Homework Assignment 1

246PP - Advanced Econometrics 2020/2021

This is an R Markdown Notebook. Please, complete your answer in the boxes below and place hw.css in the same directory of your file.

This homework assignment is due back on Thursday, March 25 (before 23:59) using the procedure described in the Homework Assignments Teams Channel.

1. Use `R` to generate $n = 200$ observations from the following model

$$y_i = \beta_0 + \beta_1 x_i + u_i, \quad n = 1, \ldots, 200$$

   where x_i and u_i are independent and (u_i, x_i) is independent of (u_j, x_j) for $i \neq j$;

$$u_i \sim \chi^2_3, \quad x_i \sim N(0, 1),$$

   and $\beta_0 = 0.5$ and $\beta_1 = 0.3$. (χ^2_p is the chi squared distribution with p degrees of freedoms).

2. Using the data generated in the previous model to estimate the β_0 and β_1 using OLS.

3. Calculate the variance of the asymptotic distribution of $\hat{\beta}_1$ that is appropriate for the assumption of the model. (*(Note: You can use the output from `lm`, `summary` and other methods in packages, e.g. `sandwich`, but you will get extra credit if you try to calculate the variance without relying on them.))*

4. Construct a 90% confidence interval for β_1.

5. Since you generated the data yourself, you know which assumptions hold for the model above. Answer the question below providing a brief, but meaningful, justification:

   1. Is OLS unbiased for β_0?
   2. Is OLS unbiased for β_1?
   3. Is the model conditionally homoscedastic?
   4. Is $\hat{\beta}_1$ estimating the partial effect?
   5. Can you derive the small sample distribution of $\hat{\beta}_1$ or you need more assumptions?

6. Suppose that instead of running a regression of y_i on x_i, you run the regression of x_i and y_i, that is you switch the dependent and independent variables

$$x_i = \gamma_0 + \gamma_1 y_i + \eta_i.$$

   Report your estimates of γ_1.

7. What is γ_1 estimating, i.e. can you give the probability limit of $\hat{\gamma}_1$?

8. Find the asymptotic distribution of the estimator defined as $\hat{\xi} = 1/\hat{\beta}_1$ where $\hat{\beta}_1$ is the OLS estimator of point (2). (*Hint: use the Delta Method.*)

9. How the answers to Question 5 would change if you simulated the data from

$$y_i = \beta_0 + \beta_1 x_i + u_i, \quad n = 1, \ldots, 200$$

where now x_i and u_i are generate according to

$$x_i \sim N(0,1), \quad u_i = \sqrt{x_i^2} \times \varepsilon_i, \quad \varepsilon_i \sim \chi_3^2,$$

and the parameters are $\beta_0 = 0.5$ and $\beta_1 = 0.3$? Give the OLS estimates and the appropriate standard errors.