MODULE *Sequences*

LOCAL INSTANCE *Naturals*

Imports the definitions from *Naturals*, but doesn't export them.

$Len(s) \triangleq$ CHOOSE $n \in Nat$: DOMAIN $s = (1 \,..\, n)$

$s \circ t \triangleq [i \in 1 \,..\, (Len(s) + Len(t)) \mapsto$ IF $i \leq s$ THEN $s[i]$
ELSE $t[i - Len(s)]]$

$Append(s,\, e) \triangleq s \circ \langle e \rangle$

$Seq(S) \triangleq$ UNION $\{[1 \,..\, n \to S] : n \in Nat\}$

$Head(s) \triangleq s[1]$

$Tail(s) \triangleq$ CASE $s \neq \langle\rangle \to [i \in 1 \,..\, (Len(s) - 1) \mapsto s[i + 1]]$

The "CASE $s \neq \langle\rangle \to$" just ensures that Tail(s) is undefined if s is the empty sequence.

$SubSeq(s,\, m,\, n) \triangleq [i \in 1 \,..\, (1 + n - m) \mapsto s[i + m - 1]]$

The sequence $\langle [m], s[m + 1], \ldots, s[n] \rangle$.

$SelectSeq(s,\, test(\_)) \triangleq$
LET $F[i \in 0 \,..\, Len(s)] \triangleq$
IF $i = 0$ THEN $\langle\rangle$
ELSE IF $test(s[i])$ THEN $Append(F[i - 1],\, s[i])$
ELSE $F[i - 1]$
IN $F[Len(s)]$

The subsequence of s consisting of all elements $s[i]$ such that $Test(s[i])$ is true.

CLOSE