

$$\textcircled{2} \quad \sigma = (1 \ 2 \ \dots \ 7)(8 \ \dots \ 14) \quad S_{14}$$

$$T \in S_{14} \text{ ai } T^2 = \sigma$$

$$\text{sgn}(\sigma) = (-1)^6 \cdot (-1)^6 = 1 \quad \text{par}$$

$$T^2 = (1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7)(8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14)$$

$$T = C_{i_1} C_{i_2} \dots C_{i_h} \quad | \quad i_1 + i_2 + \dots + i_h \leq 14$$

(=14=2
8)

$$T^2 = C_{i_1}^2 C_{i_2}^2 \dots C_{i_h}^2$$

$$\begin{array}{l} \xrightarrow{2|h} C_h^2 = \text{prod de 2 cicluri de lungime } \frac{h}{2} \\ \xrightarrow{2 \nmid h} C_h^2 = \text{un ciclu de lungime } h \end{array}$$

$$\Rightarrow h=2, \quad i_1=7, \quad i_2=7$$

$$C_7^2 = (1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7) \Rightarrow (C_7^2)^4 = C_7^7 \cdot C_7$$

$$\Rightarrow C_7^2 = (1 \ 5 \ 3 \ 7 \ 2 \ 4 \ 6)$$

$$\text{Analog } (8 \ 10 \ 12 \ 14 \ 11 \ 13)$$

$$\sigma = (1 \ 5 \ 2 \ 6 \ 3 \ 7 \ 4)(8 \ 12 \ 10 \ 13 \ 14 \ 11)$$

micro

③ $7^{17^{17}} \pmod{29}$

$$a^{\phi(n)} \equiv 1 \pmod{n}$$

$$a^{p-1} \equiv 1 \pmod{p}$$

$$a^p \equiv a \pmod{p}$$

$$(7, 29) = 1 \Rightarrow 7^{28} \equiv 1 \pmod{29}$$

But a calcula $7^{17^{17}}$ e mf ra dle $7^{17} \pmod{28}$

$$7^{10} \equiv 1 \pmod{28}$$

$$17^{17} \pmod{29}$$

$$17^{28} \equiv 1 \pmod{28}$$

$$17^{10} \equiv 1 \pmod{28}$$

Neuh

④ $(\mathbb{Z}_2, +) \times (\mathbb{Z}_3, +)$ ord 24

$$\{(\hat{a}, \bar{b}) \in (\mathbb{Z}_2 \times \mathbb{Z}_3) \mid \text{ord}(\hat{a}, \bar{b}) = 24\}$$

$$\text{ord}((\hat{a}, \bar{b})) = [\text{ord}(\hat{a}), \text{ord}(\bar{b})] = 24$$

$$\text{ord}(\bar{b}) \mid 3$$

$$\text{ord}(\bar{b}) \in \Delta_{24}^+ \Rightarrow \text{ord}(\bar{b}) = 1 \Rightarrow \text{ord}(\hat{a}) = 24$$

$$24 \nmid 27 \quad \times$$

~~$$21 \in (\mathbb{Z}_{27})$$~~

~~$$\text{ord}(\hat{a}) = 21$$~~

$$\text{ord}(\bar{b}) = 3$$

$$\text{ord}(\hat{a}) = 8$$

$$\text{ord}(\hat{a}) = 8 = \frac{2^7}{(a, 2^7)}$$

$$(a, 2^7) = 16$$

$$a \in \{16, 48, 80, 112\}$$

$$\text{ord}(\bar{b}) = 3 = \frac{3^7}{(b, 3^7)}$$

$$(b, 3^7) = 3^6$$

$$b \in \{729, 1458\}$$

$$(\hat{a}, \bar{b}) \in \{(\hat{16}, \overline{729}), \dots, (\hat{112}, \overline{1458})\}$$

$$| \nearrow | = 8$$

$$\textcircled{5} \quad x \sim y \quad \wedge \quad x = y \quad \text{non } x + y = 14$$

~ schi

$$\hat{7}, \overline{2022}$$

$$5 \subset \mathbb{R}$$

$$f: \mathbb{R}/p \rightarrow \mathbb{R}$$

$$f(x) = 4x^2 - 56x + 200 \quad \circ \quad f \text{ - } \text{Range def?}$$

$$\textcircled{a} \text{ 1) refl } x \sim x \quad x = y \checkmark$$

$$2) \text{ sim } x \sim y \rightarrow y \sim x \checkmark$$

$$3) \text{ trans } \checkmark$$

\Rightarrow g. rel. de ochi,

$$\hat{f} = \{x \in \mathbb{R} / x = 4 \text{ sau } x + 4 = 14\}$$

$$\hat{f} = \{4\}$$

$$\widehat{2022} = \{2022, -2008\}$$

$$SC\mathbb{R}: [2, \infty)$$

$$\begin{aligned} f(14-x) &= \cancel{4}(14-x) - \cancel{28}x + 56x + 200 \\ &= 4x^2 - 112x + 56x + 200 \\ &= 4x^2 - 56x + 200 = f(x) \quad \checkmark \end{aligned}$$

$$\textcircled{6} f: \mathbb{R} \rightarrow \mathbb{R}^2$$

$$f(x) = \begin{cases} 2x-7 & ; x < -2 \\ 3x^2+6x-18 & ; x \geq -2 \end{cases}$$

f inj, surj, bij

$$f^{-1}([-8, 8]) \quad \text{si} \quad f([-3, 0])$$

inj

$$\textcircled{I} x < -2$$

$$2x-7 = 7-7 \Rightarrow x = 4 \quad \checkmark$$

$$\textcircled{II} x \geq -2$$

$$3x^2+6x-18 = 3y^2+6y-18 \Rightarrow x^2=y^2 \quad \times$$

$$f \text{ inj } x < -2$$

$$f \text{ sur } \text{ e inj } x \geq -2$$

surj

$$(\forall) y \in \mathbb{R} (\exists) x \in \mathbb{R} \text{ cu } f(x) = y$$

$$\textcircled{I} A = (-\infty, -2)$$

$$7x - 7 = y \Rightarrow x = \frac{7 + y}{7}$$

$$y = 0 \Rightarrow x = 1 \textcircled{I} \quad \times$$

$$\textcircled{II} A = [-2, \infty)$$

$$3x^2 + 6x - 8 = y$$

$$\Delta = \left(-\frac{b}{2a}, 1 - \frac{\Delta}{4a}\right) = (-1, -11)$$

$$a > 0 \Rightarrow \text{Im } f = [-11, \infty) \neq \mathbb{R} \quad \times$$

f nu e surj nici pe $(-\infty, -2)$, nici pe $[-2, \infty)$

big f nu e big nicăieri

$$f([-3, 0]) = f([-3, -2] \cup [-2, 0]) = f([-3, -2]) \cup f([-2, 0])$$

$$f(-3) = -28$$

$$\lim_{x \rightarrow -2} f(x) = -21$$

$$[-28, -21) \cup [-21, -18]$$

$$f'(x) = 6x + 6$$

x	-2	-1	0
f'(x)	-6	0	6
f(x)	-28	-21	-18

$$f^{-1}([-8, 8]) = \{x \in \mathbb{R} / f(x) \in [-8, 8]\}$$

$$\textcircled{\text{I}} \quad -8 \leq 7x - 7 \leq 8$$

$$-1 \leq 7x \leq 15$$

$$-\frac{1}{7} \leq x \leq \frac{15}{7}$$

$$x \in (-\infty, -2) \quad \left| \Rightarrow x \in \emptyset \right.$$

$$\textcircled{\text{II}} \quad -8 \leq 3x^2 + 6x - 18 \leq 8$$

$$10 \leq 3x^2 + 6x \leq 26$$

$$\textcircled{\text{a}} \quad 3x^2 + 6x - 10 \geq 0$$

$$\Delta = 36 + 120 = 156$$

$$x_{1,2} = \frac{-6 \pm \sqrt{156}}{6} = \frac{-1 \pm \sqrt{39}}{3}$$

x	$-1 - \sqrt{39}/3$	$-1 + \sqrt{39}/3$
f(x)	+	+
	+	+
	0	0
	-	-
	-	-
	0	0
	+	+

$$x \in \left[-\infty, -1 - \frac{\sqrt{39}}{3} \right] \cup \left[-1 + \frac{\sqrt{39}}{3}, \infty \right)$$

$$\textcircled{\text{b}} \quad 3x^2 + 6x - 26 \leq 0$$

$$\Delta = 36 + 312 = 348$$

$$x_{1,2} = \frac{-6 \pm 2\sqrt{87}}{6} = -1 \pm \frac{\sqrt{87}}{3}$$

$$x \in \left[-1 - \frac{\sqrt{87}}{3}, -1 + \frac{\sqrt{87}}{3} \right]$$

$$-1 - \frac{\sqrt{87}}{3} < -1 - \frac{\sqrt{39}}{3} < -1 + \frac{\sqrt{39}}{3} < -1 + \frac{\sqrt{87}}{3}$$

$$\underbrace{\qquad\qquad\qquad}_{< -2} \quad \left| \quad x \in \left[\frac{1}{3}(-3 + \sqrt{39}), \frac{1}{3}(-3 + \sqrt{87}) \right] \right.$$

② $(\mathbb{Z}_8, +)$ $(\mathbb{Z}_8, +)$ total morf.
Care sunt inj ?

$f: (\mathbb{Z}_8, +) \rightarrow (\mathbb{Z}_8, +)$ morf.

$$f(x+y) = f(x) + f(y)$$

$$f(0) = \bar{0}$$

$$\text{Nr de morf} = (8, 8) = 8$$

$$\text{Deci } f(x) = ax + b$$

~~$$ax + b = ax + b$$~~

Kun zeiden

(8)

$$\begin{cases} x \equiv 3 \pmod{5} \\ x \equiv 2 \pmod{7} \\ x \equiv 8 \pmod{9} \end{cases}$$

$$N = 5 \cdot 7 \cdot 9 = 315$$

$$N_1 = 63 \quad N_2 = 45 \quad N_3 = 35$$

$$63x_1 \equiv 1 \pmod{5} \Rightarrow x_1 = 2$$

$$45x_2 \equiv 1 \pmod{7} \Rightarrow x_2 = 5$$

$$35x_3 \equiv 1 \pmod{9} \Rightarrow x_3 = 8$$

$$x = 3 \cdot 2 \cdot 63 + 2 \cdot 5 \cdot 45 + 8 \cdot 8 \cdot 35 \pmod{315}$$

$$x = 378 + 450 + 2240 = \cancel{2968} 3068$$

$$x \pmod{315} = \boxed{233}$$