

THEOREM  $\text{Induction} \Rightarrow \text{Inv} / \text{Next} \Rightarrow \text{Inv}$

<1> 1.  $\text{SUFFICES} \text{ ASSUME } \text{Inv}, \text{Next}$

PROVE  $\text{Inv}$

OBVIOUS

<1> 2.  $\text{CASE} / \text{Case } x > y$

$x = x - y$

$y = y$

<2> 1.  $\text{TypeOK}$

BY <1> 1, <1> 2,  $\text{SimpleArithmetic\_DEF\_Inv}, \text{TypeOK}$

<2> 2.  $\text{GCDInv}$

<3> 1.  $\text{GCD}(y, x) = \text{GCD}(y', x')$

BY <1> 1, <1> 2,  $\text{GCD3\_DEF\_Inv}, \text{TypeOK}$

<3> 2.  $\text{QED}$

BY <1> 1, <3> 1, <2> 1,  $\text{GCD2\_DEF\_Inv}, \text{TypeOK}, \text{GCDInv}$

<2> 3.  $\text{QED}$

BY <2> 1, <2> 2,  $\text{DEF\_Inv}$

<1> 3.  $\text{CASE} / \text{Case } y > x$

$y = y - x$

$x = x$

<2> 1.  $\text{TypeOK}$

BY <1> 1, <1> 3,  $\text{SimpleArithmetic\_DEF\_Inv}, \text{TypeOK}$

<2> 2.  $\text{GCDInv}$

BY <1> 1, <1> 3,  $\text{GCD3\_DEF\_Inv}, \text{TypeOK}, \text{GCDInv}$

<2> 3.  $\text{QED}$

BY <2> 1, <2> 2,  $\text{DEF\_Inv}$

<1> 4.  $\text{QED}$

BY <1> 1, <1> 2, <1> 3,  $\text{DEF\_Next}$