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MAT-120

Anika Ferdous — AFE

Howard Anthon [Integration]

Daniesh Zeel [Differential Integration]

[Central Exam]

UB9 [10th Floor]

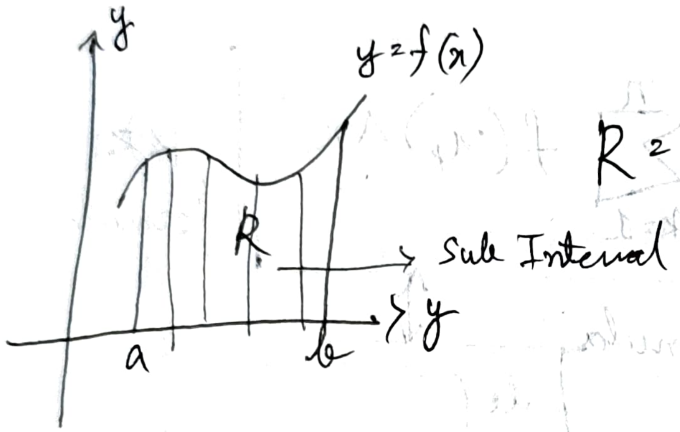
Quiz → 3/4 [n-1 will dee counted]

Howard Anthon

5.4 The Definition of Area And Limit

$\boxed{\text{Page - 343}}$

⊛ A definition of area



$R = ?$

Number of subinterval বেশি হলে Error কম হবে।

Area of Rectangle $= l \times w$

$= f(x_k) \times \Delta x$

$\Delta x = \dfrac{b-a}{n}$

$\boxed{\cancel{l \times w \times \Delta x}}$ Area

$\cancel{\Delta x \cdot f(x_k)}$

$$\text{Area} \approx \sum_{k=1}^{n} f(x_k)\,\Delta x$$

ডেল x ⇒ Δx

$\boxed{\Delta x \to 0}$ Target

when $n \to +\infty$

$$\boxed{\text{Area, } A = \lim_{n \to +\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x}$$

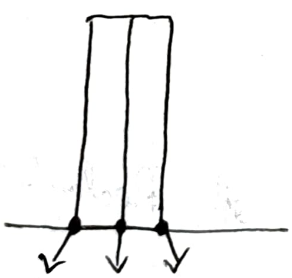
Under a curve

formula

Reimen Sum Formula

$$\int_a^b f(x)\,dx$$

Integration used for finding area under a curve



৩টা Point এ f(x) এর Value বের করা যায়

Left ~~[illegible]~~ end Point, Right end Point, Middle Point

সবগুলোতে একই Same কোনো Point

(*) (**) n এর value দেওয়া থাকলে $\Delta x = \frac{b-a}{n}$ formula use করবো।

$[a, b] \rightarrow$ interval এর Value দেওয়া থাকবে।

$\boxed{k \rightarrow 1 \text{ to } n}$

~~0+1=1~~

~~1+1=2~~

(**) n এর Value দেওয়া না থাকলে –

Left End Point $\Rightarrow$ $x_k = x_{k-1} = \boxed{a + (k-1)\Delta x}$

Right End Point $\Rightarrow$ $x_k = \boxed{a + k\Delta x}$

Mid Point $\Rightarrow$ $x_k = \frac{1}{2}(x_{k-1} + x_k)$

$$\boxed{= a + \left(k - \frac{1}{2}\right)\Delta x}$$

Summation / Sigma Notation $\Rightarrow \sum$

(a) $1+2+\cdots+n = \dfrac{n(n+1)}{2}$

(b) $1^2+2^2+\cdots+n^2 = \dfrac{n(n+1)(2n+1)}{6}$

$\boxed{5.4.4}$

Question: Use definition 5.4.3 with x_k as the midpoint of each subinterval to find the area under the parabola $y=f(x)=9-x^2$ and over the interval $[0,3]$

Solve:

$$\Delta x = \frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n}$$

~~length of the sub~~

$$x_k \text{ for Midpoint} = a+\left(k-\tfrac{1}{2}\right)\Delta x$$

$$= 0+\left(k-\tfrac{1}{2}\right)\frac{3}{n}$$

$$= \left(k-\tfrac{1}{2}\right)\frac{3}{n}$$

$$\boxed{\begin{aligned} f(x) &= x^2 \\ f(2) &= 2^2 = 4 \end{aligned}}$$

$$f(x_k)\,\Delta x = 9-(x_k)^2 \times \frac{3}{n} = \left[9-\left(k-\tfrac{1}{2}\right)^2 \times \frac{9}{n^2}\right] \times \frac{3}{n}$$

$$\boxed{f(x_k) = 9-(x_k)^2}$$

$$= \left[9-\left(k-\tfrac{1}{2}\right)^2 \times \frac{9}{n^2}\right] \frac{3}{n}$$

$$= \left[9-\left(k^2-k+\tfrac{1}{4}\right)\frac{9}{n^2}\right] \times \frac{3}{n}$$

$$= \frac{27}{n} - \frac{27}{n^3}k^2 + \frac{27k}{n^3} - \frac{27}{4n^3}$$

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{27}{n} - \frac{27}{n^3}k^2 + \frac{27k}{n^3} - \frac{27}{4n^3}\right)$$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} 27\left(\frac{1}{n} - \frac{k^2}{n^3} + \frac{k}{n^3} - \frac{1}{4n^3}\right)$$

$$= \lim_{n\to\infty} 27\left[\sum_{k=1}^{n}\frac{1}{n} - \sum_{k=1}^{n}\frac{k^2}{n^3} + \sum_{k=1}^{n}\frac{k}{n^3} - \sum_{k=1}^{n}\frac{1}{4n^3}\right]$$

n is not dependent upon k.

$$= \lim_{n\to\infty} 27\left[\frac{1}{n}\sum_{k=1}^{n} 1 - \frac{1}{n^3}\sum_{k=1}^{n} k^2 + \frac{1}{n^3}\sum_{k=1}^{n} k - \frac{1}{4n^3}\sum_{k=1}^{n} 1\right]$$

$$= \lim_{n\to\infty} 27\left[\frac{1}{n}\cdot n - \frac{1}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\cdot\frac{1}{n^3} - \frac{1}{4n^3}\cdot n\right]$$

$$= \lim_{n\to\infty} 27\left[1 - \frac{1}{n^2}\frac{(n+1)(2n+1)}{6} + \frac{n+1}{2n^2} - \frac{1}{4n^2}\right]$$

$$= \lim_{n\to\infty} 27\left[1 - \frac{1}{\cancel{n^2}}\frac{\cancel{n}(n+\frac{1}{n})\cdot\cancel{n}(2+\frac{1}{n})}{6} + \frac{1}{\cancel{n^2}\, n}\cdot\frac{\cancel{n}(1+\frac{1}{n})}{2} - \frac{1}{4n^2}\right]$$

$$= \lim_{n\to\infty} 27\left[1 - \frac{(1+\frac{1}{n})(2+\frac{1}{n})}{6} + \frac{1+\frac{1}{n}}{2n} - \frac{1}{4n^2}\right]$$

$$= 27\left[1 - \frac{(1+0)(2+0)}{6} + 0 - 0\right]$$

$$= 27\left[1 - \frac{\cancel{2}1}{\cancel{6}3}\right] = 27 - \frac{\cancel{27}^{9}}{\cancel{3}} = 27 - 9$$

$$= 18$$

$$\int_a^b f(x)\,dx$$

$$= \int_0^3 (9-x^2)\,dx$$

$$= \int_0^3 9\,dx - \int_0^3 x^2\,dx$$

$$= 9\left[x\right]_0^3 - \left[\frac{x^3}{3}\right]_0^3$$

$$= 9(3-0) - \frac{1}{3}(27-0)$$

$$= 27 - 9 = 18$$

Ans: 18.

Formula

$$\boxed{\begin{array}{l} \int dx = x \\ \int x^n\,dx = \dfrac{x^{n+1}}{n+1} \end{array}}$$

$n = 3$

$$\Delta x = \frac{3-0}{3} = 1$$

n এর Value define করা হলে Error বেশি হবে।

এখন প্রত্যেক end Point এর Result প্রত্যেকটির আলাদা।

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5.2

ANTIDERIVATIVES

$y = f(x) \rightarrow \underline{F(x)}$

antiderivatives

$\boxed{F'(x) = f(x)}$

$F(x) = \frac{1}{3}x^3$ is an antiderivative of $f(x) = x^2$ on the interval $(-\infty, +\infty)$ because for each x in the interval

Sol: $F(x) = \frac{1}{3}x^3 + c$

$F'(x) = \frac{d}{dx}\left[\frac{1}{3}x^3\right]$

$= \frac{3}{3}x^{3-1} = x^2 = f(x)$

Reverse Process Proof:

Anti deferenciation means Integration

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$F'(x) = f(x)$

$\Rightarrow \frac{d}{dx}\left[F(x)\right] = f(x)$

$\Rightarrow \int \frac{d}{dx}\left[F(x)\right]dx = \int f(x)\,dx$

$\Rightarrow F(x) = \int f(x)\,dx$

Open interval

$\circledast$ $\int x^2 dx = \frac{1}{3}x^3 + c$ [Indefinite Integral এ always c add করতে হবে]

Table 5.2.1

Definite Integral

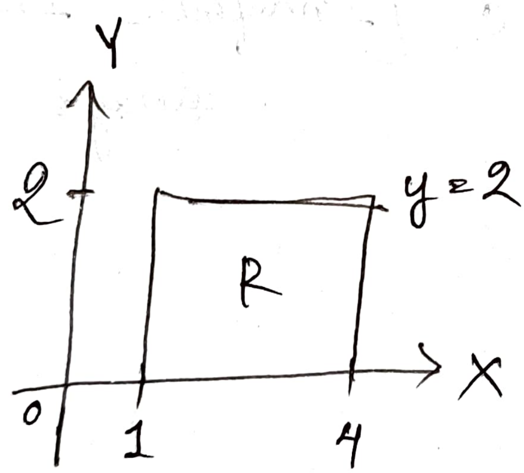
Use Geometric formula to find areas

Q: Sketch the region whose area is represented by the definite integral and evaluate the integral using an appropriate ~~form~~ formula from geometry.

(a) $\int_1^4 2\,dx$

$f(x) = y$

$\therefore y = 2$



Area of Rectangle $= L \times W$

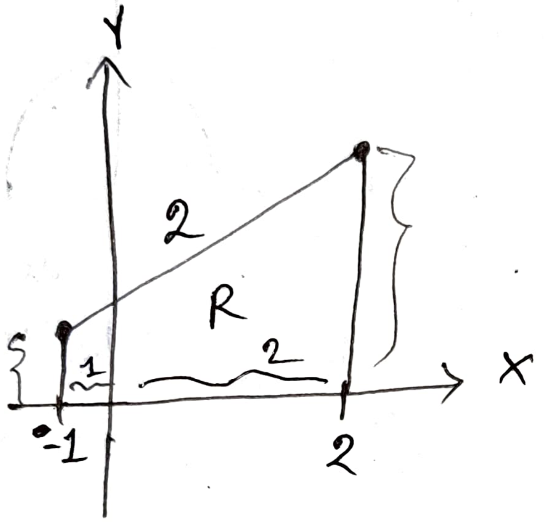
$= 2 \times (4-1)$

$= 2 \times 3$

$= 6$

(k) $\int_{-1}^{2} (x+2)\,dx$

Solve:



Area of Trapezoid $= \frac{1}{2}$(Sum of two parallel sides × distance between two parallel sides)

$= \frac{1}{2}(4+1) \times 3 = \frac{15}{2}$

$y = x+2$

$x = 2 \quad y = 4$

$x = -1 \quad y = 1$

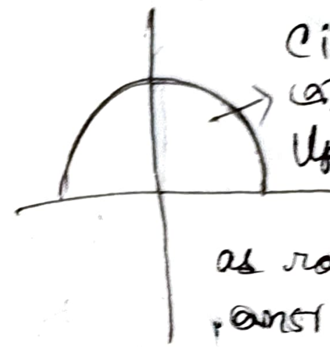
$$= \int_0^1 \sqrt{1-x^2}\, dx$$

$y = \sqrt{1-x^2}$

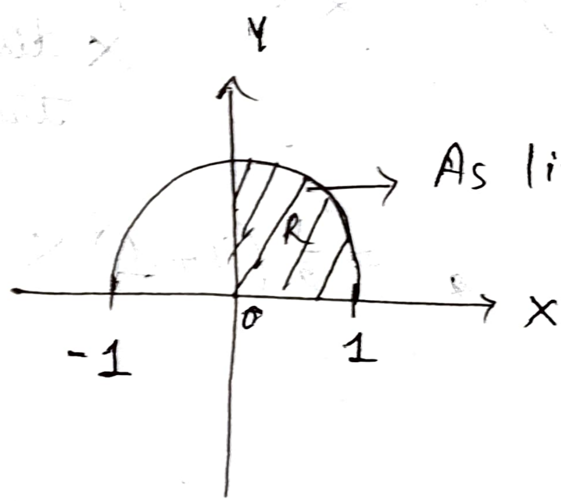
$\therefore y^2 = 1 - x^2$

$\therefore x^2 + y^2 = 1 \rightarrow y = \pm\sqrt{1-x^2}$

$r = 1, \quad (h, k) = (0, 0)$



Circle এর Upper half as root এর আগে Minus sign নাই



As limit (0 - 1)

Area of Circle

$$\frac{1}{4}(\text{area of Circle}) = \frac{1}{4}\pi r^2 = \frac{1}{4} \times \pi \times 1^2$$

$$= \frac{\pi}{4}$$

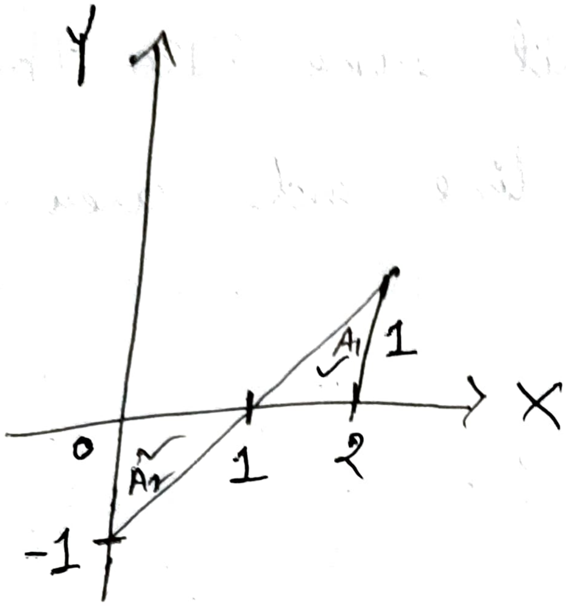
Ans: $\frac{\pi}{4}$.

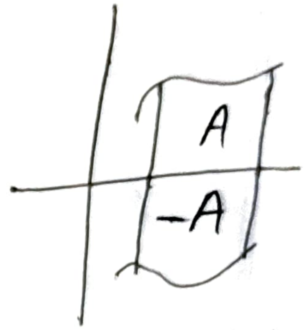
Ex. $\int_0^2 (x-1)\,dx$

x Intercept,

$y=0,\ x-1=0$

$\therefore x=1$





$A_1 - A_2$ [Negative sign for being below x-axis]

$y=x-1$
$x=2$ হলে,
$y=1$

Base $= 2-1=1$

$A_1 - A_2 = \frac{1}{2}(2-1)\times 1 - \left\{\frac{1}{2}(1-0)\times|-1|\right\}$

$= \frac{1}{2} - \frac{1}{2}$

$= 0$

~~Though~~ Net area 0

5.5.3

Upper and lower limit same হলে that will be a straight line and area will be 0.

5.5.5

5.6 [The Fundamental Theorem of Calculus]

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Indefinite Integrals

$$\int f(x)\,dx = F(x) + C$$

$$\int_a^b f(x)\,dx = \Big[F(x)+C\Big]_a^b = F(b) + \cancel{C} - F(a) - \cancel{C}$$

$$= F(b) - F(a)$$

Fundamental Theorem এর জন্য Definite Integral এ (+C) Cancle হয়ে যায়।

5.3 Integration By Substitution

Chain Rule Concept of Deferentiation

$$\frac{d}{dx}[F\, g(x)] = F'(g(x))\, g'(x)$$

$$\frac{d}{dx}\left[F(g(x))\right] = F'(g(x))\, g'(x)$$

$$\Rightarrow \int \frac{d}{dx}\left[F(g(x))\right]dx = \int F'(g(x))\, g'(x)\,dx$$

$$\Rightarrow \int F'(g(x))g'(x)\,dx = F(\underbrace{g(x)}_{u}) + c$$

$$\Rightarrow \int F'(u)\,du = F(u) + c$$

$$\Rightarrow \int f(u)\,du = F(u) + c$$

Let,

$u = g(x)$

$\frac{du}{dx} = g'(x)$

$\therefore du = g'(x)dx$

$F'(x) = f(x)$

Ex: Evaluate:

$$\int x^2\sqrt{x-1}\,dx$$

Solve: Let, $u = x-1 \Rightarrow x = u+1$

$x^2 = (u+1)^2$

$= u^2+2u+1$

$\Rightarrow \frac{du}{dx} = 1$

$\Rightarrow du = dx$

$$\int (u^2+2u+1)\sqrt{u}\,du$$

$$= \int (u^2+2u+1)\,u^{\frac{1}{2}}\,du$$

$$= \int \left(u^{\frac{5}{2}}+2u^{\frac{3}{2}}+u^{\frac{1}{2}}\right)du$$

$$= \frac{u^{\frac{5}{2}+1}}{\frac{5}{2}+1} + 2\frac{u^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$= \frac{2}{7}u^{\frac{7}{2}} + \frac{4}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}} + c$$

$$= \frac{2}{7}(x-1)^{\frac{7}{2}} + \frac{4}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + c$$

Ans:

5.6.23

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Integration By Substitution:

(*) $\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

$e^{2x} = (e^x)^2$

Solve:

Let, $u = e^x$

$\Rightarrow du = e^x\,dx$

$\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

$= \int \frac{du}{\sqrt{1-u^2}}$

$= \sin^{-1} u + c$

$= \sin^{-1}(e^x) + c$

Ans: $\sin^{-1}(e^x) + c$.

(*) $\int_0^{\pi/8} \sin^5 2x \cos 2x \, dx$ [Substitution method or Reduction method]

Usually Power এর স্থলে u দিয়ে Replace করি।

Let $u = \sin 2x$

$\Rightarrow du = 2\cos 2x \, dx$

x	0	$\pi/8$
u	0	$\frac{1}{\sqrt{2}}$

$\int_0^{1/\sqrt{2}} u^5 \left(\frac{1}{2}\right) du$

$= \frac{1}{2} \int_0^{1/\sqrt{2}} u^5 \, du$

$= \frac{1}{2} \left[\frac{u^6}{6}\right]_0^{1/\sqrt{2}} = \frac{1}{2}\left[\frac{\left(\frac{1}{\sqrt{2}}\right)^6}{6} - 0\right]$

$= \cancel{\frac{1}{96}} \quad \frac{1}{2} \times \frac{1}{48}$

$= \frac{1}{96}$

Ans. $\frac{1}{96}$.

Chapter-7 [Integration By Parts]

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Liate

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Formula for Integration By Parts

$$\int \underbrace{f(x)}_{u}\,\underbrace{g(x)\,dx}_{dv}$$

⊛ যে Function কে Defferentiate করা Easier সেগুলো U ধরবো।

LIATE → Method

L → Log

I → Inverse Trigono

A → Algebric

T → Trigono

E → Exponential

(*) $\int \underset{u}{\underline{x^2}}\, \underset{dv}{\underline{e^{-x} dx}}$ [Have to find v, du]

Let, $u = x^2$ and $\underline{e^{-x} dx = dv}$

$\Rightarrow du = 2x\,dx$ $\Rightarrow \int dv = \int e^{-x} dx$

$\therefore v = -e^{-x}$

$\int x^2 e^{-x} dx$

$= -x^2 e^{-x} + 2\int \underline{e^{-x} x\,dx}$ —(A) [Evaluate With Integration By Parts]

$\int x e^{-x} dx$

$u = x$ $dv = e^{-x}$

$du = dx$ $v = -e^{-x}$

$\int x e^{-x} dx = -xe^{-x} + \int e^{-x} dx$

$= -xe^{-x} - e^{-x}$

$= -x^2 e^{-x} + 2\left[-xe^{-x} - e^{-x}\right] + c$

$= -x^2 e^{-x} - 2xe^{-x} - 2e^{-x} + c = -e^{-x}(x^2 + 2x + 2) + c$

(Ans)

7.5 Integrating Rational Function By Partial Fractions

Rational Function is Ratio of 2 Polynomial Functions.

আংশিক ভগ্নাংশ [Partial Fraction]

$$\boxed{R(x) = \frac{P(x)}{Q(x)}}$$

~~Neu~~ Numerator
Denominator

In the rational function ~~Partial~~ Proper or Improper?
Identify them.

$$\boxed{P(x) < Q(x)}$$

Proper Rational Function:

$$\frac{x+2}{x^2+2x+3}, \quad \frac{x+2}{(x-1)(x+3)(x-4)}$$

For these, Partial Function is applicable.

(*) Is $Q(x)$ Linear or Quadratic ?

For Linear:

$(ax+b)^m$

For Quadratic;

$(ax^2+bx+c)^m$

$\boxed{ax+b}$ is the equation of straight line where $x=1$.

Linear:

$$\frac{}{(ax+b)^m} = \frac{A_1}{ax+b} + \frac{A_2}{(ax+b_2)^2} + \dots + \frac{A_m}{(ax+b)^m}$$

Quadratic:

$$\frac{}{(ax^2+bx+c)^m} = \frac{A_1x+B_1}{ax^2+bx+c} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \dots + \frac{A_mx+B_m}{(ax^2+bx+c)^m}$$

System of Equation In Calculator

Evaluate:

$$\int \frac{2x+4}{x^3-2x^2}\,dx$$

$$= \int \frac{2x+4}{x^2(x-2)}\,dx$$

$x^2 \to (ax+b)^2$

$= (1\cdot x+0)^2$

$= \frac{A}{ax+b} + \frac{B}{(ax+b)^2}$

$$\frac{2x+4}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$$

$$= \frac{-2}{x} - \frac{2}{x^2} + \frac{2}{x-2}$$

$\Rightarrow 2x+4 = Ax(x-2) + B(x-2) + Cx^2$

$\Rightarrow 2x+4 = Ax^2 - 2Ax + Bx - 2B + Cx^2$

$\Rightarrow 0\cdot x^2 + 2x + 4 = x^2(A+C) + x(B-2A) - 2B$

Equating the Co-efficient of Both sides.

$A+C=0$	$B-2A=2$	$-2B=4$
$C=2$	$B = 2+2A$	$\therefore B=-2$
	$2A = -2-2$	
	$\Rightarrow 2A = -4$	
	$\therefore A = -2$	

$$\int \frac{2x+4}{x^2(x-2)}\,dx$$

$$= -\int \frac{2}{x}\,dx - \int \frac{2}{x^2}\,dx + \int \frac{2}{x-2}\,dx$$

$$= -2\ln|x| + \frac{2}{x} + 2\int \frac{1}{\underbrace{x-2}_{u}}\,dx$$

$$= -2\ln|x| + \frac{2}{x} + 2\ln|x-2| + c$$

(Ans)

$\int \frac{1}{x}\,dx = \ln|x|$

Always positive as logarithm function's Domain 0 to ∞

$x-2 = u$

$dx = du$

$\int \frac{du}{u} = \ln|u|$

$= \ln|x-2)$

Quadratic Factor:

$$\frac{A_1 x + B_1}{ax^2+bx+c} \begin{matrix} \rightarrow \text{Linear Form} \\ \rightarrow \text{Quadratic Form} \end{matrix}$$

7.1 Formula

* $\cancel{\int \frac{x^2+x-2}{3x^3-x^2+3x-1}}$

* $\int \frac{x}{x^2+1}\,dx$

$$\boxed{\begin{aligned} u &= x^2+1 \\ du &= 2x\,dx \\ \therefore x\,dx &= \tfrac{1}{2}du \end{aligned}}$$

$$= \int \frac{\frac{1}{2}du}{u}$$

$$= \frac{1}{2}\int \frac{du}{u}$$

$$= \frac{1}{2}\ln|u|$$

$$= \frac{1}{2}\ln|x^2+1|$$

Long division formula দিয়ে Improper কে Proper এ Convert করতে হবে।

7.8. Improper Integrals।

Discontinuous

(i) define থাকবে

(ii) Limit exist নাও থাকবে।

(iii) Limit আর derivative equal থাকবে

Infinite integral & discontinuous

(i) $\int_{1}^{+\infty} \frac{dx}{x^2}$ $\int_{-\infty}^{0} e^x dx$ $\int_{-\infty}^{+\infty} \frac{dx}{1+x^2}$ [limit infinite]

(ii) $\int_{-3}^{3} \frac{dx}{x^2}$, $\int_{1}^{2} \frac{dx}{x-1}$ $\int_{0}^{\pi} \tan x \, dx$

[limit finite]

[but there is discontinuity]

(iii) $\int_{0}^{+\infty} \frac{dx}{\sqrt{x}}$, $\int_{-\infty}^{+\infty} \frac{dx}{x^2-9}$, $\int_{1}^{+\infty} \sec x \, dx$

[combination of (i) & (ii)]

Integrals over Infinite Intervals:

Infinity limit কে finite limit দিয়ে Replace করবো।

$$\boxed{\int_a^{+\infty} f(x)\,dx = \lim_{b\to\infty} \int_a^{b} f(x)\,dx}$$

Finite value → converge

Infinite value → diverge [There is no specific value]

(ক) $\int_a^b \frac{1}{x^3}\,dx$

$= \int_a^b x^{-3}\,dx$

$= \left[\frac{x^{-3+1}}{-3+1}\right]_1^b$

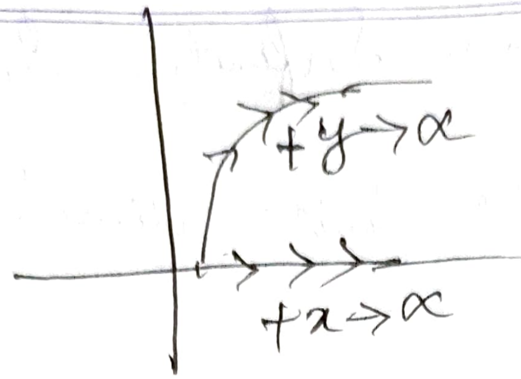
$= \left[\frac{-1}{2x^2}\right]_1^b$

$= \left[-\frac{1}{2b^2} + \frac{1}{2}\right]_1^b$

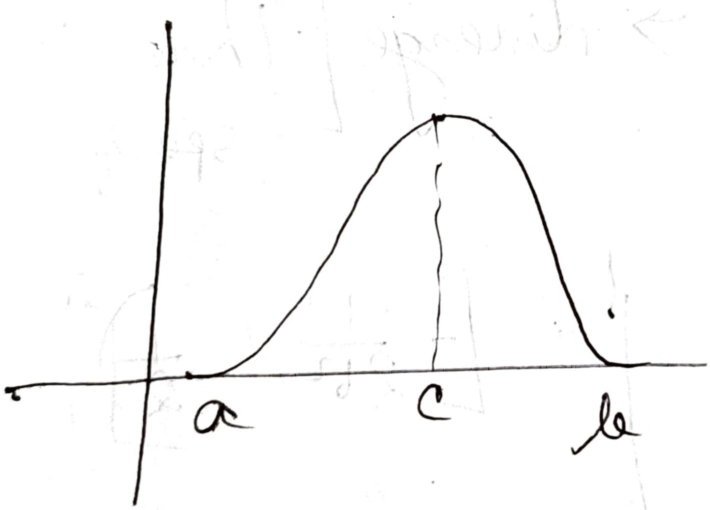
$= \left[-\cancel{\frac{1}{\infty}}^{\,0} + \frac{1}{2}\right] = \left[-\cancel{\frac{1}{2}} + \frac{1}{2}\right]$

$= \frac{1}{2}$

Ans: converges and it's value is $\frac{1}{2}$



$$\boxed{\int_{-\infty}^{b} f(x)\,dx = \lim_{a \to -\infty} \int_a^b f(x)\,dx}$$



$$\boxed{\int_a^c + \int_c^b = \int_a^b}$$ $\Longrightarrow$ Apply when 2 limits are infinity

$$\boxed{\int_{-\infty}^{+\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_{c}^{+\infty} f(x)\,dx}$$

c এর Value এমন নেয়া যাবে না যাতে Function Discontinuous হয়ে যায়।

2 converge → converge

1 diverge → diverge

$$\int_{-\infty}^{+\infty} \frac{dx}{1+x^2}$$

$\tan(-x) = -\tan x$

$$= \int_{-\infty}^{0} \frac{dx}{1+x^2} + \int_{0}^{+\infty} \frac{dx}{1+x^2}$$

$$= \lim_{a\to-\infty} \int_{a}^{0} \frac{dx}{1+x^2} + \lim_{b\to\infty} \int_{0}^{b} \frac{dx}{1+x^2}$$

$$= \lim_{a\to-\infty} \left[\tan^{-1}x\right]_{a}^{0} + \lim_{b\to\infty} \left[\tan^{-1}x\right]_{0}^{b}$$

$$= \lim_{a\to-\infty} \left[\tan^{-1}(0) - \tan^{-1}(a)\right] + \lim_{b\to\infty} \left[\tan^{-1}(b) - \tan^{-1}(0)\right]$$

$$= \lim_{a\to-\infty} \left[-\tan^{-1}(a)\right] + \lim_{b\to\infty} \left[\tan^{-1}(b)\right]$$

$$= -\tan^{-1}(-\infty) + \tan^{-1}(\infty)$$

$$= \frac{\pi}{2} + \frac{\pi}{2}$$

$$= 2\frac{\pi}{2} = \pi$$

~~the integral~~ Converges and its value is π.

Ans: π.

Case-1 [Interval finite] {Upper limit a discontinuity}

* যেখানে discontinuity থাকবে সেখানে Replace করবে।

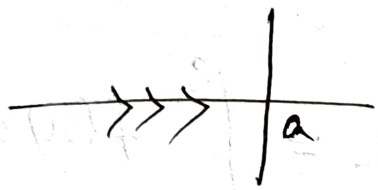
$$\boxed{\int_a^b f(x)\,dx = \lim_{k \to b^-} \int_a^k f(x)\,dx}$$

Concept of one sided limit b^-, a^-

Here given that –

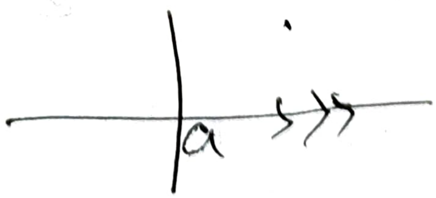
for this function discontinuity at b so, b is replaced by k.

$$\lim_{x \to a^-} f(x) =$$



a^- means a এর কম value গুলো a এর দিকে approach করবে।

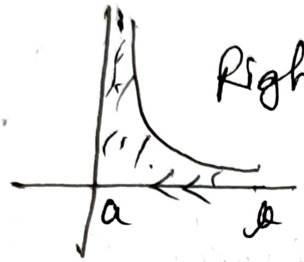
$$\lim_{x \to a^+} f(x) =$$



a^- means values less than a.
a^+ means values greater than a.

Case - 2 [Lower limit is Discontinuity]

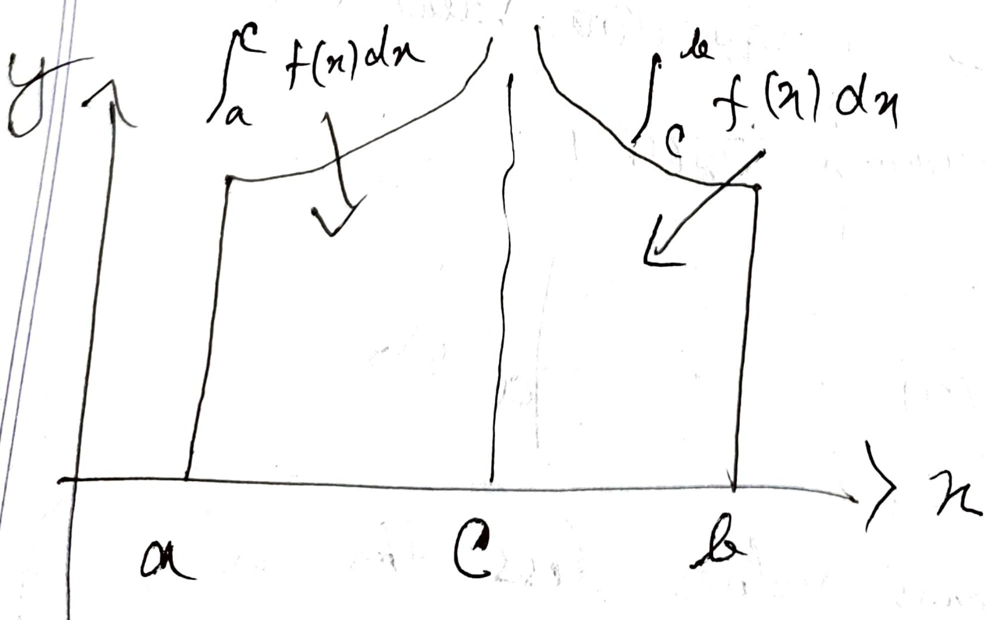
$$\int_a^b f(x)\,dx = \lim_{k \to a^+} \int_k^b f(x)\,dx$$



Right side Limit

Case - 3 interval [a, b] discontinuity at point c

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$



(*) $\int_1^4 \frac{dx}{(x-2)^{2/3}}$ [$2-2=0$, so it is discontinuous at 2]

$= \int_1^2 \frac{dx}{(x-2)^{2/3}} + \int_2^4 \frac{dx}{(x-2)^{2/3}}$

$= \lim_{k \to 2^-} \int_1^k \frac{dx}{(x-2)^{2/3}} + \lim_{k \to 2^+} \int_k^4 \frac{dx}{(x-2)^{2/3}}$

Substitution method

$u = x-2$

$du = dx$

$\int \frac{du}{u^{2/3}}$

$= \left[\frac{u^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} \right]$

$= \frac{u^{1/3}}{1/3} = 3u^{1/3}$

$$= \lim_{k\to 2^-} \left[3(x-2)^{1/3}\right]_1^k + \lim_{k\to 2^+} \left[3(x-2)^{1/3}\right]_k^4$$

$$= \lim_{k\to 2^-} \left[3(k-2)^{1/3} - 3(1-2)^{1/3}\right] + \lim_{k\to 2^+} \left[3(4-2)^{\frac{1}{3}} - 3(k-2)^{1/3}\right]$$

$$= \lim_{k\to 2^-} \left[3(k-2)^{1/3} + 3\right] + \lim_{k\to 2^+} \left[3\cdot 2^{\frac{1}{3}} - 3(k-2)^{1/3}\right]$$

$$= 3(2-2)^{1/3} + 3 + 3\cdot 2^{\frac{1}{3}} - 3(2-2)^{\frac{1}{3}}$$

$$= 3 + 3\cdot(2)^{\frac{1}{3}}$$

$$= 6.78$$

~~Ans~~ $\therefore$ The integral converges and its value is 6.78.