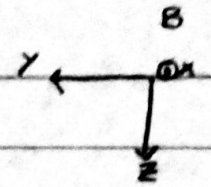
X: Roll Axis (Long Axis)
Y: Pitch Axis
Z: Yaw Axis



G: Gimbal Frame

B: Body Fixed Frame (x, y, z)

I: Inertial Frame (X, Y, Z)

Inner Gimbal rotates y
Outer Gimbal rotates z

Dynamics

$[\underline{T}]_G = [T\ 0\ 0]^T$ 1

${}^G C^B = C_x(\theta_x) C_z(\theta_y)$ 2

$[\underline{T}]_B = {}^B C^G [\underline{T}]_G$ Used For Translational Dynamics Sim 3

$[\underline{T}]_I = {}^I C^B [\underline{T}]_B$ 4

$[\underline{r}]_B = [-l\ 0\ 0]^T$ Moment Arm 5

$[\underline{M}]_B = [\underline{r}]_B \times [\underline{T}]_B$ Used In Attitude Dynamics Sim 6

$[\mathbb{I}]_B [\dot{\omega}]_B + [\underline{\omega}\times]_B [\mathbb{I}]_B [\underline{\omega}]_B = [\underline{M}]_B$ Attitude Dynamics 7

$m[\ddot{x}]_I = [\underline{T}]_G - \underline{g}$ Translational Dynamics 8

${}^B\dot{C}^I = {}^B C^I \tilde{\omega}$ 9

${}^B\frac{d}{dt} q = \frac{1}{2} q \odot \begin{bmatrix} {}^I\omega^B \\ 0 \end{bmatrix} \quad q = \begin{bmatrix} \underline{\varepsilon} \\ \varepsilon_4 \end{bmatrix}$ Orientation Update (Must normalize after) 10

Control Law

$[\underline{M}]_B = -K_p \underline{\varepsilon} - K_d\, {}^I\underline{\omega}^B - K_i \int_0^t \underline{\varepsilon}\, d\tau$

PID to determine desired moment

Ignore roll moment. Back calculate to determine $[\underline{T}]_B$ using G. Use 3 and 2 to determine θ_y and θ_z.