$$\text{─────── MODULE } Sequences \text{ ───────}$$

LOCAL INSTANCE *Naturals*
> Imports the definitions from *Naturals*, but doesn't export them.

$Len(s) \;\triangleq\; \text{CHOOSE } n \in Nat : \text{DOMAIN } s = (1 \mathinner{.\,.} n)$

$s \circ t \quad\triangleq\; [i \in 1 \mathinner{.\,.} (Len(s) + Len(t)) \mapsto \text{IF } i \leq s \text{ THEN } s[i]$
$\text{ELSE } t[i - Len(s)]]$

$Append(s,\, e) \;\triangleq\; s \circ \langle e \rangle$

$Seq(S) \;\triangleq\; \text{UNION } \{[1 \mathinner{.\,.} n \to S] : n \in Nat\}$

$Head(s) \;\triangleq\; s[1]$

$Tail(s) \quad\triangleq\; \text{CASE } s \neq \langle \rangle \to [i \in 1 \mathinner{.\,.} (Len(s) - 1) \mapsto s[i+1]]$
> The "CASE $s \neq \langle \rangle \to$" just ensures that Tail(s) is undefined if s is the empty sequence.

$SubSeq(s,\, m,\, n) \;\triangleq\; [i \in 1 \mathinner{.\,.} (1 + n - m) \mapsto s[i + m - 1]]$
> The sequence $\langle [m], s[m+1], \ldots, s[n] \rangle$.

$SelectSeq(s,\, test(\_)) \;\triangleq\;$
  LET $F[i \in 0 \mathinner{.\,.} Len(s)] \;\triangleq\;$
      IF $i = 0$ THEN $\langle \rangle$
              ELSE IF $test(s[i])$ THEN $Append(F[i-1],\, s[i])$
                              ELSE $F[i-1]$
  IN $F[Len(s)]$
> The subsequence of s consisting of all elements s[i] such that $Test(s[i])$ is true.

CLOSE