Overview of Stochastic Processes

Monday, August 31, 2015

3:11 PM

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In terms of abstract mathematics, a stochastic process is a random function that maps a parameter domain into a state space.

Random: The value of the mathematical object depends not only on the explicit variables, but also on some uncertain influences.

Parameter domain: The set describing the explicit independent variable of the stochastic process, usually time, but can be space.

State space: range of the random variable(s) of interest.

Stochastic process $X(t)$ or X_t

where $t \in T$ (parameter domain)

and

$X \in S$ (state space)

The most basic classification of stochastic processes is in terms of the nature of their parameter domains and state space.

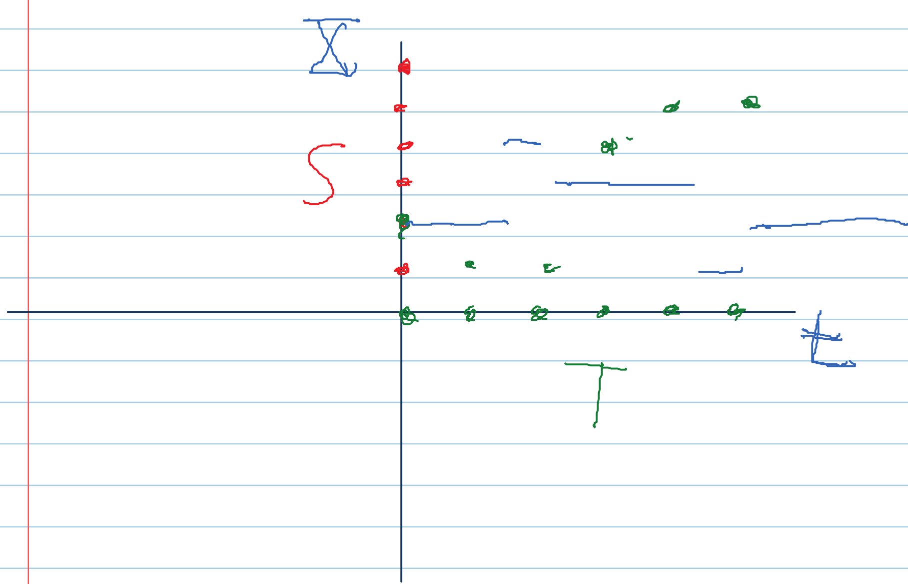
- Discrete: finite or countably infinite, typical examples are finite sets like
  - $\mathbb{Z}_m = \{1,2,\ldots,m\}$, state of one agent (i.e., susceptible, infected, exposed, recovered, immune individual)
  - $\mathbb{Z}_m^k = \{1,2,\ldots,m\} \times \{1,2,\ldots,m\} \times \cdots \times \{1,2,\ldots,m\}$, state of k agents

  or countably infinite sets like
  - $\mathbb{Z}_{\geq 0} = \{0,1,2,3,..,\}$, population or
  - $\mathbb{Z}_{\{\geq 0\}}^k = \{0,1,2,\ldots\} \times \{0,1,2,\ldots\} \ldots \times \{0,1,2,\ldots.\}$, number of molecules of k different reacting chemical species
  - modifications like $0.25\, \mathbb{Z}_{\{\geq 0\}} = \{0,0.25,0.5,0.75,1.0,1.25,\ldots\}$, the possible values of the US federal reserve interest rate
- Continuous: uncountable, typically continuous subsets of $\mathbb{R}^d$ or $\mathbb{C}^d$
  - prices of d assets/commodities
  - location or location/orientation of swimming organism

This class will only consider discrete state spaces.

As for the parameter domain, we will focus only on one-dimensional parameter domains, with the independent variable typically being time.

- could also in principle apply to one-dimensional space
- multi-dimensional spatial random functions (turbulence): require different stochastic frameworks, the theory of random fields, i..e, Yaglom, Correlation Theory of Stationary and Related Random Functions
- we will consider both discrete and continuous one-dimensional parameter domains, starting with discrete parameter domain.



Discrete time

Continuous time