ME591 - Assignment 5

Herman Kolstad Jakobsen
20196493

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Problem 1a)

The weights for a 20th order direct FIR low-pass filter with filter frequency of 1 Hz is shown in fig. 1. The stopband-edge frequency was not specified in the task, so it was arbitrary chosen to be 2 Hz.

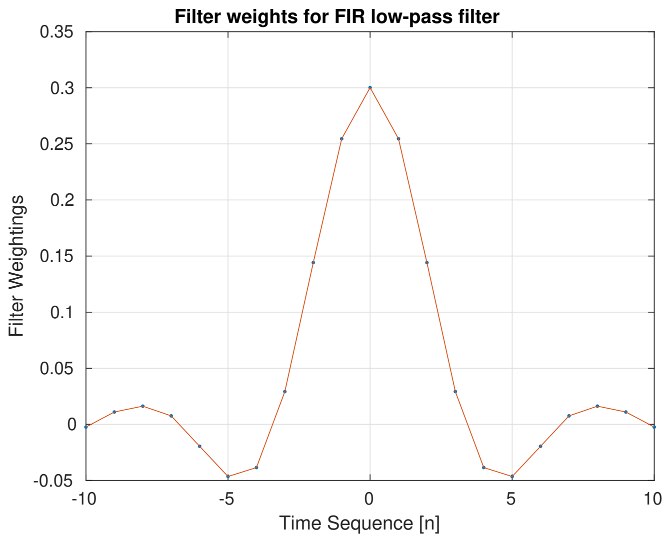

Figure 1: Filter weights for direct FIR low-pass filter

1b)

The filter frequency response function is shown in fig. 2.

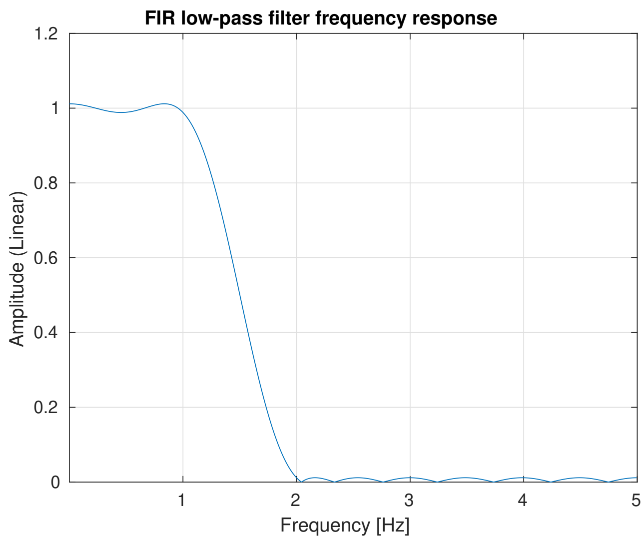

Figure 2: Frequency response of direct FIR low-pass filter

1c)

The low-pass filtered output is compared with the original signal in fig. 3.

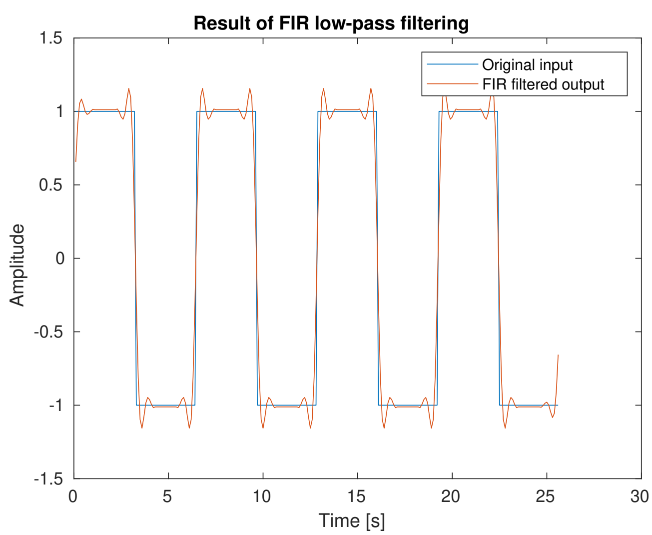

Figure 3: Comparison of FIR low-pass filtered output and original signal

1d)

The Fourier coefficient of the filtered output is plotted in fig. 4.

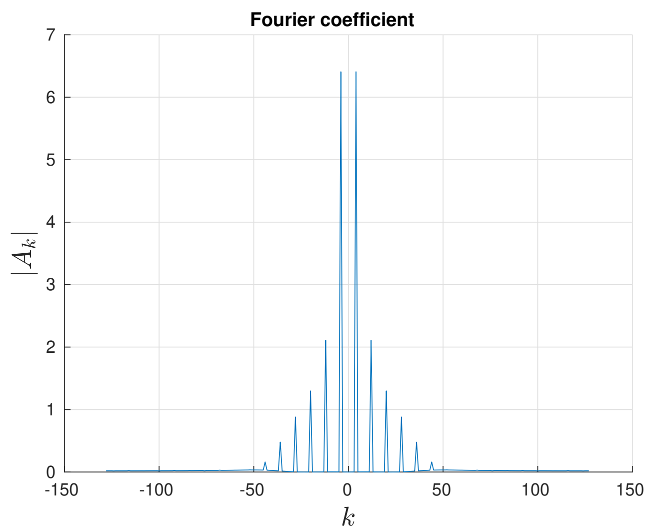

Figure 4: Fourier coefficient for FIR low-pass filtered output

1e)

A characteristic trait of a symmetric FIR filter is that the filter transfer function is purely real. This means that the filter will not impose any phase shift (time delay) to the filtered output. By looking at fig. 3, it can be verified that the filtered output has no time delay compared to the original signal.

The trade-off is that the output will not be optimally filtered. From fig. 2 it can be seen that the frequency response has a rather wide transition band together with some ripples in both the pass- and the stop band. Hence, not all the frequencies over the filter frequency will be removed. This can be seen in fig. 3 and fig. 4, where the filtered output still contains high frequency components which makes the filtered output able to obtain constant values.

Problem 2a)

The filter frequency response for a 7th order Butterworth low-pass filter with filter frequency of 1 Hz is shown in fig. 5.

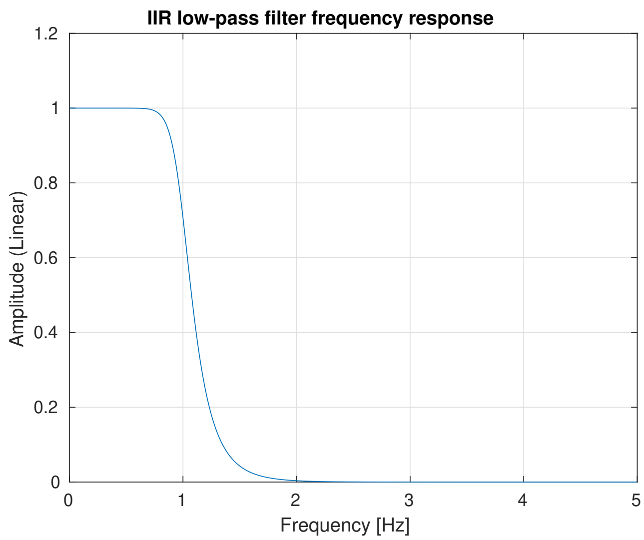

Figure 5: Frequency response of Butterworth low-pass filter

2b)

The low-pass filtered output is compared with the original signal in fig. 3.

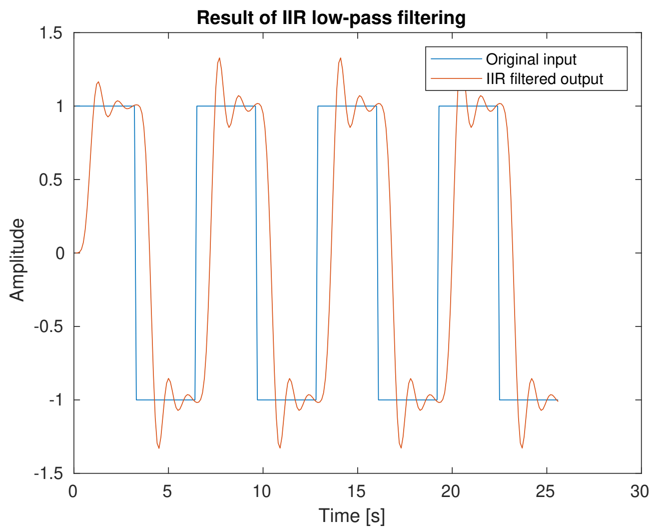

Figure 6: Comparison of Butterworth low-pass filtered output and original signal

2c)

The Fourier coefficient of the filtered output is plotted in fig. 7.

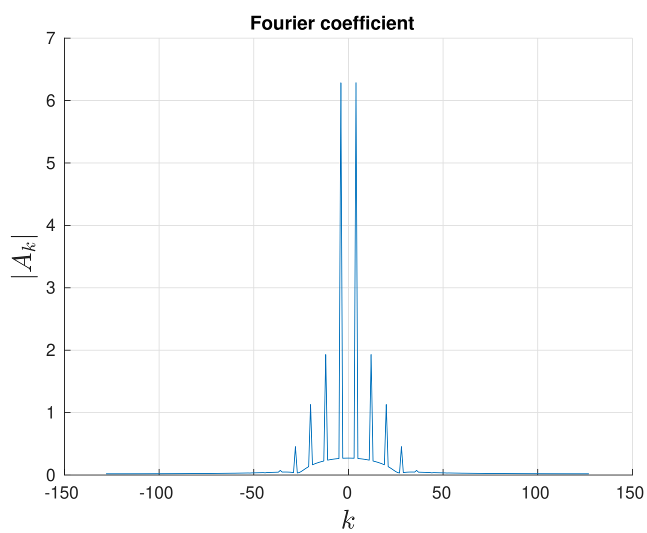

Figure 7: Fourier coefficient for Butterworth low-pass filtered output

2d)

From fig. 5 it can be seen that the filtered has a time delay compared to the original input, which is characteristic for an IIR filter.

On the other hand, the frequencies are nearly ideally filtered. By looking at fig. 5, the filter has a narrow transition band with no ripples in neither the pass- nor the stop band. This is further verified in fig. 7, where the higher frequency components are removed. It should be noted that the plot of the coefficients do not consist of pure spikes. The small "dome" under the spikes is a consequence of the imaginary part of the coefficients, which represents the phase shift in the output signal.

2e)

Comparing fig. 3 with fig. 6, it is clearly seen that a difference between an FIR filter and an IIR filter is the time delay in the filtered output. The IIR filter imposes a time delay in the filtered output.

By further comparing fig. 2 and fig. 4 with fig. 5 and fig. 7, respectively, it can be concluded that the IIR filter has a more ideal frequency response and is therefore more effective in filtering out the unwanted frequencies. It should also be noted that the IIR filter has a much lower order compared to the FIR filter. Hence, more ideal filtering can be obtained using a lower order IIR filter with the cost of imposing a time delay to the filtered output, compared to an FIR filter.

Problem 3a)

The information has been specified in table 1

Table 1: Filter characteristics

Filter Characteristics	
Filter type (FIR/IIR, low/high/band-pass/reject)	FIR, low-pass
Filter order	$n = 10$
Sampling frequency, Hz	$df = \frac{1}{dt} = \frac{1}{10^{-3}s} = 1000$ Hz
Pass band frequency, Hz	$F_p = 0.43 \cdot f_c = 0.43 \cdot 500$ Hz $= 215$ Hz
Stop band frequency, Hz	$F_s = 0.57 \cdot f_c = 0.57 \cdot 500$ Hz $= 285$ Hz
Transition width, Hz	$F_s - F_p = 285 - 215\ Hz = 70$ Hz
Pass band ripple, dB	$\delta_p = 1.1 - 1 = 0.1$ $\pm 20 \log_{10}(1 - \delta_p) = \mp 0.915$ dB
Stop band attenuation, dB	$\delta_s = 0.1 \implies 20 \log_{10}(\delta_s) = -20$ dB

3b)

Advantages in performance:

- The filter is an FIR, a type of filter which has several advantages. For instance, the filter has an exact linear phase and is always stable.
- The filter has a relative low order. This results in a smaller computational cost, compared to a higher order FIR filter.

Disadvantages in performance:

- The filter has a relatively big pass band ripple. This will distort the frequencies under the filter frequency, which is an unwanted effect.
- The filter has a significant stop band attenuation, which means that some of the frequencies over the filter frequency will not be completely filtered out.
- The transition width is pretty large, with the size of 1/3 of the pass band. The output will be influenced by frequencies in this frequency band which is unwanted. Ideally, all frequencies over F_s should be filtered out.

Problem 4a)

Spatial filtering is done by convolving an image with the filter. Applying two spatial filters in series would be equivalent to convolve the two filter masks with each other, and then convolve the image with this new filter mask. Given the following filter mask

$$w = \frac{1}{16} \cdot \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array} \tag{1}$$

Applying this filter in series (repeated two times) will then give the following filter mask

$$h = w \circledast w = \begin{array}{|c|c|c|c|c|} \hline 0.0039 & 0.0156 & 0.0234 & 0.0156 & 0.0039 \\ \hline 0.0156 & 0.0625 & 0.0938 & 0.0625 & 0.0156 \\ \hline 0.0234 & 0.0938 & 0.1406 & 0.0938 & 0.0234 \\ \hline 0.0156 & 0.0625 & 0.0938 & 0.0625 & 0.0156 \\ \hline 0.0039 & 0.0156 & 0.0234 & 0.0156 & 0.0039 \\ \hline \end{array} \tag{2}$$

4b)

The filter masks given in eq. (1) and eq. (2) are approximations of a Gaussian function. Applying this filter will therefore result in Gaussian smoothing. Hence, if the filter is used in series many times, a <u>low-pass filter</u> is expected.

4c)

Since the filter is a low-pass filter it will effectively remove high-frequency components, such as noise, by blurring out the image. However, details in images are also represented as high-frequency components. This means that also details will be blurred out. An image that is heavily influenced by noise would probably benefit from applying this low-pass filter. On the other hand, if the image contains many essential details, these details would be removed by the filter. Before applying this type of filter it is therefore worth deciding the importance of keeping details versus removing noise.