied by the spatial sampling, are:

$$\begin{cases} u/f = x_w/(h - z_w) \\ v/f = -y_w/(h - z_w) \end{cases} ; \quad \begin{cases} u = f x_w/(h - z_w) \\ v = -f y_w/(h - z_w) \end{cases}$$

Let us suppose also the robot motion to always be inside the field of view of the camera.

In this exercise you have to use a Kalman filter to estimate the robot pose, using the images of the robot obtained by the camera. In particular, assume the sensor to include both the camera hardware and some software, executed on the computer that receives the camera images, which is capable to detect the origin of the robot frame (called P_1) as well a second robot point P_2. P_2 is at distance d (a parameter whose value is known, e.g., $0.6m$ forward) from P_1 along the x_r axis. Differently from Figure 1, consider x_r to be heading forward, and y_r to be heading leftward on the robot, hence P_2 is on x_r with coordinates ($P_2^r = (d, 0, 0)$). The two image points have coordinates (u_1,v_1), (u_2,v_2).

Onboard the robot an odometric system is running. The odometric data are in the form of the length of the arcs constituted by the path of the point of contact between the wheel and the ground. A noisy measurement of such arcs become available after each time interval, i.e., at time t the data will refer to the travel of the contact points from time $t - 1$ to time t. Such arcs will be used as control input to the system, according to the so-called *odometry model*.

The state transition of the system can be as simple as: the change of state value is the one induced by the control input, in other words the system has no free motion, if the control is null the state keeps its value.

For those who did not attend the Robotics course, an exercise is made available, whose aim is to determine the relationship between the control and the change to the state value in case of a simple differential kinematics mobile robot. A required parameter for such relationship is the distance l between the contact points of the 2 wheels; l is considered known also in this exercise, e.g., $0.5m$.

Summarizing, the data in input to the system are:

- configuration of the system:
  - camera position w.r.t. the world;
  - initial (guess on the) pose of the robot w.r.t. the world;

- data acquired by the sensors:
  - odometric data, used according to the odometry model (relative motion between the poses at two consecutive times);
  - image data, the 2 image points $p_1 = (u_1, v_1)$ and $p_2 = (u_2, v_2)$, acquired at the same time of the odometric datum.