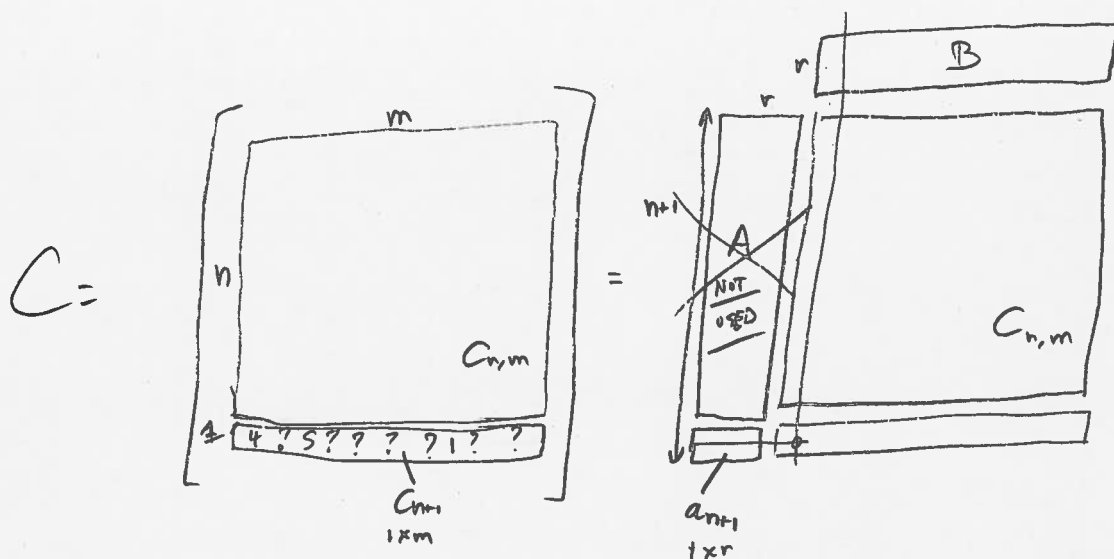New songs recommendation

$$\min_{A,B \geq 0} \quad \| \Omega \circ (C \| AB) \|_{KL} + \gamma_A \|A\|_{TV} + \gamma_B \|B\|_{TV}$$

(1) Fast recommendation   (Least-square)

$C = $ matrix $n \times m$, $C_{n,m}$, bottom row C_{n+1} ($1 \times m$): 4 5 ? ? ? ? 1 ? ?

$= A$ (NOT USED) $\cdot B$, a_{n+1} ($1 \times r$), $C_{n,m}$

$$\min_{\substack{a_{n+1} \\ 1 \times r}} \quad \left\| \underset{1 \times m}{\Omega_{n+1}} \circ \left(\underset{1 \times m}{C_{n+1}} - \underset{1 \times r}{a_{n+1}} \underset{r \times m}{B} \right) \right\|_2^2 + \epsilon \|a_{n+1}\|_2^2$$

$\underbrace{\qquad}_{1 \times m}$ $\underset{\text{to make the problem well-posed}}{}$

$$\downarrow \quad \begin{cases} \hat{a} = a_{n+1}^T \\ \hat{c} = c_{n+1}^T \\ \hat{B} = B^T \end{cases} \qquad \underset{1 \times m}{\Omega_{n+1}} \to \underset{m \times m}{\hat{\Lambda}}$$

$$\min_{\substack{\hat{a} \\ r \times 1}} \quad \left\| \underset{m \times m}{\hat{\Lambda}} \cdot \left(\underset{m \times 1}{\hat{c}} - \underset{m \times r}{\hat{B}} \underset{r \times 1}{\hat{a}} \right) \right\|_2^2 + \epsilon \|\hat{a}\|_2^2$$

$$\min_{\hat{a}} \quad \| \hat{\Lambda} \hat{c} - \hat{\Lambda} \hat{B} \hat{a} \|_2^2$$

$\Downarrow$ optimality

$$(\hat{\Lambda}\hat{B})^T (\hat{\Lambda}\hat{B}\hat{a} - \hat{\Lambda}\hat{c}) + \epsilon \hat{a}$$

$$\Rightarrow \quad \boxed{ \underset{r \times 1}{\hat{a}} = \left(\hat{B}^T \hat{\Lambda} \hat{B} + \epsilon I \right)^{-1} \left(\hat{B}^T \hat{\Lambda} \hat{c} \right) }$$

$\to$ very fast because r is small !