# A Proof of GCD3

1. It suffices to assume that $m$ and $n$ are positive integers and $d$ is any integers, and to prove that $d$ divides both $m$ and $n$ iff $d$ divides both $m$ and $n - m$.

   PROOF: Since the gcd of two numbers is the largest integer that divides both of them, it suffices to show that $m$ and $n$ have the same common divisors as $m$ and $n - m$.

2. If $d$ divides both $m$ and $n$, then $d$ divides both $m$ and $n - m$.

   PROOF: That $d$ divides $m$ follows from the assumptions; that it divides $n - m$ follows from the assumptions and Lemma Div.

3. If $d$ divides both $m$ and $n - m$, then $d$ divides both $m$ and $n$.

   PROOF: That $d$ divides $m$ follows from the assumptions; that it divides $n$ follows from the assumptions, Lemma Div, and the simple algebraic relation: $n = m + (n - m)$.

4. Q.E.D.

   PROOF: GCD3 follows from steps 1, 2, and 3.

CLOSE