Stochastic Optimization in Machine Learning

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Outline

Introduction (1)

Challenges in Machine Learning

- massive amounts of training data
- construction of very large models
- how to handle the high memory/computational demands?

$$\Downarrow$$

Stochastic Methods: Update on small amounts of training data!

Introduction (2)

Optimization Problem

$$\min_{w \in \mathbb{R}^n} F(w) = \mathbb{E}[f(w;\xi)],$$

where $f(w;\xi) = f(w;x_i,z_i) = \mathcal{L}(h(w;x_i);z_i))$.

Empirical Form of Objective Function

$$F(w) = \frac{1}{N}\sum_{i=1}^{N} f(w;x_i,z_i)$$

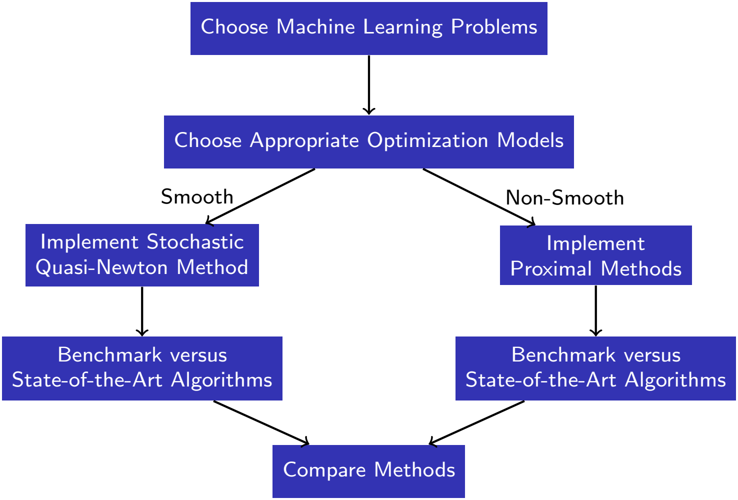
Introduction (3)

Mini-batch Stochastic Gradient

Consider small subset $\mathcal{S} \subset \{1, \ldots, N\}$, with $b := |S| \ll N$

Construct
$$\widehat{\nabla} F(w) = \frac{1}{b} \sum_{i \in \mathcal{S}} \nabla f(w; x_i, z_i)$$

Structure of this Case Study



Machine Learning Problems and Optimization Models

Possible Applications:

- Face recognition
- Text classification
- Speech recognition

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Possible Optimization Models:

- Linear Regression: $\min_w \frac{1}{N} \sum_{i=1}^{N} \|z_i - x_i w\|_2^2$
- Binary Classification:

$$f(w; x_i, z_i) = z_i \log(c(w; x_i)) + (1 - z_i) \log(1 - c(w; x_i))$$

  with $c(w; x_i) = \frac{1}{1+\exp(-x_i^T w)}$

- Neural Nets: Back propagation

Stochastic Quasi-Newton Method (1)

Problem:

- Incorporating second-order information via full Hessian too expensive for large-scale problems
- Use of curvature information highly beneficial for algorithm performance

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Idea:

- Adapt BFGS method to stochastic framework
- Employ limited memory version of BFGS algorithm (L-BFGS)
- Compute gradient based on sample $\mathcal{S}$ of training set
- Compute Hessian update at regular intervals of length L based on small subsample $\mathcal{S}_H$ of training set

Stochastic Quasi-Newton Method (2)

Iteration

$$w_{k+1} = w_k - \alpha_k H_t \widehat{\nabla} F(w_k)$$

Hessian-Update

Choose

$$s_t = \bar{w}_t - \bar{w}_{t-1} \qquad y_t = \widehat{\nabla}^2 F(\bar{w}_t) s_t,$$

with $\bar{w}_t := \sum_{i=k-L}^{k} w_i$ and $\widehat{\nabla}^2 F(w) := \frac{1}{b_H} \sum_{i \in \mathcal{S}_H} \nabla^2 f(w; x_i, z_i)$.

Compute

$$H_{t+1} = (I - \rho_t s_t y_t^T) H_t (I - \rho_t y_t s_t^T) + \rho_t s_t s_t^T,$$

with $\rho_t = \frac{1}{y_t^T s_t}$.

Stochastic Quasi-Newton Method (3)

Stochastic L-BFGS Algorithm

Initialize w_1, H_1, step-length sequence $\alpha_k > 0$
for $k = 1, \ldots,$ **do**
    Choose a sample $\mathcal{S} \subset \{1, \ldots, N\}$
    Compute $w_{k+1} = w_k - \alpha^k H_t \widehat{\nabla} F(w^k)$
    if $\mod (k, L) = 0$ **then**
        Choose a sample $\mathcal{S}_H \subset \{1, \ldots, N\}$
        Compute H_t
    end if
end for

Benchmarking of Stochastic Quasi-Newton Method

Challenge: Economical implementation of Algorithm is necessary for meaningful benchmarking

- Memory-efficient sparse coding
- Calculation of Hessian-Vector Product without storing the Hessian
- Computation of BFGS-Update via two-loop recursion

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Benchmarking:

- Comparison to Stochastic Gradient Descent Method, Standard L-BFGS Method, (Stochastic) Conjugate Gradient Descent
- Comparison of run-time, accuracy, access-points etc. under different parameter regimes and objective functions

Inducing Sparsity

Dictionary Learning

$$\min_{D,\alpha} \frac{1}{N} \sum_{i=1}^{N} \|x_i - D\alpha_i\|_2^2 + \lambda\|\alpha_i\|_1$$

- control on D and α
- better convergence
- modifications of the algorithms

Sparsity: General Formulation

Problem

$$\min \; f(x) + g(x)$$

Proximal Gradient Method

$$prox_{\lambda f}(v) := argmin_x f(x) + \frac{1}{2\lambda}\|x - v\|^2$$

$$x^{k+1} := prox_{\lambda^k g}\left(x^k - \lambda^k \nabla f(x^k)\right)$$

Sparse Formulation

Proximal Gradient Method

Given x^k, λ^{k-1}, $\beta \in (0, 1)$
Let $\lambda := \lambda^{k-1}$
Repeat:

1. Let $z := prox_{\lambda g}\left(x^k - \lambda \nabla f(x^k)\right)$
2. Break if $f(z) \leq \hat{f}_\lambda(z, x^k)$
3. Update $\lambda := \beta\lambda$

Return $\lambda^k := \lambda, x^{k+1} := z$

Conclusion

Situation

- Increasing amount of data in Machine Learning applications
- Need for robust and fast algorithms for smooth and non smooth optimization

Stochastic Second-Order Methods

- Faster convergence through curvature information
- Moderate computational cost through mini-batches