

Find the coefficients of x , x^2 , and x^3 , respectively, in the expansions of each of the following expressions.

$$(x-1)^5 \times (2x-1)^2$$

$$x: \quad \binom{5}{1} \binom{2}{0} (-1)^4 (-1)^2 + \binom{5}{0} \binom{2}{1} (-1)^5 \cdot 2 (-1) = 9$$

$$x^2: \quad \binom{5}{2} (-1)^{3+2} + \binom{5}{1} \binom{2}{1} \cdot 2 \cdot (-1)^{4+1} + 2^2 \cdot (-1)^5 = -34$$

$$x^3: \quad \binom{5}{3} (-1)^{2+2} + \binom{5}{2} \binom{2}{1} \cdot (-1)^{3+1} \cdot 2 + \binom{5}{1} \cdot 2^2 \cdot (-1)^4 = 70$$

$$(2x^2 - 1)^5 \times (x+1)$$

$$x: \quad (-1)^5 = -1$$

$$x^2: \quad \binom{5}{1} \times 2 \times (-1)^4 = 10$$

$$x^3: \quad \binom{5}{1} \times 2 \times (-1)^4 = 10$$

$$(x-1)^8 \times (2x+1)^2$$

$$x: \quad \binom{8}{1} \cdot (-1)^7 + (-1)^8 \cdot \binom{2}{1} \cdot 2 = -4$$

$$x^2: \quad \binom{8}{2} (-1)^6 + \binom{8}{1} (-1)^7 \binom{2}{1} \cdot 2 + (-1)^8 \binom{2}{2} \cdot 2^2 = 0$$

$$x^3: \quad \binom{8}{3} (-1)^5 + \binom{8}{2} (-1)^6 \binom{2}{1} \cdot 2 + \binom{8}{1} (-1)^7 \binom{2}{2} \cdot 2^2 = 24$$

$$(x^3 - 1)^7 \times (x + 2)^3$$

x: $(-1)^7 \cdot \binom{3}{1} \cdot 2^2 = -12$

x^2 : $(-1)^7 \cdot \binom{3}{2} \cdot 2 = -6$

x^3 : $\binom{7}{1} (-1)^6 \cdot 2^3 + (-1)^7 = 55$

$$(x - 1)^7 \times (2x^3 + 1)^4$$

x: $\binom{7}{1} (-1)^6 = 7$

x^2 : $\binom{7}{2} (-1)^5 = -21$

x^3 : $\binom{7}{3} (-1)^4 + (-1)^7 \binom{4}{1} \cdot 2 = 27$

$$(x^2 + 1)^6 \times (x - 2)^5$$

x: $\binom{5}{1} (-2)^4 = 80$

x^2 : $\binom{6}{1} \cdot (-2)^5 + \binom{5}{2} (-2)^3 = -272$

x^3 : $\binom{6}{1} \binom{5}{1} (-2)^4 + \binom{5}{3} (-2)^2 = 520$

$$(x^2 + 1)^9 \times (x + 2)^2$$

$$x: \quad \binom{2}{1} \cdot 2 = 4$$

$$x^2: \quad \binom{9}{1} \cdot 2^2 + 1 = 37$$

$$x^3: \quad \binom{9}{1} \cdot \binom{2}{1} \cdot 2 = 36$$

$$(x + 1)^2 \times (2x^2 - 1)$$

$$x: \quad -2$$

$$x^2: \quad -1 + 2 = 1$$

$$x^3: \quad 2 \times 2 = 4$$

$$(x^2 - 1)^6 \times (2x - 1)^2$$

$$x: \quad (-1)^6 \cdot \binom{2}{1} \cdot 2 \cdot (-1) = -4$$

$$x^2: \quad \binom{6}{1} (-1)^5 + (-1)^6 \cdot 4 = -2$$

$$x^3: \quad \binom{6}{1} (-1)^5 \cdot \binom{2}{1} \cdot 2 \cdot (-1) = 24$$

$$(2x - 1)^7 \times (x - 1)$$

x: $\binom{7}{1} \cdot 2 \cdot (-1)^6 \cdot (-1) + (-1)^7 = -15$

$$x^2: \binom{7}{2} \cdot 2^2 \cdot (-1)^5 \cdot (-1) + \binom{7}{1} \cdot 2 \cdot (-1)^6 = 98$$

$$x^3: \binom{7}{3} \cdot 2^3 \cdot (-1)^4 \cdot (-1) + \binom{7}{2} \cdot 2^2 \cdot (-1)^5 = -364$$

$$(x + 1)^5 \times (2x^3 + 1)^3$$

x: 5

$x^2:$ 10

$$x^3: \binom{5}{3} + \binom{3}{1} \cdot 2 = 16$$

$$(2x^3 + 1)^7 \times (x + 1)^3$$

x: 3

$x^2:$ 3

$$x^3: \binom{7}{1} \cdot 2 + 1 = 15$$

$$(x^3 + 1)^7 \times (2x - 1)$$

$$x: 2$$

$$x^2: 0$$

$$x^3: -7$$

$$(2x + 1)^2 \times (x + 1)^2$$

$$x: 4+2=6$$

$$x^2: 4+4\times2+1 = 13$$

$$x^3: 4\times2+4 = 12$$

$$(x - 1)^8 \times (2x - 1)$$

$$x: 2+8=10$$

$$x^2: \binom{8}{2}(-1) + \binom{8}{1}(-1)\cdot2 = -44$$

$$x^3: \binom{8}{2}\cdot2 + \binom{8}{3} = 112$$

$$(2x+1)^4 \times (x+1)^4$$

$$x: 2^4 + 4 = 12$$

$$x^2: 2^2 \binom{4}{2} + 2 \times 4 \times 4 + \binom{4}{2} = 62$$

$$x^3: 2^3 \binom{4}{3} + 2^2 \binom{4}{2} \binom{4}{1} + 2 \binom{4}{1} \binom{4}{2} + \binom{4}{3}$$

$$= 32 + 96 + 48 + 4 = 180$$

$$(2x+1)^6 \times (x-1)^4$$

$$x: 2^6 + (-4) = 8$$

$$x^2: 2^2 \binom{6}{2} + 2 \binom{6}{1} (-1) \binom{4}{1} + \binom{4}{2}$$

$$= 60 - 48 + 6 = 18$$

$$x^3: 2^3 \binom{6}{3} + 2^2 \binom{6}{2} (-1) \binom{4}{1} + 2 \binom{6}{1} \binom{4}{2} + (-1) \binom{4}{3}$$

$$= 160 - 240 + 72 - 4 = -12$$

$$(x-1)^8 \times (x^2 - 2)$$

$$x: -8 \cdot (-2) = 16$$

$$x^2: \binom{8}{2} (-2) + 1 = -55$$

$$x^3: -\binom{8}{3} (-2) + (-1) \binom{8}{1} \cdot 1$$

$$= 112 - 8 = 104$$

$$(2x - 1)^7 \times (x^3 - 1)$$

$$x: 2 \cdot 7 \cdot (-1) = -14$$

$$x^2: 2^2 \cdot \binom{7}{2} \cdot (-1) \cdot (-1) = 84$$

$$x^3: 2^3 \binom{7}{3} \cdot (-1) + (-1) = -28$$

$$(x - 1)^9 \times (2x^2 - 1)^2 = (x-1)^9 (4x^4 - 4x^2 + 1)$$

$$x: 9$$

$$x^2: -\binom{9}{2} + (-1) \cdot (-4) = -32$$

$$x^3: \binom{9}{3} + \binom{9}{1} (-4) = 48$$

$$(x^3 + 1)^6 \times (x - 2)^5$$

$$x: 5 \cdot (-2)^4 = 80$$

$$x^2: \binom{5}{2} \cdot (-2)^3 = -80$$

$$x^3: \binom{6}{1} \cdot (-2)^5 + \binom{5}{3} \cdot (-2)^2 = -152$$

$$(2x^3 + 1) \times (x - 1)$$

$$x: \quad |$$

$$x^2: \quad 0$$

$$x^3: \quad -2$$

$$(x^2 + 1)^4 \times (x - 2)^3$$

$$x: \quad 3 \times (-2)^2 = 12$$

$$x^2: \quad 4 \cdot (-2)^3 + \binom{3}{2}(-2) = -38$$

$$x^3: \quad \binom{4}{1} \cdot \binom{3}{1} \cdot (-2)^2 + 1 = 49$$

$$(x - 1)^7 \times (x + 2)^3$$

$$x: \quad 7 \times 2^3 + (-1) \cdot \binom{3}{1} \cdot 2^2 = 44$$

$$\begin{aligned} x^2: \quad & \binom{7}{2}(-1) \cdot 2^3 + \binom{7}{1} \cdot \binom{3}{1} \cdot 2^2 + (-1) \binom{3}{2} \cdot 2 \\ & = -224 + 84 - 6 = -146 \end{aligned}$$

$$\begin{aligned} x^3: \quad & \binom{7}{3} \cdot 2^3 - \binom{7}{2} \cdot \binom{3}{1} \cdot 2^2 + \binom{7}{1} \cdot \binom{3}{2} \cdot 2 - 1 \\ & = 280 - 252 + 42 - 1 = 69 \end{aligned}$$

$$(x^3 - 1)^5 \times (2x + 1)$$

$$x: -2$$

$$x^2: 0$$

$$x^3: 5$$

$$(x - 1)^3 \times (x^3 - 2)$$

$$x: 3(-2) = -6$$

$$x^2: 3 \cdot (-1) \cdot (-2) = 6$$

$$x^3: 1 \cdot (-1) - 1 = -3$$

$$(x + 1)^8 \times (x^2 + 2)$$

$$x: 8 \times 2 = 16$$

$$x^2: \binom{8}{2} \times 2 + 1 = 57$$

$$x^3: \binom{8}{3} \times 2 + \binom{8}{1} = 120$$

$$(2x+1)^6 \times (x+1)^4$$

$$x: 6 \times 2 + 4 = 16$$

$$x^2: \binom{6}{2} \cdot 2^2 + \binom{6}{1} \cdot 2 \cdot \binom{4}{1} + \binom{4}{2}$$

$$= 60 + 48 + 6 = 114$$

$$x^3: \binom{6}{3} \cdot 2^3 + \binom{6}{2} \cdot 2^2 \cdot \binom{4}{1} + \binom{6}{1} \cdot 2 \cdot \binom{4}{2} + \binom{4}{3}$$

$$= 160 + 240 + 72 + 4 = 476$$

$$(2x^3 + 1)^9 \times (x - 1)$$

$$x: 1$$

$$x^2: 0$$

$$x^3: \binom{9}{1} \cdot 2 \cdot (-1) = -18$$

$$(x-1)^6 \times (2x+1)^3$$

$$x: -6 + 3 \times 2 = 0$$

$$x^2: \binom{6}{2} - \binom{6}{1} \cdot \binom{3}{1} \cdot 2 + \binom{3}{2} \cdot 2^2$$

$$= 15 - 36 + 12 = -9$$

$$x^3: \binom{6}{3} \cdot (-1) + \binom{6}{2} \cdot \binom{3}{1} \cdot 2 + \binom{6}{1} \cdot (-1) \binom{3}{2} \cdot 2^2 + 8$$

$$= -20 + 90 - 72 + 8 = 6$$

$$(x^3 + 1)^2 \times (2x + 1)$$

$$x: 2$$

$$x^2: 0$$

$$x^3: 2$$

$$(2x^3 - 1)^7 \times (x - 1)$$

$$x: -1$$

$$x^2: 0$$

$$x^3: \binom{7}{1} \cdot 2 \cdot (-1) = -14$$

$$(x + 1)^4 \times (x + 2)^2$$

$$x: 4 \times 4 + 4 = 20$$

$$x^2: \binom{4}{2} \times 4 + \binom{4}{1} \times 4 + 1 = 41$$

$$x^3: \binom{4}{3} \times 4 + \binom{4}{2} \times 4 + \binom{4}{1} = 44$$

$$(x^2 - 1)^4 \times (x + 2)^2$$

$$x: 4$$

$$x^2: -4 \cdot 4 + 1 = -15$$

$$x^3: -4 \times 4 = -16$$

$$(2x+1)^5 \times (x^2 - 1)^2$$

$$x: 5 \times 2 = 10$$

$$x^2: \binom{5}{2} \cdot 2^2 - 2 = 38$$

$$x^3: \binom{5}{3} \cdot 2^3 - \binom{5}{1} \cdot 2 \cdot (-2)$$

$$= 80 + 20 = 100$$

$$(x^2 - 1)^9 \times (2x + 1)$$

$$x: -2$$

$$x^2: 9$$

$$x^3: 9 \times 2 = 18$$

$$(x-1)^9 \times (x^3 - 2)$$

$$x: \quad 9 \cdot (-2) = -18$$

$$x^2: \quad \binom{9}{2} \cdot (-1) \cdot (-2) = 72$$

$$x^3: \quad \binom{9}{3} \cdot (-2) + (-1) = -169$$

$$(x-1)^5 \times (2x-1)^5$$

$$x: \quad \binom{5}{1} \cdot (-1) + (-1) \cdot \binom{5}{1} \cdot 2 = -15$$

$$x^2: \quad \binom{5}{2} \cdot (-1) \cdot (-1) + \binom{5}{1} \binom{5}{1} \cdot 2 + (-1) \binom{5}{2} \cdot 2^2 \cdot (-1) \\ = 10 + 50 + 40 = 100$$

$$x^3: \quad \binom{5}{3} \cdot (-1) + \binom{5}{2} \cdot (-1) \cdot \binom{5}{1} \cdot 2 + \binom{5}{1} \cdot \binom{5}{2} \cdot 2^2 \cdot (-1) + (-1) \binom{5}{3} \cdot 2^3 \\ = -10 - 100 - 200 - 80 = -390$$

$$(x+1)^3 \times (2x^3 - 1)^3$$

$$x: \quad -3$$

$$x^2: \quad -3$$

$$x^3: \quad -1 + \binom{3}{1} \cdot 2 = 5$$

$$(2x^2 + 1)^4 \times (x - 1)^2 = (2x^2 + 1)^4 (x^2 - 2x + 1)$$

$$x: -2$$

$$x^2: \binom{4}{1} \cdot 2 + 1 = 9$$

$$x^3: \binom{4}{1} \cdot 2 \cdot (-2) = -16$$

$$(x^2 - 1)^5 \times (2x - 1)^3$$

$$x: (-1) \binom{3}{1} \cdot 2 = -6$$

$$\begin{aligned} x^2: & \binom{5}{1} \cdot (-1) + (-1) \cdot \binom{3}{2} \cdot 2^2 \cdot (-1) \\ & = -5 + 12 = 7 \end{aligned}$$

$$x^3: \binom{5}{1} \cdot \binom{3}{1} \cdot 2 + (-1) \cdot 8 = 22$$

$$(x^3 + 1)^6 \times (x + 2)$$

$$x: 1$$

$$x^2: 0$$

$$x^3: \binom{6}{1} \times 2 = 12$$

$$(x-1)^8 \times (x^2+2)^3$$

$$x: \quad \binom{8}{1} \cdot (-1) \cdot 8 = -64$$

$$x^2: \quad \binom{8}{2} \cdot 8 + \binom{3}{1} \cdot 2^2 = 224 + 12 = 236$$

$$x^3: \quad \binom{8}{3} \cdot (-1) \cdot 8 + \binom{8}{1} \cdot (-1) \cdot \binom{3}{1} \cdot 2^2$$

$$= -448 - 96 = -544$$

$$(2x-1)^3 \times (x+1)^3$$

$$x: \quad \binom{3}{1} \cdot 2 + (-1) \cdot 3 = 3$$

$$x^2: \quad \binom{3}{2} \cdot 2^2 \cdot (-1) + \binom{3}{1} \cdot 2 \cdot \binom{3}{1} + (-1) \cdot \binom{3}{2}$$

$$= -12 + 18 - 3 = 3$$

$$x^3: \quad 8 + \binom{3}{2} \cdot 2^2 \cdot (-1) \cdot 3 + \binom{3}{1} \cdot 2 \cdot 3 + (-1)$$

$$= 8 - 36 + 18 - 1 = -11$$

$$(x-1)^2 \times (x+2) = (x^2 - 2x + 1)(x+2)$$

$$x: \quad -3$$

$$x^2: \quad 0$$

$$x^3: \quad 1$$

$$(x^3 - 1)^7 \times (2x + 1)$$

$$x: -2$$

$$x^2: 0$$

$$x^3: 7$$

$$(2x - 1)^8 \times (x^3 - 1)^3 = (2x - 1)^8 \cdot (x^9 - 3x^6 + 3x^3 - 1)$$

$$x: \binom{8}{1} \cdot 2 \cdot (-1) \cdot (-1) = 16$$

$$x^2: \binom{8}{2} \cdot 2^2 \cdot (-1) = -112$$

$$x^3: \binom{8}{3} \cdot 2^3 \cdot (-1) \cdot (-1) + 3 = 451$$

$$(x - 1)^9 \times (2x - 1)$$

$$x: \binom{9}{1} \cdot (-1) + (-1) \cdot 2 = -11$$

$$x^2: \binom{9}{1} \cdot 2 + \binom{9}{2} \cdot (-1) \cdot (-1) \cdot (-1) = 54$$

$$x^3: \binom{9}{3} \cdot (-1) + \binom{9}{2} \cdot (-1) \cdot 2$$

$$= -84 - 72 = -156$$

$$(x+1)^8 \times (2x^2 - 1)^2$$

$$x: \binom{8}{1} = 8$$

$$x^2: \binom{8}{2} + (-4) = 24$$

$$x^3: \binom{8}{3} + \binom{8}{1} \cdot (-4) = 56 - 32 = 24$$

$$(2x-1)^7 \times (x^2 - 1)^3$$

$$x: \binom{7}{1} \cdot 2 \cdot (-1) = -14$$

$$x^2: \binom{7}{2} \cdot 2^2 \cdot (-1) \cdot (-1) + (-1) \cdot \binom{3}{1} \\ = 84 - 3 = 81$$

$$x^3: \binom{7}{3} \cdot 2^3 \cdot (-1) + \binom{7}{1} \cdot 2 \cdot \binom{3}{1} \\ = -280 + 42 = -238$$