State space control using LQR method for a cart-inverted pendulum linearised model

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The Cart-Inverted Pendulum System (CIPS) is a classical benchmark control problem. Its dynamics resembles with that of many real world systems of interest like missile launchers, pendubots, human walking and segways and many more. The control of this system is challenging as it is highly unstable, highly non-linear, non-minimum phase system and underactuated. Furthermore, the physical constraints on the track position also pose complexity in its control design. This paper presents a control method to stabilise the unstable CIPS within the different physical constraints such as in track length and control voltage. A novel cart-inverted pendulum model is proposed where mechanical transmission and a dc motor mathematical model have been included which resembles the real inverted pendulum. Therefore problems emerged in realtime implementation can be minimised. A systematic the state feedback design method by choosing weighting matrices key to the Linear Quadratic Regulator (LQR) design is presented. Simulation experiments have been conducted to verify the controller's performances. From the obtained simulation and experiments it is seen that the proposed method can perform well stabilising the pendulum at the upright angle position while maintaining the cart at the desired position.

Index Terms—**Cart-Inverted Pendulum, Linear Quadratic Regulator, Optimal Control, Non Linear System**

I. Introduction

CONTROLLING a Cart-Inverted Pendulum System (CIPS) is a challenging problem which is widely used as benchmark for testing control algorithms such as PID controllers [1], neural networks [2], [3], fuzzy control [4], genetic algorithms [5]. The CIPS features as higher order, nonlinear, strong coupling and multivariate system, which has been studied by many researchers. It is used to model the field of robotics and aerospace field, and so has important significance both in the field of the theoretical study research and practice. It has good practical applications right from missile launchers to segways, human walking, earthquake resistant building design etc. The CIPS dynamics resembles the missile or rocket launcher dynamics as its center of gravity is located behind the centre of drag causing aerodynamic instability. The CIPS has two equilibrium points [6], one of them is stable while the other is unstable. The stable equilibrium corresponds to a state in which the pendulum is pointing downwards. The control challange is to maintain the pendulum at the unstable equilibrium point where the pendulum upwards, with minimum control energy. In recent times optimal control provides the best possible solution to process control problems for a given set of performance objectives. Detail review on optimal control has been presented in [7]. Survey on optimal control approaches and their applications have been conducted, for instance, an optimal control approach for inventory systems [8] and energy optimisation [9].

This paper investigates the application of the Linear Quadratic Regulator (LQR) to stabilise the CIPS at the unstable equilibrium point. The organisation of this paper is structured as follows: The dynamic modelling using Lagrangian approach firstly discussed in Section II In this section, the state space representation of the CIPS is presented where the DC motor and its transmission system are considered. Section III discusses the LQR method in detail, followed by simulation and results of the LQR control approach applied on CIPS problem presented in Section IV Conclusion is presented in Section V

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II. System Modeling

A. Lagrange's Equation

Lagrange's equation is used to describe the motion equation of a complex system dynamic in a very efficient way. It reduces the need for complicated vector analysis that usually required for discribing forces applied on a mechanical system.

The fundamental principle of Lagrane's equation is the representation of the system by a set of generalised coordinate $\mathbf{q} = \{q_1, \cdots, q_i, \cdots, q_n\}$, where n is the total assigned generalised coordinate. q_i is an independent degree of freedom of true system which completely incorporate the contraints unique to that system, i.e., the interconnections between parts of the system.

The Lagrangian function $\mathcal{L}$ is expressed by the kinetic energy $\mathcal{K}$ and the potential energy $\mathcal{P}$ as discribed as follows

$$\mathcal{L} = \mathcal{K}(\mathbf{q}, \dot{\mathbf{q}}) - \mathcal{P}(\mathbf{q}) \tag{1}$$

where the kinetic energy function in terms of the geralised coordinate $\mathbf{q}$ and its derivative $\dot{\mathbf{q}}$. The potential energy function is expressed in terms of only the generalised coordinate.

The desired motion equations are derived using

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = \mathcal{Q}_i \tag{2}$$