Initial Sizing of 2020 ICLR hybrid rocket

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1 Introduction

This document provides an overview of the sizing analysis and results for the 10k SRAD hybrid entry by Imperial College London Rocketry.

This is a working document, and as more information is fed into the system, the parameters may be updated.

2 Flight performance requirement

The first challenge was to estimate the performance requirements of the hybrid rocket to able to attain the target altitude with some margin. A separate document details the analysis, but a summary is provided here.

We assume the rocket follows a bang-off control scheme - the rocket thrusts at its main engine's max thrust, F_{max} Newtons for t_{burn} seconds and then coasts the remaining part of the journey. In this scenario, the final altitude of the rocket, $h_f = \hat{h}_f c^2/g$, is entirely dependent on four non-dimensional parameters[1]:

1. The thrust to initial weight ratio, $\hat{F} = (F_{max})/(m_0 g)$
2. A drag parameter defined as[2], $x \equiv (\frac{1}{2}\rho c^2 c_d A)/(m_0 g)$
3. Propellant mass fraction, $MR \equiv m_p/m_0$
4. Atmosphere parameter, $\hat{\beta} = \beta c^2/g$

Plugging in suitable parameters, we find the required propellant mass fraction as a function of the thrust to weight ratio, as in figure 1.

From this, we can see that we need a propellant mass fraction of approximate 17%. The fact that the lines are near vertical above $T/W > 2$ suggests that a slow, low thrust burn is roughly equivalent to a fast, high thrust burn. Therefore, since we have the ability to control our burn during the flight, a long, slow burn allows us to turn off the thrust closer to apogee, and with greater certainty of success.

For design purposes, and to give ourselves some flexibility, we can therefore design our rocket to have a $T/W > 2$ and a propellant mass fraction of at least 20%.

[1] Assumes exponential atmosphere, constant drag coefficient, perfectly vertical flight

[2] Note: This parameter is like the drag to weight ratio (except it uses c as the velocity and m_0 as the mass)

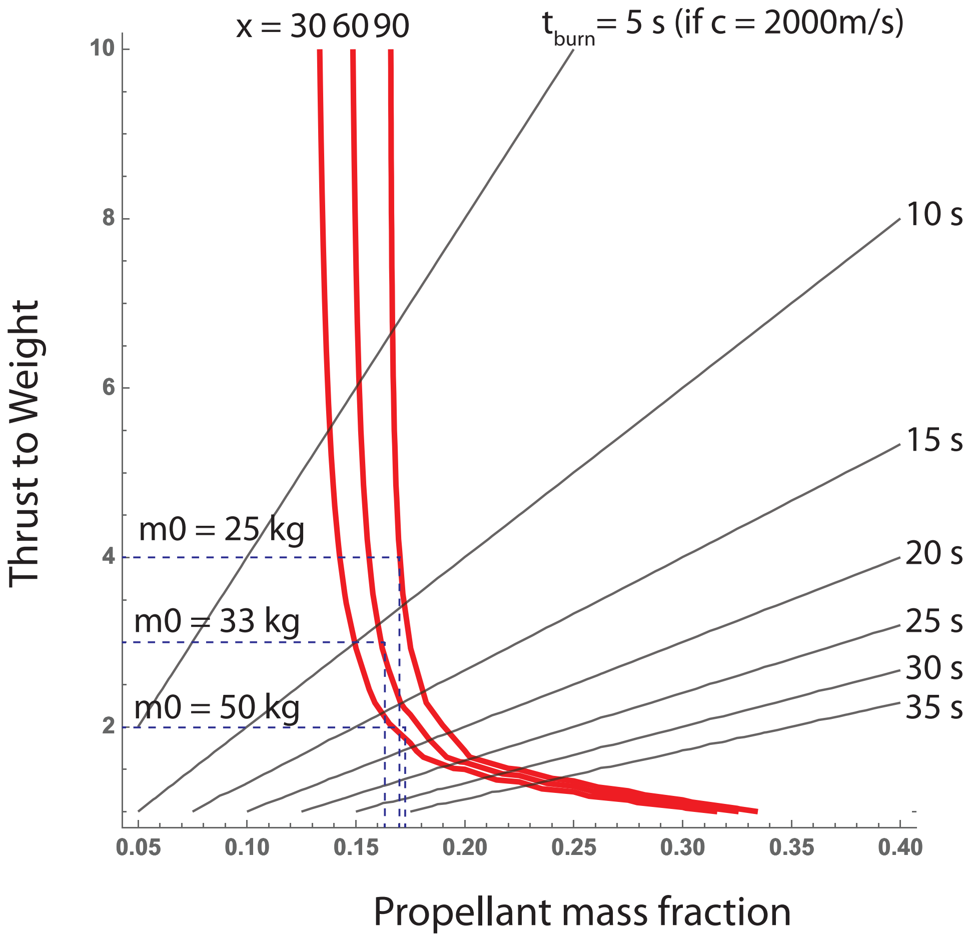

Figure 1: Required thrust to weight ratio as a function of the propellant mass fraction

Detailed analysis with accurate drag coefficients, the mach dependence, and optimal control should be performed next to ensure these performance targets are sufficient and robust to future design changes.

3 Vehicle Architecture

The chosen system breakdown of the rocket is:

1. Payload
   - 4 kg payload
   - 4.5 kg allocated, to allow for mounting
2. Avionics
   - Includes all flight computers and sensing equipment, switchboards and mounting hardware
   - Includes interface wiring to other components with electronics
   - Does not include mass of electronics for other subsystems
   - Allocated 2 kg, based on previous IREC reports.
3. Recovery System
   - Includes main and drogue parachute, parachute lines and deployment mechanism mass
   - includes mass of black powder and associated electronics.
   - Allocated 3 kg, based on previous IREC reports.
4. Main Engine
   - Includes oxidiser tank, fuel tank
   - ox mass, fuel mass,
   - valves assembly with electronics (allocated 1 kg), and nozzle assembly (allocated 1 kg)
   - Detailed engine sizing was not performed, and needs to be throughly verified.
5. Boosters
   - Since the T/W of the main engine is around 2, it is not enough to clear the launch rail with the required speed. As such, solid boosters are to be used launch the rocket, sized to provide $T/W = 10$ for the duration needed to clear the launch rails.
   - Includes motor dry mass, but not mounting structural mass
6. Structures
   - Includes nose cone (allocated 0.5 kg)
   - fins (1.2 kg)

- body tube with internal bulkheads and couplers (4 kg)
- booster mounting structure (0.3 kg)
- overall structural mass allowed is 6 kg.

4 Sizing approach

GPkit[3] was used to perform the sizing study. GPkit allows the user to define variables describing the vehicle, the constraints relating the variables (either due to physics, performance requirements or due to design requirements), and an objective function to optimise. It will then perform a global optimisation and return the optimised parameters of the design. GPkit also allows for easy compartmentalisation, by allowing the user to define these variables within classes, and thus separating the different parts of the design.

I have created a basic framework that should be general enough to allow more detail to be added into the model, as we develop it.

At the top level, a `rocket` class is defined. The six components above are created, each defined in a separate python file. These classes inherit from `gpkit.Model` which allows gpkit to interpret the variables and the constraints, and exposes a `solve` method to perform the optimisation.

For ease of visualisation and interpretation, a jupyter notebook instantiates the `rocket` and calls the signomial solver, `localsolve`. Due to the structure of the rocket unfortunately, the geometric globally optimal solver cannot be called, but a local signomial solver must be used. That said, in most scenarios, this solver is sufficiently robust to return a good, and viable solution.

The jupyter notebook also has result printing code blocks to allow for easy debugging of models.

Note, when there are changes to the python files where the relationships are described, jupyter must re-import the classes. The best way to ensure this is accurately done is by clicking the `Restart & Run-All` button.

The most important relationships used in this sizing are listed at the end of the document, but the most up-to-date relationships are only available in the python files.

5 Sizing results

The solve took 5 GP solves, and 1.54 seconds.

Total rocket mass: 27.89 kg

The results are more easily interpreted in the form of a diagram, on the last page.

[3] `https://gpkit.readthedocs.io/en/latest/`

SORTED BY LINEAGE, SENSITIVITY (solved on 05-11 19:18)

key	lineage	value	unit	sens	Description
PMF	Rocket	0.220	-	0.814	Propellant Mass Fraction required
v_{launch}	Rocket	30.000	m/s	0.125	Velocity off launch rail
L_{launch}	Rocket	5.200	m	-0.029	Length of launch rail
g	Rocket	9.810	m/s^2	0.011	Acceleration due to gravity
a_{launch}	Rocket	86.538	m/s^2	*	Acceleration off launch rail
m	Rocket	27.891	kg	*	Mass of Rocket
TW_{main, min}	Rocket	2.500	-	0.000	Main engine thrust to take off weight
min_a	Rocket	86.538	m/s^2	*	minimum launch acceleration
m	Rocket/Avionics	1.000	kg	0.069	Mass of Avionics
DMF	Rocket/Boosters	0.700	-	0.157	Dry mass fraction of boosters
c	Rocket/Boosters	2000.000	m/s	-0.067	boosters exhaust speed
m	Rocket/Boosters	0.975	kg	*	Mass of Boosters
m_{prop}	Rocket/Boosters	0.292	kg	*	Propellant mass of boosters
m_{dry}	Rocket/Boosters	0.682	kg	*	Dry mass of boosters
t_{burn}	Rocket/Boosters	0.347	s	*	Booster burn time
F	Rocket/Boosters	1687.261	N	*	Boosters cumulative thrust
m	Rocket/Payload	4.000	kg	0.275	Mass of Payload
m	Rocket/Recovery	3.500	kg	0.241	Mass of Recovery
\rho_{ox, tank}	Rocket/SimpleEngine	8000.000	kg/m^3	0.391	Density of ox tank, steel
Tank P	Rocket/SimpleEngine	60.000	bar	0.391	Max Ox Tank pressure
SF	Rocket/SimpleEngine	3.000	-	0.391	Wall thickness safety factor
\sigma_{max}	Rocket/SimpleEngine	585.000	MPa	-0.391	Max stress of tank, steel
rho_{ox}	Rocket/SimpleEngine	650.000	kg/m^3	-0.282	density of liquid ox *ROUGH*
rho_{wax}	Rocket/SimpleEngine	900.000	kg/m^3	-0.109	Density of fuel
m_{valves}	Rocket/SimpleEngine	1.000	kg	0.069	Mass of valves and plumbing
m_{nozzle}	Rocket/SimpleEngine	1.000	kg	0.069	Mass of nozzle assembly
OF	Rocket/SimpleEngine	7.500	-	-0.063	Ox to fuel ratio

F	Rocket/SimpleEngine	1000.000	N	-0.040	Engine thrust
m	Rocket/SimpleEngine	13.816	kg	*	Mass of Engine
m_{grain tank}	Rocket/SimpleEngine	1.580	kg	*	Mass of combustion chamber
m_{ox tank}	Rocket/SimpleEngine	4.101	kg	*	Mass of ox tank
m_{prop}	Rocket/SimpleEngine	6.136	kg	*	Mass of Propellant
m_{dry}	Rocket/SimpleEngine	7.680	kg	*	Dry mass of engine
m_{ox}	Rocket/SimpleEngine	5.414	kg	*	ox mass
m_{fuel}	Rocket/SimpleEngine	0.722	kg	*	fuel mass
d_ox	Rocket/SimpleEngine	6.000	in	0.000	Diameter of ox tank
t_{wall}	Rocket/SimpleEngine	2.345	mm	*	Wall Thickness of ox tank
L_{ox}	Rocket/SimpleEngine	0.457	m	*	Length of ox tank
v_{fuel}	Rocket/SimpleEngine	802.096	cm^3	*	Volume of fuel
L_{grain}	Rocket/SimpleEngine	0.176	m	*	Length of the grain
V_{ox}	Rocket/SimpleEngine	8329.458	cm^3	*	Volume of ox tank
A_{grain}	Rocket/SimpleEngine	45.581	cm^2	*	cross section area of grain
m	Rocket/Structures	4.600	kg	0.317	Mass of Structures

Note, the sensitivity is the logarithmic sensitivity, ie, sensitivity $= \frac{d\log(\text{cost})}{d\log(\text{var})}$. A positive number indicates that increasing the variable will increase the cost. Note, the star indicates a zero sensitivity, since this is a variable that gpkit has solved for. As such, it represents the minima of the function and thus has zero sensitivity, similar to how a function has zero gradient wrt to a variable when it is optimized.

6 Next steps

- Verify structural mass allocations
- Verify stability requirements - ie, ensure fins are large enough
- Perform detailed drag accounting
- Perform detailed controls analysis
- more accurate tank sizing needed, especially considering manufacturability, sourceability, cost.
- tank ullage not accounted for
- very simplified thrust curve needs to be improved

Design modification if the engine performance is poorer than expected: bigger boosters. Therefore, the booster mounts need to be flexible enough to allow different booster designs. Could look into dropping boosters after their work is done, but this is complicated.

7 Sizing relationships used

NOTE: the values in this section are probably wrong, the method should be roughly accurate. Consult the github for the values used.

Most constraints are fairly straightforward. Here are the key ones

- m = sum of mass of components
- propellant mass fraction > 20%
- Launch requirements:
  - v off launch rail > 30m/s
  - launch accel = (booster thrust + main engine thrust - mg)/m
  - launch accel > min accel = (launch v)^2/(2 launch rail L)
  - booster burn time such that burn out occurs at 5 m/s
- Components:
  - Payload: m = 4.5 kg
  - Avionics: m = 2 kg
  - Recovery: m = 3 kg
  - Engine:
    * OF = 6
    * m fuel, m ox based on m prop and OF
    * d = 150 mm
    * P tank < 60 bar

* wall thickness based on hoop stress and safety factor of 5, assumes Al-7075 due to high yield strength (double of Al 6061), sealing and welding to be determined more thoroughly
* ox is fully liquid, at critical density of 490 kg/m3 $\rightarrow$ determine length of ox tank
* mass of ox tank based on cylinder material thickness, end caps not accounted for
* grain tank is similarly sized, assumes the grain is only occupied in half the cross sectional area (needs to be refined), and same wall thickness as ox tank. No liner material considered. Carbon overwrap of tank tube would save lots of mass if possible.
* *regression rates, motor dynamics, etc not accounted for*
* m valves = 1 kg
* m nozzle = 1 kg
* assumed F= 1000 N
* c = 1800 m/s (needs to be verified)

- Boosters:
  * propellant mass such that total impulse can be delivered
  * dry mass fraction is 70%. Needs to be refined by picking a motor, assumed c = 2000m/s
- Structures:
  * m = sum of components
  * m fins = 1.2 kg
  * m nose cone = 0.5 kg
  * m tube = 4 kg
  * m booster struc = 0.3 kg

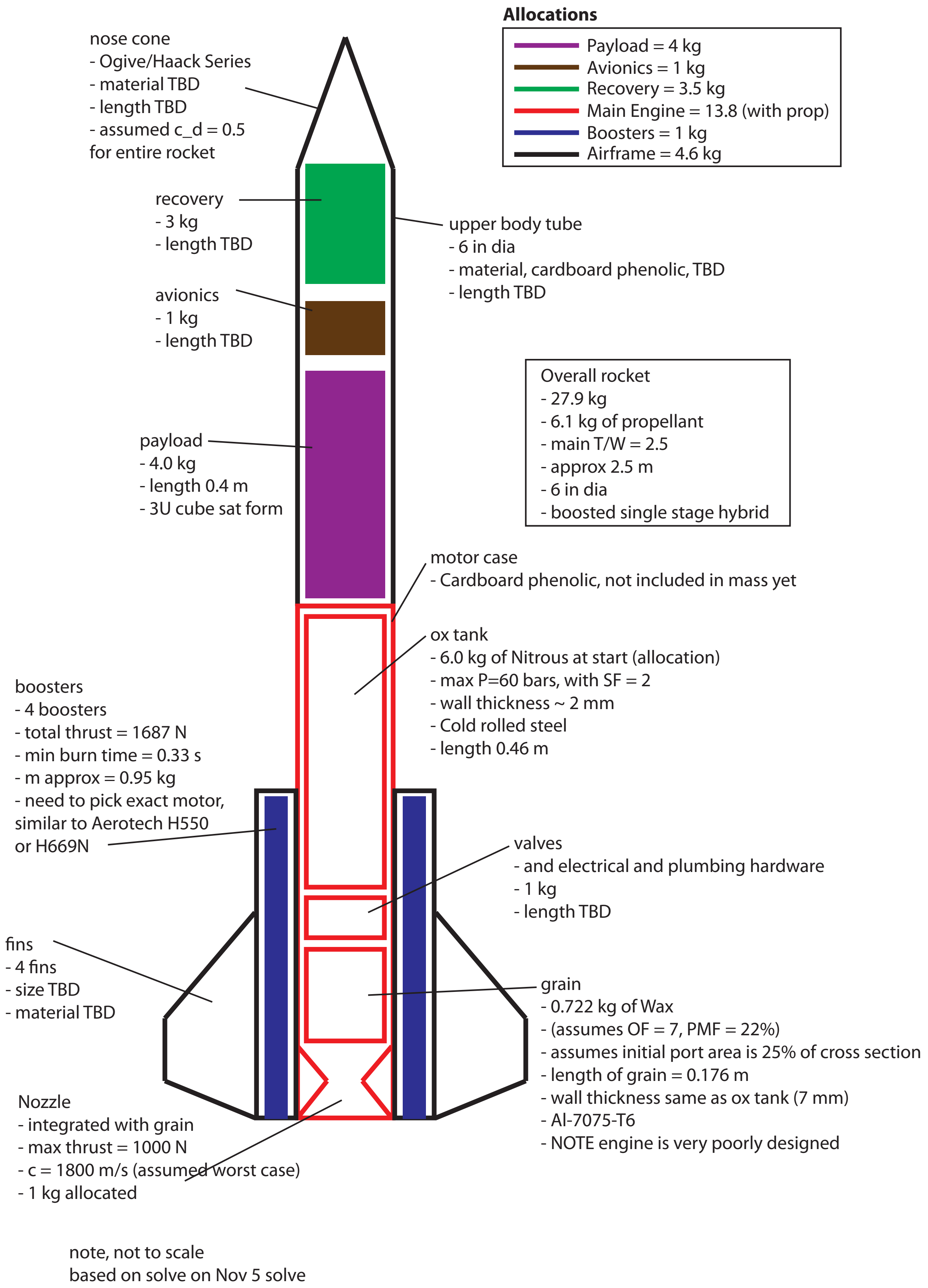

Figure 2: Summary of Sizing results