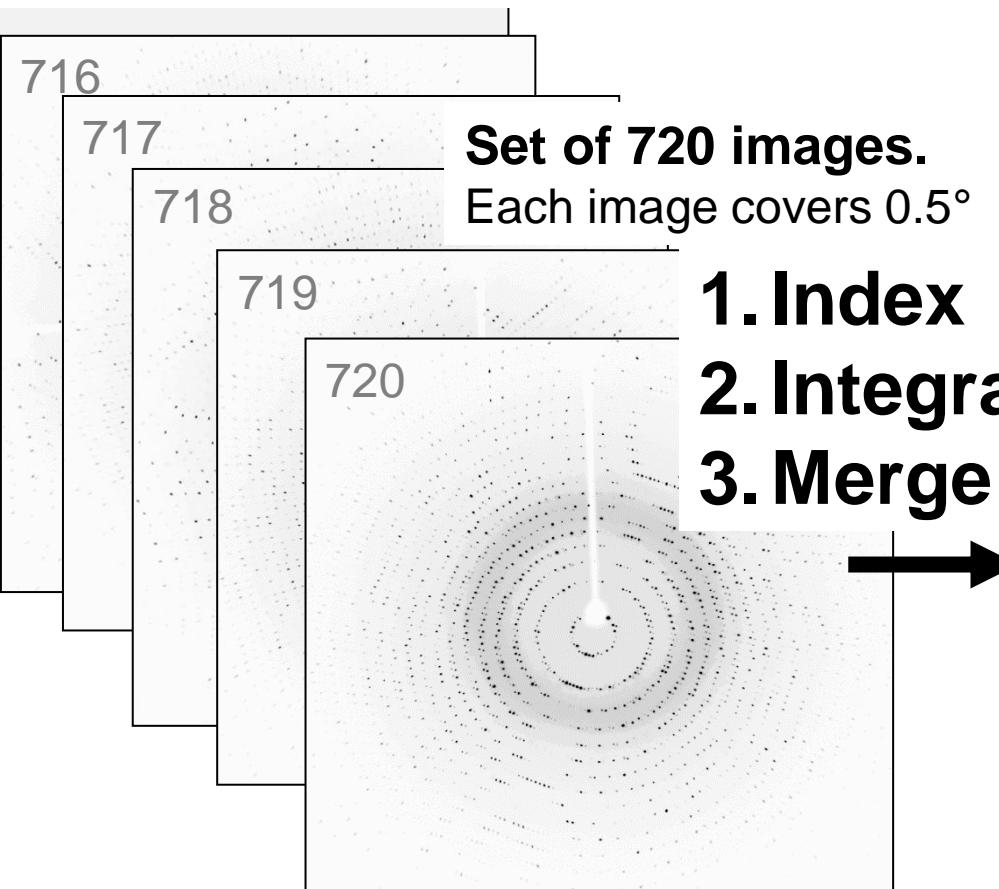


# Data Reduction Goals

From a series of diffraction images, obtain a file containing the intensity ( $I_{hkl}$ ) and standard deviation ( $\sigma(I_{hkl})$ ) for each reflection,  $hkl$ .



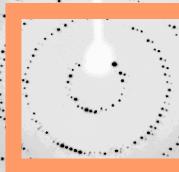
1. Index
2. Integrate
3. Merge

## Final intensities

H	K	L	I	$\sigma$
0	0	4	3295.4	174.0
0	0	8	482.1	28.7
0	0	12	9691.0	500.7
0	0	16	1743.9	67.4
0	0	20	5856.0	221.0
0	0	24	14066.5	436.2
0	0	28	9936.3	311.7
0	0	36	8409.8	273.4
0	0	40	790.5	32.8
0	0	44	103.4	18.4
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
37	7	0	28.5	16.2
37	7	1	110.1	10.9
37	7	2	337.4	13.3
37	7	3	98.5	10.6
37	7	4	25.9	10.7

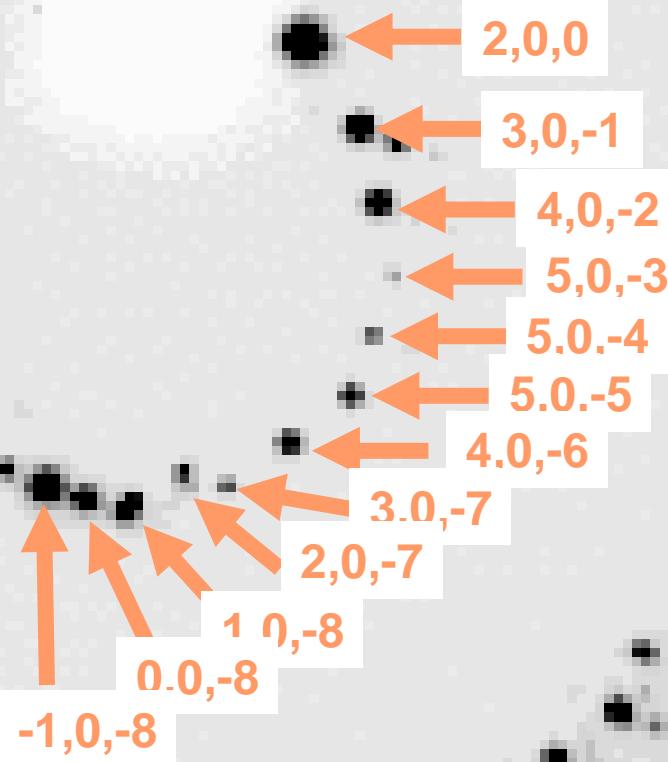
**8 x 10<sup>5</sup> bytes**

# Indexing

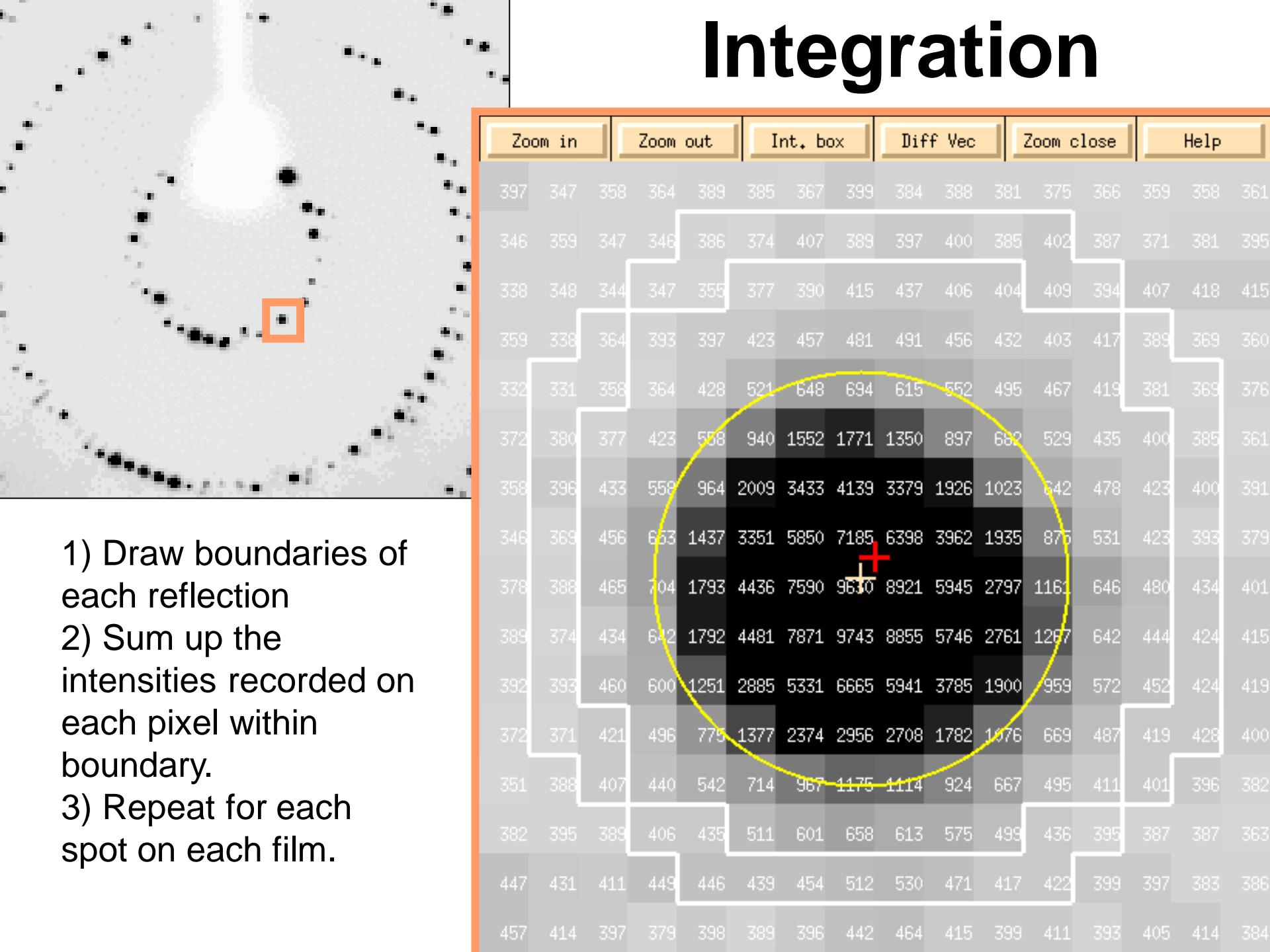


Assign an  $h,k,l$  coordinate to each reflection of the first image.

Indices  $h,k,l$  are coordinates of the reflections, analogous to how atom positions are described by coordinates  $x,y,z$ .

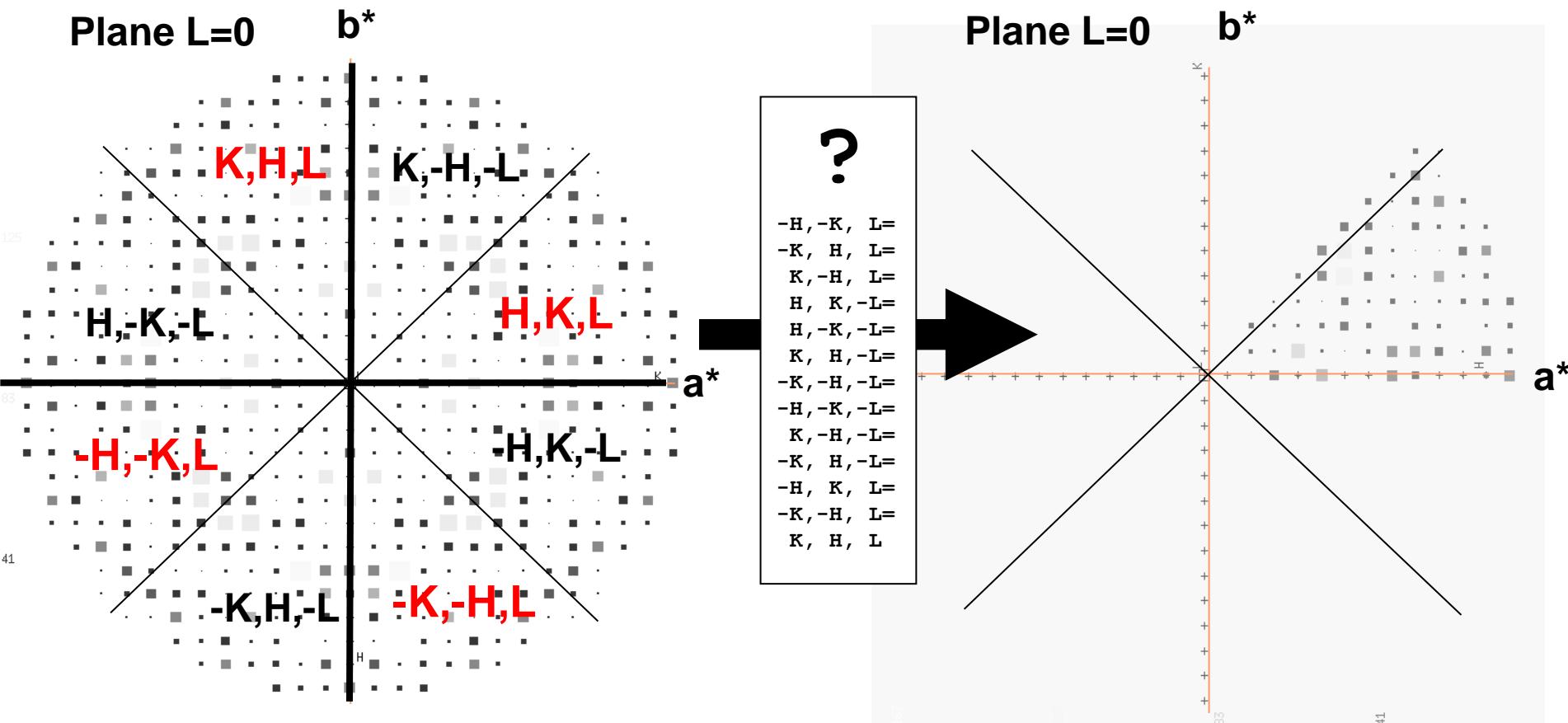


# Integration



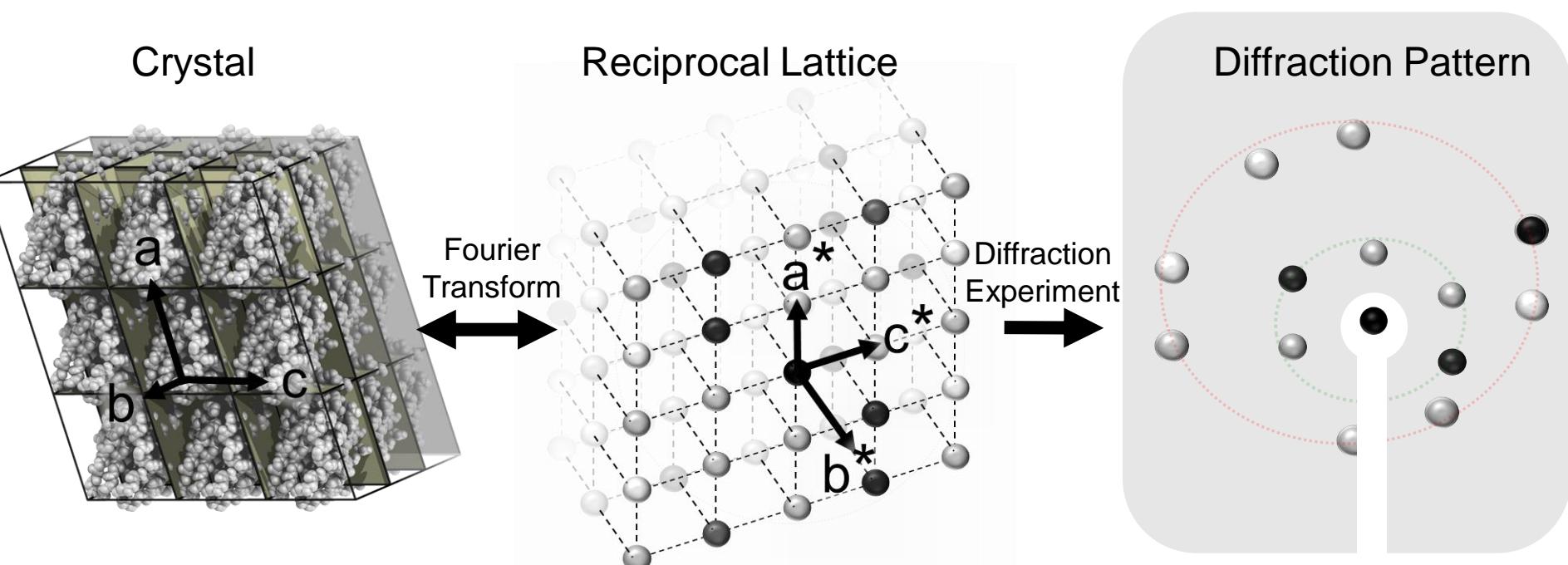
- 1) Draw boundaries of each reflection
- 2) Sum up the intensities recorded on each pixel within boundary.
- 3) Repeat for each spot on each film.

# Merging



Make a hypothesis about symmetry laws obeyed in data.  
 Average together  $I_{hkl}$  values for  $h, k, l$  indices related by putative symmetry.  
 Verify symmetry hypothesis by calculating  $R_{\text{merge}}$

# Understand the relationship between the crystal, the reciprocal lattice and the diffraction pattern



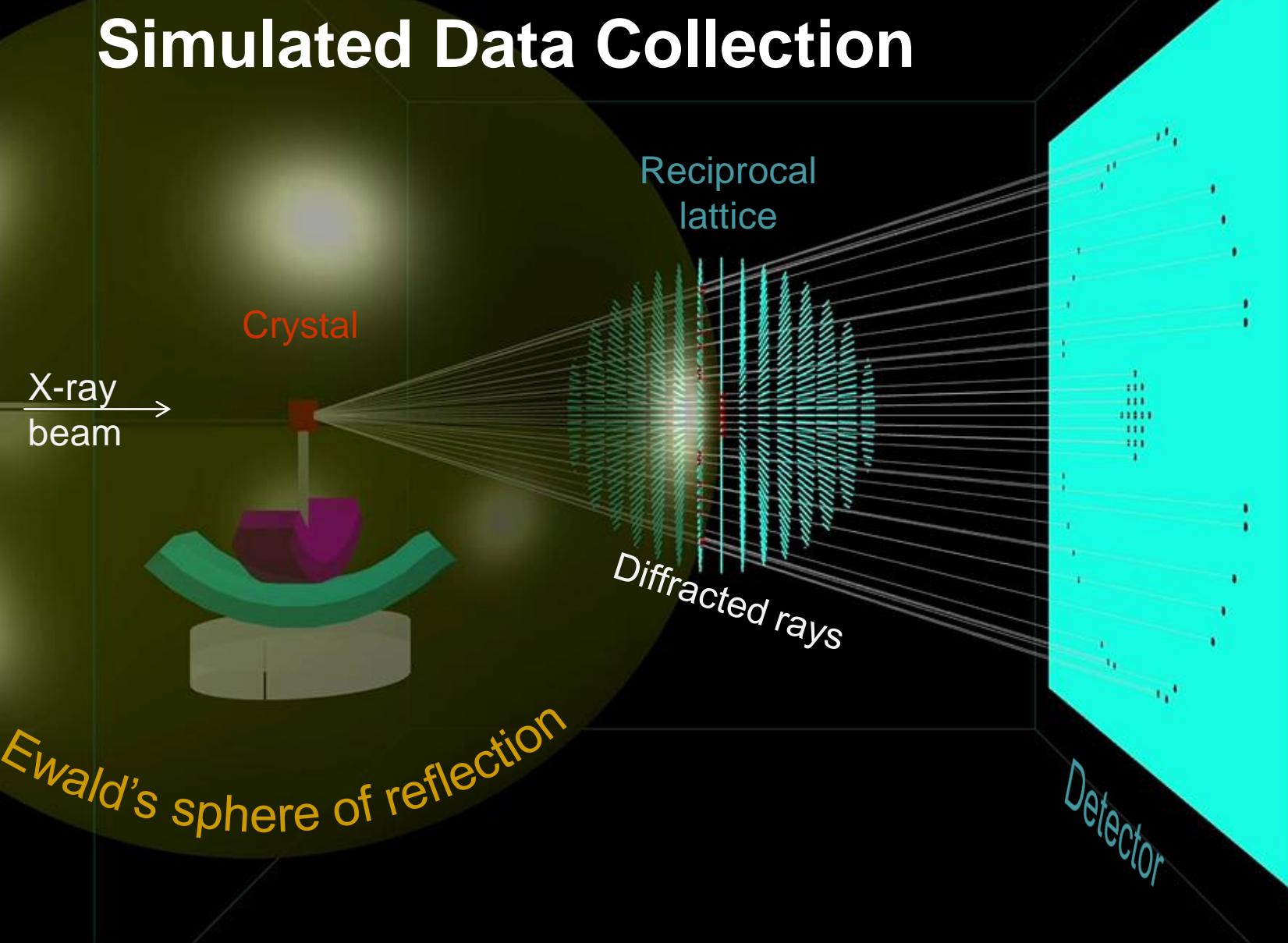
Unit cell lengths  $a, b, c$   
Atom coordinates  $x, y, z$

Reciprocal cell lengths  $a^*, b^*, c^*$   
Reflection coordinates  $h, k, l$

A 2D projection of  
a 3D reciprocal  
lattice

Goal of data reduction is to measure  
intensity at each lattice point,  $h, k, l$ .

# Simulated Data Collection



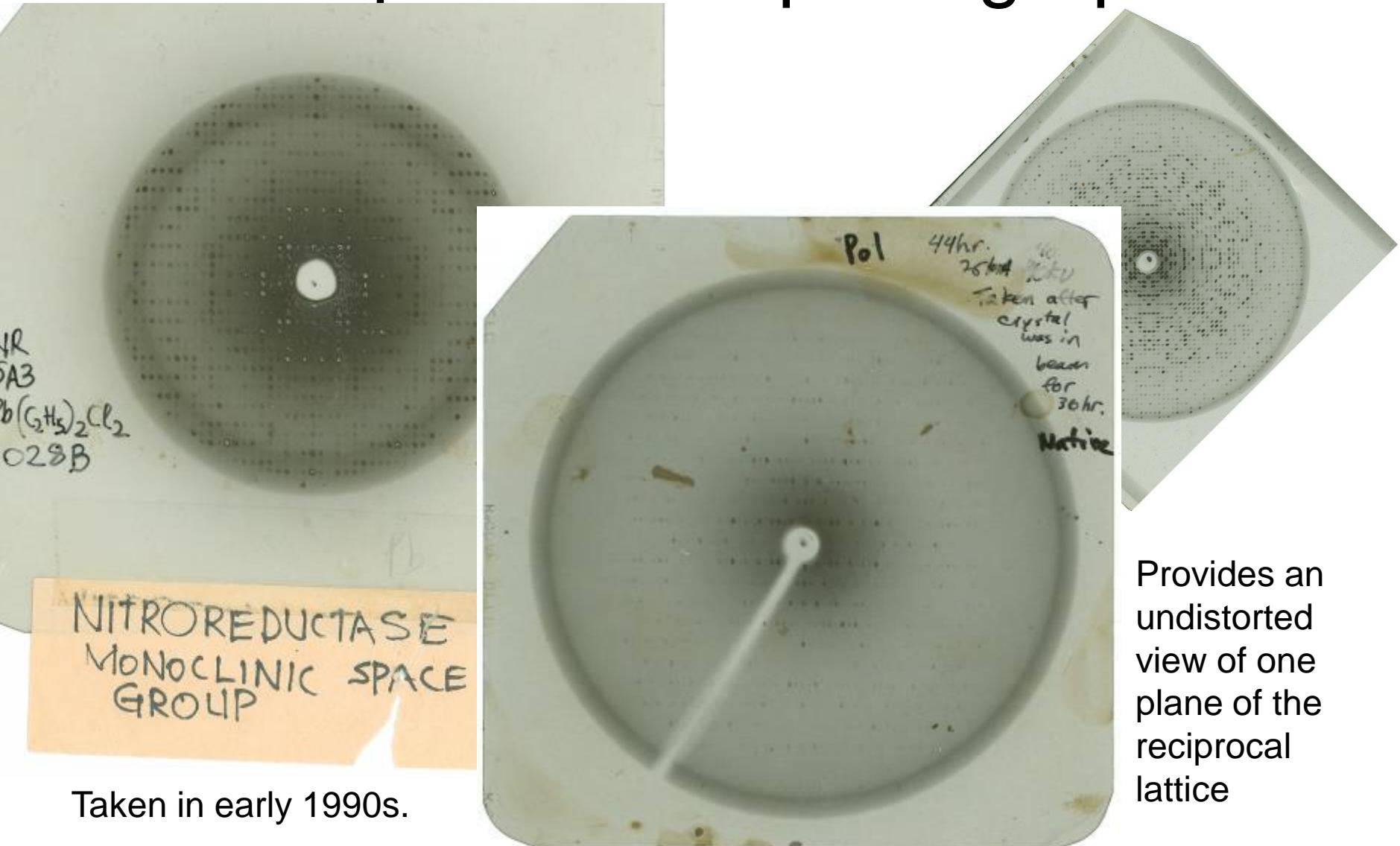
Ewald's sphere of reflection

Shutter Open

0-5 degrees

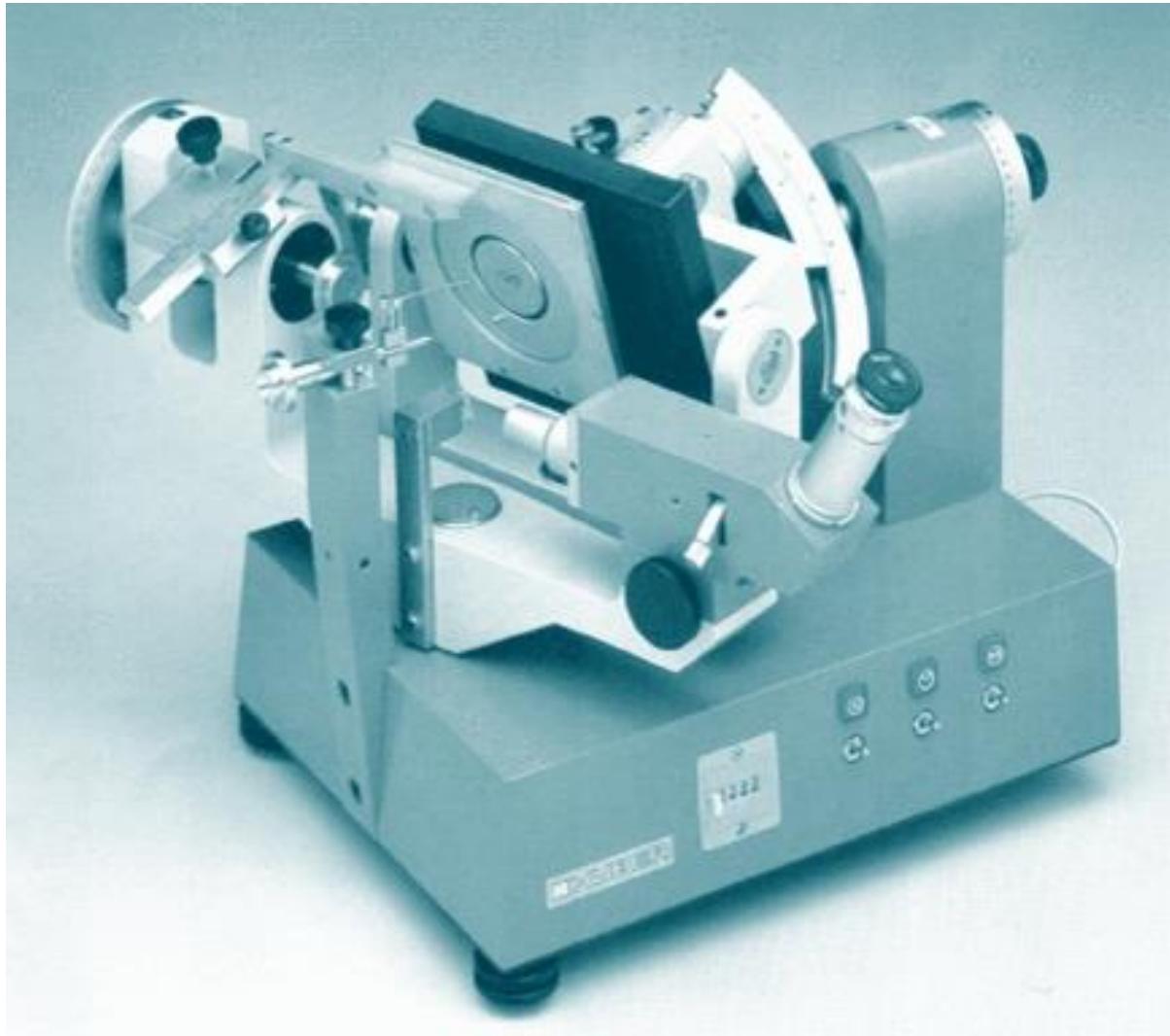
Image 001

# Indexing is conceptually simple with precession photographs



Provides an undistorted view of one plane of the reciprocal lattice

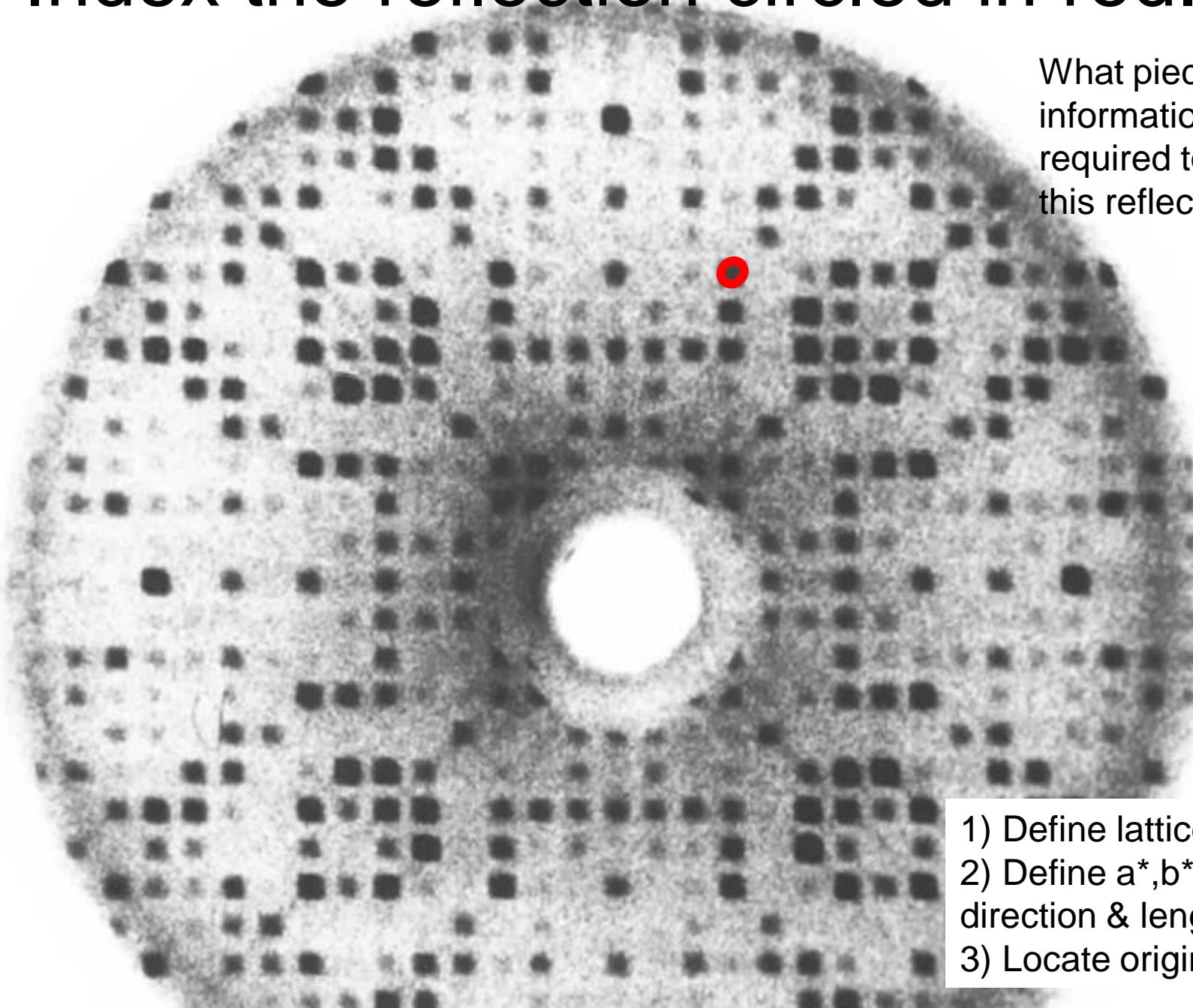
# Buerger Precession Camera



# How to Index by Inspection

- Recall that spots are arrayed on a reciprocal lattice.
- Count how many steps the given reflection is displaced from the origin.
  - $h$ =Number of steps of  $a^*$ .
  - $k$ =Number of steps of  $b^*$ .
  - $l$ =Number of steps of  $c^*$ .

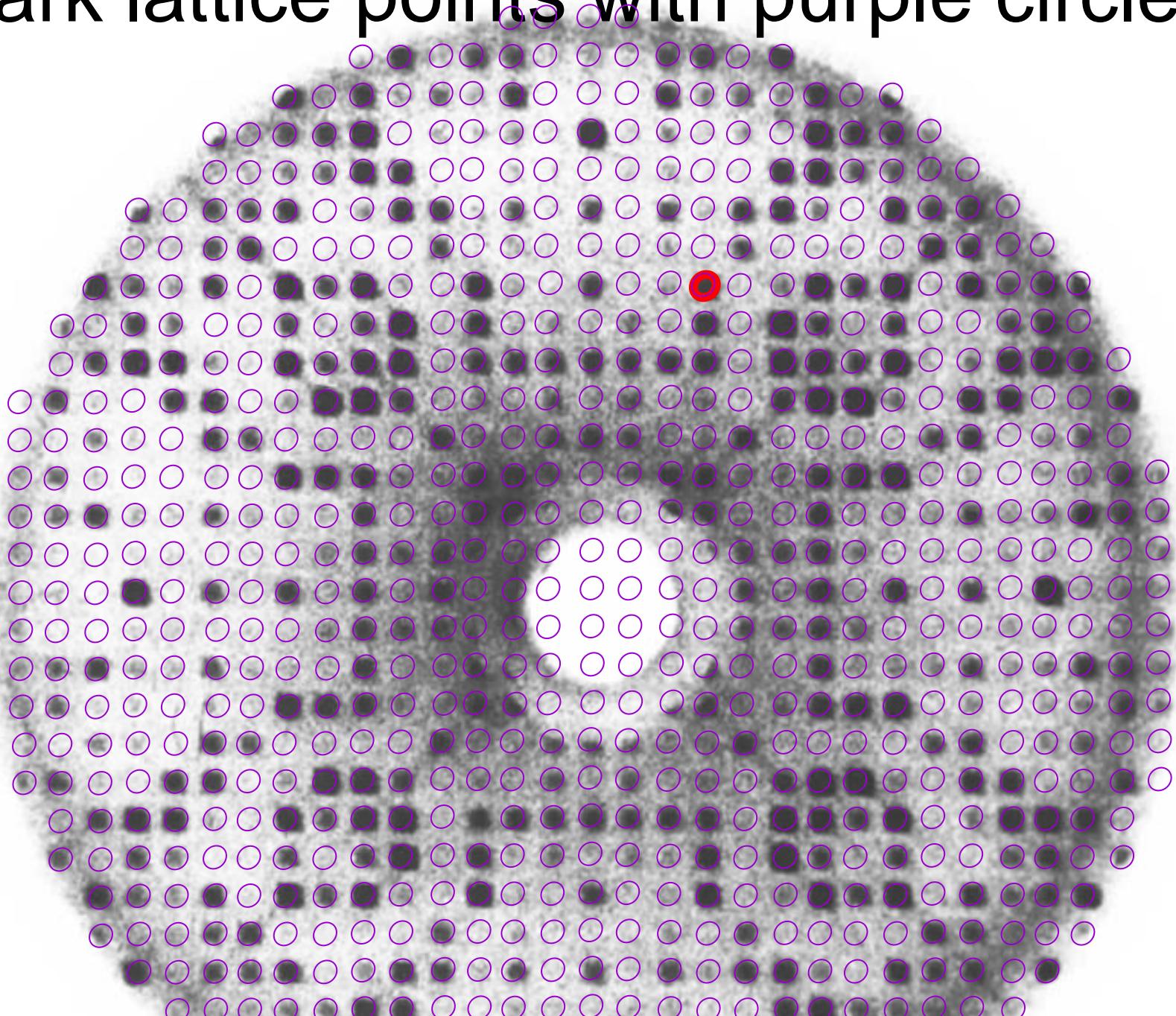
# Index the reflection circled in red.



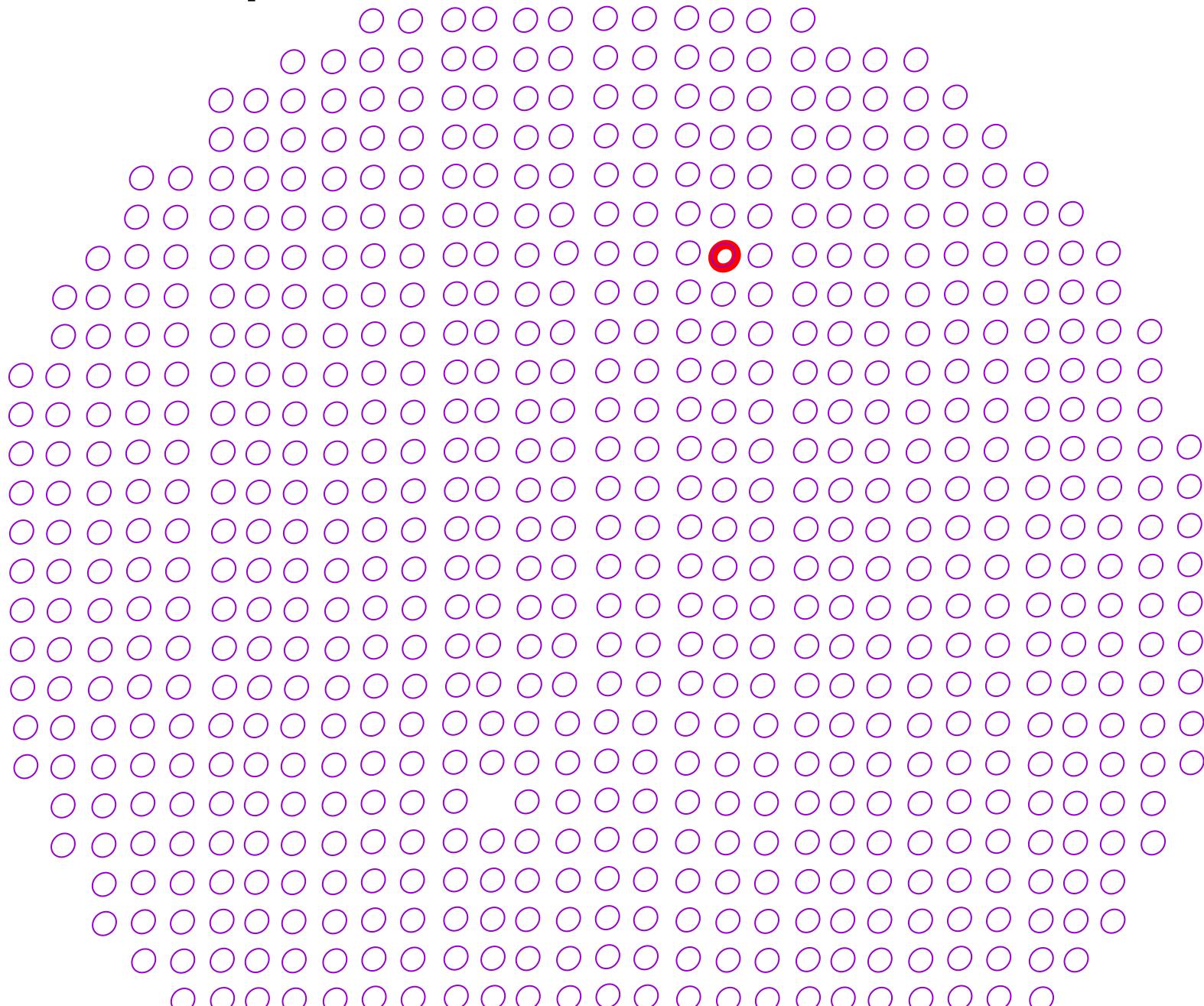
What pieces of information are required to index this reflection?

- 1) Define lattice points
- 2) Define  $a^*$ ,  $b^*$ ,  $c^*$  direction & length
- 3) Locate origin.

# Mark lattice points with purple circles.



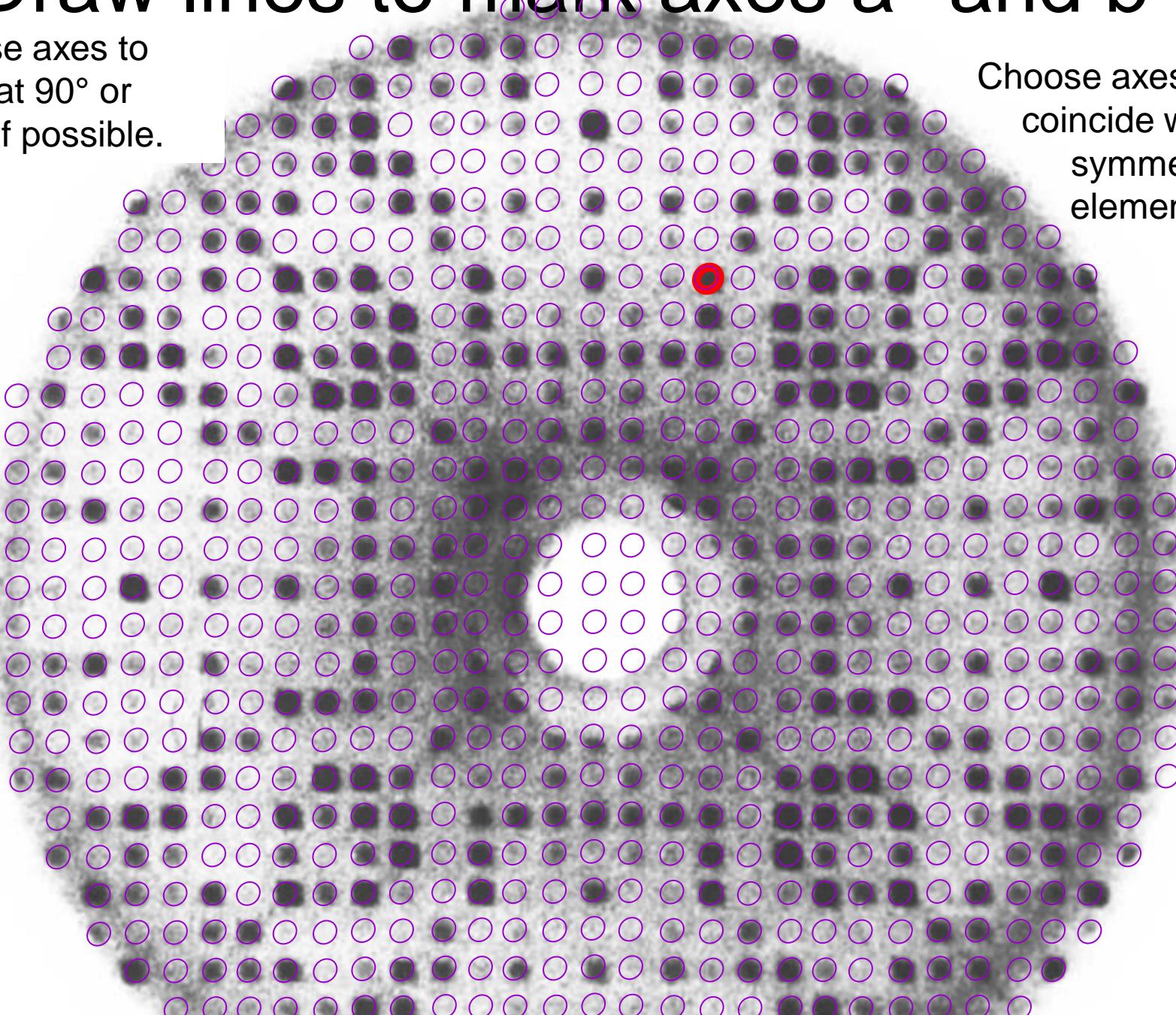
# Reciprocal Lattice is defined.



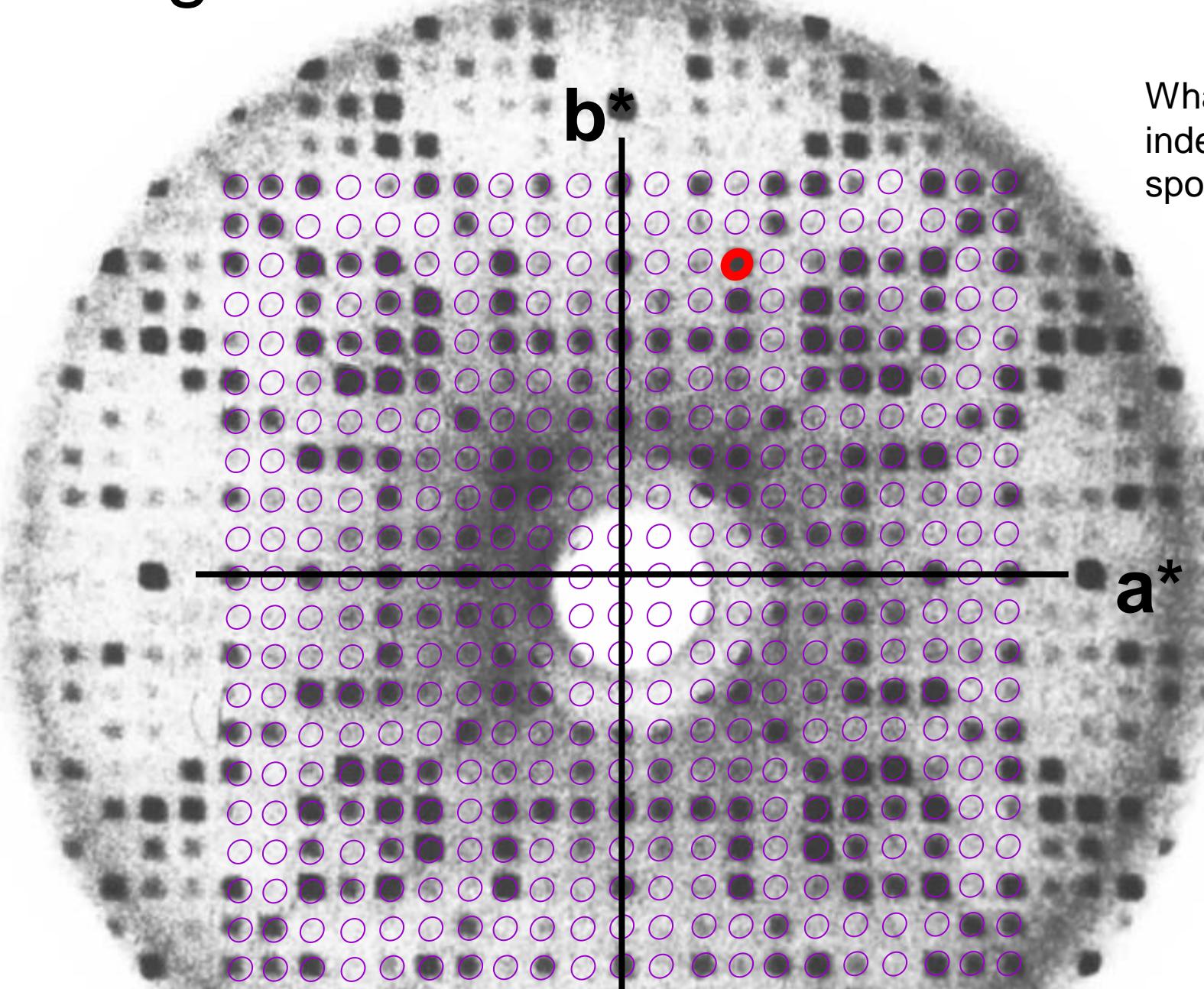
# Draw lines to mark axes $a^*$ and $b^*$ .

Choose axes to cross at  $90^\circ$  or  $120^\circ$ , if possible.

Choose axes to coincide with symmetry elements.



# Origin is defined. Label $a^*$ & $b^*$ .



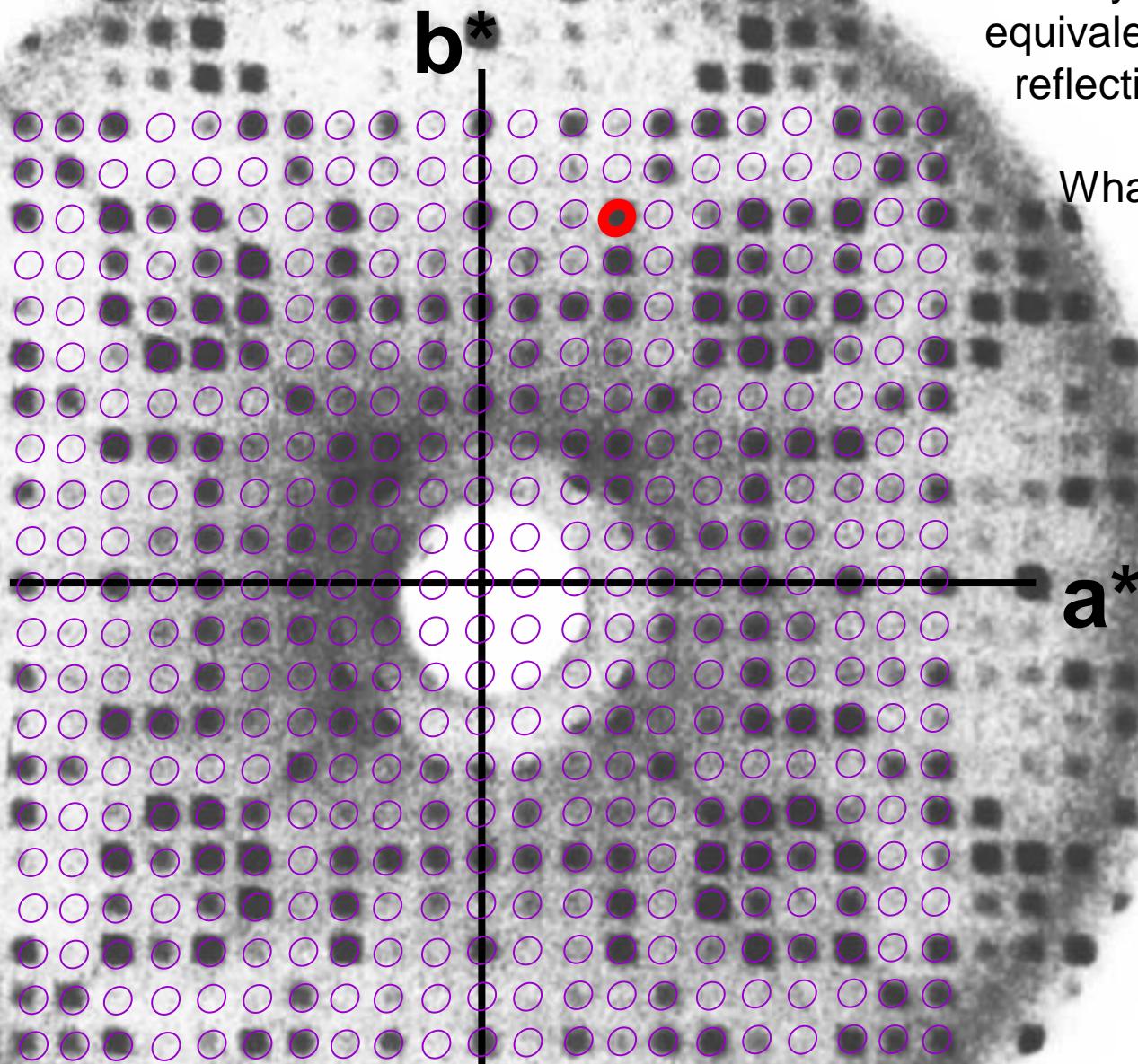
What is the index of the spot?

# Reflection $h,k,l$ is 3,8,0

3<sup>rd</sup> column on  
the right side of  
the origin.

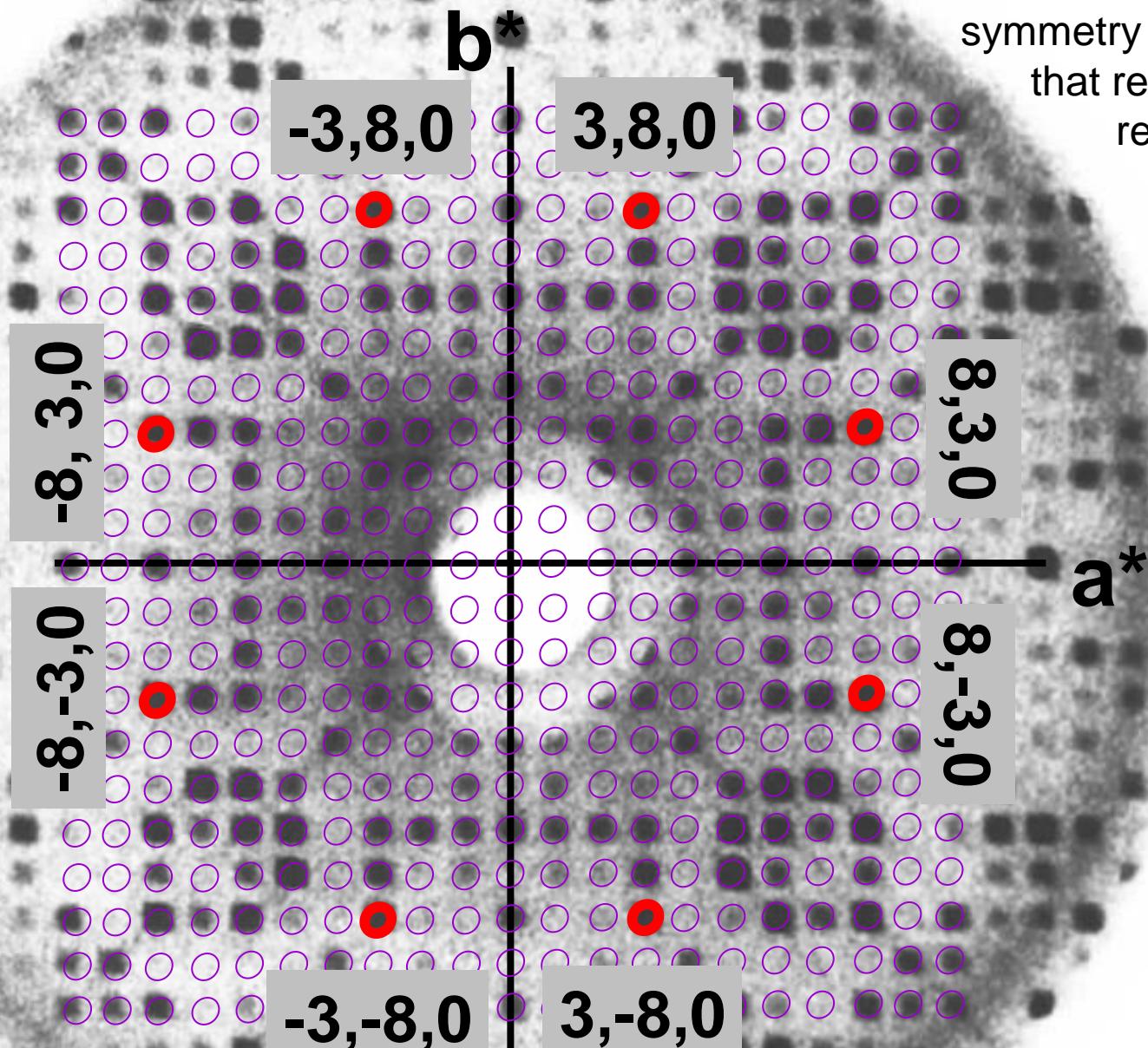
8<sup>th</sup> row above  
the origin

How many symmetry  
equivalents of this  
reflection do you  
see?  
What are their  
indices?

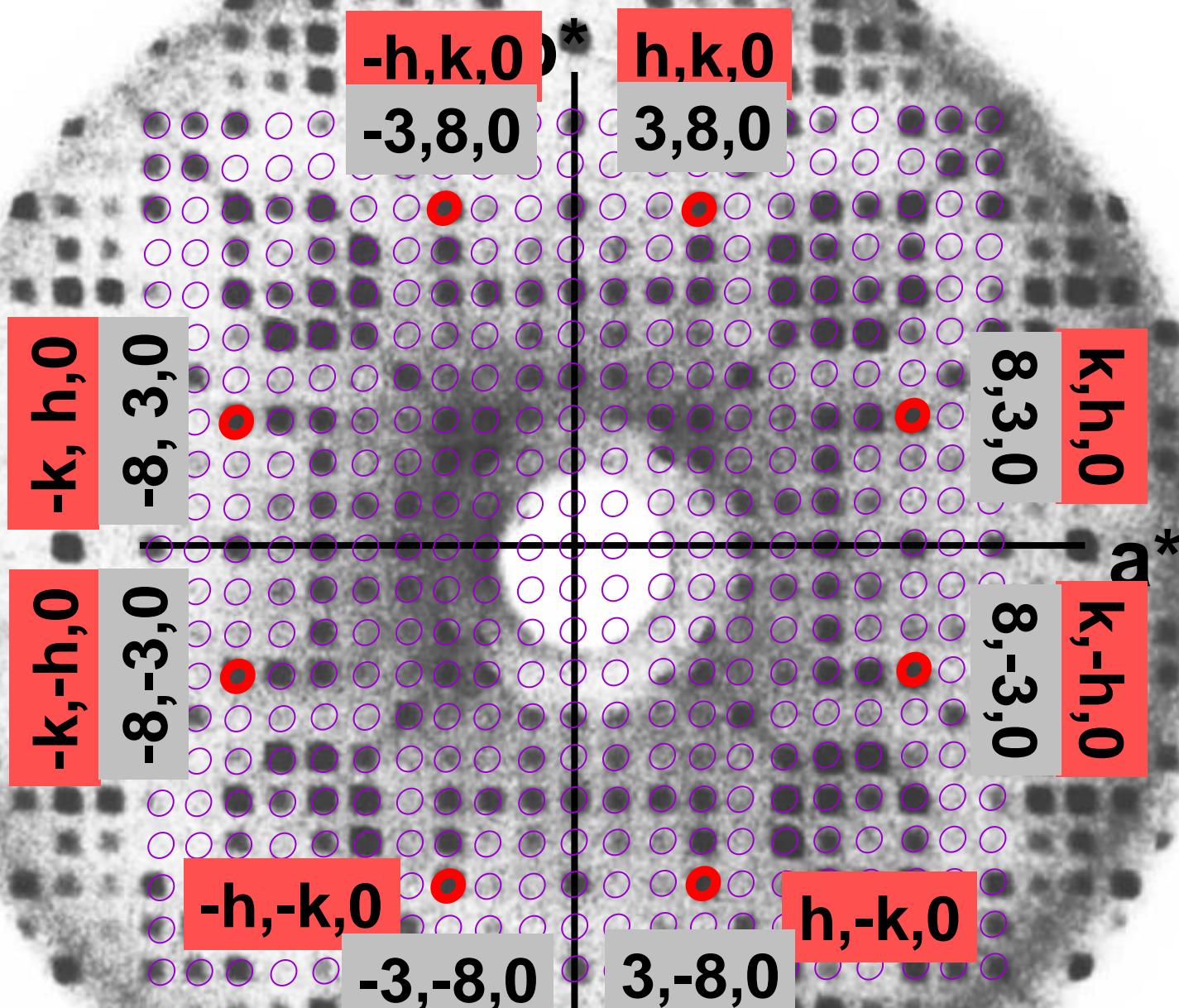


# 8 symmetry equivalents

What are the symmetry operators that relate these reflections?

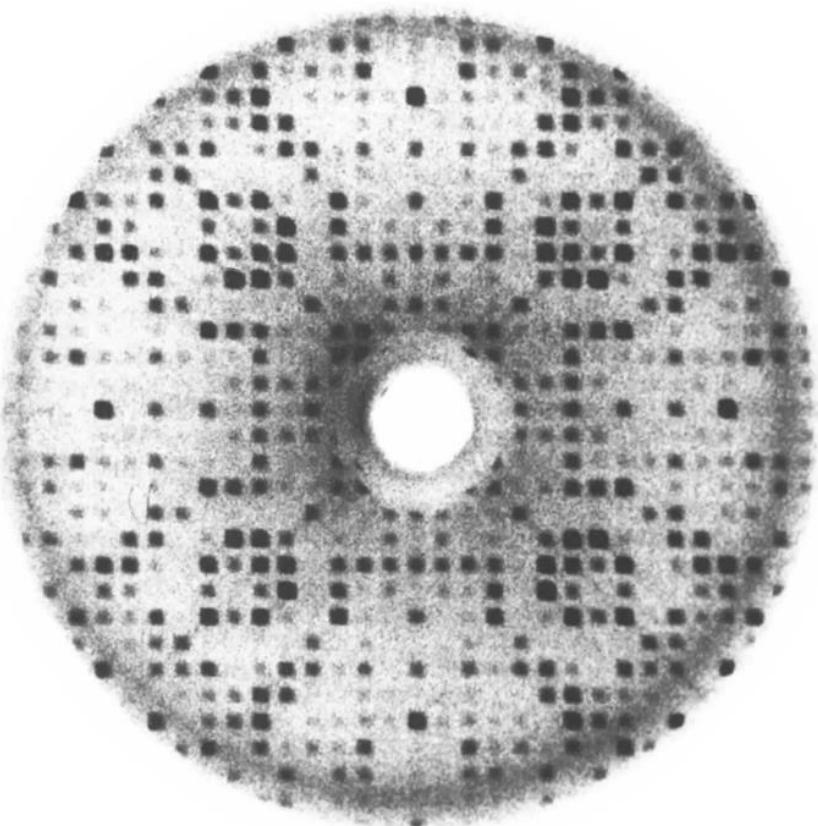


# 8 symmetry equivalents

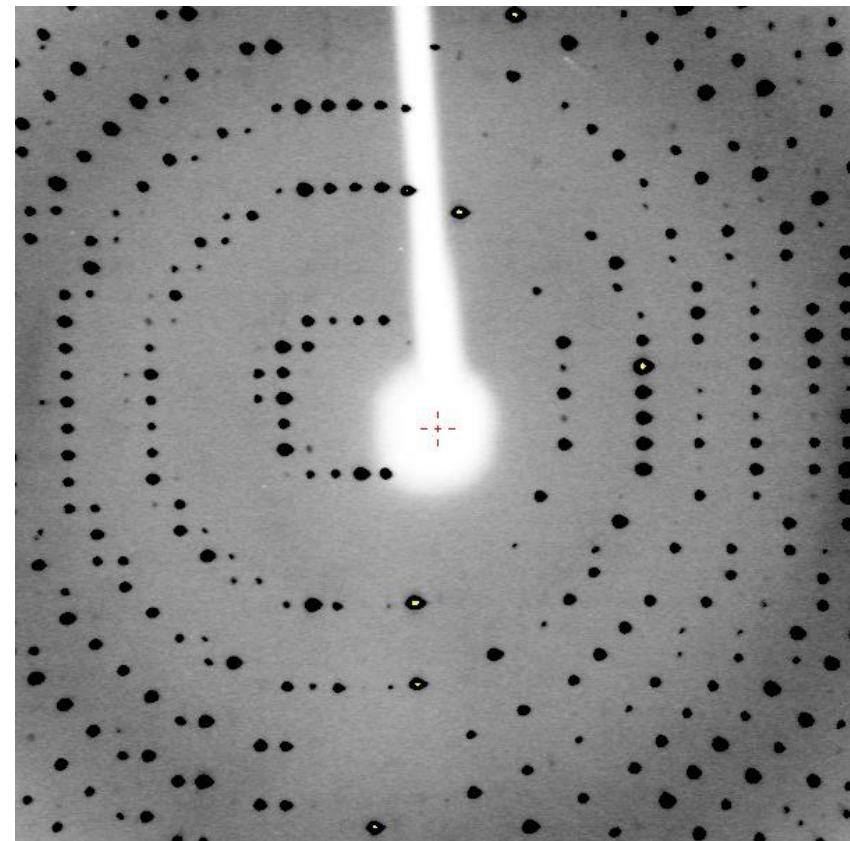


H, K, 0  
-H, K, 0  
H, -K, 0  
-H, -K, 0  
K, H, 0  
-K, H, 0  
K, -H, 0  
-K, -H, 0

Next, index spots on an oscillation image.



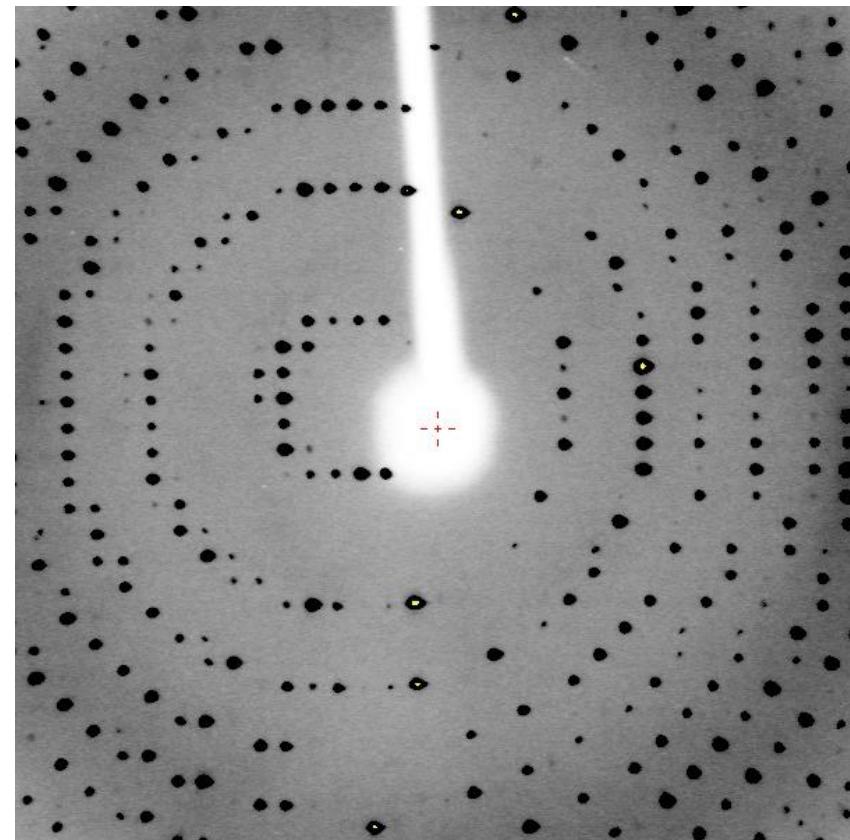
This is easy!



But, what about this?

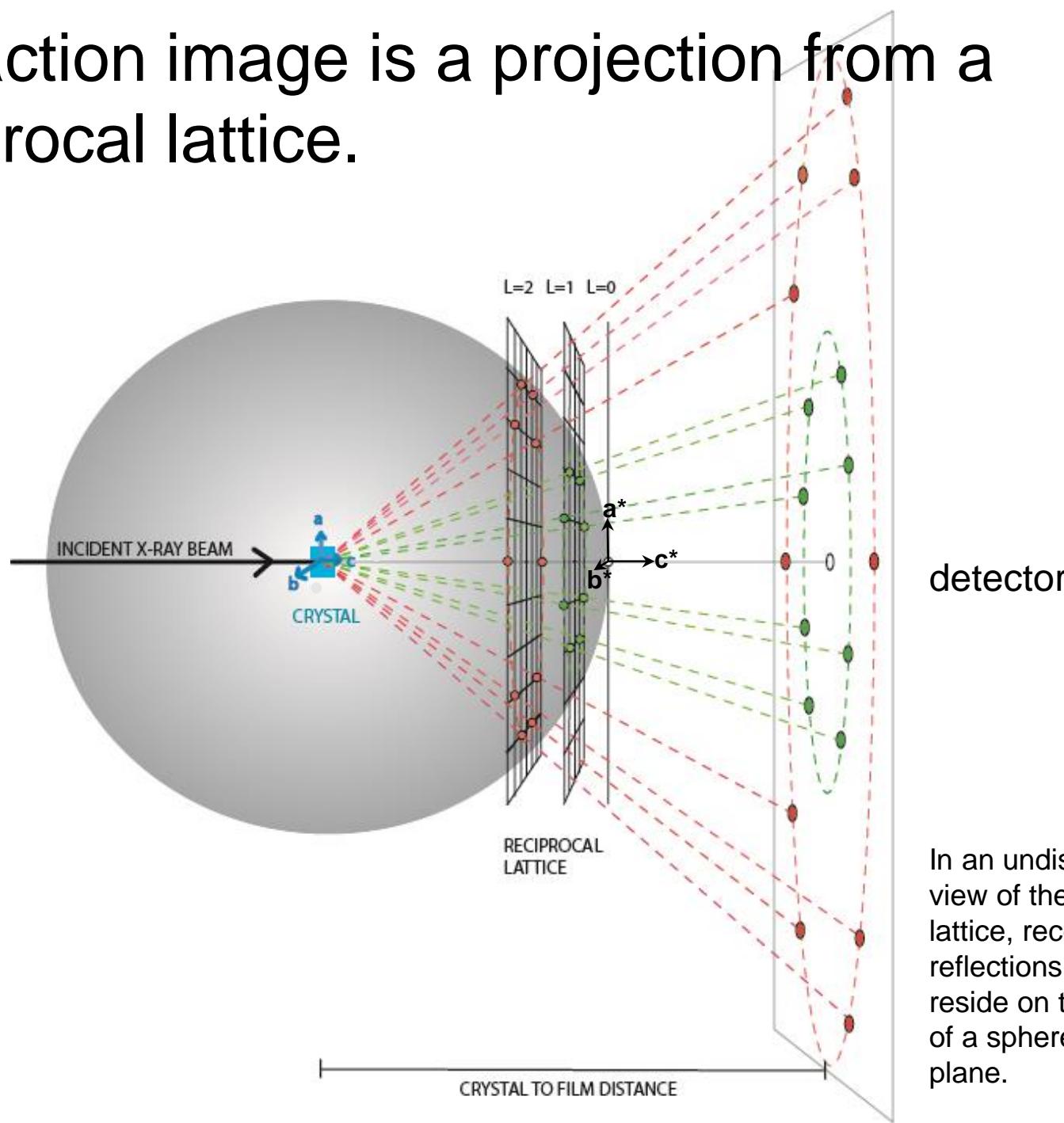
# Challenges in indexing an oscillation image

1. Orientation of reciprocal cell vectors  $a^*$ ,  $b^*$ ,  $c^*$  is random --- determined by the orientation of the crystal in the loop.
2. Can't see symmetry in the pattern..
3. Multiple planes of the reciprocal lattice are in view, not just a single plane.
4. The oscillation image is a 2D projections of a 3D lattice—a dimension of depth is lost.



Oscillation image.

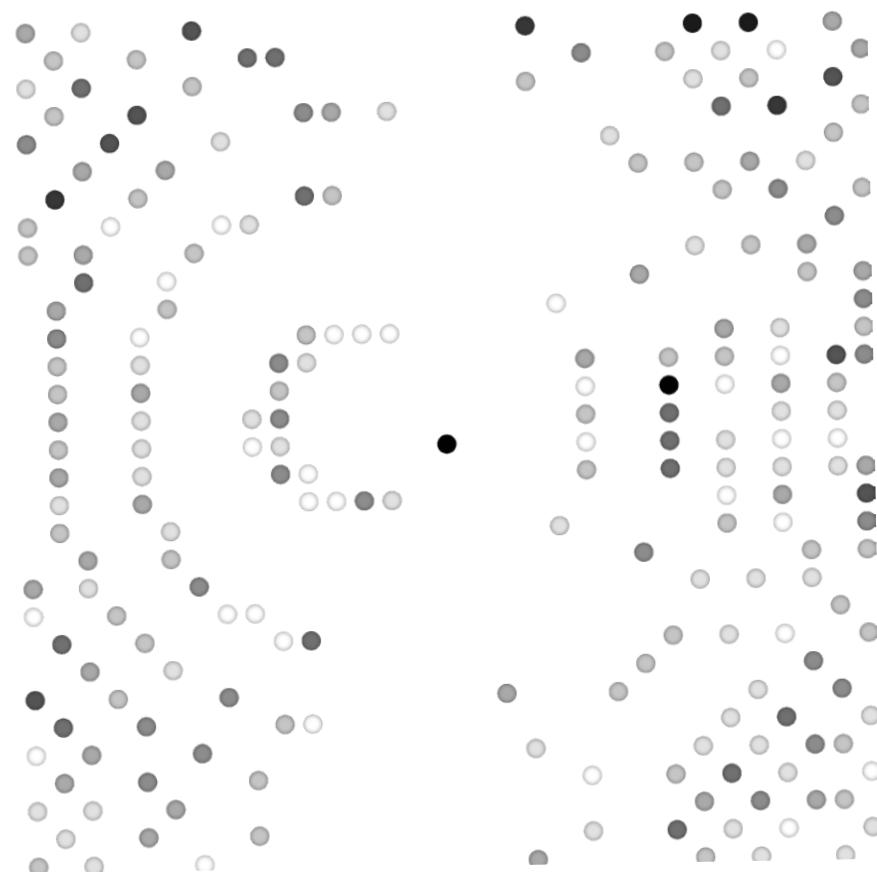
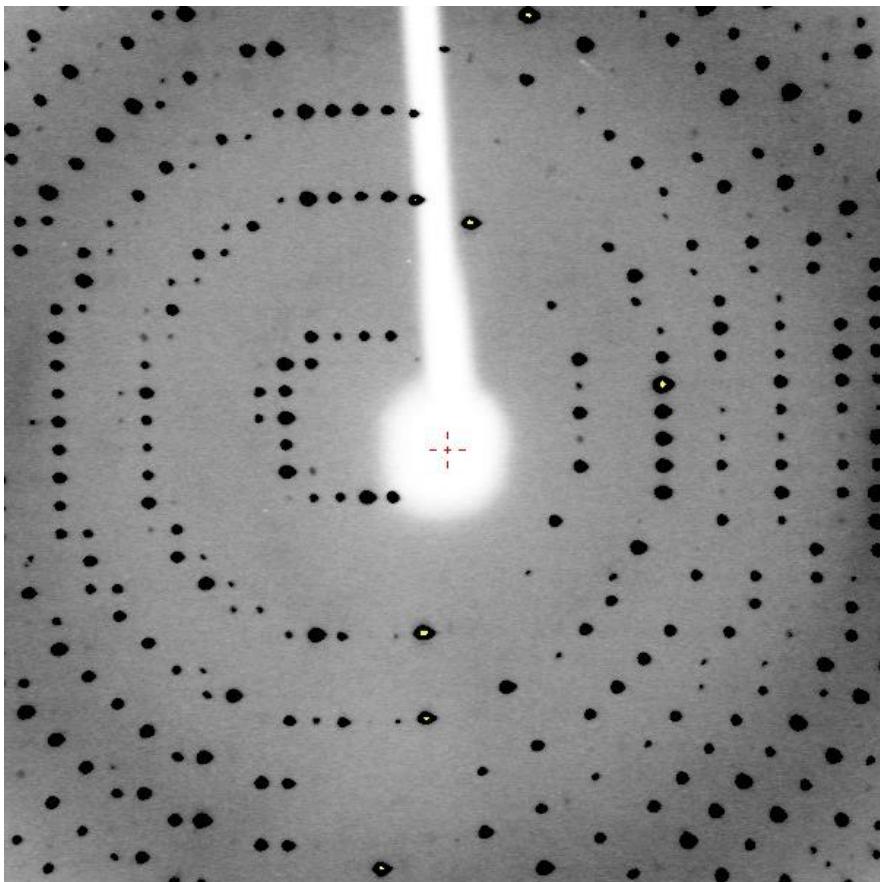
# 2D diffraction image is a projection from a 3D reciprocal lattice.



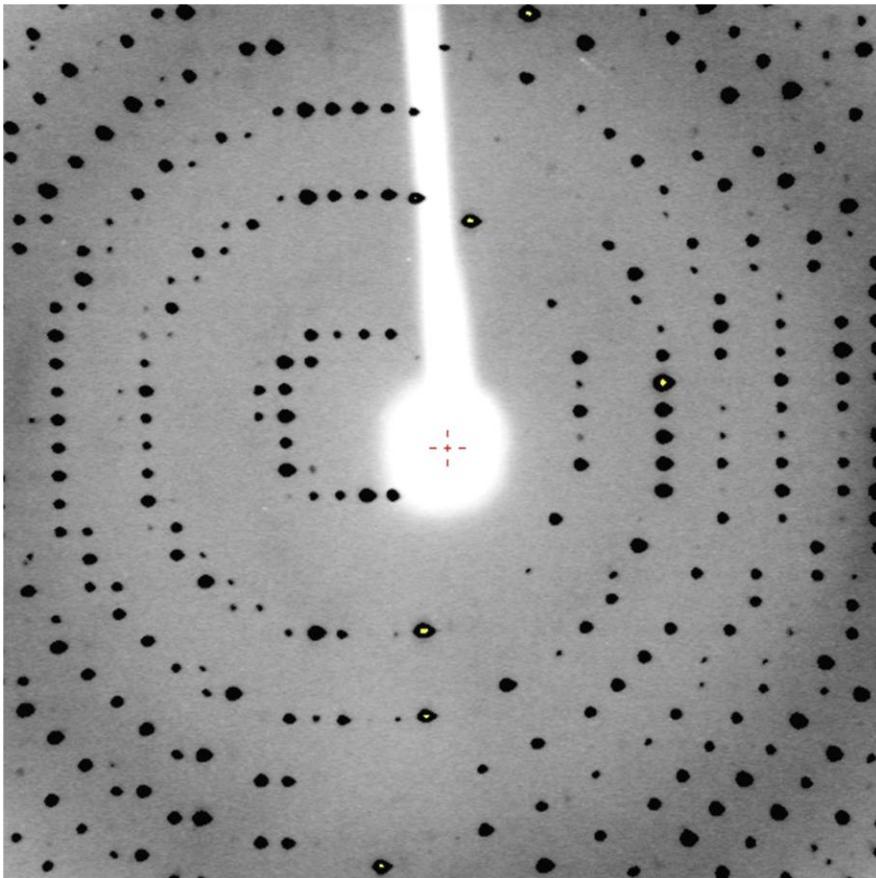
In an undistorted view of the reciprocal lattice, recorded reflections would reside on the surface of a sphere, not a plane.

# A distortion-corrected representation of the reflections

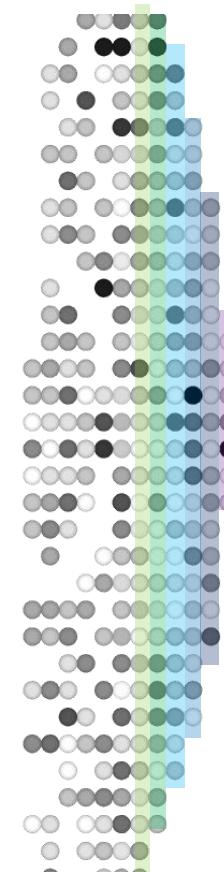
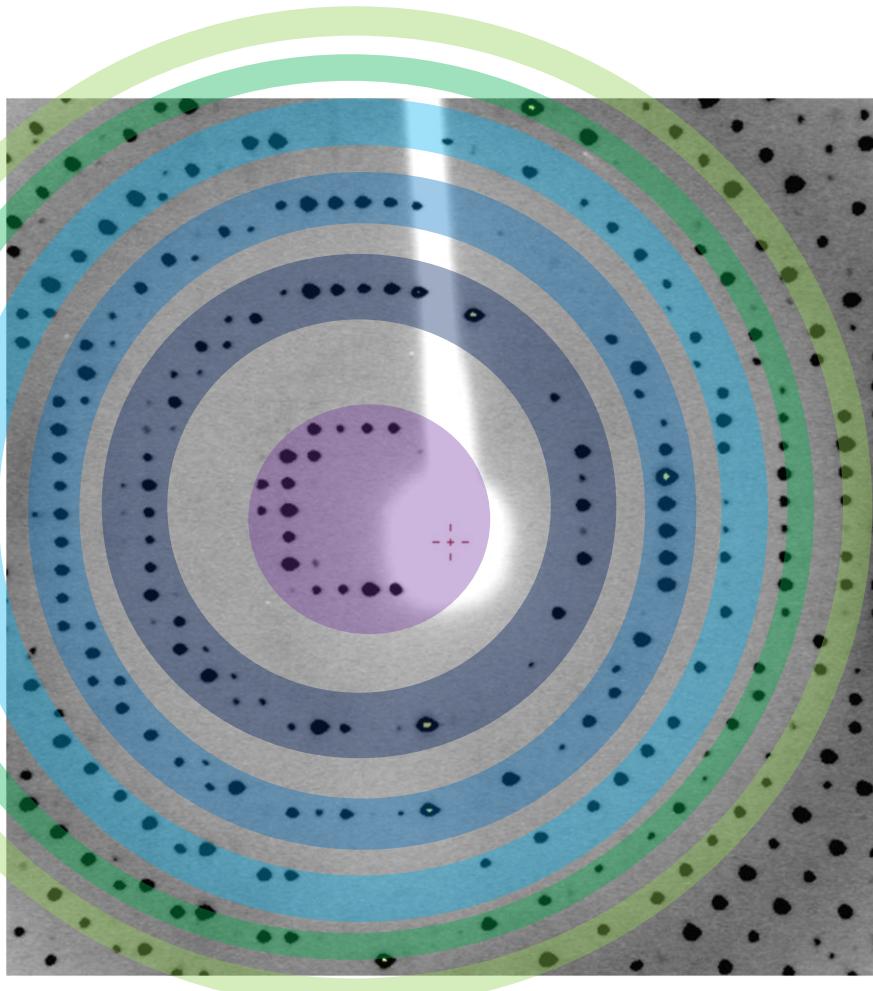
a single image reveals all 6 unit cell parameters



Restored depth of the diffraction pattern  
is evident from an orthogonal view

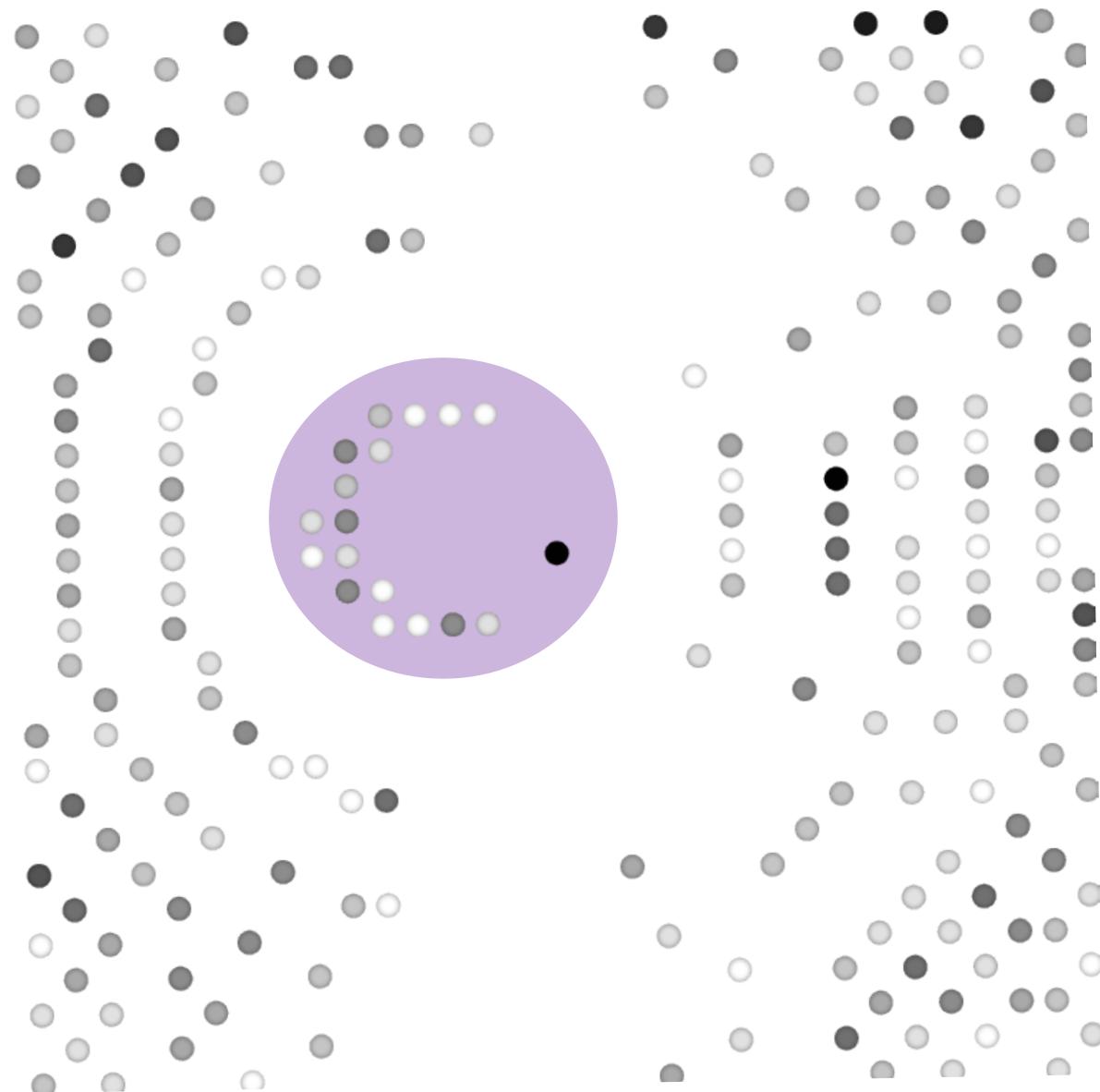


# Each circle corresponds to a different reciprocal lattice plane

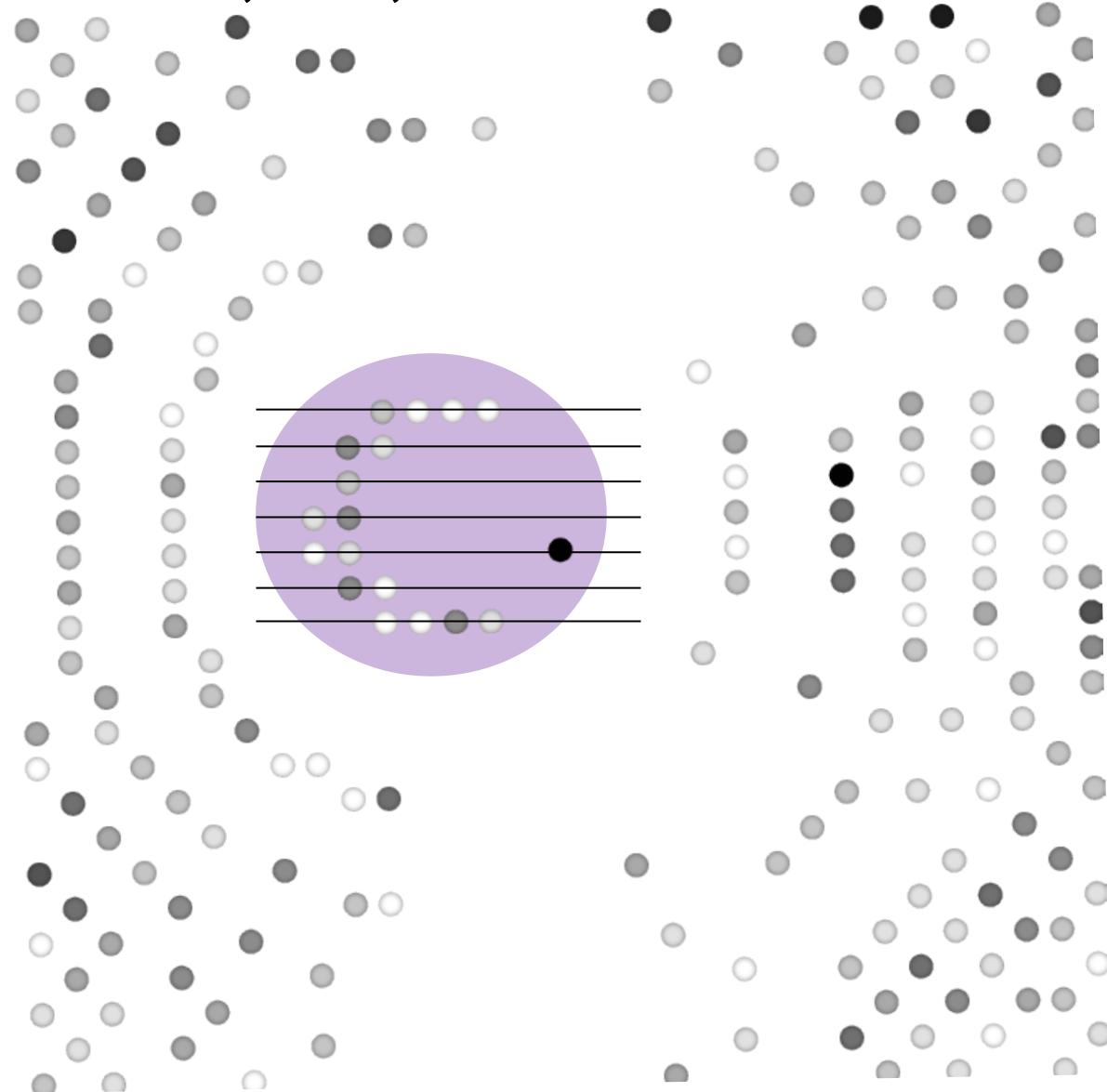


**Start indexing process using reflections within the same plane and lying near the origin.**

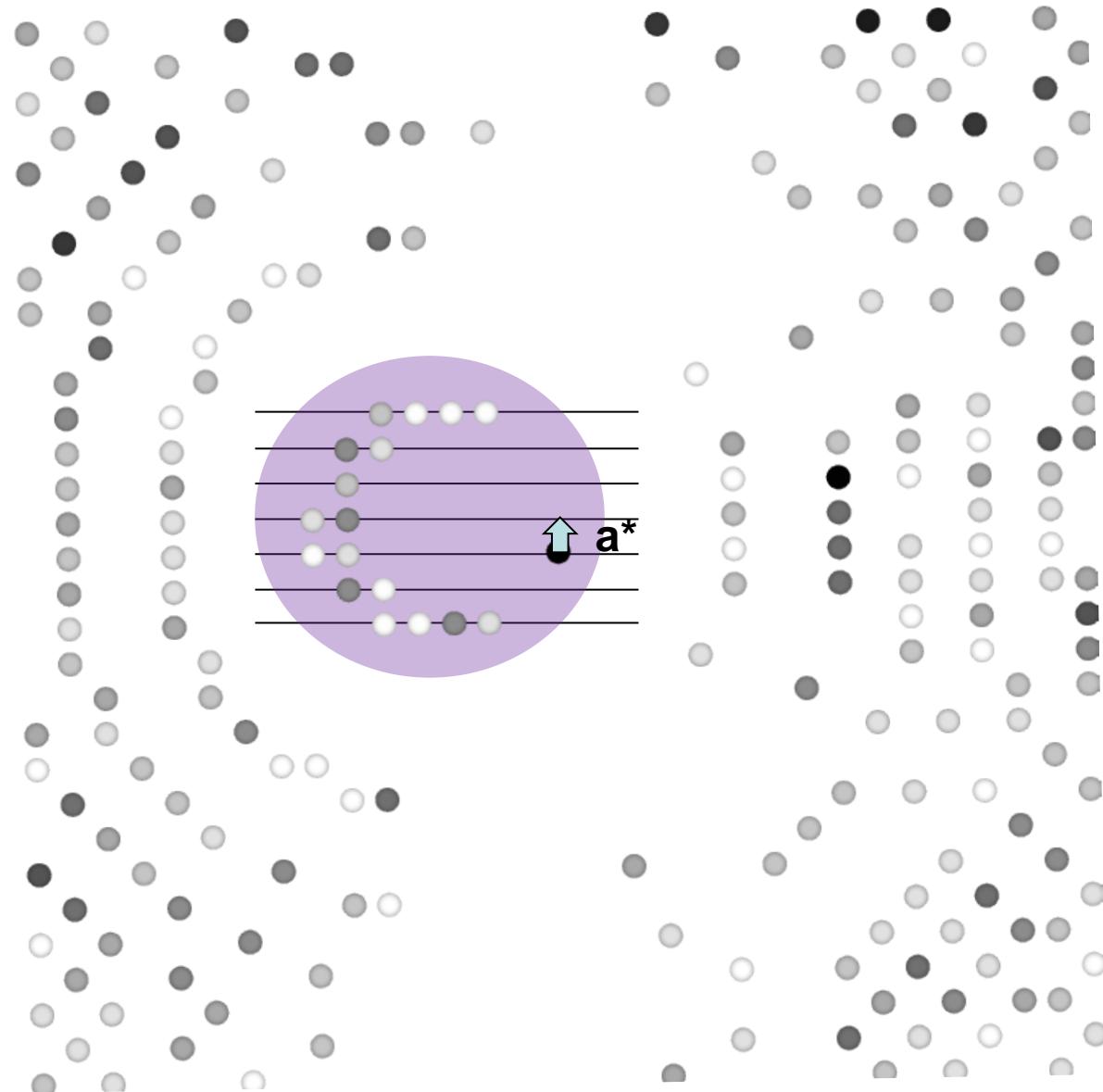
# Draw a set of evenly spaced rows



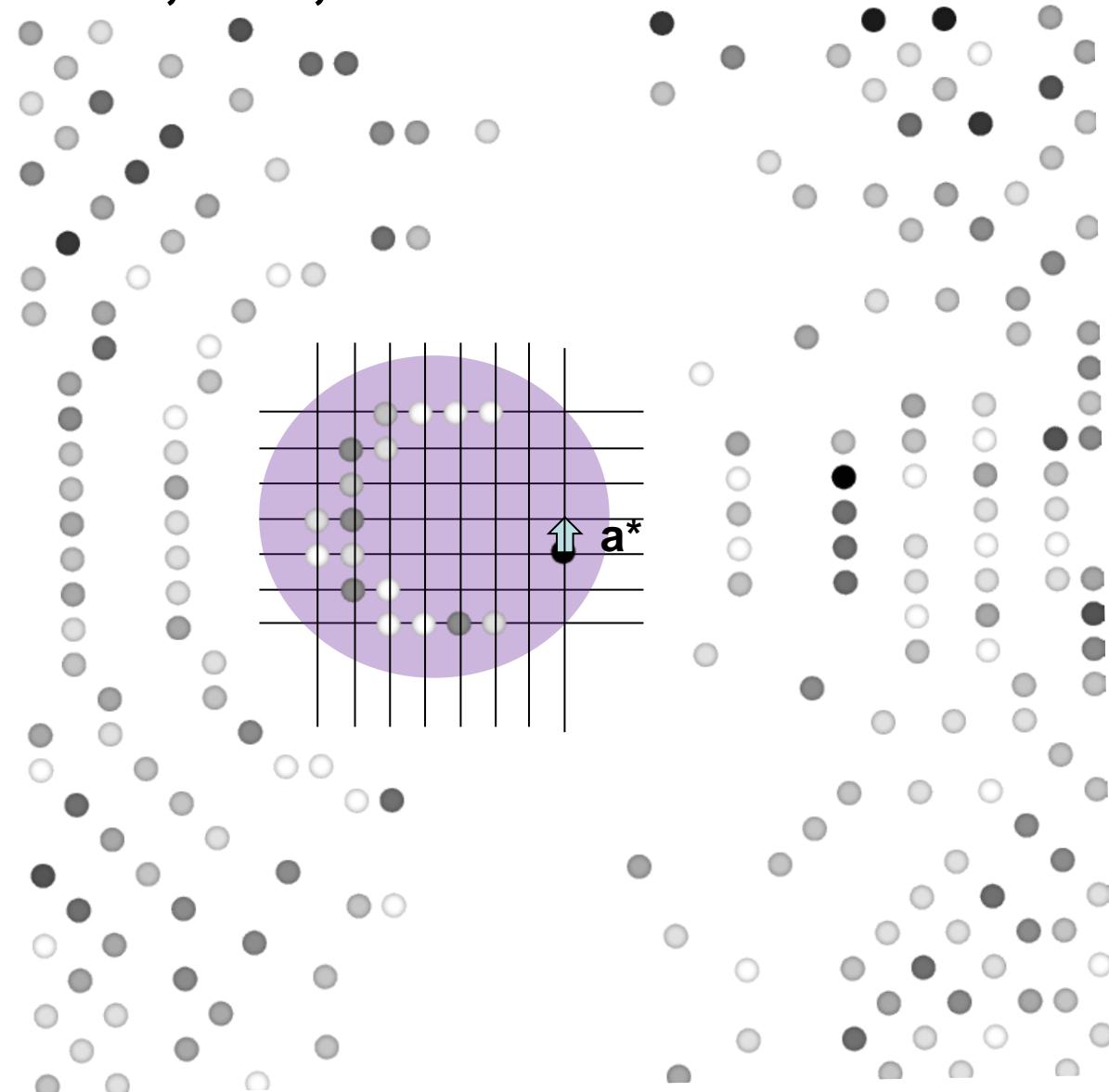
Draw the vector representing this repeat distance,  $a^*$ , between rows



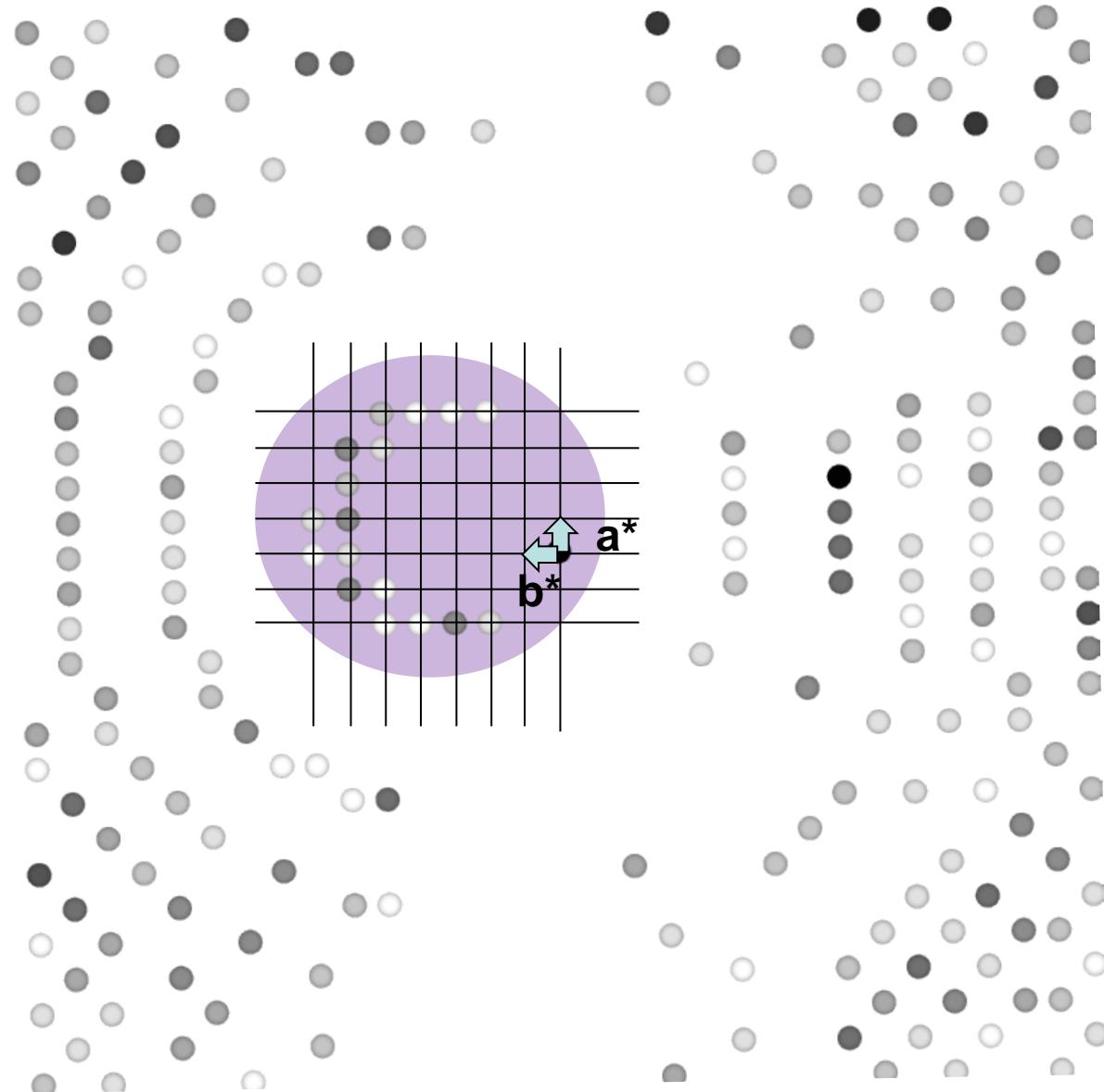
# Draw a set of evenly spaced columns



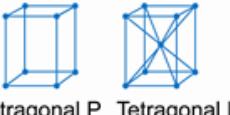
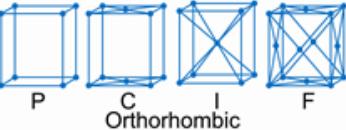
Draw the vector representing this  
repeat distance,  $b^*$ , between columns



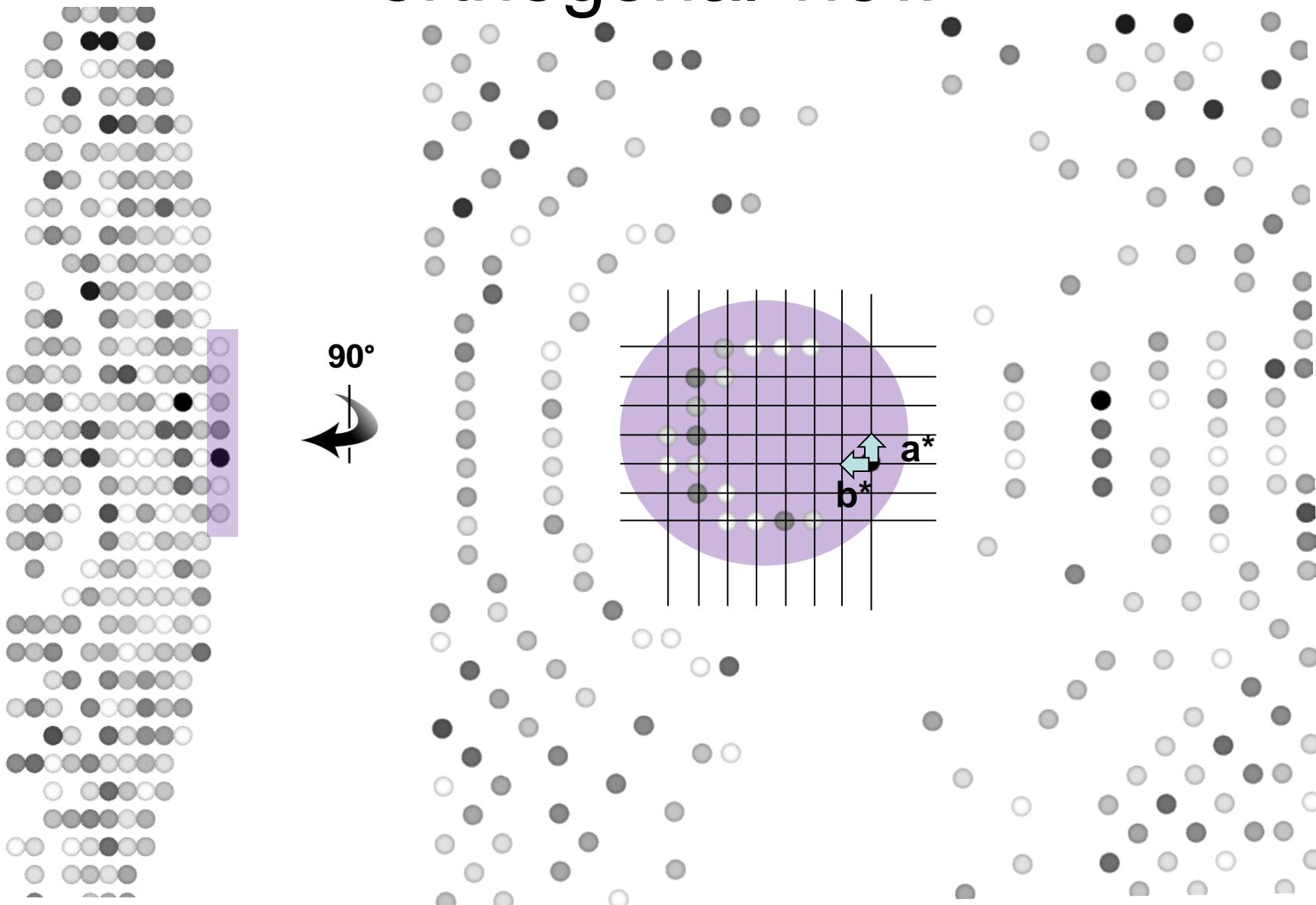
# What is the angle between $a^*$ and $b^*$ ?



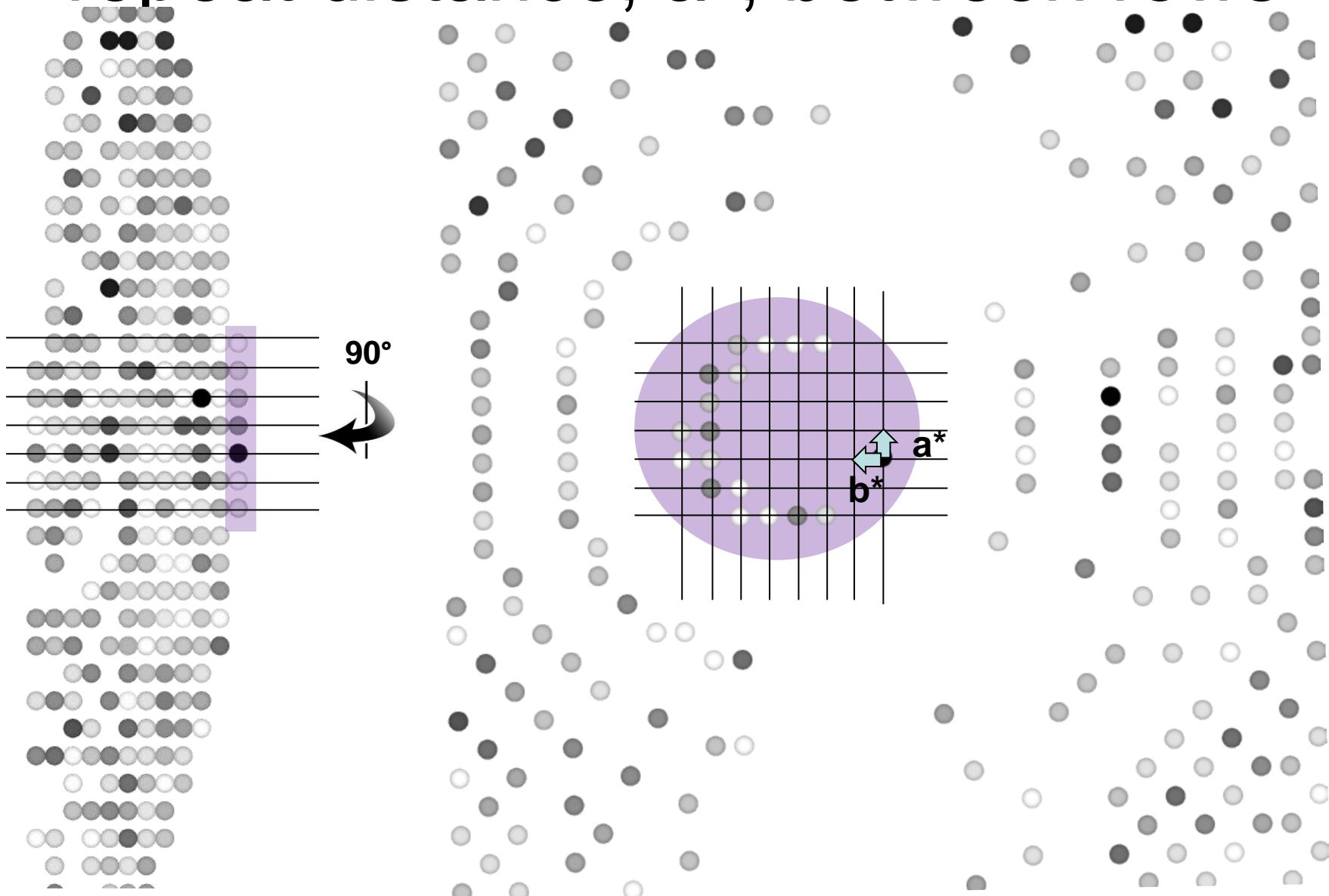
# Which of 14 Bravais Lattices has $a=b$ and $\gamma=90^\circ$

<b>Cubic</b>		Three axes at right angles; all equal: $a=b=c; \alpha=\beta=\gamma=90^\circ$
<b>Rhombohedral</b>		Three axes equally inclined, not at right angles; all equal: $a=b=c; \alpha=\beta=\gamma \neq 90^\circ$
<b>Hexagonal/Trigonal</b>		Three axes, two pairs at right angles, one at $120^\circ$ . $a=b \neq c; \alpha=\beta=90^\circ, \gamma=120^\circ$
<b>Tetragonal</b>		Three axes at right angles; two equal: $a=b \neq c; \alpha=\beta=\gamma=90^\circ$
<b>Orthorhombic</b>		Three axes at right angles; all unequal: $a \neq b \neq c; \alpha=\beta=\gamma=90^\circ$
<b>Monoclinic</b>		Three axes, one pair not at right angles, all unequal: $a \neq b \neq c; \alpha=\gamma=90^\circ \neq \beta$
<b>Triclinic</b>		Three axes not at right angles; all unequal: $a \neq b \neq c; \alpha \neq \beta \neq \gamma \neq 90^\circ$

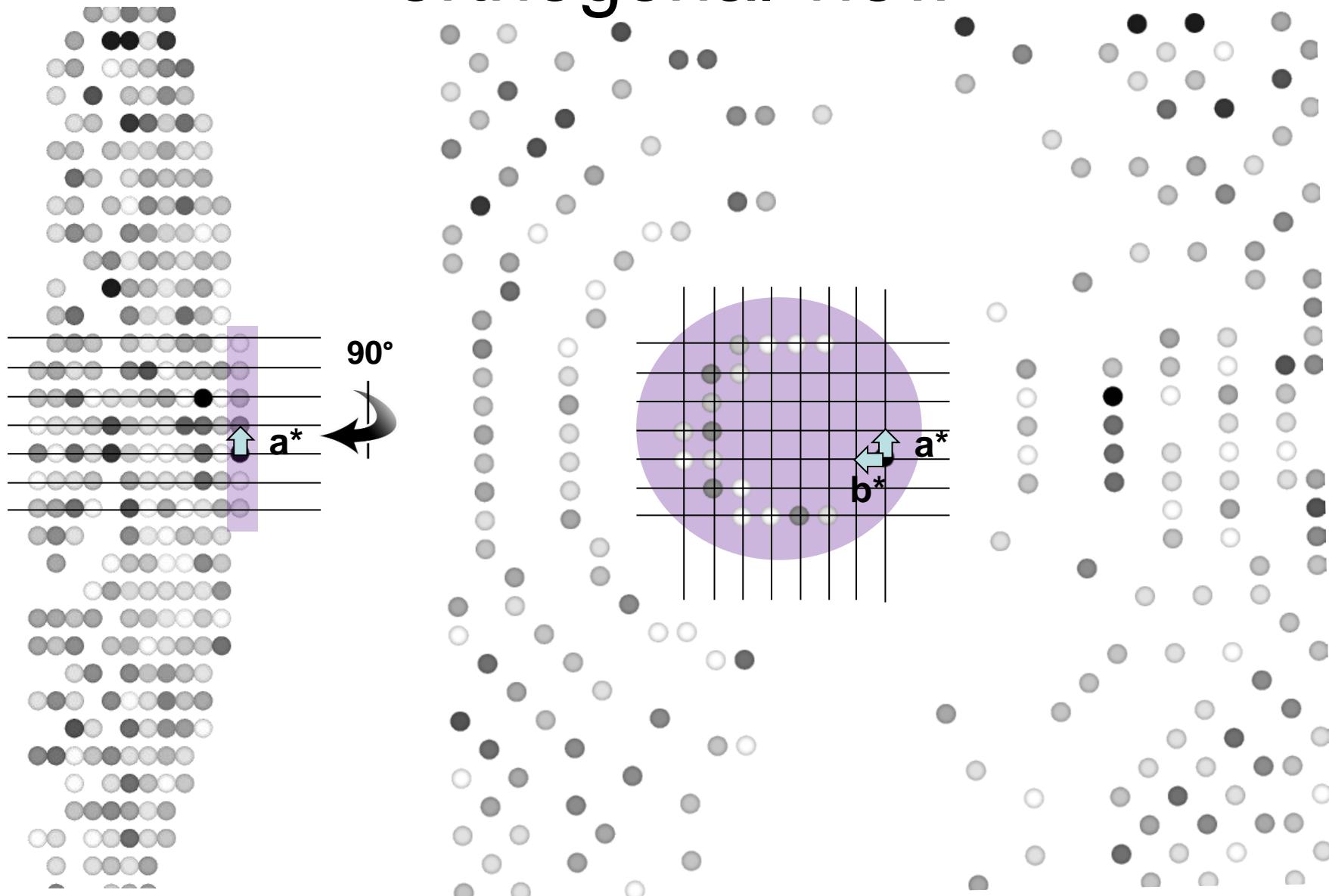
# Draw a set of evenly spaced rows in orthogonal view



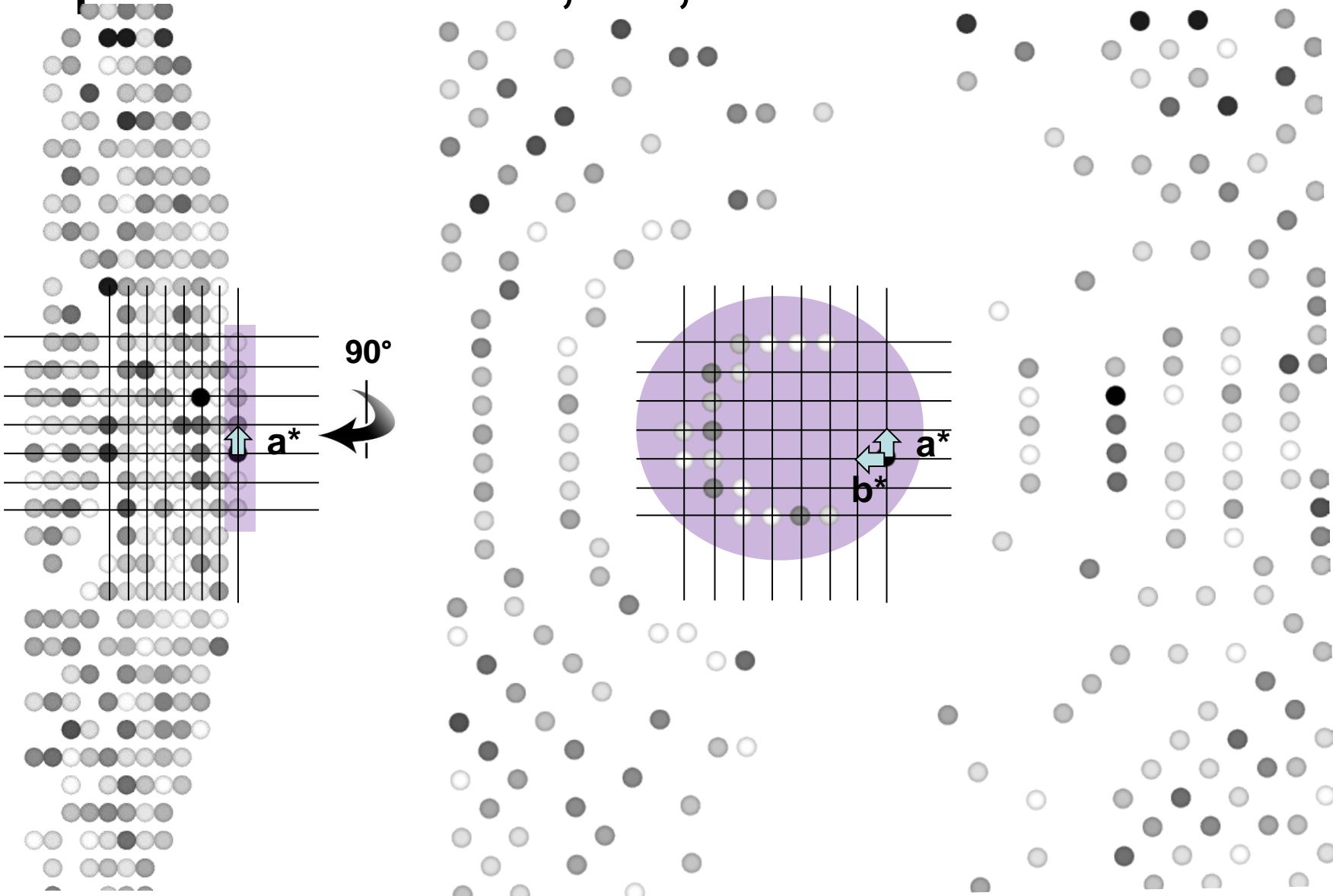
# Draw the vector representing this repeat distance, $a^*$ , between rows



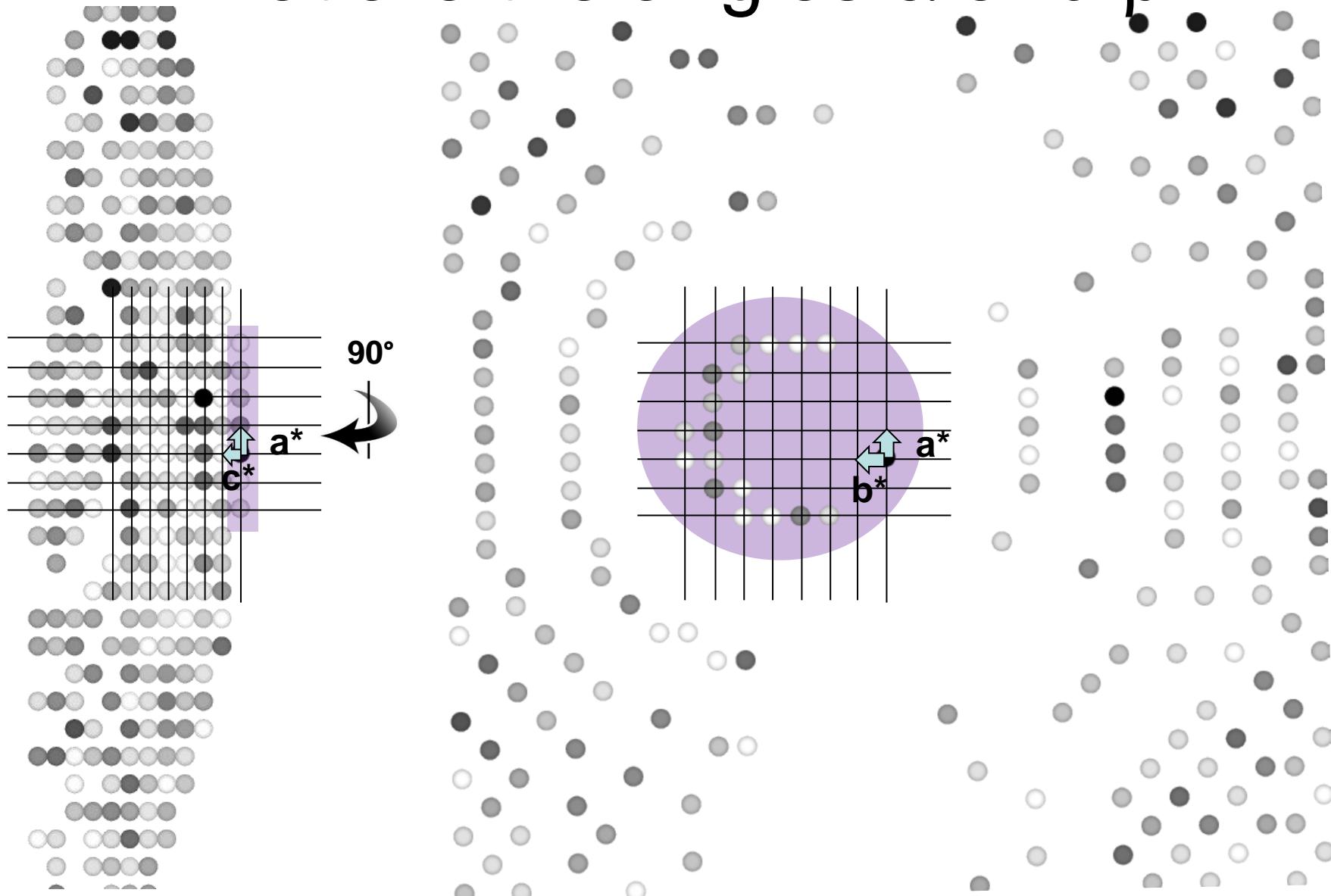
# Draw a set of evenly spaced columns in orthogonal view



# Draw the vector representing this repeat distance, $c^*$ , between columns



Is the length of  $c^*$  related to  $a^*$  and  $b^*$ ?  
What are the angles  $\alpha$  and  $\beta$ ?



# Which of 14 Bravais Lattices has $a=b\neq c$ and $\alpha=\beta=\gamma=90^\circ$

Cubic

			Three axes at right angles; all equal: $a=b=c; \alpha=\beta=\gamma=90^\circ$
---	---	--	--

Rhombohedral

	Rhombohedral	Three axes equally inclined, not at right angles; all equal: $a=b=c; \alpha=\beta=\gamma\neq 90^\circ$
---	--------------	---

Hexagonal/Trigonal

	Hexagonal	Three axes, two pairs at right angles, one at $120^\circ$ . $a=b\neq c; \alpha=\beta=90^\circ, \gamma=120^\circ$
---	-----------	---

Tetragonal

		Three axes at right angles; two equal: $a=b\neq c; \alpha=\beta=\gamma=90^\circ$
---	--	--

Orthorhombic

		Three axes at right angles; all unequal: $a\neq b\neq c; \alpha=\beta=\gamma=90^\circ$
--	--	--

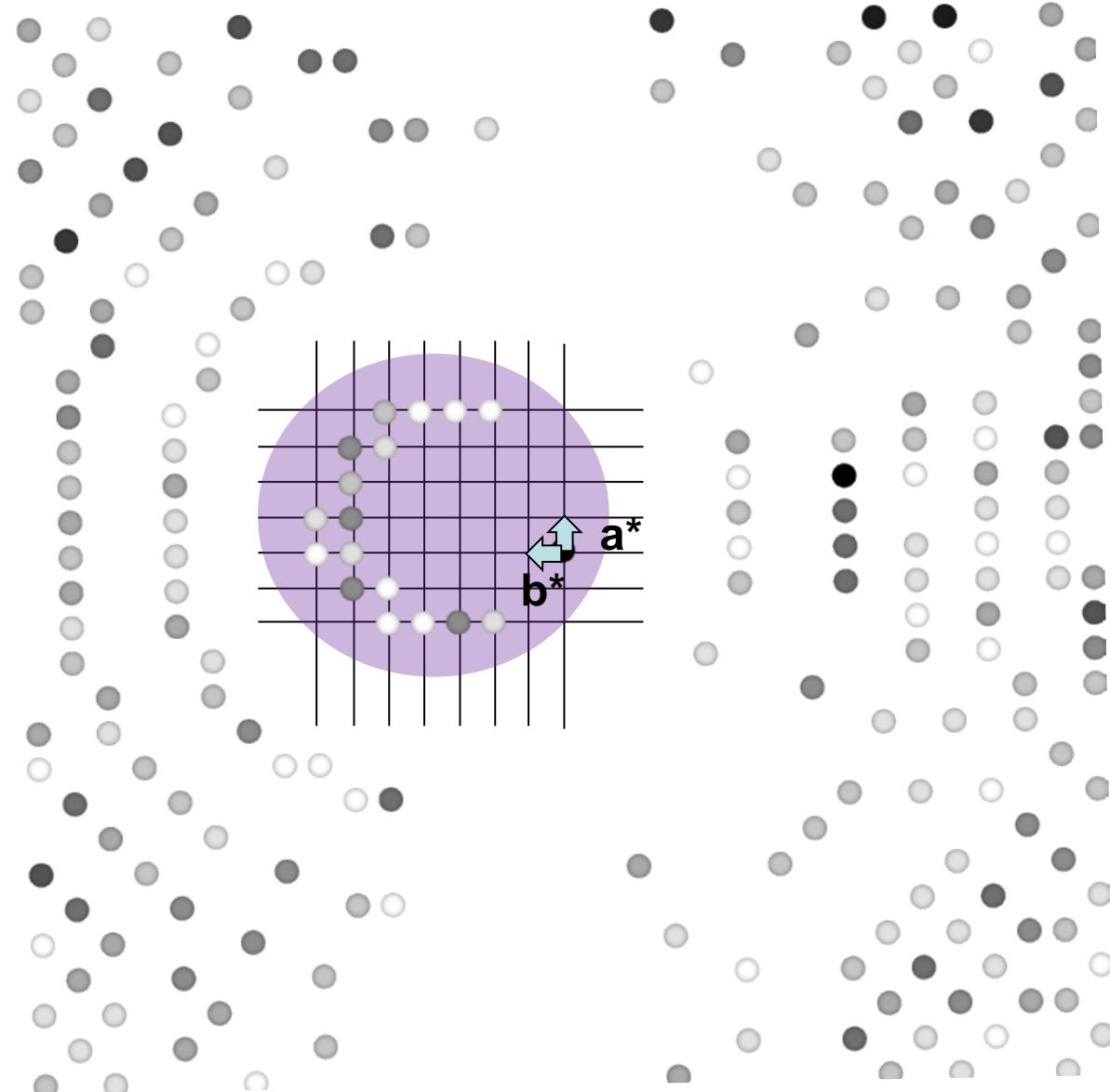
Monoclinic

		Three axes, one pair not at right angles, all unequal: $a\neq b\neq c; \alpha=\gamma=90^\circ \neq \beta$
---	--	--

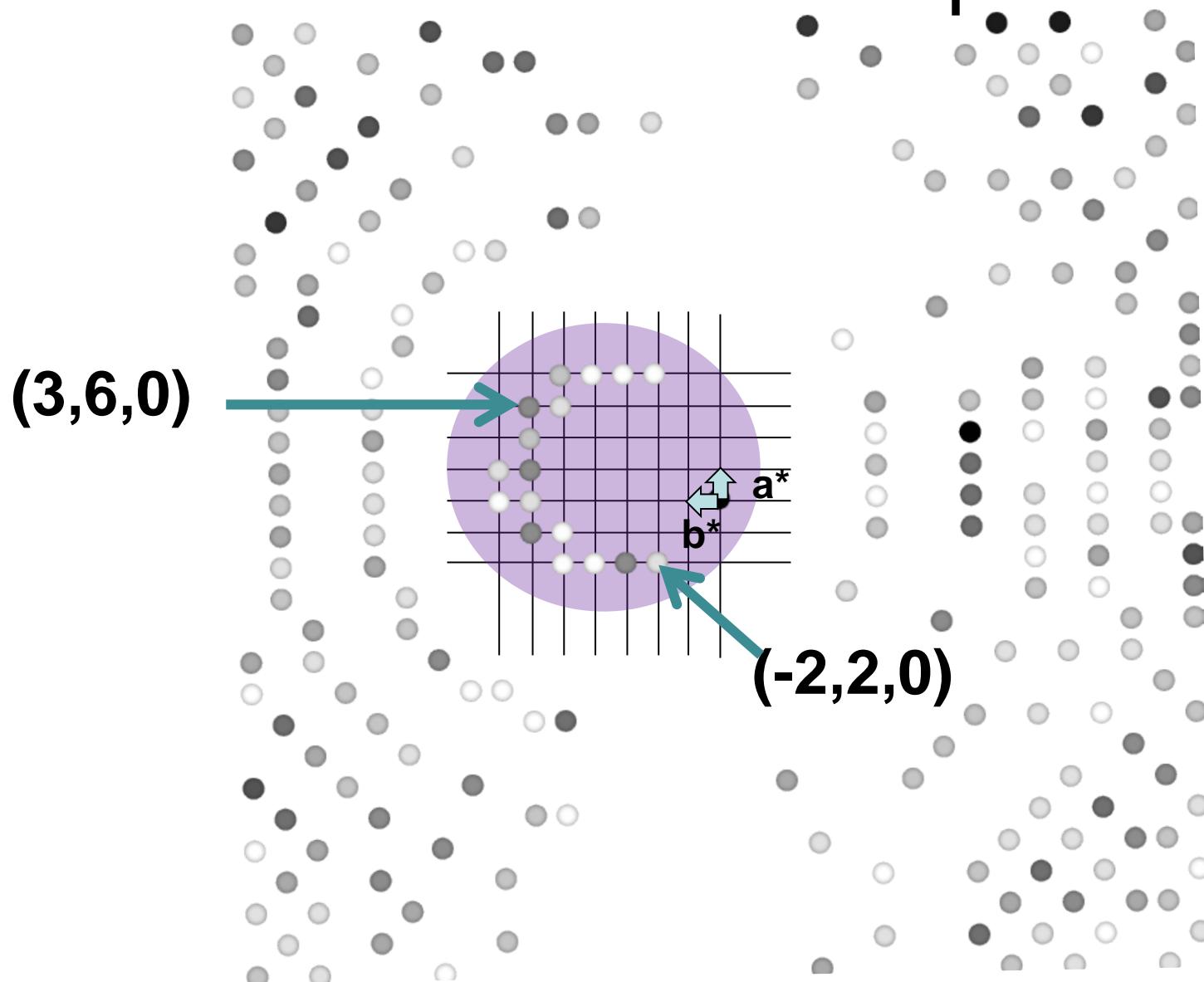
Triclinic

	Triclinic P	Three axes not at right angles; all unequal: $a\neq b\neq c; \alpha\neq\beta\neq\gamma\neq 90^\circ$
---	-------------	--

# What is the index of the lowest resolution reflection?



# What is the index of the highest resolution reflection in the $l=0$ plane?



# Which experimental parameters were required to index a spot?

Coordinates (X,Y) for the spot position

Coordinates of the direct beam, (X,Y)

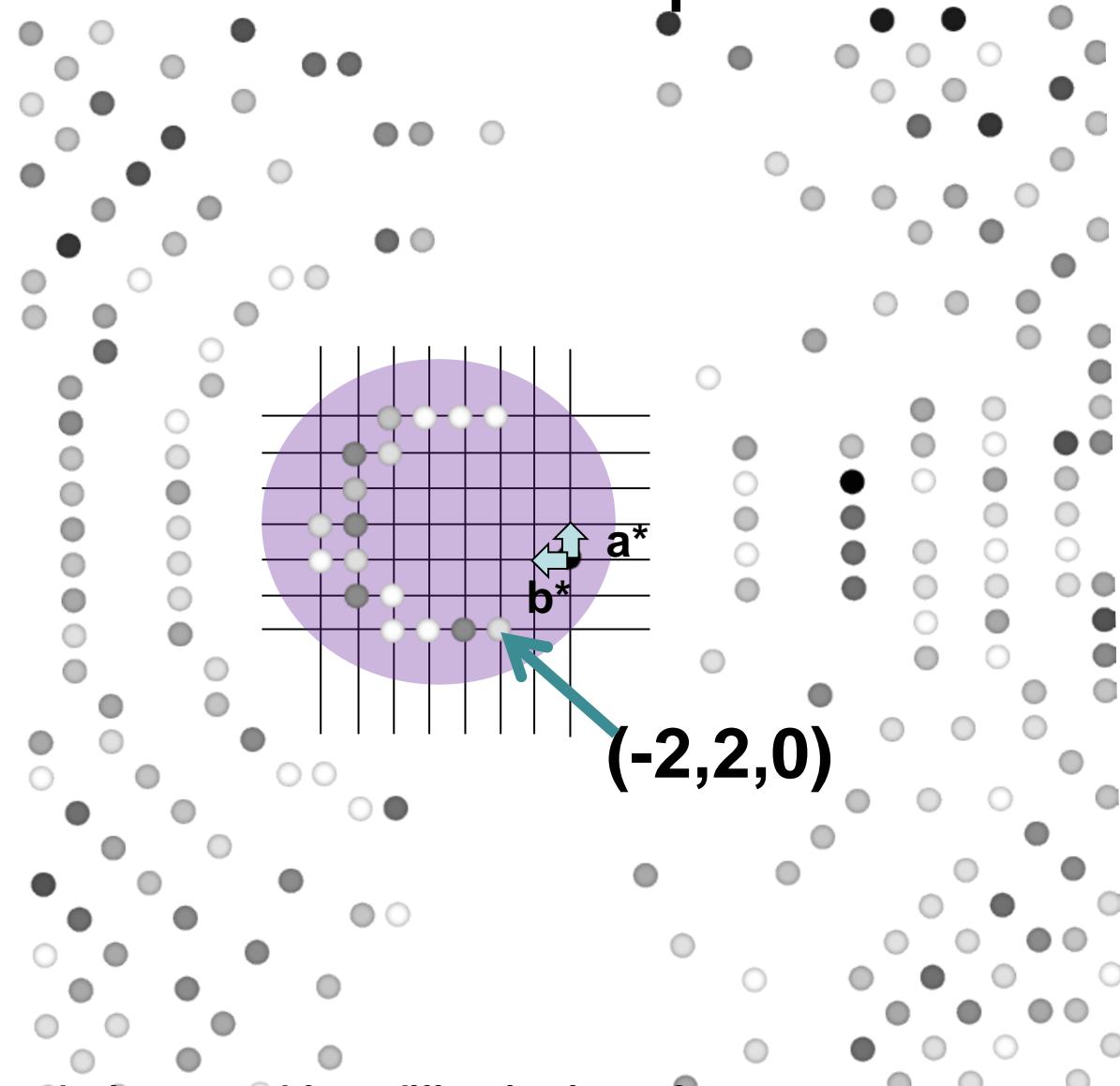
Reciprocal cell parameters  
 $a^*, b^*, c^*, \alpha^*, \beta^*, \gamma^*$

Orientation of the unit cell axes w.r.t. the laboratory axes ( $\phi\psi\kappa$ ).

To determine real cell parameters?

The wavelength of the incident radiation

Crystal-to-detector distance



Which experimental parameters must be interpreted from diffraction image?

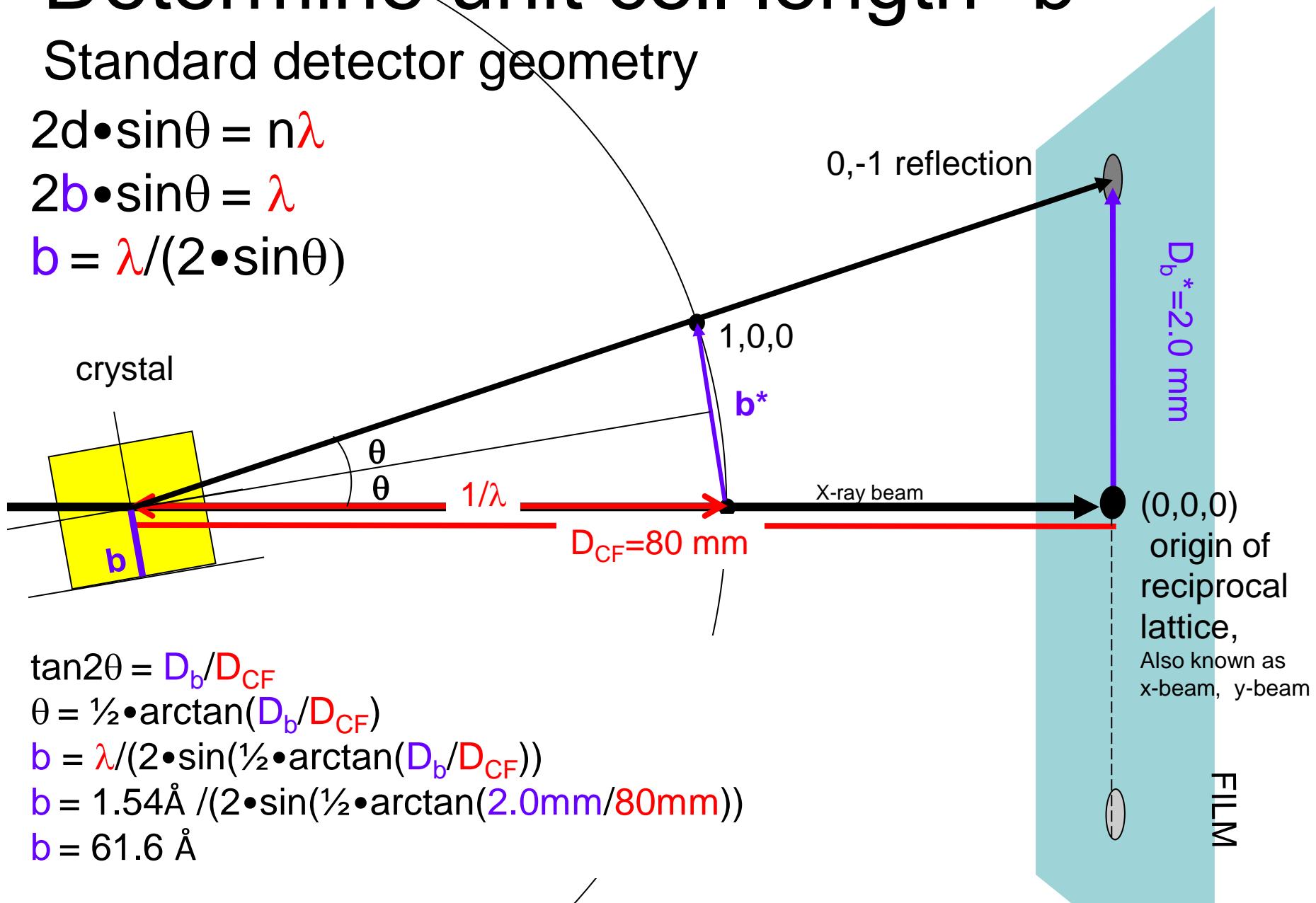
# Determine unit cell length “b”

Standard detector geometry

$$2d \cdot \sin\theta = n\lambda$$

$$2b \cdot \sin\theta = \lambda$$

$$b = \lambda / (2 \cdot \sin\theta)$$



$$\tan 2\theta = D_b^* / D_{CF}$$

$$\theta = \frac{1}{2} \cdot \arctan(D_b^* / D_{CF})$$

$$b = \lambda / (2 \cdot \sin(\frac{1}{2} \cdot \arctan(D_b^* / D_{CF})))$$

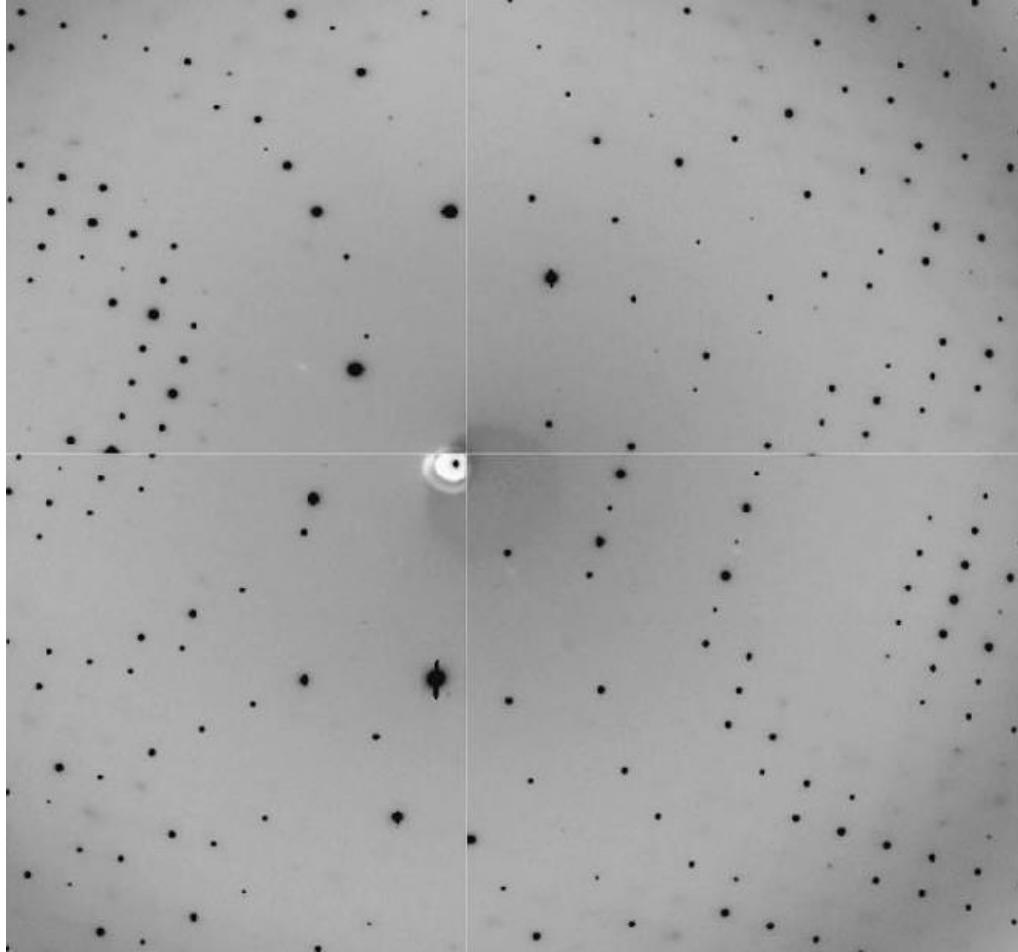
$$b = 1.54 \text{ \AA} / (2 \cdot \sin(\frac{1}{2} \cdot \arctan(2.0 \text{ mm} / 80 \text{ mm})))$$

$$b = 61.6 \text{ \AA}$$

$D_b^* = 2.0 \text{ mm}$   
(0,0,0)  
origin of reciprocal lattice,  
Also known as x-beam, y-beam

FILM

Need a program that can automatically identify spot positions, interpret crystal orientation, and unit cell dimensions.



# Automatic indexing algorithm explained

research papers

Acta Cryst. (1999), D55, 1690-1695

Acta Crystallographica Section D  
**Biological  
Crystallography**

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## The Rossmann Fourier autoindexing algorithm in *MOSFLM*

Harold R. Powell

MRC-LMB, MRC Centre, Hills Road, Cambridge  
CB2 2QU, England

Correspondence e-mail:  
hamy@mrc-lmb.cam.ac.uk

The fast Fourier transform (FFT) autoindexing routines written by the Rossmann group at Purdue University have been incorporated in *MOSFLM*, providing a rapid and reliable method of indexing oscillation images. This is a procedure which extracts direct-space information about the unit cell from the FFT. The method and its implementation in *MOSFLM* are discussed.

Received 8 February 1999

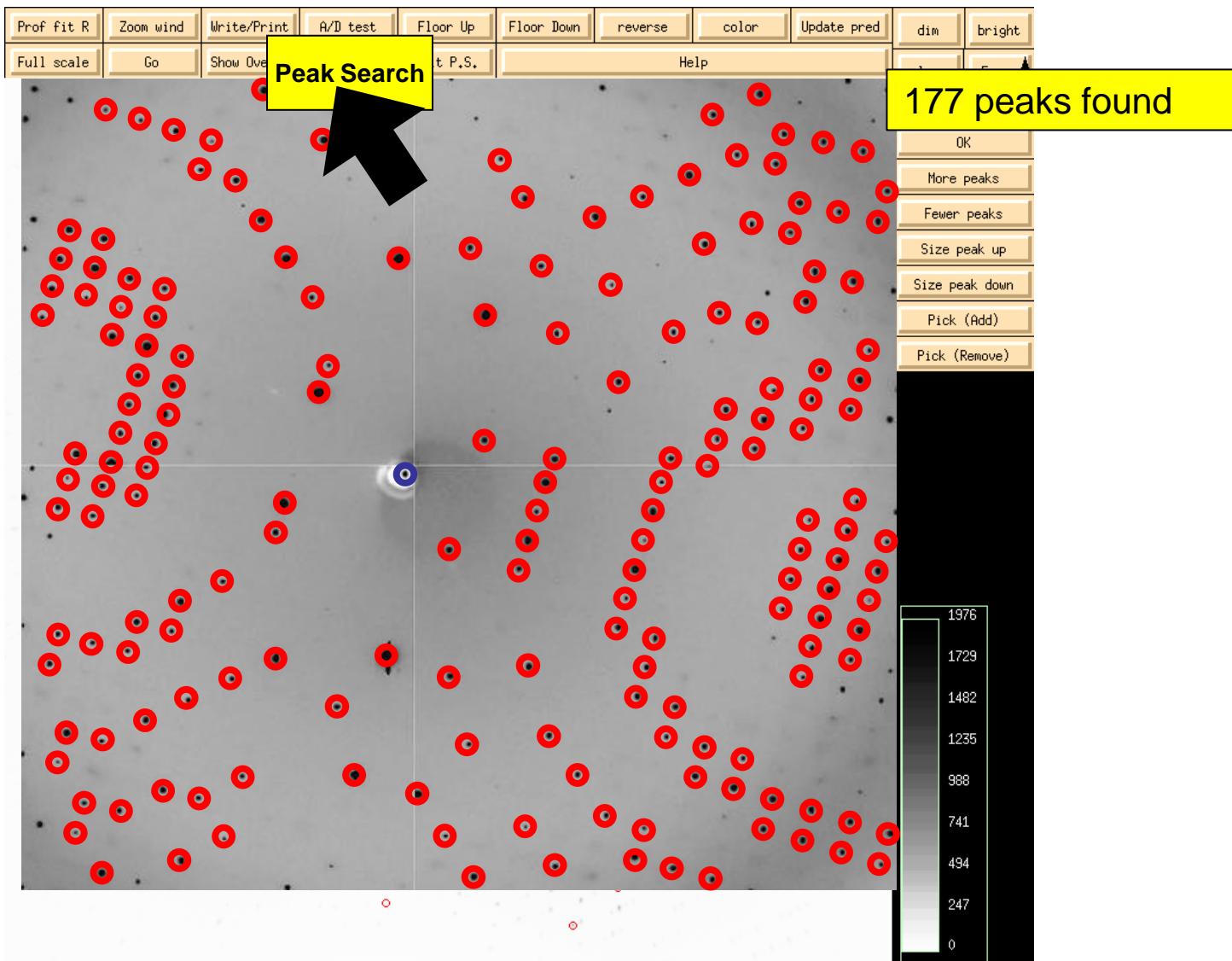
Accepted 9 July 1999

### 1. Introduction

Most newcomers to crystallography and diffraction-image processing probably view indexing as a ‘black-box process’ (Fig. 1): a diffraction image is presented to a program (often only available without the source code), which performs the process automatically in a matter of seconds; some programs are more powerful and reliable than others, and it has become

# Locate reflections positions (peaks of high intensity)

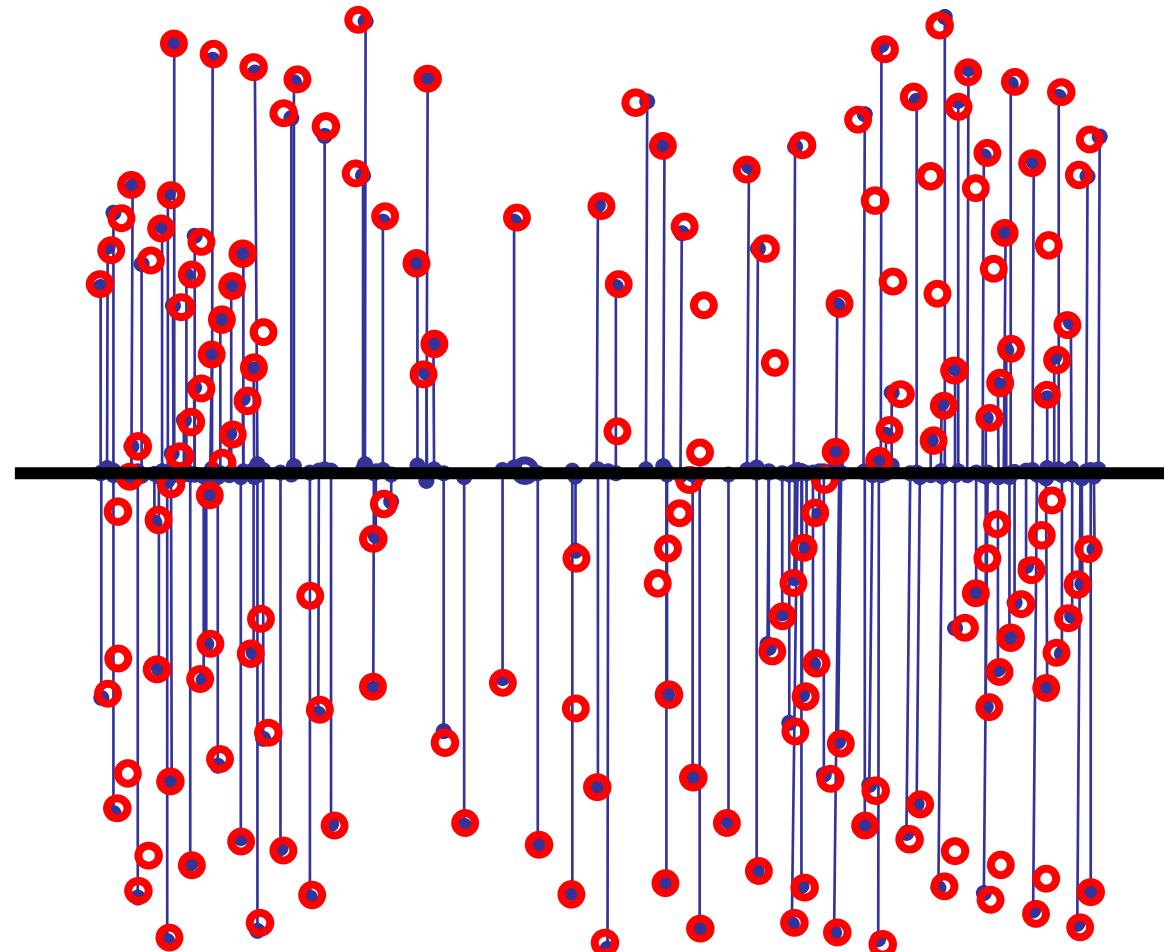
- 1) Display first image in your data set with
- 2) Press “Peak Search”. Red circles indicate position of prominent peaks (spots).
- 3) Evaluate whether you need more or fewer peaks.
- 4) Press “OK”
- 5) Spot positions (x,y) are written to a file “peaks.file.”



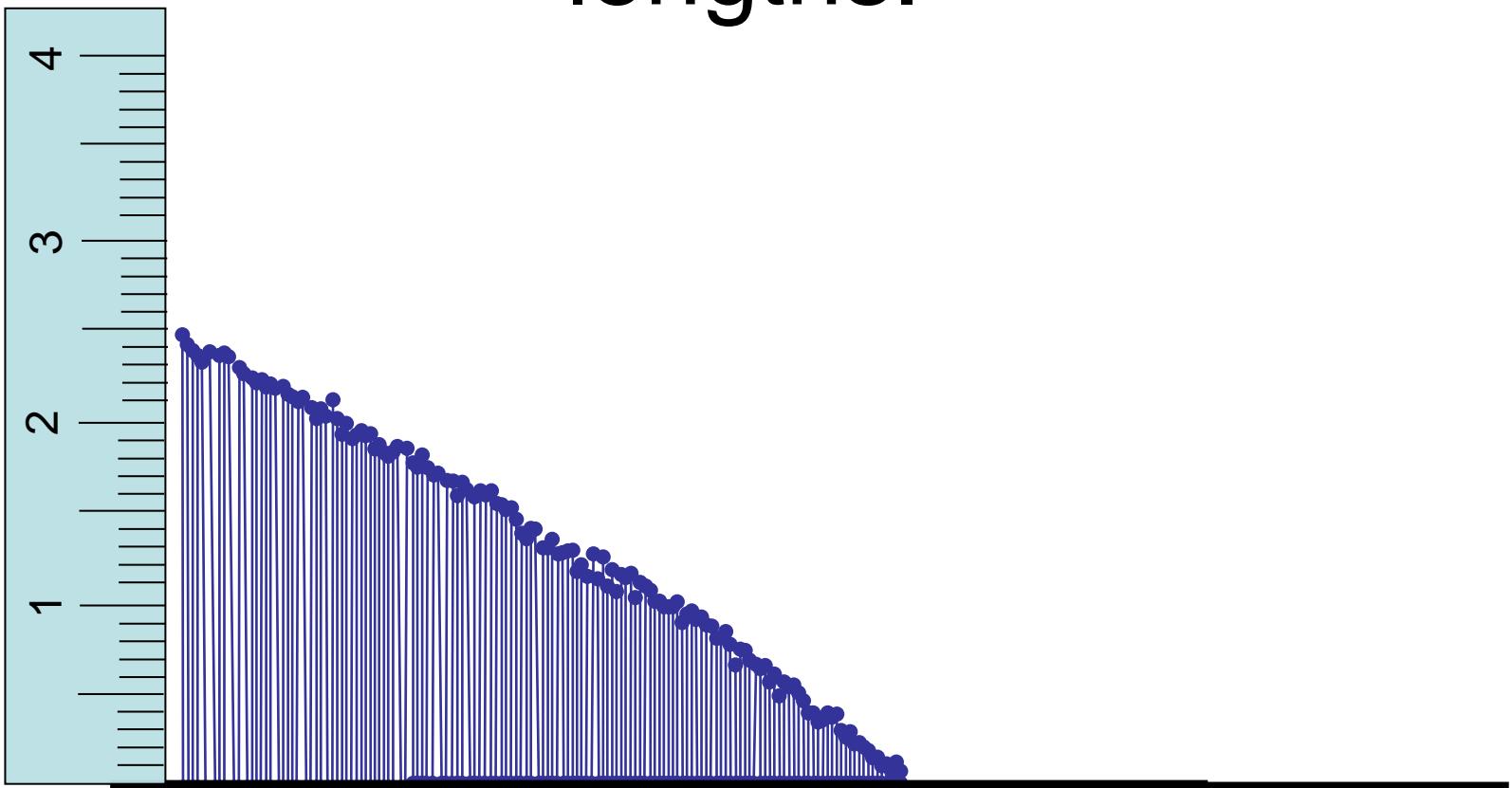
# Peaks.file

•	7777	0.0	0.0	1	1
	<b>height</b>	<b>x</b>	<b>y</b>		<b>frame</b>
•	13	2695.7	1350.5	1	1
•	27	2669.5	1062.4	1	1
•	16	2570.6	1143.5	1	1
•	26	2569.4	1302.4	1	1
•	30	2562.5	1592.5	1	1
•	32	2554.5	1902.4	1	1
•	32	2524.5	1103.4	1	1
•	22	2514.5	1523.8	1	1
•	12	2503.4	1316.6	1	1
•	21	2494.5	1949.5	1	1
•	15	2492.5	1923.4	1	1
•	35	2488.5	1721.5	1	1
•	17	2483.5	1870.6	1	1
•	12	2479.4	1212.5	1	1
•	32	2465.5	1452.5	1	1
•	15	2456.4	638.4	1	1
•	13	2444.7	900.7	1	1
•	14	2437.6	1183.4	1	1
•	23	2436.4	1969.4	1	1
•	Etc.....				

Project vectors onto a line.  
Measure the length of each projection.

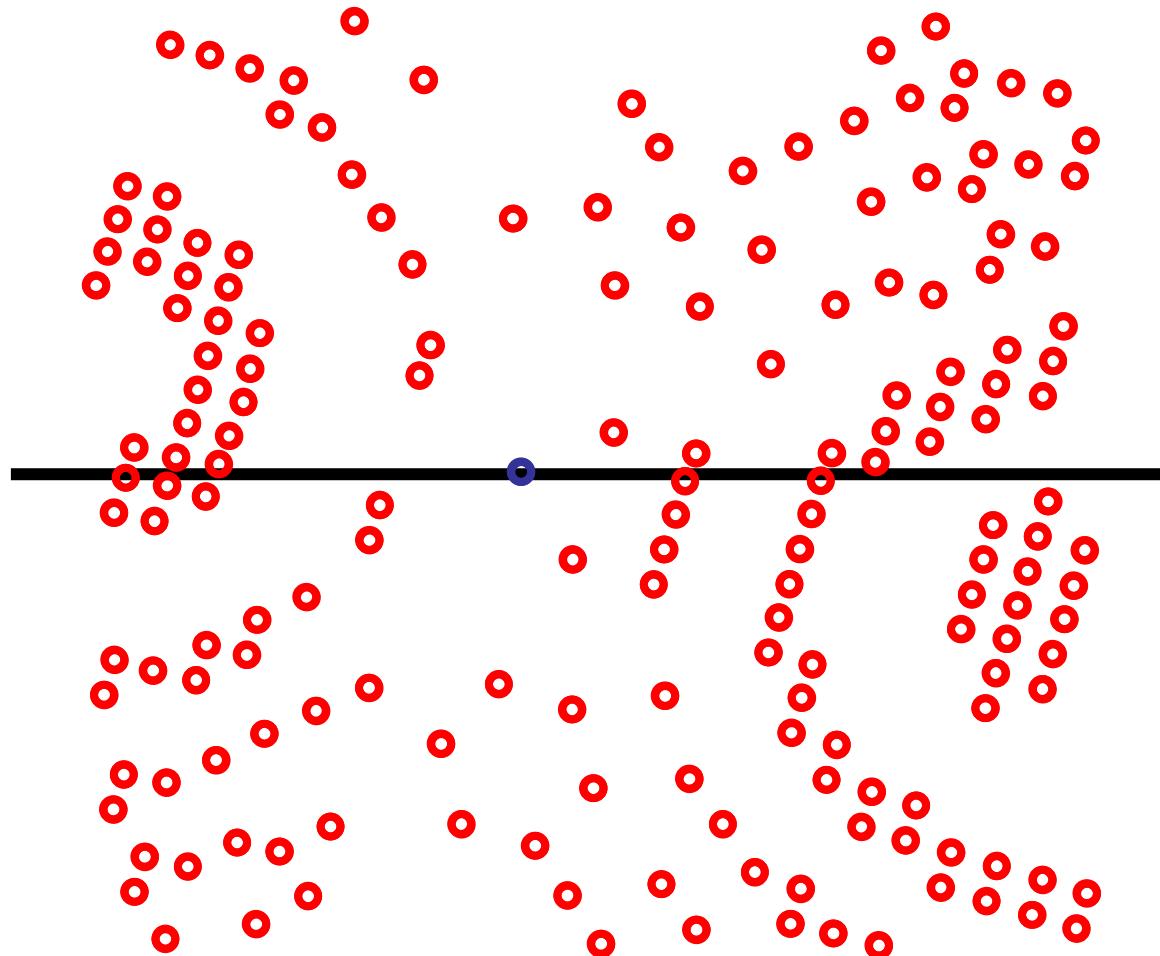


# Look for incremental differences in lengths.

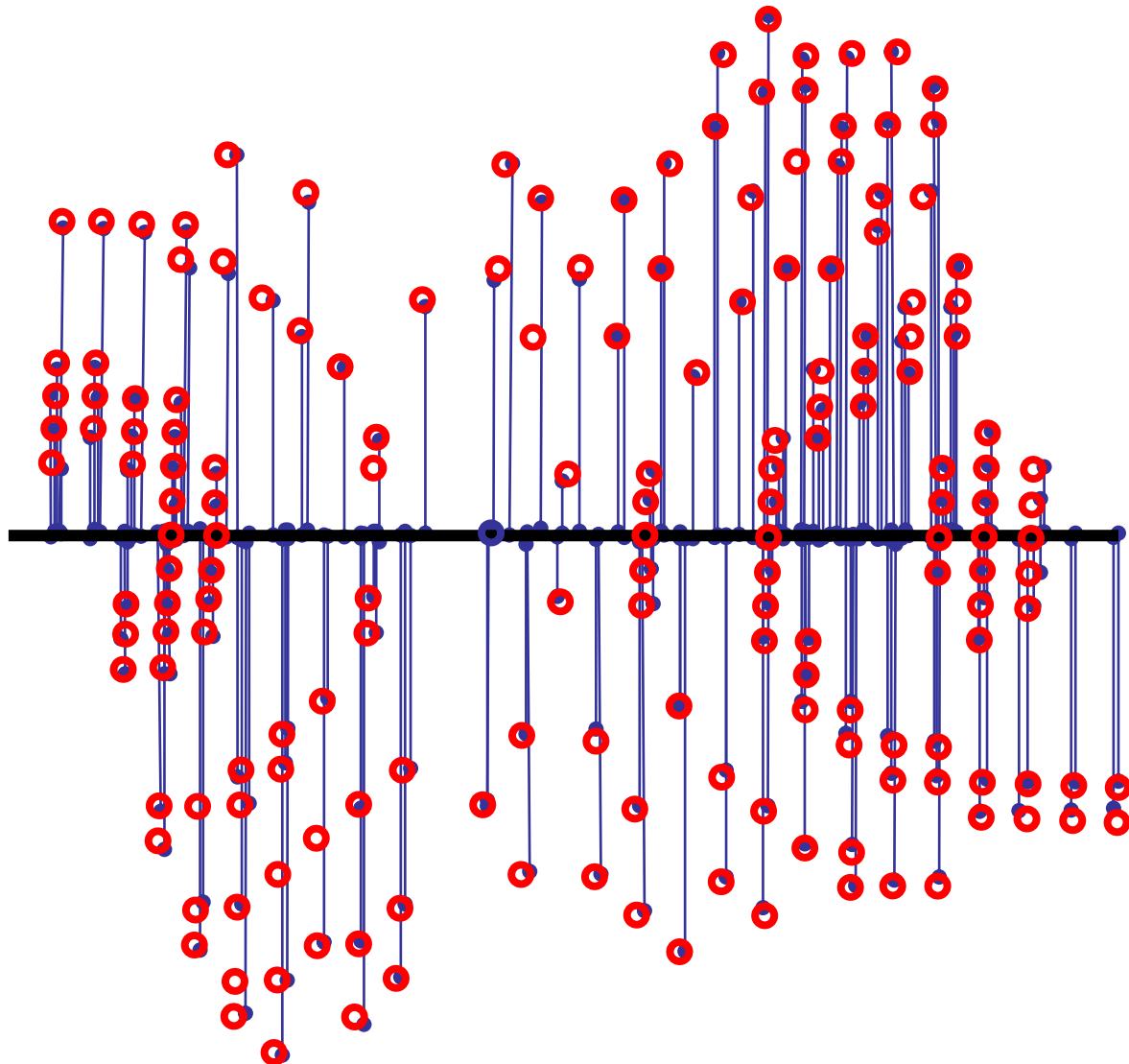


**Distribution of lengths is not incremental, it is continuous**

Rotate, 7300 orientations tested.

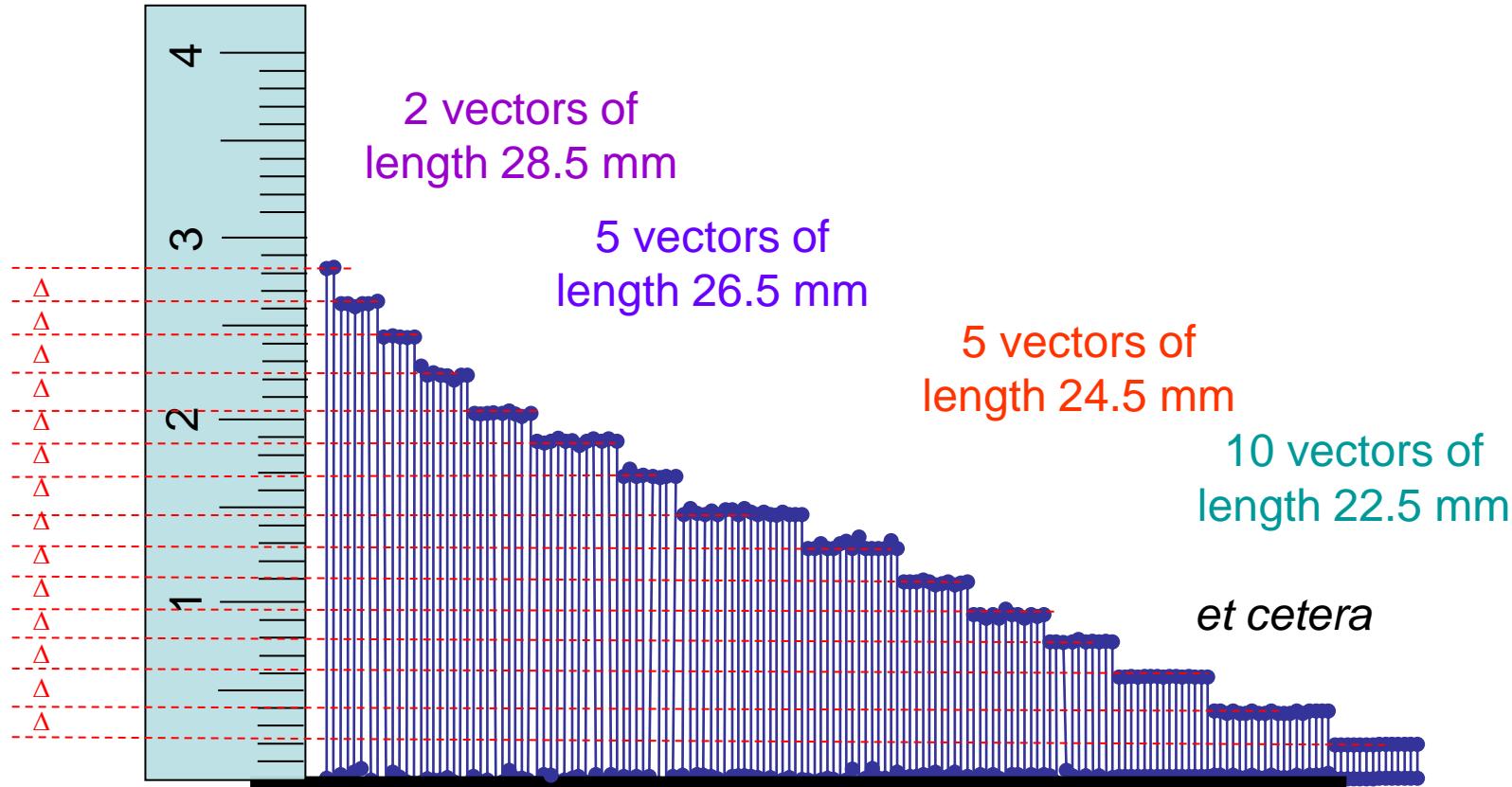


# Projected vectors for rotated image.



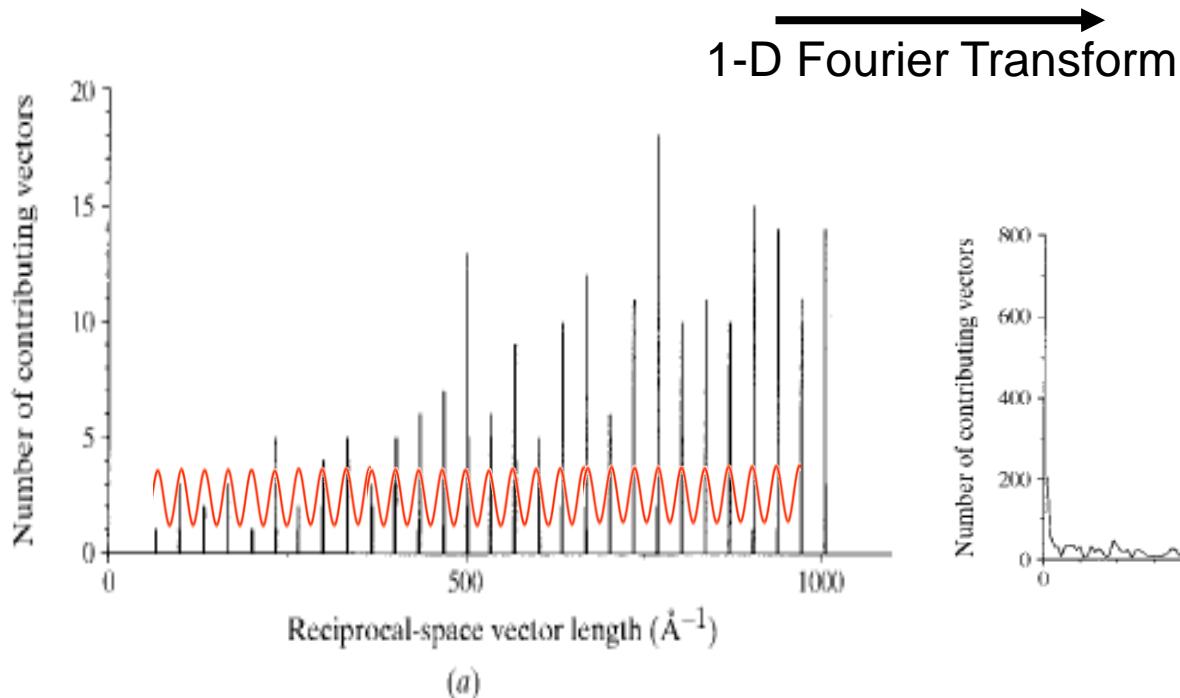
# Sort the vector projections by length.

Count the number of observations of each length.

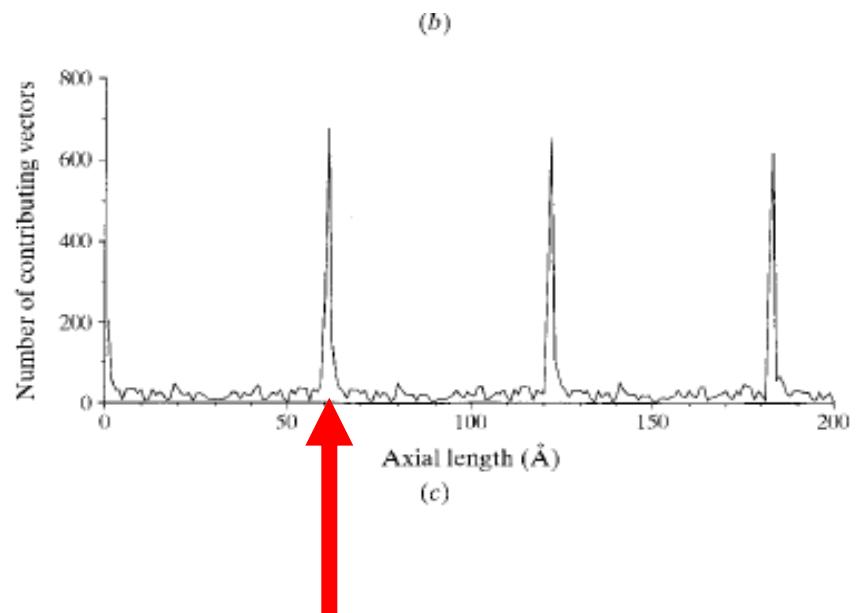


- 1) Note: Projected vectors have a quantized values (distribution looks like steps).
- 2) The incremental difference  $\Delta$  is proportional to the reciprocal cell length

# Fourier analysis of length histogram reveals cell dimension.



A cosine wave with periodicity of  $1/62.5 \text{ \AA}$  is a major contributor to the 1-D FT.



Unit cell length =  $62.5 \text{ \AA}$

# Run autoindexing script

The autoindexing script  
is simply named “a.”

Type “denzo” to start  
the program.

Then type @a to pass  
instructions to Denzo.

## AUTOINDEXING SCRIPT

this script is simply named “a”

```
format ccd adsc unsupported-q4
TITLE 'DENZO autoindexing CCD'
overload value 65000      [less than 65535 (16 bits)]
error positional 0.040     [1/2 of an 80-micron Q4 pixel.]
spindle axis 1 0 0        [establish lab coords.]
vertical axis 0 1 0       [for crystal before cryst rotx, etc.]
film rotation 0
error systematic 5.0 partiality 0.1    [default value]
wavelength 1.5418          [ Cu Edge]
error density 0.12         [ lambda-dependent detector gain]
                           [ This is probably close enough for ]
                           [ lambda ~ 1 A]
air absorption length 3145  [ lambda-dependen absorption, check Int. tbl.]
                           [ or use the 'airabs' utility.]
```

```
weak level 1.0
peak search file 'peaks.file'
profile fitting radius 20.0
[ALIGNED by D.C. 12/28/2001]
x beam 100.987 y beam 91.60
```

```
[*****CHANGE INFO BELOW*****]
sector 1
raw data file 'images/prok_hgcl1_1_###.img'
oscillation range 1.0 start 1
resolution limits 90.0 2.3
mosaicity 0.8
space group P1
distance 100
[*****]
```

```
box print 2.0 2.0
spot elliptical 0.5 0.5 0.0
background elliptical 0.6 0.6 0.0

overlap spot
go
write predictions
print statistics
list
```

# INDEXING REFLECTIONS

continued

**Distortion Index-**  
the amount of distortion  
of the observed lattice  
required to fit a perfect  
Bravais lattice of a particular  
type. We want the lowest  
possible distortion with  
the highest symmetry.  
Acceptable distortion  
indexes are below 1%.  
Here, primitive tetragonal  
appears the best choice.

Lattice	Metric tensor distortion index	Best cell (symmetrized)								Best cell (without symmetry restraints)							
primitive cubic	11.20%	67.97	67.87	101.96	90.03	90.05	90.06	79.27	79.27	79.27	90.00	90.00	90.00	90.00	90.00	90.00	
I centred cubic	24.53%	122.49	96.00	122.46	67.01	46.20	66.94	113.65	113.65	113.65	90.00	90.00	90.00	90.00	90.00	90.00	
F centred cubic	24.68%	140.11	139.98	140.10	57.98	121.96	93.45	140.06	140.06	140.06	90.00	90.00	90.00	90.00	90.00	90.00	
primitive rhombohedral	11.20%	67.87	101.96	67.97	90.05	90.06	90.03	79.27	79.27	79.27	90.05	90.05	90.05	109.31	109.31	139.98	90.00
primitive hexagonal	13.58%	67.97	67.87	101.96	90.03	90.05	90.06	67.92	67.92	101.96	90.00	90.00	90.00	120.00	120.00	120.00	120.00
primitive tetragonal	0.05%	67.97	67.87	101.96	90.03	90.05	90.06	67.92	67.92	101.96	90.00	90.00	90.00	67.92	67.92	101.96	90.00
I centred tetragonal	11.08%	67.97	67.87	225.31	72.51	72.51	90.06	67.92	67.92	225.31	90.00	90.00	90.00	67.92	67.92	225.31	90.00
primitive orthorhombic	0.03%	67.87	67.97	101.96	90.05	89.97	89.94	67.87	67.97	101.96	90.00	90.00	90.00	67.87	67.97	101.96	90.00
C centred orthorhombic	0.04%	96.00	96.10	101.96	90.01	89.95	90.08	96.00	96.10	101.96	90.00	90.00	90.00	96.00	96.10	101.96	90.00
I centred orthorhombic	11.08%	67.87	67.97	225.31	72.51	107.49	89.94	67.87	67.97	225.31	90.00	90.00	90.00	67.87	67.97	225.31	90.00
F centred orthorhombic	11.07%	96.00	96.10	225.31	89.98	115.17	90.08	96.00	96.10	225.31	90.00	90.00	90.00	96.00	96.10	225.31	90.00
primitive monoclinic	0.02%	67.87	101.96	67.97	90.05	90.06	90.03	67.87	101.96	67.97	90.00	90.06	90.00	67.87	101.96	67.97	90.00
C centred monoclinic	0.03%	96.00	96.10	101.96	89.99	90.05	90.08	96.00	96.10	101.96	90.00	90.05	90.00	96.00	96.10	101.96	90.00
primitive triclinic	0.00%	67.87	67.97	101.96	90.05	90.03	90.06										
crystal rotx, roty, rotz -60.845 -44.897 85.491																	

1208 reflections indexed  
Initial goodness of fit, chi\*\*2  
should be less than 2.00 in  
x and y Last number is the  
predicted improvement in fit  
in future refinement cycles.  
It is zero at the end of  
refinement.

Autoindex Xbeam, Ybeam 150.73 150.59

position 1208 chi\*\*2 x 4.06 y 3.16 pred. decrease: 0.000 \* 1208 = 0.0

HIGHER SYMMETRY

14 BRAVAI LATTICES

LOWER SYMMETRY

Best fit cell (without symmetry  
restraints). For the correct  
choice of lattice, these  
dimensions should be very  
close to the symmetrized cell  
above.

Best cell (symmetrized)  
i.e. the best fit perfect Bravais  
lattice.

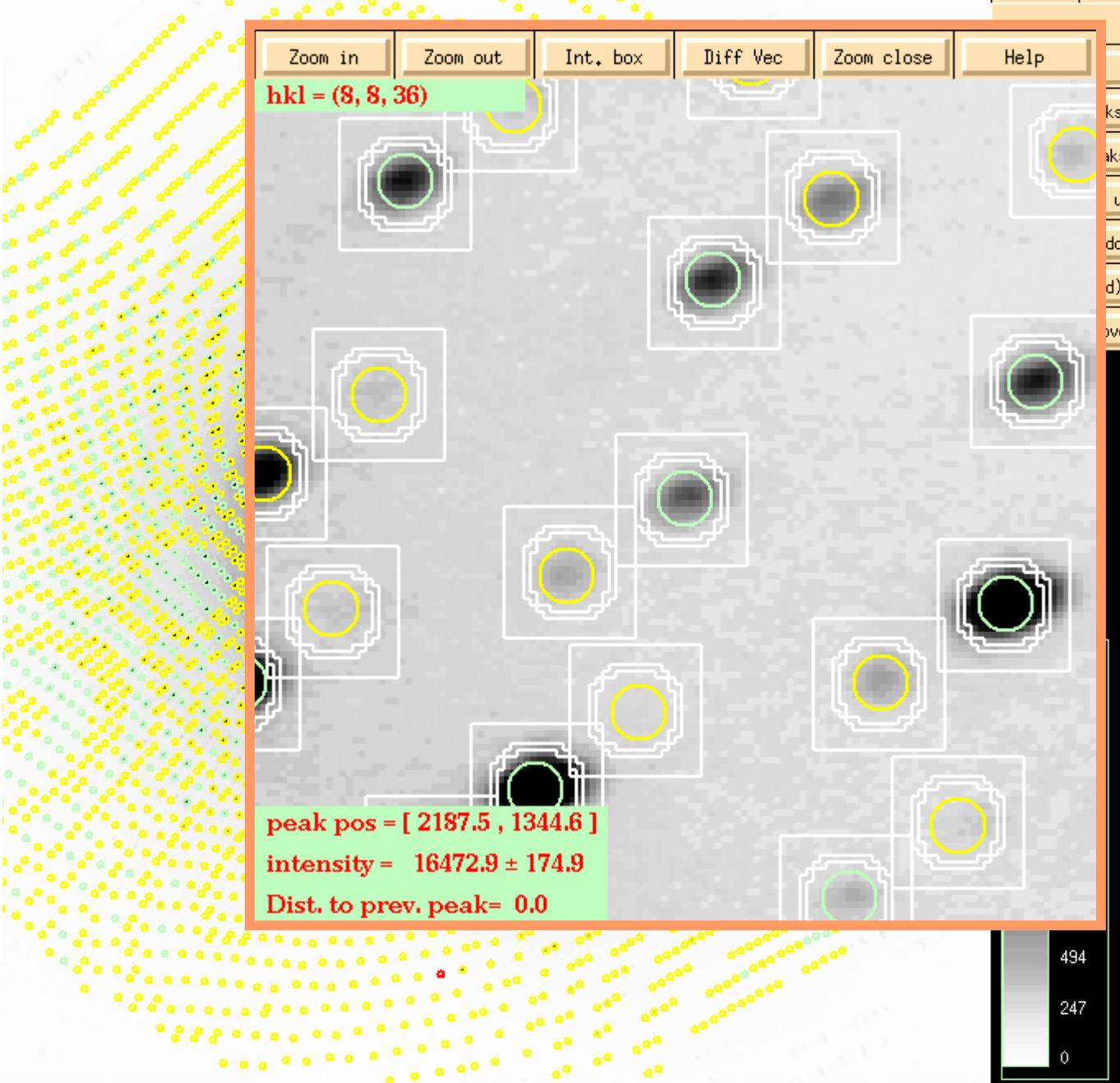
Initial crystal orientation



Select a space group with desired Bravais Lattice  
(e.g. new space group P4)

Predicted pattern should match observed diffraction pattern.

“go” to refine



Insert refined unit cell and crystal orientation parameters into integration script (integ.dat). Type “list” to obtain refined parameters..

Paste this information into “integ.dat” at the indicated location.

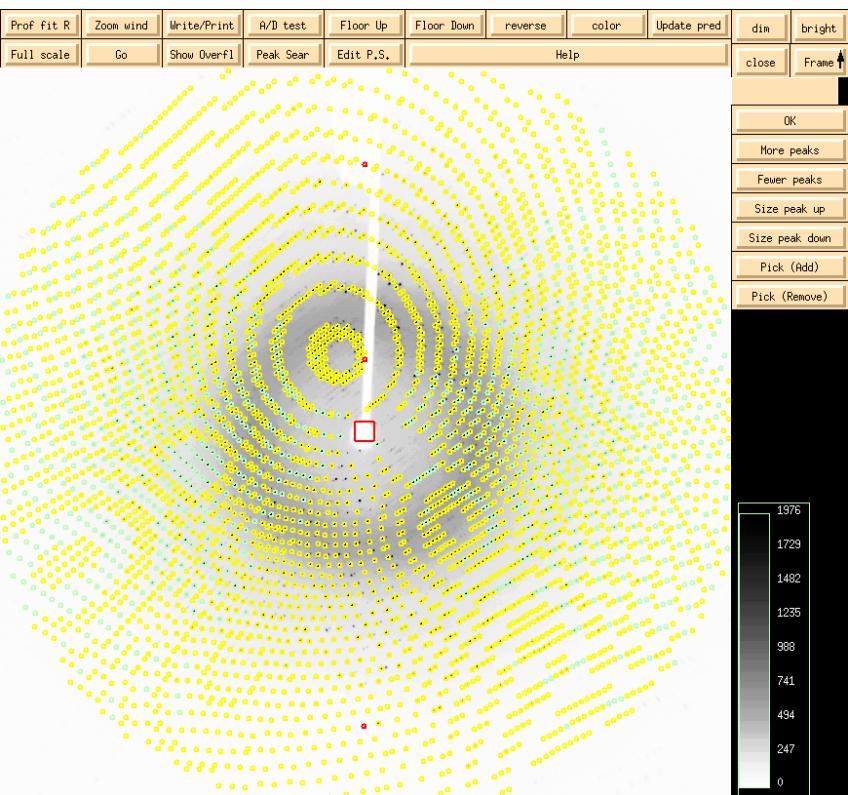
```
cassette rotx -0.04 roty -0.04 rotz 0.00 2 theta 0.00  
distance 100.38  
x beam 150.749 y beam 150.617  
y scale 0.99849  
film rotation 180.000  
skew 0.00013  
crossfire y 0.221 x 0.260 xy 0.028  
goniostat single axis  
goniostat orientation 0.000 0.000  
motor axis 0.000000 1.000000 0.000000  
profile fitting radius 30.00  
resolution limits 90.0 2.00  
wavelength 1.54180  
monochromator 0.000  
oscillation start 1.00 end 2.00  
mosaicty 0.500  
spindle axis -1.0 0.0 vertical axis 0.0 0.1  
unit cell 68.098 68.098 102.196 90.000 90.000 90.000  
crystal rotx -153.921 roty -3.305 rotz -45.111
```

delete this line

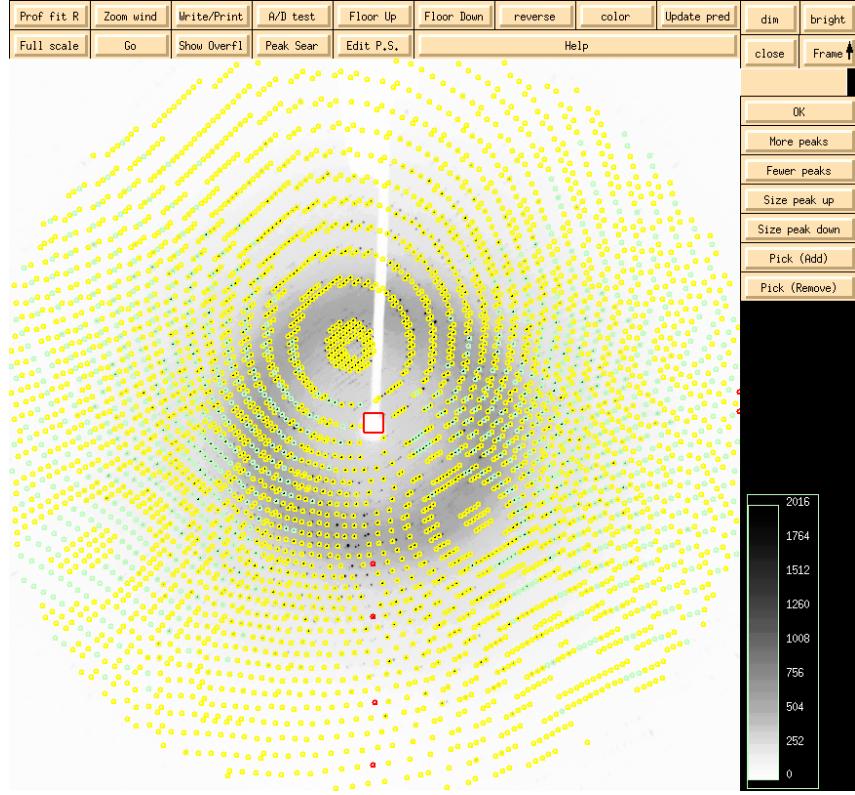
```
title refine_all_images  
format raxis4 100  
  
error density 0.6  
error systematic 5.0 partiality 0.1 positional 0.050  
weak level 5.0  
profile fitting radius 30.0  
  
[*****INSERT HERE BLOCK OF DATA FROM AUTOINDEXING****]  
  
cassette rotx -0.04 roty -0.04 rotz 0.00 2 theta 0.00  
distance 100.38  
x beam 150.749 y beam 150.617  
y scale 0.99849  
film rotation 180.000  
skew 0.00013  
crossfire y 0.221 x 0.260 xy 0.028  
goniostat single axis  
goniostat orientation 0.000 0.000  
motor axis 0.000000 1.000000 0.000000  
profile fitting radius 30.00  
resolution limits 90.0 2.00  
wavelength 1.54180  
monochromator 0.000  
mosaicty 0.500  
spindle axis -1.0 0.0 vertical axis 0.0 0.1  
unit cell 68.098 68.098 102.196 90.000 90.000 90.000  
crystal rotx -153.921 roty -3.305 rotz -45.111  
  
[*****CHANGE INFO BELOW *****]  
  
space group P4  
oscillation start 1.0 range 1.0  
raw data file 'images/prok_nat2####.osc'  
film output file 'x/prok_nat2####.x'  
sector 1 to 360  
box 2.5 2.5  
spot elliptical .75 .75 0.  
background elliptical .85 0.85 0  
  
[*****]  
  
overlap spot  
start refinement  
print no profiles  
fit all  
resolution limits 100.0 5.0  
go go go go  
write predictions  
list
```

Paste parameters into integration script (integ.dat).

# Spot positions on all following images can be predicted from info learned from first image.

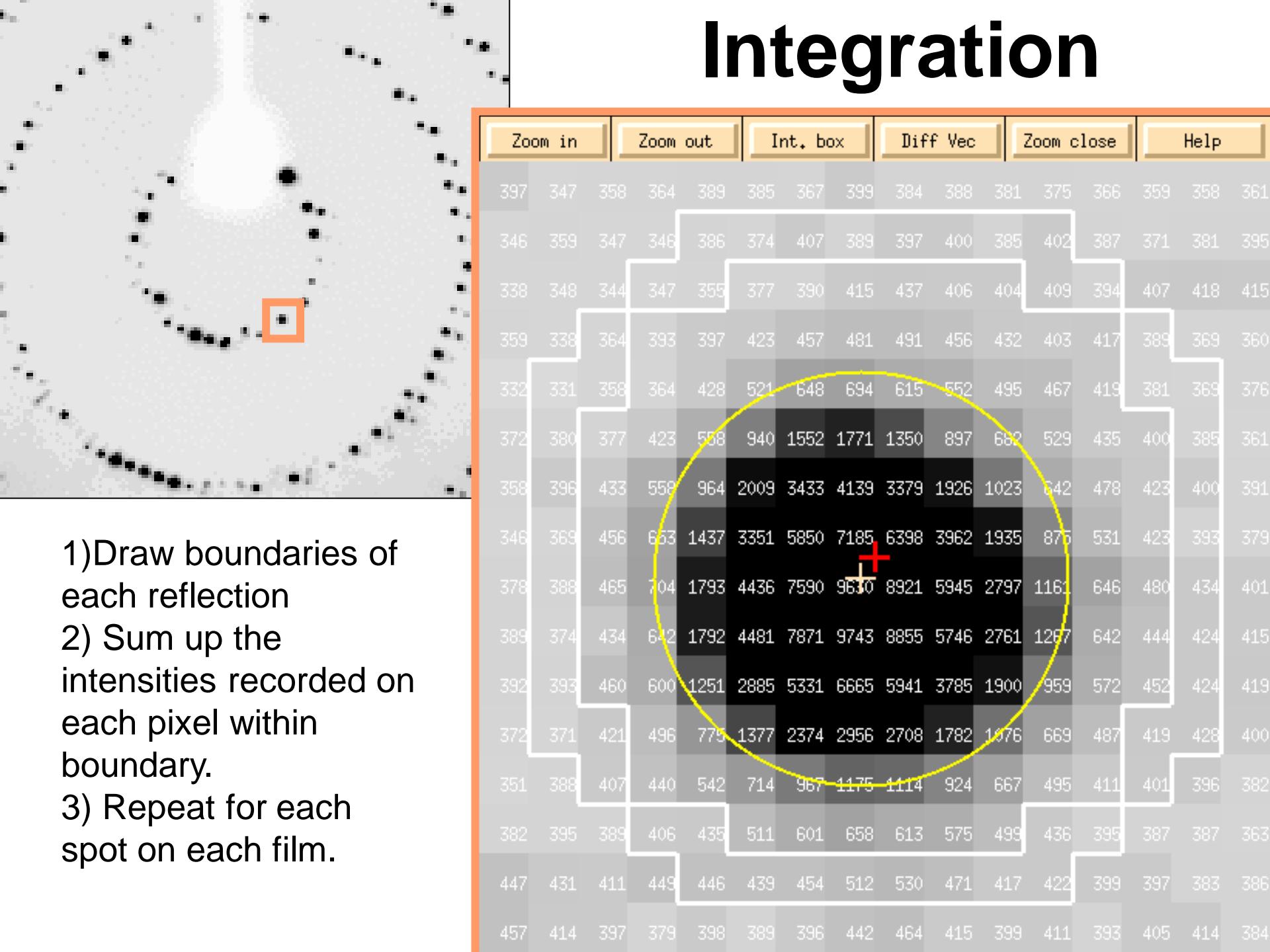


Film 1, exposed over 1 to 2 degrees



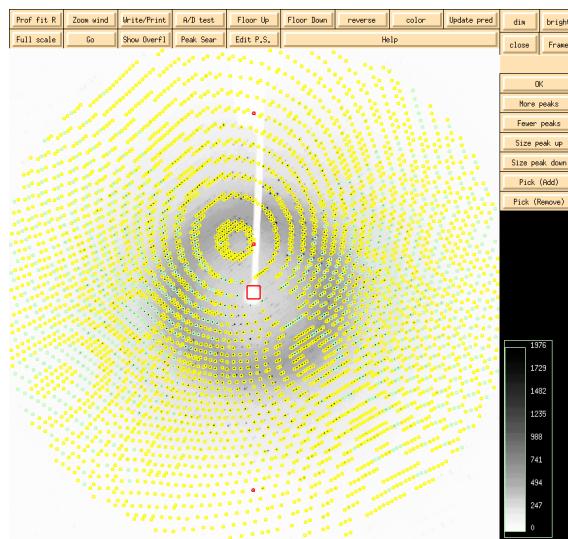
Film 2, exposed over 2 to 3 degrees

# Integration



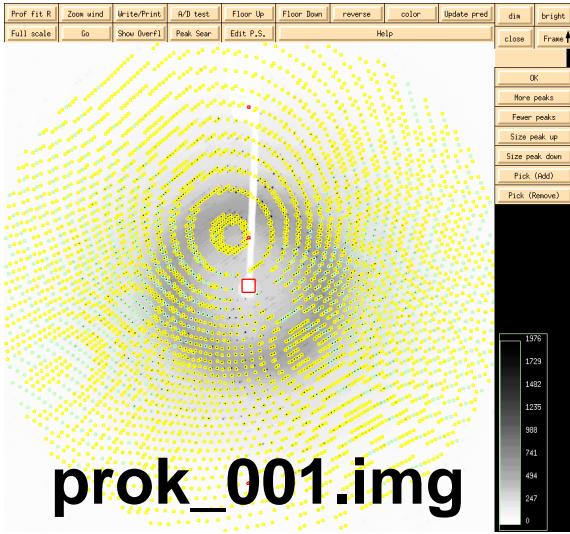
- 1) Draw boundaries of each reflection
- 2) Sum up the intensities recorded on each pixel within boundary.
- 3) Repeat for each spot on each film.

# Integrated intensities are written to .x files

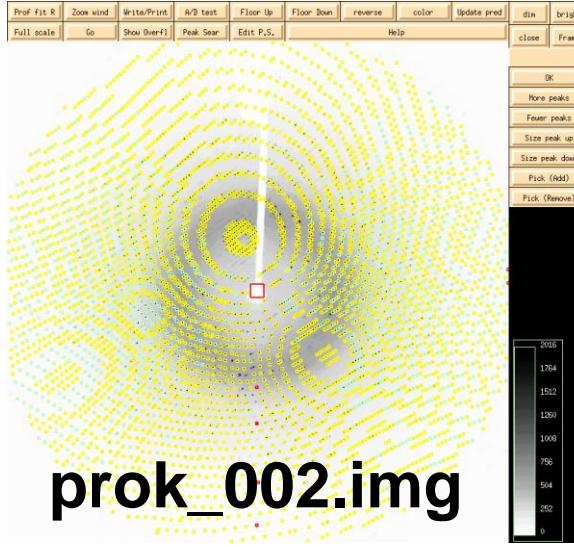


Film 1, exposed over  
1 to 2 degrees

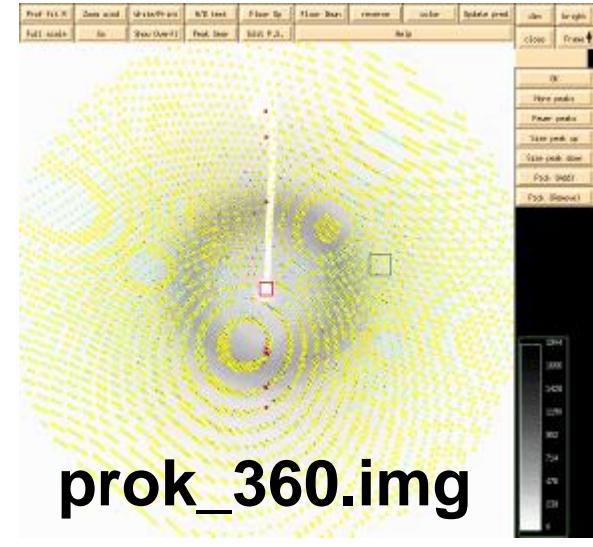
# One .x file for each film



Film 1, exposed over  
1 to 2 degrees



Film 2, exposed over  
2 to 3 degrees



Film 360, exposed over  
360 to 361 degrees



<b>h</b>	<b>k</b>	<b>l</b>	<b>flag</b>	<b>I(profit)</b>	<b>I(prosum)</b>	<b><math>\chi^2</math></b>	<b><math>\sigma(I)</math></b>	<b>cos incid.</b>	<b>X pix</b>	<b>Y pix</b>
29	20	-33	1	202.3	200.8	1.36	17.4	0.556	6.4	1353.0
29	21	-31	1	102.1	105.0	1.08	7.7	0.560	16.8	1421.5
30	26	-19	1	1291.2	1323.2	1.19	50.0	0.554	23.9	1808.7
31	28	-11	1	1554.0	1618.7	1.26	95.1	0.536	24.2	2061.6
29	22	-29	1	24.0	25.2	1.29	1.2	0.564	28.0	1489.1
29	20	-33	1	202.3	200.8	1.36	17.4	0.556	6.4	1353.0
29	21	-31	1	102.1	105.0	1.08	7.7	0.560	16.8	1421.5
30	26	-19	1	1291.2	1323.2	1.19	50.0	0.554	23.9	1808.7
31	28	-11	1	1554.0	1618.7	1.26	95.1	0.536	24.2	2061.6
29	21	-31	1	102.1	105.0	1.08	7.7	0.560	16.8	1421.5
30	26	-19	1	1291.2	1323.2	1.19	50.0	0.554	23.9	1808.7
31	28	-11	1	1554.0	1618.7	1.26	95.1	0.536	24.2	2061.6
29	21	-31	1	102.1	105.0	1.08	7.7	0.560	16.8	1421.5
30	26	-19	1	1291.2	1323.2	1.19	50.0	0.554	23.9	1808.7
31	28	-11	1	1554.0	1618.7	1.26	95.1	0.536	24.2	2061.6

**prok\_001.x**

<b>h</b>	<b>k</b>	<b>l</b>	<b>flag</b>	<b>I(profit)</b>	<b>I(prosum)</b>	<b><math>\chi^2</math></b>	<b><math>\sigma(I)</math></b>	<b>cos incid.</b>	<b>X pix</b>	<b>Y pix</b>
29	20	-33	1	52.3	50.8	1.36	17.4	0.556	6.4	1353.0
29	21	-31	1	102.1	105.0	1.08	7.7	0.560	16.8	1421.5
30	26	-19	1	1291.2	1323.2	1.19	50.0	0.554	23.9	1808.7
31	28	-11	1	1554.0	1618.7	1.26	95.1	0.536	24.2	2061.6
29	22	-29	1	24.0	25.2	1.29	1.2	0.564	28.0	1489.1
29	20	-33	1	202.3	200.8	1.36	17.4	0.556	6.4	1353.0
29	21	-31	1	102.1	105.0	1.08	7.7	0.560	16.8	1421.5
30	26	-19	1	1291.2	1323.2	1.19	50.0	0.554	23.9	1808.7
31	28	-11	1	1554.0	1618.7	1.26	95.1	0.536	24.2	2061.6
29	21	-31	1	102.1	105.0	1.08	7.7	0.560	16.8	1421.5
30	26	-19	1	1291.2	1323.2	1.19	50.0	0.554	23.9	1808.7
31	28	-11	1	1554.0	1618.7	1.26	95.1	0.536	24.2	2061.6

**prok\_002.x**

<b>h</b>	<b>k</b>	<b>l</b>	<b>flag</b>	<b>I(profit)</b>	<b>I(prosum)</b>	<b><math>\chi^2</math></b>	<b><math>\sigma(I)</math></b>	<b>cos incid.</b>	<b>X pix</b>	<b>Y pix</b>
29	20	-33	1	212.3	220.8	1.36	17.4	0.556	6.4	1353.0
29	21	-31	1	102.1	105.0	1.08	7.7	0.560	16.8	1421.5
30	26	-19	1	1291.2	1323.2	1.19	50.0	0.554	23.9	1808.7
31	28	-11	1	1554.0	1618.7	1.26	95.1	0.536	24.2	2061.6
29	22	-29	1	24.0	25.2	1.29	1.2	0.564	28.0	1489.1
29	20	-33	1	202.3	200.8	1.36	17.4	0.556	6.4	1353.0
29	21	-31	1	102.1	105.0	1.08	7.7	0.560	16.8	1421.5
30	26	-19	1	1291.2	1323.2	1.19	50.0	0.554	23.9	1808.7
31	28	-11	1	1554.0	1618.7	1.26	95.1	0.536	24.2	2061.6
29	21	-31	1	102.1	105.0	1.08	7.7	0.560	16.8	1421.5
30	26	-19	1	1291.2	1323.2	1.19	50.0	0.554	23.9	1808.7
31	28	-11	1	1554.0	1618.7	1.26	95.1	0.536	24.2	2061.6

**prok\_360.x**

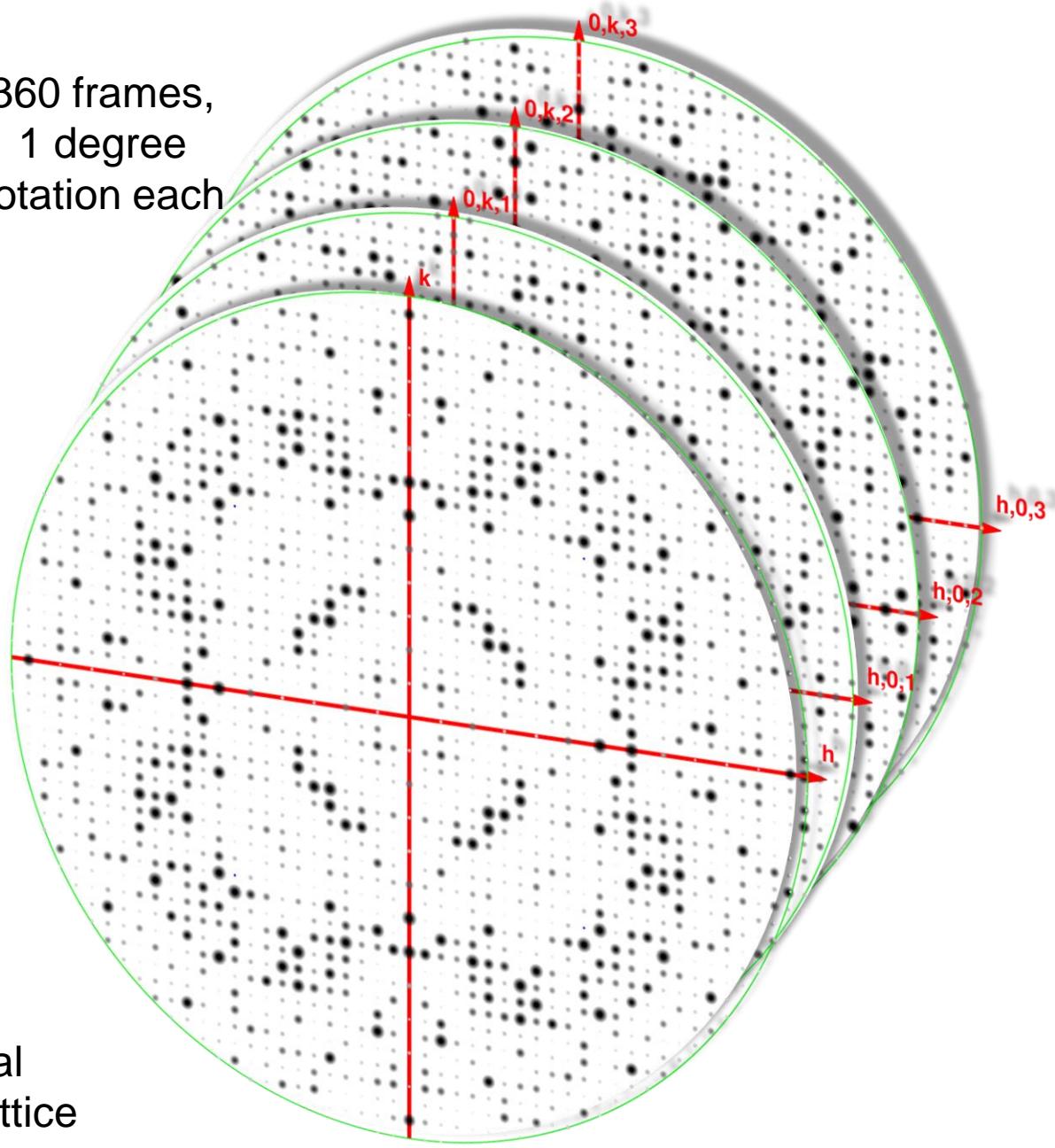
# **prok\_001 -> 360.x**



Intensities from .x files can be mapped onto the reciprocal lattice for visual inspection.

- 1) Accuracy will improve if we merge symmetry-related observations.
  - 2) But, we must test if rotational symmetry exists between lattice points.

360 frames,  
1 degree  
rotation each

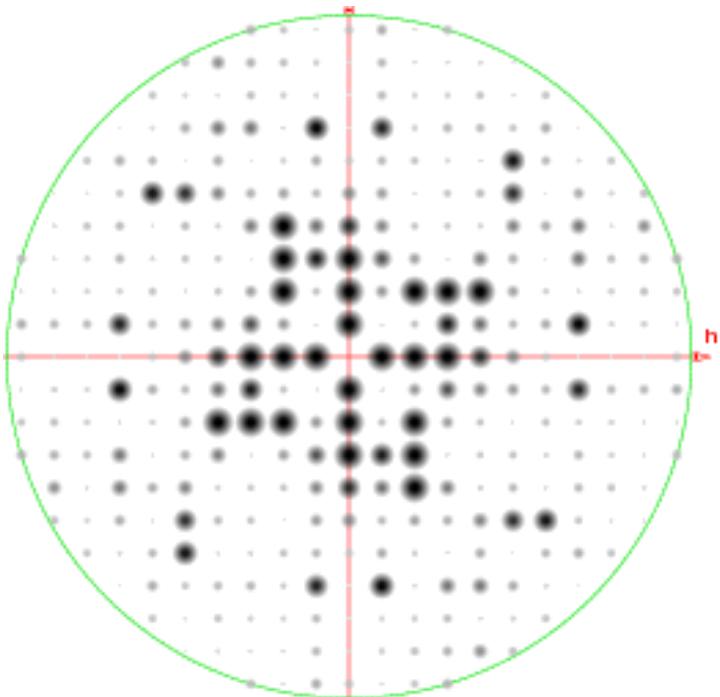


# Choose point group symmetry (4 or 422)

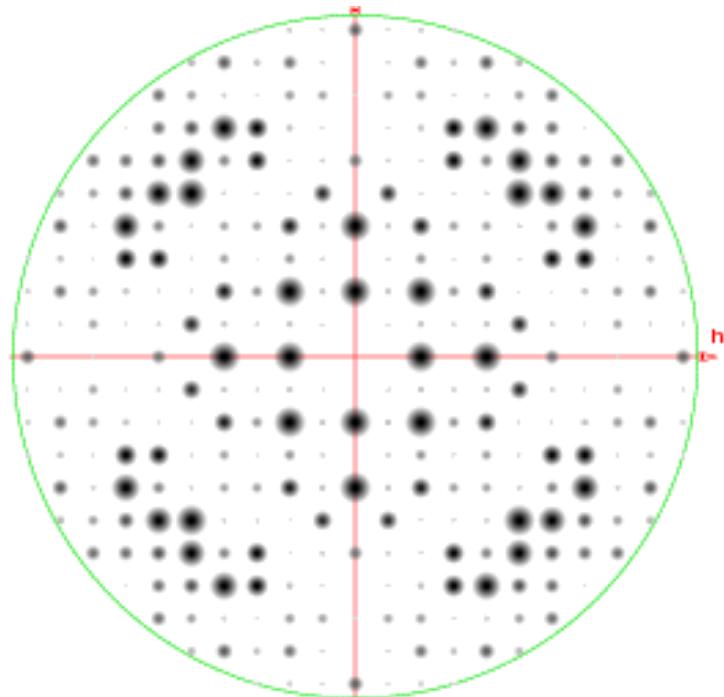
	Tetragonal P
	Three axes at right angles; two equal: $a=b \neq c$ ; $\alpha=\beta=\gamma=90^\circ$

System	Bravais lattice	Point group	65 Chiral Space Groups					
Triclinic	P	1	P1					
Monoclinic	P	2	P2	P <sub>2</sub> <sub>1</sub>				
	C		C2					
Orthorhombic	P	222	P222	P <sub>2</sub> 2 <sub>2</sub> <sub>1</sub>	P <sub>2</sub> <sub>1</sub> 2 <sub>1</sub> 2	P <sub>2</sub> <sub>1</sub> 2 <sub>1</sub> 2 <sub>1</sub>		
	C		C222 <sub>1</sub>	C222				
	F		F222					
	I		I222	I <sub>2</sub> 2 <sub>1</sub> 2 <sub>1</sub>				
Tetragonal	P	4	P4	P <sub>4</sub> <sub>1</sub>	P <sub>4</sub> <sub>2</sub>	P <sub>4</sub> <sub>3</sub>		
	I		I4	I <sub>4</sub> <sub>1</sub>				
	P	422	P422	P4 <sub>2</sub> ,2	P <sub>4</sub> ,22	P <sub>4</sub> ,2 <sub>1</sub> 2	P <sub>4</sub> ,22	P <sub>4</sub> ,2 <sub>1</sub> 2
			P4 <sub>3</sub> 22	P4 <sub>3</sub> 2 <sub>1</sub> 2				
	I		I422	I <sub>4</sub> 22				
Trigonal	P	3	P3	P31	P32			
	R		R3					
	P	32	P312	P321	P3 <sub>1</sub> 12	P3 <sub>1</sub> 21	P3 <sub>2</sub> 12	P3 <sub>2</sub> 21
	R		R32					
Hexagonal	P	6	P6	P6 <sub>1</sub>	P6 <sub>2</sub>	P6 <sub>3</sub>	P6 <sub>4</sub>	P6 <sub>5</sub>
		622	P622	P6 <sub>1</sub> 22	P6 <sub>2</sub> 22	P6 <sub>3</sub> 22	P6 <sub>4</sub> 22	P6 <sub>5</sub> 22
Cubic	P	23	P23	P <sub>2</sub> ,3				
	F		F23					
	I		I23	I <sub>2</sub> ,3				
	P	432	P432	P4 <sub>2</sub> 32	P4 <sub>3</sub> 2	P4 <sub>1</sub> 32		
	F		F432	F4 <sub>2</sub> 32				
	I		I432	I4 <sub>1</sub> 32				

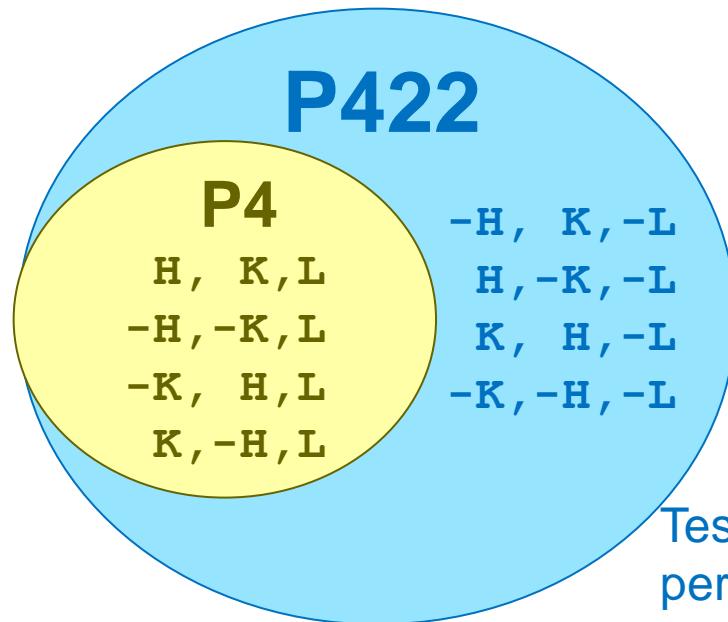
**Point group 4**



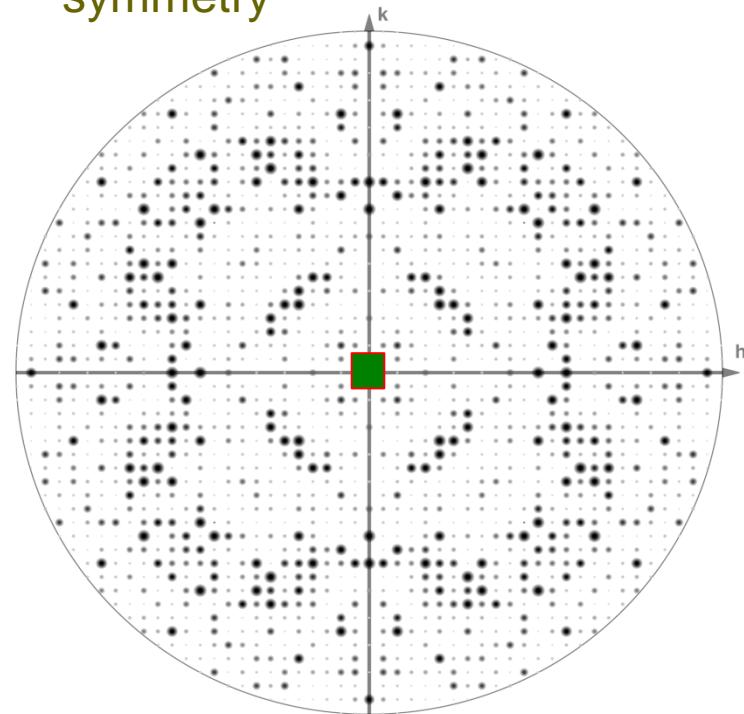
**Point group 422**



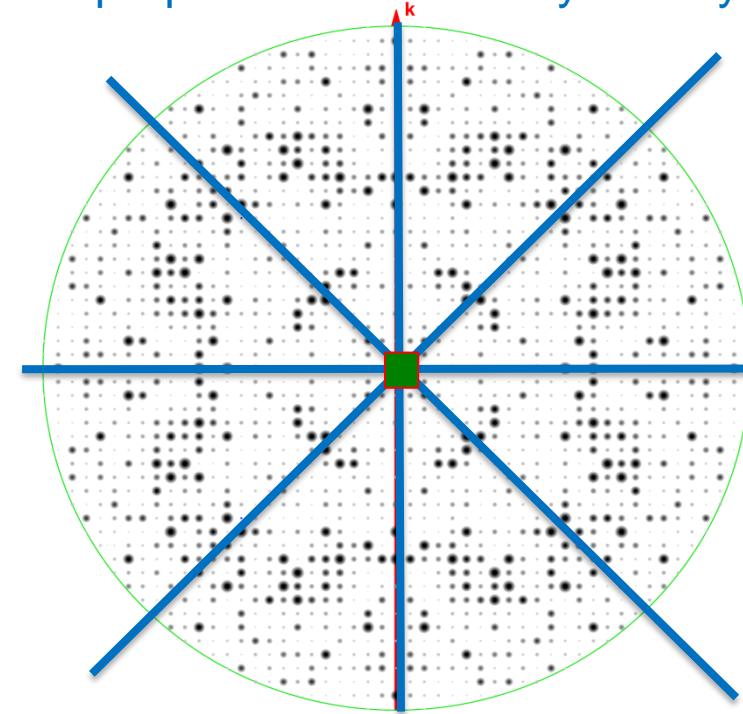
# Is it Point group 4 or Point group 422 ?



Test existence of 4-fold symmetry



Test existence of 4-fold & perpendicular 2-fold symmetry



j observations of the reflection 30 22 6

j	H	K	L	I
1	30	22	6	550
2	-30	-22	6	500
3	30	-22	-6	543

?

H, K, L =

-H, -K, L =

H, -K, -L

**Discrepancy**  
between symmetry related reflections

$$R_{\text{merge}} = \frac{\sum |I_j - \langle I \rangle|}{\sum I_j}$$

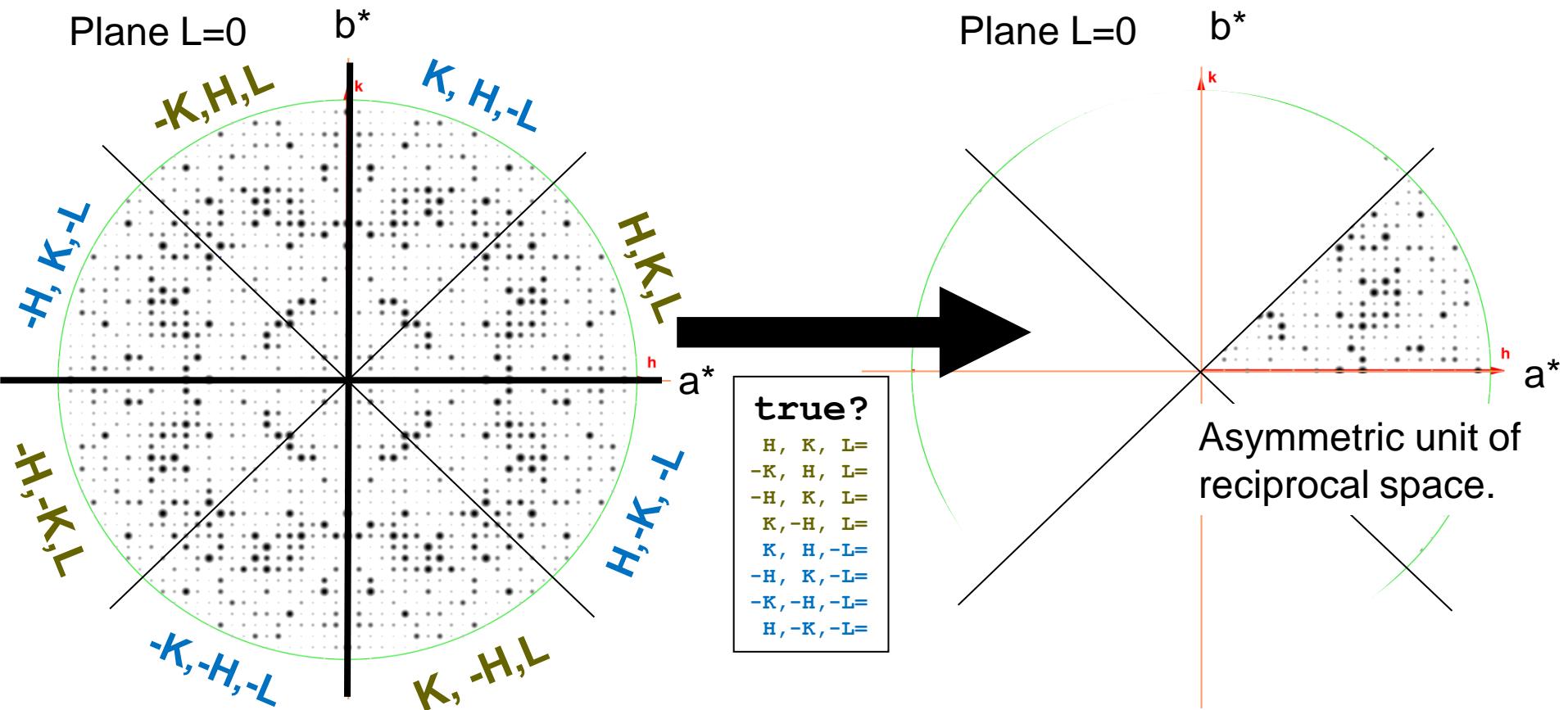
$$\begin{aligned} \langle I \rangle &= (550 + 500 + 543) / 3 \\ &= 531 \end{aligned}$$

$$\begin{aligned} \sum |I_j - \langle I \rangle| &= |550 - 531| + |500 - 531| + |543 - 531| \\ &= 19 + 31 + 12 \\ &= 62 \end{aligned}$$

$$\begin{aligned} \sum I_j &= 550 + 500 + 543 \\ &= 1593 \end{aligned}$$

$$R_{\text{merge}} = 62 / 1593 = 0.039 = 3.9\%$$

# Merge

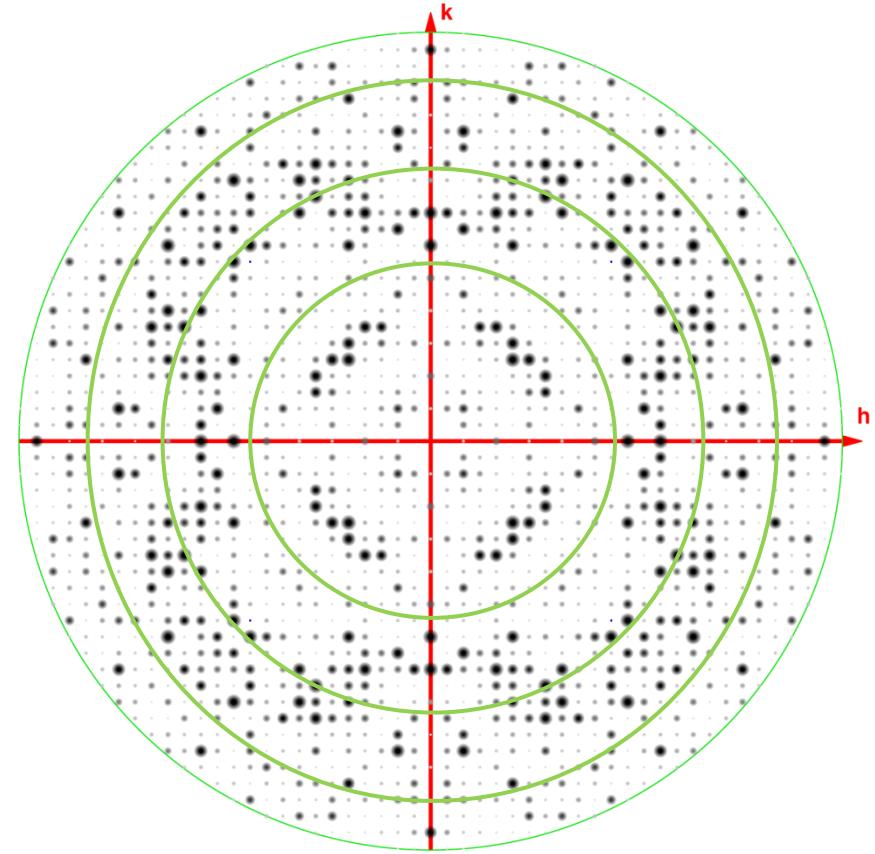


Merge together (average) reflections related by these symmetry operators in reciprocal space.

# Statistics are analyzed as a function of resolution (N shells).

Discrepancy between symmetry related reflections ( $R_{\text{merge}}$ ) increases with increasing resolution. Why?

Shell	$R_{\text{merge}}$
100-5.0 Å	0.04
5.0-3.0 Å	0.06
3.0-2.0 Å	0.08
2.0-1.7 Å	0.15

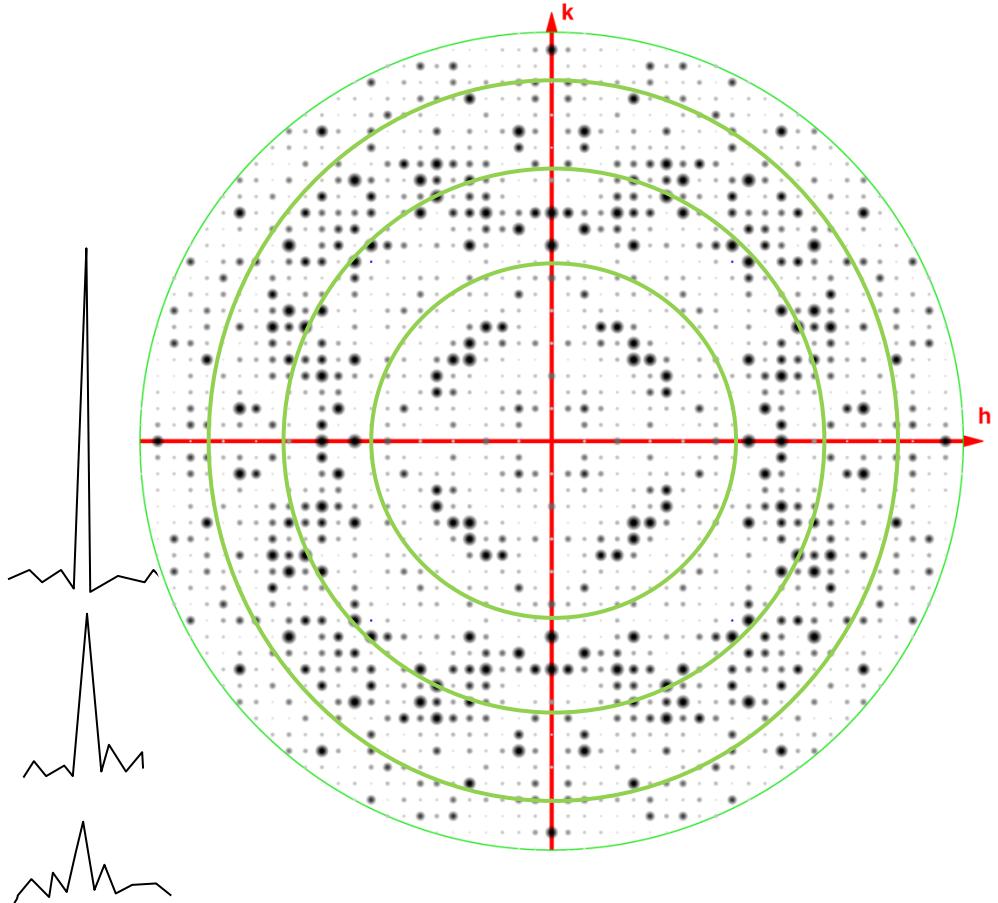


# SIGNAL TO NOISE RATIO ( $I/\sigma$ )

Average  $I/\sigma$  decreases with increasing resolution

High resolution shells with  $I/\sigma < 2$  should be discarded.

Shell	$I/\sigma_I$
100-5.0 Å	20.0
5.0-3.0 Å	10.0
3.0-2.0 Å	7.0
2.0-1.7 Å	3.0



# COMPLETENESS?

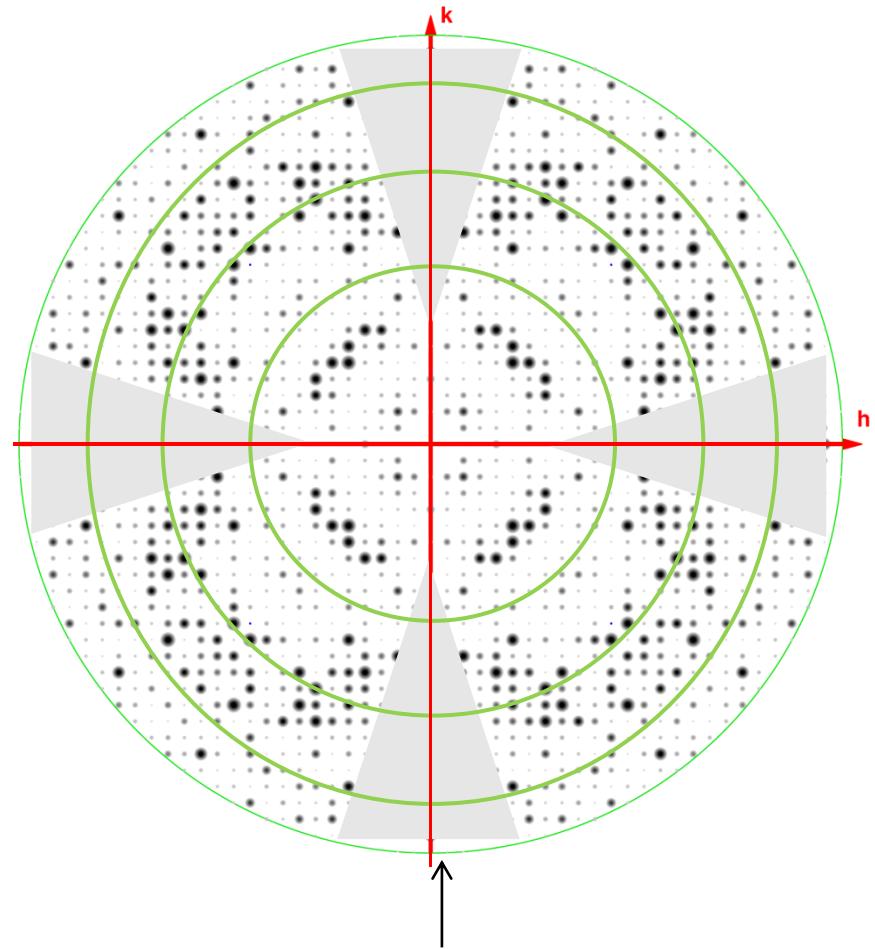
What percentage of reciprocal Lattice was measured for a given Resolution limit?

Better than 90% I hope.

Shell	completeness
100-5.0Å	99.9%
5.0-3.0Å	95.5%
3.0-2.0Å	89.0%
2.0-1.7Å	85.3%

Overall

92.5%



Missing wedges of data.

# Assignment

Table 1. Crystal and X-Ray Diffraction Data of Native, Heavy-Atom Derivatives of Aa-RNase III

	Native	Au Derivative	U Derivative
Resolution (Å)	2.15	3.00	2.90
Space group	P2 <sub>1</sub> 2 <sub>1</sub> 2 <sub>1</sub>	P2 <sub>1</sub> 2 <sub>1</sub> 2 <sub>1</sub>	P2 <sub>1</sub> 2 <sub>1</sub> 2 <sub>1</sub>
Unit cell dimensions: a (Å)	46.16	46.40	46.49
b (Å)	51.12	51.89	51.60
c (Å)	123.47	124.72	118.40
Measured reflections	68,832	23,588	28,927
Unique reflections	16,542	11,071	12,404
Overall completeness (%)	99.5	96.2	98.4
Last shell <sup>a</sup> completeness (%)	99.9	89.7	93.8
Overall R <sub>sym</sub> <sup>b</sup>	0.038	0.054	0.054
Last shell R <sub>sym</sub>	0.370	0.235	0.446
Overall I/σ(I)	23.4	18.2	11.3
Last shell I/σ(I)	3.3	3.5	1.7

<sup>a</sup>2.23–2.15 Å for native, 3.11–3.00 Å for the Au derivative, 3.00–2.90 Å for the U derivative

<sup>b</sup>R<sub>sym</sub> =  $\sum |(I - \langle I \rangle)| / \sum (I)$ , where I is the observed intensity.