EWOK specification

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INTRODUCTION

Code EWOK includes 2D relativistic PIC module and single-component SVEA, paraxial Maxwell-solver.

We approach the Vlasov kinetic equation numerically with help of the "particle-in-cell" method. The electron distribution function (EDF) is presented discretely as a number of macro-particles, which follow the equations of motion with Lorentz or ponderomotive force. Calculating the density of particles' coordinates or velocities, one may find the electromagnetic and electrostatic fields from using Ampere-Poisson equations.

In the 2D simplified numerical model, where we consider x- and y-components electron motion. The electron dynamics is described by the relativistic motion equations

$$\partial_t \mathbf{p} = -m_e c^2/(4\gamma_e)\nabla\left(|\bar{a}|^2 - 4\phi\right) \tag{1}$$

where $\bar{a}$ is an enveloped electromagnetic field, which defines a ponderomotive potential, and $\phi = e\Phi/mc^2$ is a normalized electrostatic potential. Electrostatic potential can be calculated with Poisson equation:

$$\nabla^2\phi = k_0^2 n_e/n_c\,. \tag{2}$$

The enveloped electromagnetic field is defined as $a = \mathrm{Re}[\bar{a}\,\mathrm{e}^{-i(\omega_0 t - k_0 z)}]$, where k_0 and ω_0 are the wave-vector and frequency of the laser field. Considering only waves propagating in the positive direction of z-axis, we may linearize electromagnetic equation:

$$(\partial_t^2 - c^2\nabla^2)a = -(4\pi e^2/m)n\,a\,,$$

neglecting second order derivatives over time and z-coordinate and Fourier transform over x-coordinate:

$$\omega_0^{-1}(\partial_t + c\partial_z)\langle\bar{a}_s\rangle_{kx} + i\,(k_x/k_0)^2\,\langle\bar{a}_s\rangle_{kx} = -i\,|\langle\bar{a}_s + \bar{a}_L\rangle_{kx}|\,n_e/n_c\,, \tag{3}$$

where $n_c = m\omega_0^2/(4\pi e^2)$ is the critical density for laser radiation. Field of the optical lattice reads,

$$\bar{a}_L = 2a_0(t)\sin(k_{0\perp}x)\mathrm{e}^{-i(k_0 + k_{0\parallel})z}\,,$$

and its temporal profile, $a_0(t)$, allows one to model electron beam injection.

The signal wave may be injected at the left boundary of rectangular simulation domain. Since solution to eq. (3) is represented by Fourier harmonics along x-axis, the horizontal boundaries are periodic for electromagnetic waves, however an absorbing-layer technique may be applied. Vertical boundaries may be either semi-periodic or absorbing for electromagnetic waves propagating along z-axis. Counter-propagating waves are not considered. The particles are initially spread within the rectangular simulation domain homogeneously and may have the Gaussian or "water-bag" distributions of momentums. The boundaries may be either absorbing or periodic for the particles.

With such an algorithm we may study temporal amplification of the signal wave in the periodic system as well as injection of electrons into the lattice. The vertical size of simulation domain present one potential channel width $\gamma_b\lambda_0/2$, while horizontal dimension should be chosen to be integer number of the wavelengths of longitudinal potential $a_s a_L* \sim \mathrm{e}^{i(k_{0\parallel}+k_s)z}$ as well as enveloped signal wave $\bar{a}_s \sim \mathrm{e}^{i(k_s - k_0)z}$. Electrostatic force violates the periodicity and should be neglected or considered qualitatively. Large-scale simulations of non-periodic system may be used for the study of electrostatic effects.

FIELD EQUATIONS

Calculation of electromagnetic field is based on the equation for vector potential $\mathbf{A}$, which defines electric and magnetic components as

$$\mathbf{E} = -c^{-1}\partial_t\mathbf{A} - \nabla\phi\,,\quad \mathbf{B} = \nabla\times\mathbf{A}\,.$$

Using Ampere's law and choosing the Coulomb gauge $\nabla \cdot \mathbf{A} = 0$, we can derive wave equation for vector potential with a source given by only transverse current:

$$(c^{-2}\partial_t^2 - \nabla^2)A_y = (4\pi/c)J_{tr}$$

The transverse current is calculated as $J_{tr} = -eu_y n_e$, where y-component of averaged velocity u_y depends only on the field magnitude in coordinate (z, x) and can be found using the motion equation along z-axis, with a dominate transverse force:

$$\partial_t u_y = (e/mc\gamma_e)\partial_t A_y \, ,$$

where γ_e is an average relativistic factor in this location.

Considering no macroscopic currents along y-direction we can finally write the equation in a dimensionless form:

$$(\partial_\tau^2 - \partial_\rho^2)a = -(2\pi)^2(\tilde{n}/\gamma_e)a \, , \tag{4}$$

where $\tau = ct/\lambda_0$ and $\rho = (\xi, \chi) = (z, x)/\lambda_0$ are time and coordinates in laser units and electron density $\tilde{n} = n_e/n_c$ is normalized to its critical value for laser light $n_c = \pi m_e c^2/(e\lambda_0)^2$.

Distribution of $(\tilde{n}/\gamma_e)$ is calculated with account for each particles location and relativistic factor and it is done in the "particle pusher" part.

Field in the code is split into the laser and signal waves $a = a_L + a_s$. Laser field is given by the sum of two pulses with amplitude a_0, wave-vectors

$$k_{0\|} = \beta_b k_0 \, , \quad k_{0\perp} = \pm k_0/\gamma_b \, ,$$

where β_b is a dimensionless velocity of electrons in laboratory reference system and $\gamma_b = (1 - \beta_b)^{-1/2}$. This filed defines the optical lattice and can be written in complex form

$$a_L = 2a_0 \sin(k_{0\perp}x)\mathrm{e}^{-i(\omega_0 t + k_{0\|}z)} = 2a_0 \sin(2\pi\chi/\gamma_b)\mathrm{e}^{-2\pi i(\tau + \beta_b \xi)} \, ,$$

and considering no dispersion of laser pump it satisfies the gauge

$$(\partial_\tau^2 - \partial_\rho^2)a_L \equiv 0$$

EWOK solves eq. (4) for enveloped signal wave $\hat{a}_s$, defined as $a_s = \bar{a}_s \mathrm{e}^{-i(\omega_0 t - k_0 z)}0$. The order of eq. (4) is reduced by neglecting second order derivatives ∂_τ^2 and ∂_ξ^2 of slow varying envelop and Fourier transform along x-axis $\partial_\chi^2 \bar{a}_s \to -(2\pi)^2 \tilde{k}_x^2 \langle \bar{a}_s \rangle_{k_x}$, where dimensionless wave number is normalized to $\tilde{k}_x = k_x \lambda_0$. Final form of equation to solve reads:

$$i(\partial_\tau + \partial_\xi)\langle \bar{a}_s \rangle_{k_x} - \pi \tilde{k}_x^2 \langle \bar{a}_s \rangle_{k_x} = \pi \langle (\bar{a}_s + \bar{a}_L)\tilde{n}/\gamma_e \rangle_{k_x} \tag{5}$$

This equation can be approached using hybrid (explicit with implicit $\tilde{k}_x^2 \bar{a}_s$) finite difference numerical dispersion free (NDF) scheme on the grid (ξ_i, χ_j) with a time step equal to cell size $d\tau = d\rho$, which gives:

$$(1 + i\pi k_j^2 d\tau)\hat{a}_{i+1,j} = a_{i,j} + \pi \langle (\bar{a}_s + \bar{a}_L)\tilde{n}/\gamma_e \rangle_{i,k_j}$$

where $\hat{a}$ stands for the value of vector potential on the next time step and some notations are omitted for simplicity. Periodic boundary condition along x-direction in a discrete form reads:

$$\hat{a}_{0,j} = \hat{a}_{N_x,j}$$

where index N_x corresponds to a last element along x-grid.

PARTICLE PUSHER

Electrons are pushed by the forces of ponderomotive and electrostatic potentials. The relativistic motion equations reads:

$$\partial_\tau(\beta_z, \beta_x) = -1/(4\gamma_e)(\partial_\xi, \partial_\chi)\left[|\bar{a}_s + \bar{a}_L|^2 - 4e\phi\right] \tag{6}$$

where particles relativistic factor γ_e is with account for electromagnetic potential at its position

$$\gamma_e = \sqrt{(1 + |\bar{a}_s + \bar{a}_L|^2)/(1 - \beta_z^2 - \beta_x^2)}$$

Distribution of $(\tilde{n}/\gamma_e)$ on the grid in done by a third order interpolation scheme, while projection of potentials to the particle position implies the second order. (*sometime potential in γ_e is divided by 2 due to the temporal averaging, though it makes no difference in our case*)

ELECTROSTATIC POTENTIAL

Poisson equation is linearized and solved by a Fourier transform:

$$\langle\phi\rangle_{k_x,k_y} = -e\langle\tilde{n}\rangle_{k_x,k_y}/(\tilde{k}_x^2 + \tilde{k}_y^2)\,,$$

We remove the pole at zero wave-number as

$$\langle\phi\rangle_{k_x=0,k_y=0} = \Big(\langle\phi\rangle_{1,0} + \langle\phi\rangle_{-1,0} + \langle\phi\rangle_{0,1} + \langle\phi\rangle_{0,-1}\Big)/4$$