VARIABILE ALEATOARE

$$X \sim \begin{pmatrix} -1 & 7 \\ 0,18 & 0,82 \end{pmatrix} \qquad Y \sim \begin{pmatrix} -5 & 7 \\ p_1 & p_2 \end{pmatrix}$$

a) $\mathbb{P}(X=-1, Y=7) = 0,045 \qquad E[X \mid Y=7] = 3$

X\Y	-5	7	Σ
-1	0,135	0,045	0,18
7	$p_1 - 0,135$ 0,775	$p_2 - 0,045$ 0,045	0,82
Σ	$p_1 =$ 0,91	$p_2 =$ 0,09	

$$X \mid Y=7 \sim \begin{pmatrix} -1 & 7 \\ \frac{0,045}{p_2} & \frac{p_2 - 0,045}{p_2} \end{pmatrix}$$

$$-1 \cdot \frac{0,045}{p_2} + 7 \cdot \frac{p_2 - 0,045}{p_2} = 3 \quad / p_2$$

$$-0,045 + 7p_2 - 0,315 = 3p_2$$

$$4p_2 = +0,36$$

$$p_2 = 0,09$$

$p_1 = 1 - p_2$

$p_1 = 1 - 0,09$

$p_1 = 0,91$

b) $$X \sim \begin{pmatrix} -1 & 7 \\ 0,18 & 0,82 \end{pmatrix} \qquad Y \sim \begin{pmatrix} -5 & 7 \\ 0,91 & 0,09 \end{pmatrix}$$

$\mathbb{P}\{X+Y = (-1)+(-5) = -6\} = 0,135$

$\mathbb{P}\{X+Y = (-1)+7 = 6\} = 0,045$

$\mathbb{P}\{X+Y = 7+(-5) = 2\} = 0,775$

$\mathbb{P}\{X+Y = 7+7 = 14\} = 0,045$

$$X+Y \sim \begin{pmatrix} -6 & 2 & 6 & 14 \\ 0,135 & 0,775 & 0,045 & 0,045 \end{pmatrix}$$

$\mathbb{P}\{X-Y = (-1)-(-5) = 4\} = 0,135$

$\mathbb{P}\{X-Y = (-1)-7 = -8\} = 0,045$

$\mathbb{P}\{X-Y = 7-(-5) = 12\} = 0,775$

$\mathbb{P}\{X-Y = 7-7 = 0\} = 0,045$

$$X-Y \sim \begin{pmatrix} -8 & 0 & 4 & 12 \\ 0,045 & 0,045 & 0,135 & 0,775 \end{pmatrix}$$

$$x^2 \sim \begin{pmatrix} 1 & 49 \\ 0{,}18 & 0{,}82 \end{pmatrix} \qquad y^2 \sim \begin{pmatrix} 25 & 49 \\ 0{,}91 & 0{,}09 \end{pmatrix}$$

$$4x^2 \sim \begin{pmatrix} 4 & 196 \\ 0{,}18 & 0{,}82 \end{pmatrix} \qquad 7y^2 \sim \begin{pmatrix} 175 & 343 \\ 0{,}91 & 0{,}09 \end{pmatrix}$$

NOUL TABEL

$4x^2 \backslash 7y^2$	175	343	Σ
4	0,135	0,045	
196	0,775	0,045	
Σ			

$$\mathbb{P}\{4x^2+7y^2=4+175=179\}=0{,}135$$
$$\mathbb{P}\{4x^2+7y^2=4+343=347\}=0{,}045$$
$$\mathbb{P}\{4x^2+7y^2=196+175=371\}=0{,}775$$
$$\mathbb{P}\{4x^2+7y^2=196+343=539\}=0{,}045$$

$$4x^2+7y^2 \sim \begin{pmatrix} 179 & 347 & 371 & 539 \\ 0{,}135 & 0{,}045 & 0{,}775 & 0{,}045 \end{pmatrix}$$

$$\mathbb{E}[x] = (-1)\cdot 0{,}18 + 7\cdot 0{,}82 = 5{,}56$$
$$\mathbb{E}[y] = (-5)\cdot 0{,}91 + 7\cdot 0{,}09 = -3{,}92$$
$$\mathbb{E}[x^2] = (-1)^2\cdot 0{,}18 + 7^2\cdot 0{,}82 = 40{,}36$$
$$\mathbb{E}[y^2] = (-5)^2\cdot 0{,}91 + 7^2\cdot 0{,}09 = 27{,}16$$

$$\mathrm{COV}(x,y) = \mathbb{E}[xy] - \mathbb{E}[x]\cdot\mathbb{E}[y] = -24{,}56 - 5{,}56\cdot(-3{,}92) = -2{,}7648$$

$$\mathrm{Var}(x) = \mathbb{E}[x^2] - \mathbb{E}[x]^2 = 40{,}36 - 30{,}9136 = 9{,}4464$$
$$\mathrm{Var}(y) = \mathbb{E}[y^2] - \mathbb{E}[y]^2 = 27{,}16 - 15{,}3664 = 11{,}79\cancel{36}$$

$$\begin{aligned}
\mathrm{Var}(7x-2y+8) &= \mathrm{Var}(7x-2y) \\
&= \mathrm{Var}(7x) + \mathrm{Var}(-2y) + 2\cdot 7\cdot(-2)\cdot\mathrm{COV}(x,y) \\
&= 49\cdot\mathrm{Var}(x) + 4\cdot\mathrm{Var}(y) + 2\cdot 7\cdot(-2)\cdot(-2{,}7648) \\
&= 49\cdot 9{,}4464 + 4\cdot 11{,}7936 + 77{,}4144 \\
&= 587{,}4624
\end{aligned}$$

$$xy \sim \begin{pmatrix} 5 & -7 & -35 & 49 \\ 0{,}135 & 0{,}045 & 0{,}775 & 0{,}045 \end{pmatrix}$$

$$\mathbb{E}[xy] = 5\cdot 0{,}135 + (-7)\cdot 0{,}045 + (-35)\cdot 0{,}775 + 49\cdot 0{,}045 = -24{,}56$$

$$\rho(X,Y) = \frac{\operatorname{cov}(X,Y)}{\sqrt{\operatorname{Var}(X)} \cdot \sqrt{\operatorname{Var}(Y)}}$$

$$= \frac{-2{,}7648}{\sqrt{9{,}4464} \cdot \sqrt{11{,}7936}}$$

$$= -0{,}2619433\ldots$$

VARIABILE ALEATOARE INDEPENDENTE

3) l-am rezolvat mai înainte

$$X \sim \begin{pmatrix} -1 & 7 \\ 0{,}18 & 0{,}82 \end{pmatrix} \qquad Y \sim \begin{pmatrix} -5 & 7 \\ 0{,}91 & 0{,}09 \end{pmatrix}$$

$\mathbb{P}\{X+Y = (-1)+(-5) = -6\} = \mathbb{P}(\{X=-1\} \cap \{Y=-5\}) = \mathbb{P}\{X=-1\} \cdot \mathbb{P}\{Y=-5\}$
$= 0{,}18 \cdot 0{,}91 = 0{,}1638$

$\mathbb{P}\{X+Y=6\} = \mathbb{P}(\{X=-1\} \cap \{Y=7\}) = \mathbb{P}\{X=-1\} \cdot \mathbb{P}\{Y=7\} = 0{,}18 \cdot 0{,}09 = 0{,}0162$

$\mathbb{P}\{X+Y=2\} = \mathbb{P}(\{X=7\} \cap \{Y=-5\}) = \mathbb{P}\{X=7\} \cdot \mathbb{P}\{Y=-5\} = 0{,}82 \cdot 0{,}91 = 0{,}7462$

$\mathbb{P}\{X+Y=14\} = \mathbb{P}(\{X=7\} \cap \{Y=7\}) = \mathbb{P}\{X=7\} \cdot \mathbb{P}\{Y=7\} = 0{,}82 \cdot 0{,}09 = 0{,}0738$

$$X+Y \sim \begin{pmatrix} -6 & 2 & 6 & 14 \\ 0{,}1638 & 0{,}0162 & 0{,}7462 & 0{,}0738 \end{pmatrix}$$

$\mathbb{P}\{X-Y=4\} = \mathbb{P}\{X=-1\} \cdot \mathbb{P}\{Y=-5\} = 0{,}1638$

$\mathbb{P}\{X-Y=-8\} = \mathbb{P}\{X=-1\} \cdot \mathbb{P}\{Y=7\} = 0{,}0162$

$\mathbb{P}\{X-Y=12\} = \mathbb{P}\{X=7\} \cdot \mathbb{P}\{Y=-5\} = 0{,}7462$

$\mathbb{P}\{X-Y=0\} = \mathbb{P}\{X=7\} \cdot \mathbb{P}\{Y=7\} = 0{,}0738$

$$X-Y \sim \begin{pmatrix} -8 & 0 & 4 & 12 \\ 0{,}0162 & 0{,}0738 & 0{,}1638 & 0{,}7462 \end{pmatrix}$$

$$4X^2 \sim \begin{pmatrix} 4 & 196 \\ 0{,}18 & 0{,}82 \end{pmatrix} \qquad 7Y^2 \sim \begin{pmatrix} 175 & 343 \\ 0{,}91 & 0{,}09 \end{pmatrix}$$

$\mathbb{P}\{4X^2+7Y^2=179\} = 0{,}18 \cdot 0{,}91 = 0{,}1638$

$\mathbb{P}\{4X^2+7Y^2=347\} = 0{,}18 \cdot 0{,}09 = 0{,}0162$

$\mathbb{P}\{4X^2+7Y^2=371\} = 0{,}82 \cdot 0{,}91 = 0{,}7462$

$\mathbb{P}\{4X^2+7Y^2=539\} = 0{,}82 \cdot 0{,}09 = 0{,}0738$

$$4X^2+7Y^2 \sim \begin{pmatrix} 179 & 347 & 371 & 539 \\ 0{,}1638 & 0{,}0162 & 0{,}7462 & 0{,}0738 \end{pmatrix}$$

$\mathbb{E}[X] = 5{,}56$

$\mathbb{E}[Y] = -3{,}92$

$\mathbb{E}[X^2] = 40{,}36$

$\mathbb{E}[Y^2] = 27{,}16$

$Var(X) = 9{,}4464$

$Var(Y) = 11{,}7936$

$\mathbb{P}\{X \cdot Y = 5\} = \mathbb{P}\{X = -1\} \cdot \mathbb{P}\{X = -5\} = 0{,}18 \cdot 0{,}91$
$= 0{,}1638$

$\mathbb{P}\{X \cdot Y = -7\} = 0{,}0162$

$\mathbb{P}\{X \cdot Y = -35\} = 0{,}7462$

$\mathbb{P}\{X \cdot Y = 49\} = 0{,}0738$

$$X \cdot Y \sim \begin{pmatrix} 5 & -7 & -35 & 49 \\ 0{,}1638 & 0{,}0162 & 0{,}7462 & 0{,}0738 \end{pmatrix}$$

$\mathbb{E}[XY] = 5 \cdot 0{,}1638 + (-7) \cdot 0{,}0162 + (-35) \cdot 0{,}7462 + 49 \cdot 0{,}0738$

$\mathbb{E}[XY] = -21{,}7952$

$$\begin{aligned}
cov(X,Y) &= \mathbb{E}[X,Y] - \mathbb{E}[X] \cdot \mathbb{E}[Y] \\
&= -21{,}7952 - (5{,}56 \cdot (-3{,}92)) \\
&= 0
\end{aligned}$$

$$\begin{aligned}
Var(7X - 2Y + 8) &= Var(7X - 2Y) \\
&= 49\, Var(X) + 4\, Var(Y) + 2 \cdot 7 \cdot (-2) \cdot cov(X,Y) \\
&= 49 \cdot Var(X) + 4\, Var(Y) + 0 \\
&= 49 \cdot 9{,}4464 + 4 \cdot 11{,}7936 \\
&= 510{,}048
\end{aligned}$$

$$\rho(X,Y) = \frac{cov(X,Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}} = \frac{0}{\sqrt{\ } \cdot \sqrt{\ }} = 0$$