Image Super-Resolution using Bayesian Methods

Course Project for *CS 736: Medical Image Computing*, Spring 2021

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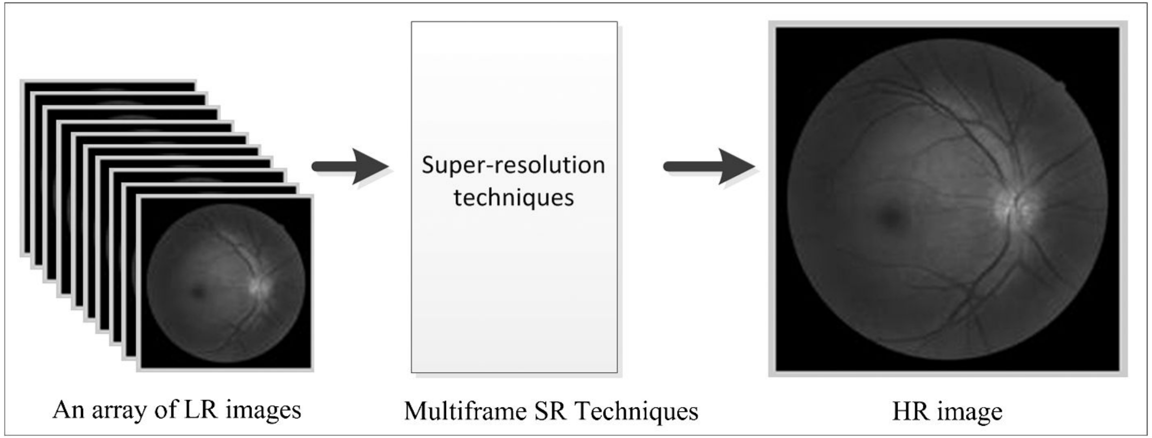
Problem Introduction

- Given a set of non-aligned low-resolution (LR) images of the same scene, estimate a single high-resolution (HR) image of the scene
- Also known as the "*Multi-Frame Super-Resolution*" (MFSR) problem



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Motivation

- Many vision applications like satellite imaging, medical imaging etc. require HR images which usually exceed the abilities of available digital cameras
- HR cameras are limited by high price, large size, or sensor limitations
- The set of LR images can also correspond consecutive frames in a LR video
- Thus, the use of image processing techniques to estimate the HR image from a set of low-quality LR images can act as a powerful and cheaper alternative

Background

- Tipping et al. suggested a Bayesian approach that estimates the registration parameters by marginalizing the likelihood over the unknown HR image
- Pickup et al. improved upon this by marginalizing over the registration parameters, which allowed the use of more suitable priors
- Our approach largely follows the approach suggested by Tipping et al.

Bayesian Image Super-Resolution

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Bayesian Image Super-resolution, Continued

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Bayesian Formulation

Observation / Acquisition Model

$$y_k = W_k x + \epsilon_k$$

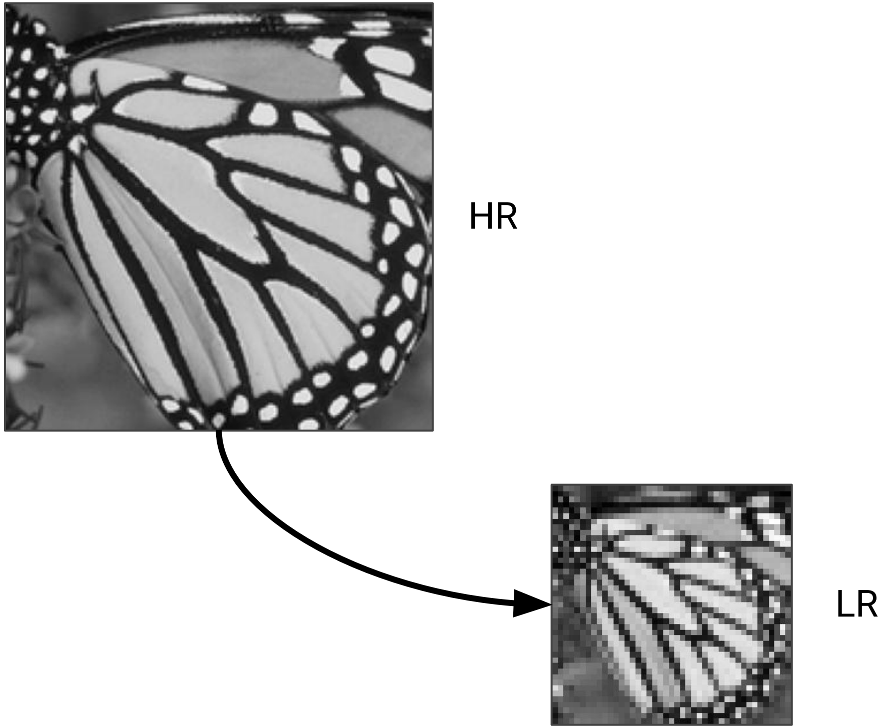
$$\epsilon_k(i) \sim \mathcal{N}(0, \sigma^2)$$

$$W_{ij} = \frac{W'_{ij}}{\sum_{j=1}^{N} W'_{ij}}$$

$$W'_{ij} = \exp\left(-\frac{\|v_j - u_i\|^2}{\gamma^2}\right)$$

$$u_i = R_k(v_i - \bar{v}) + \bar{v} + s_k$$

$$R_k = \begin{pmatrix} \cos\theta_k & \sin\theta_k \\ -\sin\theta_k & \cos\theta_k \end{pmatrix}$$



Likelihood

From the observation model, we get:

$$p(y_k|x, s_k, \theta_k, \gamma) = \frac{1}{(2\pi\sigma^2)^{M/2}} \exp\left(-\frac{1}{2\sigma^2}\|y_k - W_k x\|^2\right)$$

Thus, the log likelihood becomes:

$$\log \mathcal{L} = \sum_{k=1}^{K} \log p(y_k|x, s_k, \theta_k, \gamma)$$

$$= -\frac{KM}{2} \log\left(2\pi\sigma^2\right) - \frac{1}{2\sigma^2} \sum_{k=1}^{K} \|y_k - W_k x\|^2$$

Prior

Gaussian Prior

$$p(x) = \frac{1}{(2\pi \det(Z_x))^{\frac{N}{2}}} \exp\left(-\frac{1}{2}x^T Z_x^{-1} x\right)$$

$$\log p(x) = -\frac{N}{2}\log\left(2\pi \det\left(Z_x\right)\right) - \frac{1}{2}x^T Z_x^{-1} x$$

$$Z_x(i,j) = A \cdot \exp\left(-\frac{\|v_i - v_j\|^2}{r^2}\right)$$

MRF Prior

$$p(x) = \frac{1}{Z}\exp\left(-\frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}(i)} V(x_i - x_j)\right)$$

$$\log p(x)) = -\log Z - \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}(i)} V(x_i - x_j)$$

Posterior

$$\mathcal{P} = p(x \mid \{y_k, s_k, \theta_k\}_{k=1}^{K}, \gamma)$$

$$\log \mathcal{P} \propto (1-\beta) \log p(x) + \beta \log \mathcal{L}$$

Using the Gaussian prior, we can exactly compute the posterior, as both the likelihood and the prior are Gaussians, thus the posterior is also a Gaussian with the covariance and mean as follows

$$C = \left(Z_x^{-1} + \frac{1}{\sigma^2} \sum_{k=1}^{K} W_k^T W_k \right)^{-1}$$

$$\mu = \frac{C}{\sigma^2} \sum_{k=1}^{K} W_k^T y_k$$

Marginal Likelihood

When using the Gaussian prior, we know the posterior, prior and the likelihood in closed form. Thus, we can get the marginal likelihood in closed form.

$$p(y \mid \{s_k, \theta_k\}_{k=1}^K, \gamma) = \mathcal{N}(0, Z_y)$$

$$Z_y = \sigma^2 I + W Z_x W^T$$

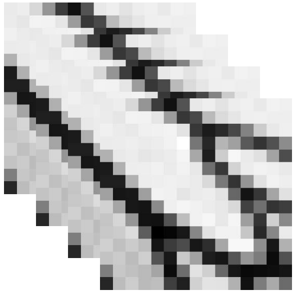
$$\log \mathcal{M} = \log p(\{y_k\}_{k=1}^K \mid \{s_k\}_{k=1}^K, \{\theta_k\}_{k=1}^K, \gamma)$$

$$= -\frac{1}{2}\left(\frac{1}{\sigma^2} \sum_{k=1}^K \|y_k - W_k \mu\|^2 + \mu^T Z_x^{-1} \mu + \log\left(\det\left(Z_x\right)\right) - \log\left(\det\left(C\right)\right) + 2KM \log\left(\sigma\right) \right)$$

Optimization Strategy

Set of LR Images

Extract patches



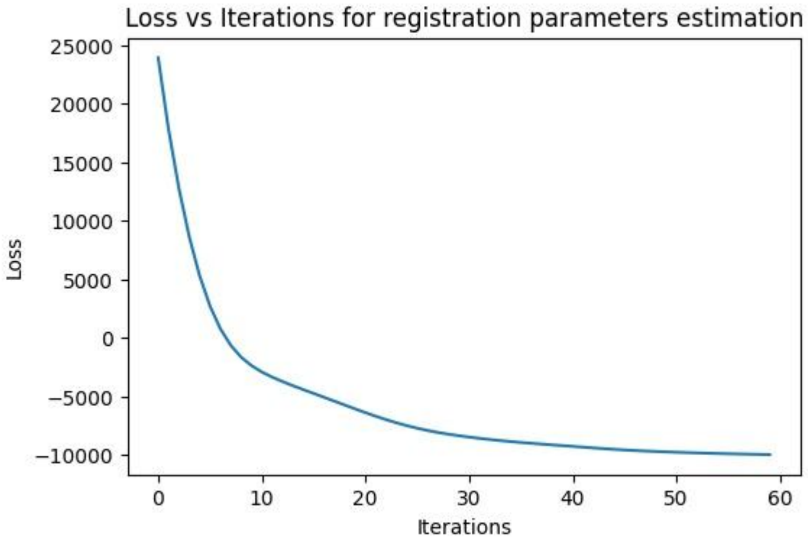
Minimize negative log marginal to estimate the registration parameters

Minimize the negative log posterior to estimate the HR image, keeping the registration parameters fixed

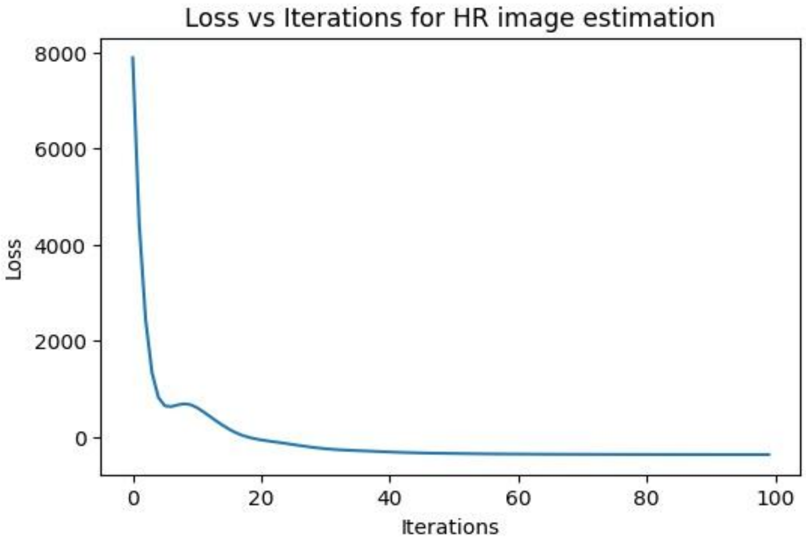
Estimated HR Image

Optimization performed using AdamW and PyTorch Autograd

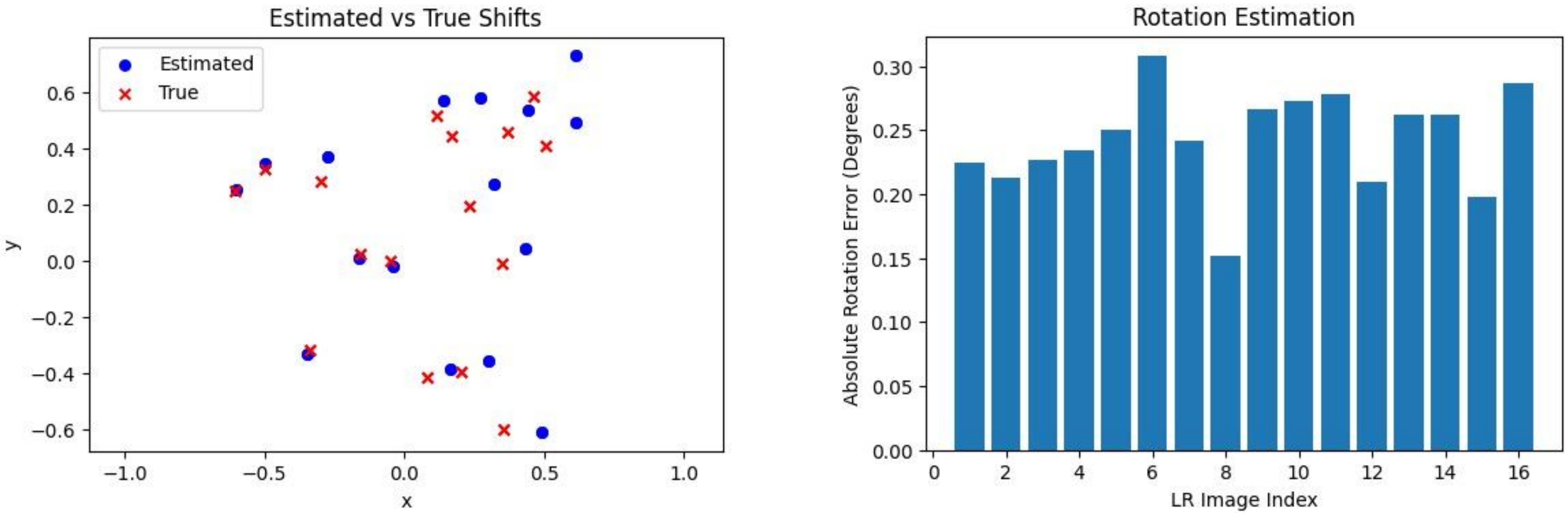
Results



Loss vs Iterations for registration parameters estimation
25000
20000
15000
10000
5000
0
−5000
−10000
Loss
0
10
20
30
40
50
60
Iterations

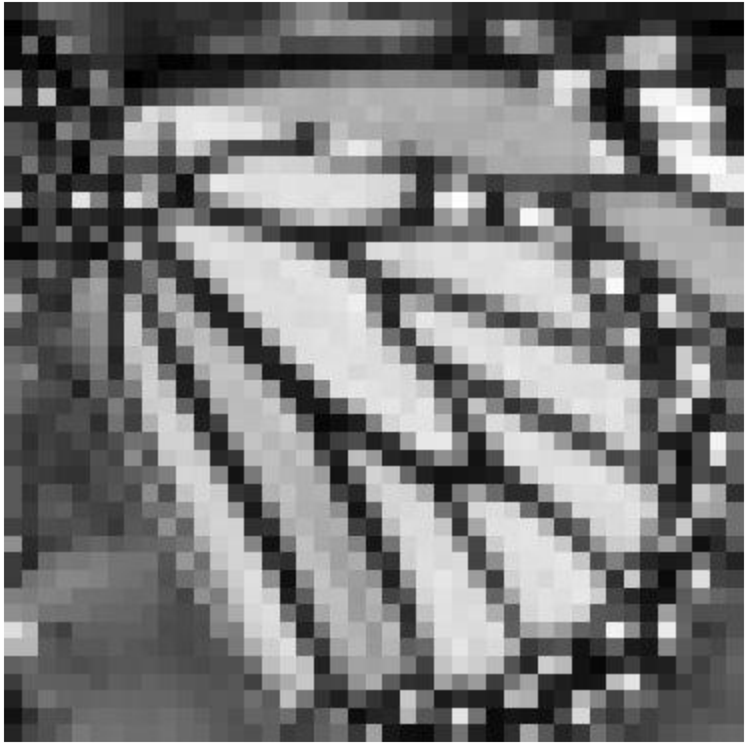


Loss vs Iterations for HR image estimation
8000
6000
4000
2000
0
Loss
0
20
40
60
80
100
Iterations

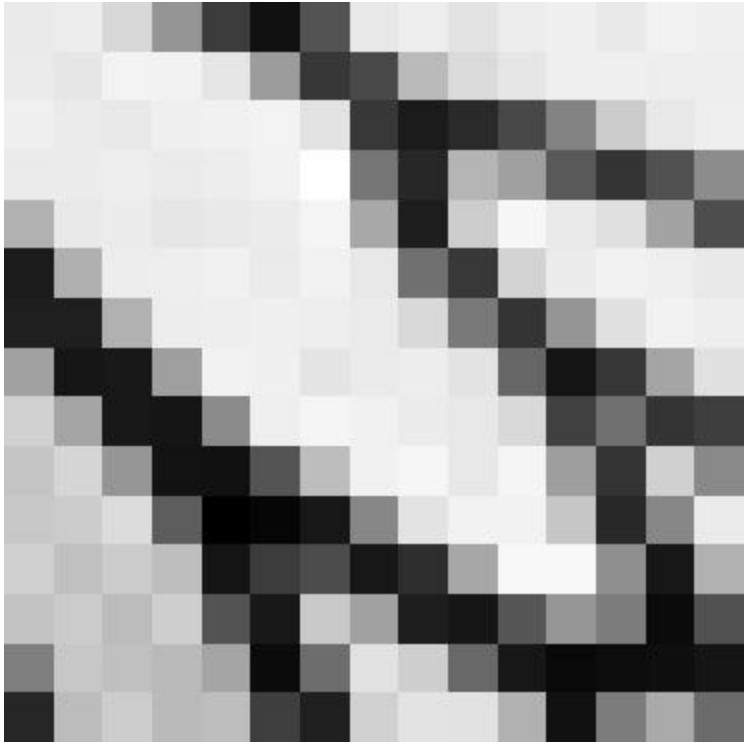

Estimated vs True Shifts
Estimated
True
y
x
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−1.0
−0.5
0.0
0.5
1.0
Rotation Estimation
Absolute Rotation Error (Degrees)
LR Image Index
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
2
4
6
8
10
12
14
16

Original HR Image

One of the LR Images



One of the patches used for estimating the registration parameters



Initial Estimate for the HR Image (Using Bicubic Interpolation)

PSNR: 15.57

Using Gaussian Prior

PSNR: 18.62

Using MRF Prior

PSNR: 22.05

Original HR Image

Using MRF Prior

PSNR: 22.05

Challenges Faced

- Memory requirements: The marginal requires computing C, the covariance matrix of the posterior. If the original HR image has N pixels, C is of size NxN. Even for small images (eg. 300x300), this is projected to occupy > 50 GB!
    - Solution 1: Since the marginal is used only for estimating the registration parameters, use small sized patches instead of the entire image
    - Solution 2: Use sparse matrices for operations, but PyTorch Autograd currently doesn’t work with sparse matrices
    - Solution 3: Divide the original images into different regions and run the algorithm per region, then combine the super-resolved regions
- The HR image estimated using the Gaussian prior has some periodic artifacts which we were unable to debug/remove

Conclusion

- We formulated and analysed the MFSR problem in a Bayesian way
- Using a Gaussian prior for computing the marginal likelihood allows us to:
  - Estimate the registration parameters more accurately
  - Use better priors (eg. MRF) while estimating the HR image, which improve the visual quality of the estimated image by a lot

Thank You