**C2.**  $c \succeq^+ d$  holds, for any elements c and d of  $\mathcal{CW}$ .

1. Let c be an arbitrary element of  $\mathcal{CW}$  and define

$$D \triangleq \{d \in \mathcal{CW} : \neg(c \succeq^+ d)\}$$

It suffices to assume D is nonempty and obtain a contradiction.

PROOF: Obvious.

2.  $d \succ c$  holds, for all  $d \in D$ .

PROOF:  $\neg(c \succeq^+ d)$  implies  $\neg(c \succeq d)$  (because  $c \succeq d$  implies  $c \succeq^+ d$  by definition of the transitive closure), and  $\neg(c \succeq d)$  equals  $d \succ c$  by definition of  $\succeq$ .

- 3. For all  $d \in D$  and all  $e \in \mathcal{CW} \setminus D : d \succ e$ .
  - 3.1. It suffices to assume  $d \in D$ ,  $e \in \mathcal{CW} \setminus D$ , and  $\neg (d \succ e)$  and obtain a contradiction

PROOF: Obvious.

3.2.  $c \succ^{+} e$ 

PROOF: By the assumption of 3.1, which implies  $e \notin D$ , and the definition of D.

3.3.  $e \succeq d$ 

PROOF: By the assumption of 3.1, which asserts  $\neg(d \succ e)$ , and the definition of *succeq*.

3.4. Q.E.D.

Steps 3.2 and 3.3 and the definition of the transitive closure imply  $c \succeq^+ d$ , which by the definition of D contradicts the step 3.1 assumption  $d \in D$ .

4. *D* is a dominating set.

PROOF: We must prove that if  $d \in D$ , then  $d \succ e$  for any candidate e not in D. If e is not in  $\mathcal{CW}$ , this follows because  $\mathcal{CW}$  is a dominating set. If  $e \in \mathcal{CW}$ , then  $e \in \mathcal{CW} \setminus D$  and this follows from step 3.

5. Q.E.D.

PROOF: D is a subset of  $\mathcal{CW}$  by definition. It is a proper subset of  $\mathcal{CW}$  because  $c \in \mathcal{CW}$  by the step 1 assumption and step 2 implies  $c \notin D$ . Therefore, step 4 implies that  $\mathcal{CW}$  is not the smallest dominating set, which is a contradiction.