EXAMEN PS
-restanta-

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Bobac Ștefan
grupa 241

Exercițiul 1

$$X \sim \begin{pmatrix} -7 & 4 & 6 \\ 0{,}44 & 0{,}24 & 0{,}32 \end{pmatrix} \qquad Y \sim \begin{pmatrix} 0 & 2 & 7 \\ 0{,}05 & 0{,}65 & 0{,}3 \end{pmatrix}$$

$P\{X+Y=-7+0=-7\} = P(\{X=-7\} \cap \{Y=0\}) = P\{X=-7\} \cdot P\{Y=0\} = 0{,}44 \cdot 0{,}05 = 0{,}022$

$P\{X+Y=-7+2=-5\} = P(\{X=-7\} \cap \{Y=2\}) = P\{X=-7\} \cdot P\{Y=2\} = 0{,}44 \cdot 0{,}65 = 0{,}286$

$P\{X+Y=-7+7=0\} = P(\{X=-7\} \cap \{Y=7\}) = P\{X=-7\} \cdot P\{Y=7\} = 0{,}44 \cdot 0{,}3 = 0{,}132$

$P\{X+Y=4+0=4\} = P(\{X=4\} \cap \{Y=0\}) = P\{X=4\} \cdot P\{Y=0\} = 0{,}24 \cdot 0{,}05 = 0{,}012$

$P\{X+Y=4+2=6\} = P(\{X=4\} \cap \{Y=2\}) = P\{X=4\} \cdot P\{Y=2\} = 0{,}24 \cdot 0{,}65 = 0{,}156$

$P\{X+Y=4+7=11\} = P(\{X=4\} \cap \{Y=7\}) = P\{X=4\} \cdot P\{Y=7\} = 0{,}24 \cdot 0{,}3 = 0{,}072$

$P\{X+Y=6+0=6\} = P(\{X=6\} \cap \{Y=0\}) = P\{X=6\} \cdot P\{Y=0\} = 0{,}32 \cdot 0{,}05 = 0{,}016$

$P\{X+Y=6+2=8\} = P(\{X=6\} \cap \{Y=2\}) = P\{X=6\} \cdot P\{Y=2\} = 0{,}32 \cdot 0{,}65 = 0{,}208$

$P\{X+Y=6+7=13\} = P(\{X=6\} \cap \{Y=7\}) = P\{X=6\} \cdot P\{Y=7\} = 0{,}32 \cdot 0{,}3 = 0{,}096$

$$X+Y \sim \begin{pmatrix} -7 & -5 & 0 & 4 & 6 & 8 & 11 & 13 \\ 0{,}022 & 0{,}286 & 0{,}132 & 0{,}012 & 0{,}172 & 0{,}208 & 0{,}072 & 0{,}096 \end{pmatrix}$$

$P\{X-Y=(-7)-0=-7\} = P\{X=-7\} \cdot P\{Y=0\} = 0{,}022$

$P\{X-Y=(-7)-2=-9\} = P\{X=-7\} \cdot P\{Y=2\} = 0{,}286$

$P\{X-Y=(-7)-7=-14\} = P\{X=-7\} \cdot P\{Y=7\} = 0{,}132$

$P\{X-Y=4-0=4\} = P\{X=4\} \cdot P\{Y=0\} = 0{,}012$

$P\{X-Y=4-2=2\} = P\{X=4\} \cdot P\{Y=2\} = 0{,}156$

$P\{X-Y=4-7=-3\} = P\{X=4\} \cdot P\{Y=7\} = 0{,}072$

$P\{X-Y=6-0=6\} = P\{X=6\} \cdot P\{Y=0\} = 0{,}016$

$P\{X-Y=6-2=4\} = P\{X=6\} \cdot P\{Y=2\} = 0{,}208$

$P\{X-Y=6-7=-1\} = P\{X=6\} \cdot P\{Y=7\} = 0{,}096$

$$X-Y \sim \begin{pmatrix} -14 & -9 & -7 & -3 & -1 & 2 & 4 & 6 \\ 0{,}132 & 0{,}286 & 0{,}022 & 0{,}072 & 0{,}096 & 0{,}156 & 0{,}22 & 0{,}016 \end{pmatrix}$$

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$$x^2 \sim \begin{pmatrix} 49 & 16 & 36 \\ 0{,}44 & 0{,}24 & 0{,}32 \end{pmatrix} \qquad y^2 \sim \begin{pmatrix} 0 & 4 & 49 \\ 0{,}05 & 0{,}65 & 0{,}3 \end{pmatrix}$$

$$4x^2 \sim \begin{pmatrix} 196 & 64 & 144 \\ 0{,}44 & 0{,}24 & 0{,}32 \end{pmatrix} \qquad 2y^2 \sim \begin{pmatrix} 0 & 8 & 98 \\ 0{,}05 & 0{,}65 & 0{,}3 \end{pmatrix}$$

$\mathbb{P}\{4x^2+2y^2 = 196+0 = 196\} = 0{,}022$

$\mathbb{P}\{4x^2+2y^2 = 196+8 = 204\} = 0{,}286$

$\mathbb{P}\{4x^2+2y^2 = 196+98 = 294\} = 0{,}132$

$\mathbb{P}\{4x^2+2y^2 = 64+0 = 64\} = 0{,}012$

$\mathbb{P}\{4x^2+2y^2 = 64+8 = 72\} = 0{,}156$

$\mathbb{P}\{4x^2+2y^2 = 64+98 = 162\} = 0{,}072$

$\mathbb{P}\{4x^2+2y^2 = 144+0 = 144\} = 0{,}016$

$\mathbb{P}\{4x^2+2y^2 = 144+8 = 152\} = 0{,}208$

$\mathbb{P}\{4x^2+2y^2 = 144+98 = 242\} = 0{,}096$

$$4x^2+2y^2 \sim \begin{pmatrix} 64 & 72 & 144 & 152 & 162 & 196 & 204 & 242 & 294 \\ 0{,}012 & 0{,}156 & 0{,}016 & 0{,}208 & 0{,}072 & 0{,}022 & 0{,}286 & 0{,}096 & 0{,}132 \end{pmatrix}$$

$\mathbb{E}[x] = (-7) \cdot 0{,}44 + 4 \cdot 0{,}24 + 6 \cdot 0{,}32 = -0{,}2$

$\mathbb{E}[y] = 0 \cdot 0{,}05 + 2 \cdot 0{,}65 + 7 \cdot 0{,}3 = 3{,}4$

$\mathbb{E}[x^2] = 49 \cdot 0{,}44 + 16 \cdot 0{,}24 + 36 \cdot 0{,}32 = 36{,}92$

$\mathbb{E}[y^2] = 0 \cdot 0{,}05 + 4 \cdot 0{,}65 + 49 \cdot 0{,}3 = 17{,}3$

$\mathrm{Var}(x) = \mathbb{E}[x^2] - \mathbb{E}[x]^2 = 36{,}88$

$\mathrm{Var}(y) = \mathbb{E}[y^2] - \mathbb{E}[y]^2 = 5{,}74$

$$\begin{aligned}
Var(8x-7y+15) &= Var(8x-7y) \\
&= Var(8x) + Var(-7y) \\
&= 64\,Var(x) + 49\,Var(y) \\
&= 64 \cdot 36{,}88 + 49 \cdot 5{,}74 \\
&= 2641{,}58
\end{aligned}$$

$$\begin{aligned}
Var(2xy+3) &= Var(2xy) \\
&= 4\,Var(xy) \\
&= 4 \cdot 638{,}2536 \\
&= 2553{,}0144
\end{aligned}$$

$$\begin{aligned}
&\mathbb{P}\{x\cdot y = 0\} = 0{,}022 \\
&\mathbb{P}\{x\cdot y = -14\} = 0{,}286 \\
&\mathbb{P}\{x\cdot y = -49\} = 0{,}132 \\
&\mathbb{P}\{x\cdot y = 12\} = 0{,}012 \\
&\mathbb{P}\{x\cdot y = 8\} = 0{,}156 \\
&\mathbb{P}\{x\cdot y = 28\} = 0{,}072 \\
&\mathbb{P}\{x\cdot y = 0\} = 0{,}016 \\
&\mathbb{P}\{x\cdot y = 12\} = 0{,}208 \\
&\mathbb{P}\{x\cdot y = 42\} = 0{,}086
\end{aligned}$$

$$x\cdot y \sim \begin{pmatrix} -49 & -14 & 0 & 8 & 12 & 28 & 42 \\ 0{,}132 & 0{,}286 & 0{,}05 & 0{,}156 & 0{,}208 & 0{,}072 & 0{,}086 \end{pmatrix}$$

$$(x\cdot y)^2 \sim \begin{pmatrix} 2401 & 196 & 0 & 64 & 144 & 784 & 1764 \\ 0{,}132 & 0{,}286 & 0{,}05 & 0{,}156 & 0{,}208 & 0{,}072 & 0{,}086 \end{pmatrix}$$

$$\mathbb{E}[x\cdot y] = (-49)\cdot 0{,}132 + (-14)\cdot 0{,}286 + 0 + 8\cdot 0{,}156 + 0{,}208\cdot 12 + 28\cdot 0{,}072 + 42\cdot 0{,}08$$

$$\mathbb{E}[x\cdot y] = -0{,}68$$

$$\mathbb{E}[(xy)^2] = 638{,}716$$

$$Var(xy) = \mathbb{E}[(xy)^2] - E[xy]^2 = 638{,}2536$$

$Cov(X,Y) = 0$ (deoarece sunt v.a. independente)

$$\rho = \frac{cov(X,Y)}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}} = \frac{0}{\sqrt{Var(X)} \cdot \sqrt{Var(Y)}} = 0$$

Exercitiul 2

X - nr. de succese înainte de al 6-lea eșec

~~$P(X=k) = \binom{k-1}{5} \cdot (1-p)^{k-6} \cdot p^6$~~

~~$P(X=k) = \binom{k-1}{5} \cdot (1-0,93)^{k-6} \cdot (0,93)^6$~~

~~$P(X=k) = \binom{k-1}{5} \cdot (0,07)^{k-6} \cdot (0,93)^6$~~

~~$P(X=k) =$~~

$$P(X=k-6) = \binom{k-1}{5} \cdot (1-p)^6 \cdot p^{k-6}$$

$$P(X=k-6) = \binom{k-1}{5} \cdot (1-0,93)^6 \cdot (0,93)^{k-6}$$

$$P(X=k-6) = \binom{k-1}{5} \cdot (0,07)^6 \cdot (0,93)^{k-6}$$

$$P(X=k-6) = \binom{k-1}{5} \cdot 0,000000011764\,9 \cdot (0,93)^{k-6}$$

$$E(X) = \frac{p \cdot r}{1-p} = \frac{0,93 \cdot 6}{1-0,93} = \frac{5,58}{0,07} = 79,714285\,7142\ldots$$

$$E[6X-8] = 6 \cdot E[X] - 8 = 6 \cdot 79,714285\,7142 - 8 =$$

$$= 470,285714\,2852$$

$$Var(3X+8) = Var(3X)$$
$$= 9\ Var(X)$$
$$= 9 \cdot 6,72208167$$
$$= 60,49873503$$

$$Var(X) = \frac{p \cdot r}{(1-p)^2} = \frac{0,93 \cdot 6}{(0,07)^2} = \frac{5,58}{0,8301\ldots} = 6,72208167\ldots$$

Exercitiul 3

Y - ~~nr. de~~ capete.

X - puncte

1.

	H	T
H	$(0,19)^2$ 0,0361	$(0,19 \cdot 0,81)$
T	$(0,19 \cdot 0,81)$ 0,1539	$(0,81)^2$ 0,6561

$H = 0,19$

$T = 1 - 0,19 = 0,81$

a) $P(Y=0 \mid X=2) = 0,81 \cdot 0,81 = 0,6561.$

2. $P(Y=1 \cup Y=2) = P(X=1) \cdot P(Y=1 \cup Y=2 \mid X=1) +$
$P(X=2)\ P(Y=1 \cup Y=2 \mid X=2) +$
$\ldots$
$P(X=6)\ P(Y=1 \cup Y=2 \mid X=6)$

$P(X=1) = P(X=2) = \ldots = P(X=6) = 1/6.$

$P(Y=1 \cup Y=2 \mid X=1) = P(H=1 \mid X=1) = P(H=1) = 0,19.$

$P(Y=1 \cup Y=2 \mid X=2) = P(H=1 \cup H=2 \mid X=2) = P(H \neq 1 \cap H=1) + P(HT) + P(TH)$

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$= 0,0361 + 0,1538 + 0,1538 = 0,3438.$

$P(Y=1 \cup Y=2 \mid X=3) = P(HHH) \cdot P(HHT) \cdot P(HTH) \cdot P(HTT)$

$C_n^k p^k (1-p)^{n-k}$

Exercitiul 4

X_n = nr. puncte negru : $p_n = \frac{1}{3}$

X_a = nr. puncte alb : $p_a = \frac{1}{3}$.

1. repetitie comuna

$X_n \backslash X_a$	1	2	3	Σ puncte
1	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{3}$
2	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{3}$
3	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{3}$
Σ puncte	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	

$P(X_n = 1, X_a = 1) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

$P(X_n = 1, X_a = 2) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

$P(X_n = 1, X_a = 3) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

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