A Partially Expanded Proof of $\langle 4\rangle 3$

$\langle 1\rangle 1.$ ASSUME: Inv, $Producer$
PROVE: $C!Send$

$\langle 2\rangle 1.$ $Len(chBar) \neq N$

$\langle 2\rangle 2.$ $\exists v \in Msg : chBar' = Append(chBar, v)$

$\langle 3\rangle 1.$ Pick $v \in Msg$ such that $buf' = [buf \text{ EXCEPT } ![p \,\%\, N] = v]$

$\langle 3\rangle 2.$ $chBar' = Append(chBar, v)$

$\langle 4\rangle 1.$ $p \ominus c \in 0 \,..\, (N-1)$

$\langle 4\rangle 2.$ $p' \ominus c' = (p \ominus c) + 1$

$\langle 4\rangle 3.$ Q.E.D.

$\langle 5\rangle 1.$ $Append(chBar, v) =$

$$[\, i \in 1 \,..\, ((p \ominus c)+1) \mapsto \text{IF } i \in 1 \,..\, (p \ominus c) \text{ THEN } buf[(c \oplus (i-1)) \,\%\, N] \text{ ELSE } v \,]$$

PROOF: By definition of $chBar$ and $Append$, since Inv implies $p \ominus c$ is in Nat.

$\langle 5\rangle 2.$ $chBar' = [\, i \in 1 \,..\, ((p \ominus c)+1) \mapsto buf'[(c \oplus (i-1)) \,\%\, N]\,]$

PROOF: By definition of $chBar$, $\langle 4\rangle 2$, and definition of $Producer$, which implies $c' = c$.

Remember that priming an expression means priming all the variables in it.

$\langle 5\rangle 3.$ ASSUME: NEW $i \in 1 \,..\, ((p \ominus c)+1)$
PROVE: $chBar'[i] = Append(chBar, v)[i]$

$\langle 6\rangle 1.$ CASE: $i \in 1 \,..\, (p \ominus c)$

$\langle 6\rangle 2.$ CASE: $i = (p \ominus c) + 1$

$\langle 6\rangle 3.$ Q.E.D.

PROOF: By the $\langle 5\rangle 3$ assumption, $\langle 6\rangle 1$, $\langle 6\rangle 2$, and Inv, which implies $p \ominus c$ is in Nat, so $1 \,..\, ((p \ominus c)+1)$ equals $(1 \,..\, (p \ominus c)) \cup \{(p \ominus c)+1\}$.

$\langle 5\rangle 4.$ Q.E.D.

PROOF: By $\langle 5\rangle 1$, $\langle 5\rangle 2$, and $\langle 5\rangle 3$.

$\langle 3\rangle 3.$ Q.E.D.

$\langle 2\rangle 3.$ Q.E.D.

$\langle 1\rangle 2.$ ASSUME: Inv, $Consumer$
PROVE: $C!Rcv$

$\langle 1\rangle 3.$ Q.E.D.

CLOSE