The Meaning of $Divides(p,\ n)$

$Divides(p,\ n)$ means

$$\exists\, q \in Int \ :\ n = q * p$$

regardless of what p and n equal. If p is an integer and n is not (for example, if n equals $\sqrt{2}$), then it equals FALSE. (Do you see why?) For completely arbitrary values of p and n, the important thing we can say about its value is that it is a Boolean—that is, it equals either TRUE or FALSE. This is because

- $n = q * p$ is a Boolean. Even if we don't know whether n and $q * p$ are equal, the assertion that they are equal must be true or false.

- *Int* is a set. Actually, $Divides(p, q)$ would be a Boolean even if we replaced *Int* by "abc" in the definition. TLA$^+$ is based on Zermelo-Fraenkel set theory, in which any value is a set. Thus, the string "abc" is a set, although the semantics of TLA$^+$ don't tell us what its elements are.

If n or p is not a number, then a close examination of the definition of $*$ might tell us more about the value of $Divides(p,\ n)$. However, such an examination is pointless because we should not use the expression $Divides(p,\ n)$ in any context in which we care what its value is when p and n are not both numbers.

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