Leonard Euler rolls three fair standard six-sided dice. The results are D_1, D_2, and D_3 respectively. Find the probability of each event below:

$D_1 \cdot D_2$ is a perfect square

$1^2 \sim 6^2$, 1×4, 4×1 $\Rightarrow \frac{8}{36} = \frac{2}{9}$

$D_1 > D_2 > D_3$

$$\left.\begin{array}{l} P(D_1=D_2=D_3) = \frac{1}{36} \\ P(D_1=D_2\neq D_3) = \frac{5}{36} \end{array}\right\} \Rightarrow P(D_1\neq D_2, D_1\neq D_3, D_2\neq D_3) = 1-\frac{1}{36}-3\times\frac{5}{36} = \frac{20}{36}$$

$\therefore P(D_1>D_2>D_3) = \frac{1}{6}\times\frac{20}{36} = \frac{5}{54}$

$D_1 + D_2$ is prime

$1+1$, $1+2$, $2+1$, $1+4 \sim 4+1$, $1+6 \sim 6+1$, $5+6$, $6+5$

$\therefore \frac{1+2+4+6+2}{36} = \frac{5}{12}$

$D_1 + D_2 + D_3$ is prime

$C(=3) = 1$

$C(=5) = \binom{4}{2} = 6$

$C(=7) = \binom{6}{2} = 15$

$C(=11) = C(D_1+D_2=5\sim10) = 4+5+6+5+4+3 = 27$

$C(=13) = C(D_1+D_2=7\sim12) = 6+5+\cdots+1 = 21$

$C(=17) = 3$

$\therefore \frac{1+6+15+27+21+3}{216} = \frac{73}{216}$

$|D_1 + D_2 - D_3|$ is a perfect square

$$C(=0) = C(D_1+D_2=2\sim6) = 1+2+\cdots+5 = 15$$

$$C(=|1|) = C(D_1+D_2=2\sim7) = 1+2+\cdots+6 = 21$$

$$C(=|-1|) = C(D_1+D_2=2\sim5) = 1+2+3+4 = 10$$

$$C(=|4|) = C(D_1+D_2=5\sim10) = 4+5+6+5+4+3 = 27$$

$$C(=|-4|) = 1 \qquad C(=9) = C(D_1+D_2=10\sim12) = 3+2+1 = 6$$

$$\Rightarrow \frac{15+21+10+27+1+6}{216} = \frac{80}{216} = \frac{10}{27}$$

$|D_1 + D_2 - D_3|$ is prime

$$C(=|2|) = C(D_1+D_2=3\sim8) = 2+3+4+5+6+5 = 25$$

$$C(=|-2|) = C(D_1+D_2=2\sim4) = 1+2+3 = 6$$

$$C(=|3|) = C(D_1+D_2=4\sim9) = 3+4+5+6+5+4 = 27$$

$$C(=|-3|) = C(D_1+D_2=2,3) = 1+2 = 3$$

$$C(=|5|) = C(D_1+D_2=6\sim11) = 5+6+5+4+3+2 = 25$$

$$C(=|7|) = C(D_1+D_2=8\sim12) = 5+4+\cdots+1 = 15$$

$$C(=|11|) = C(D_1+D_2=12) = 1$$

$$\therefore \frac{25+6+27+3+25+15+1}{216} = \frac{17}{36}$$

$D_1 \cdot D_2 \cdot D_3$ is prime

$$P(1\times1\times\text{prime}) = \frac{1}{6}\times\frac{1}{6}\times\frac{1}{2} = \frac{1}{72}$$

$$\therefore P(1\times\text{prime}\times1) = P(\text{prime}\times1\times1) = \frac{1}{72}$$

$$\therefore \frac{1}{72}\times3 = \frac{1}{24}$$

$D_1 = D_2 = D_3$

$$\frac{1}{36}$$

$|D_1 + D_2 - D_3|$ is composite

$$C(=|4|) = C(D_1+D_2=5\sim10) = 4+5+6+5+4+3 = 27$$

$$C(=|-4|) = C(D_1+D_2=2) = 1$$

$$C(=6) = C(D_1+D_2=7\sim12) = 6+5+\cdots+1 = 21$$

$$C(=8) = C(D_1+D_2=9\sim12) = 4+3+2+1 = 10$$

$$C(=9) = C(D_1+D_2=10\sim12) = 3+2+1 = 6$$

$$C(=10) = C(D_1+D_2=11\sim12) = 2+1 = 3$$

$$\therefore \frac{27+1+21+10+6+3}{216} = \frac{17}{54}$$

$D_1 = D_2$ $\frac{1}{6}$

$D_1 \cdot D_2$ is odd $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$D_1 > |D_1 - D_2|$

$P(D_1 \geq D_2 \text{ or } D_2 > D_1 > \frac{D_2}{2})$

$= P(D_1 > \frac{D_2}{2}) = P(D_1 = 1\sim6, 2D_1 > D_2)$

$= \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{3}{6} + \frac{1}{6} \times \frac{5}{6} + \frac{1}{6} \times 1 \times 3 = \frac{3}{4}$

$|D_1 - D_2|$ is a perfect square

$C(=0) = 6$ $C(=4) = 2 \times 2 = 4$

$C(=1) = 5 \times 2 = 10$ $\therefore \frac{6+4+10}{36} = \frac{5}{9}$

$|D_1 + D_2 - D_3|$ is odd

$\frac{1}{2}$

$D_1 + D_2 + D_3$ is odd

$\frac{1}{2}$

$D_1 = |D_1 - D_2|$

$P(D_2 = 0 \text{ or } D_2 = 2D_1)$

$= P(D_2 = 2D_1) = \frac{3}{36} = \frac{1}{12}$

$|D_1 - D_2|$ is prime

$C(=2) = 4 \times 2 = 8$ $\quad C(=5) = 2$

$C(=3) = 3 \times 2 = 6$

$\therefore \dfrac{8+2+6}{36} = \dfrac{4}{9}$

D_1 is prime

$\dfrac{3}{6} = \dfrac{1}{2}$

$D_1 + D_2 + D_3$ is a perfect square

$C(=4) = 3$

$C(=9) = \binom{8}{2} - 3$ (stars-and-bars, removing 1,1,7)

$C(=16) = \underset{(4,6,6)}{3} + \underset{(5,5,6)}{3} = 6$

$\therefore \dfrac{3+\binom{8}{2}-3+6}{216} = \dfrac{17}{108}$

$D_1 + D_2 + D_3$ is composite

$C(=3) = 1$ $\quad C(=5) = \binom{4}{2} = 6$ $\quad C(=7) = \binom{6}{2} = 15$

$C(=11) = C(D_1 + D_2 = 5 \sim 10) = 4+5+6+5+4+3 = 27$

$C(=13) = C(D_1 + D_2 = 7 \sim 12) = 6+5+\cdots+1 = 21$

$C(=17) = 3$

$\therefore \dfrac{216-1-6-15-27-21-3}{216} = \dfrac{143}{216}$

$D_1 \cdot D_2$ is composite

prime: $1\times 2,\ 2\times 1,\ 1\times 3,\ 3\times 1,\ 1\times 5,\ 5\times 1$

1 : 1×1

$\therefore \dfrac{36-6-1}{36} = \dfrac{29}{36}$

$D_1 > D_2$

$P(D_1 = D_2) = \frac{1}{6} \quad \therefore \quad \frac{1-\frac{1}{6}}{2} = \frac{5}{12}$

$(D_1 + D_2) | D_3$

$C(D_3=2,\ D_1+D_2=2 \qquad = 1$

or $D_3=3,\ D_1+D_2=3 \qquad + 2$

or $D_3=4,\ D_1+D_2=2,4 \qquad + 1+3$

or $D_3=5,\ D_1+D_2=5 \qquad + 4$

or $D_3=6,\ D_1+D_2=2,3,6) \qquad + 1+2+5 = 19 \qquad \therefore \frac{19}{216}$

$D_1 \cdot D_2 \cdot D_3$ is odd

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

$D_1 + D_2$ is composite

$C(= 2, 3, 5, 7, \text{or } 11)$

$= 1 + 2 + 4 + 6 + 2 = 15 \qquad \therefore \frac{36-15}{36} = \frac{7}{12}$

$D_1 \cdot D_2 \cdot D_3$ is a perfect square

$C(=1) = C(\{1,1,1\}) = 1$

$C(=4) = C(\{1,1,4\}, \text{or } \{1,2,2\}) = 3+3 = 6$

$C(=9) = C(\{1,3,3\}) = 3$

$C(=16) = C(\{1,4,4\}, \{2,2,4\}) = 3+3 = 6$

$C(=25) = C(\{1,5,5\}) = 3$

$C(=36) = C(\{1,6,6\}, \{4,3,3\}, \{2,3,6\}) = 3+3+6 = 12$

$C(=64) = C(\{4,4,4\}) = 1$

$C(=100) = C(\{4,5,5\}) = 3$

$C(=144) = C(\{4,6,6\}) = 3$

$\therefore \frac{1+6+3+6+3+12+1+3+3}{216} = \frac{19}{108}$

$D_1 + D_2$ is a perfect square

$$\left.\begin{aligned} C(=4) &= 3 \\ C(=9) &= 4 \end{aligned}\right\} \Rightarrow \frac{3+4}{36} = \frac{7}{36}$$

$D_1 \mid D_2$

$C\big((1,1) \sim (6,6)\big) = 6$

$C\big((1,2) \sim (1,6)\big) = 5$

$C\big((2,4), (2,6), (3,6)\big) = 3$

$\therefore \dfrac{6+5+3}{36} = \dfrac{7}{18}$

D_1 and D_2 are coprime

$C(D_1=1) = 6$ $\quad C(D_1=4) = 3$

$C(D_1=2) = 3$ $\quad C(D_1=5) = 5$

$C(D_1=3) = 4$ $\quad C(D_1=6) = 2$

$\therefore \dfrac{6+3+4+3+5+2}{36} = \dfrac{23}{36}$

$D_1 \cdot D_2$ is prime

$1\times2,\ 2\times1,\ 1\times3,\ 3\times1,\ 1\times5,\ 5\times1$

$\therefore \dfrac{6}{36} = \dfrac{1}{6}$

$(D_1 - D_2) \mid D_1$

$C(D_1 - D_2 = \pm1) = 5\times2 = 10$

$C(D_1 - D_2 = \pm2,\ D_1 = 2,4,6) = C\big(\{2,4\}, \{4,2\}, \{4,6\}, \{6,4\}\big) = 4$

$C(D_1 - D_2 = \pm3,\ D_1 = 3,6) = C\big(\{3,6\}, \{6,3\}\big) = 2$

$C(D_1 - D_2 = \pm4,\ D_1 = 4) = 0$

$\therefore \dfrac{10+4+2}{36} = \dfrac{4}{9}$

$(D_1 \cdot D_2)|D_3$

$C(D_1D_2=1, D_3=1\sim6) = 1\times6=6$

$C(D_1D_2=2, D_3=2,4,6) = 2\times3=6$

$C(D_1D_2=3, D_3=3,6) = 2\times2=4$

$C(D_1D_2=4, D_3=4) = 3\times1=3$

$C(D_1D_2=5, D_3=5) = 2\times1=2$

$C(D_1D_2=6, D_3=6) = 4\times1=4$

$\therefore \dfrac{6+6+4+3+2+4}{216} = \dfrac{25}{216}$

$|D_1 - D_2|$ is composite

$= P(D_1-D_2=\pm4) = \dfrac{2\times2}{36} = \dfrac{1}{9}$

D_1 is odd $\frac{1}{2}$

D_1 is composite $\frac{2}{6} = \frac{1}{3}$

$|D_1 - D_2|$ is odd

$\frac{1}{2}$

$D_1 \cdot D_2 \cdot D_3$ is composite

$C(=1) = C(\{1,1,1\}) = 1$

$C(=\text{prime}) = C(\{1,1,\text{prime}\}) = 3\times3=9$

$\therefore \dfrac{216-1-9}{216} = \dfrac{103}{108}$

$D_1 + D_2$ is odd

$\frac{1}{2}$

D_1 is a perfect square

$\frac{2}{6} = \frac{1}{3}$