MAT 120 Suggested Exercise

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Vector Calculus [Double, Triple, Multiple (Jacobean) Integral]

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1. Evaluated the iterated integrals: DOUBLE Integral

(a) $\int_{\frac{\pi}{2}}^{\pi}\int_{0}^{x^2} \frac{1}{x}\cos\frac{y}{x}\,dy\,dx$ (b) $\int_0^1\int_0^1 \frac{x}{(xy+1)^2}dy\,dx$ (c) $\int_1^2\int_0^{y^2} e^{\frac{x}{y^2}}\,dx\,dy$ (d) $\int_0^2\int_0^{\sqrt{4-x^2}} e^{\sqrt{x^2+y^2}}\,dy\,dx$.

2. DOUBLE Integral

(a) Find the area of the region inside the circle r = 4sinθ and outside the circle r = 2.

(b) $\iint_R \frac{1}{x^2+y^2+1}dA$, where R is the sector in the first quadrant bounded by

$y = 0$, $y = x$ and $x^2 + y^2 = 4$.

3. Use polar coordinates to evaluate the double integral $\int_{-a}^{a}\int_{0}^{\sqrt{a^2-x^2}} \left(x^2+y^2\right)^{1/2}dydx$. (DOUBLE Int.)

4. (a) Find the volume of the solid that is bounded by the cylinder $y = x^2$ and by the planes $y + z = 4$ and $z = 0$. (TRIPLE Int)

(b) Find the volume of the surface enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$. (TRIPLE Int)

5. Evaluated the iterated integral by converting to polar coordinates: (DOUBLE Int)

(a) $\int_0^1\int_0^{\sqrt{1-x^2}}\left(x^2+y^2\right)dydx$ (b) $\int_0^2\int_0^{\sqrt{2x-x^2}}\sqrt{x^2+y^2}\;dydx$

(c) $\int_0^a\int_0^{\sqrt{a^2-x^2}}\frac{dydx}{\left(1+x^2+y^2\right)^{\frac{3}{2}}}$ $(a>0)$.

6. (a) Evaluate $\int_{y=0}^{4}\int_{x=\frac{y}{2}}^{x=\frac{y}{2}+1}\frac{2x-y}{2}\,dxdy$ by applying transformation T :

where $u=\frac{2x-y}{2}$, $v=\frac{y}{2}$ and integrating over an appropriate region in uv–plane. (JACOBEAN)

(b) Evaluate $\iint_R \frac{x-y}{x+y}\,dA$, where R is the region enclosed by the lines

$x-y=0$, $x-y=1$, $x+y=1$ & $x+y=3$, using the transformation.

(JACOBEAN)

Double Integral

Exercise- 14.1- 1-16.

EXERCISE SET 14.1 C CAS

1–12 Evaluate the iterated integrals.

1. $\int_0^1 \int_0^2 (x+3)\,dy\,dx$

2. $\int_1^3 \int_{-1}^1 (2x-4y)\,dy\,dx$

3. $\int_2^4 \int_0^1 x^2 y\,dx\,dy$

4. $\int_{-2}^0 \int_{-1}^2 (x^2+y^2)\,dx\,dy$

5. $\int_0^{\ln 3} \int_0^{\ln 2} e^{x+y}\,dy\,dx$

6. $\int_0^2 \int_0^1 y\sin x\,dy\,dx$

7. $\int_{-1}^0 \int_2^5 dx\,dy$

8. $\int_4^6 \int_{-3}^7 dy\,dx$

9. $\int_0^1 \int_0^1 \frac{x}{(xy+1)^2}\,dy\,dx$

10. $\int_{\pi/2}^{\pi} \int_1^2 x\cos xy\,dy\,dx$

11. $\int_0^{\ln 2} \int_0^1 xye^{y^2x}\,dy\,dx$

12. $\int_3^4 \int_1^2 \frac{1}{(x+y)^2}\,dy\,dx$

13–16 Evaluate the double integral over the rectangular region R.

13. $\iint_R 4xy^3\,dA;\ R = \{(x,y) : -1 \le x \le 1, -2 \le y \le 2\}$

14. $\iint_R \frac{xy}{\sqrt{x^2+y^2+1}}\,dA;$
$R = \{(x,y) : 0 \le x \le 1, 0 \le y \le 1\}$

15. $\iint_R x\sqrt{1-x^2}\,dA;\ R = \{(x,y) : 0 \le x \le 1, 2 \le y \le 3\}$

16. $\iint_R (x\sin y - y\sin x)\,dA;$
$R = \{(x,y) : 0 \le x \le \pi/2, 0 \le y \le \pi/3\}$

Exercise- 14.2- 1-26.

EXERCISE SET 14.2 Graphing Utility C CAS

1–8 Evaluate the iterated integral. ■

1. $\int_0^1 \int_{x^2}^{x} xy^2\,dy\,dx$ **2.** $\int_1^{3/2} \int_y^{3-y} y\,dx\,dy$

3. $\int_0^3 \int_0^{\sqrt{9-y^2}} y\,dx\,dy$ **4.** $\int_{1/4}^1 \int_{x^2}^{x} \sqrt{\frac{x}{y}}\,dy\,dx$

5. $\int_{\sqrt{\pi}}^{\sqrt{2\pi}} \int_0^{x^3} \sin\frac{y}{x}\,dy\,dx$ **6.** $\int_{-1}^1 \int_{-x^2}^{x^2} (x^2 - y)\,dy\,dx$

7. $\int_0^1 \int_0^x y\sqrt{x^2 - y^2}\,dy\,dx$ **8.** $\int_1^2 \int_0^{y^2} e^{x/y^2}\,dx\,dy$

FOCUS ON CONCEPTS

9. Let R be the region shown in the accompanying figure. Fill in the missing limits of integration.

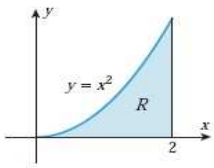
(a) $\iint_R f(x, y)\,dA = \int_\square^\square \int_\square^\square f(x, y)\,dy\,dx$

(b) $\iint_R f(x, y)\,dA = \int_\square^\square \int_\square^\square f(x, y)\,dx\,dy$

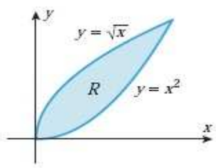
10. Let R be the region shown in the accompanying figure. Fill in the missing limits of integration.

(a) $\iint_R f(x, y)\,dA = \int_\square^\square \int_\square^\square f(x, y)\,dy\,dx$

(b) $\iint_R f(x, y)\,dA = \int_\square^\square \int_\square^\square f(x, y)\,dx\,dy$



▲ Figure Ex-9



▲ Figure Ex-10

11. Let R be the region shown in the accompanying figure. Fill in the missing limits of integration.

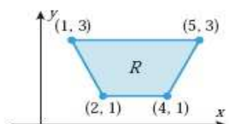
(a) $\iint_R f(x, y)\,dA = \int_1^2 \int_\square^\square f(x, y)\,dy\,dx + \int_2^4 \int_\square^\square f(x, y)\,dy\,dx + \int_4^5 \int_\square^\square f(x, y)\,dy\,dx$

(b) $\iint_R f(x, y)\,dA = \int_\square^\square \int_\square^\square f(x, y)\,dx\,dy$

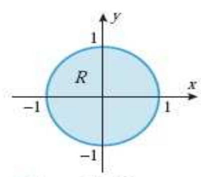
12. Let R be the region shown in the accompanying figure. Fill in the missing limits of integration.

(a) $\iint_R f(x, y)\,dA = \int_\square^\square \int_\square^\square f(x, y)\,dy\,dx$

(b) $\iint_R f(x, y)\,dA = \int_\square^\square \int_\square^\square f(x, y)\,dx\,dy$



▲ Figure Ex-11



▲ Figure Ex-12

13. Evaluate $\iint_R xy\,dA$, where R is the region in

(a) Exercise 9 (b) Exercise 11.

14. Evaluate $\iint_R (x + y)\,dA$, where R is the region in

(a) Exercise 10 (b) Exercise 12.

15–18 Evaluate the double integral in two ways using iterated integrals: (a) viewing R as a type I region, and (b) viewing R as a type II region. ■

15. $\iint_R x^2\,dA$; R is the region bounded by $y = 16/x$, $y = x$, and $x = 8$.

16. $\iint_R xy^2\,dA$; R is the region enclosed by $y = 1$, $y = 2$, $x = 0$, and $y = x$.

17. $\iint_R (3x - 2y)\,dA$; R is the region enclosed by the circle $x^2 + y^2 = 1$.

18. $\iint_R y\,dA$; R is the region in the first quadrant enclosed between the circle $x^2 + y^2 = 25$ and the line $x + y = 5$.

19–24 Evaluate the double integral. ■

19. $\iint_R x(1 + y^2)^{-1/2}\,dA$; R is the region in the first quadrant enclosed by $y = x^2$, $y = 4$, and $x = 0$.

20. $\iint_R x\cos y\,dA$; R is the triangular region bounded by the lines $y = x$, $y = 0$, and $x = \pi$.

21. $\iint_R xy\,dA$; R is the region enclosed by $y = \sqrt{x}$, $y = 6 - x$, and $v = 0$.

20. $\iint_R x\cos y\,dA$; R is the triangular region bounded by the lines $y = x$, $y = 0$, and $x = \pi$.

21. $\iint_R xy\,dA$; R is the region enclosed by $y = \sqrt{x}$, $y = 6 - x$, and $y = 0$.

22. $\iint_R x\,dA$; R is the region enclosed by $y = \sin^{-1} x$, $x = 1/\sqrt{2}$, and $y = 0$.

23. $\iint_R (x - 1)\,dA$; R is the region in the first quadrant enclosed between $y = x$ and $y = x^3$.

24. $\iint_R x^2\,dA$; R is the region in the first quadrant enclosed by $xy = 1$, $y = x$, and $y = 2x$.

25. Evaluate $\iint_R \sin(y^3)\,dA$, where R is the region bounded by $y = \sqrt{x}$, $y = 2$, and $x = 0$. [*Hint:* Choose the order of integration carefully.]

26. Evaluate $\iint_R x\,dA$, where R is the region bounded by $x = \ln y$, $x = 0$, and $y = e$.

Exercise- 14.3-   1-12, 23-34.

1. $\int_0^{\pi/2}\int_0^{\sin\theta} r\cos\theta\, dr\, d\theta$ **2.** $\int_0^{\pi}\int_0^{1+\cos\theta} r\, dr\, d\theta$

3. $\int_0^{\pi/2}\int_0^{a\sin\theta} r^2\, dr\, d\theta$

4. $\int_0^{\pi/6}\int_0^{\cos 3\theta} r\, dr\, d\theta$

5. $\int_0^{\pi}\int_0^{1-\sin\theta} r^2\cos\theta\, dr\, d\theta$

6. $\int_0^{\pi/2}\int_0^{\cos\theta} r^3\, dr\, d\theta$

7–10 Use a double integral in polar coordinates to find the area of the region described.

7. The region enclosed by the cardioid $r = 1 - \cos\theta$.

8. The region enclosed by the rose $r = \sin 2\theta$.

9. The region in the first quadrant bounded by $r = 1$ and $r = \sin 2\theta$, with $\pi/4 \le \theta \le \pi/2$.

10. The region inside the circle $x^2 + y^2 = 4$ and to the right of the line $x = 1$.

FOCUS ON CONCEPTS

11–12 Let R be the region described. Sketch the region R and fill in the missing limits of integration.

$$\iint_R f(r,\theta)\, dA = \int_{\square}^{\square}\int_{\square}^{\square} f(r,\theta) r\, dr\, d\theta$$

11. The region inside the circle $r = 4\sin\theta$ and outside the circle $r = 2$.

12. The region inside the circle $r = 1$ and outside the cardioid $r = 1 + \cos\theta$.

13–16 Express the volume of the solid described as a double integral in polar coordinates.

23. $\iint_R \sin(x^2 + y^2)\, dA$, where R is the region enclosed by the circle $x^2 + y^2 = 9$.

24. $\iint_R \sqrt{9 - x^2 - y^2}\, dA$, where R is the region in the first quadrant within the circle $x^2 + y^2 = 9$.

25. $\iint_R \frac{1}{1 + x^2 + y^2}\, dA$, where R is the sector in the first quadrant bounded by $y = 0$, $y = x$, and $x^2 + y^2 = 4$.

26. $\iint_R 2y\, dA$, where R is the region in the first quadrant bounded above by the circle $(x-1)^2 + y^2 = 1$ and below by the line $y = x$.

27–34 Evaluate the iterated integral by converting to polar coordinates.

27. $\int_0^1\int_0^{\sqrt{1-x^2}} (x^2 + y^2)\, dy\, dx$

28. $\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} e^{-(x^2+y^2)}\, dx\, dy$

29. $\int_0^2\int_0^{\sqrt{2x-x^2}} \sqrt{x^2 + y^2}\, dy\, dx$

30. $\int_0^1\int_0^{\sqrt{1-y^2}} \cos(x^2 + y^2)\, dx\, dy$

31. $\int_0^a\int_0^{\sqrt{a^2-x^2}} \frac{dy\, dx}{(1 + x^2 + y^2)^{3/2}}$ $(a > 0)$

32. $\int_0^1\int_y^{\sqrt{y}} \sqrt{x^2 + y^2}\, dx\, dy$

33. $\int_0^{\sqrt{2}}\int_y^{\sqrt{4-y^2}} \frac{1}{\sqrt{1 + x^2 + y^2}}\, dx\, dy$

34. $\int_{-4}^{0}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} 3x\, dy\, dx$

Surface Area from Double Integral

Exercise- 14.4-   1-9.

EXERCISE SET 14.4   Graphing Utility   C CAS

1–4 Express the area of the given surface as an iterated double integral, and then find the surface area.

1. The portion of the cylinder $y^2 + z^2 = 9$ that is above the rectangle $R = \{(x, y) : 0 \le x \le 2, -3 \le y \le 3\}$.

2. The portion of the plane $2x + 2y + z = 8$ in the first octant.

3. The portion of the cone $z^2 = 4x^2 + 4y^2$ that is above the region in the first quadrant bounded by the line $y = x$ and the parabola $y = x^2$.

4. The portion of the surface $z = 2x + y^2$ that is above the triangular region with vertices $(0, 0)$, $(0, 1)$, and $(1, 1)$.

5–10 Express the area of the given surface as an iterated double integral in polar coordinates, and then find the surface area.

5. The portion of the cone $z = \sqrt{x^2 + y^2}$ that lies inside the cylinder $x^2 + y^2 = 2x$.

6. The portion of the paraboloid $z = 1 - x^2 - y^2$ that is above the xy-plane.

7. The portion of the surface $z = xy$ that is above the sector in the first quadrant bounded by the lines $y = x/\sqrt{3}$, $y = 0$, and the circle $x^2 + y^2 = 9$.

8. The portion of the paraboloid $2z = x^2 + y^2$ that is inside the cylinder $x^2 + y^2 = 8$.

9. The portion of the sphere $x^2 + y^2 + z^2 = 16$ between the planes $z = 1$ and $z = 2$.

Triple Integral

Exercise- 14.5- 1-12, 15-18.

EXERCISE SET 14.5 C CAS

1–8 Evaluate the iterated integral.

1. $\int_{-1}^{1}\int_{0}^{2}\int_{0}^{1}(x^2+y^2+z^2)\,dx\,dy\,dz$

2. $\int_{1/3}^{1/2}\int_{0}^{\pi}\int_{0}^{1} zx\sin xy\,dz\,dy\,dx$

3. $\int_{0}^{2}\int_{-1}^{y^2}\int_{-1}^{z} yz\,dx\,dz\,dy$

4. $\int_{0}^{\pi/4}\int_{0}^{1}\int_{0}^{x^2} x\cos y\,dz\,dx\,dy$

5. $\int_{0}^{3}\int_{0}^{\sqrt{9-z^2}}\int_{0}^{x} xy\,dy\,dx\,dz$

6. $\int_{1}^{3}\int_{x}^{x^2}\int_{0}^{\ln z} xe^{y}\,dy\,dz\,dx$

7. $\int_{0}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{-5+x^2+y^2}^{3-x^2-y^2} x\,dz\,dy\,dx$

8. $\int_{1}^{2}\int_{z}^{2}\int_{0}^{\sqrt{3}y}\frac{y}{x^2+y^2}\,dx\,dy\,dz$

9–12 Evaluate the triple integral.

9. $\iiint\limits_G xy\sin yz\,dV$, where G is the rectangular box defined by the inequalities $0\le x\le\pi,\ 0\le y\le 1,\ 0\le z\le\pi/6$.

10. $\iiint\limits_G y\,dV$, where G is the solid enclosed by the plane $z=y$, the xy-plane, and the parabolic cylinder $y=1-x^2$.

11. $\iiint\limits_G xyz\,dV$, where G is the solid in the first octant that is bounded by the parabolic cylinder $z=2-x^2$ and the planes $z=0$, $y=x$, and $y=0$.

12. $\iiint\limits_G \cos(z/y)\,dV$, where G is the solid defined by the inequalities $\pi/6\le y\le\pi/2,\ y\le x\le\pi/2,\ 0\le z\le xy$.

15–18 Use a triple integral to find the volume of the solid.

15. The solid in the first octant bounded by the coordinate planes and the plane $3x+6y+4z=12$.

16. The solid bounded by the surface $z=\sqrt{y}$ and the planes $x+y=1$, $x=0$, and $z=0$.

17. The solid bounded by the surface $y=x^2$ and the planes $y+z=4$ and $z=0$.

18. The wedge in the first octant that is cut from the solid cylinder $y^2+z^2\le 1$ by the planes $y=x$ and $x=0$.

Change of variables

Exercise- 14.7- 1-12, 21-24, 35-37.

EXERCISE SET 14.7

1–4 Find the Jacobian $\partial(x,y)/\partial(u,v)$.

1. $x=u+4v,\ y=3u-5v$
2. $x=u+2v^2,\ y=2u^2-v$
3. $x=\sin u+\cos v,\ y=-\cos u+\sin v$
4. $x=\dfrac{2u}{u^2+v^2},\ y=-\dfrac{2v}{u^2+v^2}$

5–8 Solve for x and y in terms of u and v, and then find the Jacobian $\partial(x,y)/\partial(u,v)$.

5. $u=2x-5y,\ v=x+2y$
6. $u=e^{x},\ v=ye^{-x}$
7. $u=x^2-y^2,\ v=x^2+y^2\quad(x>0,y>0)$
8. $u=xy,\ v=xy^3\quad(x>0,y>0)$

9–12 Find the Jacobian $\partial(x,y,z)/\partial(u,v,w)$.

9. $x=3u+v,\ y=u-2w,\ z=v+w$
10. $x=u-uv,\ y=uv-uvw,\ z=uvw$
11. $u=xy,\ v=y,\ w=x+z$
12. $u=x+y+z,\ v=x+y-z,\ w=x-y+z$

21. Use the transformation $u=x-2y$, $v=2x+y$ to find

$$\iint\limits_R \frac{x-2y}{2x+y}\,dA$$

(cont.)

where R is the rectangular region enclosed by the lines $x-2y=1$, $x-2y=4$, $2x+y=1$, $2x+y=3$.

22. Use the transformation $u=x+y$, $v=x-y$ to find

$$\iint\limits_R (x-y)e^{x^2-y^2}\,dA$$

over the rectangular region R enclosed by the lines $x+y=0$, $x+y=1$, $x-y=1$, $x-y=4$.

23. Use the transformation $u=\frac{1}{2}(x+y)$, $v=\frac{1}{2}(x-y)$ to find

$$\iint\limits_R \sin\tfrac{1}{2}(x+y)\cos\tfrac{1}{2}(x-y)\,dA$$

over the triangular region R with vertices $(0,0)$, $(2,0)$, $(1,1)$.

24. Use the transformation $u=y/x$, $v=xy$ to find

$$\iint\limits_R xy^3\,dA$$

over the region R in the first quadrant enclosed by $y=x$, $y=3x$, $xy=1$, $xy=4$.

35–38 Evaluate the integral by making an appropriate change of variables.

35. $\displaystyle\iint_R \frac{y-4x}{y+4x}\,dA$, where R is the region enclosed by the lines $y=4x$, $y=4x+2$, $y=2-4x$, $y=5-4x$.

36. $\displaystyle\iint_R (x^2-y^2)\,dA$, where R is the rectangular region enclosed by the lines $y=-x$, $y=1-x$, $y=x$, $y=x+2$.

37. $\displaystyle\iint_R \frac{\sin(x-y)}{\cos(x+y)}\,dA$, where R is the triangular region enclosed by the lines $y=0$, $y=x$, $x+y=\pi/4$.

Book: Elementary Calculus- Howard Anton (10th Edition), Soft Copy