

Summary of TLA⁺

The Constant Operators

Miscellaneous Constructs

Action Operators

?

Temporal Operators

←

User-Definable Operator Symbols

→

C

Precedence Ranges of Operators

I

Operators Defined in Standard Modules.

S

ASCII Representation of Typeset Symbols

The Constant Operators

Logic

$\wedge \quad \vee \quad \neg \quad \Rightarrow \quad \equiv$
 TRUE FALSE BOOLEAN [the set {TRUE, FALSE}]
 $\forall x \in S : p \quad ^{(1)} \quad \exists x \in S : p \quad ^{(1)}$
 CHOOSE $x \in S : p$ [An x in S satisfying p]

?

Sets

$= \neq \in \notin \cup \cap \subseteq \setminus$ [set difference]
 $\{e_1, \dots, e_n\}$ [Set consisting of elements e_i]
 $\{x \in S : p\} \quad ^{(2)}$ [Set of elements x in S satisfying p]
 $\{e : x \in S\} \quad ^{(1)}$ [Set of elements e such that x in S]
 SUBSET S [Set of subsets of S]
 UNION S [Union of all elements of S]

←

→

C

I

S

Functions

$f[e]$ [Function application]
 DOMAIN f [Domain of function f]
 $[x \in S \mapsto e] \quad ^{(1)}$ [Function f such that $f[x] = e$ for $x \in S$]
 $[S \rightarrow T]$ [Set of functions f with $f[x] \in T$ for $x \in S$]
 $[f \text{ EXCEPT } ![e_1] = e_2] \quad ^{(3)}$ [Function \hat{f} equal to f except $\hat{f}[e_1] = e_2$]

Records

$e.h$ [The h -field of record e]
 $[h_1 \mapsto e_1, \dots, h_n \mapsto e_n]$ [The record whose h_i field is e_i]
 $[h_1 : S_1, \dots, h_n : S_n]$ [Set of all records with h_i field in S_i]
 $[r \text{ EXCEPT } !.h = e] \quad ^{(3)}$ [Record \hat{r} equal to r except $\hat{r}.h = e$]

Tuples

$e[i]$ [The i^{th} component of tuple e]
 $\langle e_1, \dots, e_n \rangle$ [The n -tuple whose i^{th} component is e_i]
 $S_1 \times \dots \times S_n$ [The set of all n -tuples with i^{th} component in S_i]

-
- (1) $x \in S$ may be replaced by a comma-separated list of items $v \in S$, where v is either a comma-separated list or a tuple of identifiers.
 (2) x may be an identifier or tuple of identifiers.
 (3) $![e_1]$ or $!.h$ may be replaced by a comma separated list of items $!a_1 \dots a_n$, where each a_i is $[e_i]$ or $.h_i$.

Miscellaneous Constructs

IF p THEN e_1 ELSE e_2 [e_1 if p true, else e_2]
CASE $p_1 \rightarrow e_1 \square \dots \square p_n \rightarrow e_n$ [Some e_i such that p_i true]
CASE $p_1 \rightarrow e_1 \square \dots \square p_n \rightarrow e_n \square \text{OTHER} \rightarrow e$ [Some e_i such that p_i true,
or e if all p_i are false]

LET $d_1 \triangleq e_1 \dots d_n \triangleq e_n$ IN e [e in the context of the definitions]

$\wedge p_1$ [the conjunction $p_1 \wedge \dots \wedge p_n$] $\vee p_1$ [the disjunction $p_1 \vee \dots \vee p_n$]
 \vdots \vdots
 $\wedge p_n$ $\vee p_n$

?

←

→

C

I

S

Action Operators

e' [The value of e in the final state of a step]
 $[A]_e$ [$A \vee (e' = e)$]
 $\langle A \rangle_e$ [$A \wedge (e' \neq e)$]
ENABLED A [An A step is possible]
UNCHANGED e [$e' = e$]
 $A \cdot B$ [Composition of actions]

Temporal Operators

$\square F$ [F is always true]
 $\diamond F$ [F is eventually true]
 $\text{WF}_e(A)$ [Weak fairness for action A]
 $\text{SF}_e(A)$ [Strong fairness for action A]
 $F \leadsto G$ [F leads to G]

User-Definable Operator Symbols

Infix Operators

| | | | | | |
|----------------|-----------------|---------------------|---------------|---------------|----------------|
| $+^{(1)}$ | $-^{(1)}$ | $*^{(1)}$ | $/^{(2)}$ | $\circ^{(3)}$ | $++$ |
| $\div^{(1)}$ | $\%_0^{(1)}$ | $\wedge^{(1,4)}$ | $\dots^{(1)}$ | \dots | $--$ |
| $\oplus^{(5)}$ | $\ominus^{(5)}$ | \otimes | \oslash | \odot | $**$ |
| $<^{(1)}$ | $>^{(1)}$ | $\leq^{(1)}$ | $\geq^{(1)}$ | \sqcap | $//$ |
| \prec | \succ | \preceq | \succeq | \sqcup | $\hat{\hat{}}$ |
| \ll | \gg | $<:^{(6)}$ | $:>^{(6)}$ | $\&$ | $\&\&$ |
| \sqsubset | \sqsupset | $\sqsubseteq^{(5)}$ | \sqsupseteq | $ $ | $\%\%$ |
| \subset | \supset | \subseteq | \supseteq | \star | $@@^{(6)}$ |
| \vdash | \dashv | \Vdash | \Vdash | \bullet | $\#\#$ |
| \sim | \simeq | \approx | \cong | $\$$ | $\$\$$ |
| \bigcirc | $::=$ | \times | \doteq | $??$ | $!!$ |
| \propto | \wr | \uplus | | | |

?

←

→

C

I

S

Postfix Operators ⁽⁷⁾

$\hat{+}$ $\hat{*}$ $\hat{\#}$

-
- (1) Defined by the *Naturals*, *Integers*, and *Reals* modules.
(2) Defined by the *Reals* module.
(3) Defined by the *Sequences* module.
(4) $x\hat{y}$ is printed as x^y .
(5) Defined by the *Bags* module.
(6) Defined by the *TLC* module.
(7) $e\hat{+}$ is printed as e^+ , and similarly for $\hat{*}$ and $\hat{\#}$.

Precedence Ranges of Operators

The relative precedence of two operators is unspecified if their ranges overlap.
Left-associative operators are indicated by (a).

?
←
→
C
I
S

Prefix Operators

| | | | | | |
|-----------|------|------------|------|--------|-------|
| \neg | 4-4 | \square | 4-15 | UNION | 8-8 |
| ENABLED | 4-15 | \diamond | 4-15 | DOMAIN | 9-9 |
| UNCHANGED | 4-15 | SUBSET | 8-8 | — | 12-12 |

Infix Operators

| | | | | | | | |
|---------------|---------|---------------|----------|--------------|-----------|---------------|-----------|
| \Rightarrow | 1-1 | \leq | 5-5 | $<:$ | 7-7 | \ominus | 11-11 (a) |
| \supset | 2-2 | \ll | 5-5 | \backslash | 8-8 | — | 11-11 (a) |
| \equiv | 2-2 | \prec | 5-5 | \cap | 8-8 (a) | -- | 11-11 (a) |
| \leadsto | 2-2 | \succ | 5-5 | \cup | 8-8 (a) | $\&$ | 13-13 (a) |
| \wedge | 3-3 (a) | \propto | 5-5 | \dots | 9-9 | $\&\&$ | 13-13 (a) |
| \vee | 3-3 (a) | \sim | 5-5 | \dots | 9-9 | \odot | 13-13 (a) |
| \neq | 5-5 | \simeq | 5-5 | !! | 9-13 | \oslash | 13-13 |
| \vdash | 5-5 | \sqcap | 5-5 | ## | 9-13 (a) | \otimes | 13-13 (a) |
| ::= | 5-5 | \sqsubseteq | 5-5 | \$ | 9-13 (a) | * | 13-13 (a) |
| : = | 5-5 | \sqsubset | 5-5 | \$\$ | 9-13 (a) | ** | 13-13 (a) |
| $<$ | 5-5 | \sqsupset | 5-5 | ?? | 9-13 (a) | / | 13-13 |
| $=$ | 5-5 | \subset | 5-5 | \sqcap | 9-13 (a) | // | 13-13 |
| \sqsubseteq | 5-5 | \subseteq | 5-5 | \sqcup | 9-13 (a) | \bigcirc | 13-13 (a) |
| $>$ | 5-5 | \supset | 5-5 | \uplus | 9-13 (a) | \bullet | 13-13 (a) |
| \approx | 5-5 | \succeq | 5-5 | \wr | 9-14 | \div | 13-13 |
| \times | 5-5 | \supset | 5-5 | \oplus | 10-10 (a) | \circ | 13-13 (a) |
| \cong | 5-5 | \sqsupseteq | 5-5 | + | 10-10 (a) | \star | 13-13 (a) |
| \doteq | 5-5 | \vdash | 5-5 | ++ | 10-10 (a) | \wedge | 14-14 |
| \supseteq | 5-5 | \models | 5-5 | % | 10-11 | $\sim\sim$ | 14-14 |
| \gg | 5-5 | $\cdot^{(1)}$ | 5-14 (a) | %% | 10-11 (a) | $\cdot^{(2)}$ | 17-17 (a) |
| \in | 5-5 | @@ | 6-6 (a) | | 10-11 (a) | | |
| \notin | 5-5 | :> | 7-7 | | 10-11 (a) | | |

Postfix Operators

| | | | | | | | |
|---------|-------|---------|-------|----------|-------|-----|-------|
| $\sim+$ | 15-15 | $\sim*$ | 15-15 | $\sim\#$ | 15-15 | $'$ | 15-15 |
|---------|-------|---------|-------|----------|-------|-----|-------|

(1) Action composition ($\backslash\text{cdot}$).
(2) Record field (period).

Operators Defined in Standard Modules.

Modules *Naturals*, *Integers*, *Reals*

| | | | | | | | |
|--------|-----------|--------|-----------|----------------|------|---------------------------|--------------------------------|
| $+$ | $-^{(1)}$ | $*$ | $/^{(2)}$ | $\wedge^{(3)}$ | $..$ | <i>Nat</i> | <i>Real</i> ⁽²⁾ |
| \div | $\%$ | \leq | \geq | $<$ | $>$ | <i>Int</i> ⁽⁴⁾ | <i>Infinity</i> ⁽²⁾ |

(1) Only infix $-$ is defined in *Naturals*.

(2) Defined only in *Reals* module.

(3) Exponentiation.

(4) Not defined in *Naturals* module.

?

←

→

C

I

S

Module *Sequences*

| | | | |
|---------------|-------------|------------------|---------------|
| \circ | <i>Head</i> | <i>SelectSeq</i> | <i>SubSeq</i> |
| <i>Append</i> | <i>Len</i> | <i>Seq</i> | <i>Tail</i> |

Module *FiniteSets*

| | |
|--------------------|--------------------|
| <i>IsFiniteSet</i> | <i>Cardinality</i> |
|--------------------|--------------------|

Module *Bags*

| | | | |
|-----------------------|-----------------|-----------------|---------------|
| \oplus | <i>BagIn</i> | <i>CopiesIn</i> | <i>SubBag</i> |
| \ominus | <i>BagOfAll</i> | <i>EmptyBag</i> | |
| \sqsubseteq | <i>BagToSet</i> | <i>IsABag</i> | |
| <i>BagCardinality</i> | <i>BagUnion</i> | <i>SetToBag</i> | |

Module *RealTime*

| | | |
|----------------|--------------|--|
| <i>RTBound</i> | <i>RTnow</i> | <i>now</i> (declared to be a variable) |
|----------------|--------------|--|

Module *TLC*

| | | | | | |
|----------------|------|--------------|---------------|-----------------|---------------------|
| $:>$ | $@@$ | <i>Print</i> | <i>Assert</i> | <i>JavaTime</i> | <i>Permutations</i> |
| <i>SortSeq</i> | | | | | |

ASCII Representation of Typeset Symbols

| | | | | | |
|---------------|---|---------------|---|---------------|---------------------------------------|
| \wedge | <code>\w</code> or <code>\land</code> | \vee | <code>\v</code> or <code>\lor</code> | \Rightarrow | <code>=></code> |
| \neg | <code>\n</code> or <code>\lnot</code> or <code>\neg</code> | \equiv | <code><=></code> or <code>\equiv</code> | \triangleq | <code>==</code> |
| \in | <code>\in</code> | \notin | <code>\notin</code> | \neq | <code>#</code> or <code>/=</code> |
| \langle | <code><<</code> | \rangle | <code>>></code> | \square | <code>[]</code> |
| $<$ | <code><</code> | $>$ | <code>></code> | \diamond | <code><></code> |
| \leq | <code>\leq</code> or <code>=<</code> or <code><=</code> | \geq | <code>\geq</code> or <code>>=</code> | \sim | <code>~></code> |
| \ll | <code>\ll</code> | \gg | <code>\gg</code> | \rightarrow | <code>-+></code> |
| \prec | <code>\prec</code> | \succ | <code>\succ</code> | \mapsto | <code> -></code> |
| \preceq | <code>\preceq</code> | \succeq | <code>\succeq</code> | \div | <code>\div</code> |
| \subseteq | <code>\subseteq</code> | \supseteq | <code>\supseteq</code> | \cdot | <code>\cdot</code> |
| \subset | <code>\subset</code> | \supset | <code>\supset</code> | \circ | <code>\o</code> or <code>\circ</code> |
| \sqsubset | <code>\sqsubset</code> | \sqsupset | <code>\sqsupset</code> | \bullet | <code>\bullet</code> |
| \sqsubseteq | <code>\sqsubseteq</code> | \sqsupseteq | <code>\sqsupseteq</code> | \star | <code>\star</code> |
| \vdash | <code> -</code> | \dashv | <code>- </code> | \bigcirc | <code>\bigcirc</code> |
| \models | <code> =</code> | \vDash | <code>= </code> | \sim | <code>\sim</code> |
| \rightarrow | <code>-></code> | \leftarrow | <code><-</code> | \simeq | <code>\simeq</code> |
| \cap | <code>\cap</code> or <code>\intersect</code> | \cup | <code>\cup</code> or <code>\union</code> | \asymp | <code>\asymp</code> |
| \sqcap | <code>\sqcap</code> | \sqcup | <code>\sqcup</code> | \approx | <code>\approx</code> |
| \oplus | <code>(+)</code> or <code>\oplus</code> | \uplus | <code>\uplus</code> | \cong | <code>\cong</code> |
| \ominus | <code>(-)</code> or <code>\ominus</code> | \times | <code>\X</code> or <code>\times</code> | \doteq | <code>\doteq</code> |
| \odot | <code>(.)</code> or <code>\odot</code> | \wr | <code>\wr</code> | x^y | <code>x^y</code> ⁽²⁾ |
| \otimes | <code>(\X)</code> or <code>\otimes</code> | \propto | <code>\propto</code> | x^+ | <code>x^+</code> ⁽²⁾ |
| \oslash | <code>(/)</code> or <code>\oslash</code> | "s" | <code>"s"</code> ⁽¹⁾ | x^* | <code>x^*</code> ⁽²⁾ |
| \exists | <code>\E</code> | \forall | <code>\A</code> | $x^\#$ | <code>x^\#</code> ⁽²⁾ |
| \exists | <code>\EE</code> | \forall | <code>\AA</code> | $'$ | <code>,</code> |
| $]_v$ | <code>]_v</code> | \rangle_v | <code>>>_v</code> | | |
| WF_v | <code>WF_v</code> | SF_v | <code>SF_v</code> | | |

| | | | |
|----------------------------|------------------------|----------------------------|------------------------|
| $\overline{\hspace{2cm}}$ | <code>-----</code> (3) | $\overline{\hspace{2cm}}$ | <code>-----</code> (3) |
| $\underline{\hspace{2cm}}$ | <code>-----</code> (3) | $\underline{\hspace{2cm}}$ | <code>=====</code> (3) |

- (1) s is a sequence of characters.
(2) x and y are any expressions.
(3) a sequence of four or more - or = characters.