

STOCHASTIC OPTIMIZATION IN MACHINE LEARNING

CASE STUDIES IN NONLINEAR OPTIMIZATION

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*WE'RE NOT RUNNING OUT OF DATA ANYTIME
SOON. IT'S MAYBE THE ONLY RESOURCE THAT
GROWS EXPONENTIALLY.*

ANDREAS WEIGEND

1. Introduction
2. SQN: A Stochastic Quasi-Newton Method
3. Proximal Method
4. Logistic Regression: An Example
5. Dictionary Learning
6. Conclusion
7. Appendix

INTRODUCTION

INTRODUCTION: WHAT IS MACHINE LEARNING (ML) ?

Implementation of autonomously learning software for:

- Discovery of patterns and relationships in data
- Prediction of future events

Examples:

Electroencephalography (EEG)



Section 4

Image Denoising



Section 5

Training a Machine Learning model means finding optimal parameters ω :

$$\omega^* = \operatorname{argmin}_{\omega} F(\omega, X, z)$$

- F : Loss function of chosen ML-model
- X : The training data ($N := \text{\#samples} \times \text{\#features}$ matrix)
- z : Training labels (only in classification models; vector of size N)
- The dimension n of ω is model dependent, often $\text{\#features}+1$

After we have found ω^* , we can do **Prediction** on new data points:

$$\hat{z}_i := h(\omega^*, x_i)$$

- x_i : new data point with *unknown* label z_i
- h : hypothesis function of the ML model

- Massive amounts of training data
- Construction of very large models
- Handling high memory/computational demands

Ansatz: Stochastic Methods

$$F(\omega) := \mathbb{E} [f(\omega, \xi)]$$

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- ξ : Random variable; takes the form of an input-output-pair (x_i, z_i)

$$F(\omega) := \mathbb{E} [f(\omega, \xi)] = \frac{1}{N} \sum_{i=1}^N f(\omega, x_i, z_i)$$

- ξ : Random variable; takes the form of an input-output-pair (x_i, z_i)
- f : Partial loss function corresponding to a single data point.

Gradient Method

$$\min F(\omega)$$

Stochastic Gradient Descent

$$\min \mathbb{E} [f(\omega, \xi)]$$

Gradient Method

$$\min F(\omega)$$

$$\omega^{(k+1)} := \omega^k - \alpha_k \nabla F(\omega^k)$$

Stochastic Gradient Descent

$$\min \mathbb{E} [f(\omega, \xi)]$$

Gradient Method

$$\min F(\omega)$$

$$\omega^{(k+1)} := \omega^k - \alpha_k \nabla F(\omega^k)$$

Stochastic Gradient Descent

$$\min \mathbb{E} [f(\omega, \xi)]$$

$$\omega^{k+1} := \omega^k - \alpha_k \nabla \hat{F}(\omega^k)$$

with

$$\nabla \hat{F}(\omega^k) := \frac{1}{b} \sum_{i \in \mathcal{S}_k} f(\omega, x_i, z_i)$$

where $\mathcal{S}_k \subset [N]$, $b := |\mathcal{S}_k| \ll N$
"Mini Batch"

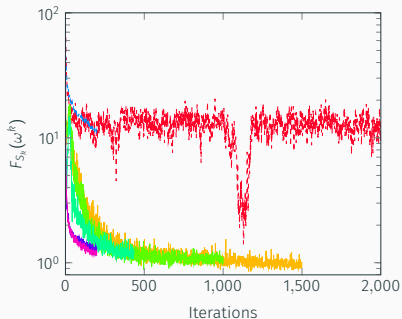
SQN: A STOCHASTIC QUASI-NEWTON METHOD

- Stochastically use second-order information
- Based on BFGS-method.
- Basic idea:

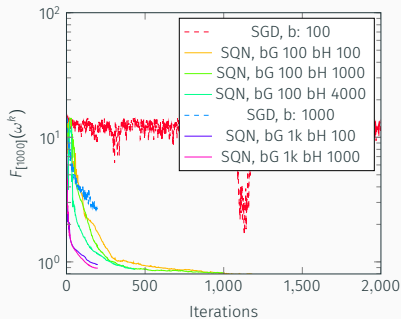
$$\omega^{k+1} = \omega^k - \alpha_k H_t \nabla \hat{F}(\omega^k)$$

- t running on slower time-scale than k .
- H_t update in $\mathcal{O}(n)$ time and constant memory, using several tricks

EEG: Sample Objective vs. Iterations



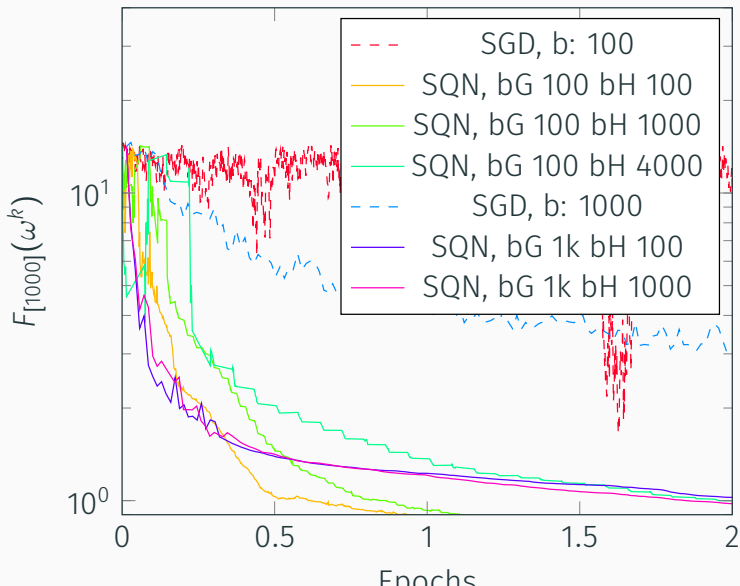
EEG: Fixed Subset Objective vs. Iterations



Performance on EEG Dataset, Problem size: 69550×600

Armijo-stepsizes, Further SQN-parameters: $L = 10, M = 5$

EEG: Fixed Subset Objective vs. Accessed Data Points



- Can be faster than SGD on appropriate Datasets
- Requires tedious, manual tuning of hyperparameters to be efficient!

PROXIMAL METHOD

Problem

$$\min_x F(x) := \underbrace{f(x)}_{\text{smooth}} + \underbrace{h(x)}_{\text{non-smooth}}$$

Proximity Operator

$$\text{prox}_f(v) = \arg\min_x \left(f(x) + \frac{1}{2} \|x - v\|_2^2 \right)$$

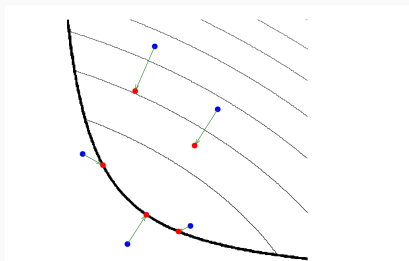


Figure 1: Evaluating a proximal operator at various points. *N Parikh, S Boyd, Proximal Methods, Foundations and Trends in Optimization 1, 2014*

Traditional Proximal Gradient Step:

$$x_{k+1} = \text{prox}_{\lambda_k h}(x_k - \lambda_k \nabla f(x_k))$$

Quasi-Newton Proximal Step:

$$x_{k+1} = \text{prox}_h^{B_k}(x_k - B_k^{-1} \nabla f(x_k)),$$

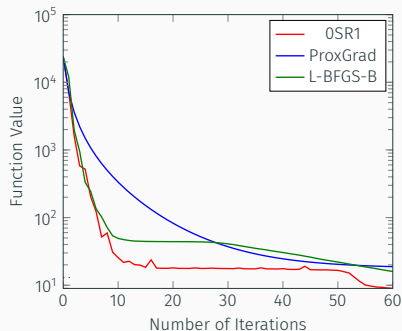
$$\text{with } B_k = \underbrace{D_k}_{diag} + \underbrace{u_k}_{\in \mathbb{R}^n} u_k^T.$$

PROXIMAL METHOD

$$F(x) = \|Ax - b\| + \lambda \|x\|_1$$

$$A \in \mathbb{R}^{1500 \times 3000}, b \in \mathbb{R}^{1500}$$

$$A_{ij}, b_i \sim \mathcal{N}(0, 1), \lambda = 0.1$$



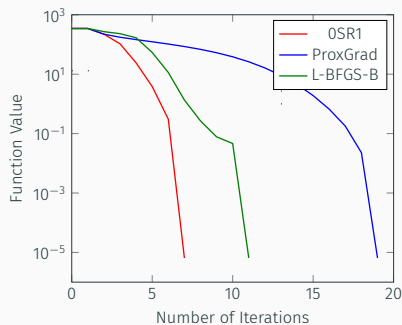
	OSR1	ProxGrad	L-BFGS-B
Iterations	1,822	135,328	1,989
Run-Time	68 s	1,144 s	56 s

$$F(x) = \|Ax - b\| + \lambda \|x\|_1$$

$$A \in \mathbb{R}^{2197 \times 2197}, b \in \mathbb{R}^{2197}$$

A: Discretization of 3D Laplacian

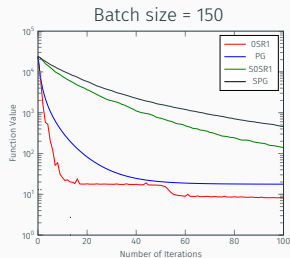
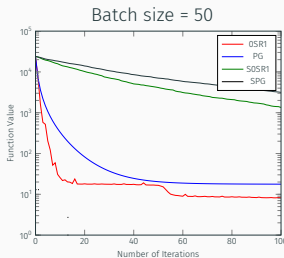
$$\lambda = 1$$



	OSR1	ProxGrad	L-BFGS-B
Iterations	7	18	10
Run-Time	0.037 s	0.004 s	0.022 s

High-dimensional data: Extension to stochastic framework

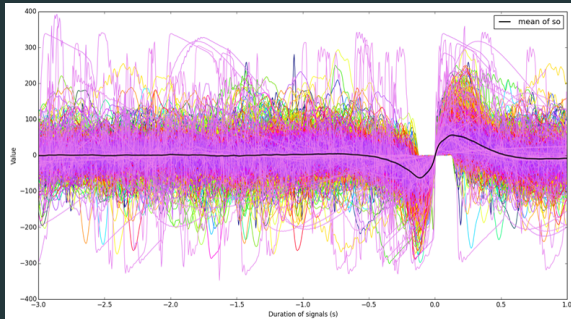
Effect of batch size



LOGISTIC REGRESSION: AN EXAMPLE

ELECTROENCEPHALOGRAPHY (EEG)

HOW DEEP IS YOUR SLEEP?



SLEEPING PATIENT / 20 NIGHTS OF EEG RECORDINGS

PREDICT NEXT SLOW WAVE

RESULTS

Nice table with SQN, SGD (no reg, L2), (Lasso,) Prox (L1) showing Obj. value in found optimum, CPU time, Iterations, F1 score of prediction model

	$F(\omega^*)$	Model Score	Cost
No regularization			
SGD	0.01	96%	x sec, y AP
SQN	0.5	96%	x sec, y AP
Prox	0.01	96%	x sec, y AP
L1			
LASSO	.71	55%	blablabla
Prox	0.01	96%	x sec, y AP
L2			
SGD	.71	55%	blablabla
SQN	0.01	96%	x sec, y AP

DICTIONARY LEARNING

DICTIONARY LEARNING

CAN WE RECOVER THE IMAGE?



IMAGE IS PARTIALLY DESTROYED

RECONSTRUCT IMAGE

bla

CONCLUSION



QUESTIONS?



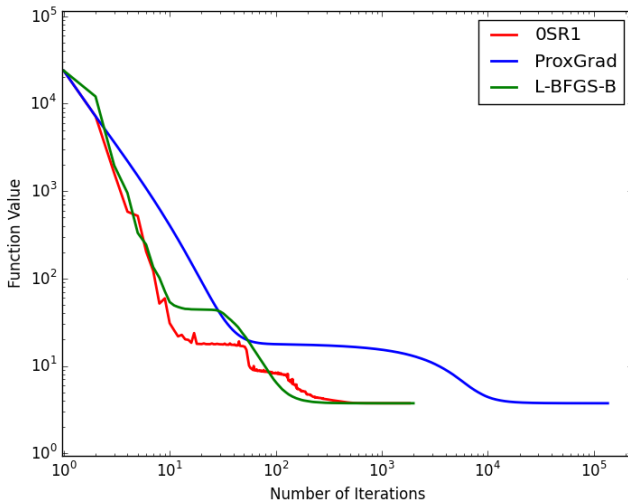
S. Becker and J. Fadili.

A quasi-newton proximal splitting method.

In Advances in Neural Information Processing Systems, pages 2618–2626, 2012.

APPENDIX

PROXIMAL METHOD

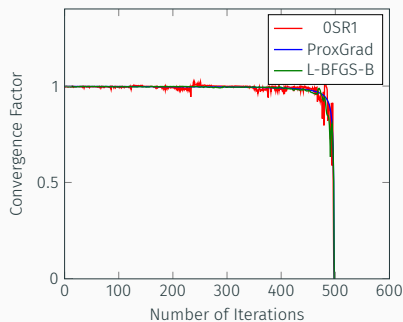


PROXIMAL METHOD

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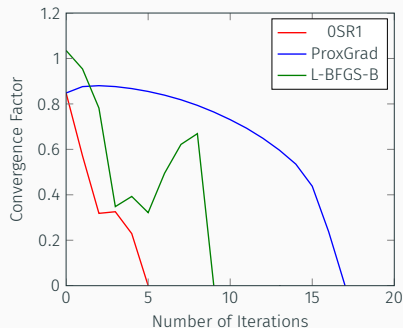


$$F(x) = \|Ax - b\| + \lambda \|x\|_1$$

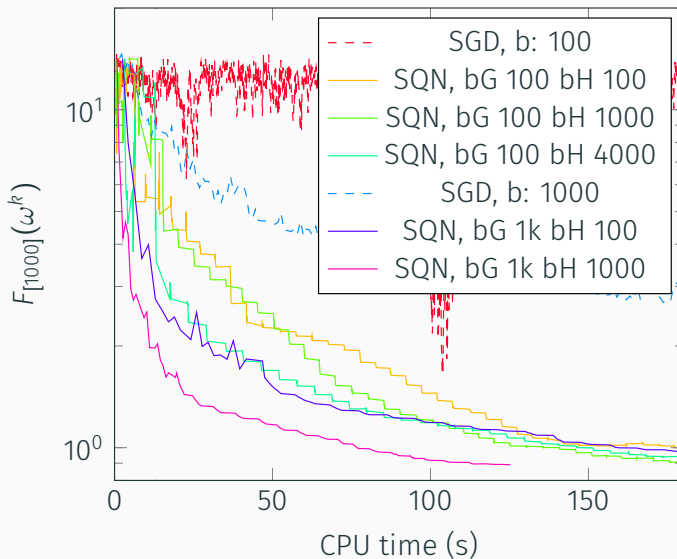
$$A \in \mathbb{R}^{2197 \times 2197}, b \in \mathbb{R}^{2197}$$

A: Discretization of 3D Laplacian

$$\lambda = 1$$



EEG: Fixed Subset Objective vs. CPU time



PROXIMAL METHOD

