

A Subtle Point

The formula

$$\begin{aligned} &\wedge \forall i \in \text{DOMAIN } A : A'[i] = (\text{IF } i = 3 \text{ THEN } 42 \text{ ELSE } A[i]) \\ &\wedge (\text{DOMAIN } A') = (\text{DOMAIN } A) \end{aligned}$$

does not imply that A' is a function. If v is not a function, then the values of $\text{DOMAIN } v$ and $v[x]$ for some number x are not specified. The semantics of TLA^+ does not rule out the possibility that this formula is satisfied if A' equals $\sqrt{43}$. (The semantics also does not say whether or not $\sqrt{43}$ is a function.)

To turn this formula into a correct specification of the assignment statement, we have to add the requirement that A' is a function. This requirement is expressed by the formula

$$A' = [i \in \text{DOMAIN } A' \mapsto A'[i]]$$

Can you see why?