Computer Simulation of Electromagnetic Space Tethers

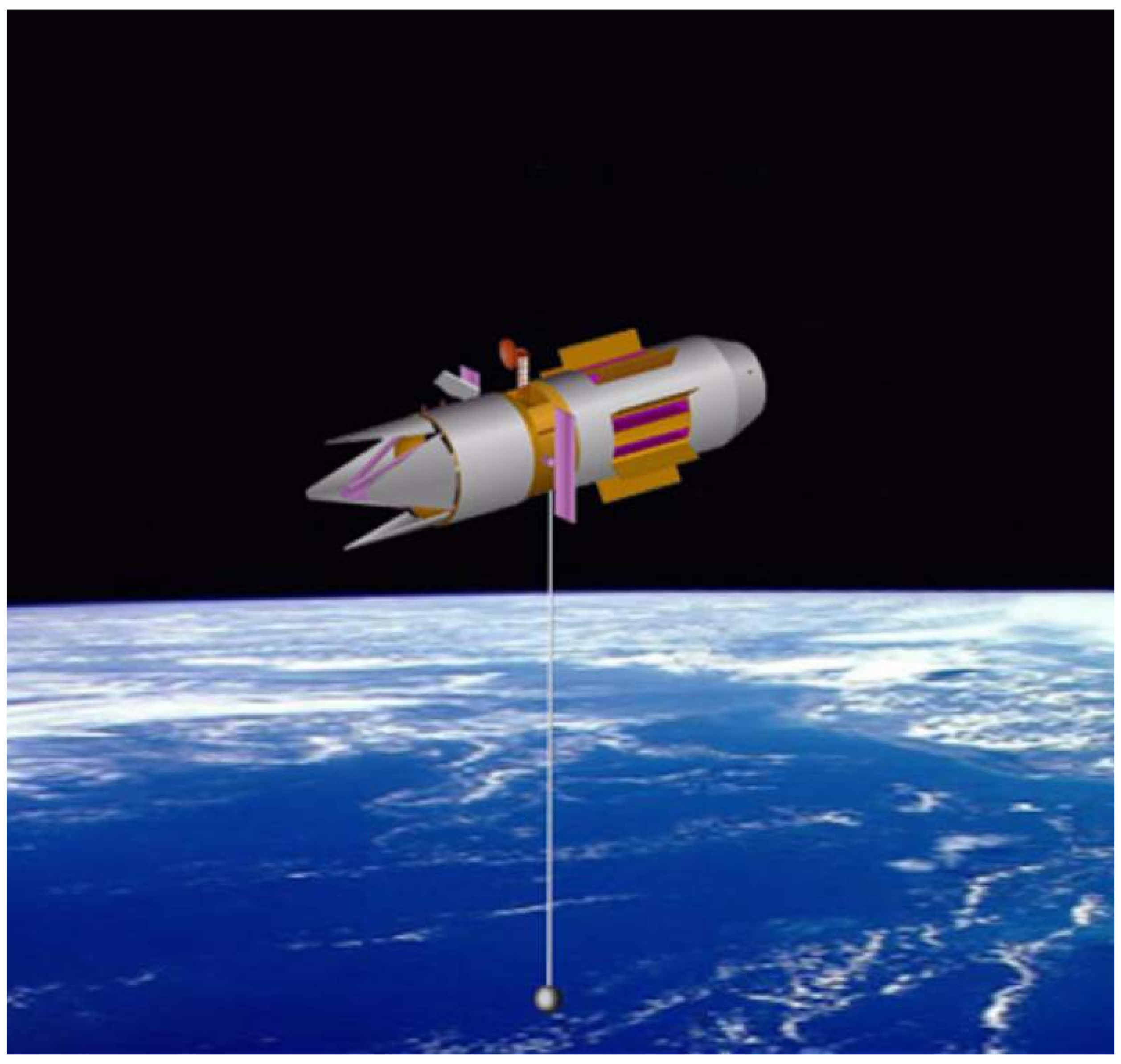
PAVEL SOLIN, JAKUB CERVENY, LENKA DUBCOVA
Department of Mathematical Sciences, The University of Texas at El Paso (UTEP)
http://hpfem.math.utep.edu/

About Space Tethers

Space tether is a long cable used to couple spacecrafts to each other or to other masses, such as spent booster rockets, space stations, or asteroids. Space tethers are usually made of thin strands of high-strength fibers or conducting wires. The tether can provide a mechanical connection between two space objects that enables the transfer of energy and momentum from one object to the other, and as a result they can be used to provide space propulsion without consuming propellant. Additionally, conductive space tethers can interact with the Earth's magnetic field and ionospheric plasma to generate thrust or drag forces without expending propellant.

A Look into History

The first mission of the Tethered Satellite System (TSS-1), launched 31 July 1992 aboard the space shuttle Atlantis, deployed an electrically conductive 1.6 m diameter satellite that was tethered to the Orbiter by a conductive tether. The second Tethered Satellite System (TSS-1R) was launched February 22, 1996 aboard the Space Shuttle Columbia. However, the tether broke during the deployment. It was approximately 12.2 miles long (19.7 km) when it broke, close to the maximum deploy length of 12.8 miles (20.7 km). These missions have provided valuable insight for planning the mission ProSEDS and they are a guide for future missions.



A satellite with the electromagnetic space tether. Courtesy NASA.

Applications

Space tethers can provide a number of applications:

- *Momentum-Exchange Tethers* allow momentum and energy to be transferred between objects in space, enabling a tether system to toss spacecraft from one orbit to another.
- *Formation Flying Tether Systems* can enable groups of satellites to fly in tight formation for applications such as long-baseline interferometry and synthetic aperture radar.
- *Electrostatic Tethers* may enable remediation of the Earth's radiation belts.
- *Electrodynamic Tethers* interact with the Earth's magnetosphere to generate power or propulsion without consuming propellant. They can be used, for example, to bring old satellites into atmosphere for thermal destruction.

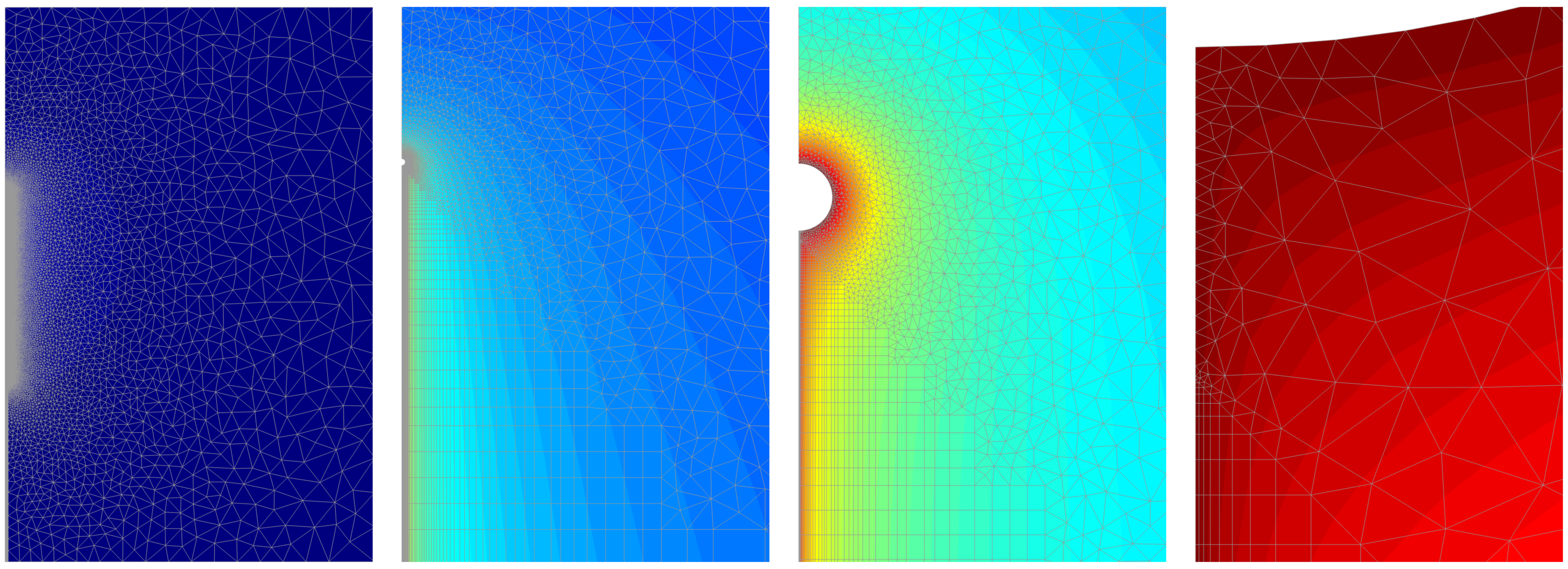
Computer Simulation

We are concerned with reliable computer simulation of Electrodynamic Tethers, which is challenging for a number of reasons. The length of a tether can vary from several meters to 20–30 kilometers, while the width of the induced boundary layer ranges from millimeters to centimeters. Typically, commercial codes do not provide error estimates, and therefore results obtained with various codes may vary significantly.

Traditional FEM

The goal of the model computation is to find the electric potential in the vicinity of a 10 km long conductive tether whose diameter is 1 mm. The potential along the tether itself is prescribed to change gradually from zero to 1000 V. A conductive sphere of a radius 20 cm is attached to the end of the tether. The size of the computational domain is 20 km × 30 km.

The lowest-order mesh consists of 1,478,160 linear triangular and bilinear quadrilateral elements. The employment of very thin quadrilateral elements turned out to be mandatory for the resolution of the boundary layer along the tether. The mesh was generated by combining Triangle (triangular part) with our own software.



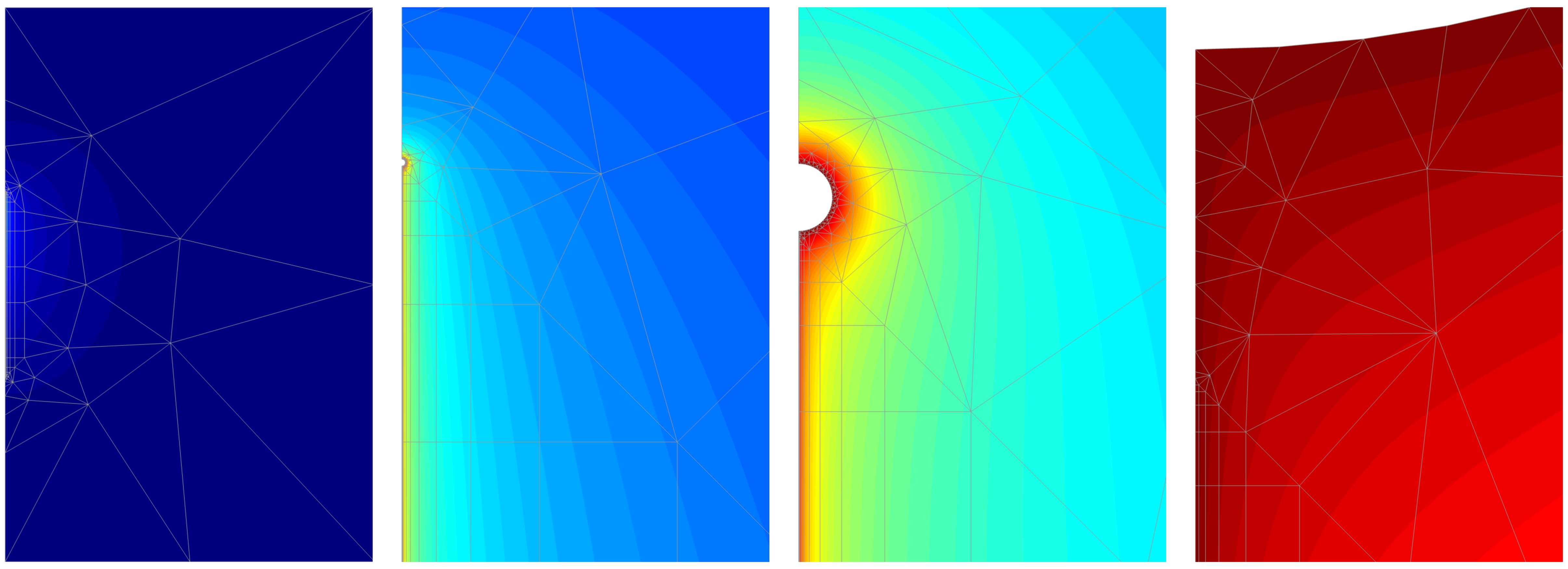
Lowest-order FEM solution. Zoom 1, 1000, 10000, and 400000.

hp-FEM

The hp-FEM mesh consists of 675 triangular and quadrilateral elements equipped with polynomial degrees ranging from 2 to 8. The diameter of finite elements in the mesh ranges from 0.5 mm to 20 km.

While the largest elements equipped with the highest polynomial degrees are used for the approximation of the farfield, the smallest elements equipped with lowest polynomial degrees are employed to resolve changes occuring on small scales in the vicinity of the tether and the sphere at its end.

On quadrilateral elements, we allow for different polynomial degrees in both directions in order to resolve efficiently the one-dimensional boundary layer along the tether.



hp-FEM solution. Zooms 1, 1000, 10000, and 400000.

Comparison of Results

	FEM, p=1	FEM, p=2	hp-FEM
Elements	1,478,160	119,997	675
Unknowns	1,236,885	404,385	4011
Error est.	0.7969%	0.2855%	0.1103%
Iterations	1759	532	58
CPU time	942 s	100 s	0.11 s

This table compares the performance of the piecewise-linear and piecewise-quadratic FEM to the hp-FEM. The traditional FEM with quadratic elements is much more efficient than the piecewise-linear version. However, the hp-FEM beats both of them by a factor more than 100 in both the number of DOFs and CPU time.

Error Estimation

HERMES calculates a-posteriori error estimates in various norms based on reference solutions. Reference solution u_{ref} is an approximation of the exact solution u, which is obtained with a higher-order of accuracy than the original approximation $u_{h,p}$. We use an enriched polynomial space for the computation of u_{ref}. The enrichment is done in a hierarchic fashion, by increasing polynomial degrees of all elements in the mesh. In this way, the stiffness matrix of the original problem is part of the stiffness matrix for the reference solution. The reference solution u_{ref} can be computed efficiently using various methods such as algebraic domain decomposition, algebraic multigrid, or a two-grid solver. The true error $e_{h,p} = u - u_{h,p}$ is approximated by $e_{h,p} \approx \hat{e}_{h,p} = u_{ref} - u_{h,p}$.

HERMES

HERMES (Hierarchic Modular hp-FEM System) is a modular framework for the hp-FEM discretization of nonlinear coupled problems, which is being developed by the hp-FEM group at the University of Texas at El Paso. In HERMES, various physical quantities are approximated using generally different types of finite elements. Each physical field is moreover approximated on an individual mesh, so that specific features such as singularities or boundary layers can be resolved most efficiently. The project is supported by the Department of Defense Award No. ONR 05PR07548-00, and by the NSF Grant No. DMS 0532645.