

Lab 05

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1 Preliminaries

Question 1: Set the seed for random numbers in R with `set.seed(1991)`

See R code

Question 2: Simulate 100 data points from a normal distribution with mean $\mu = 100$ and variance $\sigma^2 = 25$. You will use these data to test the conjugate functions that you will write in section IV below. Call the data set `y`. Be careful here. R requires the standard deviation, not the variance, as a parameter.

See R code

Question 3: Write a function called `draw.mean`

See R code

Question 4: Write a function called `draw.var`

See R code

2 Writing a Sampler

Question 5: Set up a matrix for storing samples from the posterior distribution of the mean. The number of columns should equal the number of chains (3) and number of rows should equal the number of iterations (10,000). Do the same thing for storing samples from the posterior distribution of the variance.

See R code

Question 6: Assign initial values to the first row of each matrix, a different value for each of the chains. These can be virtually any value within the support of the random variable.

See R code

Question 7: Set up nested for loops to iterate from two to the total number of iterations for each of the three chains for each parameter (recall we already selected values for the first row—these were our initial values). Use the conjugate functions `draw.mean` and `draw.var` to draw a sample from the distribution of the mean using the value of the variance at the previous current iteration. Then make a draw from the variance using the value of the mean from the current iteration. Repeat. Assume vague priors for the mean and variance

See R code

3 Trace Plots and Plots of Marginal Posterior Distributions

Question 8: Discard the first 1000 iterations as burn-in. On the same figure, make three plots (one for each chain) with the value of the mean as a function of iteration number. This is called a trace plot.

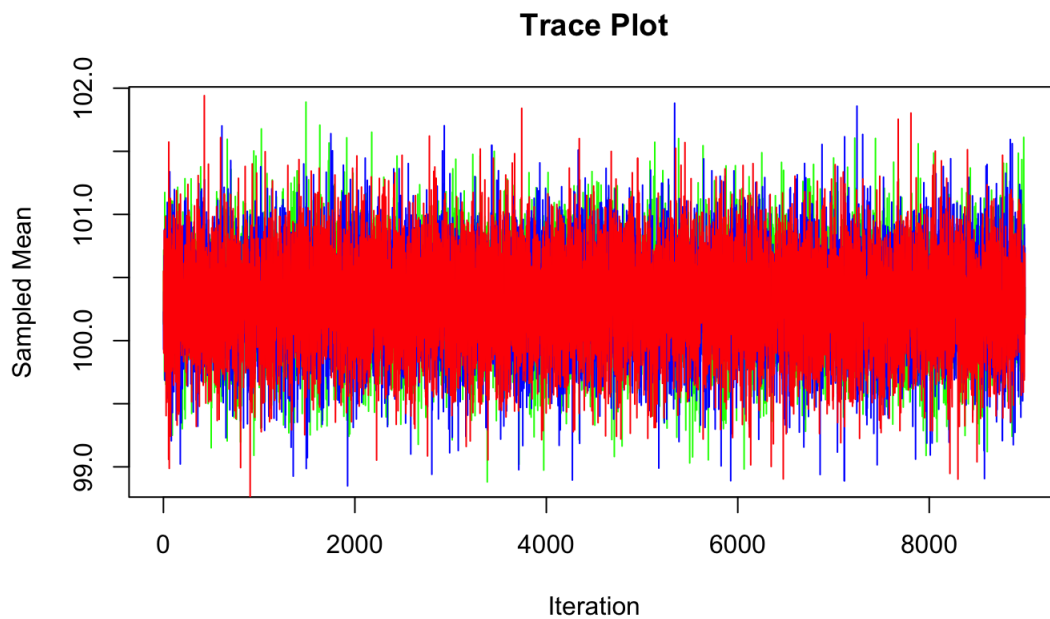


Figure 1: Trace plot of the μ chains

Question 9: For all chains combined, make a histogram of the samples of the mean retained after burn-in. Put a vertical line on the plot showing the generating value.

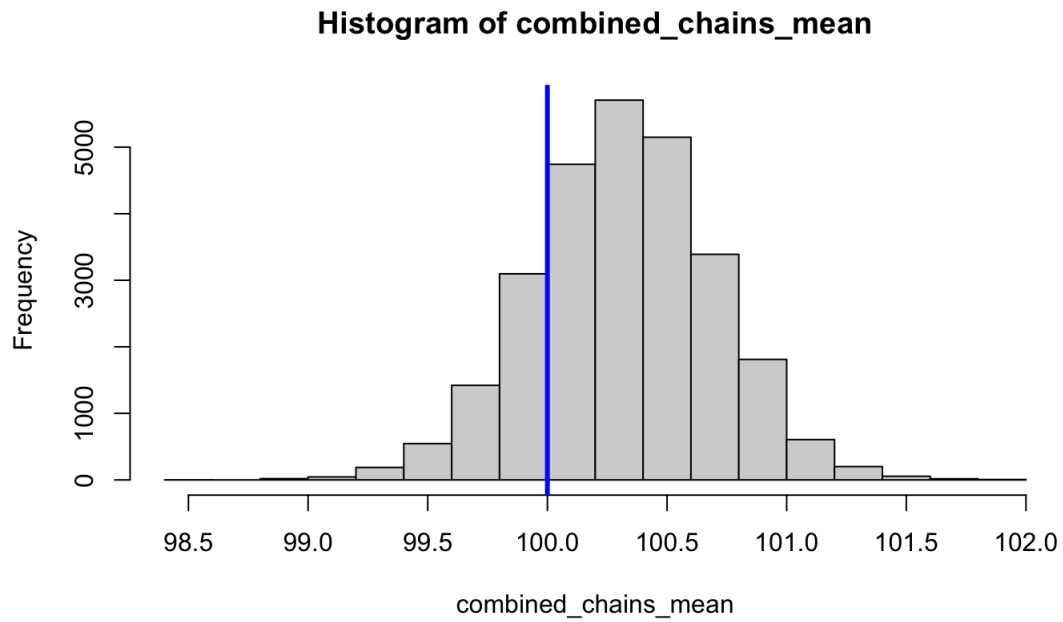


Figure 2: Histogram of the drawn values of μ for all chains

Question 10: Repeat steps 8-9 for the variance.

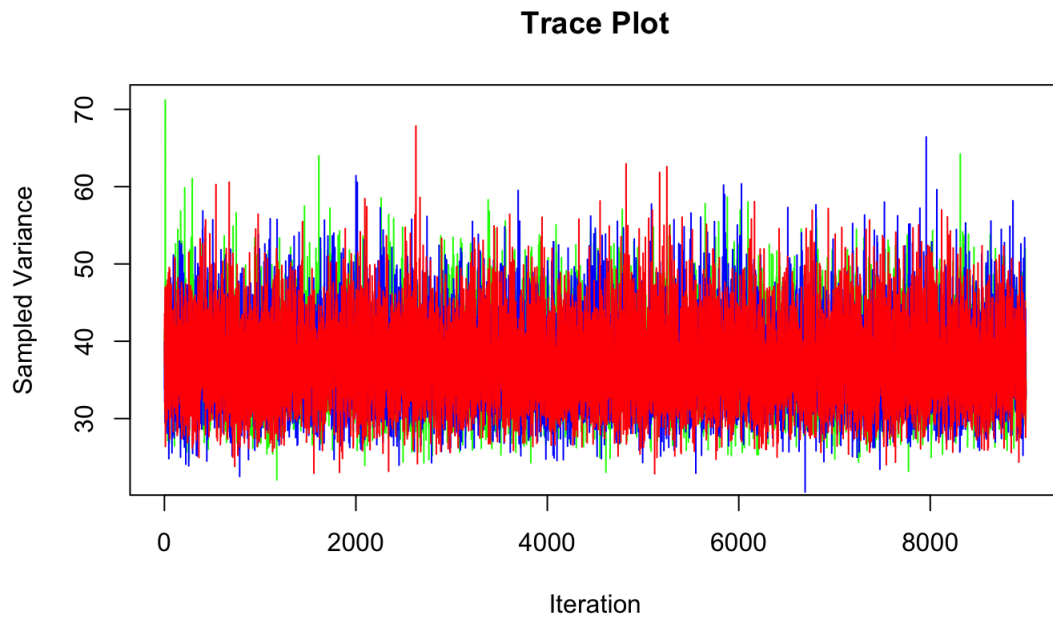


Figure 3: Trace plot of the σ^2 chains

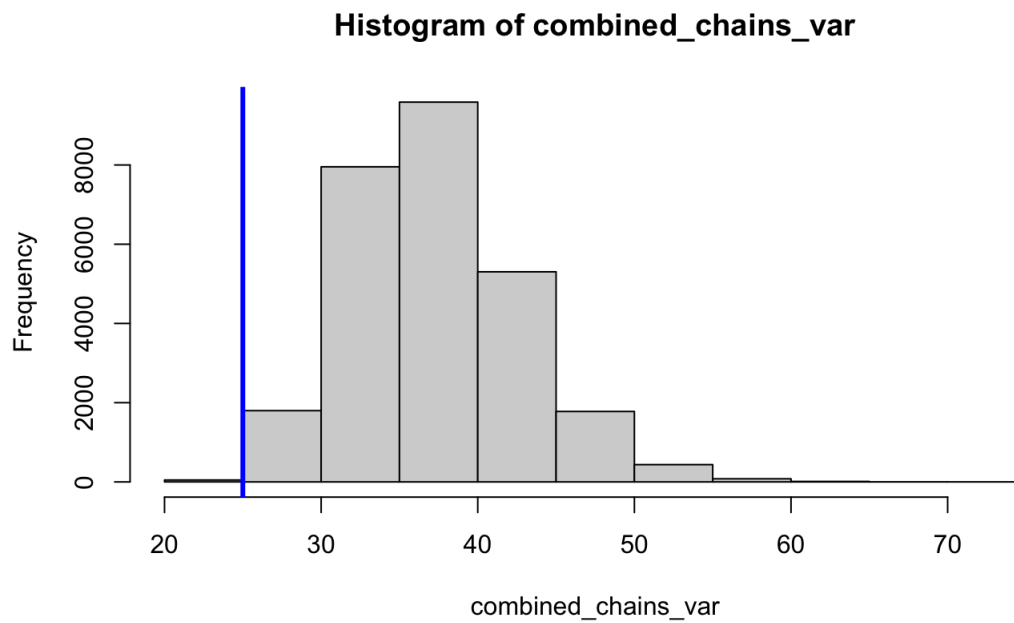


Figure 4: Histogram of all drawn values for σ^2 for all chains

Question 11: For both μ and σ^2 , calculate the mean of all the chains combined and its standard deviation. Interpret these quantities.

The mean of the μ combined chain was 100.3186 while the SD was 0.3781. The mean of the σ^2 combined chain was 37.2907 while the SD was 5.3945.

Given that the mean of the μ chain is very similar to the mean of our simulated dataset, we can say that this iterative process of sampling from the posterior distribution of μ recovers the mean of our simulated dataset very well. We can also say that we can be very confident in knowing where the true value of μ is given that the SD of the combined μ chain is very low. However, the mean of our combined σ^2 is far apart from the variance of our simulated dataset, which may mean that we either need more iterations to recover the true value of σ^2 or that we need to test our MCMC algorithm on multiple simulated datasets to get a better understanding of the true value of σ^2 . The standard deviation of our combined σ^2 chain is also high, indicating that we cannot have a high level of confidence in knowing the true value of σ^2 .

Question 12: Compare the standard deviation of the posterior distribution of μ with an approximation using the standard deviation of the data divided by the square root of the sample size. What is this approximation called in the frequentist world?

The value of the standard deviation of the data divided by the square root of n is 0.6065, which is reasonably similar to our standard deviation of our posterior distribution. In frequentist statistics, this is referred to as the standard error.

Question 13: Vary the number of values in the simulated data set, e.g., $n = 10$, 100, 1000. What happens to the mean and variance of the posterior distributions as n gets large? Explain why you think this happens.

As n gets larger and larger, our mean and variance of our posterior distributions gradually shift closer and closer to the mean and variance of our simulated dataset. We believe this happens because as we increase our n , the random numbers in our simulated dataset become less meaningful individually in terms of their effect on the variance of the dataset. Because of this, an iterative chain in an MCMC algorithm will also be less subjected to variance and will be more likely to recover the true value from the posterior distributions of our parameters.

Question 14: Make the burn-in = 1 instead of 1000. Does this change your results? Why or why not?

Yes, this changes our results, but only slightly. While our mean is mostly still recovered by the MCMC algorithm except when $n = 10$, the variance is recovered but with a relatively high amount of uncertainty even when $n = 100$. Only when $n = 1000$ does the variance begin to be recovered with lower levels of uncertainty. This is likely because keeping more of the initial samples allows for potentially higher levels of variability before the MCMC algorithm narrows in on more likely range of values containing the true variance.

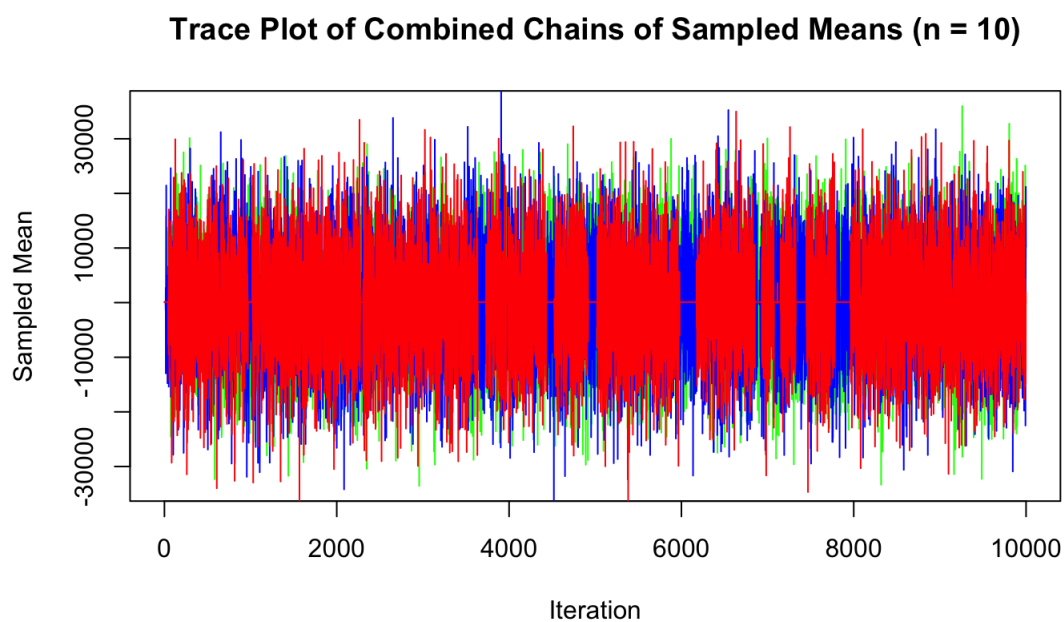


Figure 5: Trace plot when $n = 10$

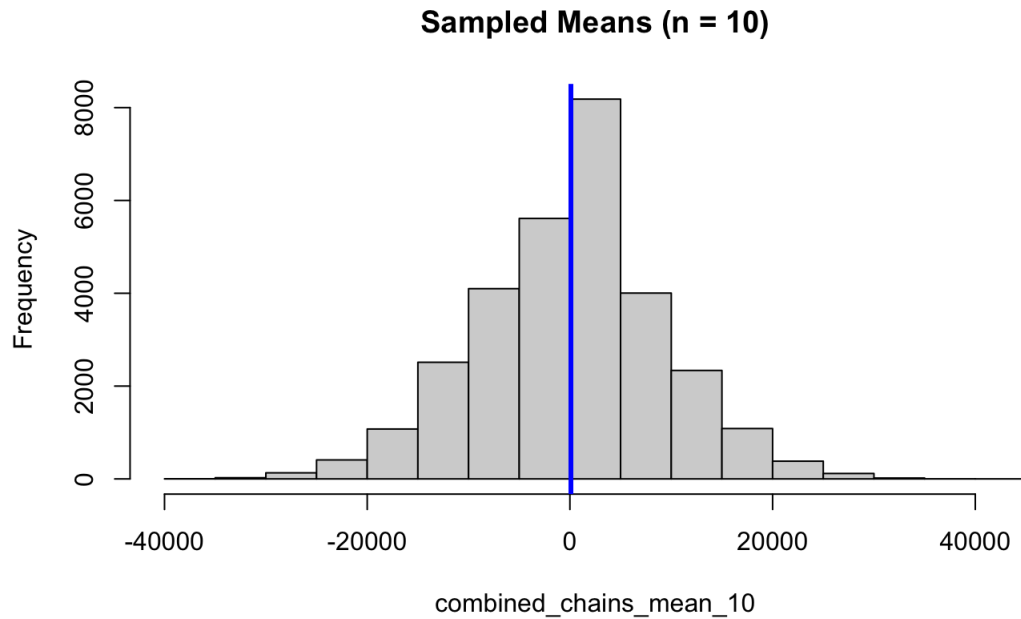


Figure 6: Our sampled means displayed high variance when $n = 10$

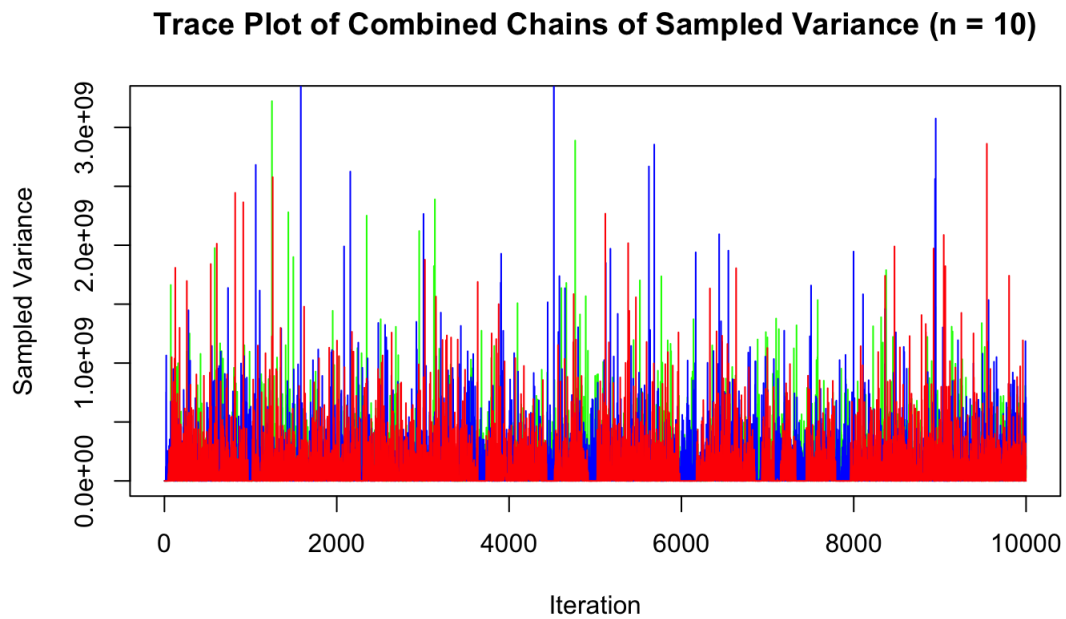


Figure 7: Trace plot of sampled variance when $n = 10$

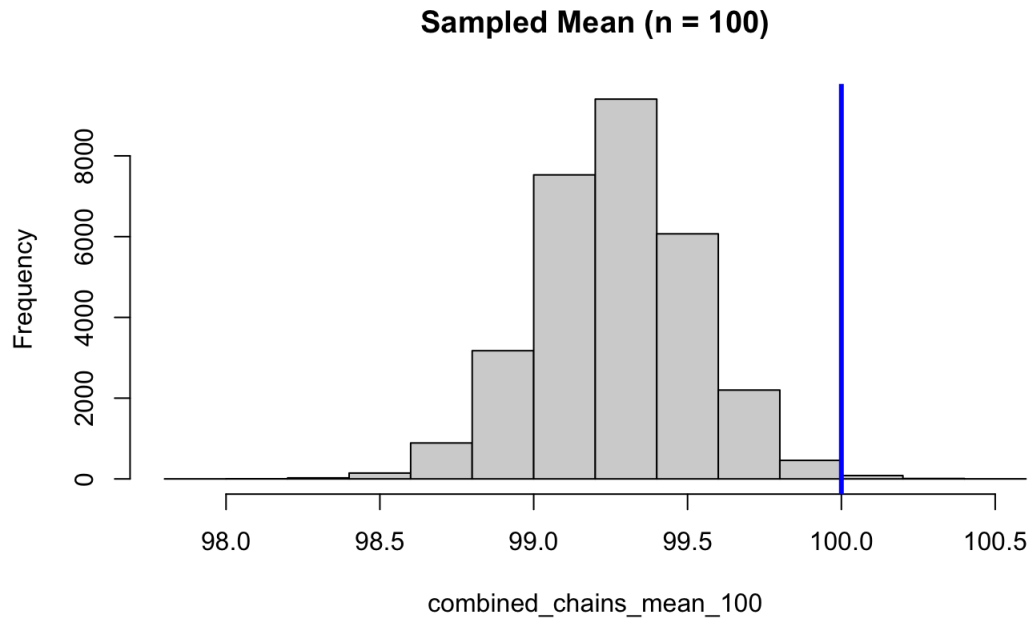


Figure 8: Sampled means when $n = 100$ displayed much less variance

Question 15: Reverse the order of the conjugate functions in step 7 so that the variance is drawn first followed by the mean. Be careful, this involves a bit more than simply reversing the order of the functions in the loop. Does this reordering have an effect on the posteriors? Why or why not?

It has very little effect on the posteriors, probably due to the large number of iterations in our chain and the vague priors we set in question 7. We also continue to sample within our sequence rather than sampling values out of sequence, so the order has at most a minimal impact.

4 Appendix: R Code

```

1  ““{r}
2  library(pacman)
3  p_load(tidyverse, here, invgamma)
4  ““
5
6  ## Section 1: Preliminaries
7  ““{r}
8  ## Question 1
9  set.seed(1991)
10 ## Question 2
11 ## simulating 100 data points with mean mu = 100 and variance = 25
12 y <- rnorm(100, 100, sqrt(25))
13
14 ## Question 3
15 draw.mean <- function(data, mu_0, sigma_0, variance){
16   mu_1 <- ((mu_0 / sigma_0^2) + (sum(data) / variance)) / ((1/sigma_0^2) + (
17     length(data)/variance))
18   sigma_sqd_1 <- 1 / ((1 / sigma_0^2) + length(data) / variance)
19   random_meandraw <- rnorm(1, mu_1, sigma_sqd_1)
20   return(random_meandraw)
21 }
22
23 ## Question 4
24 draw.var <- function(data, alpha_0, beta_0, mu){
25   alpha_1 <- alpha_0 + (length(data) / 2)
26   beta_1 <- beta_0 + (sum((data - mu)^2) / 2)
27   random_vardraw <- rinvgamma(1, shape = alpha_1, rate = beta_1)
28   return(random_vardraw)
29 }
30 ““
31
32 ## Section 2: Making a Sampler
33 ““{r}
34 ## Question 5
35 ## creating matrices to store samples from posterior distributions of mean and
36   variance
37 sample_mean_matrix <- matrix(nrow = 10000, ncol = 3)
38 sample_var_matrix <- matrix(nrow = 10000, ncol = 3)
39
40 ## Question 6
41 sample_mean_matrix[1,] <- 20

```



```

41 sample_var_matrix[1,] <- 15
42
43 ## Question 7
44 for (chain in 1:ncol(sample_mean_matrix)){
45   for (iter in 2:nrow(sample_mean_matrix)){
46     prev_var <- sample_var_matrix[iter-1,chain]
47     sample_mean_matrix[iter,chain] <- draw.mean(y, 0, sqrt(10000), prev_var)
48     prev_mean <- sample_mean_matrix[iter,chain]
49     sample_var_matrix[iter, chain] <- draw.var(y, 0.01, 0.01, prev_mean)
50   }
51 }
52 ''',
53
54
55 ## Section 3: Trace Plots and Plots of Marginal Posterior Distributions
56 ''',{r}
57 ## Question 8
58 # discarding first 1000 rows as burn-in/warmup
59 post_burn_mean <- sample_mean_matrix[1001:nrow(sample_mean_matrix),]
60 # hist(post_burn_mean)
61
62 # making trace plots
63 plot(seq(1, nrow(post_burn_mean)), post_burn_mean[,1], xlab = "Iteration",
64       ylab = "Sampled Mean", type = 'l', main = 'Trace Plot', col = 'green')
65 lines(post_burn_mean[,2], col = 'blue')
66 lines(post_burn_mean[,3], col = 'red')
67
68 ## Question 9
69 # combining chains
70 combined_chains_mean <- c(post_burn_mean[,1], post_burn_mean[,2], post_burn_
71   mean[,3])
72 hist(combined_chains_mean)
73 abline(v = 100, col = 'blue', lwd = 3)
74
75 ## Question 10
76 # repeating 8 and 9 for the variance
77 post_burn_var <- sample_var_matrix[1001:nrow(sample_var_matrix),]
78
79 # making trace plots
80 plot(seq(1, nrow(post_burn_var)), post_burn_var[,1], xlab = "Iteration", ylab
81   = "Sampled Variance", type = 'l', main = 'Trace Plot', col = 'green')
82 lines(post_burn_var[,2], col = 'blue')
83 lines(post_burn_var[,3], col = 'red')
84
85 # combining chains
86 combined_chains_var <- c(post_burn_var[,1], post_burn_var[,2], post_burn_var
87   [,3])
88 hist(combined_chains_var)
89 abline(v = 25, col = 'blue', lwd = 3)
90
91 ## Question 11
92 # mean and SD of combined chains for mu
93 mean_combined_chains_mean <- mean(combined_chains_mean)
94 sd_combined_chains_mean <- sd(combined_chains_mean)

```

```

91 # mean and SD of combined chains for sigma^2
92 mean_combined_chains_var <- mean(combined_chains_var)
93 sd_combined_chains_var <- sd(combined_chains_var)
94 # printing values
95 mean_combined_chains_mean
96 sd_combined_chains_mean
97 mean_combined_chains_var
98 sd_combined_chains_var
99
100 ## Question 12
101 se_data <- sd(y) / sqrt(length(y))
102 se_data
103 ```
104
105 ## Question 13, n = 10
106 ```{r}
107 # simulating new dataset
108 y_10 <- rnorm(10, 100, sqrt(25))
109
110
111 # creating matrices to store samples from posterior distributions of mean and
    variance
112 sample_mean_matrix_10 <- matrix(nrow = 10000, ncol = 3)
113 sample_var_matrix_10 <- matrix(nrow = 10000, ncol = 3)
114
115 # initializing values
116 sample_mean_matrix_10[1,] <- 20
117 sample_var_matrix_10[1,] <- 15
118
119 # sampling from posterior distributions
120 for (chain in 1:ncol(sample_mean_matrix_10)){
121   for (iter in 2:nrow(sample_mean_matrix_10)){
122     prev_var <- sample_var_matrix_10[iter-1,chain]
123     sample_mean_matrix_10[iter,chain] <- draw.mean(y_10, 0, sqrt(10000), prev_
    var)
124     prev_mean <- sample_mean_matrix_10[iter,chain]
125     sample_var_matrix_10[iter, chain] <- draw.var(y_10, 0.01, 0.01, prev_mean)
126   }
127 }
128
129 # discarding first 1000 rows as burn-in/warmup
130 post_burn_mean_10 <- sample_mean_matrix_10[1001:nrow(sample_mean_matrix_10),]
131 # hist(post_burn_mean_10)
132
133 # making trace plots
134 plot(seq(1, nrow(post_burn_mean_10)), post_burn_mean_10[,1], xlab = "Iteration
    ", ylab = "Sampled Mean", type = 'l', main = 'Trace Plot', col = 'green')
135 lines(post_burn_mean_10[,2], col = 'blue')
136 lines(post_burn_mean_10[,3], col = 'red')
137
138 # combining chains
139 combined_chains_mean_10 <- c(post_burn_mean_10[,1], post_burn_mean_10[,2],
    post_burn_mean_10[,3])
140 hist(combined_chains_mean_10)

```

```

141 abline(v = 100, col = 'blue', lwd = 3)
142
143 # repeating for the variance
144 post_burn_var_10 <- sample_var_matrix_10[1001:nrow(sample_var_matrix_10),]
145
146 # making trace plots
147 plot(seq(1, nrow(post_burn_var_10)), post_burn_var_10[,1], xlab = "Iteration",
      ylab = "Sampled Variance", type = 'l', main = 'Trace Plot', col = 'green'
    )
148 lines(post_burn_var_10[,2], col = 'blue')
149 lines(post_burn_var_10[,3], col = 'red')
150
151 # combining chains
152 combined_chains_var_10 <- c(post_burn_var_10[,1], post_burn_var_10[,2], post_
      burn_var_10[,3])
153 hist(combined_chains_var_10)
154 abline(v = 25, col = 'blue', lwd = 3)
155
156
157 # mean and SD of combined chains for mu
158 mean_combined_chains_mean_10 <- mean(combined_chains_mean_10)
159 sd_combined_chains_mean_10 <- sd(combined_chains_mean_10)
160 # mean and SD of combined chains for sigma^2
161 mean_combined_chains_var_10 <- mean(combined_chains_var_10)
162 sd_combined_chains_var_10 <- sd(combined_chains_var_10)
163 # printing values
164 mean_combined_chains_mean_10
165 sd_combined_chains_mean_10
166 mean_combined_chains_var_10
167 sd_combined_chains_var_10
168 ''
169
170 ## Question 13, n = 100
171 ''{'r}
172 # creating new simulated data
173 y_100 <- rnorm(100, 100, sqrt(25))
174
175 # creating matrices to store samples from posterior distributions of mean and
      variance
176 sample_mean_matrix_100 <- matrix(nrow = 10000, ncol = 3)
177 sample_var_matrix_100 <- matrix(nrow = 10000, ncol = 3)
178
179 # initializing values
180 sample_mean_matrix_100[1,] <- 20
181 sample_var_matrix_100[1,] <- 15
182
183 # sampling from posterior distributions
184 for (chain in 1:ncol(sample_mean_matrix_100)){
185   for (iter in 2:nrow(sample_mean_matrix_100)){
186     prev_var <- sample_var_matrix_100[iter-1,chain]
187     sample_mean_matrix_100[iter,chain] <- draw.mean(y_100, 0, sqrt(10000),
      prev_var)
188     prev_mean <- sample_mean_matrix_100[iter,chain]

```

```

189     sample_var_matrix_100[iter, chain] <- draw.var(y_100, 0.01, 0.01, prev_
190     mean)
191 }
192 }
193 # discarding first 1000 rows as burn-in/warmup
194 post_burn_mean_100 <- sample_mean_matrix_100[1001:nrow(sample_mean_matrix_100)
195     ,]
196 # hist(post_burn_mean_100)
197 # making trace plots
198 plot(seq(1, nrow(post_burn_mean_100)), post_burn_mean_100[,1], xlab = "
199     Iteration", ylab = "Sampled Mean", type = 'l', main = 'Trace Plot', col =
200     'green')
201 lines(post_burn_mean_100[,2], col = 'blue')
202 lines(post_burn_mean_100[,3], col = 'red')
203 # combining chains
204 combined_chains_mean_100 <- c(post_burn_mean_100[,1], post_burn_mean_100[,2],
205     post_burn_mean_100[,3])
206 hist(combined_chains_mean_100)
207 abline(v = 100, col = 'blue', lwd = 3)
208 # repeating for the variance
209 post_burn_var_100 <- sample_var_matrix_100[1001:nrow(sample_var_matrix_100),]
210 # making trace plots
211 plot(seq(1, nrow(post_burn_var_100)), post_burn_var_100[,1], xlab = "Iteration
212     ", ylab = "Sampled Variance", type = 'l', main = 'Trace Plot', col = '
213     green')
214 lines(post_burn_var_100[,2], col = 'blue')
215 lines(post_burn_var_100[,3], col = 'red')
216 # combining chains
217 combined_chains_var_100 <- c(post_burn_var_100[,1], post_burn_var_100[,2],
218     post_burn_var_100[,3])
219 hist(combined_chains_var_100)
220 abline(v = 25, col = 'blue', lwd = 3)
221 # mean and SD of combined chains for mu
222 mean_combined_chains_mean_100 <- mean(combined_chains_mean_100)
223 sd_combined_chains_mean_100 <- sd(combined_chains_mean_100)
224 # mean and SD of combined chains for sigma^2
225 mean_combined_chains_var_100 <- mean(combined_chains_var_100)
226 sd_combined_chains_var_100 <- sd(combined_chains_var_100)
227 # printing values
228 mean_combined_chains_mean_100
229 sd_combined_chains_mean_100
230 mean_combined_chains_var_100
231 sd_combined_chains_var_100
232 ""
233 ## Question 13, n = 1000
234 ""{r}

```

```

235 # creating new simulated data
236 y_1000 <- rnorm(1000, 100, sqrt(25))
237
238 # creating matrices to store samples from posterior distributions of mean and
    variance
239 sample_mean_matrix_1000 <- matrix(nrow = 10000, ncol = 3)
240 sample_var_matrix_1000 <- matrix(nrow = 10000, ncol = 3)
241
242 # initializing values
243 sample_mean_matrix_1000[1,] <- 20
244 sample_var_matrix_1000[1,] <- 15
245
246 # sampling from posterior distributions
247 for (chain in 1:ncol(sample_mean_matrix_1000)){
248   for (iter in 2:nrow(sample_mean_matrix_1000)){
249     prev_var <- sample_var_matrix_1000[iter-1,chain]
250     sample_mean_matrix_1000[iter,chain] <- draw.mean(y_1000, 0, sqrt(10000),
    prev_var)
251     prev_mean <- sample_mean_matrix_1000[iter,chain]
252     sample_var_matrix_1000[iter,chain] <- draw.var(y_1000, 0.01, 0.01, prev_
    mean)
253   }
254 }
255
256 # discarding first 1000 rows as burn-in/warmup
257 post_burn_mean_1000 <- sample_mean_matrix_1000[1001:nrow(sample_mean_matrix_
    1000),]
258 # hist(post_burn_mean_100)
259
260 # making trace plots
261 plot(seq(1, nrow(post_burn_mean_1000)), post_burn_mean_1000[,1], xlab = "
    Iteration", ylab = "Sampled Mean", type = 'l', main = 'Trace Plot', col =
    'green')
262 lines(post_burn_mean_1000[,2], col = 'blue')
263 lines(post_burn_mean_1000[,3], col = 'red')
264
265 # combining chains
266 combined_chains_mean_1000 <- c(post_burn_mean_1000[,1], post_burn_mean_
    1000[,2], post_burn_mean_1000[,3])
267 hist(combined_chains_mean_1000)
268 abline(v = 100, col = 'blue', lwd = 3)
269
270 # repeating for the variance
271 post_burn_var_1000 <- sample_var_matrix_1000[1001:nrow(sample_var_matrix_1000)
    ,]
272
273 # making trace plots
274 plot(seq(1, nrow(post_burn_var_1000)), post_burn_var_1000[,1], xlab = "
    Iteration", ylab = "Sampled Variance", type = 'l', main = 'Trace Plot',
    col = 'green')
275 lines(post_burn_var_1000[,2], col = 'blue')
276 lines(post_burn_var_1000[,3], col = 'red')
277
278 # combining chains

```

```

279 combined_chains_var_1000 <- c(post_burn_var_1000[,1], post_burn_var_1000[,2],
    post_burn_var_1000[,3])
280 hist(combined_chains_var_1000)
281 abline(v = 25, col = 'blue', lwd = 3)
282
283 # mean and SD of combined chains for mu
284 mean_combined_chains_mean_1000 <- mean(combined_chains_mean_1000)
285 sd_combined_chains_mean_1000 <- sd(combined_chains_mean_1000)
286 # mean and SD of combined chains for sigma^2
287 mean_combined_chains_var_1000 <- mean(combined_chains_var_1000)
288 sd_combined_chains_var_1000 <- sd(combined_chains_var_1000)
289 # printing values
290 mean_combined_chains_mean_1000
291 sd_combined_chains_mean_1000
292 mean_combined_chains_var_1000
293 sd_combined_chains_var_1000
294 '""
295
296 ## Question 13, n = 100000
297 ""{r}
298 # creating new simulated data
299 y_100000 <- rnorm(100000, 100, sqrt(25))
300
301 # creating matrices to store samples from posterior distributions of mean and
    variance
302 sample_mean_matrix_100000 <- matrix(nrow = 10000, ncol = 3)
303 sample_var_matrix_100000 <- matrix(nrow = 10000, ncol = 3)
304
305 # initializing values
306 sample_mean_matrix_100000[1,] <- 20
307 sample_var_matrix_100000[1,] <- 15
308
309 # sampling from posterior distributions
310 for (chain in 1:ncol(sample_mean_matrix_100000)){
311   for (iter in 2:nrow(sample_mean_matrix_100000)){
312     prev_var <- sample_var_matrix_100000[iter-1,chain]
313     sample_mean_matrix_100000[iter,chain] <- draw.mean(y_100000, 0, sqrt
    (10000), prev_var)
314     prev_mean <- sample_mean_matrix_100000[iter,chain]
315     sample_var_matrix_100000[iter, chain] <- draw.var(y_100000, 0.01, 0.01,
    prev_mean)
316   }
317 }
318
319 # discarding first 1000 rows as burn-in/warmup
320 post_burn_mean_100000 <- sample_mean_matrix_100000[1001:nrow(sample_mean_
    matrix_100000),]
321 # hist(post_burn_mean_100)
322
323 # making trace plots
324 plot(seq(1, nrow(post_burn_mean_100000)), post_burn_mean_100000[,1], xlab = "
    Iteration", ylab = "Sampled Mean", type = 'l', main = 'Trace Plot', col =
    'green')
325 lines(post_burn_mean_100000[,2], col = 'blue')

```

```

326 lines(post_burn_mean_100000[,3], col = 'red')
327
328 # combining chains
329 combined_chains_mean_100000 <- c(post_burn_mean_100000[,1], post_burn_mean_
    100000[,2], post_burn_mean_100000[,3])
330 hist(combined_chains_mean_100000)
331 abline(v = 100, col = 'blue', lwd = 3)
332
333 # repeating for the variance
334 post_burn_var_100000 <- sample_var_matrix_100000[1001:nrow(sample_var_matrix_
    100000),]
335
336 # making trace plots
337 plot(seq(1, nrow(post_burn_var_100000)), post_burn_var_100000[,1], xlab = "
    Iteration", ylab = "Sampled Variance", type = 'l', main = 'Trace Plot',
    col = 'green')
338 lines(post_burn_var_100000[,2], col = 'blue')
339 lines(post_burn_var_100000[,3], col = 'red')
340
341 # combining chains
342 combined_chains_var_100000 <- c(post_burn_var_100000[,1], post_burn_var_
    100000[,2], post_burn_var_100000[,3])
343 hist(combined_chains_var_100000)
344 abline(v = 25, col = 'blue', lwd = 3)
345
346 # mean and SD of combined chains for mu
347 mean_combined_chains_mean_100000 <- mean(combined_chains_mean_100000)
348 sd_combined_chains_mean_100000 <- sd(combined_chains_mean_100000)
349 # mean and SD of combined chains for sigma^2
350 mean_combined_chains_var_100000 <- mean(combined_chains_var_100000)
351 sd_combined_chains_var_100000 <- sd(combined_chains_var_100000)
352 # printing values
353 mean_combined_chains_mean_100000
354 sd_combined_chains_mean_100000
355 mean_combined_chains_var_100000
356 sd_combined_chains_var_100000
357 '‘‘‘
358
359 ## Question 14
360 '‘‘{r}
361 ##### n = 10 #####
362 # simulating new dataset
363 y_10 <- rnorm(10, 100, sqrt(25))
364
365
366 # creating matrices to store samples from posterior distributions of mean and
    variance
367 sample_mean_matrix_10 <- matrix(nrow = 10000, ncol = 3)
368 sample_var_matrix_10 <- matrix(nrow = 10000, ncol = 3)
369
370 # initializing values
371 sample_mean_matrix_10[1,] <- 20
372 sample_var_matrix_10[1,] <- 15
373

```

```

374 # sampling from posterior distributions
375 for (chain in 1:ncol(sample_mean_matrix_10)){
376   for (iter in 2:nrow(sample_mean_matrix_10)){
377     prev_var <- sample_var_matrix_10[iter-1,chain]
378     sample_mean_matrix_10[iter,chain] <- draw.mean(y_10, 0, sqrt(10000), prev_var)
379     prev_mean <- sample_mean_matrix_10[iter,chain]
380     sample_var_matrix_10[iter, chain] <- draw.var(y_10, 0.01, 0.01, prev_mean)
381   }
382 }
383
384 # discarding first 1000 rows as burn-in/warmup
385 post_burn_mean_10 <- sample_mean_matrix_10[2:nrow(sample_mean_matrix_10),]
386 # hist(post_burn_mean_10)
387
388 # making trace plots
389 plot(seq(1, nrow(post_burn_mean_10)), post_burn_mean_10[,1], xlab = "Iteration",
390      ylab = "Sampled Mean", type = 'l', main = 'Trace Plot of Combined
391      Chains of Sampled Means (n = 10)', col = 'green')
392 lines(post_burn_mean_10[,2], col = 'blue')
393 lines(post_burn_mean_10[,3], col = 'red')
394
395 # combining chains
396 combined_chains_mean_10 <- c(post_burn_mean_10[,1], post_burn_mean_10[,2],
397                               post_burn_mean_10[,3])
398 hist(combined_chains_mean_10, main = "Sampled Means (n = 10)")
399 abline(v = 100, col = 'blue', lwd = 3)
400
401 # repeating for the variance
402 post_burn_var_10 <- sample_var_matrix_10[2:nrow(sample_var_matrix_10),]
403
404 # making trace plots
405 plot(seq(1, nrow(post_burn_var_10)), post_burn_var_10[,1], xlab = "Iteration",
406      ylab = "Sampled Variance", type = 'l', main = 'Trace Plot of Combined
407      Chains of Sampled Variance (n = 10)', col = 'green')
408 lines(post_burn_var_10[,2], col = 'blue')
409 lines(post_burn_var_10[,3], col = 'red')
410
411 # combining chains
412 combined_chains_var_10 <- c(post_burn_var_10[,1], post_burn_var_10[,2], post_burn_var_10[,3])
413 hist(combined_chains_var_10, main = "Sampled Variance (n = 10)")
414 abline(v = 25, col = 'blue', lwd = 3)
415
416 # mean and SD of combined chains for mu
417 mean_combined_chains_mean_10 <- mean(combined_chains_mean_10)
418 sd_combined_chains_mean_10 <- sd(combined_chains_mean_10)
419 # mean and SD of combined chains for sigma^2
420 mean_combined_chains_var_10 <- mean(combined_chains_var_10)
421 sd_combined_chains_var_10 <- sd(combined_chains_var_10)
422
423 # printing values
424 mean_combined_chains_mean_10
425 sd_combined_chains_mean_10
426 mean_combined_chains_var_10
427 sd_combined_chains_var_10

```



```

421 sd_combined_chains_var_10
422
423
424 ##### n = 100 #####
425 # creating new simulated data
426 y_100 <- rnorm(100, 100, sqrt(25))
427
428 # creating matrices to store samples from posterior distributions of mean and
    variance
429 sample_mean_matrix_100 <- matrix(nrow = 10000, ncol = 3)
430 sample_var_matrix_100 <- matrix(nrow = 10000, ncol = 3)
431
432 # initializing values
433 sample_mean_matrix_100[1,] <- 20
434 sample_var_matrix_100[1,] <- 15
435
436 # sampling from posterior distributions
437 for (chain in 1:ncol(sample_mean_matrix_100)){
438   for (iter in 2:nrow(sample_mean_matrix_100)){
439     prev_var <- sample_var_matrix_100[iter-1,chain]
440     sample_mean_matrix_100[iter,chain] <- draw.mean(y_100, 0, sqrt(10000),
    prev_var)
441     prev_mean <- sample_mean_matrix_100[iter,chain]
442     sample_var_matrix_100[iter,chain] <- draw.var(y_100, 0.01, 0.01, prev_
    mean)
443   }
444 }
445
446 # discarding first 1000 rows as burn-in/warmup
447 post_burn_mean_100 <- sample_mean_matrix_100[2:nrow(sample_mean_matrix_100),]
448 # hist(post_burn_mean_100)
449
450 # making trace plots
451 plot(seq(1, nrow(post_burn_mean_100)), post_burn_mean_100[,1], xlab = "
    Iteration", ylab = "Sampled Mean", type = 'l', main = 'Trace Plot (n =
    100)', col = 'green')
452 lines(post_burn_mean_100[,2], col = 'blue')
453 lines(post_burn_mean_100[,3], col = 'red')
454
455 # combining chains
456 combined_chains_mean_100 <- c(post_burn_mean_100[,1], post_burn_mean_100[,2],
    post_burn_mean_100[,3])
457 hist(combined_chains_mean_100, main = "Sampled Mean (n = 100)")
458 abline(v = 100, col = 'blue', lwd = 3)
459
460 # repeating for the variance
461 post_burn_var_100 <- sample_var_matrix_100[2:nrow(sample_var_matrix_100),]
462
463 # making trace plots
464 plot(seq(1, nrow(post_burn_var_100)), post_burn_var_100[,1], xlab = "Iteration
    ", ylab = "Sampled Variance", type = 'l', main = 'Trace Plot', col = '
    green')
465 lines(post_burn_var_100[,2], col = 'blue')
466 lines(post_burn_var_100[,3], col = 'red')

```

```

467
468 # combining chains
469 combined_chains_var_100 <- c(post_burn_var_100[,1], post_burn_var_100[,2],
    post_burn_var_100[,3])
470 hist(combined_chains_var_100, main = "Sampled Variance (n = 100)")
471 abline(v = 25, col = 'blue', lwd = 3)
472
473 # mean and SD of combined chains for mu
474 mean_combined_chains_mean_100 <- mean(combined_chains_mean_100)
475 sd_combined_chains_mean_100 <- sd(combined_chains_mean_100)
476 # mean and SD of combined chains for sigma^2
477 mean_combined_chains_var_100 <- mean(combined_chains_var_100)
478 sd_combined_chains_var_100 <- sd(combined_chains_var_100)
479 # printing values
480 mean_combined_chains_mean_100
481 sd_combined_chains_mean_100
482 mean_combined_chains_var_100
483 sd_combined_chains_var_100
484
485
486 ##### n = 1000 #####
487 # creating new simulated data
488 y_1000 <- rnorm(1000, 100, sqrt(25))
489
490 # creating matrices to store samples from posterior distributions of mean and
    variance
491 sample_mean_matrix_1000 <- matrix(nrow = 10000, ncol = 3)
492 sample_var_matrix_1000 <- matrix(nrow = 10000, ncol = 3)
493
494 # initializing values
495 sample_mean_matrix_1000[1,] <- 20
496 sample_var_matrix_1000[1,] <- 15
497
498 # sampling from posterior distributions
499 for (chain in 1:ncol(sample_mean_matrix_1000)){
500   for (iter in 2:nrow(sample_mean_matrix_1000)){
501     prev_var <- sample_var_matrix_1000[iter-1,chain]
502     sample_mean_matrix_1000[iter,chain] <- draw.mean(y_1000, 0, sqrt(10000),
        prev_var)
503     prev_mean <- sample_mean_matrix_1000[iter,chain]
504     sample_var_matrix_1000[iter, chain] <- draw.var(y_1000, 0.01, 0.01, prev_
        mean)
505   }
506 }
507
508 # discarding first 1000 rows as burn-in/warmup
509 post_burn_mean_1000 <- sample_mean_matrix_1000[2:nrow(sample_mean_matrix_1000)
    ,]
510 # hist(post_burn_mean_100)
511
512 # making trace plots
513 plot(seq(1, nrow(post_burn_mean_1000)), post_burn_mean_1000[,1], xlab = "
    Iteration", ylab = "Sampled Mean", type = 'l', main = 'Trace Plot', col =
    'green')

```

```

514 lines(post_burn_mean_1000[,2], col = 'blue')
515 lines(post_burn_mean_1000[,3], col = 'red')
516
517 # combining chains
518 combined_chains_mean_1000 <- c(post_burn_mean_1000[,1], post_burn_mean_
    1000[,2], post_burn_mean_1000[,3])
519 hist(combined_chains_mean_1000)
520 abline(v = 100, col = 'blue', lwd = 3)
521
522 # repeating for the variance
523 post_burn_var_1000 <- sample_var_matrix_1000[2:nrow(sample_var_matrix_1000),]
524
525 # making trace plots
526 plot(seq(1, nrow(post_burn_var_1000)), post_burn_var_1000[,1], xlab = "
    Iteration", ylab = "Sampled Variance", type = 'l', main = 'Trace Plot',
    col = 'green')
527 lines(post_burn_var_1000[,2], col = 'blue')
528 lines(post_burn_var_1000[,3], col = 'red')
529
530 # combining chains
531 combined_chains_var_1000 <- c(post_burn_var_1000[,1], post_burn_var_1000[,2],
    post_burn_var_1000[,3])
532 hist(combined_chains_var_1000)
533 abline(v = 25, col = 'blue', lwd = 3)
534
535 # mean and SD of combined chains for mu
536 mean_combined_chains_mean_1000 <- mean(combined_chains_mean_1000)
537 sd_combined_chains_mean_1000 <- sd(combined_chains_mean_1000)
538 # mean and SD of combined chains for sigma^2
539 mean_combined_chains_var_1000 <- mean(combined_chains_var_1000)
540 sd_combined_chains_var_1000 <- sd(combined_chains_var_1000)
541 # printing values
542 mean_combined_chains_mean_1000
543 sd_combined_chains_mean_1000
544 mean_combined_chains_var_1000
545 sd_combined_chains_var_1000
546
547
548 ##### n = 100000 #####
549 # creating new simulated data
550 y_100000 <- rnorm(100000, 100, sqrt(25))
551
552 # creating matrices to store samples from posterior distributions of mean and
    variance
553 sample_mean_matrix_100000 <- matrix(nrow = 10000, ncol = 3)
554 sample_var_matrix_100000 <- matrix(nrow = 10000, ncol = 3)
555
556 # initializing values
557 sample_mean_matrix_100000[1,] <- 20
558 sample_var_matrix_100000[1,] <- 15
559
560 # sampling from posterior distributions
561 for (chain in 1:ncol(sample_mean_matrix_100000)){
562   for (iter in 2:nrow(sample_mean_matrix_100000)){

```

```

563     prev_var <- sample_var_matrix_100000[iter-1,chain]
564     sample_mean_matrix_100000[iter,chain] <- draw.mean(y_100000, 0, sqrt
(10000), prev_var)
565     prev_mean <- sample_mean_matrix_100000[iter,chain]
566     sample_var_matrix_100000[iter, chain] <- draw.var(y_100000, 0.01, 0.01,
prev_mean)
567   }
568 }
569
570 # discarding first 1000 rows as burn-in/warmup
571 post_burn_mean_100000 <- sample_mean_matrix_100000[2:nrow(sample_mean_matrix_
100000),]
572 # hist(post_burn_mean_100)
573
574 # making trace plots
575 plot(seq(1, nrow(post_burn_mean_100000)), post_burn_mean_100000[,1], xlab = "
Iteration", ylab = "Sampled Mean", type = 'l', main = 'Trace Plot of
Combined Chains of Sampled Means (n = 100000)', col = 'green')
576 lines(post_burn_mean_100000[,2], col = 'blue')
577 lines(post_burn_mean_100000[,3], col = 'red')
578
579 # combining chains
580 combined_chains_mean_100000 <- c(post_burn_mean_100000[,1], post_burn_mean_
100000[,2], post_burn_mean_100000[,3])
581 hist(combined_chains_mean_100000, main = "Sampled Means (n = 100000)")
582 abline(v = 100, col = 'blue', lwd = 3)
583
584 # repeating for the variance
585 post_burn_var_100000 <- sample_var_matrix_100000[2:nrow(sample_var_matrix_
100000),]
586
587 # making trace plots
588 plot(seq(1, nrow(post_burn_var_100000)), post_burn_var_100000[,1], xlab = "
Iteration", ylab = "Sampled Variance", type = 'l', main = 'Trace Plot of
Combined Chains of Sampled Variance (n = 100000)', col = 'green')
589 lines(post_burn_var_100000[,2], col = 'blue')
590 lines(post_burn_var_100000[,3], col = 'red')
591
592 # combining chains
593 combined_chains_var_100000 <- c(post_burn_var_100000[,1], post_burn_var_
100000[,2], post_burn_var_100000[,3])
594 hist(combined_chains_var_100000, main = "Sampled Variance (n = 100000)")
595 abline(v = 25, col = 'blue', lwd = 3)
596
597 # mean and SD of combined chains for mu
598 mean_combined_chains_mean_100000 <- mean(combined_chains_mean_100000)
599 sd_combined_chains_mean_100000 <- sd(combined_chains_mean_100000)
600 # mean and SD of combined chains for sigma^2
601 mean_combined_chains_var_100000 <- mean(combined_chains_var_100000)
602 sd_combined_chains_var_100000 <- sd(combined_chains_var_100000)
603 # printing values
604 mean_combined_chains_mean_100000
605 sd_combined_chains_mean_100000
606 mean_combined_chains_var_100000

```

```

607 sd_combined_chains_var_100000
608 ' '
609
610 ### Question 15
611 '{r}'
612 ## setting seed
613 set.seed(1991)
614
615 ## new draw mean function
616 draw.mean_new <- function(data, mu_0, sigma_0, variance){
617   mu_1 <- ((mu_0 / sigma_0^2) + (sum(data) / variance)) / ((1/sigma_0^2) + (
618     length(data)/variance))
619   sigma_sqd_1 <- 1 / ((1 / sigma_0^2) + length(data) / variance)
620   random_meandraw <- rnorm(1, mu_1, sigma_sqd_1)
621   return(random_meandraw)
622 }
623
624 ## new draw var function
625 draw.var_new <- function(data, alpha_0, beta_0, mu){
626   alpha_1 <- alpha_0 + (length(data) / 2)
627   beta_1 <- beta_0 + (sum((data - mu)^2) / 2)
628   random_vardraw <- rinvgamma(1, shape = alpha_1, rate = beta_1)
629   return(random_vardraw)
630 }
631
632 ## creating matrices to store samples from posterior distributions of mean and
633   variance
634 sample_mean_matrix <- matrix(nrow = 10000, ncol = 3)
635 sample_var_matrix <- matrix(nrow = 10000, ncol = 3)
636
637 ## initializing first mean and variance values
638 sample_mean_matrix[1,] <- 20
639 sample_var_matrix[1,] <- 15
640
641 ## drawing variance first, then mean
642 for (chain in 1:ncol(sample_mean_matrix)){
643   for (iter in 2:nrow(sample_mean_matrix)){
644     prev_mean <- sample_mean_matrix[iter-1,chain]
645     sample_var_matrix[iter, chain] <- draw.var_new(y, 0.01, 0.01, prev_mean)
646     prev_var <- sample_var_matrix[iter,chain]
647     sample_mean_matrix[iter, chain] <- draw.mean_new(y, 0, sqrt(10000), prev_
648       var)
649   }
650 }
651
652 # discarding first 1000 rows as burn-in/warmup
653 post_burn_mean <- sample_mean_matrix[1001:nrow(sample_mean_matrix),]
654 # hist(post_burn_mean)
655
656 # making trace plots
657 plot(seq(1, nrow(post_burn_mean)), post_burn_mean[,1], xlab = "Iteration",
658   ylab = "Sampled Mean", type = 'l', main = 'Trace Plot', col = 'green')
659 lines(post_burn_mean[,2], col = 'blue')
660 lines(post_burn_mean[,3], col = 'red')

```

```

657
658 # combining chains
659 combined_chains_mean <- c(post_burn_mean[,1], post_burn_mean[,2], post_burn_
    mean[,3])
660 hist(combined_chains_mean)
661 abline(v = 100, col = 'blue', lwd = 3)
662
663 # repeating burn-in removal, plots for the variance
664 post_burn_var <- sample_var_matrix[1001:nrow(sample_var_matrix),]
665
666 # making trace plots
667 plot(seq(1, nrow(post_burn_var)), post_burn_var[,1], xlab = "Iteration", ylab
    = "Sampled Variance", type = 'l', main = 'Trace Plot', col = 'green')
668 lines(post_burn_var[,2], col = 'blue')
669 lines(post_burn_var[,3], col = 'red')
670
671 # combining chains
672 combined_chains_var <- c(post_burn_var[,1], post_burn_var[,2], post_burn_var
    [,3])
673 hist(combined_chains_var)
674 abline(v = 25, col = 'blue', lwd = 3)
675
676
677 # mean and SD of combined chains for mu
678 mean_combined_chains_mean <- mean(combined_chains_mean)
679 sd_combined_chains_mean <- sd(combined_chains_mean)
680 # mean and SD of combined chains for sigma^2
681 mean_combined_chains_var <- mean(combined_chains_var)
682 sd_combined_chains_var <- sd(combined_chains_var)
683 # printing values
684 mean_combined_chains_mean
685 sd_combined_chains_mean
686 mean_combined_chains_var
687 sd_combined_chains_var
688 ' '

```