# Package ‘ahpsensitivity’

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**Title** Sensitivity analysis tools for Multicriteria Decision Making methods

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**Depends** R (>= 3.6.2)

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**Description** Contains a set of sensitivity analysis tools for two Multicriteria Decision Making (MDCM) methods: Analytic Hierarchy Process (AHP) and Weighted Sum Model (WSM). For the AHP, the probability of rank reversals and the most influential user can be obtained. Moreover, for both methods, the most critical measure of performance can be determined.

**License** GPL-3.0

**Imports** pracma,
matrixStats

**Encoding** UTF-8

**LazyData** true

**RoxygenNote** 7.0.2.9000

## R topics documented:

ahp.interval *A function to obtain the probabilities of rank reversals*

## Description

A function to obtain the probabilities of rank reversals

## Usage

```
ahp.interval(lower, upper, nmatrices, norm_test)
```

## Arguments

| | |
|---|---|
| lower | A list of dataframes objects with the lower bounds for each pairwise comparison matrix |
| upper | A list of dataframes objects with the upper bounds for each pairwise comparison matrix |
| nmatrices | Integer, defines the total number of random matrices to generate |
| norm_test | Logical, if TRUE determines if the components of the right eigenvector of all the generated random pairwise comparison matrices are normally distributed, via the kolmogorov-smirnoff test |

## Value

A list of list, for each interval pairwise comparison matrix

**matrix_list** List of random matrices (rm)

**matrices_w** List of weight of rm

**norm_matrices_w** List of normalized weights of rm

**lambda_max** List of largest eigenvalues

**ci** List of consistency indices

**cr** List of consistency ratios

**w_consistent** Dataframe of normalized weights of consistent random matrices (crm)

**maximum** Named vector, maximum value of the crm for each criteria

**minimum** Named vector, minimum value of the crm for each criteria

**mean** Named vector, mean value of the crm for each criteria

**sd** Named vector, standard deviation value of the crm for each criteria

**normality** List of list with the normality test for each criteria

**p_ij** Vector, probabilities of rank reversal between two alternatives $A_i$ and $A_j$

**p_i** Vector, probabilities that a given alternative will reverse rank with other alternative

## Details

The probabilities $p_i, i = 1, \ldots, n$ that a given alternative will reverse rank with another alternative are given by

$$p_i = 1 - \prod_{j=1}^{n}(1 - p_{ij})$$

where $p_{ij}$ is the probability of rank reversal for two alternatives, it is calculated considering different cases for the lower and upper bounds on the $i$ and $j$ components of the right eigenvector $w$ and the probability cumulative distribution $F_i(x_i)$:

| Case | Condition | $p_{ij}$ |
|---|---|---|
| 1 | $w_j^L \leq w_i^L$ and $w_i^U \leq w_j^U$ | $F_j(w_i^U) - F_j(w_i^L)$ |
| 2 | $w_i^L < w_j^L$ and $w_j^U < w_i^U$ | $F_i(w_j^U) - F_i(w_j^U)$ |
| 3 | $w_i^L < w_j^L < w_i^U < w_j^U$ | $(F_i(w_i^U) - F_i(w_j^L))(F_j(w_i^U) - F_j(w_j^L))$ |
| 4 | $w_j^L < w_i^L < w_j^U < w_i^U$ | $(F_i(w_j^U) - F_i(w_i^L))(F_j(w_j^U) - F_j(w_i^L))$ |

## References

Saaty, Thomas L. y Vargas, Luis G.: Uncertainty and rank order in the analytic hierarchy process. European Journal of Operational Research, 1987, 32(1), pp. 107–117. ISSN 0377- 2217. doi: 10.1016/0377-2217(87)90275-X.

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`consistency.index` *A function to obtain the consistency index*

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## Description

A function to obtain the consistency index

## Usage

```
consistency.index(lambda_max, n)
```

## Arguments

| | |
|---|---|
| `lambda_max` | largest eigenvalue |
| `n` | total number of criteria |

## Details

$$C.I. = \frac{\lambda_{max} - n}{n - 1}$$

where $\lambda_{max}$ is the largest eigenvalue and $n$ represents the total number of criteria

## Examples

```
consistency.index(lambda_max = 1, n = 6)
```

consistency.ratio *A function to obtain the consistency ratio*

### Description

A function to obtain the consistency ratio

### Usage

```
consistency.ratio(ci, ri)
```

### Arguments

| | |
|---|---|
| ci | Consistency index |
| ri | Random inconsistency |

### Details

$$C.R. = \frac{C.I.}{R.I.}$$

### Note

The random inconsistency number is a function of the number of criteria

### Examples

```
consistency.ratio(.05, 0.90)
```

group.sens *Group sensitivity*

### Description

This function allows you to identify the most influential expert on the group

### Usage

```
group.sens(users_list, pert)
```

### Arguments

| | |
|---|---|
| users_list | List with the pairwise comparison matrices |
| pert | Double, the perturbation magnitude for the finite differences method |

### Value

A list of lists for each user

**users_list** List with the pairwise comparison matrices

**n** Number of criteria

**users_total** Integer, the total number of users

**pert** Double, the perturbation magnitude for the finite differences method

**users_w** List with weights for the pairwise comparison matrices

**norm_users_w** List with the normalized weights for the pairwise comparison matrices

**group_matrix** Matrix with the normalized weight for all criteria (rows: criteria, columns: users)

**sensitivity** List of matrices with the sensitivity coefficients for each user

### References

Marie Ivanco, Gene Hou, Jennifer Michaeli, Sensitivity analysis method to address user disparities in the analytic hierarchy process, Expert Systems with Applications, Volume 90, 2017, Pages 111-126, ISSN 0957-4174, https://doi.org/10.1016/j.eswa.2017.08.003.

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`group.vector` *Create a group vector*

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### Description

This function allows you to create the group vector

### Usage

```
group.vector(x, users_total)
```

### Arguments

| | |
|---|---|
| `x` | a list of matrices with the normalized weight for all criteria and users (rows: critera, columns: user) |
| `users_total` | the total number of users in the group |

### Value

A list of vectors

### Details

The group aggregated vector is obtained by following

$$G_r^{(i,j_{i-1})} = \sqrt[P]{\bar{w}_r^1 \bar{w}_r^2 \dots \bar{w}_r^P}$$

where $P$ represents the total number of users in the group and $\bar{w}_r$ is the normalized weight of the rth criterion

## References

Marie Ivanco, Gene Hou, Jennifer Michaeli, Sensitivity analysis method to address user disparities in the analytic hierarchy process, Expert Systems with Applications, Volume 90, 2017, Pages 111-126, ISSN 0957-4174, https://doi.org/10.1016/j.eswa.2017.08.003.

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`norm.fun` *The normalized criteria weight*

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## Description

This function allows you to obtain the normalized criteria weight (normalized geometric mean) for one user

## Usage

```
norm.fun(x)
```

## Arguments

| | |
|---|---|
| `x` | Vector of criteria weight |

## Value

A vector

## Details

For each component $r$ of the vector the normalized weight is obtained by

$$\bar{w_r} = \frac{w_r}{\sum_{q=1}^{n} w_q}$$

where $n$ is the total number of criteria

## Examples

```
w <- c(.7,.2,.3)
norm.fun(w)
```

`pcmatrix` *A function to create a pairwise comparison matrix*

## Description

A function to create a pairwise comparison matrix

## Usage

```
pcmatrix(x, n)
```

## Arguments

| | |
|---|---|
| `x` | Vector, it contains the upper triangular elements of the matrix |
| `n` | Integer, total number of criteria |

## Details

Given a vector with the upper triangular elements of a pairwise comparison matrix, this function returns a matrix in the pairwise comparison form

$$C = \begin{bmatrix} 1 & a_{12} & \dots & a_{1m} \\ 1/a_{12} & 1 & \dots & a_{2m} \\ \dots & \dots & 1 & \dots \\ 1/a_{1m} & 1/a_{2m} & \dots & 1 \end{bmatrix}$$

## Examples

```
pcmatrix(c(1,2,3,4,5,6), 4)
```

`random.pcmatrices` *A function to obtain random matrices*

## Description

A function to obtain random matrices

## Usage

```
random.pcmatrices(data_lower, data_upper, nmatrices)
```

## Arguments

| | |
|---|---|
| `data_lower` | A list of dataframes objects with the lower bounds for each pairwise comparison matrix |
| `data_upper` | A list of dataframes objects with the upper bounds for each pairwise comparison matrix |
| `nmatrices` | Integer, total number of random matrices to generate |

### Value

A list of matrices

### Details

Given matrices

$$C^L = \begin{bmatrix} 1 & a^L_{12} & \dots & a^L_{1m} \\ 1/a^L_{12} & 1 & \dots & a^L_{2m} \\ \dots & \dots & 1 & \dots \\ 1/a^L_{1m} & 1/a^L_{2m} & \dots & 1 \end{bmatrix}, \quad C^U = \begin{bmatrix} 1 & a^U_{12} & \dots & a^U_{1m} \\ 1/a^U_{12} & 1 & \dots & a^U_{2m} \\ \dots & \dots & 1 & \dots \\ 1/a^U_{1m} & 1/a^U_{2m} & \dots & 1 \end{bmatrix}$$

with the lower and upper bounds of an interval pairwise comparison matrix, this function generates a set of random matrices within the interval. The random numbers for each comparison are generated with an uniform distribution.

### References

Saaty, Thomas L. y Vargas, Luis G.: Uncertainty and rank order in the analytic hierarchy process. European Journal of Operational Research, 1987, 32(1), pp. 107–117. ISSN 0377- 2217. doi: 10.1016/0377-2217(87)90275-X.

### See Also

runif about the uniform distribution

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## `weights.matrix` *A function to obtain the criteria weight*

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### Description

Given a pairwise comparison matrix this function allows you to obtain the criteria weight (row geometric mean) for one user and any number of criteria

### Usage

```
## S3 method for class 'matrix'
weights(x, n)
```

### Arguments

| | |
|---|---|
| `x` | a pairwise comparison matrix |
| `n` | Integer, the total number of criteria |

### Details

The weight criteria is obtained by the computation of the geometric mean

$$\bar{w}_r^{P_j} = \sqrt[m]{a_{r1} a_{r2} \dots a_{rm}}$$

where $a_{rm}$ are elements in the pairwise comparisons matrix

$$C = \begin{bmatrix} 1 & a_{12} & \dots & a_{1m} \\ 1/a_{12} & 1 & \dots & a_{2m} \\ \dots & \dots & 1 & \dots \\ 1/a_{1m} & 1/a_{2m} & \dots & 1 \end{bmatrix}$$

### References

Marie Ivanco, Gene Hou, Jennifer Michaeli, Sensitivity analysis method to address user disparities in the analytic hierarchy process, Expert Systems with Applications, Volume 90, 2017, Pages 111-126, ISSN 0957-4174, https://doi.org/10.1016/j.eswa.2017.08.003.

### Examples

```
pcm <- pcmatrix(c(1,2,3,4,5,6), 4)
weights.matrix(pcm, 4)
```

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## wsm.tau    *A function to obtain the most critical measure of performance in terms of some reference value for a set of indices*

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### Description

A function to obtain the most critical measure of performance in terms of some reference value for a set of indices

### Usage

```
wsm.tau(x_ij, w_ij, indices, rho)
```

### Arguments

| | |
|---|---|
| x_ij | A list of matrices with the standarized measures of performance for each index $i$ and each criteria $j$ |
| w_ij | A list of matrices with the importance weights for each criteria $j$ |
| indices | A list of dataframes with the preferences for each alternative |
| rho | A list with the references values for each index |

### Details

For each index $i$, the minimum change in a measure of performance $x_{ij}^h$ for a criteria $j$ that generates a rank reversal is obtained with

$$\tau_{ij}^h = \frac{V_i^h - V_i^\rho}{w_{ij} x_{ij}^h}$$

In terms of $\tau_{ij}^h$, the critical indicator value

$$\mathcal{C}_{ij} = \frac{1}{\Delta_{ij}} \times p_{ij}$$

considers the sensitivity coefficient $\Delta_{ij} = |\tau_{ij}^h|^{Q_1}$ (first quartile $Q_1$) and the probability of rank reversals $p_{ij}$. Moreover, the modified measure of performance is given by

$$\hat{x}_{ij}^h = x_{ij}^h - \tau_{ij}^h$$

**References**

Sensitivity analysis for household vulnerability assessment: a case of study from Brazil surveys

Triantaphyllou, Evangelos y Sánchez, Alfonso: A Sensitivity Analysis Approach for Some Deterministic Multi-Criteria Decision-Making Methods*. Decision Sciences, 1997, 28(1), pp. 151–194. doi: 10.1111/j.1540-5915.1997.tb01306.x. https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-5915.1997.tb01306.x

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