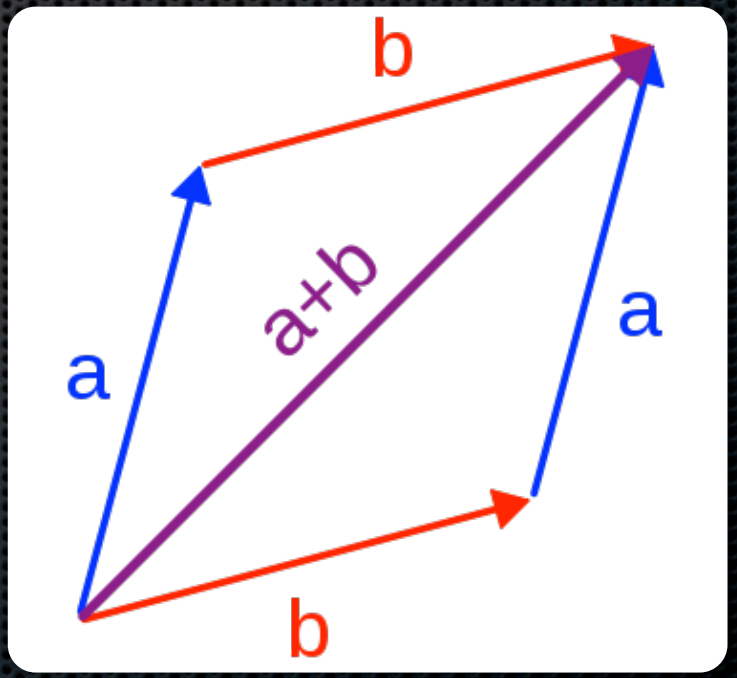
Introduction to Computer Graphics

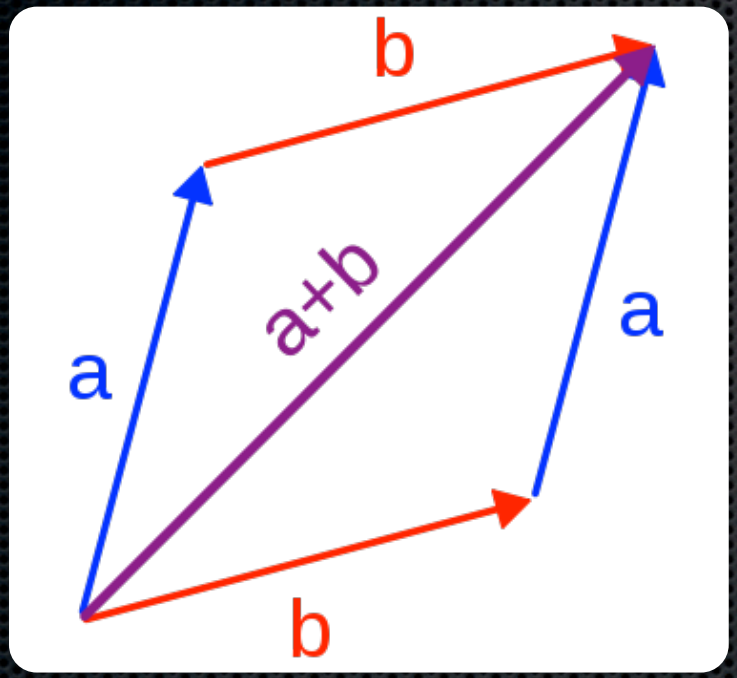
Barycentric Coordinates and Triangle Rasterization

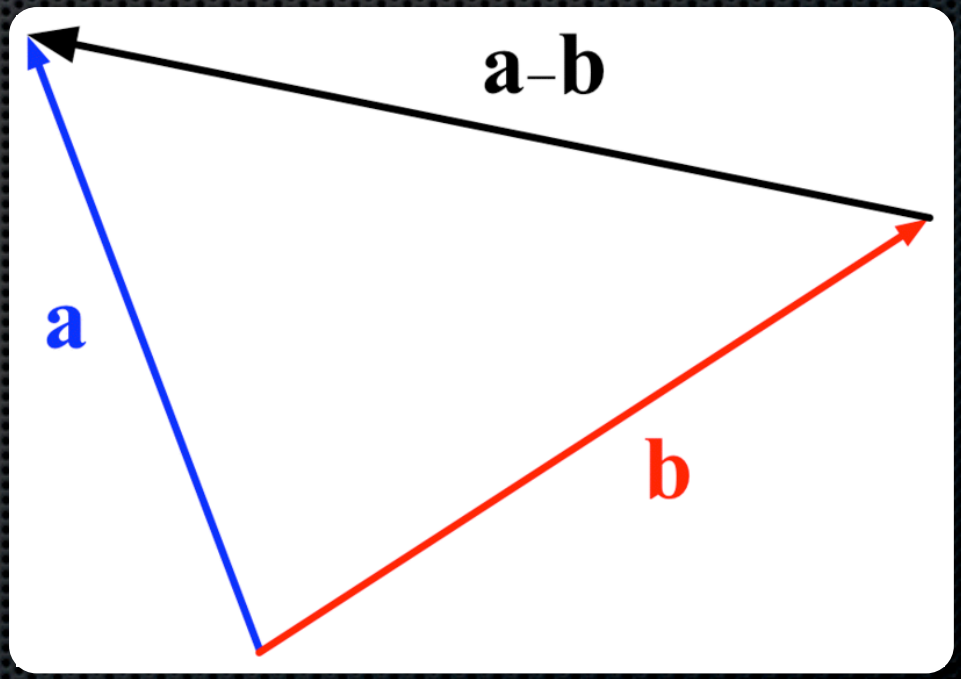
Vectors

- Vectors
  - Addition & Subtraction
  - Scalar Multiplication
  - Magnitude & Normalization
  - Dot & Cross Product

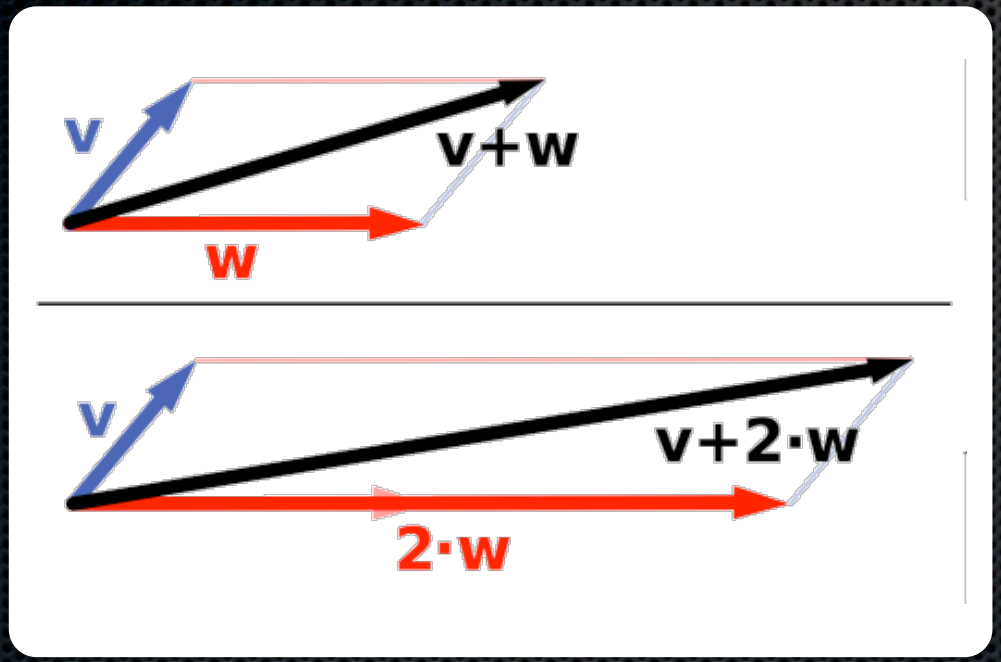


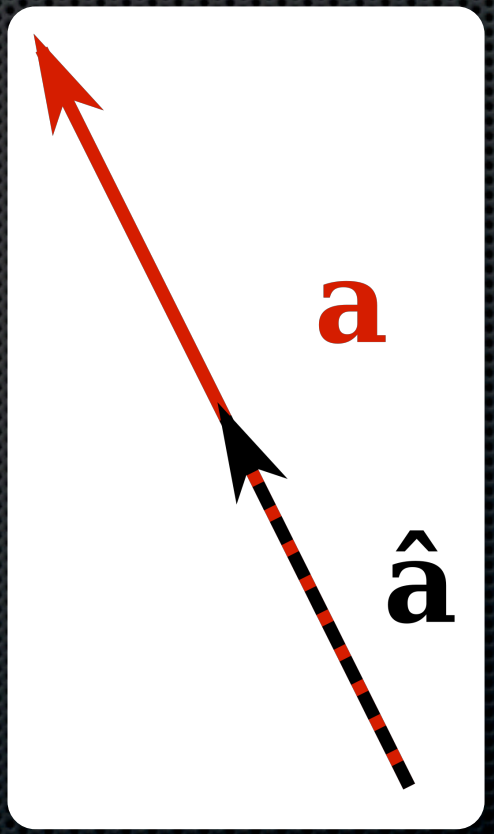
Vector Addition & Subtraction





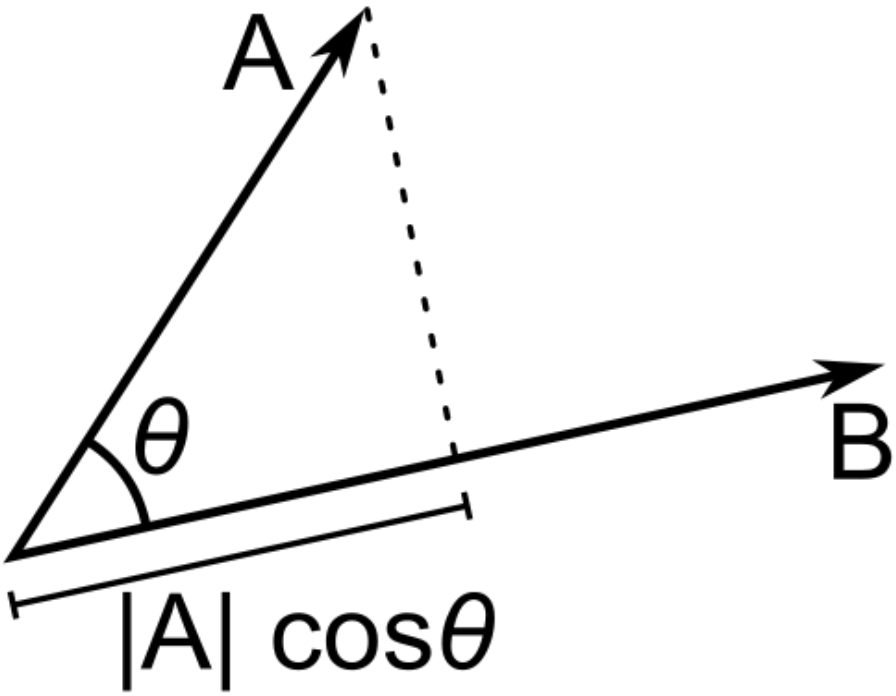
Vector-Scalar Multiplication



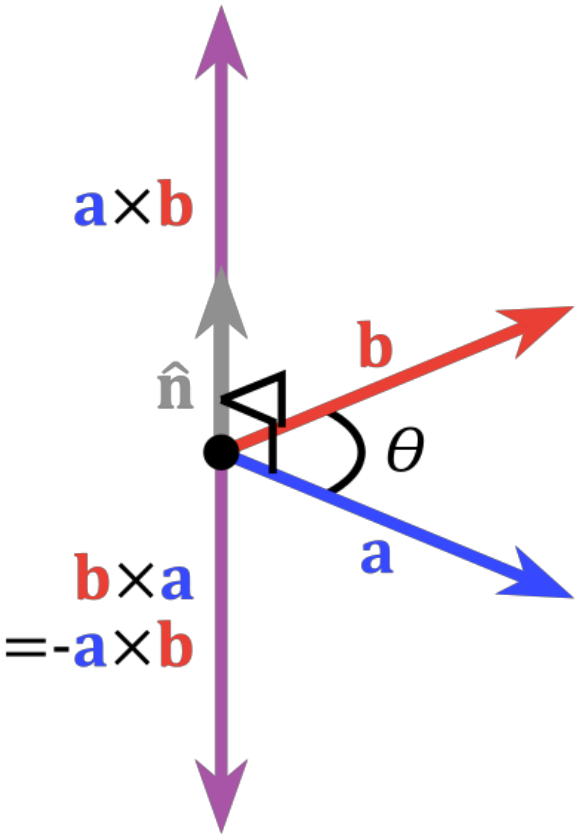


$$\hat{a} = \frac{1}{|a|} a$$

Vector Dot & Cross Product



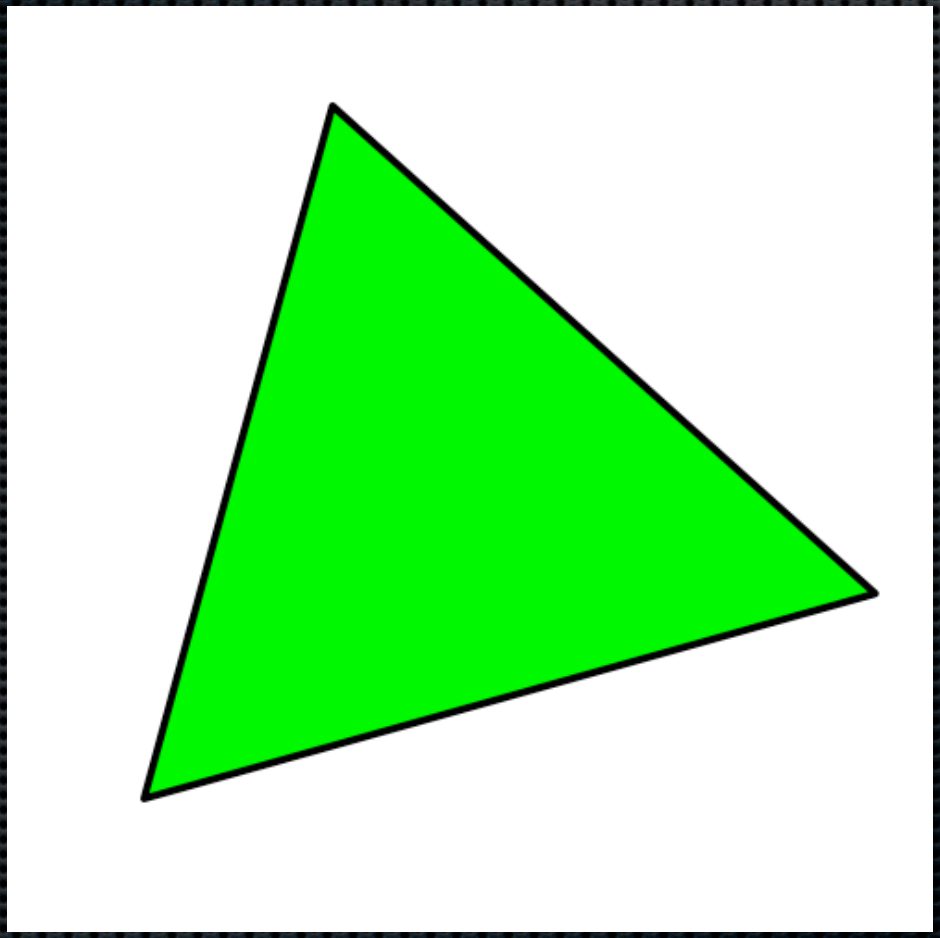
$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}|\,|\mathbf{b}| \cos\theta$$



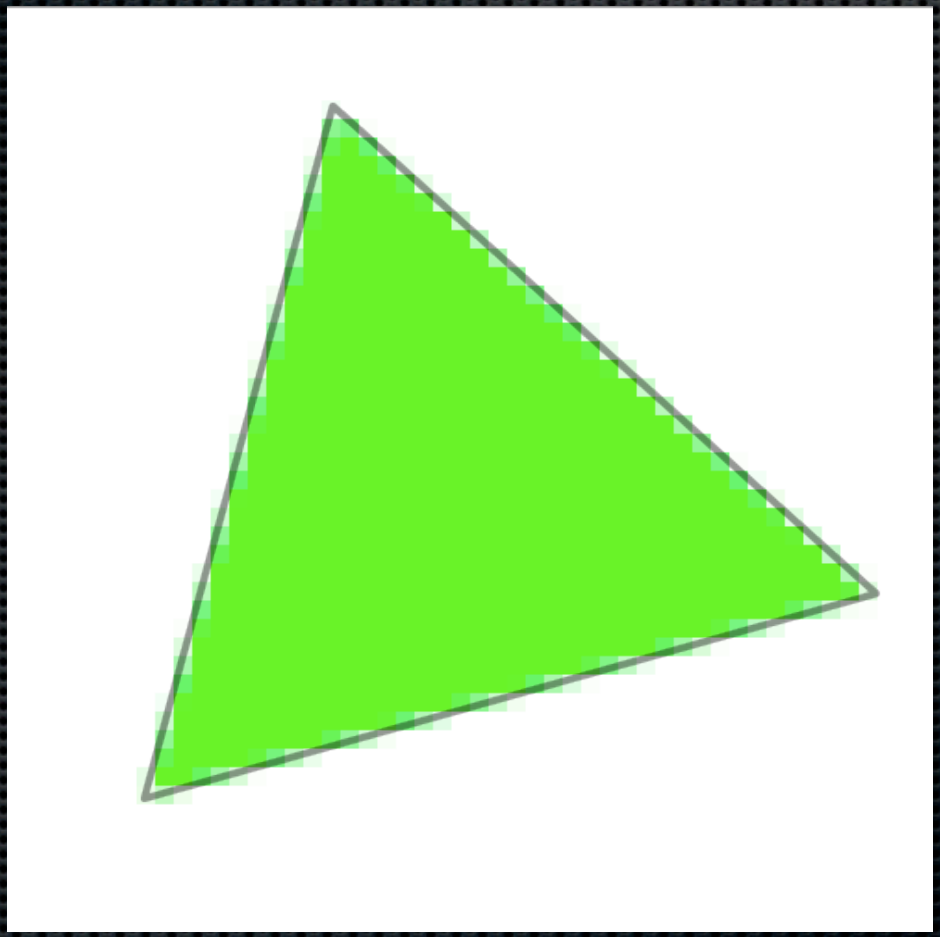
$$\mathbf{a} \times \mathbf{b} = \|\mathbf{a}\|\,\|\mathbf{b}\| \sin(\theta)\,\mathbf{n}$$

Point and Free Vectors

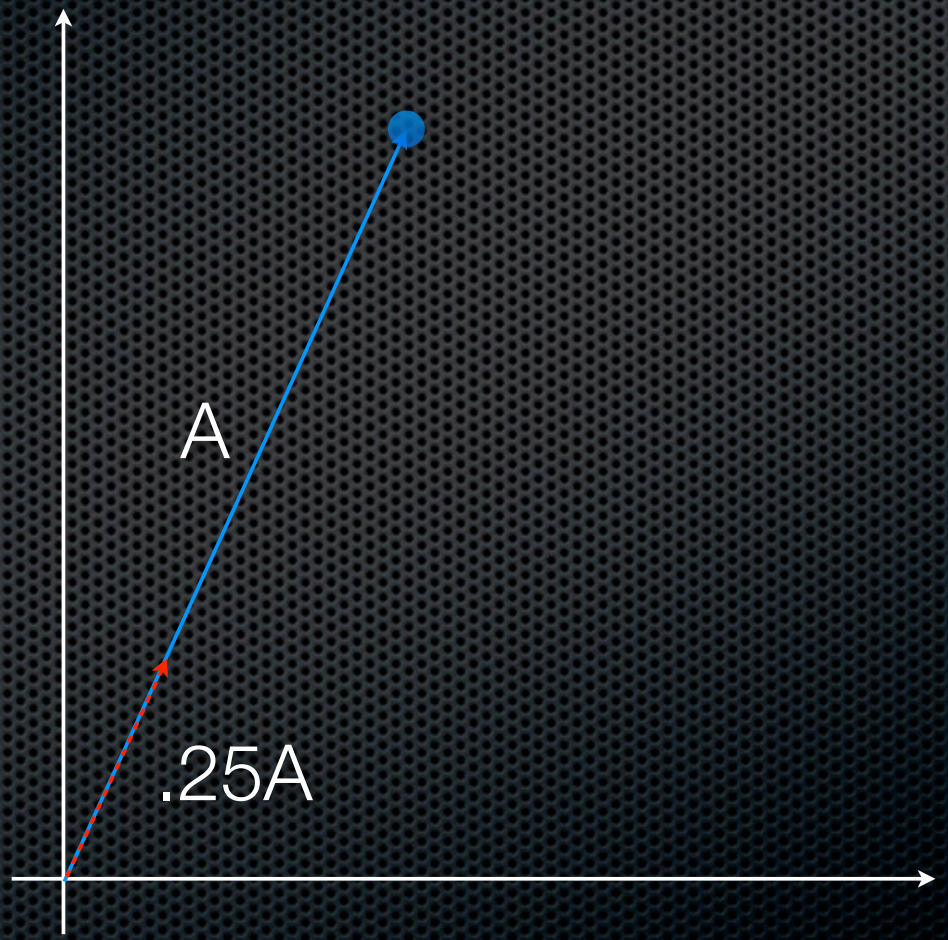
Rasterization



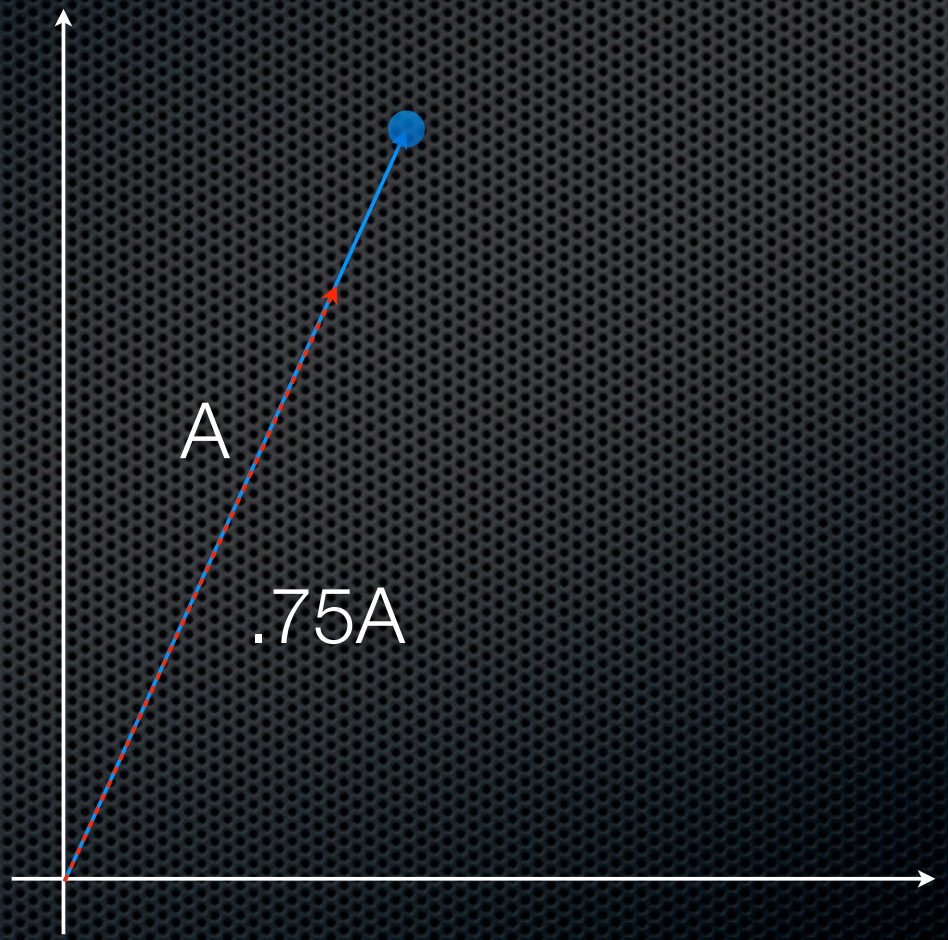
Rasterization



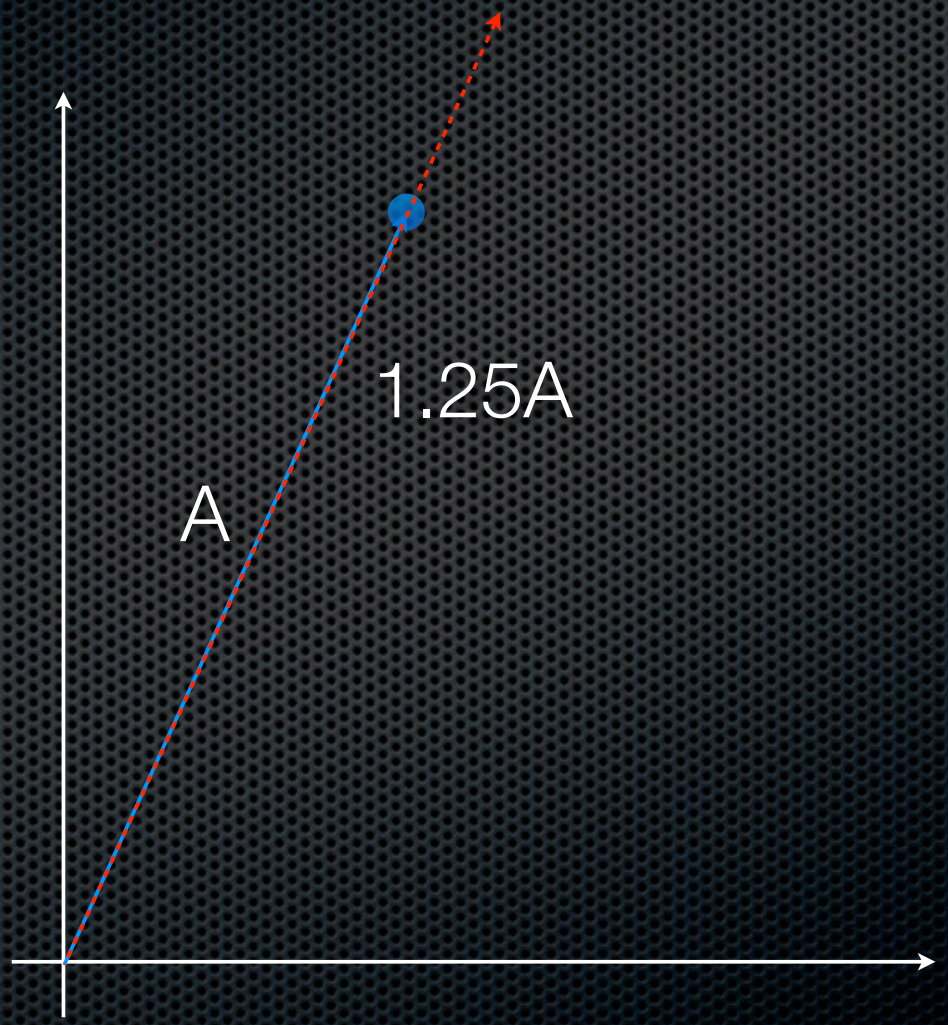
An Interesting Idea



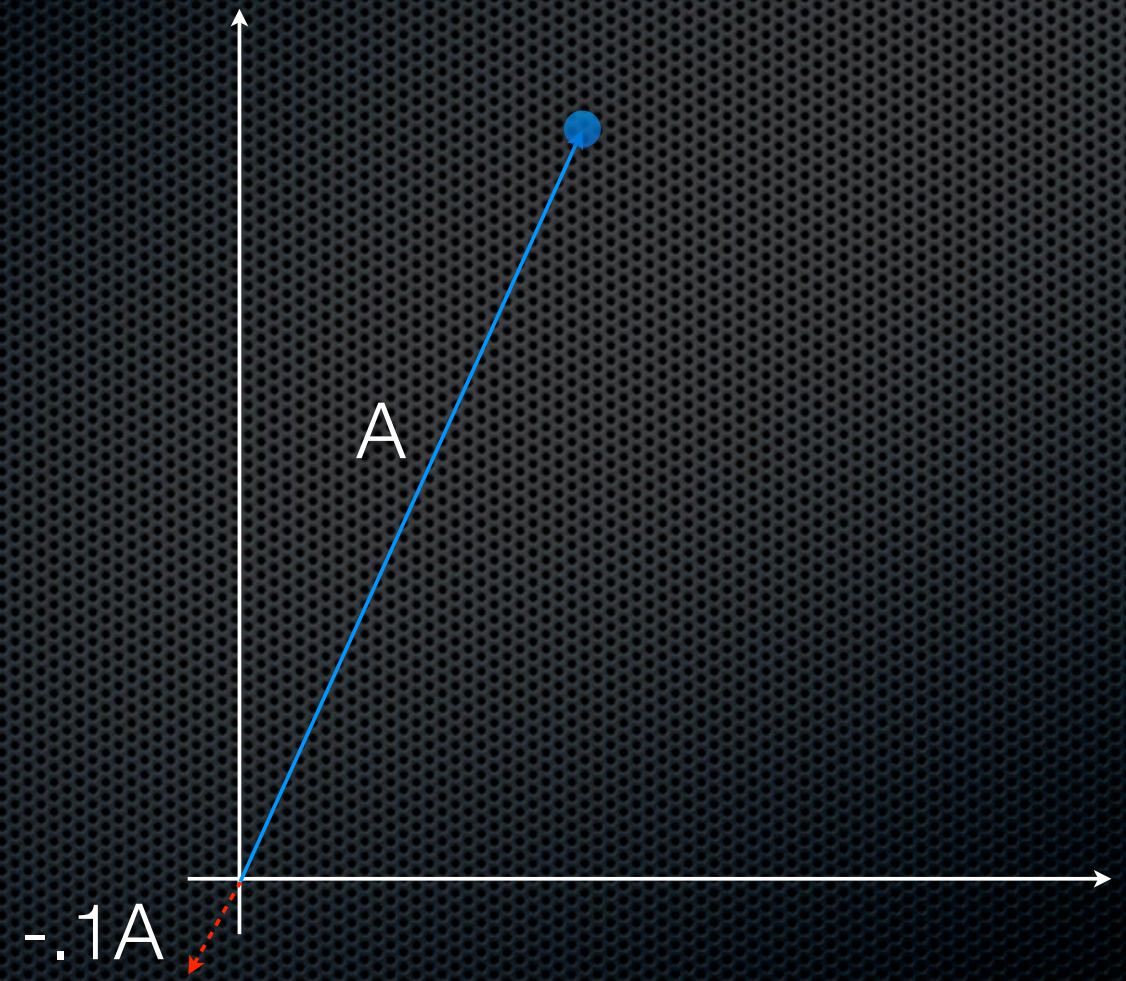
An Interesting Idea



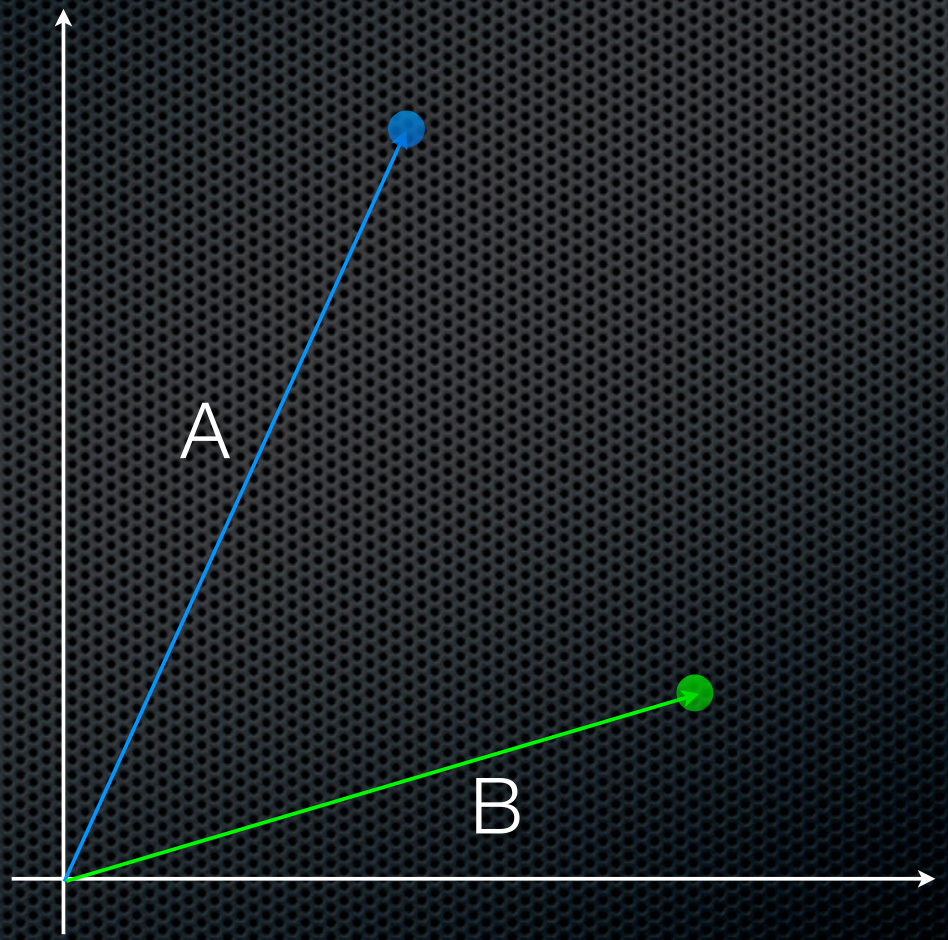
An Interesting Idea



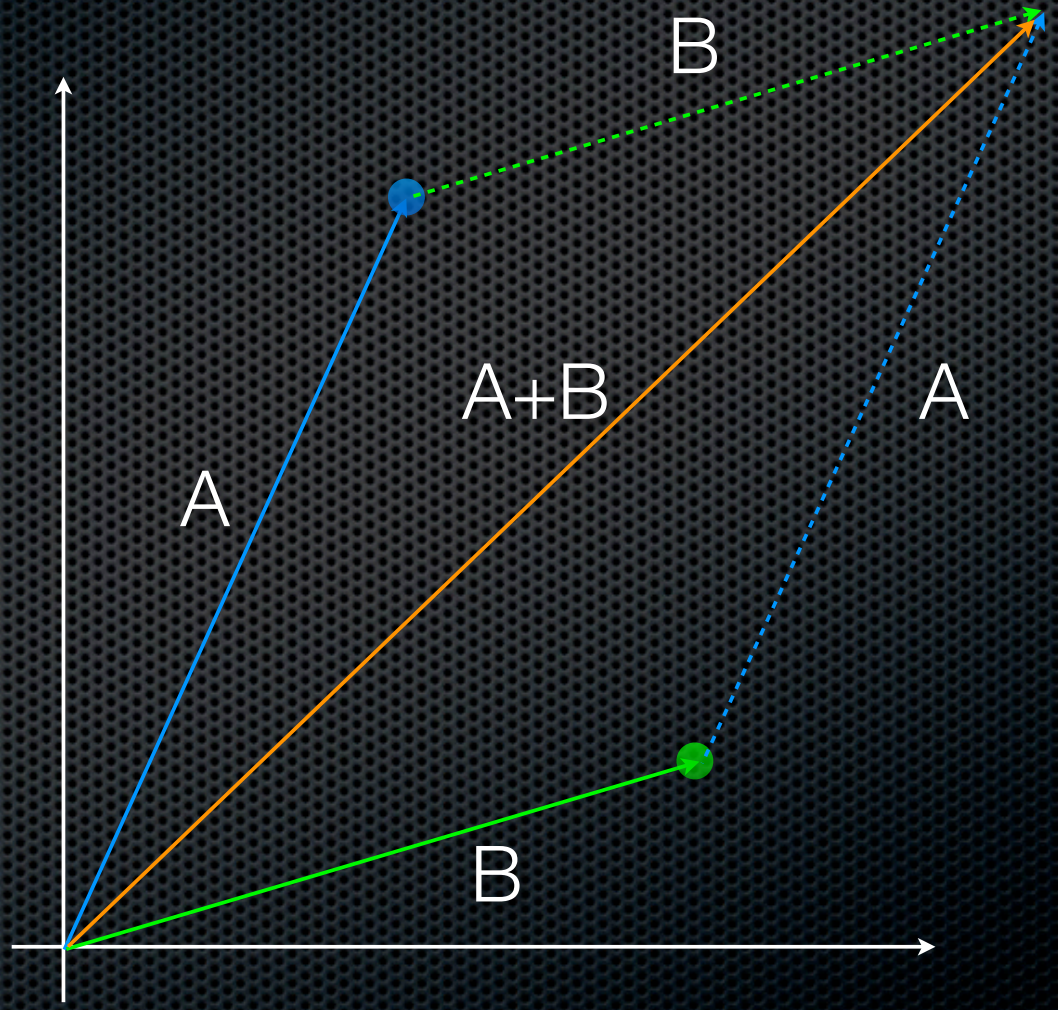
An Interesting Idea



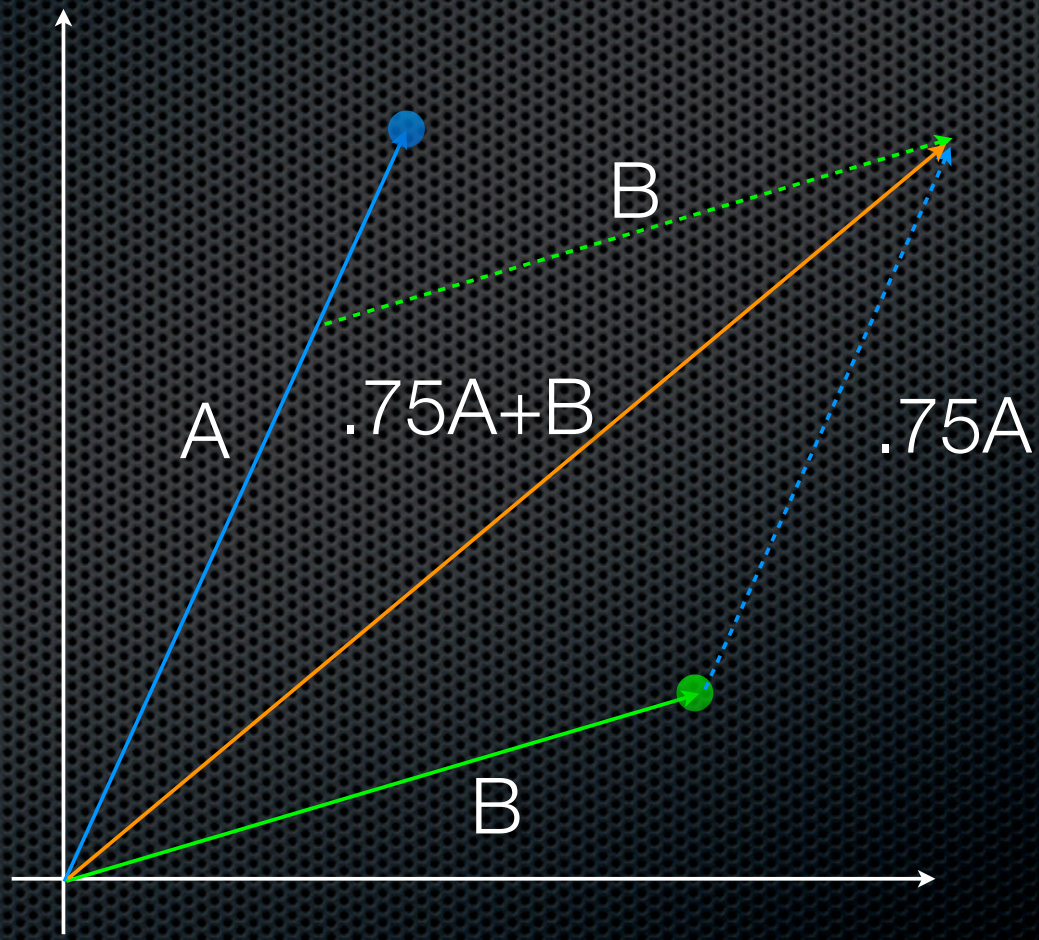
An Interesting Idea



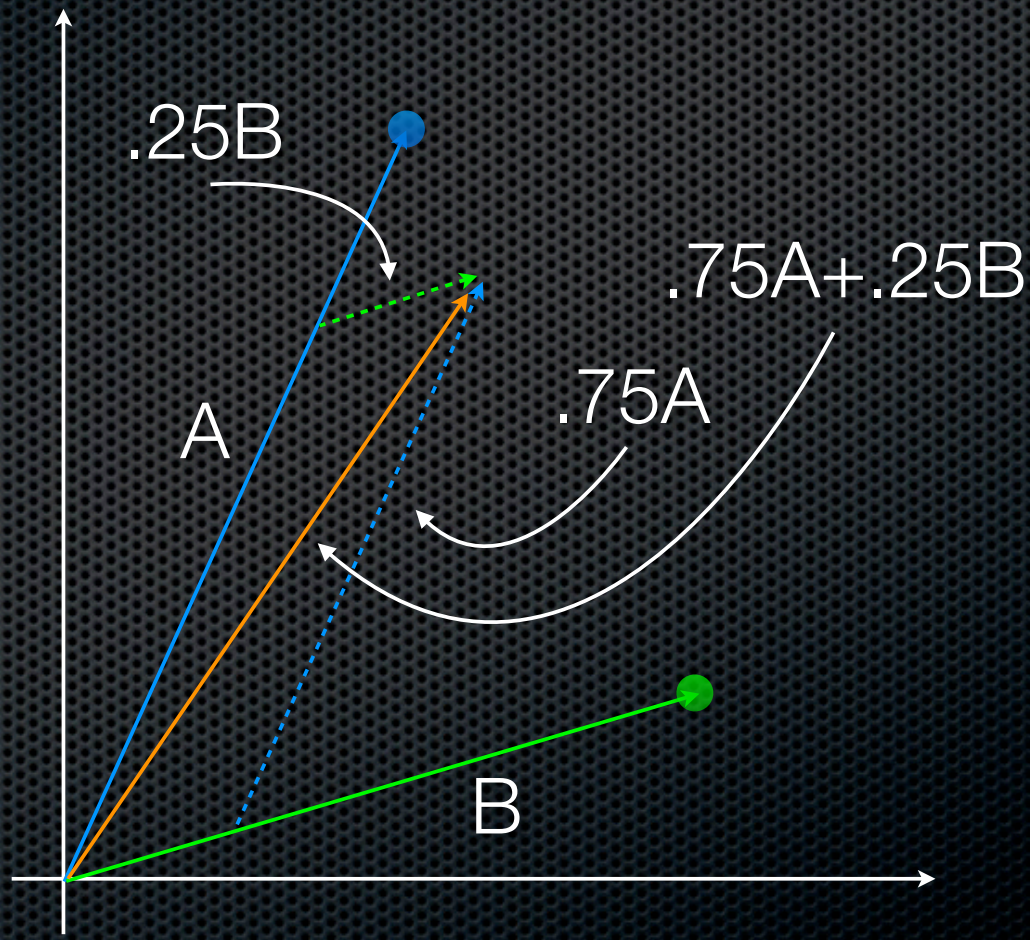
An Interesting Idea



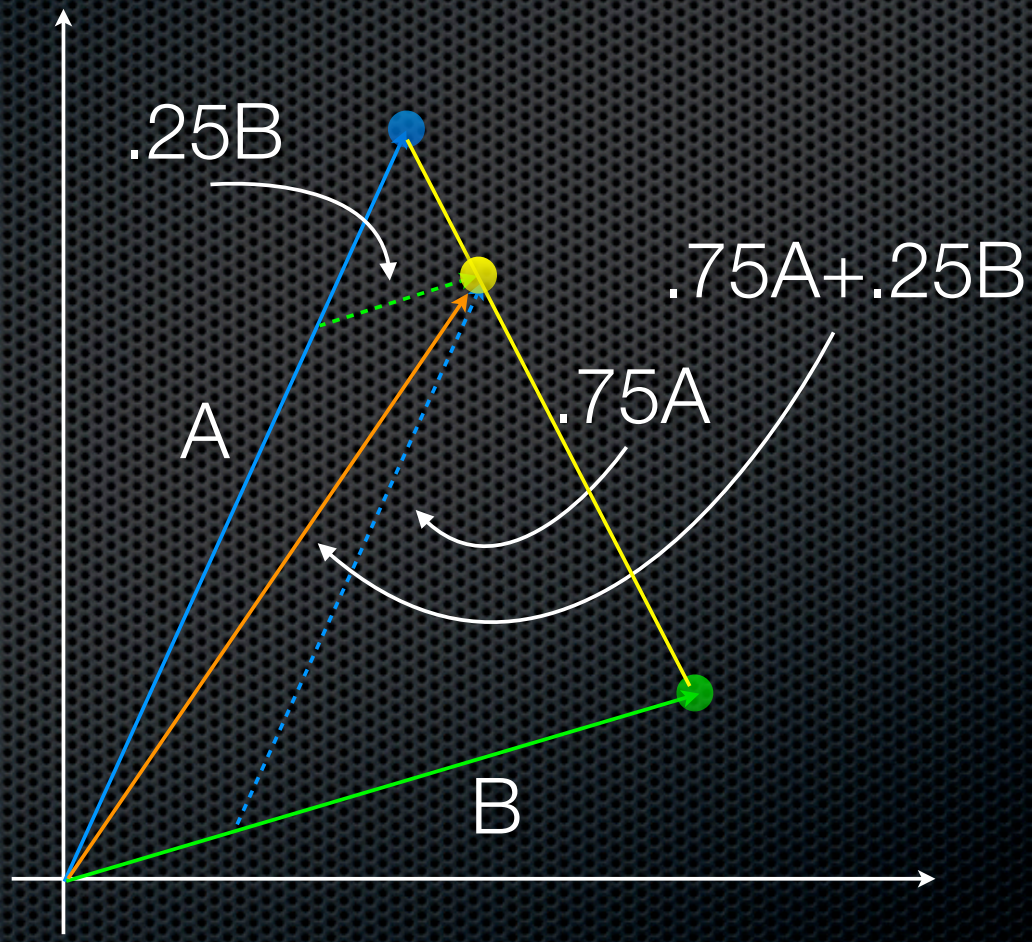
Linear Combination



Linear Combination

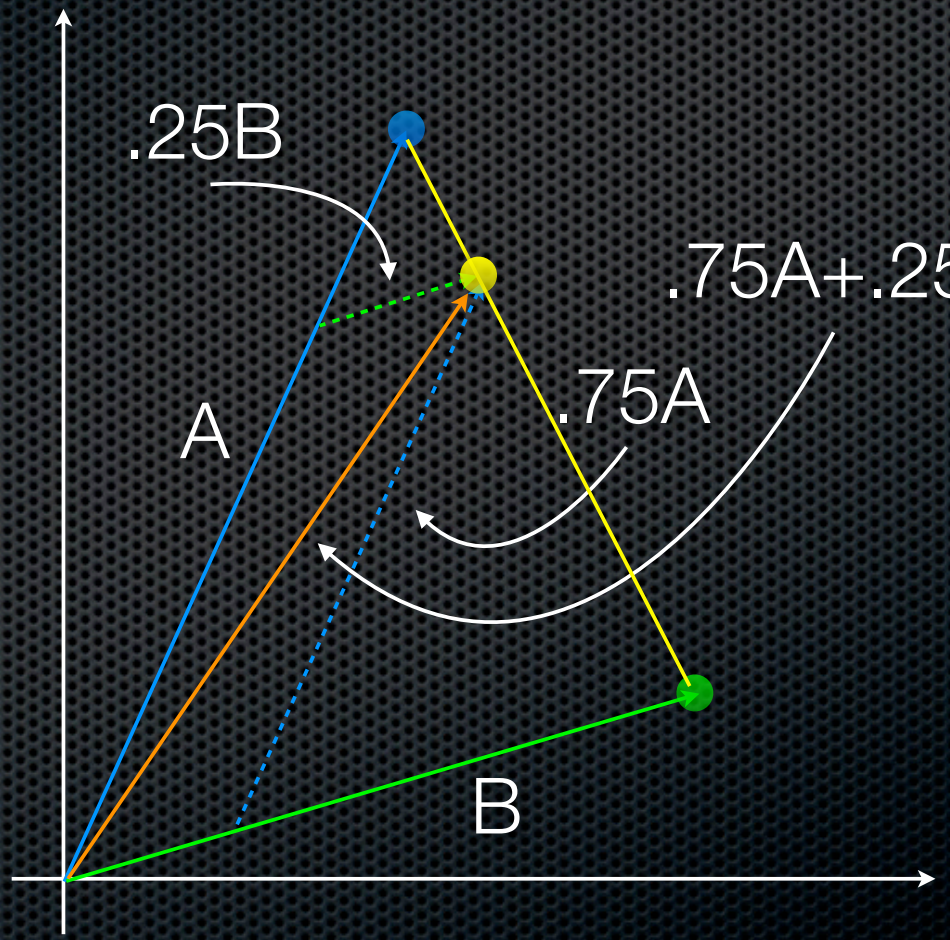


Linear Combination



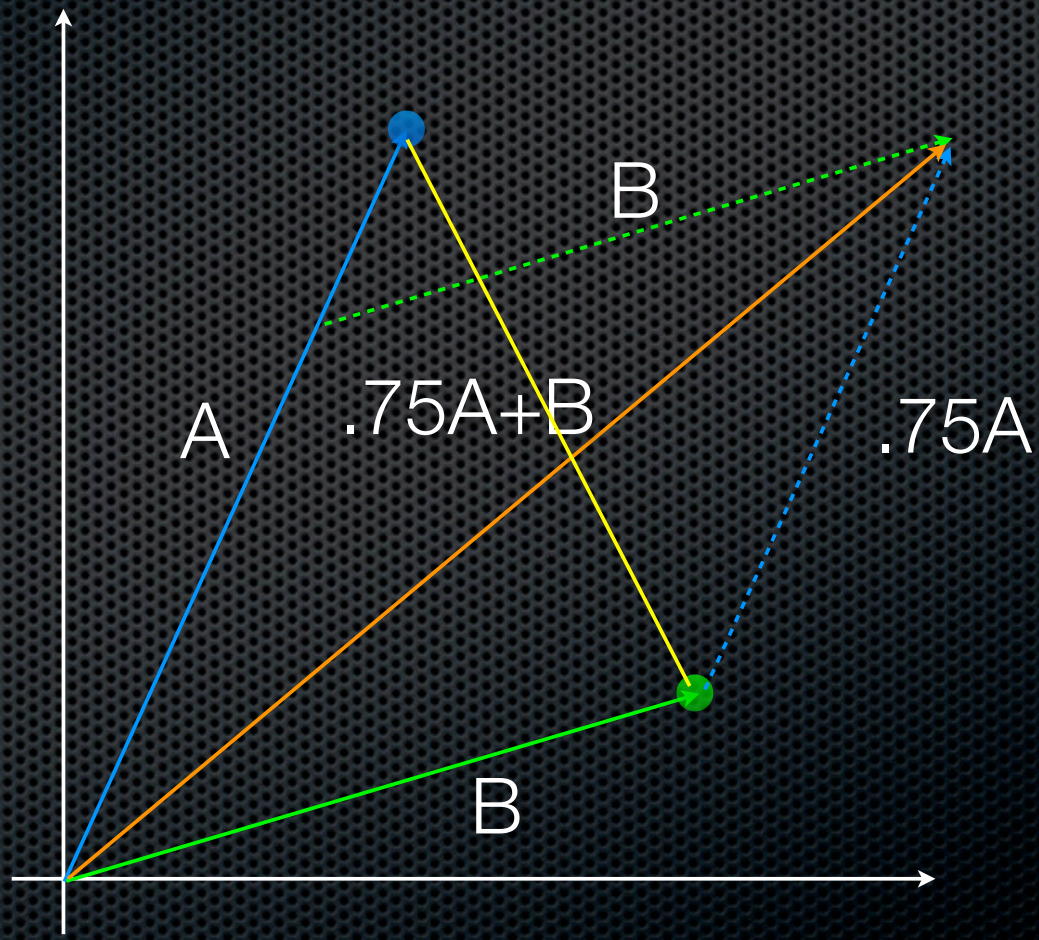
Linear Combination or Blend

.75 + .25 = 1



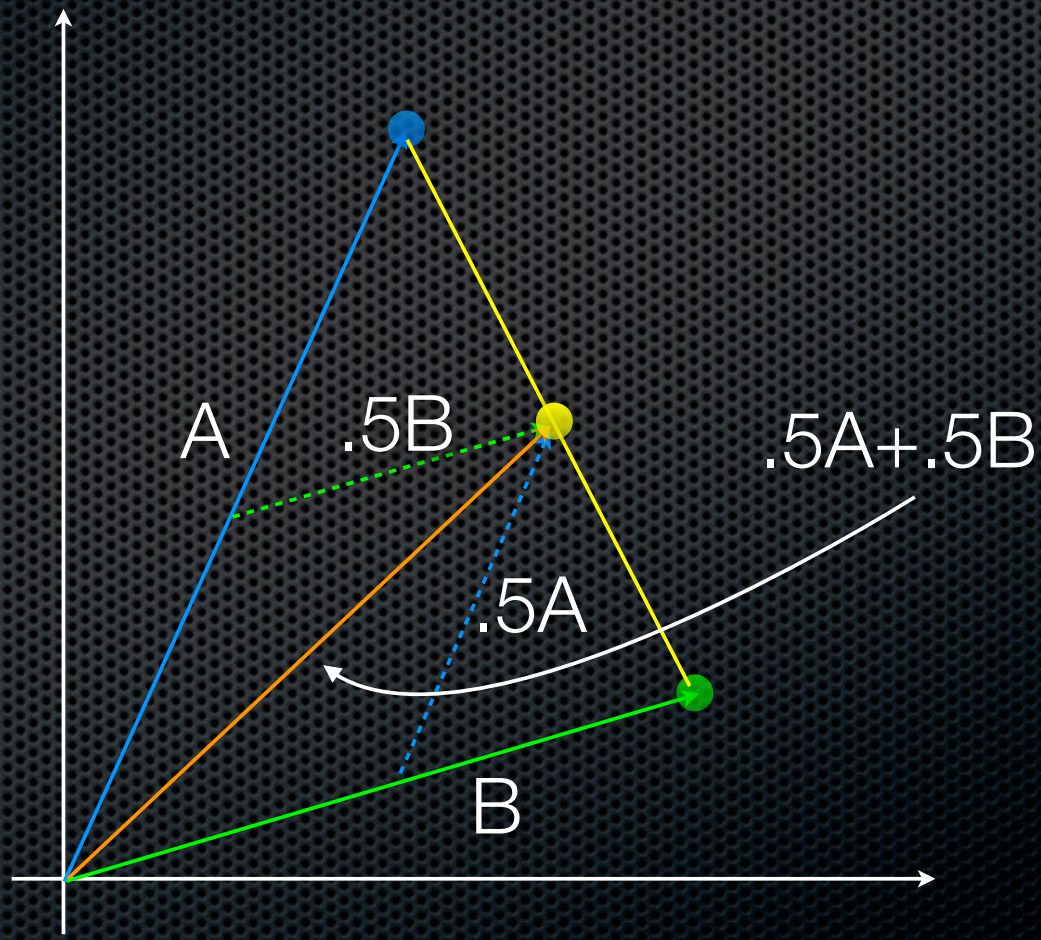
Linear Combination

.75 + 1 ≠ 1

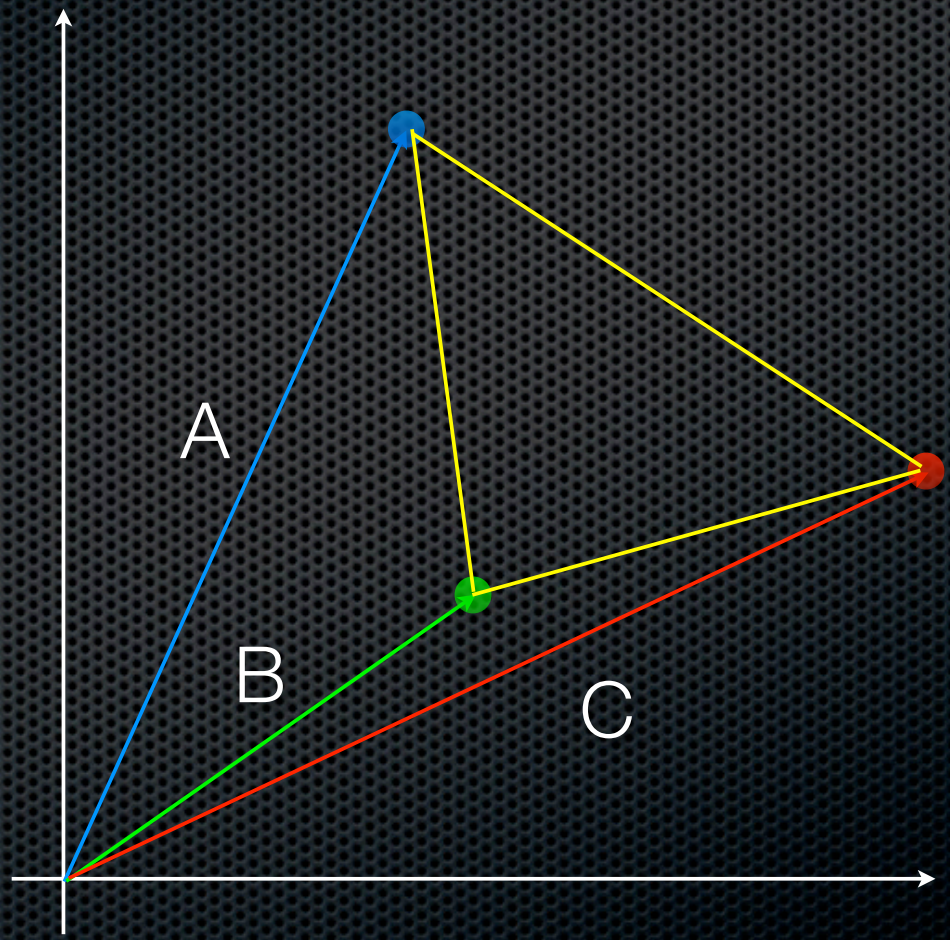


Linear Combination or Blend

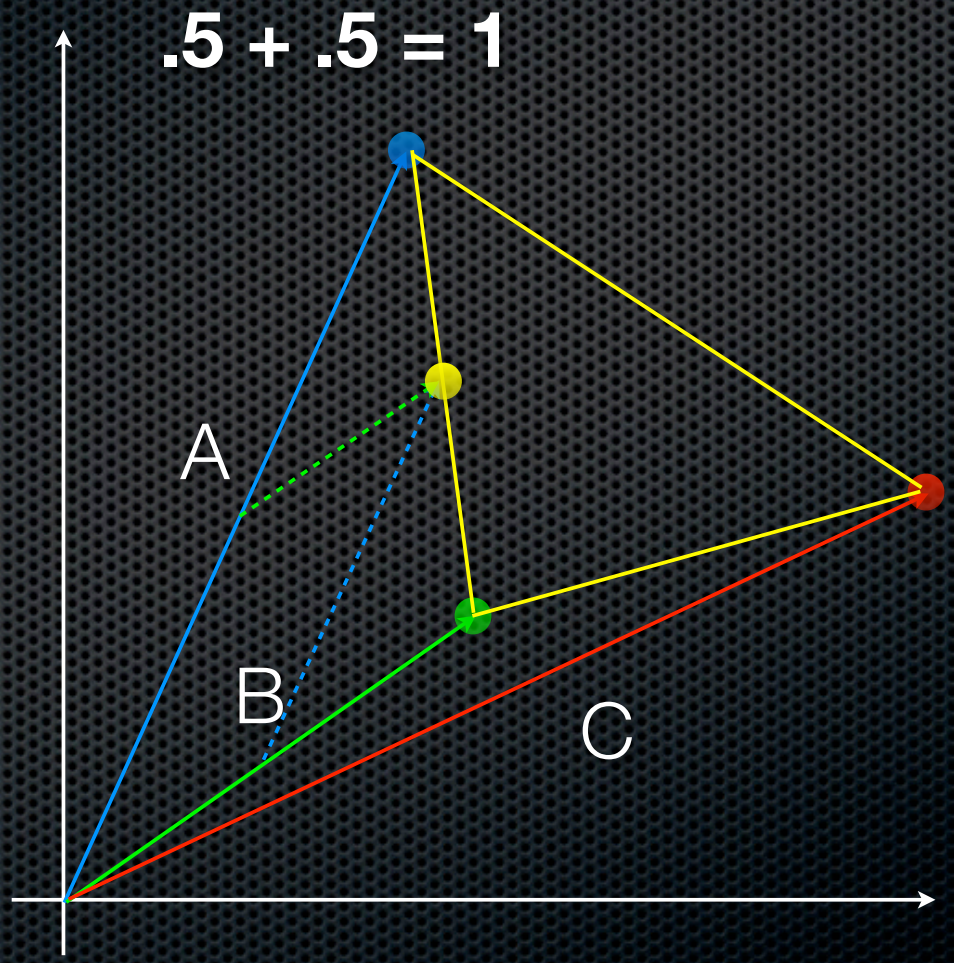
.5 + .5 = 1



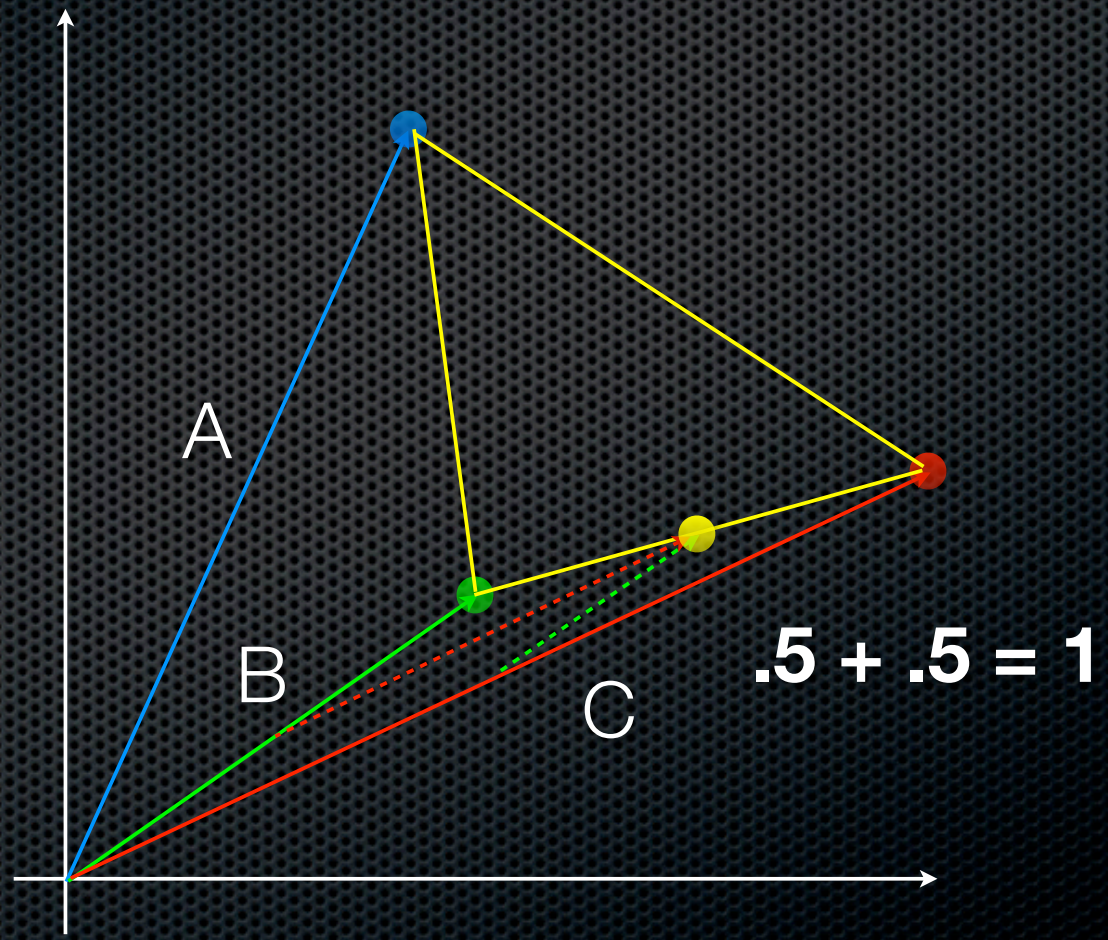
Triangle



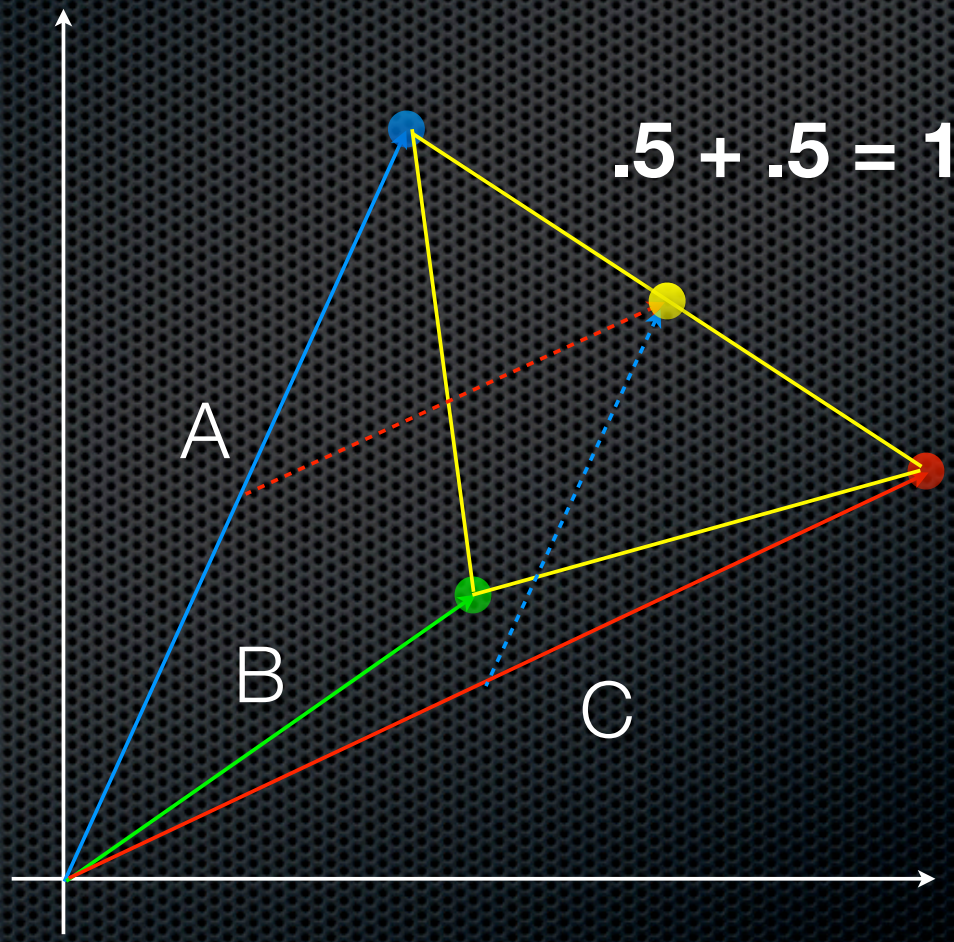
Triangle



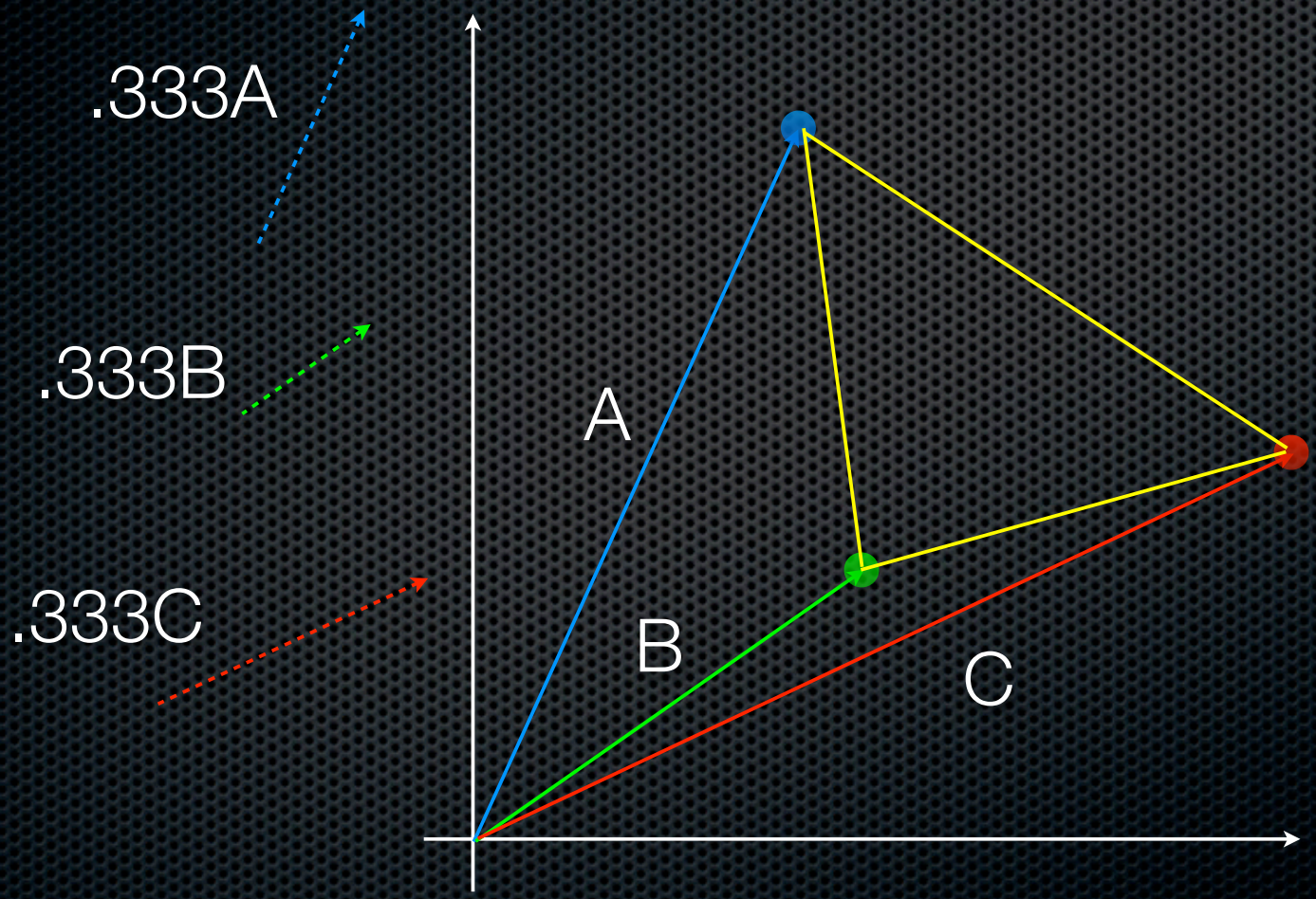
Triangle



Triangle

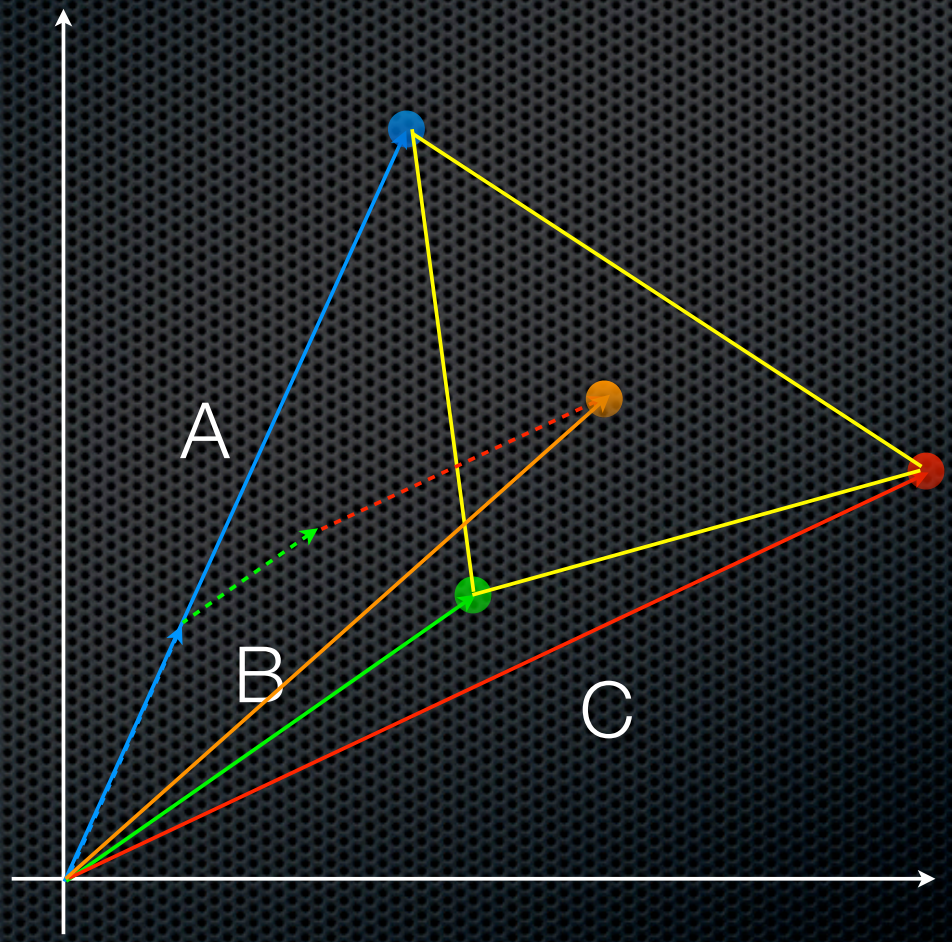


Triangle

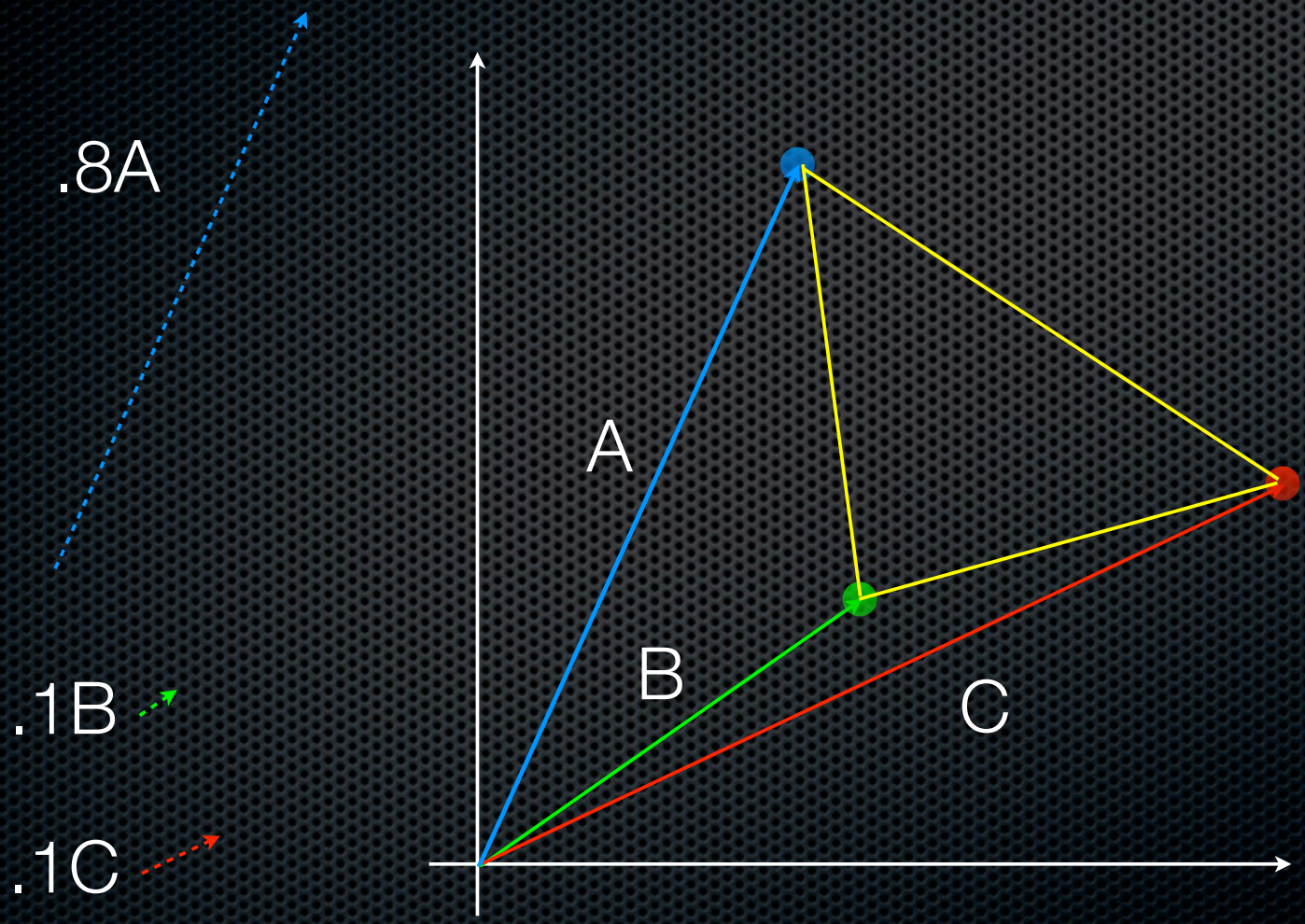


Triangle

.333 + .333 + .333 = 1

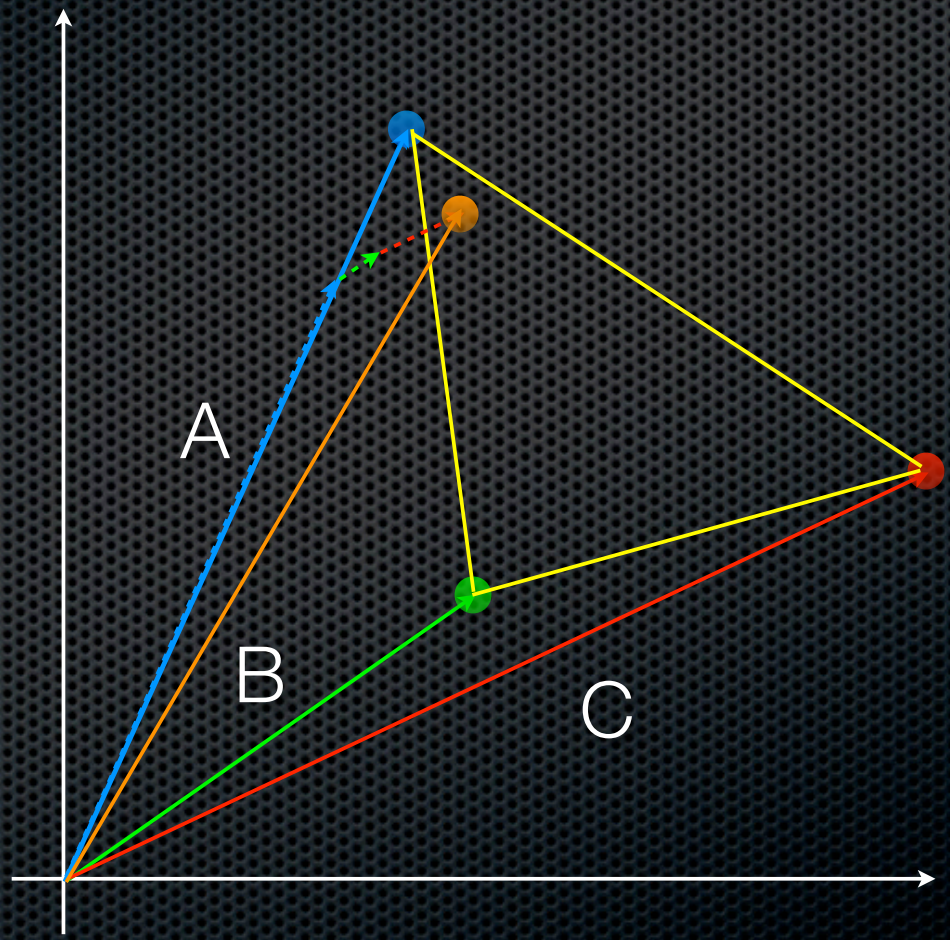


Triangle

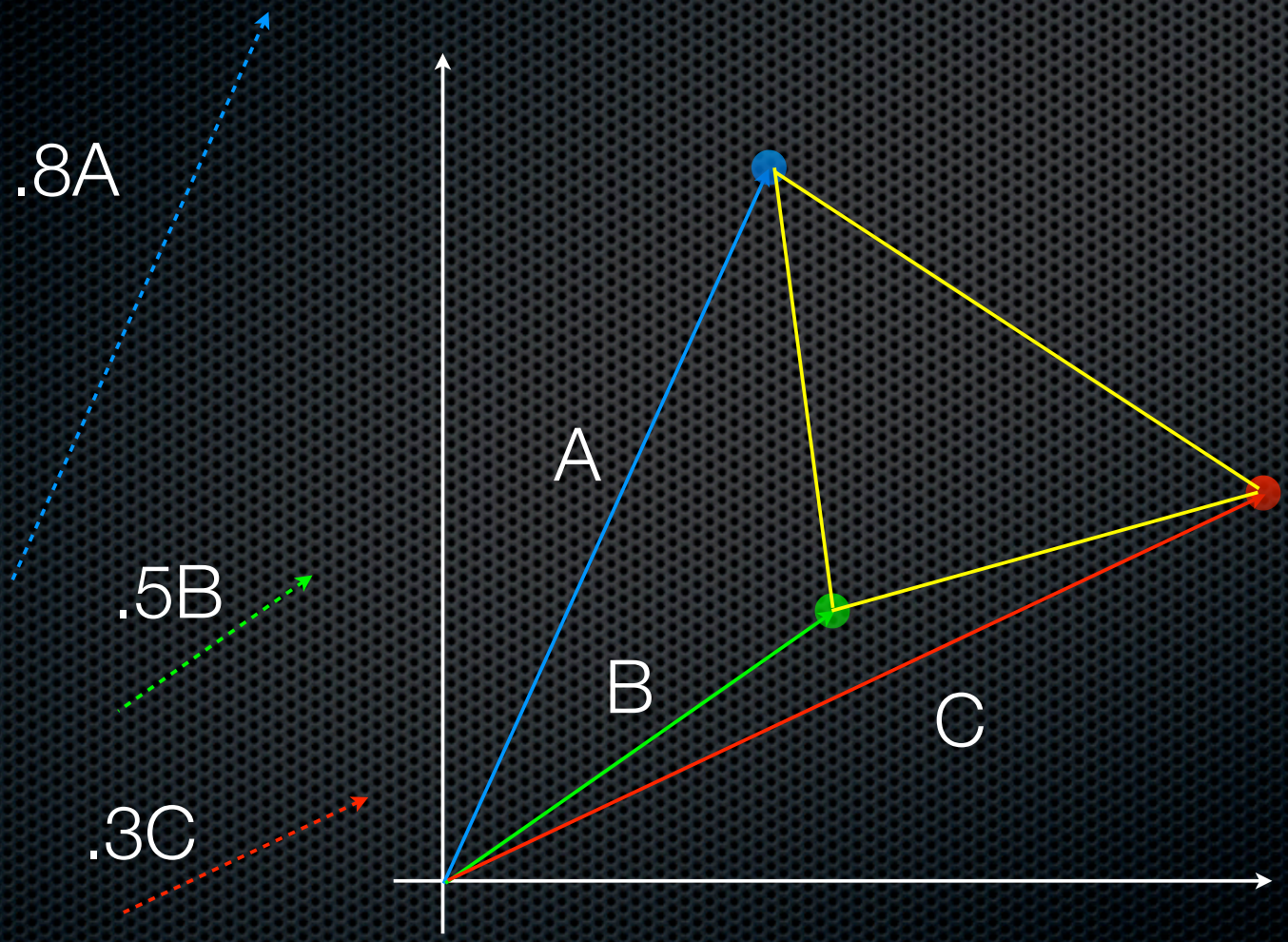


Triangle

.8 + .1 + .1 = 1

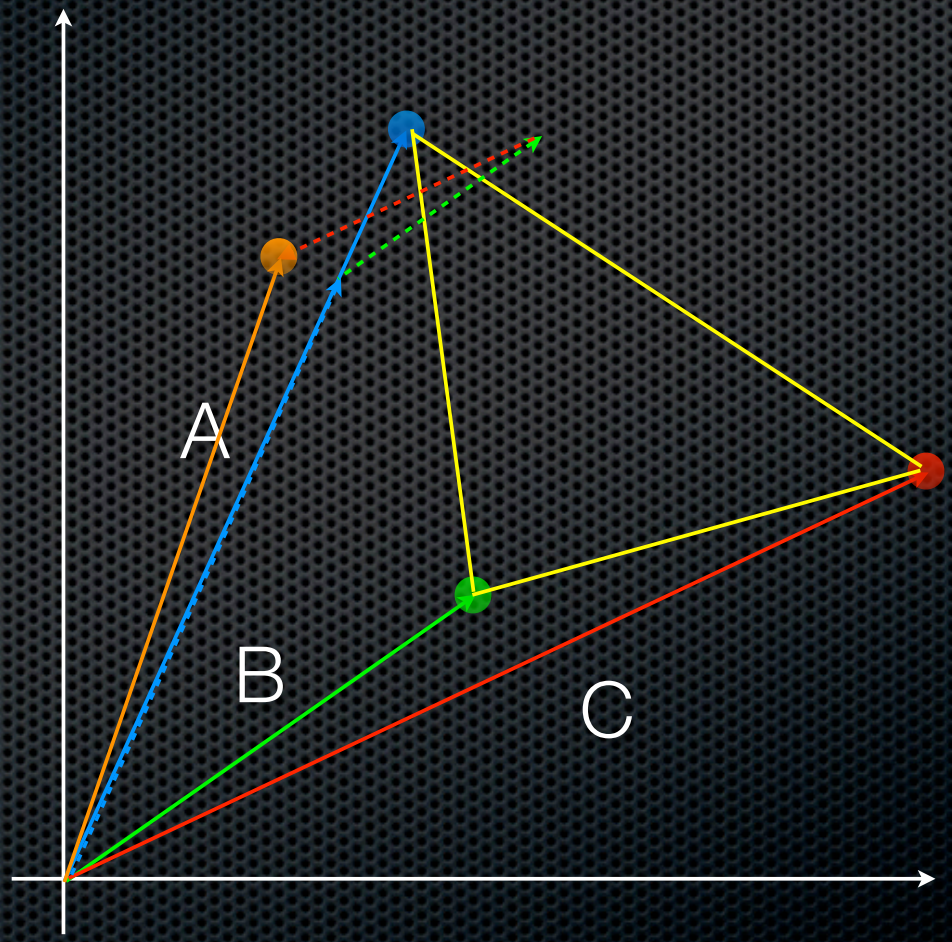


Triangle



Triangle

.8 + .5 - .3 = 1

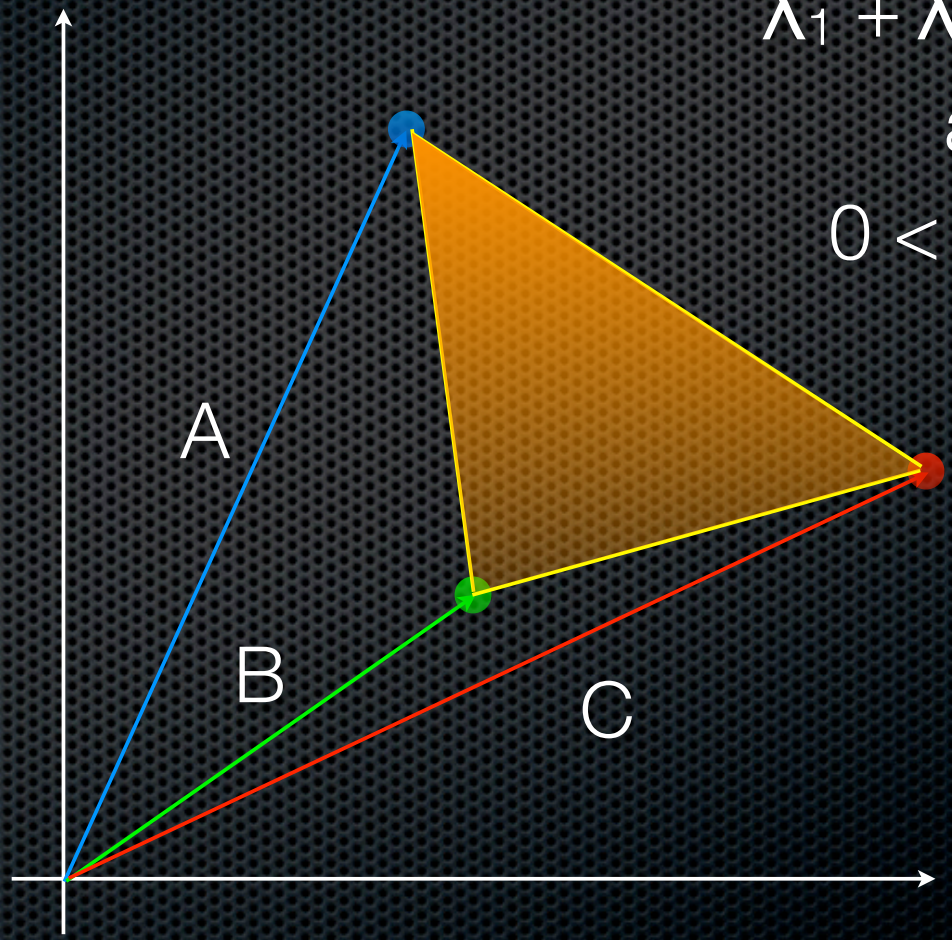


Barycentric Coordinates

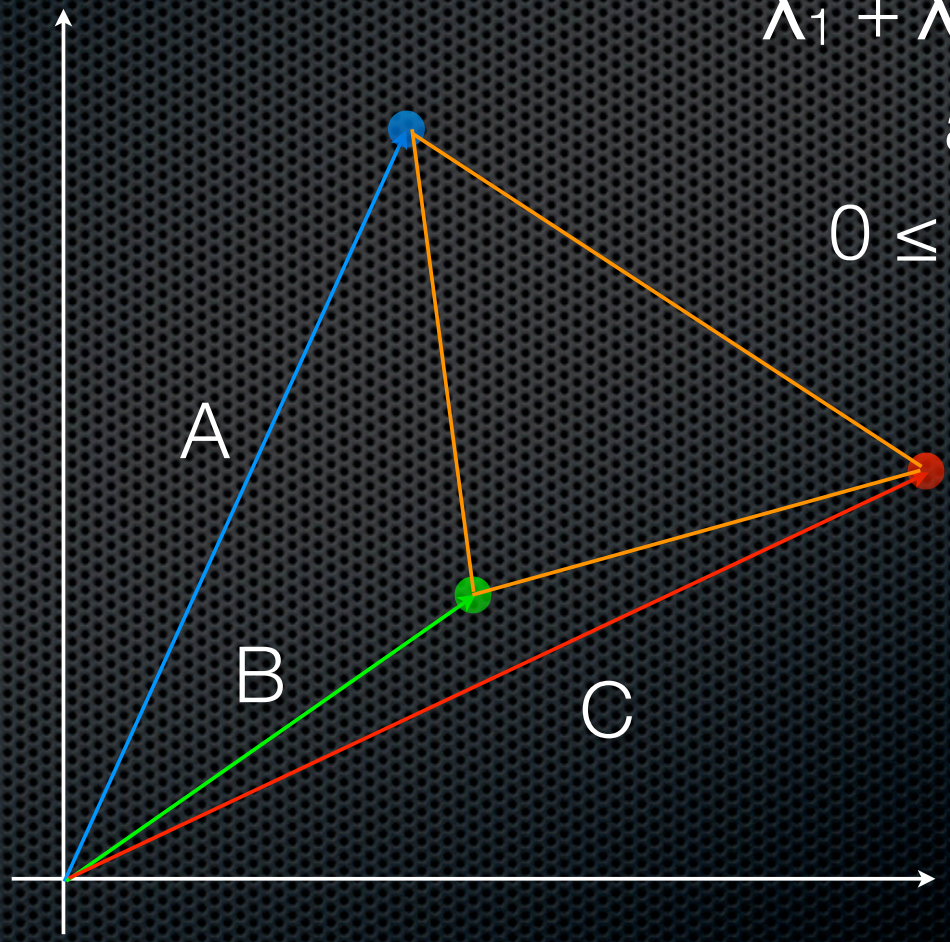
$\lambda_1 + \lambda_2 + \lambda_3 = 1$

and

$0 < \lambda_x < 1$



Barycentric Coordinates



$\lambda_1 + \lambda_2 + \lambda_3 = 1$

and

$0 \leq \lambda_x \leq 1$

and

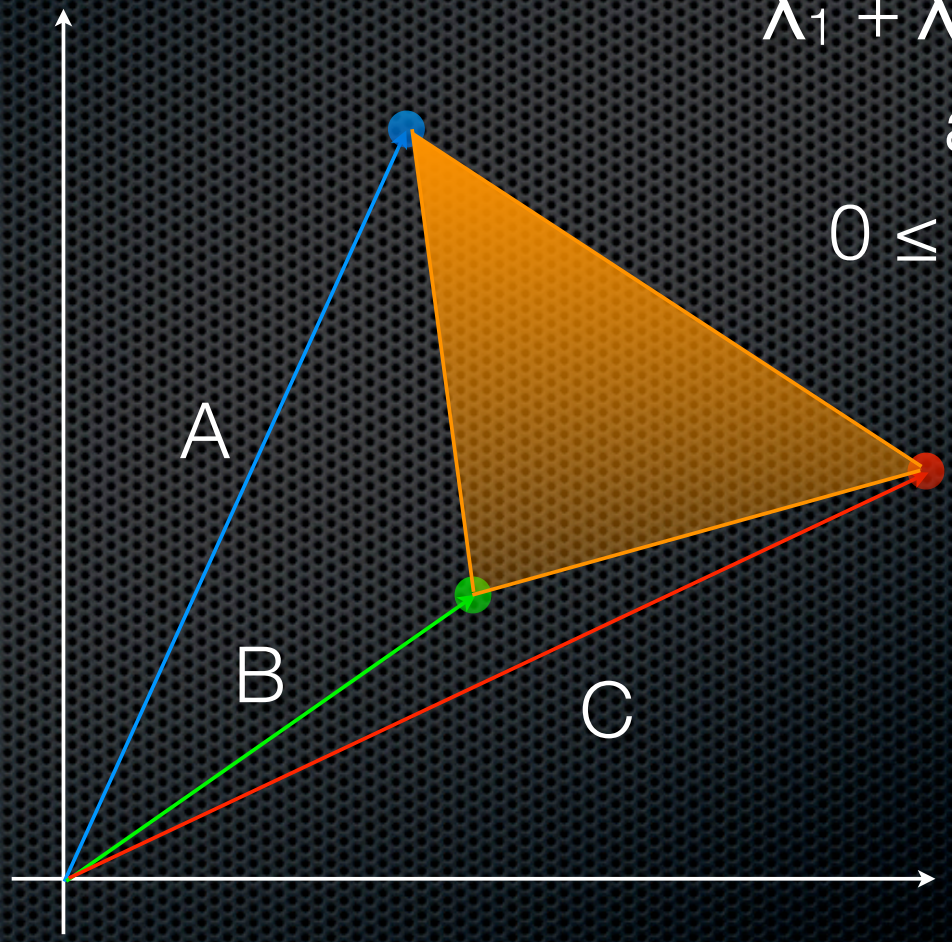
1 or 2 $\lambda_x = 0$

Barycentric Coordinates

$$\lambda_1 + \lambda_2 + \lambda_3 = 1$$

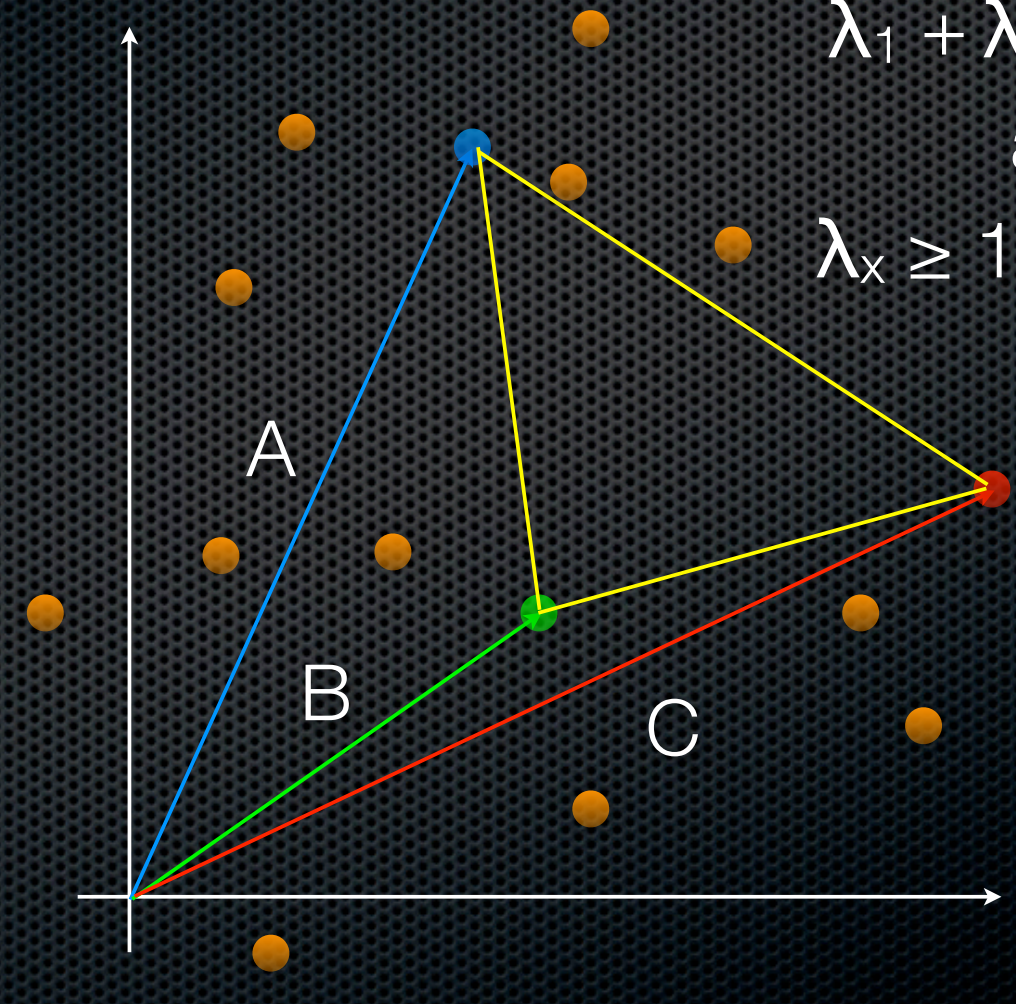
and

$$0 \leq \lambda_x \leq 1$$



Barycentric Coordinates

Outside



$\lambda_1 + \lambda_2 + \lambda_3 = 1$

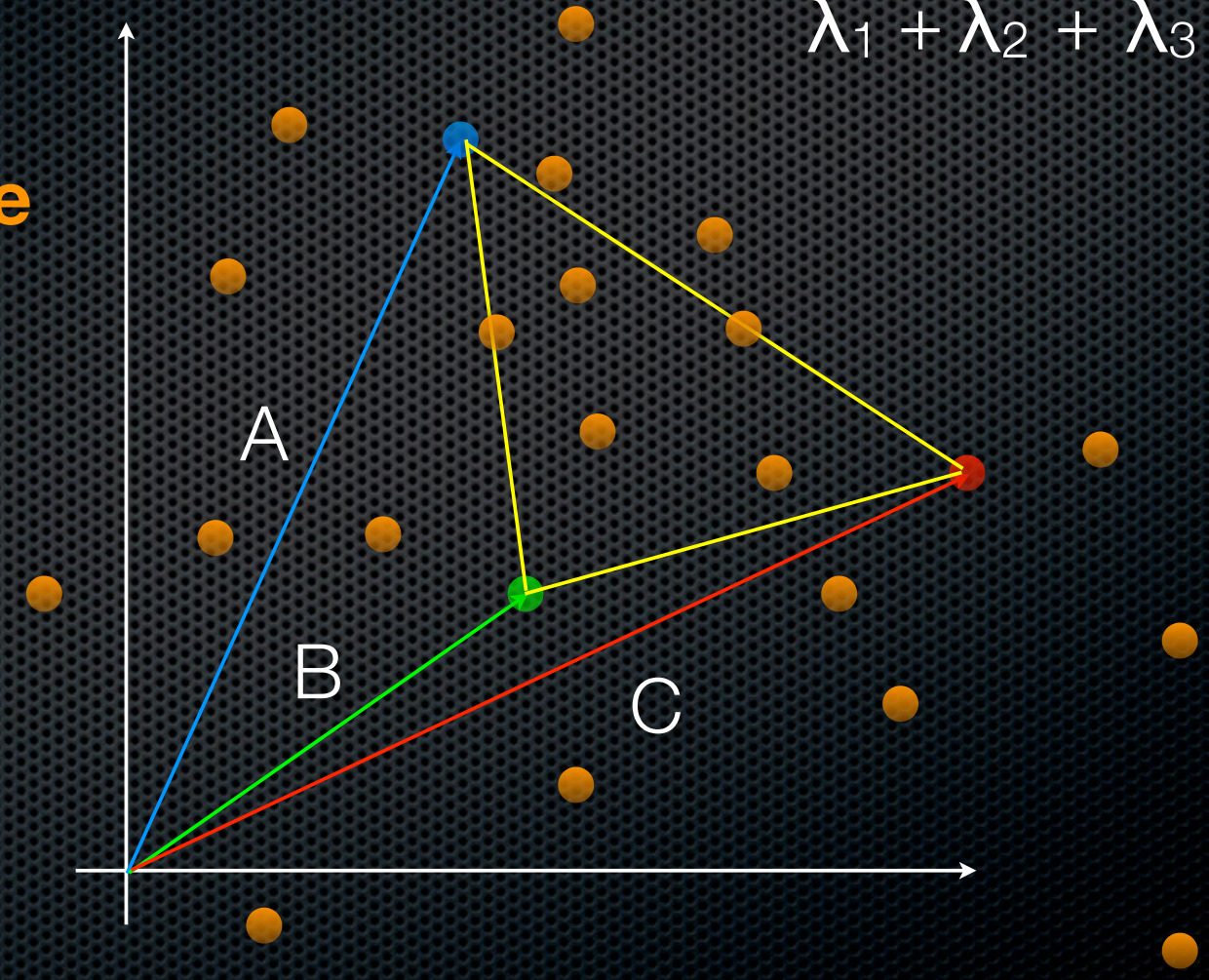
and

$\lambda_x \geq 1$ or $\lambda_x \leq 0$

Not Barycentric Coordinates

$\lambda_1 + \lambda_2 + \lambda_3 \neq 1$

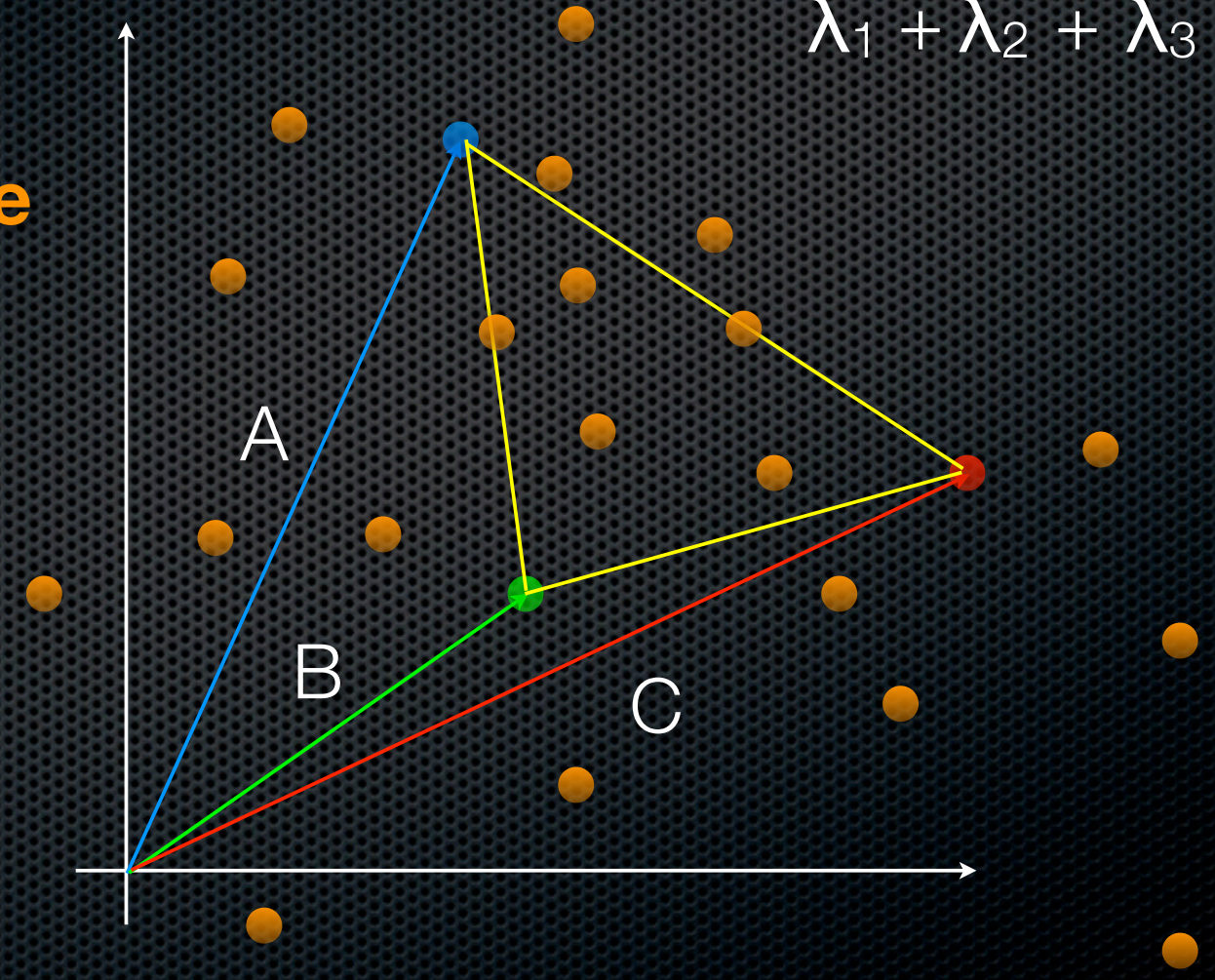
Anywhere



Linear Combination

$\lambda_1 + \lambda_2 + \lambda_3 \neq 1$

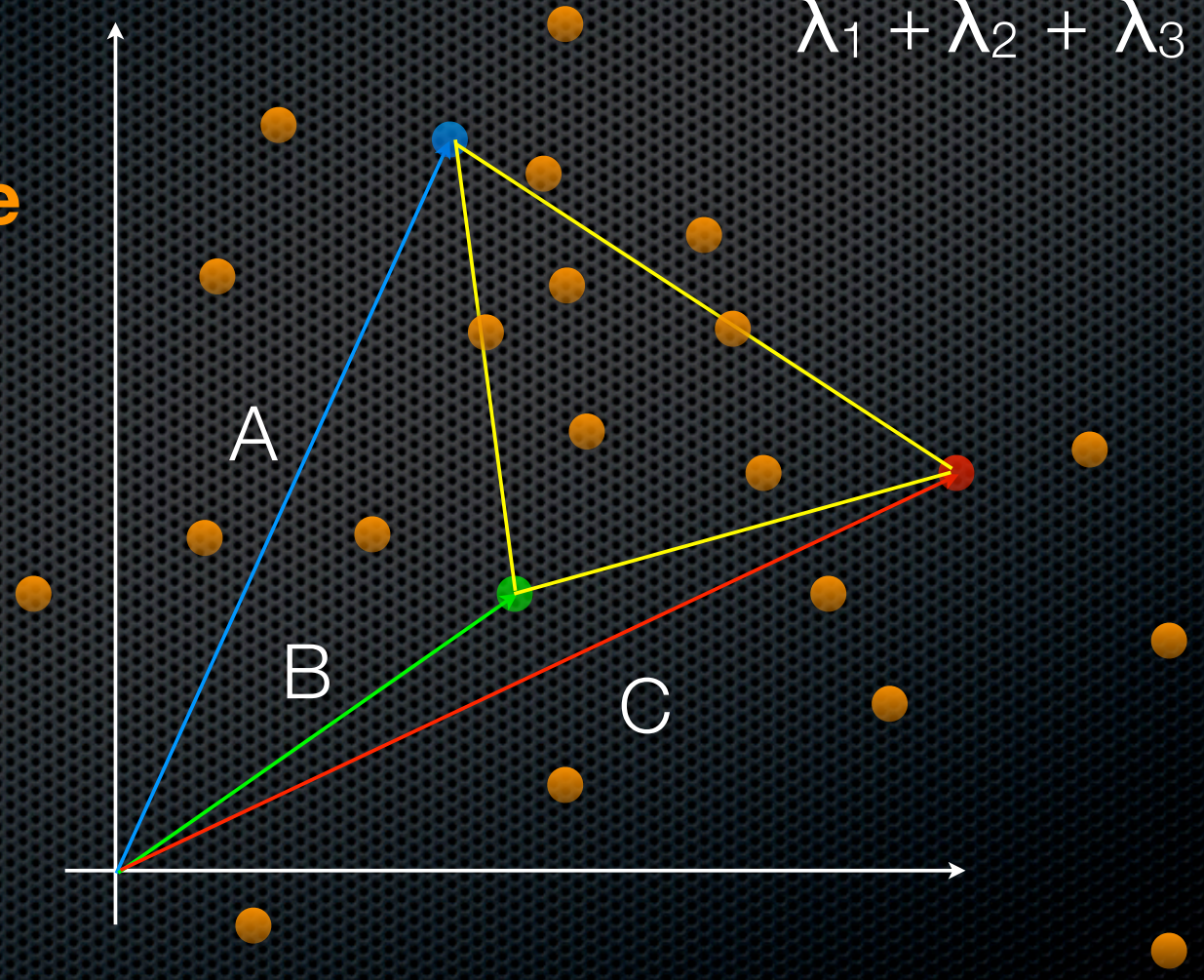
Anywhere



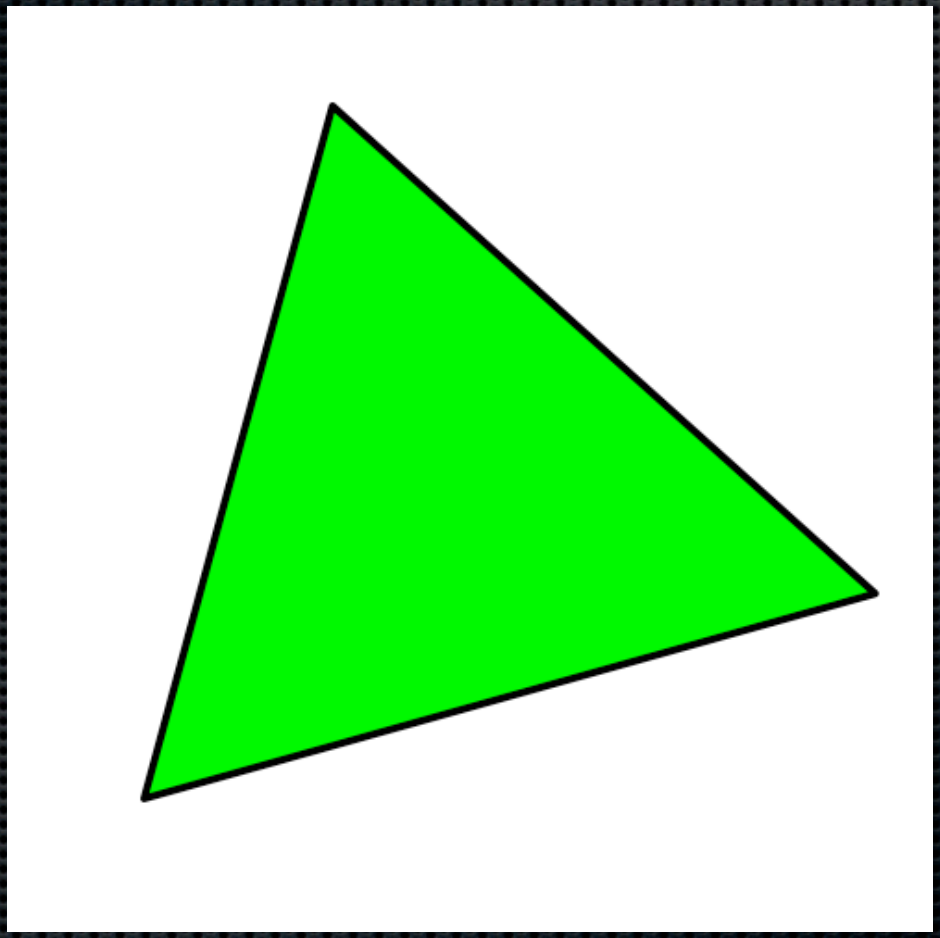
Linear Combination but **Not** a Blend

$\lambda_1 + \lambda_2 + \lambda_3 \neq 1$

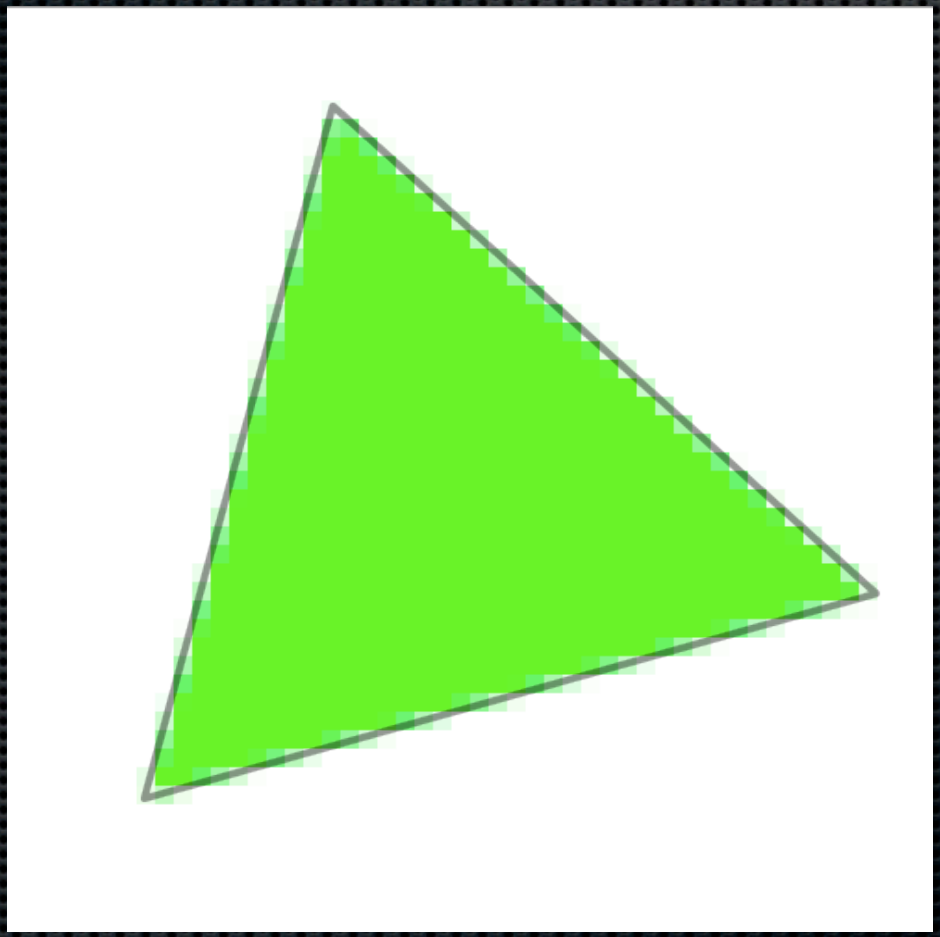
Anywhere



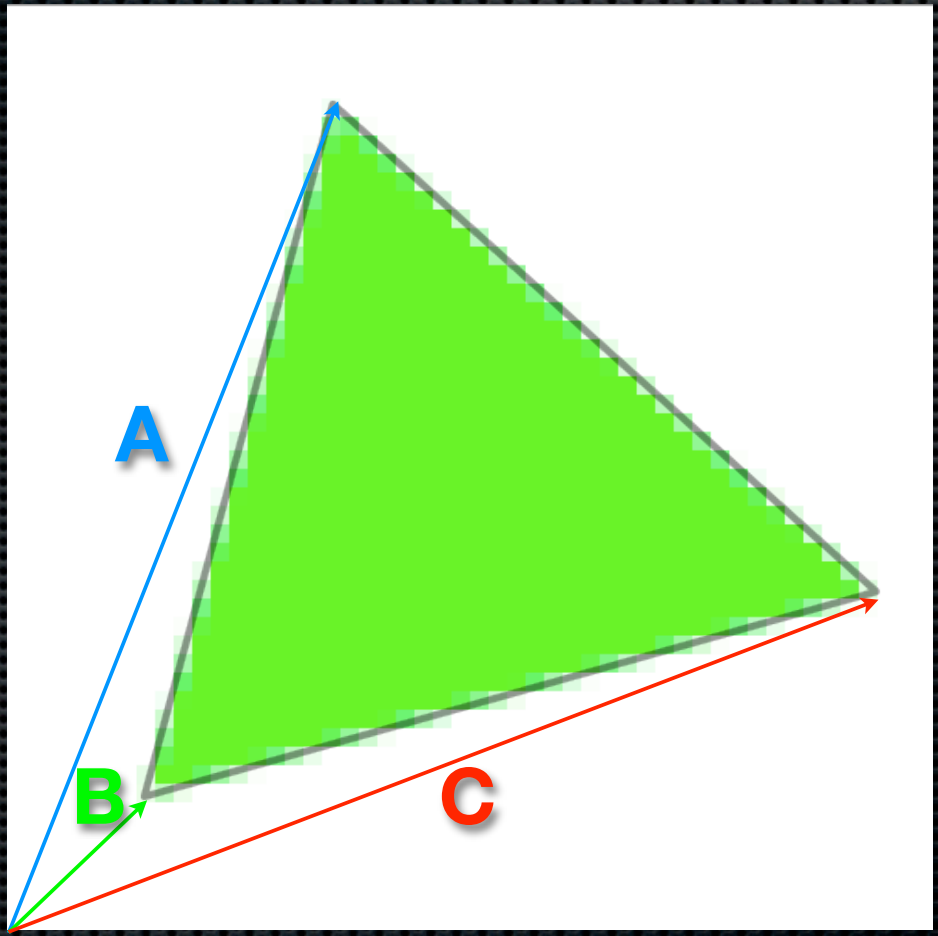
Rasterization



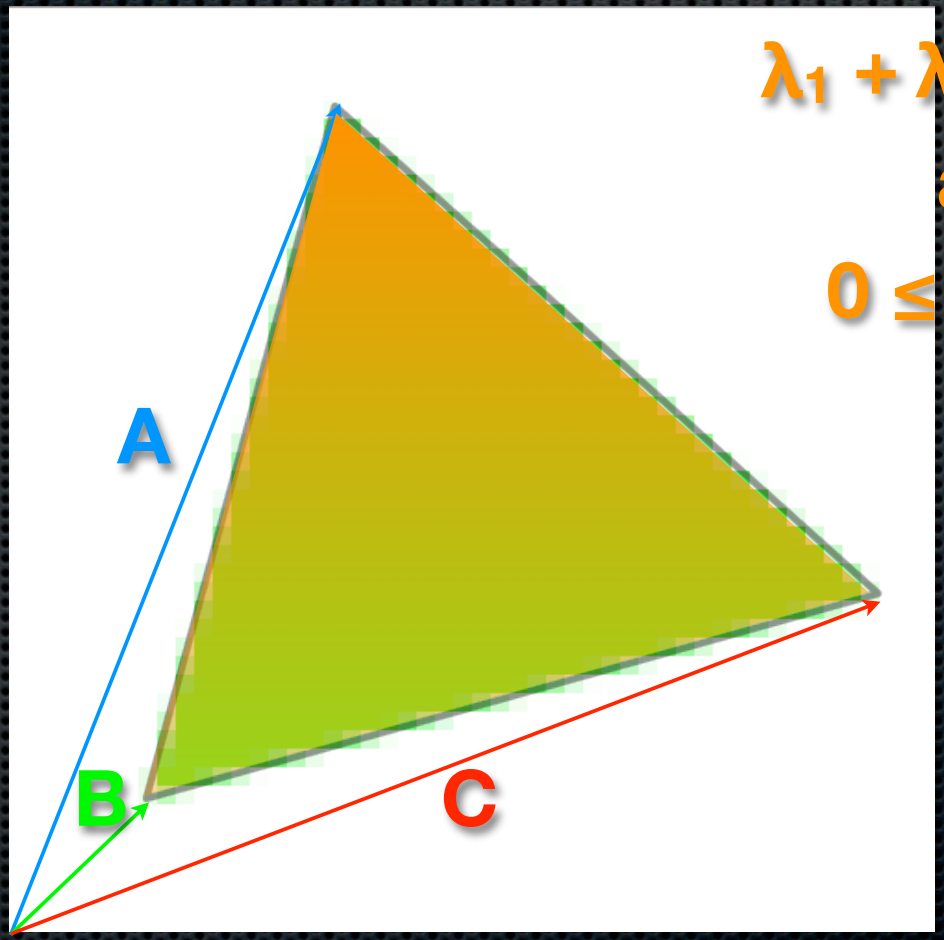
Rasterization



Rasterization



Barycentric Coordinates

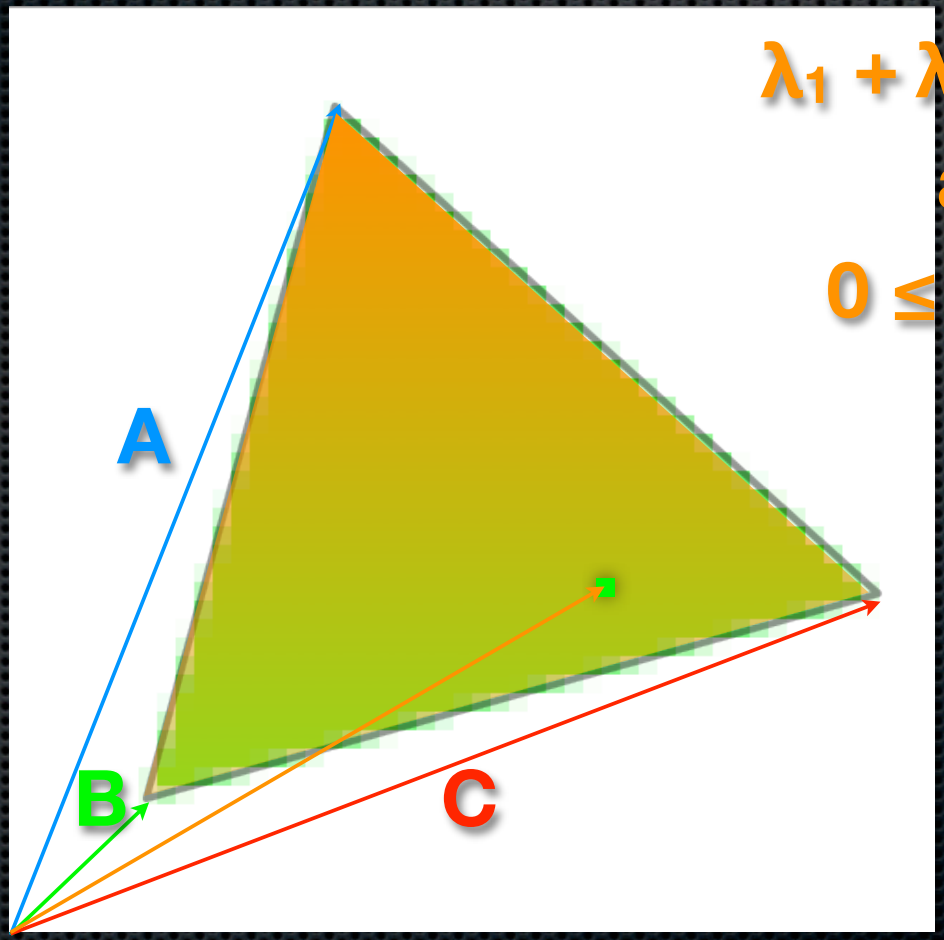


$\lambda_1 + \lambda_2 + \lambda_3 = 1$

and

$0 \leq \lambda_x \leq 1$

Barycentric Coordinates

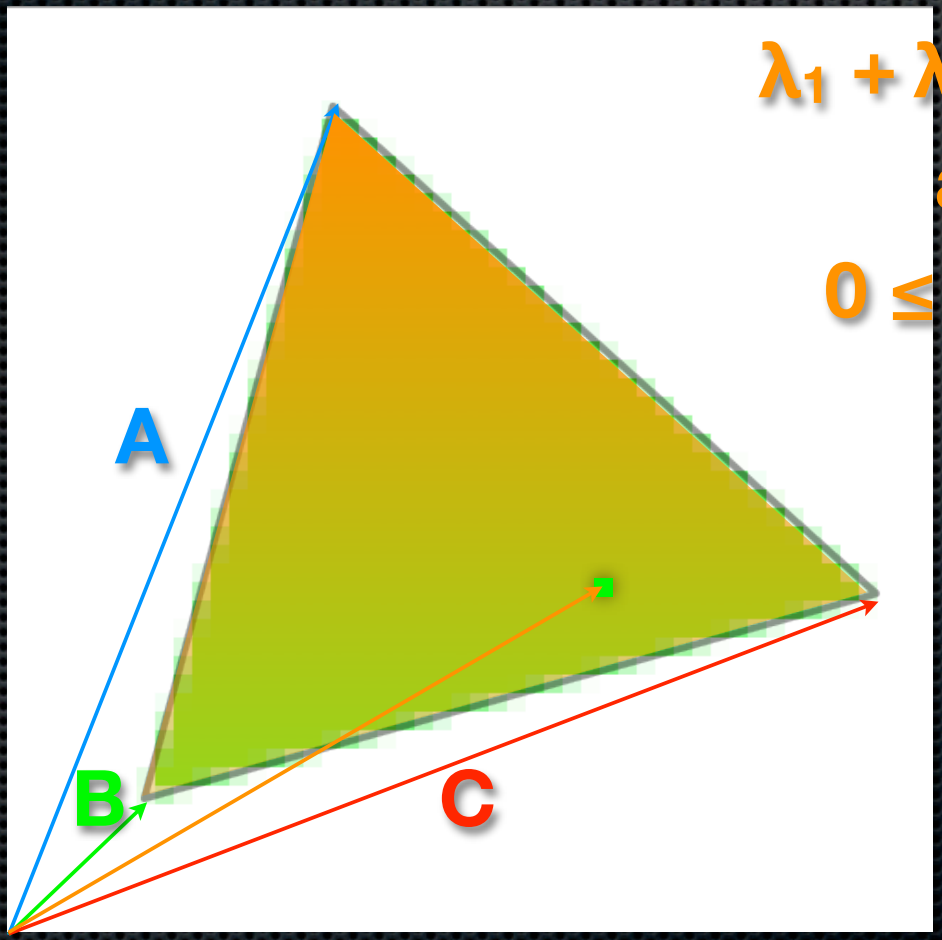


$\lambda_1 + \lambda_2 + \lambda_3 = 1$

and

$0 \leq \lambda_x \leq 1$

Barycentric Coordinates



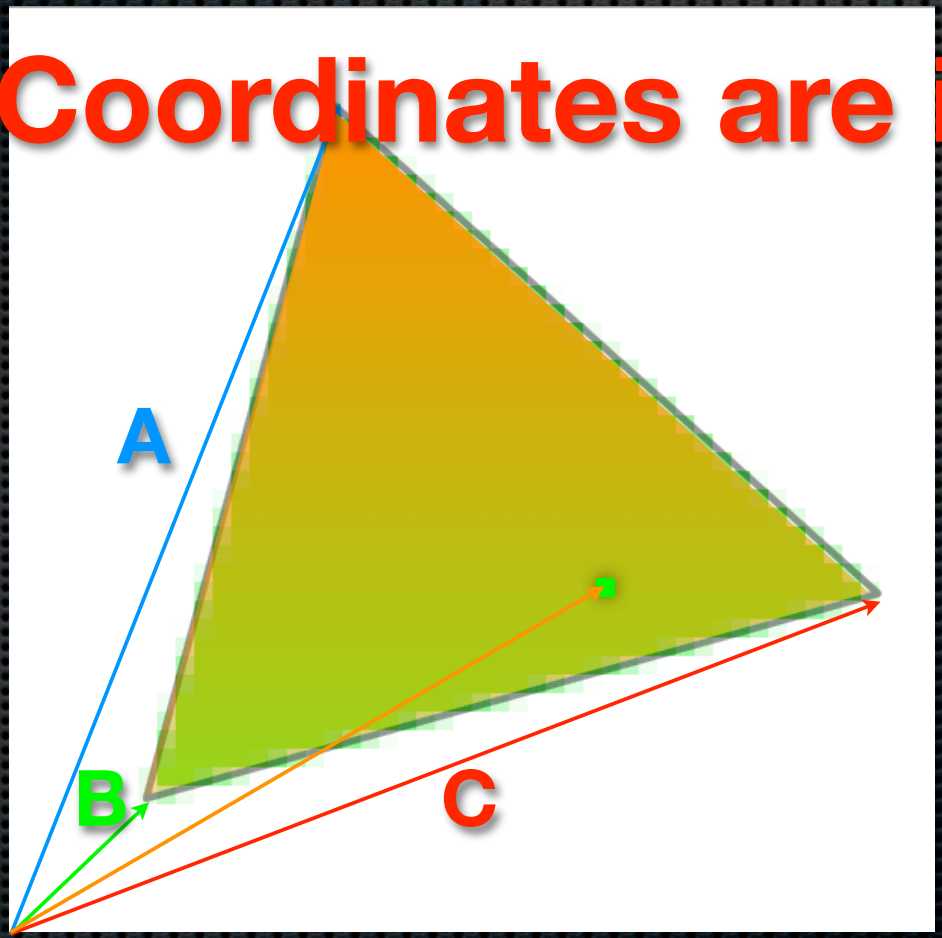
$\lambda_1 + \lambda_2 + \lambda_3 = 1$

and

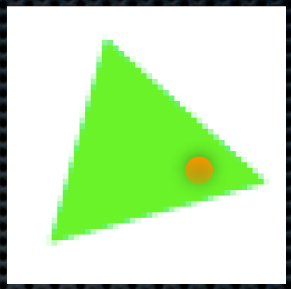
$0 \leq \lambda_x \leq 1$

Barycentric Coordinates

Pixel Coordinates are in X,Y



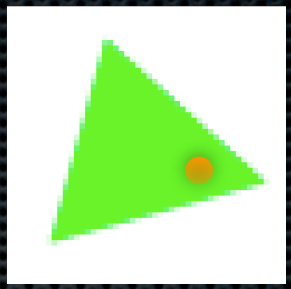
Barycentric Coordinates



$$x = \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3$$
$$y = \lambda_1 y_1 + \lambda_2 y_2 + \lambda_3 y_3$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 1$$

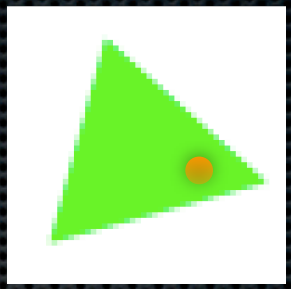
Barycentric Coordinates



$$x = \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3$$
$$y = \lambda_1 y_1 + \lambda_2 y_2 + \lambda_3 y_3$$

$$\lambda_3 = 1 - \lambda_1 - \lambda_2$$

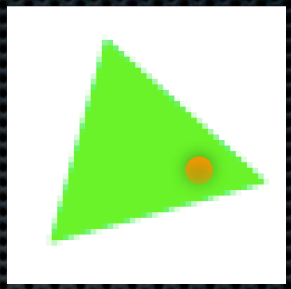
Barycentric Coordinates



$$x = \lambda_1 x_1 + \lambda_2 x_2 + (1 - \lambda_1 + \lambda_2) x_3$$
$$y = \lambda_1 y_1 + \lambda_2 y_2 + (1 - \lambda_1 + \lambda_2) y_3$$

$$\lambda_3 = 1 - \lambda_1 - \lambda_2$$

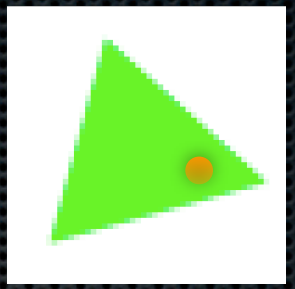
Barycentric Coordinates



Lots of rearranging...

$$\lambda_3 = 1 - \lambda_1 - \lambda_2$$

Barycentric Coordinates



$$\lambda_1 = \frac{(y_2 - y_3)(x - x_3) + (x_3 - x_2)(y - y_3)}{(y_2 - y_3)(x_1 - x_3) + (x_3 - x_2)(y_1 - y_3)}$$

$$\lambda_2 = \frac{(y_3 - y_1)(x - x_3) + (x_1 - x_3)(y - y_3)}{(y_2 - y_3)(x_1 - x_3) + (x_3 - x_2)(y_1 - y_3)}$$

$$\lambda_3 = 1 - \lambda_1 - \lambda_2$$

Barycentric Coordinates