

Recall the original (non-TLA⁺) definition:

$$R ** S \triangleq \{ \langle x, y \rangle : \exists z : (\langle x, z \rangle \in R) \wedge (\langle z, y \rangle \in S) \}$$

Since x is replacing $\langle x, y \rangle$, we can replace x and y by $x[1]$ and $x[2]$. Hence, the definition becomes

$$R ** S \triangleq \{ x \in T : \exists z : (\langle x[1], z \rangle \in R) \wedge (\langle z, x[2] \rangle \in S) \}$$

We now have to decide what the set T should be. A little thought reveals that the elements of $R ** S$ have to be pairs $\langle r, s \rangle$ with r a node of R and s a node of S . Therefore, we can take T to be the Cartesian product $NodesOf(R) \times NodesOf(S)$, to obtain:

$$\begin{aligned} R ** S \triangleq \{ x \in NodesOf(R) \times NodesOf(S) : \\ \exists z : (\langle x[1], z \rangle \in R) \wedge (\langle z, x[2] \rangle \in S) \} \end{aligned}$$

This is a legal TLA⁺ definition, but TLC can't evaluate it because it contains the unbounded quantifier $\exists z : \dots$. We need to restrict the range of the bound identifier z . The body of the quantified expression is satisfied only if z is an element of both $NodesOf(R)$ and $NodesOf(S)$. So we could write this quantified expression in any of these ways:

$$\begin{aligned} \exists z \in NodesOf(R) : \dots \\ \exists z \in NodesOf(S) : \dots \\ \exists z \in NodesOf(R) \cap NodesOf(S) : \dots \end{aligned}$$

Although longer, I find the third to be a little clearer:

$$\begin{aligned} R ** S \triangleq \{ x \in NodesOf(R) \times NodesOf(S) : \\ \exists z \in NodesOf(R) \cap NodesOf(S) : \\ (\langle x[1], z \rangle \in R) \wedge (\langle z, x[2] \rangle \in S) \} \end{aligned}$$