

C2. $c \succ^+ d$ holds, for any elements c and d of \mathcal{CW} .

1. Let c be an arbitrary element of \mathcal{CW} and define

$$D \triangleq \{d \in \mathcal{CW} : \neg(c \succ^+ d)\}$$

It suffices to assume D is nonempty and obtain a contradiction.

PROOF: Obvious.

2. $d \succ c$ holds, for all $d \in D$.

PROOF: $\neg(c \succ^+ d)$ implies $\neg(c \succeq d)$ (because $c \succeq d$ implies $c \succ^+ d$ by definition of the transitive closure), and $\neg(c \succeq d)$ equals $d \succ c$ by definition of \succ .

3. For all $d \in D$ and all $e \in \mathcal{CW} \setminus D : d \succ e$.

3.1. It suffices to assume $d \in D$, $e \in \mathcal{CW} \setminus D$, and $\neg(d \succ e)$ and obtain a contradiction

PROOF: Obvious.

3.2. $c \succ^+ e$

PROOF: By the assumption of 3.1, which implies $e \notin D$, and the definition of D .

3.3. $e \succ d$

PROOF: By the assumption of 3.1, which asserts $\neg(d \succ e)$, and the definition of *succeq*.

3.4. Q.E.D.

Steps 3.2 and 3.3 and the definition of the transitive closure imply $c \succ^+ d$, which by the definition of D contradicts the step 3.1 assumption $d \in D$.

4. D is a dominating set.

PROOF: We must prove that if $d \in D$, then $d \succ e$ for any candidate e not in D . If e is not in \mathcal{CW} , this follows because \mathcal{CW} is a dominating set. If $e \in \mathcal{CW}$, then $e \in \mathcal{CW} \setminus D$ and this follows from step 3.

5. Q.E.D.

PROOF: D is a subset of \mathcal{CW} by definition. It is a proper subset of \mathcal{CW} because $c \in \mathcal{CW}$ by the step 1 assumption and step 2 implies $c \notin D$. Therefore, step 4 implies that \mathcal{CW} is not the smallest dominating set, which is a contradiction.