

Seminar 3

Exercise 1. For each of the regular expressions below, give examples of words that are and are not in their respective languages.

- (i) a^*b^* ;
- (ii) $(a \cup b)^*$;
- (iii) $a^* \cup b^*$;
- (iv) $(aaa)^*$;
- (v) $(\epsilon \cup a)b^*$;

Exercise 2. Define regular expressions recognizing the following languages.

- (i) The words of length multiple of k over the alphabet $\{a, b\}$, for some $k \geq 1$;
- (ii) The words of odd length;
- (iii) The language $\{\epsilon, a\}$ over some alphabet Σ such that $a \in \Sigma$;
- (iv) The decimal numbers of the form $-12.456, 12.5, 66, .47, 0.14$.

Exercise 3. Let L be the language over $\{0, 1\}$ of the words not containing the subword 110.

Exercise 4. Convert the following regex into an equivalent NFA: $a^+ \cup (ab)^+$.

Exercise 5. Let Σ and Γ be two alphabets and $f : \Sigma^* \rightarrow \Gamma^*$ be a function such that $f(w_1w_2 \dots w_n) = f(w_1)f(w_2) \dots f(w_n)$ for any $w = w_1 \dots w_n \in \Sigma^*$, and such that $f(\epsilon) = \epsilon$. Prove that, if $L \subseteq \Sigma^*$ is regular, then $f(L) = \{f(w) \mid w \in L\} \subseteq \Gamma^*$ is also regular.