

Principles and Specifications of Concurrent Systems

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The *Principles* and *Specification* Tracks

1 Introduction

- 1.1 Concurrent Computation
- 1.2 Modeling Computation
- 1.3 Specification
- 1.4 Systems and Languages

If you are just starting to read this hyperbook, click [here](#).

2 The One-Bit Clock

- 2.1 The Clock's Behaviors
- 2.2 Describing the Behaviors
- 2.3 Writing the Specification
- 2.4 The Pretty-Printed Version of Your Spec
- 2.5 Checking the Specification
- 2.6 Computing the Behaviors from the Specification
- 2.7 Other Ways of Writing the Behavior Specification
- 2.8 Specifying the Clock in PlusCal

Sections colored like this have not yet been written.

3 The Die Hard Problem

- 3.1 Representing the Problem in TLA⁺
- 3.2 Applying TLC
- 3.3 Expressing the Problem in PlusCal

4 Euclid's Algorithm

- 4.1 The Greatest Common Divisor
 - 4.1.1 Divisors
 - 4.1.2 CHOOSE and the Maximum of a Set
 - 4.1.3 The GCD Operator
- 4.2 Comments
- 4.3 The Algorithm
- 4.4 The TLA⁺ Translation
- 4.5 Checking Safety
- 4.6 Checking Liveness
- 4.7 The Translation Revisited
- 4.8 The Grain of Atomicity

- 4.9 Why Euclid's Algorithm Is Correct
 - 4.9.1 Proving Invariance
 - 4.9.2 Verifying $GCD1-GCD3$
 - 4.9.3 Proving Termination
- 4.10 Euclid's Algorithm for Sets

5 The Generalized Die Hard Problem

- 5.1 The PlusCal Representation
- 5.2 Checking the Algorithm
- 5.3 The TLA⁺ Translation

6 Alternation

- 6.1 The Problem
- 6.2 The One-Bit Clock Revisited
- 6.3 Specifying Alternation: Safety
- 6.4 Specifying Alternation: Liveness
- 6.5 The Two-Phase Handshake Protocol
- 6.6 Refinement
- 6.7 Refinement and Stuttering
 - 6.7.1 Adding Steps
 - 6.7.2 Temporal Logic and Stuttering
 - 6.7.3 A Finer-Grained Algorithm
- 6.8 Temporal Logic and Refinement
- 6.9 Alternation Revisited
- 6.10 Round-Robin Synchronization
 - 6.10.1 The One-Bit Clock Revisited Again
 - 6.10.2 An N -Valued Clock
 - 6.10.3 An Implementation of the N -Valued Clock
 - 6.10.4 Round-Robin Synchronization

The *Principles* Track

7 Mutual Exclusion

- 7.1 The Problem
- 7.2 The One-Bit Protocol
 - 7.2.1 The Protocol
 - 7.2.2 An Assertional Proof
 - 7.2.3 Using TLC to Check an Inductive Invariant
- 7.3 The Two-Process One-Bit Algorithm
 - 7.3.1 The Two-Process Algorithm
 - 7.3.2 Busy Waiting Versus Synchronization Primitives
 - 7.3.3 Requirement (c)
- 7.4 Proving Liveness

- 7.5 An Informal Proof
- 7.6 A More Formal Proof
- 7.7 The N -Process One-Bit Algorithm
- 7.8 The Bakery Algorithm
 - 7.8.1 The Big-Step Algorithm
 - 7.8.2 Choosing the Grain of Atomicity
 - 7.8.3 The Atomic Bakery Algorithm
 - 7.8.4 The Real Bakery Algorithm
- 7.9 Mutual Exclusion in Modern Programs

8 The Bounded Channel and Bounded Buffer

- 8.1 The Bounded Channel
 - 8.1.1 The Specification
 - 8.1.2 Safety
 - 8.1.3 Liveness
 - 8.1.4 Implementing The Bounded Channel
- 8.2 The Bounded Buffer
 - 8.2.1 Modular Arithmetic
 - 8.2.2 The Algorithm
- 8.3 The Bounded Buffer Implements the Bounded Channel
 - 8.3.1 The Refinement Mapping
 - 8.3.2 Showing Implementation
 - 8.3.3 Liveness
- 8.4 A Finer-Grained Bounded Buffer
- 8.5 Further Refinement
- 8.6 What is a Process?

The *Specification* Track

9 An Input/Output Specification

- 9.1 The Example
- 9.2 Sorting
- 9.3 Votes
- 9.4 The Borda Ranking
- 9.5 The Condorcet Ranking
- 9.6 Transitive Closure
 - 9.6.1 A Mathematical Definition
 - 9.6.2 A Definition TLC Can Execute Faster
 - 9.6.3 Warshall's Algorithm
- 9.7 The Condorcet Ranking Revisited

The TLA⁺ Proof Track

10 About Proofs and Proving

- 10.1 About Proofs
- 10.2 About TLAPS

11 Correctness of Euclid's Algorithm

- 11.1 Proving Safety
- 11.2 Proving Properties of the GCD

12 The Proof Language

- 12.1 What a Theorem Asserts
- 12.2 The Hierarchical Structure of a Proof
 - 12.2.1 Writing Structured Proofs
 - 12.2.2 Reading Structured Proofs
- 12.3 The State of a Proof
 - 12.3.1 Steps That Can Have a Proof
 - 12.3.2 Steps That Cannot Have a Proof
- 12.4 Proof Obligations
- 12.5 Further Details
 - 12.5.1 Additional Language Features
 - 12.5.2 Importing
 - 12.5.3 Recursively Defined Functions and Operators
 - 12.5.4 The Fine Print

13 The Bounded Buffer Proof

Math

13 Arithmetic and Logic

- 13.1 Arithmetic
- 13.2 Mathematical Logic
- 13.3 Propositional Logic
 - 13.3.1 \wedge and \vee
 - 13.3.2 Other Propositional Operators
- 13.4 Predicate Logic
- 13.5 The CHOOSE Operator

14 Sets

- 14.1 An Introduction to Sets
- 14.2 Simple Set Operators
- 14.3 Set Constructors
- 14.4 SUBSET and UNION
- 14.5 Collections too Big to Be Sets

14.6 Bags

15 Functions

15.1 Functions and Their Domains

15.2 Writing Functions

15.3 Sets of Functions

15.4 The EXCEPT Construct

15.5 Tuples

15.6 Records

15.7 Strings

16 Miscellaneous Constructs

16.1 Conditional Constructs

16.1.1 IF / THEN / ELSE

16.1.2 CASE

16.2 Definitions

16.2.1 Simple Operator Definitions

16.2.2 Function Definitions

16.2.3 Recursive Operator Definitions

16.2.4 Recursive Or Inductive?

16.3 The LET / IN Construct

16.4 The LAMBDA Construct

17 Temporal Logic

17.1 Understanding Temporal Formulas

17.2 Proof Rules and Proofs

17.3 Rules for Proving Safety

17.4 Leads To

17.4.1 The Leads-To Induction Rule

17.4.2 The $\Box \leadsto$ Rule

17.4.3 Proving \leadsto Formulas by Contradiction

17.5 Fairness

17.5.1 The ENABLED Operator

17.5.2 Weak Fairness

17.5.3 Strong Fairness

17.5.4 Proving \leadsto Properties with Fairness

17.5.5 Proving Fairness

Topics

18 Variable Hiding and Auxiliary Variables

19 Reduction

20 Debugging With TLC

20.1 Print Statements

20.2 Having TLC Set and Read Values

20.3 Using LET

20.4 The Perils of Lazy Evaluation