

## Rule WF1

The following proof rule is used to deduce a  $\leadsto$  property from a weak fairness assumption. It assumes that  $P$  and  $Q$  are state formulas (contain only unprimed variables and have no temporal operators),  $N$  and  $A$  are action formulas, and  $v$  is a state expression.

$$\begin{array}{l} \text{WF1:} \quad P \wedge [N]_v \Rightarrow (P' \vee Q') \\ \quad \quad P \wedge \langle N \wedge A \rangle_v \Rightarrow Q' \\ \quad \quad P \Rightarrow \text{ENABLED } \langle A \rangle_v \\ \hline \square[N]_v \wedge WF_v(A) \Rightarrow (P \leadsto Q) \end{array}$$

It is generally applied with  $N$  the specification's next-state action and  $A$  a subaction of  $N$ , meaning that  $A$  implies  $N$ . The first hypothesis then asserts that every step that begins in a state with  $P$  true leaves  $P$  true or makes  $Q$  true. The second hypothesis asserts that a non-stuttering  $A$  step starting with  $P$  true makes  $Q$  true. The three hypotheses imply that if  $P$  ever becomes true, then it remains true and a non-stuttering  $A$  action remains enabled unless a non-stuttering  $A$  step occurs and makes  $Q$  true. Weak fairness of  $A$  therefore implies that if  $P$  ever becomes true, then  $Q$  must eventually become true.

As with all our temporal proof rules, the conclusion is true of a behavior  $\sigma$  if all of the hypotheses are true of all suffixes of  $\sigma$ . Hence, in applying the rule in a context in which  $\square Inv$  is assumed, we can assume  $Inv$  in proving the hypotheses.