

Extended Euclidean Algorithm

Find the smallest positive integer solution

$$73x - 26y = 1$$

	x	y	$73x - 26y$
①	0	1	-26
②	1	0	73
③	$① \times 3 + ②$	1	3
④	$③ \times 5 - ①$	5	14
			1

$$\therefore (5, 14)$$

$$51x - 31y = 1$$

	x	y	$51x - 31y$
①	0	1	-31
②	1	0	51
③: ①+②	1	1	20
④: ③×2+①	2	3	9
⑤: ③-④×2	-3	-5	2
⑥: ④-⑤×4	14	23	1

$$\therefore (14, 23)$$

$$37x - 93y = 1$$

x	0	1	3	-5		
y	1	0	1	-2		
$37x - 93y$	-93	37	18	1		

$$\therefore (-5 + 93, -2 + 37)$$

$$= (88, 35)$$

$$11x - 37y = 1$$

x	0	1	4	-3	7	-10		
y	1	0	1	-1	2	-3		
$11x - 37y$	-37	11	7	4	3	1		

$$\therefore (-10 + 37, -3 + 11)$$

$$= (27, 8)$$

$$25x - 61y = 1$$

$$\begin{array}{cccccc} x & 0 & 1 & 2 & 5 & 22 \end{array}$$

$$\begin{array}{cccccc} y & 1 & 0 & 1 & 2 & 9 \end{array} \quad \therefore (22, 9)$$

$$25x - 61y \quad -61 \quad 25 \quad -11 \quad 3 \quad 1$$

$$45x - 71y = 1$$

$$\begin{array}{cccccccccc} x & 0 & 1 & 1 & 2 & 3 & 8 & 11 & 30 \end{array}$$

$$\begin{array}{cccccccccc} y & 1 & 0 & 1 & 1 & 2 & 5 & 7 & 19 \end{array} \quad \therefore (30, 19)$$

$$45x - 71y \quad -71 \quad 45 \quad -26 \quad 19 \quad -7 \quad 5 \quad -2 \quad 1$$

$$85x - 38y = 1$$

$$\begin{array}{cccccc} x & 1 & 0 & 1 & 4 & 17 \end{array}$$

$$\begin{array}{cccccc} y & 0 & 1 & 2 & 9 & 38 \end{array} \quad \therefore (17, 38)$$

$$85x - 38y \quad 85 \quad -38 \quad 9 \quad -2 \quad 1$$

$$17x - 94y = 1$$

$$\begin{array}{cccccc} x & 0 & 1 & 5 & 6 & 11 \end{array}$$

$$\begin{array}{cccccc} y & 1 & 0 & 1 & 1 & 2 \end{array} \quad \therefore (-11+94, -2+17)$$

$$17x - 94y \quad -94 \quad 17 \quad -9 \quad 8 \quad -1 \quad = (83, 15)$$

$$91x - 68y = 1$$

$$\begin{array}{r} x \\ \hline 0 & 1 & 1 & 3 \end{array}$$

$$\begin{array}{r} y \\ \hline 1 & 0 & 1 & 4 \end{array} \quad \therefore (3, 4)$$

$$91x - 68y \quad -68 \quad 91 \quad 23 \quad 1$$

$$72x - 23y = 1$$

$$\begin{array}{r} x \\ \hline 0 & 1 & 1 & 8 \end{array}$$

$$\begin{array}{r} y \\ \hline 1 & 0 & 3 & 25 \end{array} \quad \therefore (8, 25)$$

$$72x - 23y \quad -23 \quad 72 \quad 3 \quad 1$$

$$57x - 43y = 1$$

$$\begin{array}{r} x \\ \hline 1 & 0 & 1 & 3 \end{array}$$

$$\therefore (-3 + 43, -4 + 57)$$

$$\begin{array}{r} y \\ \hline 0 & 1 & 1 & 4 \end{array}$$

$$= (40, 53)$$

$$57x - 43y \quad 57 \quad -43 \quad 14 \quad -1$$

$$73x - 17y = 1$$

$$\begin{array}{r} x \\ \hline 1 & 0 & 1 & 3 & 7 \end{array}$$

$$\begin{array}{r} y \\ \hline 0 & 1 & 4 & 13 & 30 \end{array}$$

$$\therefore (7, 30)$$

$$73x - 17y \quad 73 \quad -17 \quad 5 \quad -2 \quad 1$$

$$29x - 90y = 1$$

$$\begin{array}{cccccc} x & 0 & 1 & 3 & 31 \end{array}$$

$$\begin{array}{cccccc} y & 1 & 0 & 1 & 10 \end{array} \quad \therefore (-31+90, -10+29)$$

$$29x - 90y = -90 \quad 29 \quad -3 \quad -1 \quad = (59, 19)$$

$$59x - 57y = 1$$

$$\begin{array}{cccccc} x & 1 & 0 & 1 & 29 \end{array}$$

$$\begin{array}{cccccc} y & 0 & 1 & 1 & 30 \end{array} \quad \therefore (29, 30)$$

$$59x - 57y = 59 - 57 \quad 2 \quad 1$$

$$41x - 81y = 1$$

$$\begin{array}{cccccc} x & 0 & 1 & 1 & 2 \end{array}$$

$$\begin{array}{cccccc} y & 1 & 0 & 1 & 1 \end{array} \quad \therefore (z, 1)$$

$$41x - 81y = -81 \quad 41 \quad -40 \quad 1$$

$$79x - 20y = 1$$

$$\begin{array}{cccccc} x & 1 & 0 & 1 & \end{array} \quad \therefore (-1+20, -4+79)$$

$$\begin{array}{cccccc} y & 0 & 1 & 4 & \end{array} \quad = (19, 75)$$

$$79x - 20y = 79 - 20 \quad -1$$

$$40x - 63y = 1$$

$$\begin{array}{r} x \\ \hline 0 & 1 & 1 & 2 & 3 & 11 \end{array}$$

$$\begin{array}{r} y \\ \hline 1 & 0 & 1 & 1 & 2 & 7 \end{array}$$

$$\therefore (-11 + 63, -7 + 40)$$

$$40x - 63y \quad -63 \quad 40 \quad -23 \quad 17 \quad -6 \quad -1 \quad = (52, 33)$$

$$59x - 74y = 1$$

$$\begin{array}{r} x \\ \hline 0 & 1 & 1 & 5 \end{array}$$

$$\begin{array}{r} y \\ \hline 1 & 0 & 1 & 4 \end{array}$$

$$\therefore (-5 + 74, -4 + 59)$$

$$59x - 74y \quad -74 \quad 59 \quad -15 \quad -1 \quad = (69, 55)$$

$$57x - 92y = 1$$

$$\begin{array}{r} x \\ \hline 0 & 1 & 2 & -3 & 5 & -8 & 21 \end{array}$$

$$\begin{array}{r} y \\ \hline 1 & 0 & 1 & -2 & 3 & -5 & 13 \end{array}$$

$$\therefore (21, 13)$$

$$57x - 92y \quad -92 \quad 57 \quad 22 \quad 13 \quad 9 \quad 4 \quad 1$$

$$89x - 4y = 1$$

$$(1, 22)$$

$$59x - 5y = 1$$

$$(-1+5, -12+59)$$

$$= (4, 47)$$

$$67x - 92y = 1$$

$$\begin{array}{ccccccc} x & 0 & 1 & 1 & 3 & 4 & 11 \end{array}$$

$$\begin{array}{ccccccc} y & 1 & 0 & 1 & 2 & 3 & 8 \end{array} \quad \therefore (11, 8)$$

$$67x - 92y \quad -92 \quad 67 \quad -25 \quad 17 \quad -8 \quad 1$$

$$27x - 35y = 1$$

$$\begin{array}{cccccc} x & 0 & 1 & 1 & 4 & 13 \end{array}$$

$$\begin{array}{cccccc} y & 1 & 0 & 1 & 3 & 10 \end{array} \quad \therefore (13, 10)$$

$$27x - 35y \quad -35 \quad 27 \quad -8 \quad 3 \quad 1$$

$$19x - 21y = 1$$

$$\begin{array}{cccccc} x & 0 & 1 & 1 & 10 \end{array}$$

$$\begin{array}{cccccc} y & 1 & 0 & 1 & 9 \end{array} \quad \therefore (10, 9)$$

$$19x - 21y \quad -21 \quad 19 \quad -2 \quad 1$$

$$71x - 69y = 1$$

$$\begin{array}{cccccc} x & | & 0 & | & 1 & 35 \\ y & | & 0 & | & 1 & 36 \end{array} \quad \therefore (35, 36)$$

$$71x - 69y \quad 71 \quad -69 \quad 2 \quad 1$$

$$80x - 9y = 1$$

$$\begin{array}{cccccc} x & | & 0 & | & 1 \\ y & | & 0 & | & 9 \end{array} \quad \therefore (-1+9, -9+80) = (8, 71)$$

$$53x - 37y = 1$$

$$\begin{array}{cccccc} x & | & 0 & | & 1 & 2 & 7 \\ y & | & 0 & | & 1 & 3 & 10 \end{array} \quad \therefore (7, 10)$$

$$53x - 37y \quad 53 \quad -37 \quad 16 \quad -5 \quad 1$$

$$79x - 54y = 1$$

$$\begin{array}{cccccc} x & | & 0 & | & 1 & 2 & 13 \\ y & | & 0 & | & 1 & 3 & 19 \end{array} \quad \therefore (13, 19)$$

$$79x - 54y \quad 79 \quad -54 \quad 25 \quad -4 \quad 1$$

$$10x - 59y = 1$$

$$(6, 1)$$

$$81x - 46y = 1$$

$$\begin{array}{ccccccc} x & | & 0 & | & 1 & | & 4 \\ y & | & 0 & | & 1 & | & 2 \\ 81x - 46y & | & 81 & | & -46 & | & 35 \end{array}$$

$\therefore (25, 44)$

$$\begin{array}{ccccccc} & & & & -11 & | & 2 \\ & & & & | & | & \\ & & & & -11 & | & 1 \end{array}$$

$$32x - 25y = 1$$

$$\begin{array}{ccccccc} x & | & 0 & | & 1 & | & 4 \\ y & | & 0 & | & 1 & | & 5 \\ 32x - 25y & | & 32 & | & -25 & | & 7 \end{array}$$

$\therefore (-7+25, -9+32) = (18, 23)$

$$21x - 95y = 1$$

$$\begin{array}{ccccccc} x & | & 0 & | & 1 & | & 5 \\ y & | & 1 & | & 0 & | & -2 \\ 21x - 95y & | & -95 & | & 21 & | & 10 \end{array}$$

$\therefore (-9+95, -2+21) = (86, 19)$

$$67x - 10y = 1$$

$$(3, 20)$$

$$97x - 25y = 1$$

$$\begin{array}{cccccc} x & | & 0 & | & -8 \\ y & | & 1 & | & 4 & -31 \end{array}$$

$$\therefore (8, 31)$$

$$97x - 25y \quad 97 \quad -25 \quad -3 \quad -1$$

$$61x - 77y = 1$$

$$\begin{array}{cccccc} x & | & 0 & | & 1 & 5 & 101 \\ y & | & 1 & | & 0 & 4 & 80 \end{array}$$

$$\therefore (101 - 77, 80 - 61)$$

$$61x - 77y \quad -77 \quad 61 \quad -16 \quad -3 \quad 1 \quad = (24, 19)$$

$$70x - 27y = 1$$

$$\begin{array}{cccccc} x & | & 1 & 0 & 1 & 2 & -5 \\ y & | & 0 & 1 & 2 & 5 & -13 \end{array}$$

$$\therefore (-5 + 27, -13 + 70)$$

$$70x - 27y \quad 70 \quad -27 \quad 16 \quad 5 \quad 1 \quad = (22, 57)$$

$$65x - 12y = 1$$

x	1	0	1	2	5	
y	0	1	5	11	27	$\therefore (5, 27)$

$$65x - 12y \quad 65 \quad -12 \quad 5 \quad -2 \quad 1$$

$$38x - 91y = 1$$

x	0	1	3	-2	5	12	
y	1	0	1	-1	2	5	$\therefore (12, 5)$
$38x - 91y$	-91	38	23	15	8	1	

$$43x - 80y = 1$$

x	0	1	2	-13		
y	1	0	1	-7		$\therefore (-13+80, -7+43)$
$43x - 80y$	-80	43	6	1		$= (67, 36)$

$$55x - 54y = 1$$

$$(1, 1)$$

$$89x - 17y = 1$$

$$\begin{array}{cccccc} x & 1 & 0 & 1 & 4 \\ y & 0 & 1 & 5 & 21 & \therefore (-4+17, -21+89) \\ 89x - 17y & 89 & -17 & 4 & -1 & = (13, 68) \end{array}$$

$$68x - 23y = 1$$

$$(-1+23, -3+68)$$

$$= (22, 65)$$

$$13x - 69y = 1$$

$$\begin{array}{cccccc} x & 0 & 1 & 5 & 16 \\ y & 1 & 0 & 1 & 3 & \therefore (16, 3) \\ 13x - 69y & -69 & 13 & -4 & 1 \end{array}$$

$$86x - 3y = 1$$

$$\begin{aligned} & (-1+3, -29+86) \\ & = (2, 57) \end{aligned}$$

$$72x - 11y = 1$$

$$\begin{array}{cccccc} x & 1 & 0 & 1 & 2 \\ \end{array}$$

$$\begin{array}{cccccc} y & 0 & 1 & 6 & 13 \\ \end{array} \quad \therefore (2, 13)$$

$$72x - 11y \quad 72 \quad -11 \quad 6 \quad 1$$

$$47x - 80y = 1$$

$$\begin{array}{cccccc} x & 0 & 1 & 2 & -5 & -17 \\ \end{array}$$

$$\begin{array}{cccccc} y & 1 & 0 & 1 & -3 & -10 \\ \end{array} \quad \therefore (-17+80, -10+47) \\ = (63, 37)$$

$$47x - 80y \quad -80 \quad 47 \quad 14 \quad 5 \quad 1$$

$$67x - 74y = 1$$

$$\begin{array}{cccccc} x & 0 & 1 & 1 & 10 & 21 \\ \end{array}$$

$$\begin{array}{cccccc} y & 1 & 0 & 1 & 9 & 19 \\ \end{array} \quad \therefore (21, 19)$$

$$67x - 74y \quad -74 \quad 67 \quad -7 \quad 4 \quad 1$$

$$83x - 66y = 1$$

$$\begin{array}{cccccc} x & 1 & 0 & 1 & 4 & 35 \\ \end{array}$$

$$\begin{array}{cccccc} y & 0 & 1 & 1 & 5 & 44 \\ \end{array} \quad \therefore (35, 44)$$

$$83x - 66y \quad 83 \quad -66 \quad 17 \quad 2 \quad 1$$

$$83x - 3y = 1$$

$$(-1+3, -28+83)$$

$$= (2, 55)$$

$$23x - 28y = 1$$

x	0	1	1	5	11
y	1	0	1	4	9
$23x - 28y$	-28	23	-5	3	1