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1  |----- MODULE RelationUtils -----|
   |Relation related operators.
5  |EXTENDS Naturals, Sequences, FunctionUtils
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   |Basic definitions.
10  $Dom(R) \triangleq \{a : \langle a, b \rangle \in R\}$  Domain of  $R$ 
11  $Ran(R) \triangleq \{b : \langle a, b \rangle \in R\}$  Range of  $R$ 
12  $Support(R) \triangleq Dom(R) \cup Ran(R)$  Support of  $R$ 
   |Basic operations.
16  $Inverse(R) \triangleq \{\langle b, a \rangle : \langle a, b \rangle \in R\}$  Inverse of  $R$ 
17  $R|S \triangleq R \cap S \times S$  Restriction of  $R$  on  $S$ 
19  $R ** T \triangleq$  Composition of  $R$  and  $T$ 
20   LET  $SR \triangleq Support(R)$ 
21    $ST \triangleq Support(T)$ 
22   IN  $\{\langle r, t \rangle \in SR \times ST : \exists s \in SR \cap ST : (\langle r, s \rangle \in R) \wedge (\langle s, t \rangle \in T)\}$ 
24  $GT(R, a) \triangleq \{b \in Ran(R) : \langle a, b \rangle \in R\}$ 
25  $LT(R, b) \triangleq \{a \in Dom(R) : \langle a, b \rangle \in R\}$ 
   |The following definition is from https://github.com/jamesfisher/tlaplus/blob/master/examples/TransitiveClosure/TransitiveClosure.tla.
   |It also contains several other methods for computing  $TC$ .
31  $TC(R) \triangleq$  Transitive closure of  $R$ 
32   LET  $S \triangleq Support(R)$ 
33   RECURSIVE  $TCR(-)$ 
34    $TCR(T) \triangleq$  IF  $T = \{\}$ 
35   THEN  $R$ 
36   ELSE LET  $r \triangleq$  CHOOSE  $s \in T : \text{TRUE}$ 
37    $RR \triangleq TCR(T \setminus \{r\})$ 
38   IN  $RR \cup \{\langle s, t \rangle \in S \times S :$ 
39    $\langle s, r \rangle \in RR \wedge \langle r, t \rangle \in RR\}$ 
40   IN  $TCR(S)$ 
   |Example:  $SeqToRel(\langle 1, 2, 3 \rangle) = \{\langle 1, 2 \rangle, \langle 1, 3 \rangle, \langle 2, 3 \rangle\}$ 
44  $SeqToRel(s) \triangleq$  Transform a (total order) sequence to a relation
45   LET RECURSIVE  $SeqRel(-, -)$ 
46    $SeqRel(seq, rel) \triangleq$ 
47   IF  $Len(seq) \leq 1$ 
48   THEN  $rel$ 
49   ELSE LET  $h \triangleq Head(seq)$ 
50    $t \triangleq Tail(seq)$ 
51   IN  $SeqRel(t, rel \cup \{\langle h, r \rangle : r \in Range(t)\})$ 
52   IN  $SeqRel(s, \{\})$ 
   |Basic properties.
56  $Reflexive(S, R) \triangleq \forall a \in S : \langle a, a \rangle \in R$  is  $R$  reflexive?

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58 Transitive(R)  $\triangleq$  Is R transitive?
59   LET S  $\triangleq$  Support(R)
60   IN    $\forall a, b, c \in S : (\langle a, b \rangle \in R \wedge \langle b, c \rangle \in R) \Rightarrow \langle a, c \rangle \in R$ 

62 Respect(R, T)  $\triangleq$  Does total order R (represented as a sequence) respect T?
63   (T | Range(R))  $\subseteq$  SeqToRel(R)
64 |_____|

\ * Modification History
\ * Last modified Mon Jan 27 14:06:50 CST 2020 by hengxin
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