

IMMERSE Training

June 5-8, 2023

University of California, Santa Barbara



Day 23: Mixture Modeling and Latent Class Analysis (Part 2)

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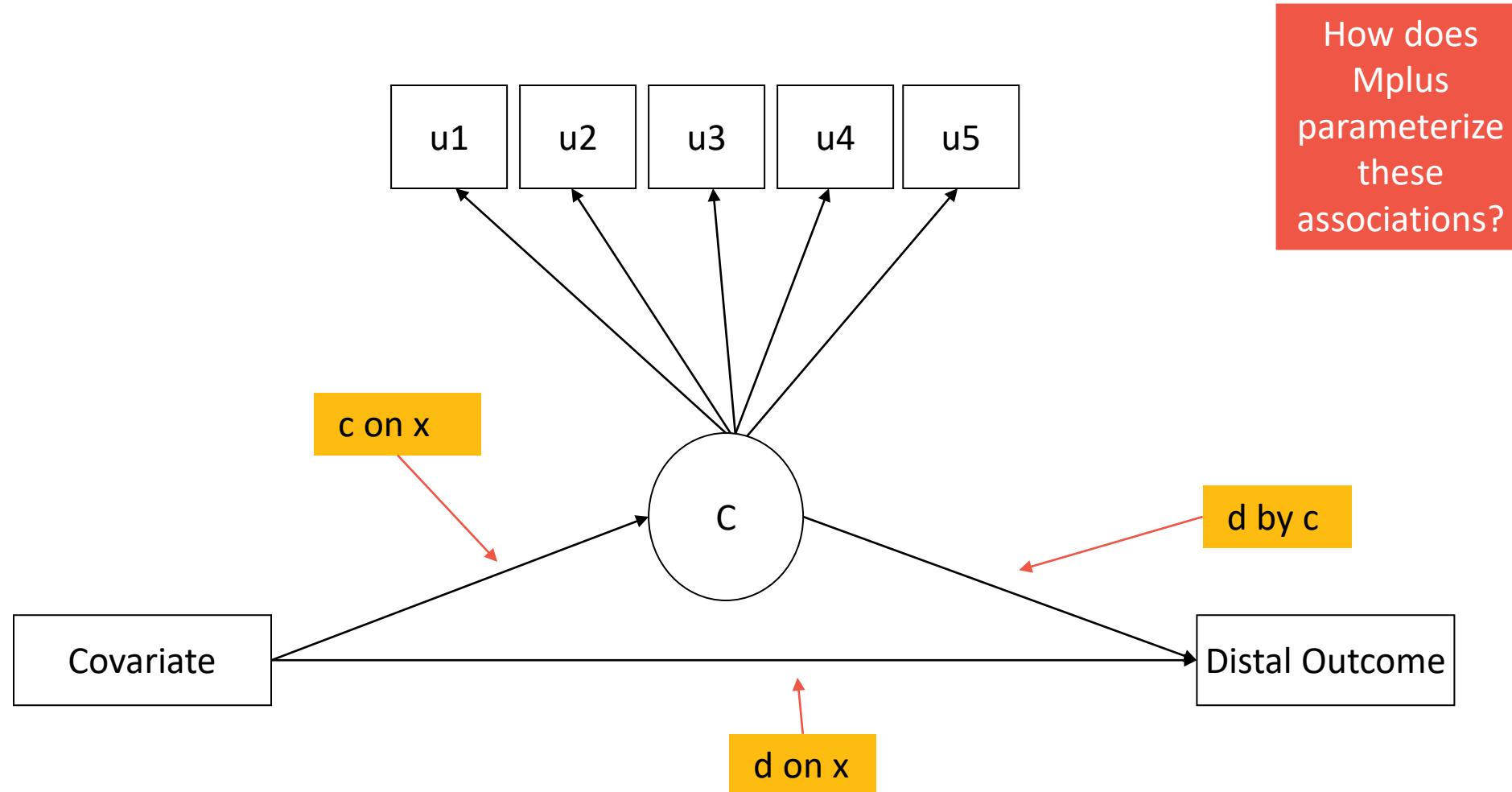


Today's Agenda

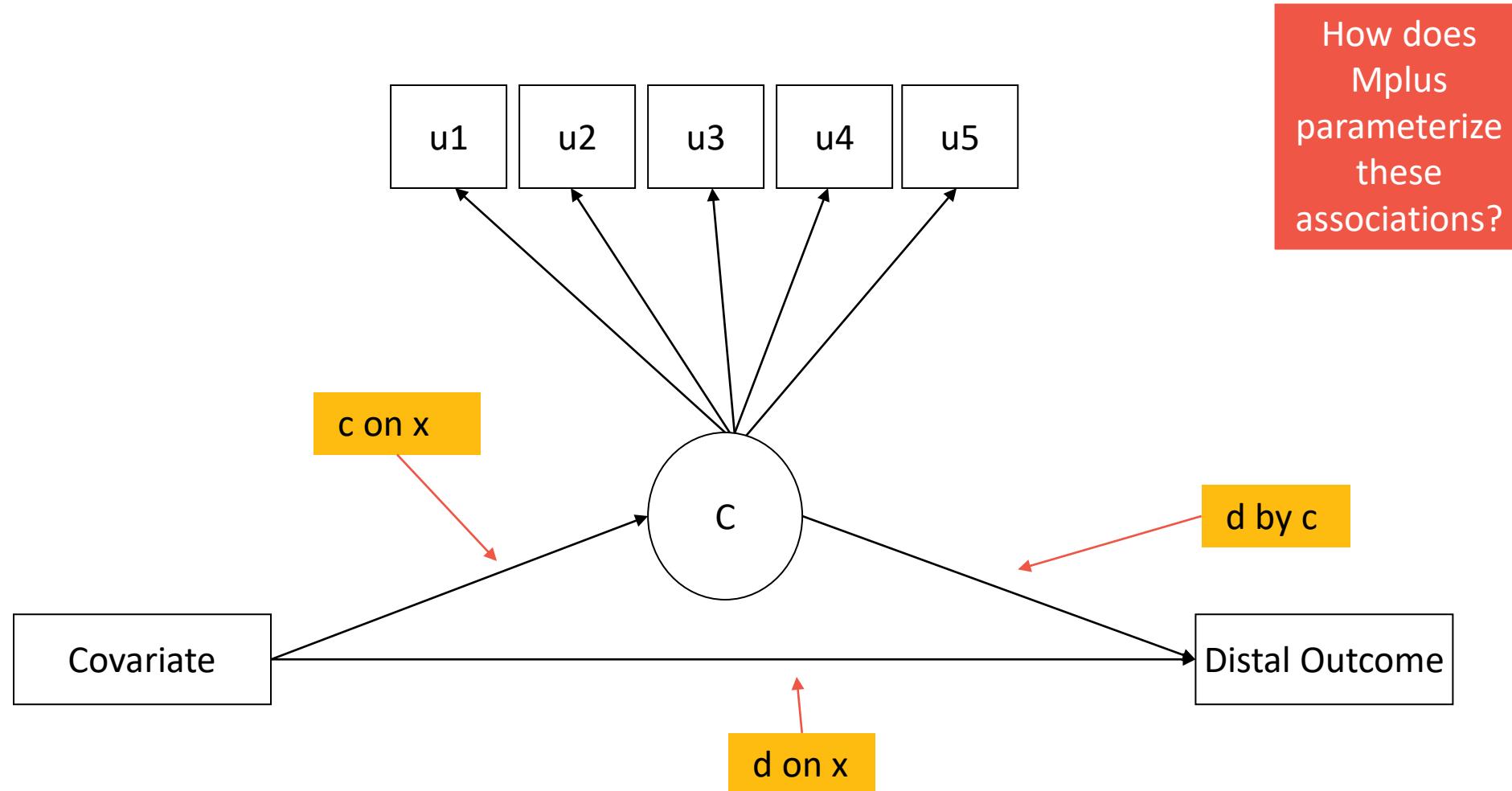
- Distal outcomes with covariates
 - Overview
 - ML 3-Step
 - BCH 3-Step
- Measurement Invariance and DIF (in brief)
- Latent Profile Analysis (in brief)
- Write-up Recommendations
- What comes next? (discussion)

Latent Class Regression with Distal Outcomes

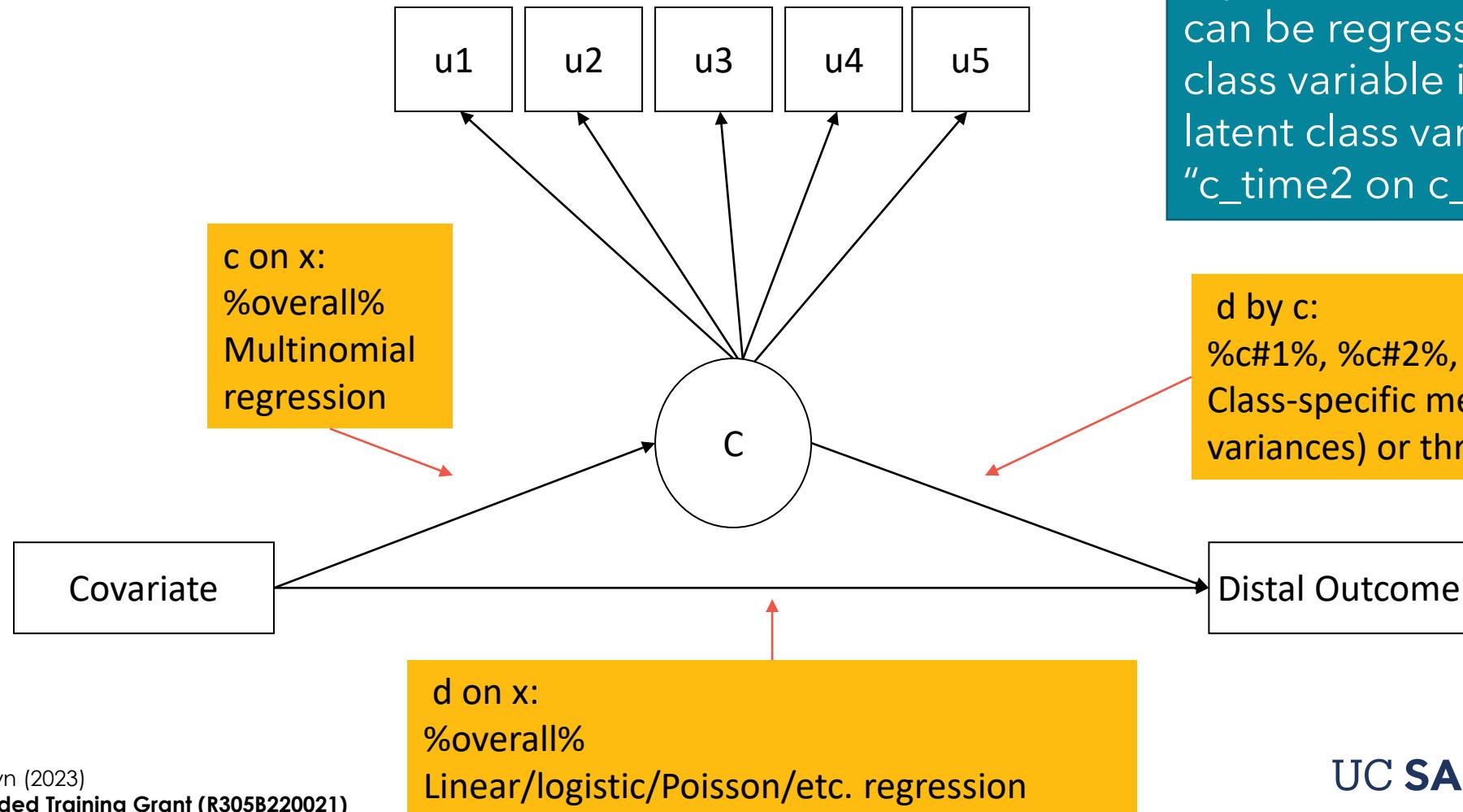
Covariates and distal outcomes in mixture models



Covariates and distal outcomes in mixture models



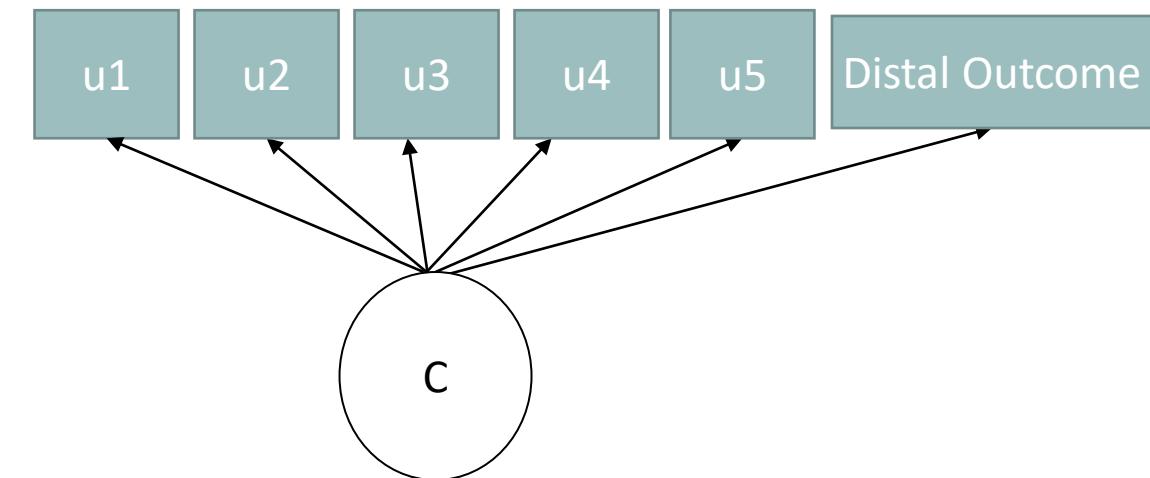
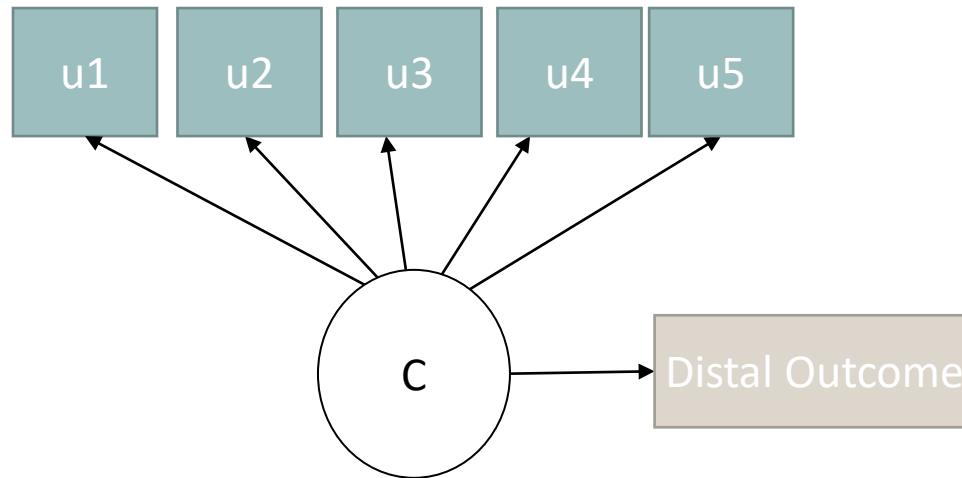
Covariates and distal outcomes in mixture models



Side Note: Distal-an-Indicator

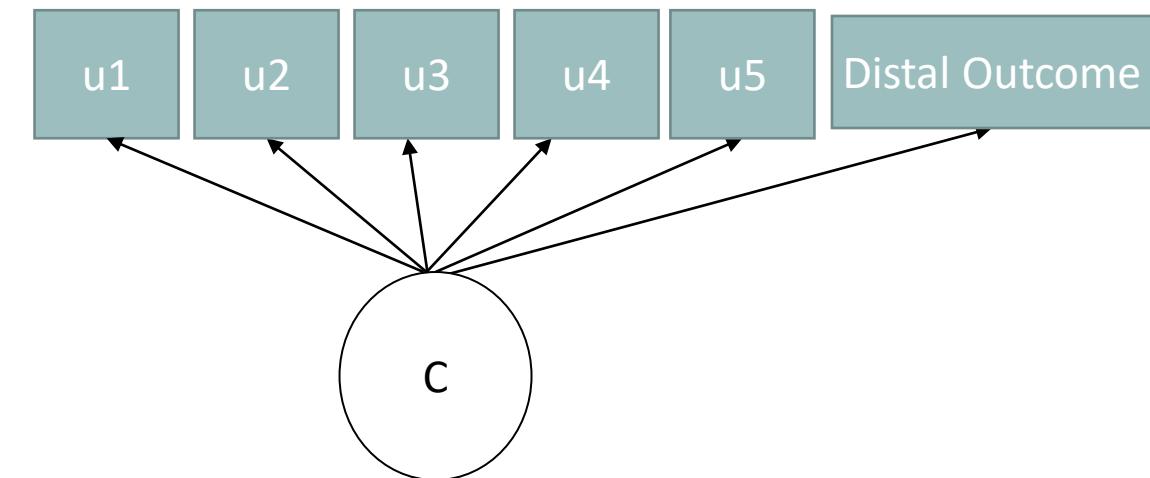
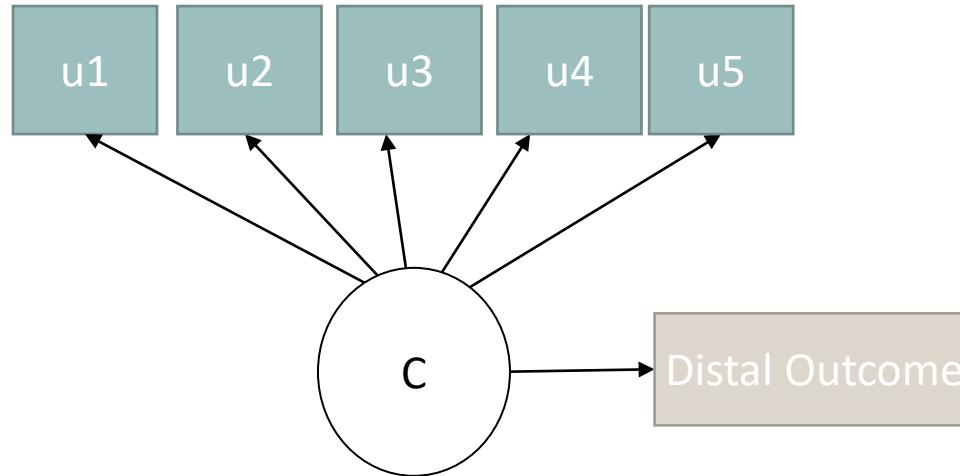
1-step approach

- Also referred to as the “distal-as-indicator” approach.
- Distal is treated as an *additional latent class indicator* if included as endogenous variable
 - This means your latent class variable is now specified as measured by all the items and the distals.
 - This may be what you intend but, if so, the distals should be included as indicators from the get-go.



1-step: Not good or bad, just maybe not what you want...

- What if you don't want your distal outcomes to characterize/measure the latent class variable?
- All the other existing approaches are an attempt to keep the distal outcome from influencing the class formation.



ML 3-step for Covariates & Distals: Example

ML 3-step with a covariate (gender) and distal (emthirtn)

```
usevariables = cmod5 emthirtn female;
Nominal = cmod5;
missing=all(9999);
classes= c(5);
idvariable = lsayid;
```

Define:
center female (grandmean);

Step 1 and 2 don't change!
We are just adding to the Step 3 model.

Why is this a good idea?



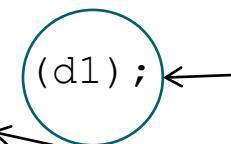
MODEL:

```
%OVERALL%
c on female;           ← c on x
emthirtn on female;   ← d on x
[emthirtn];           ← d by c
emthirtn;              ← Var(d) estimated
```

%C#1% !pro-math w/o anxiety

```
[cmod5#1@12.075];
[cmod5#2@7.715];
[cmod5#3@8.980];
[cmod5#4@8.232];
```

```
[emthirtn] (d1);
emthirtn;
```



...

Labeling this conditional mean for comparisons later
Var(d) estimated for each class.

%C#1% !pro-math w/o anxiety

```
[cmod5#1@12.075];
[cmod5#2@7.715];
[cmod5#3@8.980];
[cmod5#4@8.232];
```

```
[EMTHIRTN] emthirtn;
```

Labeling this conditional mean for comparisons later

%C#2% !pro-math w/ anxiety

```
[cmod5#1@5.784];
[cmod5#2@7.126];
[cmod5#3@4.438];
[cmod5#4@4.295];
```

```
[EMTHIRTN] EMTHIRTN;
```

Labeling this conditional mean for comparisons later

by adding this, you'd test for mean differences without assuming equal var

Model Test:

```
0= d1-d2;  
0= d1-d3;  
0= d1-d4; ←  
0= d1-d5;
```

1st: Omnibus test if all the means are equal across the classes. (Very similar to omnibus F-test in ANCOVA but without sphericity assumption!) Tests whether there is an overall association between the latent class variables and the distal outcome (adjusting for covariates).

Model constraint:

```
New (diff12 diff13 diff14 diff15  
diff23 diff24 diff25  
diff34 diff35 ←  
diff45 );  
  
diff12 = d1-d2;  
diff13 = d1-d3;  
diff14 = d1-d4;  
diff15 = d1-d5;  
diff23 = d2-d3;  
diff24 = d2-d4;  
diff25 = d2-d5; ←  
diff34 = d3-d4;  
diff35 = d3-d5;  
diff45 = d4-d5;
```

2nd : If there is a relationship (above is significant), then we can test which means are different.

We need to create new difference scores

These are all the pairwise differences of the means.
Class 1 v Class 2, Class 2 v Class 3, etc...

FINAL CLASS COUNTS AND PROPORTIONS FOR THE LATENT CLASSES BASED ON THE ESTIMATED MODEL

Latent
Classes

1	1059.24881	0.39598
2	331.28706	0.12385
3	569.14805	0.21277
4	434.78106	0.16253
5	280.53502	0.10487

FINAL CLASS COUNTS AND PROPORTIONS FOR THE LATENT CLASSES BASED ON THE ESTIMATED MODEL

Latent
Classes

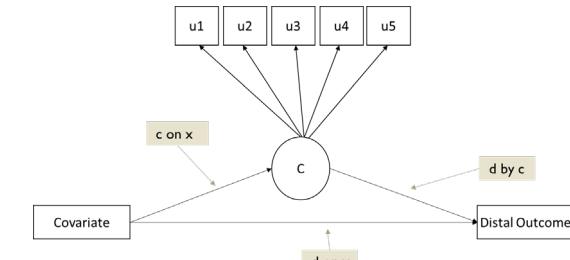
1	1060.17029	0.39633
2	329.48094	0.12317
3	568.57733	0.21255
4	436.54703	0.16320
5	280.22441	0.10476

. tabulate cmod5 cmod5cmod

cmod5	cmod5cmod					Total
	1	2	3	4	5	
0	1,128	0	0	0	0	1,128
1	0	290	0	0	0	290
2	0	0	538	0	0	538
3	0	0	0	437	0	437
4	0	0	0	0	282	282
Total	1,128	290	538	437	282	2,675

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Latent Class 1					
EMTHIRTN	ON FEMALE	0.882	0.560	1.576	0.115
Means					
CMOD5#1		12.075	0.000	999.000	999.000
CMOD5#2		7.715	0.000	999.000	999.000
CMOD5#3		8.980	0.000	999.000	999.000
CMOD5#4		8.232	0.000	999.000	999.000
Intercepts					
EMTHIRTN		63.868	0.464	137.742	0.000
Residual Variances					
EMTHIRTN		124.800	6.553	19.044	0.000
Latent Class 2					
EMTHIRTN	ON FEMALE	0.882	0.560	1.576	0.115
Means					
CMOD5#1		5.784	0.000	999.000	999.000
CMOD5#2		7.126	0.000	999.000	999.000
CMOD5#3		4.438	0.000	999.000	999.000
CMOD5#4		4.295	0.000	999.000	999.000
Intercepts					
EMTHIRTN		53.388	1.139	46.869	0.000
Residual Variances					
EMTHIRTN		148.944	12.660	11.765	0.000

d on x (does not vary by class).



This is the mean of the distal, for class 1, controlling for gender.

This is the mean of the distal, for class 2, controlling for gender.

Latent Class 3

EMTHIRTN ON
FEMALE

0.882

0.560 1.576 0.115

[edited output– deleted parts to
make it easier to view]

Intercepts
EMTHIRTN

58.671

0.782 75.000 0.000

Residual Variances
EMTHIRTN

152.628

10.275 14.854 0.000

Latent Class 4

EMTHIRTN ON
FEMALE

0.882

0.560 1.576 0.115

Intercepts
EMTHIRTN

54.076

0.755 71.599 0.000

Residual Variances
EMTHIRTN

112.470

7.968 14.115 0.000

Latent Class 5

EMTHIRTN ON
FEMALE

0.882

0.560 1.576 0.115

Intercepts
EMTHIRTN

52.335

1.025 51.055 0.000

Residual Variances
EMTHIRTN

164.270

12.504 13.137 0.000

Distal outcome differences

New/Additional Parameters

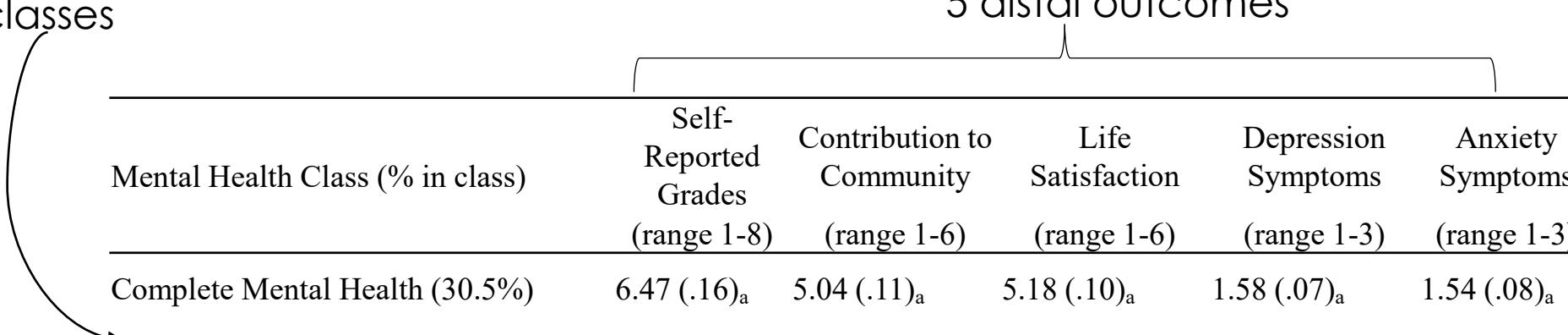
DIFF12	10.481	1.304	8.039	0.000
DIFF13	5.197	0.962	5.403	0.000
DIFF14	9.793	0.880	11.132	0.000
DIFF15	11.533	1.122	10.275	0.000
DIFF23	-5.284	1.460	-3.619	0.000
DIFF24	-0.688	1.445	-0.476	0.634
DIFF25	1.053	1.512	0.696	0.486
DIFF34	4.596	1.129	4.070	0.000
DIFF35	6.336	1.369	4.628	0.000
DIFF45	1.741	1.329	1.309	0.190

Differences are significant

The distal means for classes 2 and 4 are not significantly different from each other. Neither are 2 and 5, or 4 and 5.

Distal Mean Comparison (Multiple Distal outcomes)

Here is an example table where we have five distal outcomes and four latent classes.

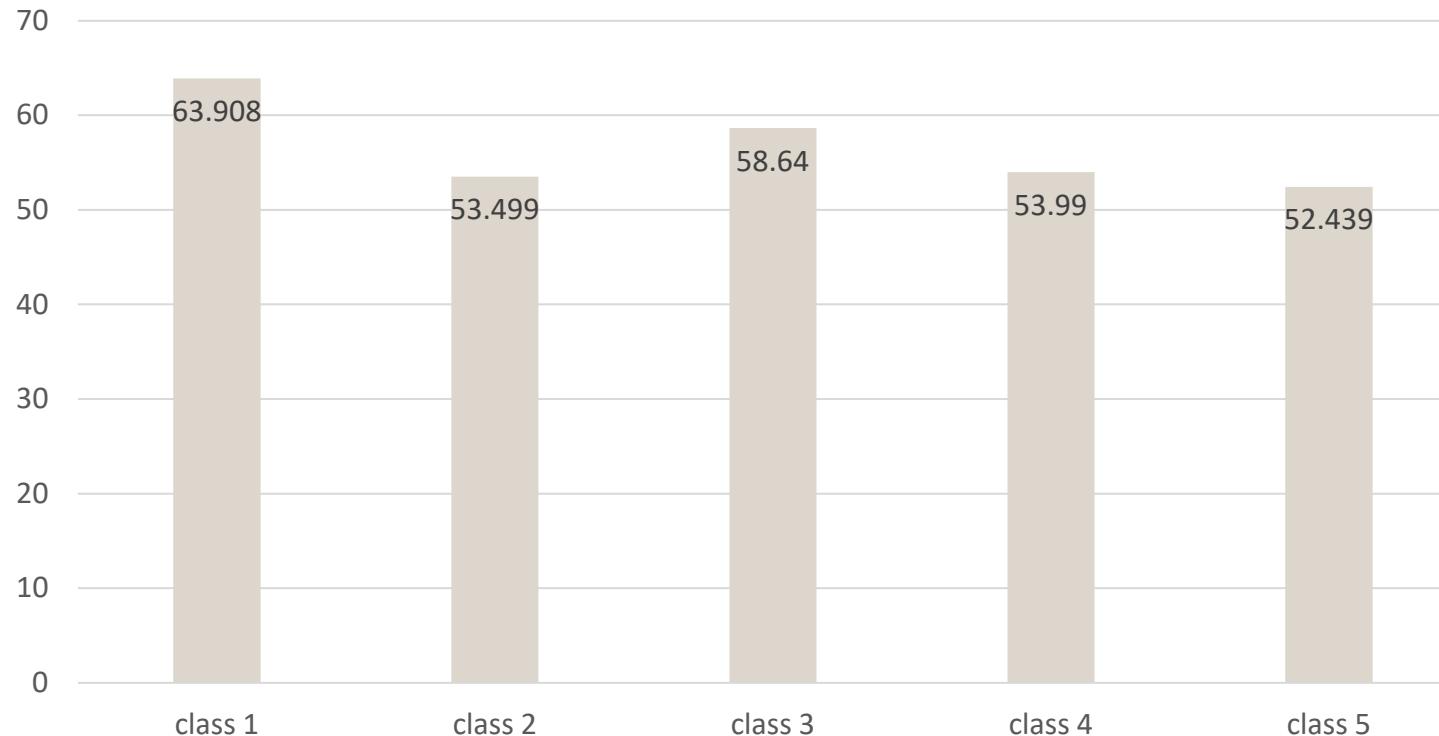


Mental Health Class (% in class)	5 distal outcomes				
	Self-Reported Grades (range 1-8)	Contribution to Community (range 1-6)	Life Satisfaction (range 1-6)	Depression Symptoms (range 1-3)	Anxiety Symptoms (range 1-3)
Complete Mental Health (30.5%)	6.47 (.16) _a	5.04 (.11) _a	5.18 (.10) _a	1.58 (.07) _a	1.54 (.08) _a
Moderately Mentally Healthy (43.4%)	6.63 (.12) _a	4.61 (.09) _a	5.07 (.07) _a	1.48 (.05) _a	1.42 (.06) _a
Symptomatic but Content (20.3%)	6.03 (.20) _a	4.37 (.13) _b	4.58 (.14) _b	1.91 (.10) _b	2.02 (.13) _b
Troubled (5.7%)	6.45 (.44) _a	4.15 (.20) _b	4.57 (.30) _{ab}	1.37 (.14) _a	1.52 (.17) _{ab}

Note. Means that do not share subscripts differ at $p < .01$.

Table from: Moore, S., Dowdy, E., Nylund-Gibson, K., Furlong, (2019). An Empirical Approach to Complete Mental Health Classification in Adolescents. *School Mental Health*, 1-16

Distal Mean Comparison (Multiple Distal outcomes)



What might be added to this graph?

What other ways could you graphically represent differences in a continuous distal outcome across classes?

Distal Mean Comparison (Multiple Distal outcomes)

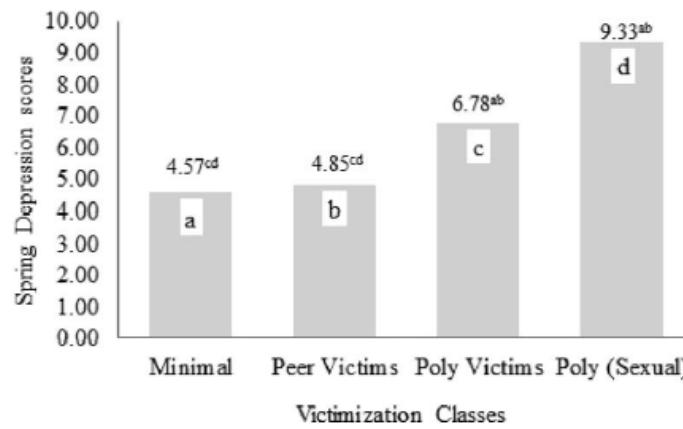


Figure 3. Mean depression scores by class. Column letters correspond to superscripts. Superscripts denote which columns are significantly different. For example, column a (*Minimal*) has a significantly lower mean of depression than columns c (*Poly Victims*) and d [*Poly (Sexual)*].

Differences in Spring Depression and Anxiety Based on Victimization Classes

Using the BCH method, we next estimated the mean spring depression and anxiety for each latent class. As described above, these models included fall depression and anxiety as covariates. To test for significant differences, we conducted a series of Wald tests to investigate whether the means of spring depression and anxiety were significantly different across victimization classes. Figures 3 and 4 provide visual summaries of these results. In terms of depression, *Poly (sexual)* had the highest mean score ($M = 9.33$), but this was not statistically different from the mean score for *Polyvictimization* ($M = 6.78$). The mean depression scores for *Peer Victimization* ($M = 4.85$) and *Minimal Victimization* ($M = 4.57$) were not statistically different from each other. However, they were significantly lower than the means for both the *Poly (sexual)* and *Polyvictimization* classes.

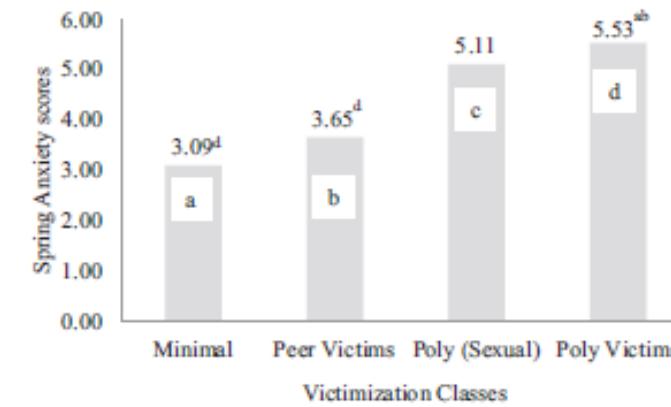


Figure 4. Mean anxiety scores by class. Column letters correspond to superscripts. Superscripts denote which columns are significantly different. For example, column a (*Minimal*) has a significantly lower mean of anxiety than column d (*Polyvictims*).

A different picture emerged when we examined spring anxiety levels. *Polyvictimization* had the highest mean anxiety score ($M = 5.53$) instead of *Poly (sexual)* ($M = 5.11$). The *Minimal Victimization* and *Peer Victimization* classes remained in the same rank order as before with means anxiety scores of 3.09 and 3.65, respectively. Statistically significant differences were more nuanced for the class-specific anxiety means compared with the class-specific depression means. Perhaps most striking was the mean for the *Poly (sexual)* class was not statistically different from any of the other means.

Holt, M. K., Felix, E., Grimm, R., Nylund-Gibson, K., Green, J. G., Poteat, V. P., & Zhang, C. (2017). A latent class analysis of past victimization exposures as predictors of college mental health. *Psychology of violence*, 7(4), 521.

Example write up with a distal outcome

Constellations of School Belonging And Complete Mental Health differences

The final step of the analysis included examining the associations between latent profiles and mental health outcomes. Specifically, class-specific means of psychological strengths and psychological distress were estimated for each of the latent profiles, at the average of the gender and ethnicity covariates.

First, an omnibus test of association was conducted between the latent profile variable and the three proximal outcomes and found to be significant indicating significant relations between the profiles and psychological strengths, $\chi^2 = 314.21$, $df = 2$, $p < .01$, and both aspects of psychological distress: emotional, $\chi^2 = 132.33$, $df = 2$, $p < .01$, and behavioural difficulties, $\chi^2 = 72.39$, $df = 2$, $p < .01$.

To understand where class differences occurred, pairwise tests were examined. Results indicated that all pairwise comparisons were significantly different for all three distal outcomes. Precisely, students in the *High School Belonging* profile had significantly higher psychological strengths than students in the *Moderate School Belonging* and *Low School Belonging* profiles. Students in the *Moderate School Belonging* profile reported significantly higher psychological strengths than students in the *Low School Belonging* profile. Concerning psychological distress, students in the *High School Belonging* profile reported significantly lower emotional and behavioural difficulties than students in the *Moderate* and *Low School Belonging* profiles. Students in the *Moderate School Belonging* profile reported significantly lower emotional and behavioural difficulties than students in the *Low School Belonging*

profile. For students in all profiles, emotional difficulties were slightly higher than behavioural difficulties.

Differences in mental health were also based on the covariates of gender and ethnic identification. Female students reported higher psychological strengths ($p = .01$) and emotional difficulties ($p < .001$) than males. Gender differences for behavioural difficulties were non-significant ($p = .165$). White students reported lower emotional difficulties than non-White students, though this difference was nonsignificant ($p = .069$). Latinx students did not significantly differ on self-reported mental health indicators from non-Latinx students. Table 4 presents the class-specific means, standard errors, and p -values for each latent profile with demographic covariates held constant.

Table 4. Model results for mean proximal outcome values within each latent school belonging profile.

Outcome	Latent Profile	Estimate	S.E.
Psychological Strengths	<i>Low School Belonging Class</i>	2.66	.04
	<i>Moderate School Belonging Class</i>	3.11	.03
	<i>High School Belonging Class</i>	3.51	.05
Emotional Difficulties	<i>Low School Belonging Class</i>	1.85	.03
	<i>Moderate School Belonging Class</i>	1.61	.03
	<i>High School Belonging Class</i>	1.38	.04
Behavioural Difficulties	<i>Low School Belonging Class</i>	1.56	.03
	<i>Moderate School Belonging Class</i>	1.35	.03
	<i>High School Belonging Class</i>	1.24	.04

All pairwise comparisons of distal outcomes are significantly different when comparing with class, $p < .001$.

Wagle, R., Dowdy, E., Nylund-Gibson, K., Sharkey, J. D., Carter, D., & Furlong, M. J. (2021). School belonging constellations considering complete mental health in primary schools. *The Educational and Developmental Psychologist*, 1-13.

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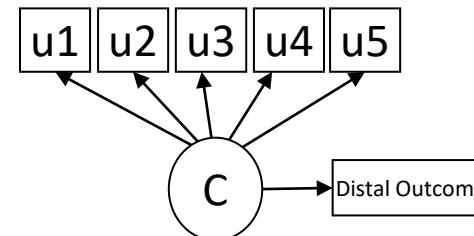
Automatic ML 3-Step

Just FYI...

ML 3-step automatic (distal)

1. Embedded in Mplus and limited to only covariates or only distal outcomes.
 - a) DU3step –distal outcome via 3-step with unequal variances
 - b) DE3stpe– distal outcome via 30step with equal variances

```
usevariables = ca28ar ca28br ca28cr ca28er ca28gr ca28hr ca28ir  
ca28kr ca28lr;  
CATEGORICAL = ca28ar ca28br ca28cr ca28er ca28gr ca28hr ca28ir  
ca28kr ca28lr;  
  
Auxiliary = EMTHIRTN (du3step); ← Distal outcome  
idvariable = lsayid; "d by c"  
missing=all(9999);  
  
classes=c(5);  
  
Analysis:  
type= mixture;  
starts=100 20;
```



Distal Mean Comparison (Distal outcomes)

Final Class Counts and Proportions for the Latent Class Patterns
 Based On Estimated Posterior Probabilities for EMTHIRTN: Step 1 vs. Step 3

Latent Classes		Step 1		Step 3
1	1059.25072	0.39598	850.39818	0.41221
2	331.28426	0.12384	241.55518	0.11709
3	569.14756	0.21277	430.28468	0.20857
4	434.78344	0.16254	325.61655	0.15784
5	280.53402	0.10487	215.14541	0.10429

Classification Probabilities for the Step 1 Most Likely Latent Class Membership (Row)
 by Step 3 Most Likely Latent Class Membership (Column) for EMTHIRTN

	1	2	3	4	5
1	1.000	0.000	0.000	0.000	0.000
2	0.000	1.000	0.000	0.000	0.000
3	0.000	0.000	1.000	0.000	0.000
4	0.000	0.000	0.000	1.000	0.000
5	0.000	0.000	0.000	0.000	1.000

ML 3-step automatic (Distal) Output

EQUALITY TESTS OF MEANS ACROSS CLASSES USING THE 3-STEP PROCEDURE
WITH 4 DEGREE(S) OF FREEDOM FOR THE OVERALL TEST

EMTHIRTN	<i>These are the means of the distal outcome, per class</i>			
	Mean	S.E.	Mean	S.E.
Class 1	63.908	0.457	Class 2	53.499
Class 3	58.640	0.782	Class 4	53.990
Class 5	52.439	1.013		

	Chi-Square	P-Value		Chi-Square	P-Value
Overall test	225.575	0.000	Class 1 vs. 2	69.686	0.000
Class 1 vs. 3	29.895	0.000	Class 1 vs. 4	123.688	0.000
Class 1 vs. 5	108.055	0.000	Class 2 vs. 3	13.325	0.000
Class 2 vs. 4	0.126	0.723	Class 2 vs. 5	0.516	0.472
Class 3 vs. 4	16.652	0.000	Class 3 vs. 5	21.018	0.000
Class 4 vs. 5	1.392	0.238			

Overall test—seeing if there are any differences at all. Think omnibus F test in ANVOA

*The rest are pairwise tests (Wald tests) of all classes.
E.g., class 2 and 5 are not significantly different from each other. But 3 and 5 are.*

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BCH 3-Step for Covariates & Distals

BCH 3-Step

- Named after authors who wrote the paper introducing the approach
 - [Bolck, A., Croon, M., & Hagenaars, J. \(2004\). Estimating latent structure models with categorical variables: One-step versus three-step estimators. Political Analysis, 12,3–27. doi:10.1093/pan/mph001](#)
- BCH is similar to the ML 3-step approach except it uses classification errors for each individual (rather than averaging across individuals with the same modal class assignment)
 - Technically, the inverse logits of those individual-level error rates are used as **weights** in Step 3 (for covariates and/or distal outcomes) rather than using the modal class assignment as an imperfect latent class indicator.
- Drawback: The weights sometimes take negative values (which is non-admissible)
 - If the entropy is large and the latent class variable is measured without error then the weight w_{ij} is 1.
 - If the entropy is low, however, the weights w_{ij} can become negative and the estimates for the auxiliary model can become inadmissible.
- In your analysis, you will get an error message that there are negative weights. If so, the closest alternative is the ML 3-Step.

BCH 3-STEP

- Can be used for distal outcomes while including predictors and controls.
- Very similar to idea to previous 3-step but rather than computing the average classification error for each class, “BCH” weights are computed for every individual, corresponding to every class:

$$\text{logit}[\Pr(\text{cmod}_i = k | c_i = j)] = \text{logit}\left[\frac{\Pr(c_i = j | \text{cmod}_i = k) \Pr(\text{cmod}_i = k)}{\Pr(c_i = j)} \right]$$

- Mplus implementation is limited but you can always do a manual BCH 3-step in order to analyze multiple distal outcomes at the same time while including covariates, potential moderators, etc.
- WARNING: The 3-step approach does not guarantee that your distal will not influence the latent class formation. Mplus checks for this now—**you** have to check yourself if using any manual 3-step. (Although BCH Step 3 classes seem more stable than other 3-step methods)
- Limitation: Can only use BCH weights if Step 3 model has only one latent class variable.

BCH 3-Step (Manual): Example

```
usevariables = ca28ar ca28br ca28cr ca28er ca28gr ca28hr ca28ir  
ca28kr ca28lr;  
  
CATEGORICAL = ca28ar ca28br ca28cr ca28er ca28gr ca28hr ca28ir  
ca28kr ca28lr;  
  
missing=all(9999);  
  
classes= c(5); ←  
  
idvariable = lsayid;  
  
auxiliary = gender eMTHIRTN; ←  
  
savedata:  
    file is lsay_c5_nosplit_bch.txt;  
    save = bchweights; ←  
    missflag = 9999;  
    format = free;
```

BCH 3-Step (Manual): Example

Save file
lsay_c5_nosplit_bch.txt

Order of variables

CA28AR
CA28BR
CA28CR
CA28ER
CA28GR
CA28HR
CA28IR
CA28KR
CA28LR
LSAYID
GENDER
EMTHIRTN
BCHW1
BCHW2
BCHW3
BCHW4
BCHW5
CPROB1
CPROB2
CPROB3
CPROB4
CPROB5
C

Here are your BCH weights

You also get “cprobs” if you have the plot command in your input syntax, otherwise you only get bchweights in the savedata file.



[LINK TO FULL OUTPUT](#)

UC SANTA BARBARA

BCH 3-Step (Manual): Example

```
data: file is lsay_c5_nosplit_bch.txt;
```

variable:

names are CA28AR
CA28BR
CA28CR
CA28ER
CA28GR
CA28HR
CA28IR
CA28KR
CA28LR
LSAYID
GENDER
EMTHIRTN
BCHW1-BCHW5
cp1-cp5
CMOD5;

Step 3
Sample Input

idvariable = lsayid;

training = BCHW1-BCHW5 (bch);

Define:

female = gender EQ 1;
center female (grandmean);

Analysis:

estimator = mlr;
type=mixture;
starts=0; ←
processors = 4;

usevariables = bchw1-bchw5 emthirtn female;

missing=all(9999);

classes= c(5);

Sample
Input, cont.

Model:

%OVERALL%

c on female (b1-b4);

emthirtn on female;

[emthirtn];

emthirtn;

%C#1% !pro-math w/o anxiety

[emthirtn] (m1);

emthirtn;

%C#2% !pro-math w/ anxiety

[emthirtn] (m2);

emthirtn;

%C#3% !math lover

[emthirtn] (m3);

emthirtn;

%C#4% !I don't like math but know it's good for me

[emthirtn] (m4);

emthirtn;

%C#5% !anti-math w/ anxiety

[emthirtn] (m5);

emthirtn;



C on X

D on X

D by C (class specific estimate of
the distal outcome)

Sample
Input, cont.

```
Model test:  
0=b1;  
0=b2;  
0=b3;  
0=b4;
```

```
!Model Test:  
! 0=m1-m2;  
! 0=m1-m3;  
! 0=m1-m4;  
! 0=m1-m5;
```

Model Constraint:

```
New (m1v2 m1v3 m1v4 m1v5 m2v3 m2v4 m2v5 m3v4  
m3v5 m4v5);  
m1v2 = m1 - m2;  
m1v3 = m1 - m3;  
m1v4 = m1 - m4;  
m1v5 = m1 - m5;  
m2v3 = m2 - m3;  
m2v4 = m2 - m4;  
m2v5 = m2 - m5;  
m3v4 = m3 - m4;  
m3v5 = m3 - m5;  
m4v5 = m4 - m5;
```

Omnibus test of if there is a relation between the covariate (female) and the latent class variable. Will produce a Wald Test (df=4).

Omnibus test to see if there is a relation between the distal outcome and the latent class variable. Will produce a Wald Test (df=4).

Remember: You can not do both at the same time. You have to run it once with the distal commented out (as here), then again commenting out the covariate.

Code to test pairwise difference of the distal outcome means (which we labeled with m's in the input— see previous slide)

We don't usually test covariate slopes in this format— we use the c on x logits and odds ratio to make covariate relations. Remember that Mplus provides post-hoc results for the LCR portion using each class (other than the last class) as the reference class for the multinomial regression.

BCH 3-Step (Manual)

Wald Test of Parameter Constraints

Value	27.683
Degrees of Freedom	4
P-Value	0.0000

MODEL RESULTS

	Estimate	S.E.	Two-Tailed Est./S.E.	P-Value
Latent Class 1				
EMTHIRTN ON FEMALE	1.022	0.562	1.820	0.069
Intercepts EMTHIRTN	63.812	0.449	141.995	0.000
Residual Variances EMTHIRTN	124.358	6.878	18.081	0.000
Latent Class 2				
Intercepts EMTHIRTN	53.423	1.114	47.936	0.000
Residual Variances EMTHIRTN	156.115	15.211	10.263	0.000

Omnibus test indicates relation between covariate and the latent class variable

Latent Class 3

Intercepts	EMTHIRTN	58.653	0.795	73.791	0.000
------------	----------	--------	-------	--------	-------

Residual Variances	EMTHIRTN	154.990	12.367	12.533	0.000
--------------------	----------	---------	--------	--------	-------

Latent Class 4

Intercepts	EMTHIRTN	53.746	0.758	70.901	0.000
------------	----------	--------	-------	--------	-------

Residual Variances	EMTHIRTN	103.593	9.781	10.592	0.000
--------------------	----------	---------	-------	--------	-------

Latent Class 5

Intercepts	EMTHIRTN	52.512	1.008	52.071	0.000
------------	----------	--------	-------	--------	-------

Residual Variances	EMTHIRTN	167.327	13.917	12.023	0.000
--------------------	----------	---------	--------	--------	-------

The intercepts are the class specific estimates of the distal outcome means

BCH 3-Step (Manual)

New/Additional Parameters				
M1V2	10.388	1.259	8.253	0.000
M1V3	5.159	0.971	5.315	0.000
M1V4	10.066	0.886	11.363	0.000
M1V5	11.300	1.100	10.271	0.000
M2V3	-5.229	1.427	-3.663	0.000
M2V4	-0.322	1.416	-0.227	0.820
M2V5	0.913	1.487	0.613	0.540
M3V4	4.906	1.154	4.251	0.000
M3V5	6.141	1.354	4.536	0.000
M4V5	1.235	1.317	0.937	0.349

These are the new parameters created in the “model constraint” code above. Show pairwise difference for latent classes.



BCH 3-Step (Manual)

Wald Test of Parameter Constraints

Value	27.683
Degrees of Freedom	3
P-Value	0.0000

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Latent Class 1				
EMTHIRTN ON FEMALE	1.022	0.562	1.820	0.069
Intercepts				
EMTHIRTN	63.812	0.449	141.995	0.000
Residual Variances				
EMTHIRTN	124.358	6.878	18.081	0.000
Latent Class 2				
EMTHIRTN ON FEMALE	1.022	0.562	1.820	0.069
Intercepts				
EMTHIRTN	53.423	1.114	47.936	0.000
Residual Variances				
EMTHIRTN	156.115	15.211	10.263	0.000

Here “d on x” does not vary by class

Model:

```
%OVERALL%
c on female (b1-b4);
emthirtn on female;

[emthirtn];
emthirtn;
```

The “d on x” is estimated for each class (but I removed it from the slide to highlight key ideas). It is estimated to be the same for each class because it was mentioned in the overall statement. If you are interested in allowing that to vary across class, you can do that.

Here “d on x” does vary by class

```
%OVERALL%
c on female (b1-b4);
emthirtn on female;

[emthirtn];
emthirtn;
```

```
%C#3% [emthirtn] (m3);
emthirtn;
emthirtn on female;
```

```
%C#4%
[emthirtn] (m4);
emthirtn;
emthirtn on female;
```

```
%C#1%
emthirtn on female;
[emthirtn] (m1);
emthirtn;
```

```
%C#2%
[emthirtn] (m2);
emthirtn;
emthirtn on female;
```

```
%C#5%
emthirtn on female;
[emthirtn] (m5);
emthirtn;
```

```
%C#1%
emthirtn on female;
[emthirtn] (m1);
emthirtn;
```

```
%C#2%
[emthirtn] (m2);
emthirtn;
emthirtn on female;
```

```
%C#3%
[emthirtn] (m3);
emthirtn;
emthirtn on female;
```

BCH 3-Step (Manual)

Categorical Latent Variables

C#1	ON				
FEMALE		0.299	0.175	1.707	0.088
C#2	ON				
FEMALE		-0.230	0.241	-0.953	0.341
C#3	ON				
FEMALE		0.163	0.215	0.761	0.447
C#4	ON				
FEMALE		0.980	0.227	4.314	0.000

C on x results. Estimates are logits.

Comparing girls (female=1) to boys, what is the log odds of being in a given class relative to the reference class.

Note that these results are similar to what we saw with the ML 3-step

Intercepts				
C#1	1.399	0.088	15.984	0.000
C#2	0.037	0.121	0.306	0.759
C#3	0.679	0.107	6.321	0.000
C#4	0.352	0.114	3.097	0.002

LOGISTIC REGRESSION ODDS RATIO RESULTS

	Estimate	S.E.	95% C.I.		
			Lower	2.5%	Upper
Categorical Latent Variables					
C#1	ON				
FEMALE		1.348	0.236	0.957	1.899
C#2	ON				
FEMALE		0.795	0.192	0.495	1.275
C#3	ON				
FEMALE		1.177	0.253	0.773	1.793
C#4	ON				
FEMALE		2.664	0.605	1.707	4.158

Mplus will provide the covariate relations in odds ratios (OR) as well. It provides the OR and the 95% CI for that value. OR =1 means no difference in odds between a given class and the reference class.

Girls, compared to boys, have significantly higher odds (OR = 2.66 [1.71 , 4.16]) of being in class 4 relative to class 5.

BCH 3-Step (Manual)

ALTERNATIVE PARAMETERIZATIONS FOR THE CATEGORICAL LATENT VARIABLE REGRESSION

		Estimate	S.E.	Est./S.E.	P-Value	Two-Tailed
Parameterization using Reference Class 1						
C#2	ON					
FEMALE		-0.529	0.209	-2.534	0.011	
C#3	ON					
FEMALE		-0.135	0.158	-0.854	0.393	
C#4	ON					
FEMALE		0.681	0.170	3.997	0.000	
C#5	ON					
FEMALE		-0.299	0.175	-1.707	0.088	
Intercepts						
C#2		-1.362	0.104	-13.044	0.000	
C#3		-0.720	0.079	-9.089	0.000	
C#4		-1.048	0.085	-12.312	0.000	
C#5		-1.399	0.088	-15.984	0.000	

ODDS RATIO FOR THE ALTERNATIVE PARAMETERIZATIONS FOR THE CATEGORICAL LATENT VARIABLE REGRESSION

		Estimate	S.E.	95% C.I.		
				Lower 2.5%	Upper 2.5%	
Parameterization using Reference Class 1						
C#2	ON					
FEMALE		0.589	0.123	0.392	0.887	
C#3	ON					
FEMALE		0.873	0.138	0.640	1.191	
C#4	ON					
FEMALE		1.976	0.337	1.415	2.761	
C#5	ON					
FEMALE		0.742	0.130	0.527	1.045	

This is the reparameterization of the covariate relation with the reference class being different.

In this example the reference class is 1. Mplus provides each in the output (didn't include all in this slide)

Girls, compared to boys are significantly more likely to be in class 4 relative to class 1. (note we didn't know that when we only considered class 5 as the reference class)

This is the reparameterization of the covariate relation in Odds Ratios too.

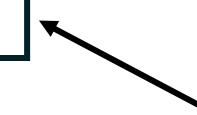
Automatic BCH 3-Step

Just FYI...

BCH 3-Step Automatic

```
usevar = ca28ar ca28br ca28cr ca28er   ca28gr ca28hr ca28ir  
ca28kr ca28lr;  
  
CATEGORICAL = ca28ar ca28br ca28cr ca28er   ca28gr ca28hr ca28ir  
ca28kr ca28lr;  
  
missing=all(999);  
idvariable = lsayid;  
classes = c(5);  
auxiliary = EMTHIRTN (bch);  
  
Analysis: type=mixture;  
starts = 100 10;
```

The Automatic BCH can **only** estimate distal outcomes relations across class

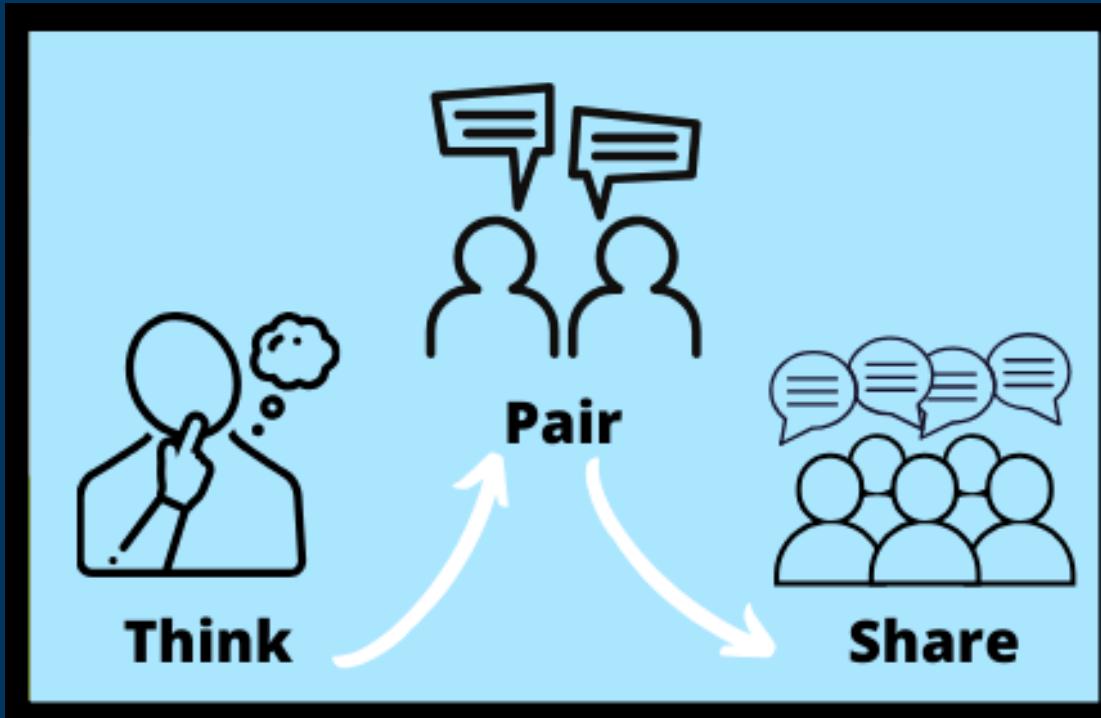


BCH Automatic

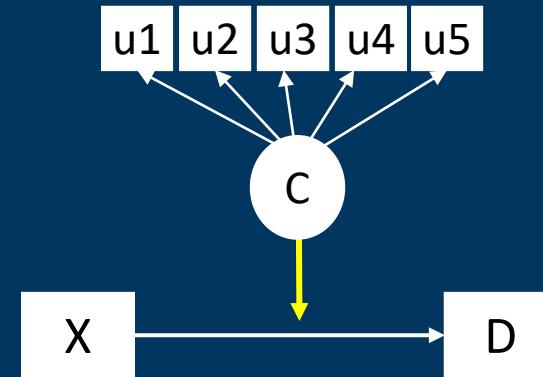
EQUALITY TESTS OF MEANS ACROSS CLASSES USING THE BCH PROCEDURE
WITH 4 DEGREE(S) OF FREEDOM FOR THE OVERALL TEST

EMTHIRTN

	Mean	S.E.		Mean	S.E.
Class 1	63.815	0.448	Class 2	53.306	1.108
Class 3	58.621	0.796	Class 4	53.910	0.750
Class 5	52.450	1.014			
	Chi-Square	P-Value		Chi-Square	P-Value
Overall test	229.161	0.000	Class 1 vs. 2	70.921	0.000
Class 1 vs. 3	28.675	0.000	Class 1 vs. 4	125.068	0.000
Class 1 vs. 5	106.398	0.000	Class 2 vs. 3	13.929	0.000
Class 2 vs. 4	0.188	0.665	Class 2 vs. 5	0.330	0.566
Class 3 vs. 4	16.790	0.000	Class 3 vs. 5	20.620	0.000
Class 4 vs. 5	1.236	0.266			



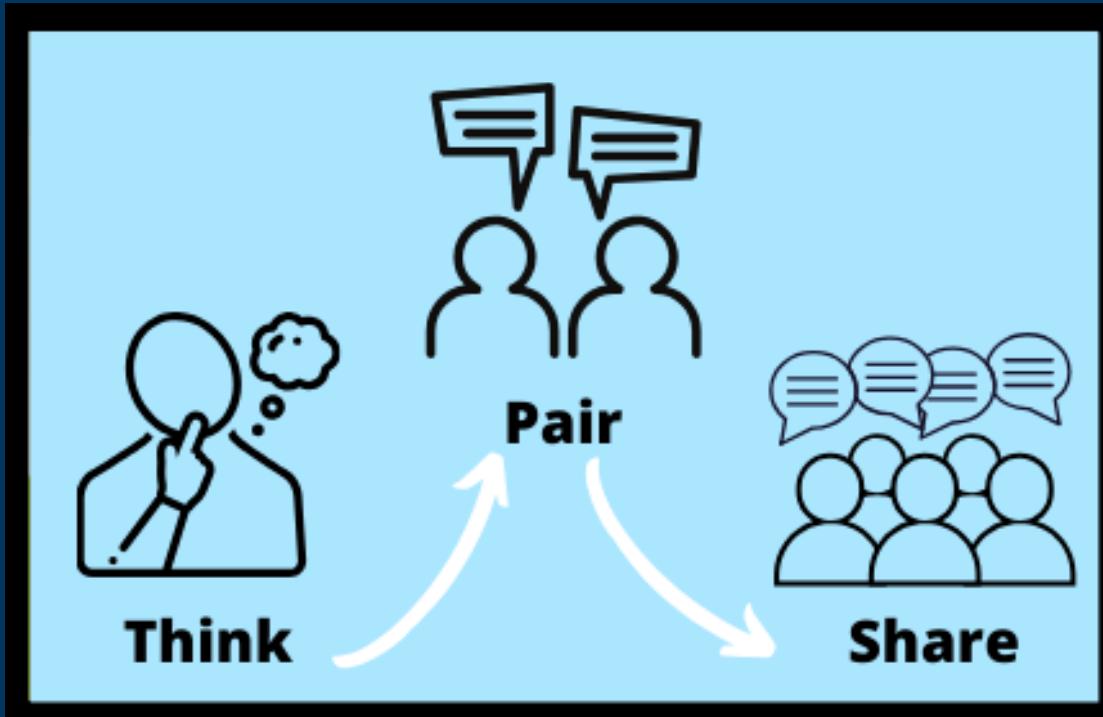
- What if you hypothesized that your latent class variable moderated the effect of a predictor, X, on an outcome, D? How would you specify that in Mplus? How would you test it?



- What if you instead hypothesized that X moderated the effect of C on D?

3-Steppin' Lab

Measurement Invariance and DIF in LCA



- Broadly, what is differential item functioning (DIF), i.e., measurement non-invariance, and why do we care about it?
- Can you think of an example LCA (real or hypothetical) for which DIF might be present? What is the latent class measuring? Which item(s) has DIF? What is the source of DIF?
- In a latent class regression, if your predictor of interest is also a source of DIF, what might be the consequences of ignoring the DIF and just modeling the impact of the predictor on class membership?

DIF in Mixtures

From Suzuki et al. (2021)

Moment 2: Decision-Making About the Role of Race in Planned Analyses:

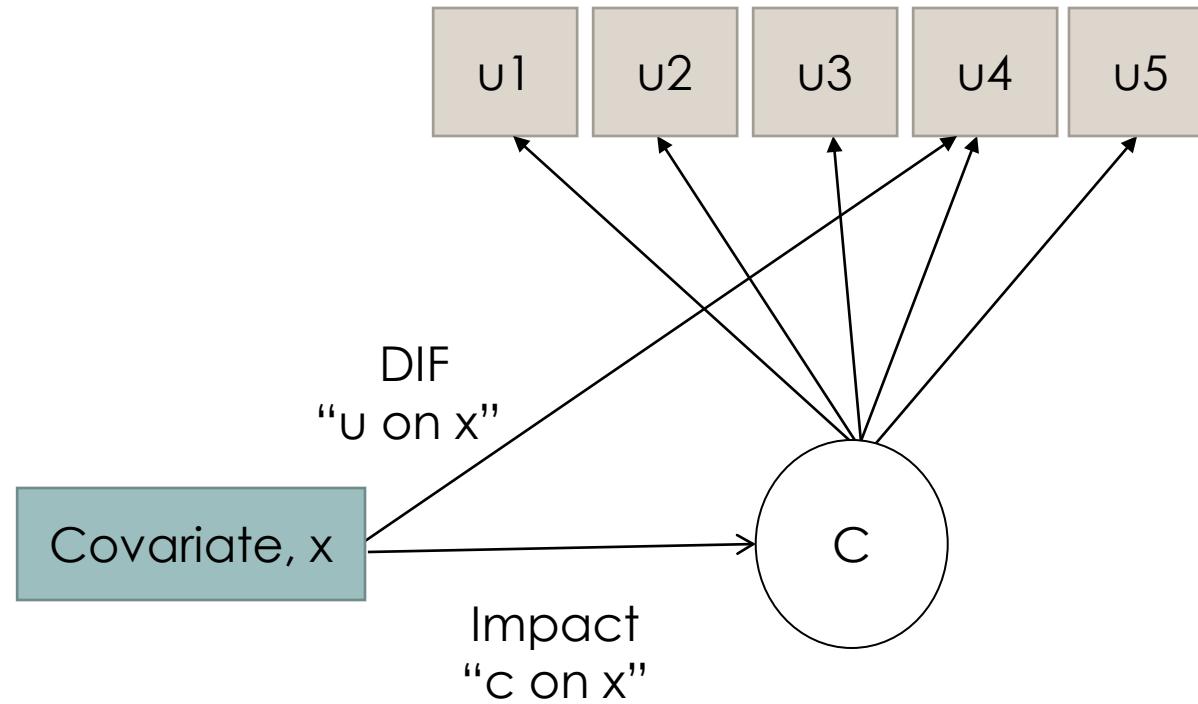
- “Our final example for this moment comes from a study of youth health disparities by Liu et al. (2018). They drew from a nationally representative sample, which included Black, Latinx, and white youth...Empirically, a test of measurement invariance found that a model which assumed identical latent classes for all groups was a poor fit to the data. **Without measurement invariance**, and without a theoretical rationale for measuring whether different racial groups score “higher” or “lower” on their outcome, **looking for such differences between the groups would not only be incorrect, it would generalize to the population conclusions about the racial groups that are taken out of context.**”

Defining DIF

$$\Pr(u_{1i}, u_{2i}, \dots, u_{Mi} | c_i = k, X_i) = \Pr(u_{1i}, u_{2i}, \dots, u_{Mi} | c_i = k), \quad \forall i, k \in \{1, 2, \dots, K\}.$$

- **No DIF:** Two people in the same class with different X values have the same expected outcomes for the latent class indicators.
- **Uniform DIF:** Two people in the same class with different X values have different expected outcomes for the latent class indicators. This difference in expected outcomes is the same for all classes.
- **Nonuniform DIF:** Two people in the same class with different X values have different expected outcomes for the latent class indicators. This difference in expected outcomes is allowed to vary across the latent classes.

Covariates and Mixture Models (LC-MIMIC)



If you ignore DIF (i.e., don't include “ u on x ” in your model when there is, in the population, DIF on u)...

Then there will be bias in your estimates of “ c on x ”.

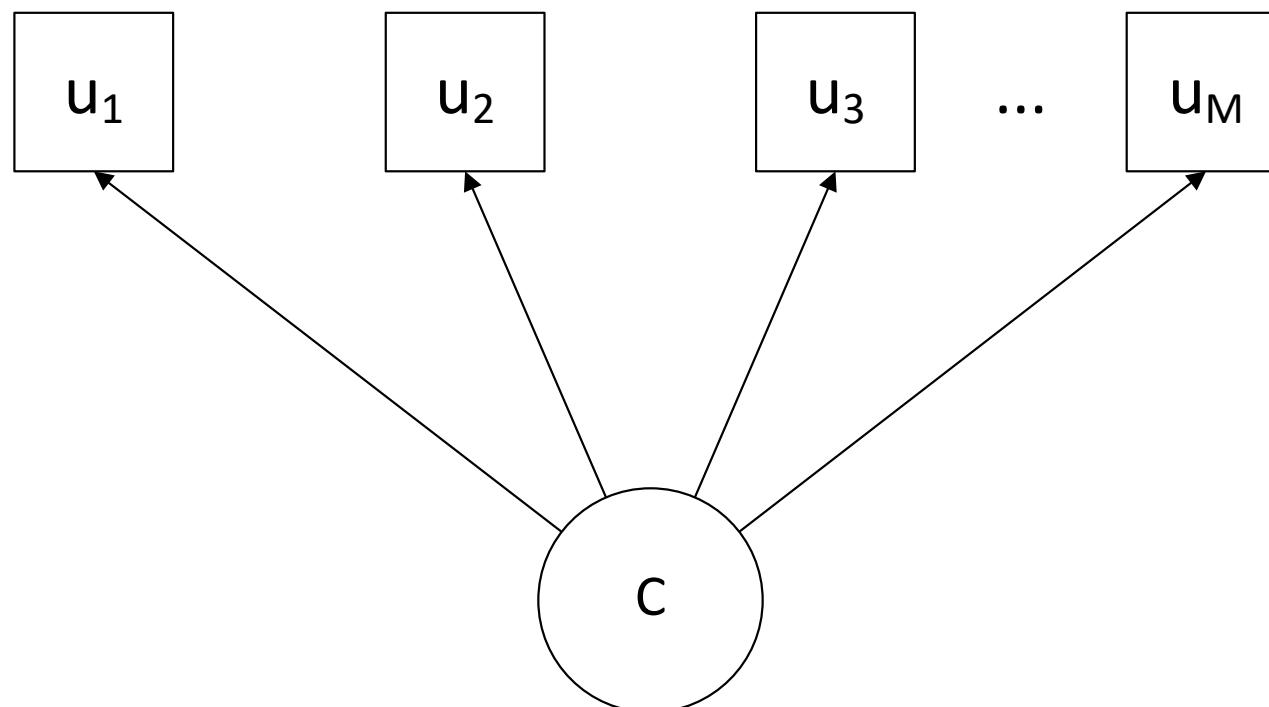
Can't I just 3-Step my way around DIF?



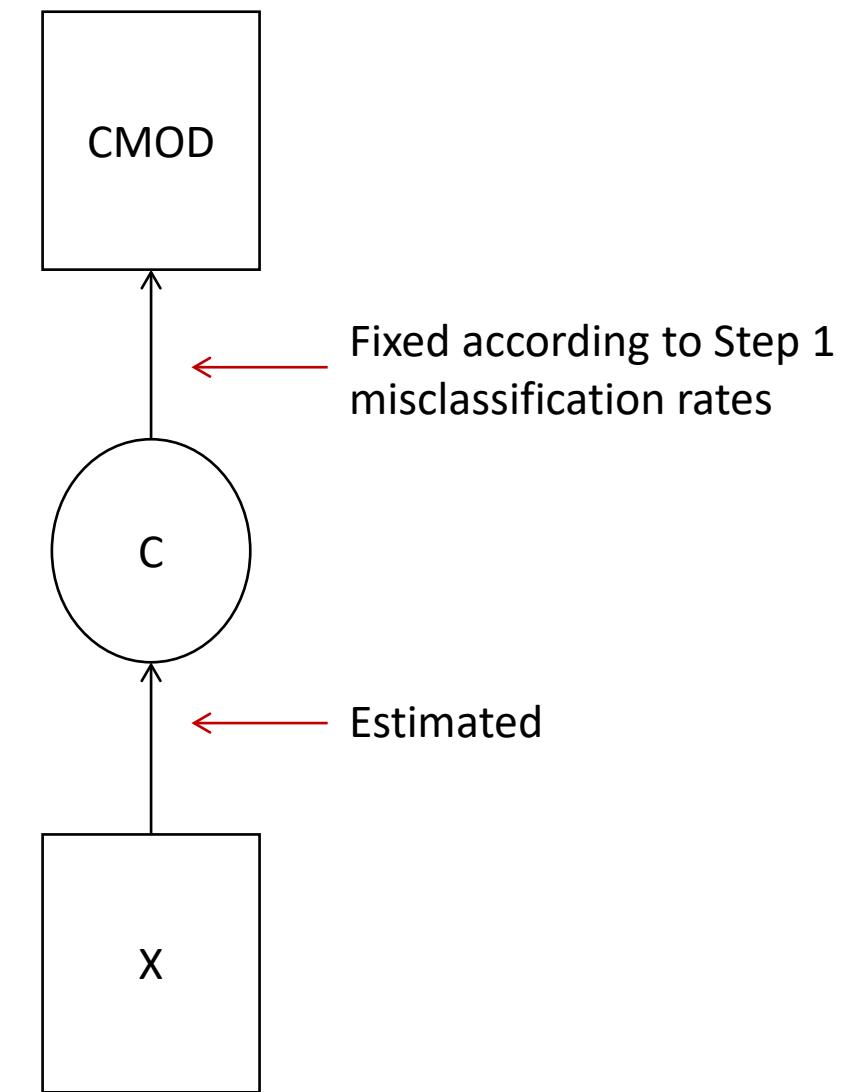
- **NO**, but...
- We certainly *thought* so when the newer stepwise procedures were first implemented. And we said so many times in the literature.



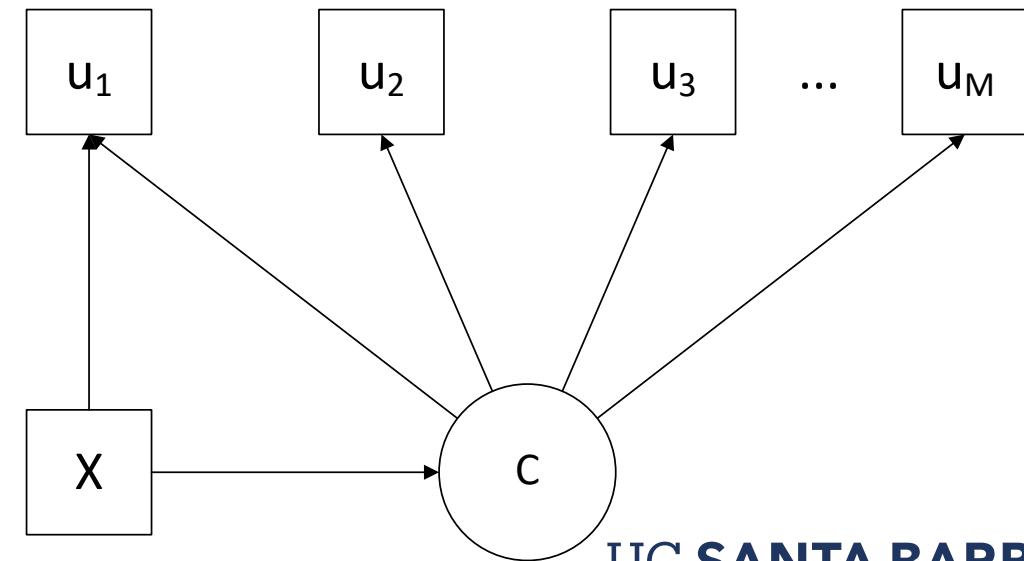
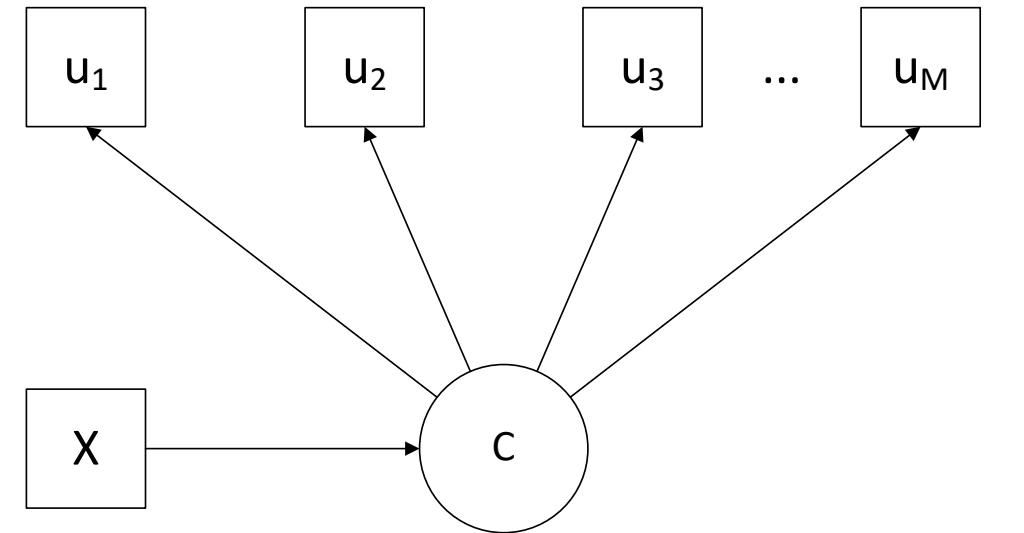
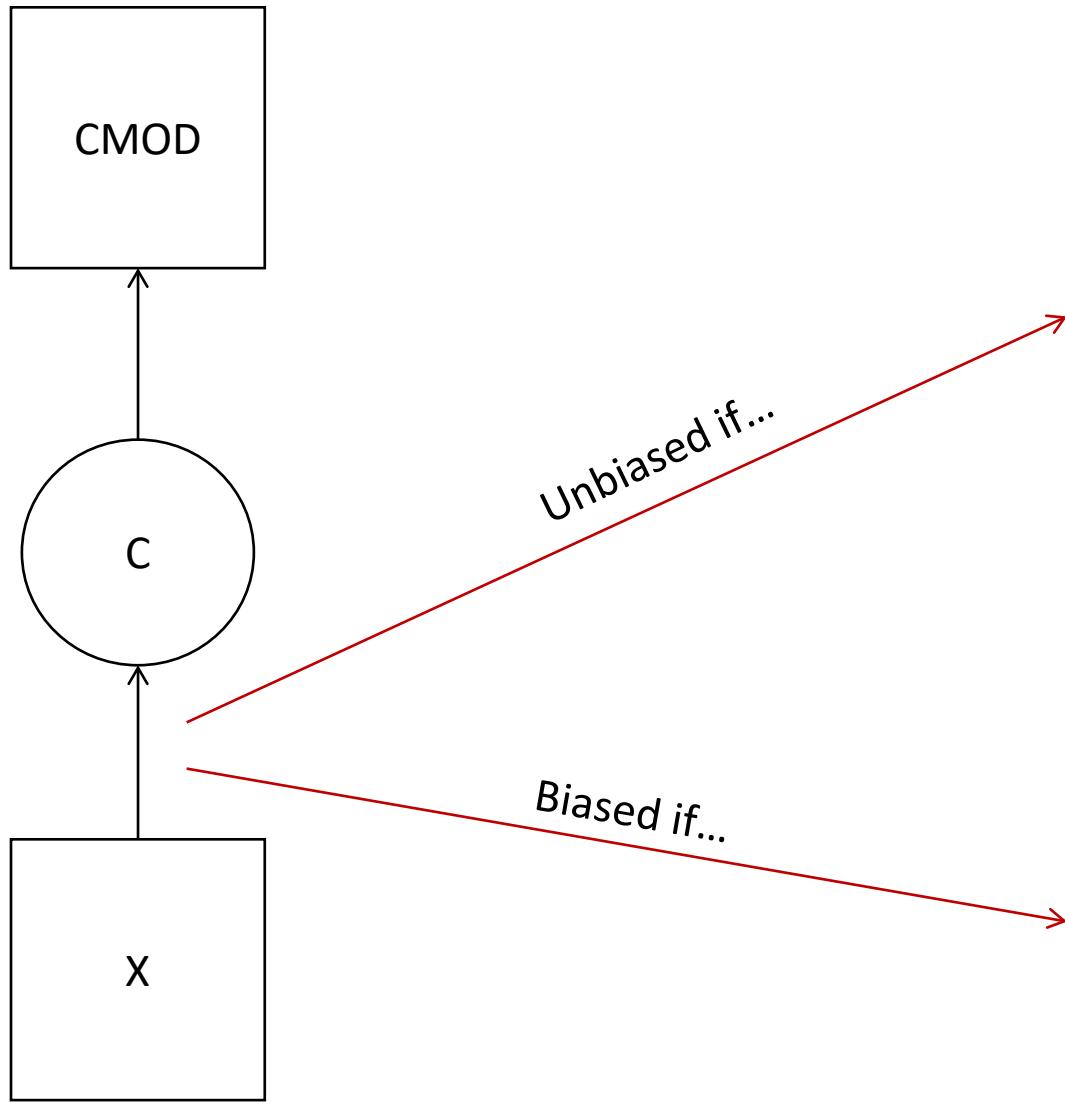
Step 1



Step 3



What we now know....



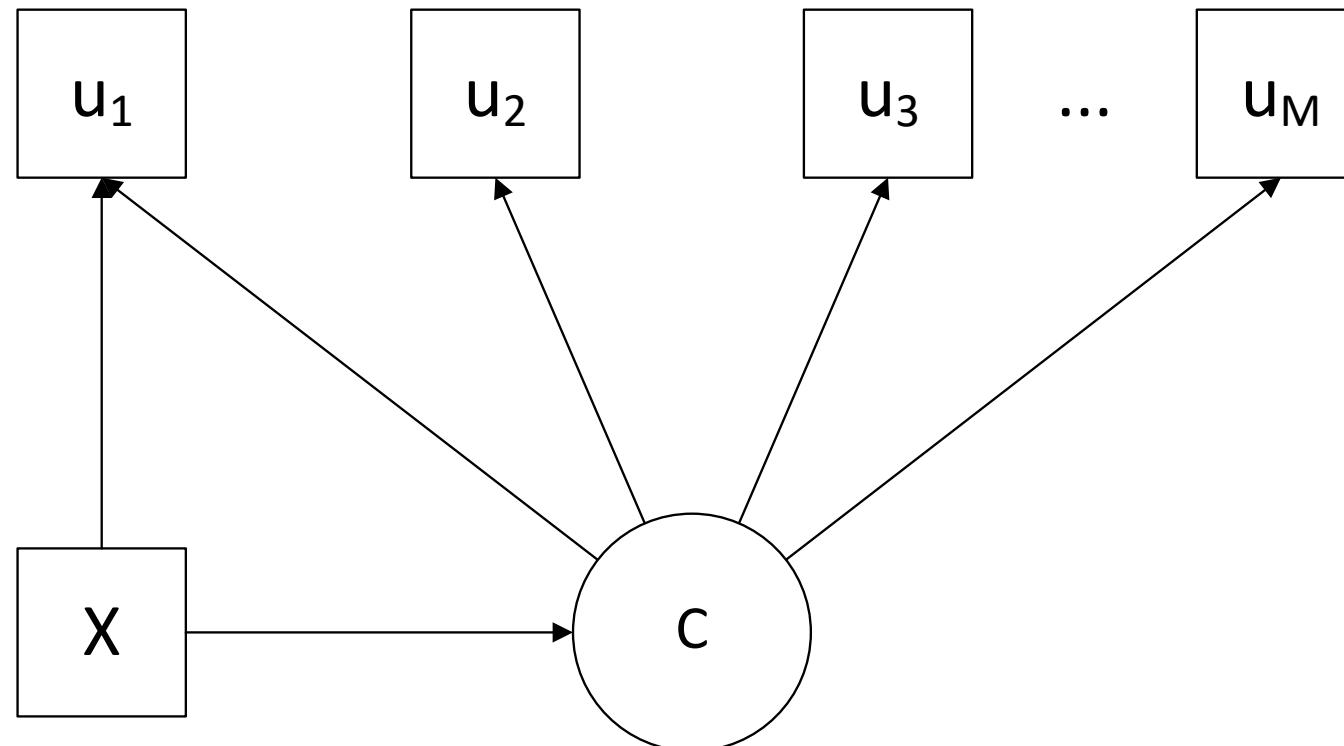
Sorry, ...



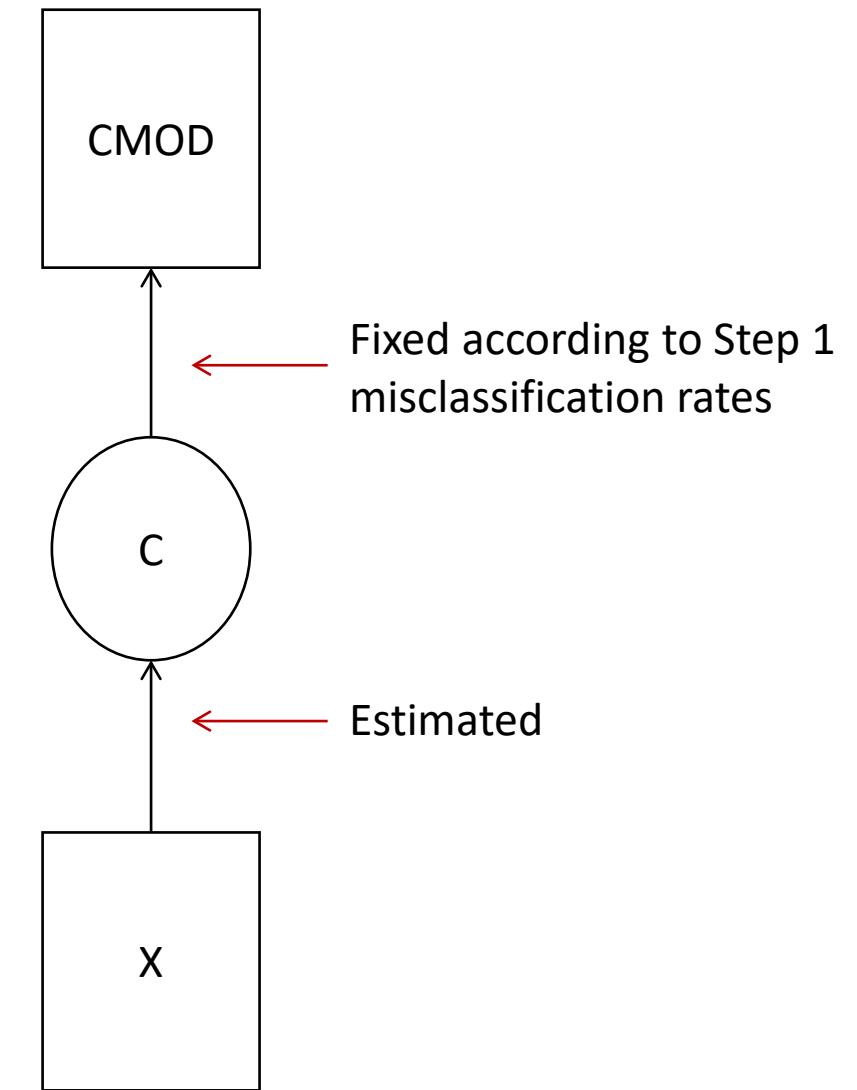
Ain't no way
around it

- You can't ignore measurement non-invariance and DIF in a latent class MIMIC model if you want unbiased structural path estimates, even if you plan to use a step-wise procedure and estimate your measurement and structural models separately.

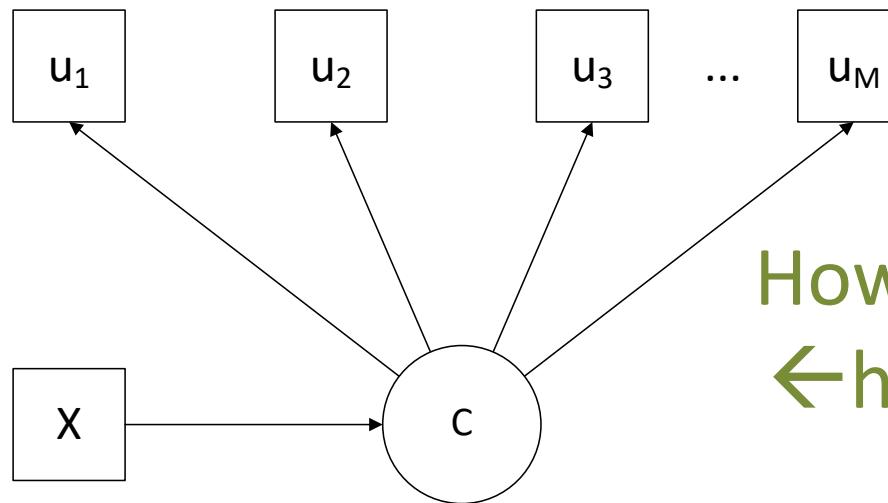
Step 1



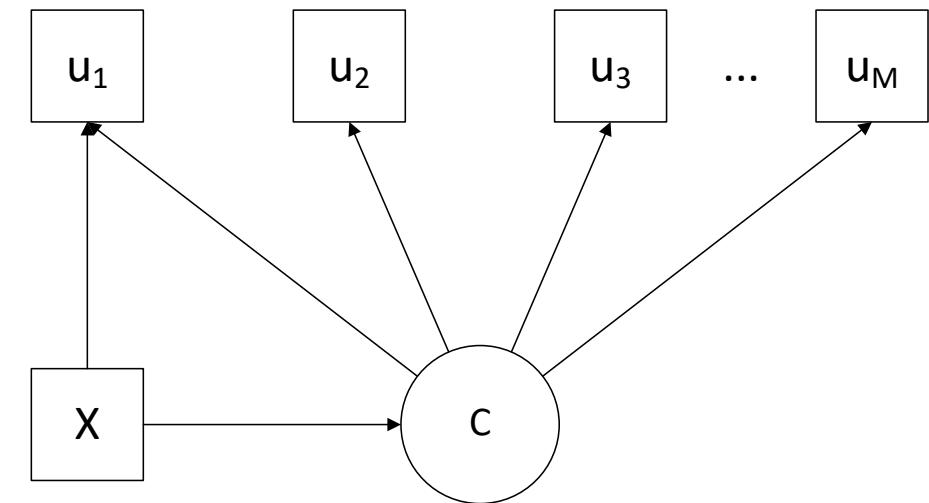
Step 3



Investigating DIF



How do we get from
←here to there? →



LCA-DIF Detection: The general Idea

- Establish the unconditional measurement model.
- Classify individuals (accounting for classification error).
- Examine each item to see if item response depends on X within each latent class (no DIF vs. Nonuniform DIF).
- For items exhibiting DIF, evaluate if DIF is uniform or nonuniform.
- Evaluate “C on X” association, accounting for DIF.

Note: There is also a process for investigating measurement non-invariance using a multiple group approach.
 (“knownclass” option in Mplus.)

Probing for DIF in Mixture Modeling

Structural Equation Modeling: A Multidisciplinary Journal, 24: 180–197, 2017
 Copyright © Taylor & Francis Group, LLC
 ISSN: 1070-5511 print / 1532-8007 online
 DOI: 10.1080/10705511.2016.1254049



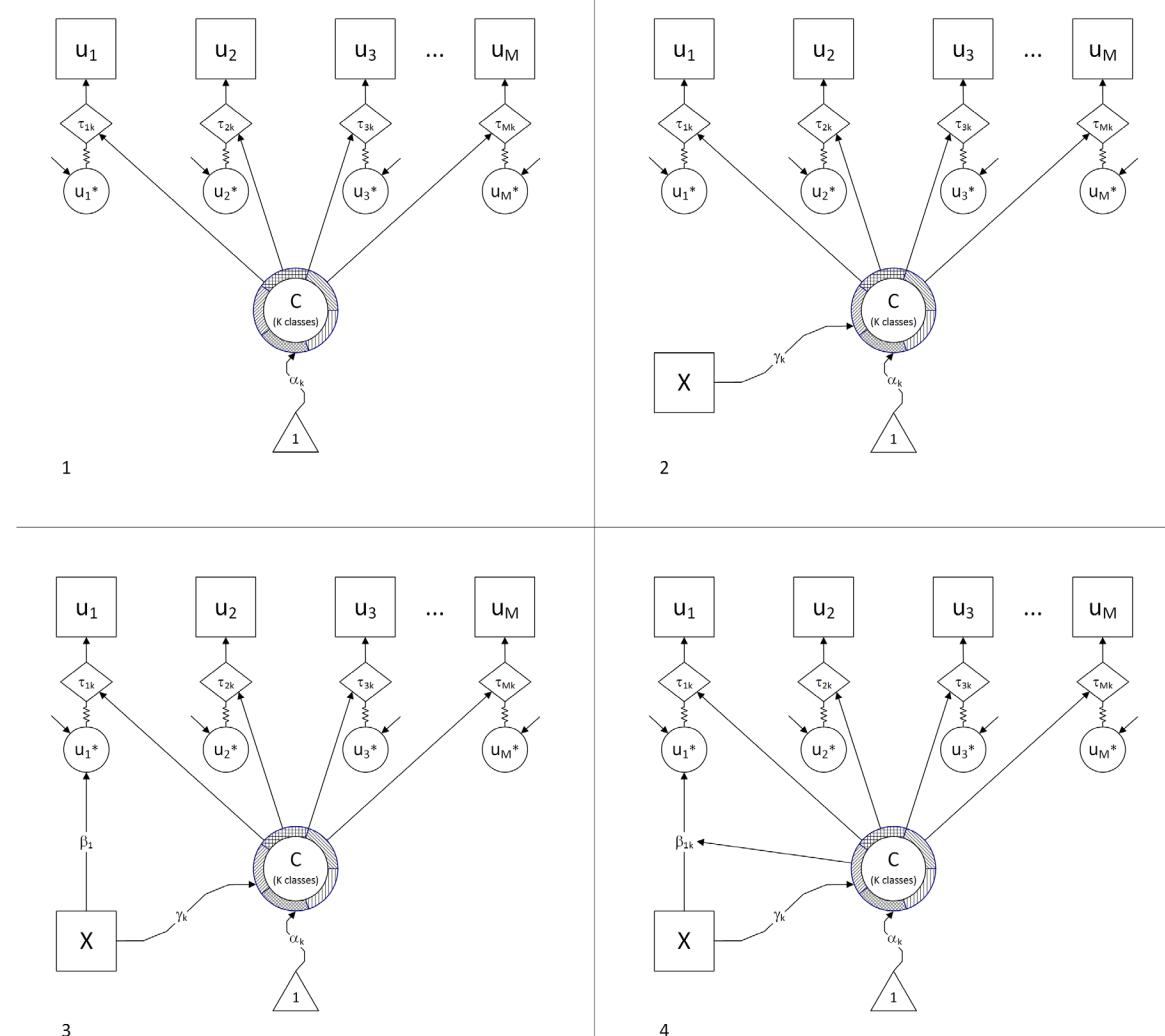
Measurement Invariance and Differential Item Functioning in Latent Class Analysis With Stepwise Multiple Indicator Multiple Cause Modeling

Katherine E. Masyn
Georgia State University

The use of latent class analysis, and finite mixture modeling more generally, has become almost commonplace in social and health science domains. Typically, research aims in mixture model applications include investigating predictors and distal outcomes of latent class membership. The most recent developments for incorporating latent class antecedents and consequences are stepwise procedures that decouple the classification and prediction models. It was initially believed these procedures might avoid the potential misspecification bias in the simultaneous models that include both latent class indicators and predictors. However, if direct effects from the predictors to the indicators are omitted in the stepwise procedure, the prediction model can yield biased estimates. This article presents a logical and principled approach, readily implemented in current software, to testing for direct effects from latent class predictors to indicators using multiple indicator multiple cause modeling. This approach is illustrated with real data and opportunities for future developments are discussed.

Keywords: differential item functioning, latent class analysis, measurement invariance, MIMIC model, mixture model

ONLINE APPENDIX
 for
 Measurement Invariance and Differential Item Functioning in Latent Class Analysis
 with Stepwise Multiple Indicator Multiple Cause Modeling
 Katherine E. Masyn
 Georgia State University School of Public Health



Masyn, K. E. (2017). Measurement invariance and differential item functioning in latent class analysis with stepwise multiple indicator multiple cause modeling. *Structural Equation Modeling: A Multidisciplinary Journal*, 24(2), 180-197.

Latent Profile Analysis (LCA with continuous indicators)

In the Beginning...



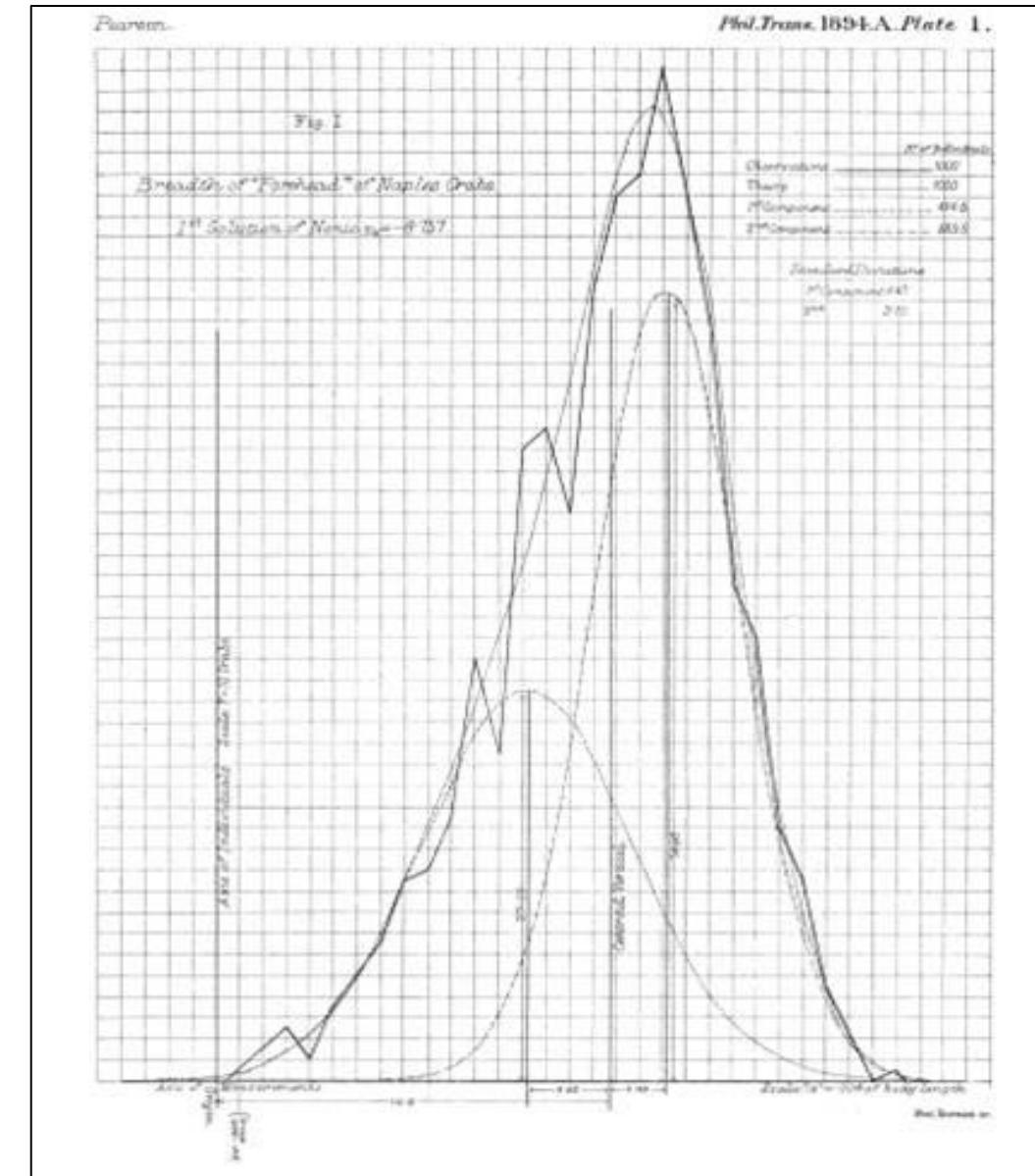
Karl Pearson (1894) – fit a mixture of two normal distributions with different means and variances to measurements of the ratio of forehead to body length of crabs to infer that the crabs had evolved into two separate species

- estimation of model parameters was accomplished with a new technique at the time called *method of moments*

Karl Pearson (1894)...

“...the asymmetry may arise from the fact that the units grouped together in the measured material are not really homogeneous. It may happen that we have a mixture of $2, 3, \dots, n$ homogenous groups, each of which deviates about its own mean symmetrically and in a manner represented with sufficient accuracy by the normal curve (p. 72).”

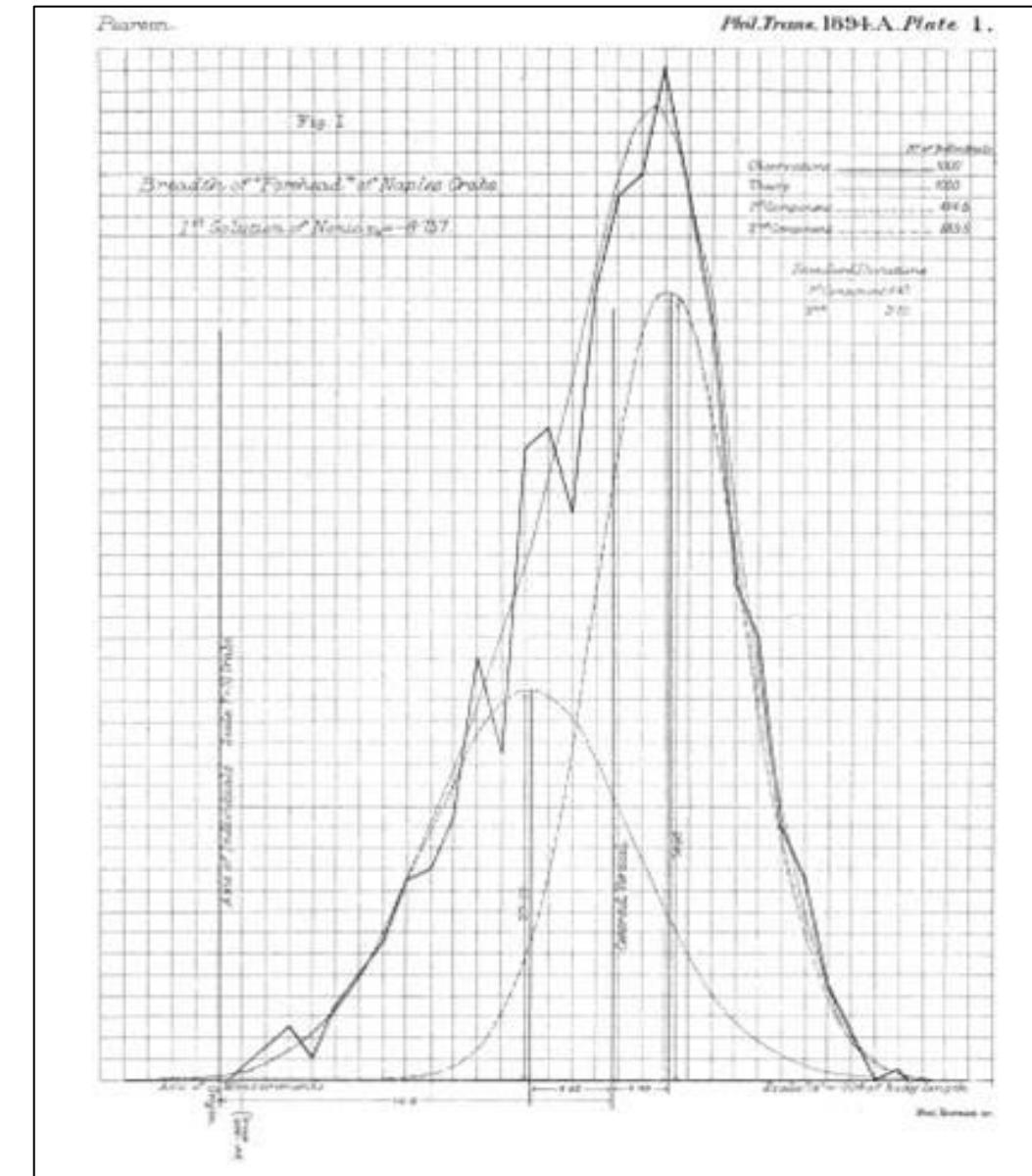
Courtesy of © J. Harring (April 2018)



Karl Pearson (1894)...

“...for the special case of $n = 2$ treated in this paper; they require us only to calculate higher moments. But the analytical difficulties, even for the case of $n = 2$, are so considerable, that it may be questioned whether the general theory could ever be applied in practice to any numerical case (p. 72).”

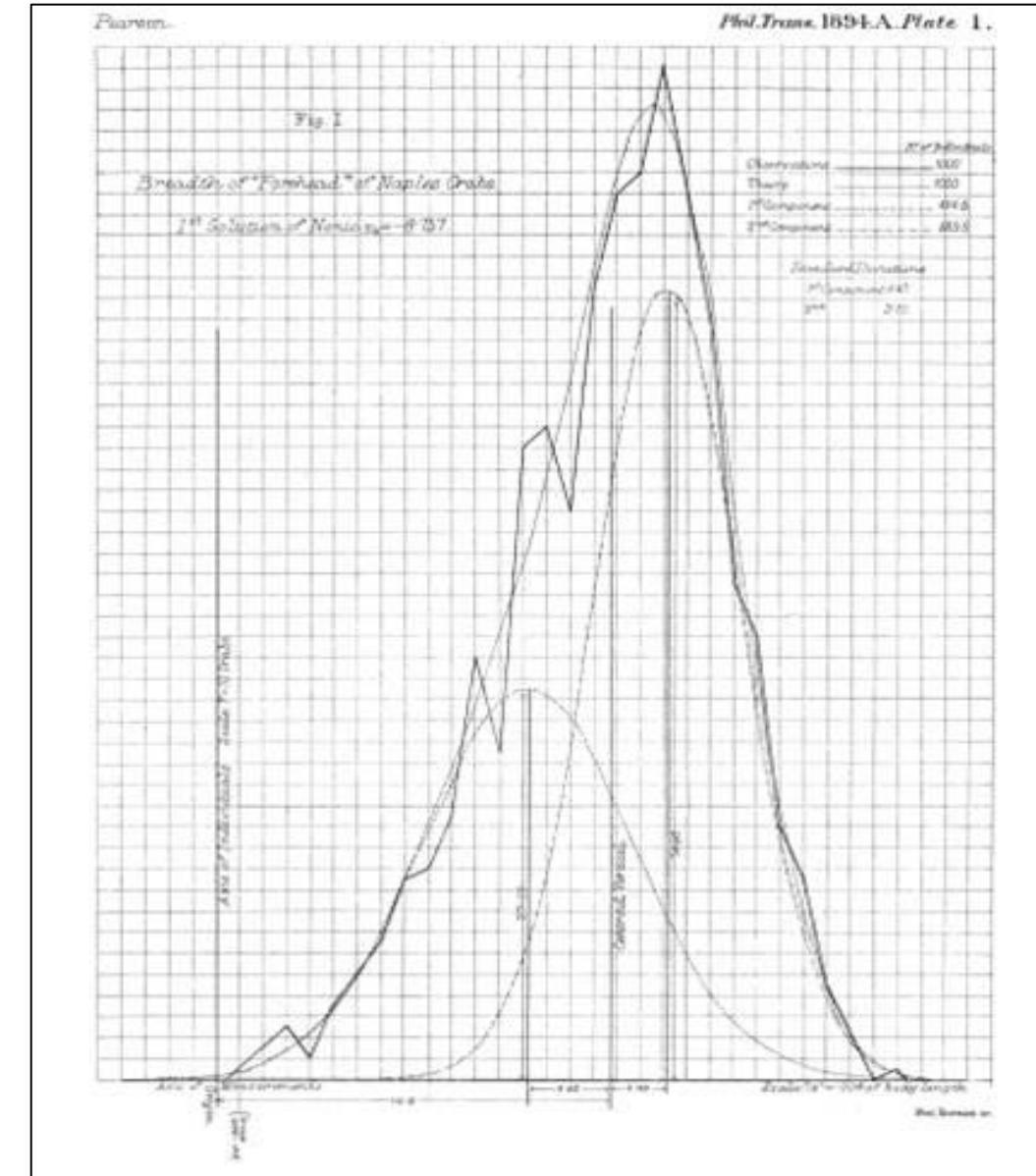
Courtesy of © J. Harring (April 2018)

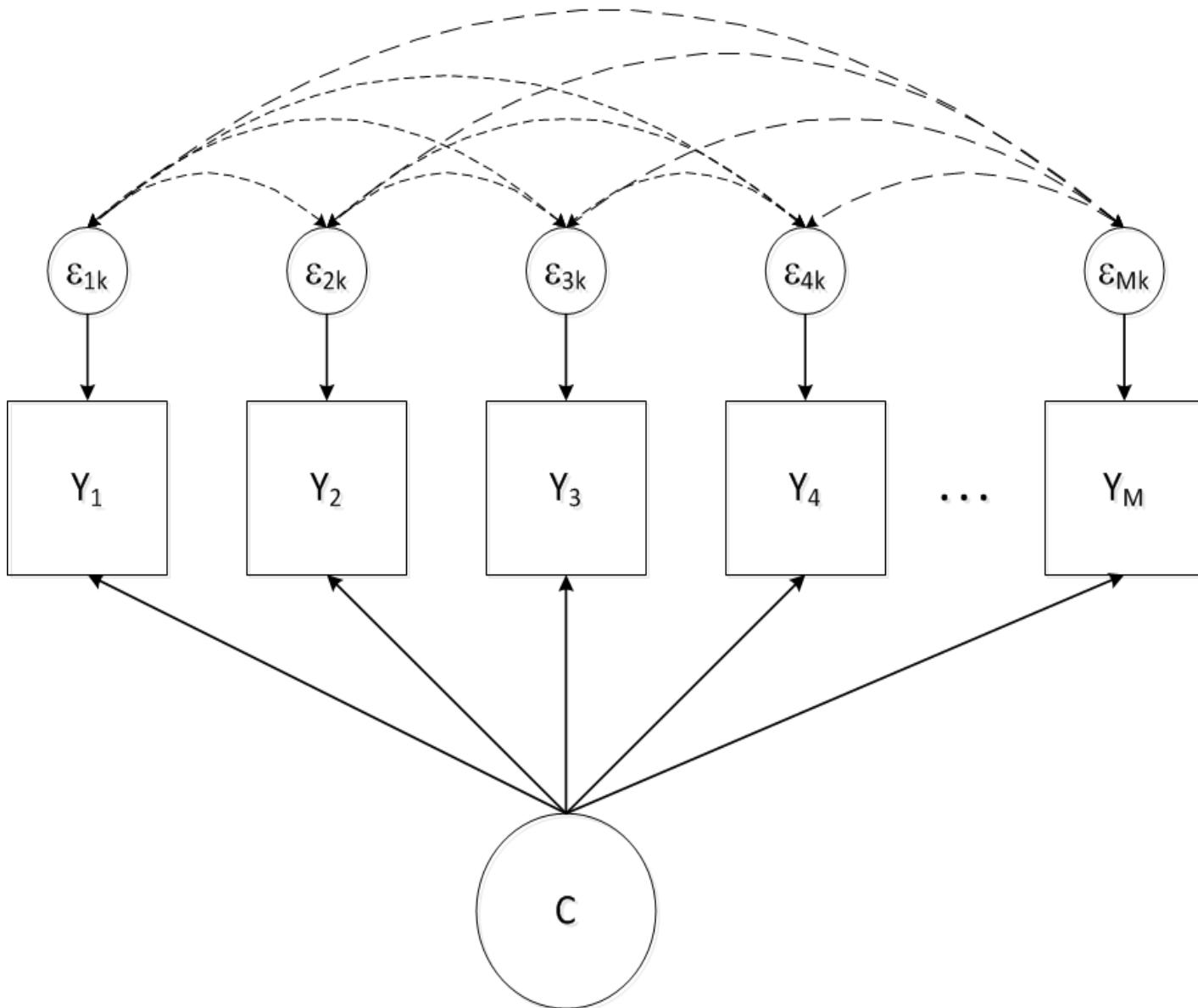


Karl Pearson (1894)...

“...on the other hand, I cannot think that for the problem of evolution the dissection of the most symmetrical curve given by the measurements is unnecessary. There will always be the problem : Is the material homogenous and a true evolution going on, or is the material a mixture? To throw the solution on the judgment of the eye in examining the graphical results is, I feel certain, quite futile (p. 99).”

Courtesy of © J. Harring (April 2018)





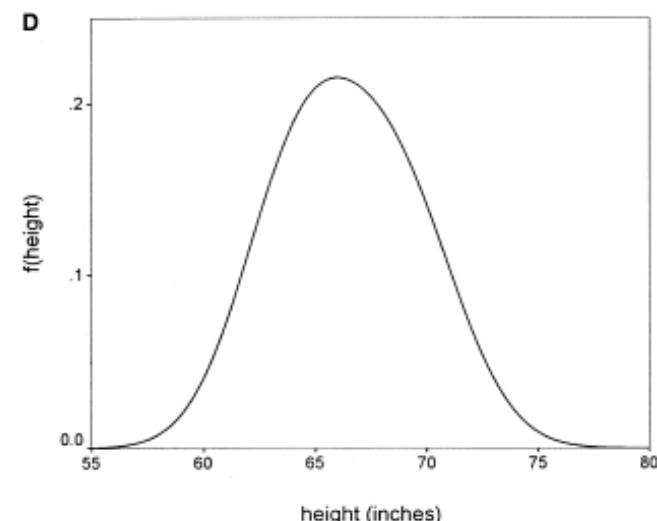
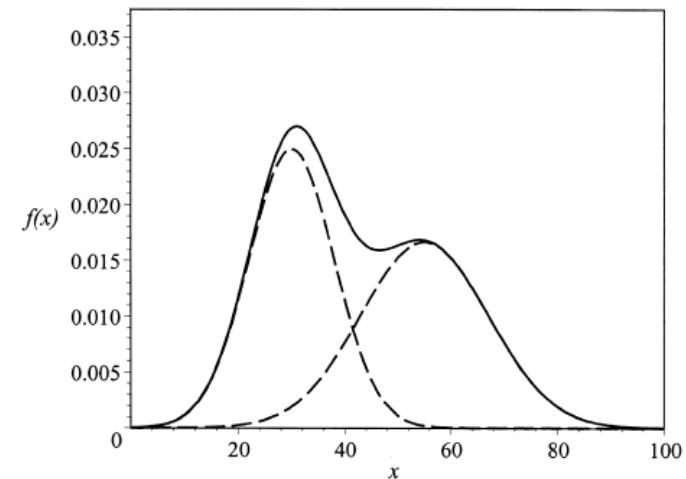
- The basic finite mixture model has the following likelihood function:

$$f(\mathbf{y}_i | \boldsymbol{\theta}) = \sum_{k=1}^K \pi_k f_k(\mathbf{y}_i | \boldsymbol{\theta}_k)$$

- Typically, f_k is assumed to be a (multivariate) normal density.
- In LPA, the measurement parameters are the class-specific:
 - **means**, μ_k
 - **variances** of the observed variables
 - **covariances** between the observed variables, Σ_k .

Model-Based Classification: Finite Mixture Models

- “[Mixture modeling] may provide an approximation to a complex but unitary population distribution of individual trajectories” (Bauer & Curran, 2003, p. 339)
- Consider two examples
 - A lognormal distribution MAY BE correctly approximated as being composed of two simpler curves
 - A normal distribution is correctly approximated as being composed of one simple curve
- “Not only is nonnormality *required* for the solution of the model to be nontrivial, it may well also be a *sufficient* condition for extracting multiple components.” (Bauer & Curran, 2003, 343)





- Some of these drawbacks can be mitigated if one abandons the belief that mixture modeling is able to recover the “true” populations that have been sampled
- Muthén (2003) writes that “there are many examples of equivalent models in statistics” (p. 376). A better approach may be to view mixture modeling as presenting a model of what populations may have been sampled
- Here’s what we really care about:
 - Is the finite mixture model solution consistent with the data (i.e., does it fit the data?)
 - Is the finite mixture model solution useful and substantively meaningful?

IMPORTANT:

The choice you make about f_k and the within-class variance/covariance structure, Σ_k , **WILL** influence the number and nature of latent classes in your final model selection → You must consider different forms for Σ_k during your model building process.



The more restrictive your Σ_k structure is, the more work the latent class variable has to do in explaining the observed var/cov and **you will probably need more classes.**

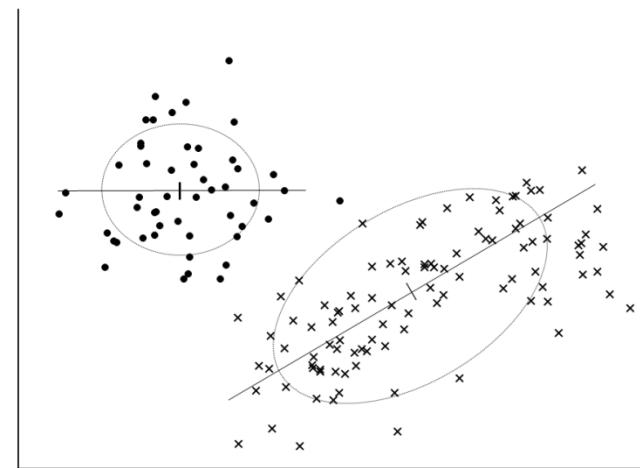
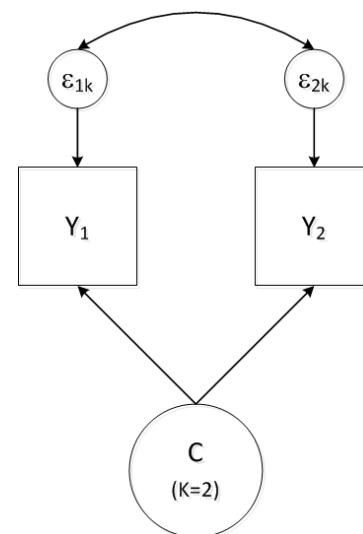
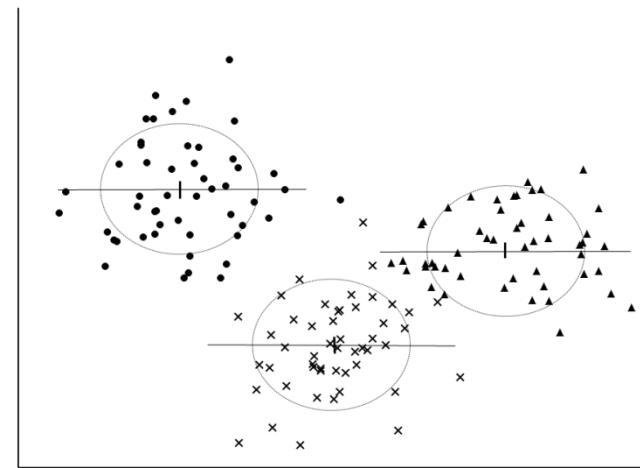
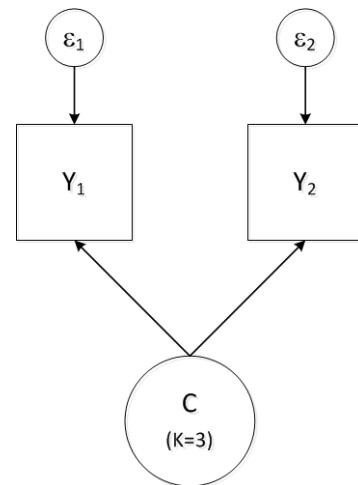
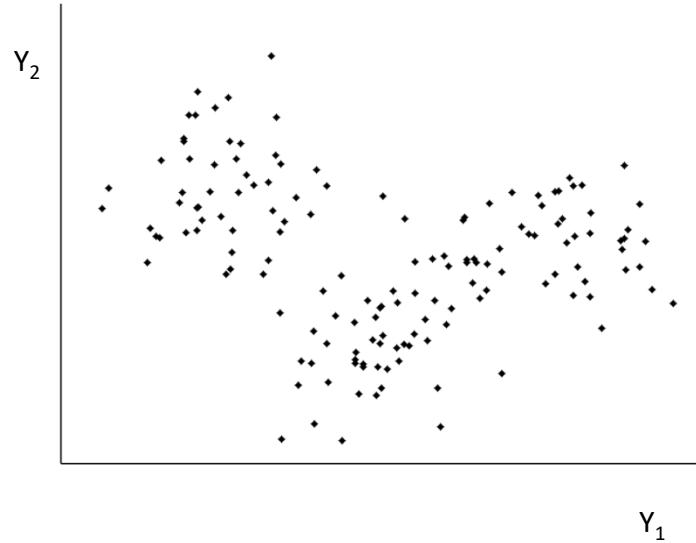


The less restrictive your Σ_k structure is, the more complicated the class profiles and interpretations become (as classes are distinguished not only by class-specific means but also class-specific var/cov).



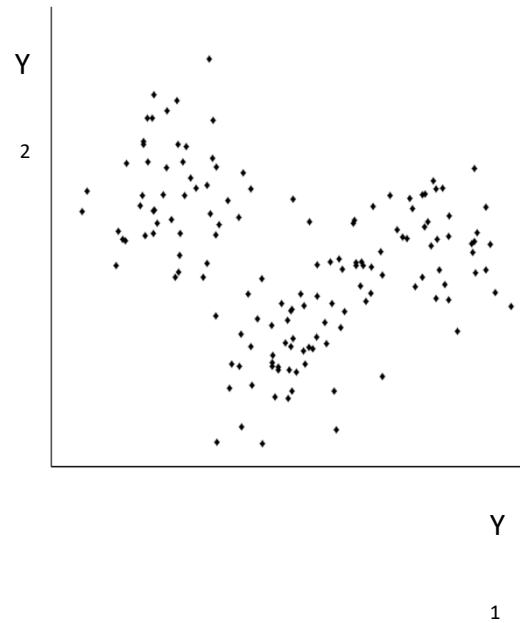
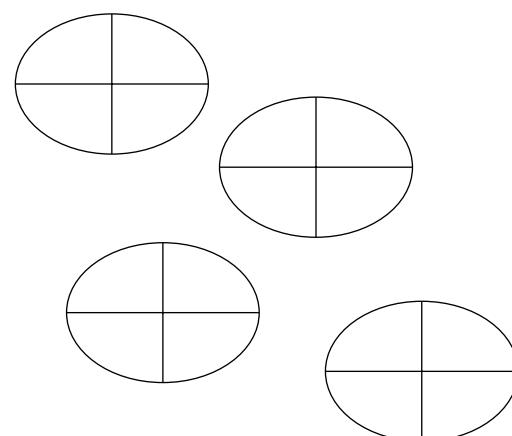
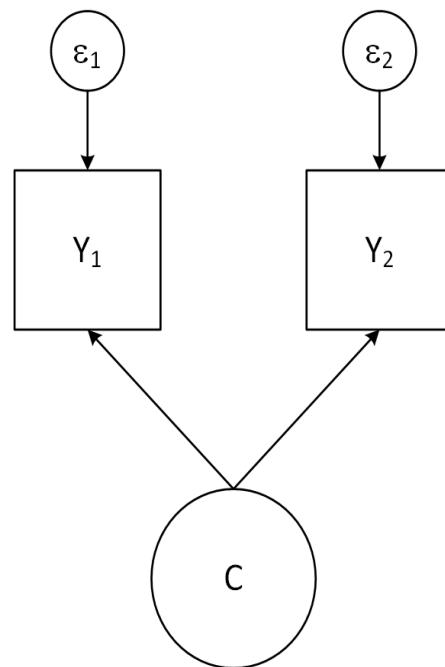
Mixture models with more classes are not always less parsimonious—that very much depends on how many parameters are permitted to be class-varying.

f(x)



What structures of Σ_k should you consider?

- Σ_k **diagonal** (conditional independence—latent class membership explains all the observed covariation) and **class-invariant**.
- Default in Mplus (Model 1)
 - Diagonal \rightarrow no item correlation
 - Invariant \rightarrow item variances are equal across class

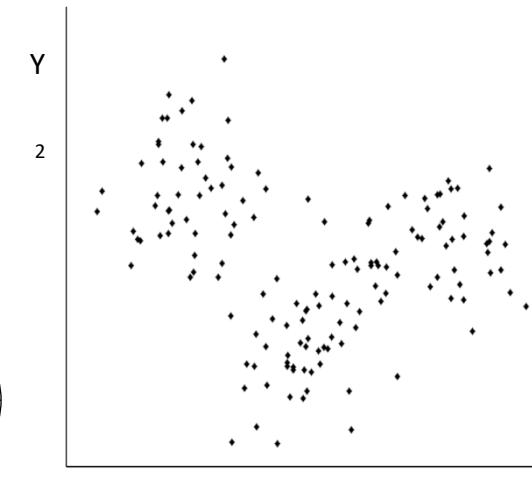
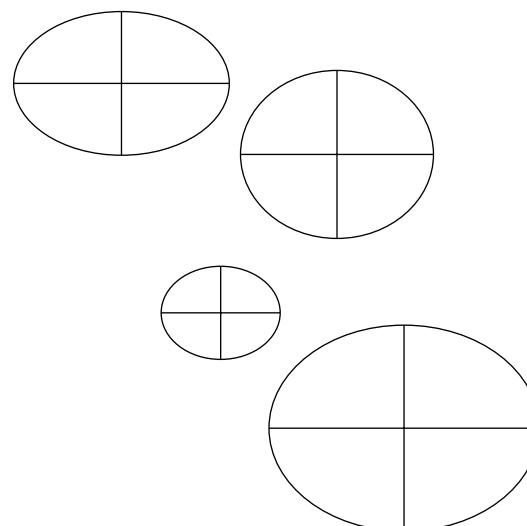
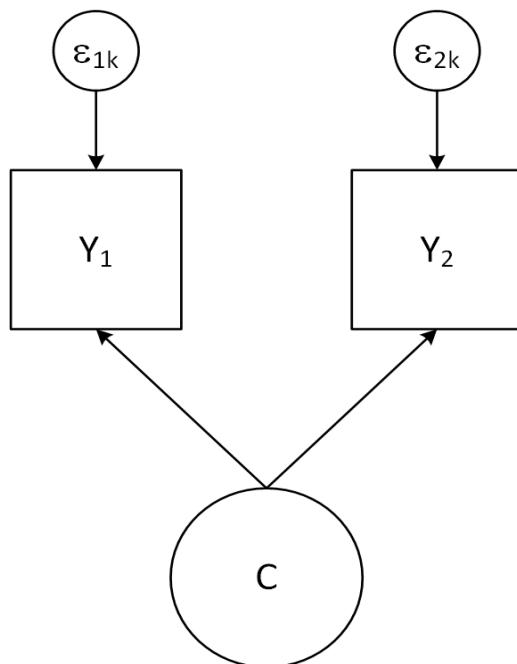


1

What structures of Σ_k should you consider?

- Σ_k **diagonal** and **class-varying** (Model 2)

- Diagonal → no item correlation
- Class varying → item variances are not held equal across class

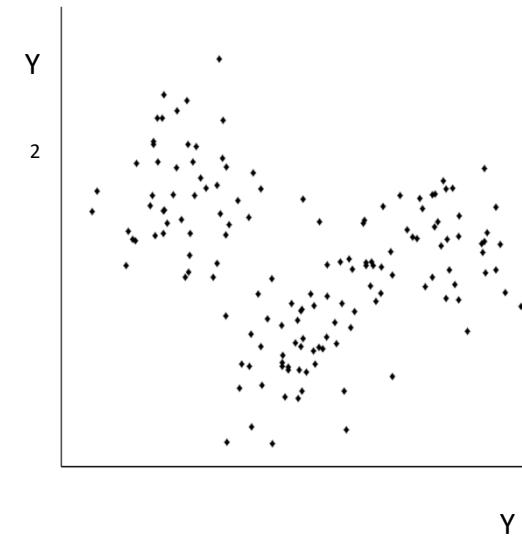
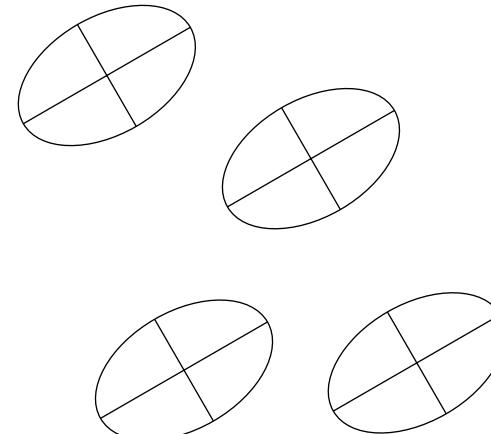
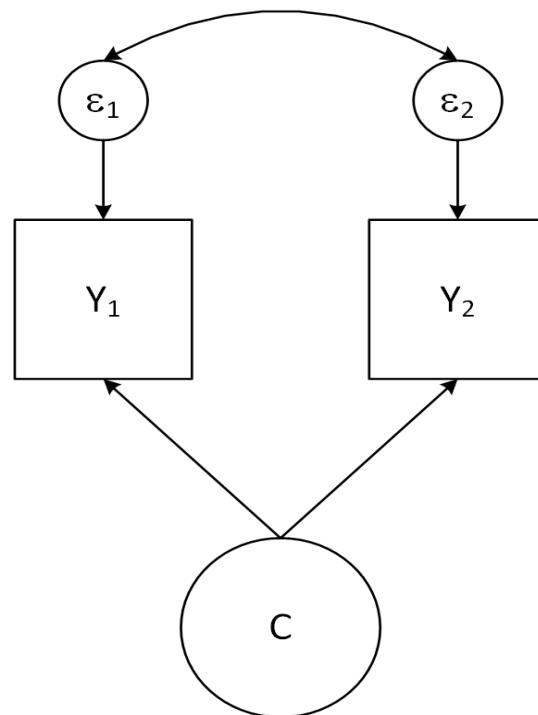


1

What structures of Σ_k should you consider?

- Σ_k **non-diagonal** and **class-invariant** (Model 3)

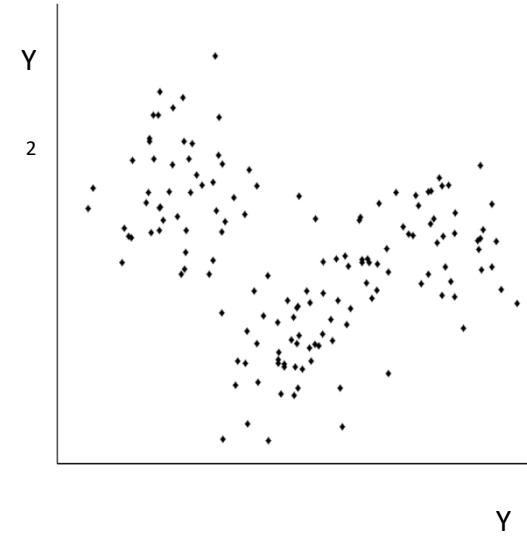
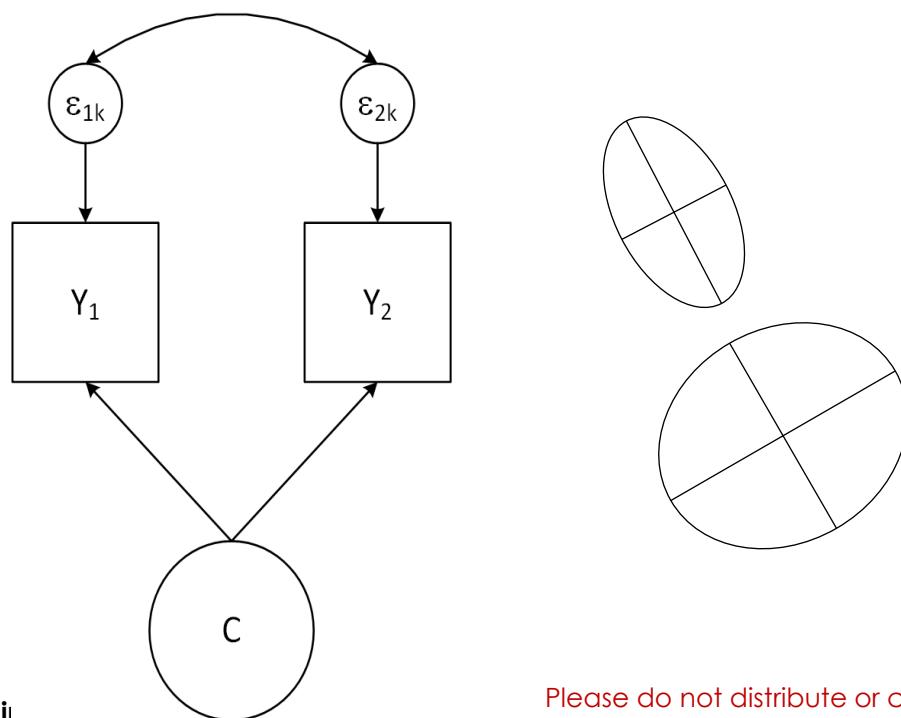
- Non-diagonal → allows for item correlation
- Invariant → item variances are equal across class



1

What structures of Σ_k should you consider?

- Σ_k **non-diagonal** and **class-varying** (Model 4)
 - Non-diagonal → allows for item correlation
 - Class varying → item variances are not held equal across class
 - This specification likely will need far fewer classes and is also likely with only a few classes to become weakly or empirically unidentified, failing to converge during estimation.



Model 1: Σ_k diagonal and class-invariant (default)

```
Data: file is LPA.dat;
Variable: Names are T1Age T1Sex T1ID2009 T1BESSC1 T2BESSC1 T3BESSC1
          T4BESSC1 T1BESBIN T2BESBIN T3BESBIN T4BESBIN T1BESCON T2BESCON
          T3BESCON T4BESCON;

usevariables are T1BESCON T2BESCON T3BESCON T4BESCON;

Missing = all (-9999);
class=c(3);

Analysis: type = mixture;
           starts=1000 100;

Output: tech1 tech11 tech14 sampstat;
Plot: type=plot3;
      series=T1BESCON-T4BESCON (*) ;
```

Model 1: Σ_k diagonal and class-invariant (default)

Estimate	S.E.	Est./S.E.	P-Value
Latent Class 1			
Means			
T1BESCON	44.241	1.425	31.056 0.000
T2BESCON	42.275	2.186	19.336 0.000
T3BESCON	45.910	6.304	7.283 0.000
T4BESCON	43.069	2.479	17.374 0.000
Variances			
T1BESCON	60.111	22.293	2.696 0.007
T2BESCON	41.847	10.226	4.092 0.000
T3BESCON	84.941	53.638	1.584 0.113
T4BESCON	60.048	12.901	4.654 0.000
Latent Class 2			
Means			
T1BESCON	56.577	5.593	10.117 0.000
T2BESCON	53.863	3.688	14.606 0.000
T3BESCON	56.766	2.405	23.599 0.000
T4BESCON	55.151	5.108	10.798 0.000
Variances			
T1BESCON	60.111	22.293	2.696 0.007
T2BESCON	41.847	10.226	4.092 0.000
T3BESCON	84.941	53.638	1.584 0.113
T4BESCON	60.048	12.901	4.654 0.000
Latent Class 3			
Means			
T1BESCON	70.324	6.904	10.186 0.000
T2BESCON	68.463	8.613	7.949 0.000
T3BESCON	71.512	2.922	24.476 0.000
T4BESCON	70.883	5.021	14.117 0.000
Variances			
T1BESCON	60.111	22.293	2.696 0.007
T2BESCON	41.847	10.226	4.092 0.000
T3BESCON	84.941	53.638	1.584 0.113
T4BESCON	60.048	12.901	4.654 0.000

Means

The variances are the same

Model 2: Σ_k diagonal and class-varying

```
data: file is LPA.dat;
Variable: Names are T1Age T1Sex T1ID2009 T1BESSC1 T2BESSC1 T3BESSC1
          T4BESSC1 T1BESBIN T2BESBIN T3BESBIN T4BESBIN T1BESCON T2BESCON
          T3BESCON T4BESCON;

usevariables are T1BESCON T2BESCON T3BESCON T4BESCON;

Missing = all (-9999);
class=c(3);

Analysis: type = mixture;
starts=1000 100;
Model:
%overall%
%c#1%
  T1BESCON T2BESCON T3BESCON T4BESCON;
%c#2%
  T1BESCON T2BESCON T3BESCON T4BESCON;
%c#3%
  T1BESCON T2BESCON T3BESCON T4BESCON;
output: tech1 tech11 tech14 sampstat;
plot: type=plot3;
series=T1BESCON-T4BESCON (*) ;
```

By mentioning the variables in class specific statements, you are telling Mplus to estimate class-specific variances (e.g., class-varying variances).

In this example, we are allowing ALL the variables variances to be free across ALL the classes. You can change this.

Model 2: Σ_k diagonal and class-varying

```
%overall%
%c#1%
  T1BESCON T2BESCON T3BESCON T4BESCON;
%c#2%
  T1BESCON T2BESCON T3BESCON T4BESCON;
%c#3%
  T1BESCON T2BESCON T3BESCON T4BESCON;
```

Notice that the item variances are all different across the latent classes.

Latent Class 1
Means

T1BESCON	44.105	1.320	33.401	0.000
T2BESCON	42.352	1.092	38.786	0.000
T3BESCON	46.729	1.588	29.425	0.000
T4BESCON	43.348	1.101	39.366	0.000

Variances

T1BESCON	45.914	12.361	3.714	0.000
T2BESCON	40.028	5.940	6.738	0.000
T3BESCON	122.321	30.379	4.027	0.000
T4BESCON	45.295	7.672	5.904	0.000

Latent Class 2
Means

T1BESCON	57.869	1.325	43.688	0.000
T2BESCON	54.590	1.083	50.395	0.000
T3BESCON	56.332	1.257	44.810	0.000
T4BESCON	56.278	1.807	31.152	0.000

Variances

T1BESCON	66.527	12.143	5.479	0.000
T2BESCON	35.010	6.625	5.285	0.000
T3BESCON	47.544	8.838	5.379	0.000
T4BESCON	101.220	26.352	3.841	0.000

Latent Class 3
Means

T1BESCON	68.601	3.068	22.360	0.000
T2BESCON	69.135	3.253	21.253	0.000
T3BESCON	74.366	3.152	23.591	0.000
T4BESCON	68.179	0.168	405.870	0.000

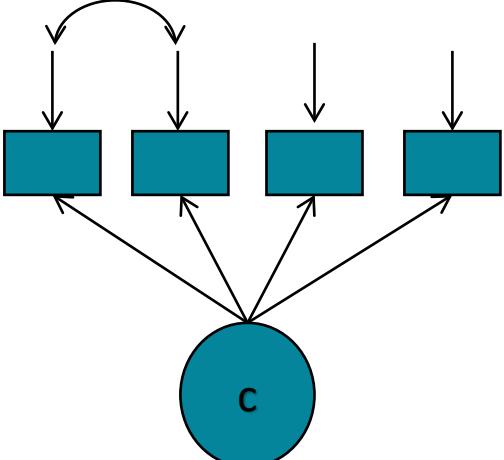
Variances

T1BESCON	113.859	53.308	2.136	0.033
T2BESCON	90.142	30.997	2.908	0.004
T3BESCON	85.316	29.706	2.872	0.004
T4BESCON	86.147	26.102	1.366	0.174

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Model 3: Σ_k non-diagonal and class-invariant

Model:
 %overall%
 %c#1%
 T1BESCON with T2BESCON; ←
 %c#2%
 T1BESCON with T2BESCON; ←
 %c#3%
 T1BESCON with T2BESCON; ←



We are adding a covariance between T1 item and T2

Latent Class 1	T1BESCON WITH T2BESCON Means	26.517	24.847	1.067	0.286
T1BESCON	44.085	1.921	22.954	0.000	
T2BESCON	41.181	3.566	11.550	0.000	
T3BESCON	40.941	4.024	10.174	0.000	
T4BESCON	40.773	1.984	20.553	0.000	
Variances					
T1BESCON	84.772	13.959	6.073	0.000	
T2BESCON	57.280	10.547	5.431	0.000	
T3BESCON	61.559	10.122	6.081	0.000	
T4BESCON	50.733	13.878	3.656	0.000	
Latent Class 2	T1BESCON WITH T2BESCON Means	40.153	10.334	3.886	0.000
T1BESCON	53.274	2.023	26.335	0.000	
T2BESCON	51.182	1.608	31.830	0.000	
T3BESCON	55.039	1.995	27.582	0.000	
T4BESCON	51.309	2.469	20.780	0.000	
Variances					
T1BESCON	84.772	13.959	6.073	0.000	
T2BESCON	57.280	10.547	5.431	0.000	
T3BESCON	61.559	10.122	6.081	0.000	
T4BESCON	50.733	13.878	3.656	0.000	
Latent Class 3	T1BESCON WITH T2BESCON Means	32.086	12.553	2.556	0.011
T1BESCON	65.679	2.914	22.541	0.000	
T2BESCON	64.072	2.652	24.162	0.000	
T3BESCON	70.656	2.736	25.826	0.000	

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UC SANTA BARBARA

But, we could get specific...

Model 3a: Getting specific about diagonal elements.

You may look at the output and think that two class-specific item correlations look similar and then constrain them to be equal.

This is a hybrid version of Model 3. Let's call it Model 3a.

Latent Class 1					
T1BESCON WITH					
T2BESCON		26.517	24.847	1.067	0.286
Means					
T1BESCON		44.085	1.921	22.954	0.000
T2BESCON		41.181	3.566	11.550	0.000
T3BESCON		40.941	4.024	10.174	0.000
T4BESCON		40.773	1.984	20.553	0.000
Variances					
T1BESCON		84.772	13.959	6.073	0.000
T2BESCON		57.280	10.547	5.431	0.000
T3BESCON		61.559	10.122	6.081	0.000
T4BESCON		50.733	13.878	3.656	0.000
Latent Class 2					
T1BESCON WITH					
T2BESCON		40.153	10.334	3.886	0.000
Means					
T1BESCON		53.274	2.023	26.335	0.000
T2BESCON		51.182	1.608	31.830	0.000
T3BESCON		55.039	1.995	27.582	0.000
T4BESCON		51.309	2.469	20.780	0.000
Variances					
T1BESCON		84.772	13.959	6.073	0.000
T2BESCON		57.280	10.547	5.431	0.000
T3BESCON		61.559	10.122	6.081	0.000
T4BESCON		50.733	13.878	3.656	0.000
Latent Class 3					
T1BESCON WITH					
T2BESCON		32.086	12.553	2.556	0.011
Means					
T1BESCON		65.679	2.914	22.541	0.000

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Model 3A: Σ_k non-diagonal (constrained) and class-invariant

```

Model:
%overall%
%c#1%
  T1BESCON with T2BESCON(1);
%c#2%
  T1BESCON with T2BESCON ;
%c#3%
  T1BESCON with T2BESCON(1);

```

In this model, instead of estimating 3 class specific covariance, we estimate only one.

Note, that since we are keeping the number of classes constant here, we could do LL difference testing if we wanted.

Latent Class 1				
T1BESCON WITH T2BESCON	Means			
30.546	13.052	2.340	0.019	
44.246	1.964	22.529	0.000	
41.419	3.198	12.951	0.000	
41.162	3.771	10.915	0.000	
40.846	1.868	21.868	0.000	
Variances				
T1BESCON	85.151	14.598	5.833	0.000
T2BESCON	57.902	10.359	5.590	0.000
T3BESCON	61.625	9.944	6.198	0.000
T4BESCON	50.194	14.352	3.497	0.000
Latent Class 2				
T1BESCON WITH T2BESCON	Means			
40.424	10.891	3.712	0.000	
53.342	1.962	27.185	0.000	
51.245	1.545	33.158	0.000	
55.147	1.868	29.529	0.000	
51.450	2.411	21.340	0.000	
Variances				
T1BESCON	85.151	14.598	5.833	0.000
T2BESCON	57.902	10.359	5.590	0.000
T3BESCON	61.625	9.944	6.198	0.000
T4BESCON	50.194	14.352	3.497	0.000
Latent Class 3				
T1BESCON WITH T2BESCON	Means			
30.546	13.052	2.340	0.019	
65.710	2.921	22.498	0.000	

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Compare Models 3 to 3a

Latent Class 1

T1BESCON WITH		
T2BESCON	26.517	24.847
Means		
T1BESCON	44.085	1.921
T2BESCON	41.181	3.566
T3BESCON	40.941	4.024
T4BESCON	40.773	1.984
Variances		
T1BESCON	84.772	13.959
T2BESCON	57.280	10.547
T3BESCON	61.559	10.122
T4BESCON	50.733	13.878
Latent Class 2		
WITH		
T2BESCON	40.153	10.334
Means		
T1BESCON	53.274	2.023
T2BESCON	51.182	1.608
T3BESCON	55.039	1.995
T4BESCON	51.309	2.469
Variances		
T1BESCON	84.772	13.959
T2BESCON	57.280	10.547
T3BESCON	61.559	10.122
T4BESCON	50.733	13.878
Latent Class 3		
WITH		
T2BESCON	32.086	12.553
Means		
T1BESCON	65.679	2.914
T2BESCON	64.072	2.652
T3BESCON	70.656	2.736
T4BESCON	70.917	1.959

Note—when comparing mixture models with the same number of classes, we can use LRT tests. So we can compare models 3 and 3a and ask, does constraining the covariance to be equal significantly increase model misfit?

Latent Class 1

T1BESCON WITH		
T2BESCON	30.546	13.052
Means		
T1BESCON	44.246	1.964
T2BESCON	41.419	3.198
T3BESCON	41.162	3.771
T4BESCON	40.846	1.868
Variances		
T1BESCON	85.151	14.598
T2BESCON	57.902	10.359
T3BESCON	61.625	9.944
T4BESCON	50.194	14.352
Latent Class 2		
WITH		
T2BESCON	40.424	10.891
Means		
T1BESCON	53.342	1.962
T2BESCON	51.245	1.545
T3BESCON	55.147	1.868
T4BESCON	51.450	2.411
Variances		
T1BESCON	85.151	14.598
T2BESCON	57.902	10.359
T3BESCON	61.625	9.944
T4BESCON	50.194	14.352
Latent Class 3		
WITH		
T2BESCON	30.546	13.052
Means		
T1BESCON	65.710	2.921
T2BESCON	64.078	2.664
T3BESCON	70.689	2.700
T4BESCON	70.950	1.940

Please do

Model 4: Σ_k non-diagonal and class-varying

```

Model:
%overall%
%c#1%
  T1BESCON with T2BESCON ;
  T1BESCON T2BESCON T3BESCON T4BESCON;
%c#2%
  T1BESCON with T2BESCON ;
  T1BESCON T2BESCON T3BESCON T4BESCON;
%c#3%
  T1BESCON with T2BESCON ;
  T1BESCON T2BESCON T3BESCON T4BESCON;

```

Latent Class 1		T1BESCON WITH T2BESCON	2.726	2.602	1.048	0.295
Means		T1BESCON	39.004	0.592	65.833	0.000
		T2BESCON	37.058	1.626	22.793	0.000
		T3BESCON	35.975	1.782	20.187	0.000
		T4BESCON	35.821	1.113	32.181	0.000
Variances		T1BESCON	4.232	1.783	2.373	0.018
		T2BESCON	16.772	10.083	1.663	0.096
		T3BESCON	30.283	8.496	3.564	0.000
		T4BESCON	10.452	4.488	2.329	0.020
Latent Class 2		T1BESCON WITH T2BESCON	37.948	7.678	4.942	0.000
Means		T1BESCON	51.832	0.978	52.992	0.000
		T2BESCON	49.477	0.874	56.586	0.000
		T3BESCON	52.414	0.980	53.487	0.000
		T4BESCON	49.204	0.920	53.455	0.000
Variances		T1BESCON	89.299	11.083	8.057	0.000
		T2BESCON	59.732	10.565	5.654	0.000
		T3BESCON	79.899	13.916	5.742	0.000
		T4BESCON	62.446	9.935	6.285	0.000
Latent Class 3		T1BESCON WITH T2BESCON	65.462	31.371	2.087	0.037
Means		T1BESCON	65.083	2.635	24.703	0.000
		T2BESCON	63.715	2.526	25.226	0.000
		T3BESCON	69.834	2.694	25.921	0.000
		T4BESCON	70.624	1.780	39.685	0.000
Variances		T1BESCON	126.783	48.808	2.598	0.009

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LPA in practice

- Reality check: Most papers that use LPA only consider the default.
- If you use LPA, best to consider at least the default and the diagonal, class-varying model
- Use your understanding of the variables and their relationships to guide model specification.

Model	Classes	LogL	Bic
1	1	--	--
1	2	--	--
1	3	--	--
1	4	--	--
1	5	--	--
2	1	--	--
2	2	--	--
2	3	--	--
2	4	--	--
3	1	--	--
3	2	--	--

Class enumeration for LPA

- Absolute fit
 - There are not widely accepted or implemented measures of absolute fit for LPA models
- Can compute absolute fit diagnostic tools:
 - Compute the overall model-estimated means, variances, covariances, univariate skewness, and univariate kurtosis of the latent class indicator variables.
 - Thus residuals for these parameters can be used.
 - These limited residuals allow at least some determination to be made about how well the model is fitting the observed data beyond the first- and second-order moments and also allow some comparisons of relative overall fit across models.

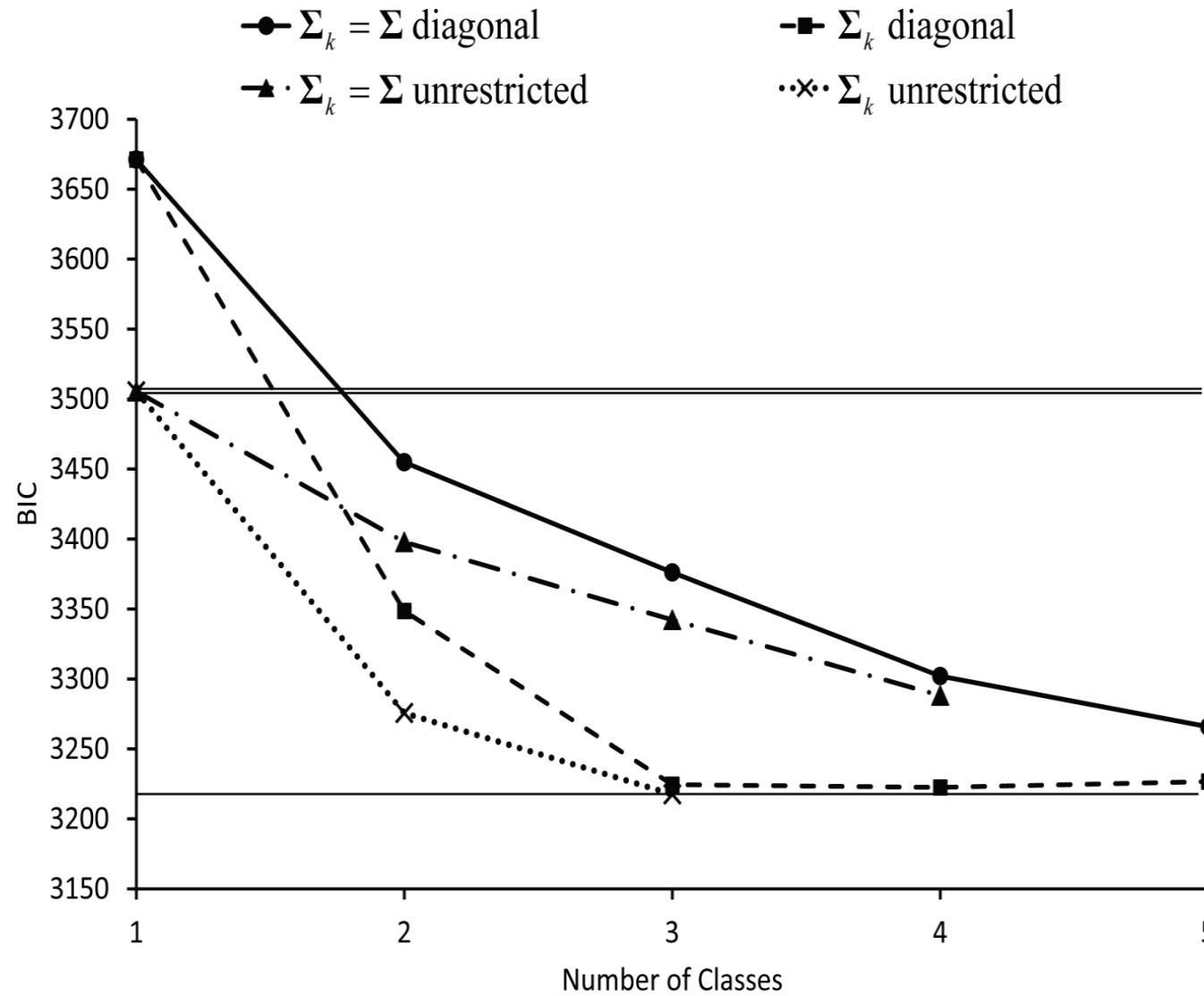
Class enumeration for LPA

- You can provide yourself with an absolute fit benchmark by estimating a fully-saturated mean and variance/covariance model that is an exact fit to the data with respect to the first- and second-order moments but assumes all higher-order moments have values of zero. This corresponds to fitting a 1-class LPA with an unrestricted specification. In the model building process, you would want to arrive at a measurement model that fit the individual data better (as ascertained by various relative fit indices) than a model only informed by the sample means and covariances.
- Relative fit: **Same as LCA.**
- Classification diagnostics: **Same as LCA**

Diabetes Example: Model Fit Indices for Exploratory Latent Profile Analysis Using Four Different Within-class Variance/Covariance Structure

Specifications (n=145)

	1	2	3	4	5	6	7	8	9	10	11
Σ_k	# of classes (K)	LL	npar*	BIC	CAIC	AWE	Adj. LMR-LRT p-value (H_0 : K classes; H_1 : K+1 classes)	$\hat{BF}_{K,K+1}$	$cm\hat{P}_K$	$cm\hat{P}_.$	
Class-invariant, diagonal $\Sigma_k = \Sigma$	1	-1820.68	6	3671.22	3677.22	3719.08	<.01	<.10	<.01	-	
	2	-1702.55	10	3454.88	3464.88	3534.64	<.01	<.10	<.01	-	
	3	-1653.24	14	3376.15	3390.15	3487.82	<.01	<.10	<.01	-	
	4	-1606.30	18	3302.18	3320.18	3445.76	.29	<.10	<.01	-	
	5	-1578.21	22	3265.90	3287.90	3441.39	-	-	>.99	<.01	
Class-varying, diagonal Σ_k	1	-1820.68	6	3671.22	3677.22	3719.08	<.01	<.10	<.01	-	
	2	-1641.95	13	3348.60	3361.60	3452.30	<.01	<.10	<.01	-	
	3	-1562.48	20	3224.49	3244.49	3384.03	<.01	0.38	.25	.03	
	4	-1544.10	27	3222.57	3249.57	3437.95	.15	7.76	.66	-	
	5	-1528.73	34	3226.67	3260.67	3497.88	-	-	.09	-	
Class-invariant, restricted $\Sigma_k = \Sigma$	1	-1730.40	9	3505.60	3514.60	3577.39	<.01	<.10	<.01	-	
	2	-1666.63	13	3397.95	3410.95	3501.65	<.01	<.10	<.01	-	
	3	-1628.86	17	3342.33	3359.33	3477.93	.19	<.10	<.01	-	
	4	-1591.84	21	3288.19	3309.19	3455.70	-	-	>.99	<.01	
Class-varying, unrestricted Σ_k	1	-1730.40	9	3505.60	3514.60	3577.39	<.01	<.10	<.01	-	
	2	-1590.57	19	3275.69	3294.69	3427.25	<.01	<.10	<.01	-	
	3	-1536.64	29	3217.61	3246.61	3448.93	-	-	>.99	.97	



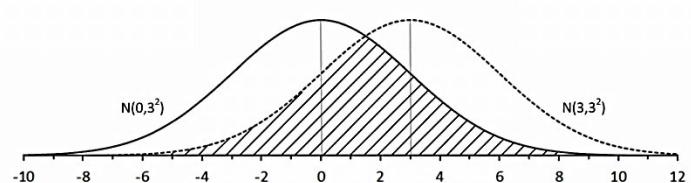
Class Homogeneity

- Individuals belonging to the same class are more similar to other members of that class than they are compared to members of other classes. That is, $\frac{\hat{\theta}_{mk}}{\hat{\theta}_m}$
 - Individuals belonging to the same class are closer to the class mean than they are to the overall population mean.
 - Within-class variance for each indicator is smaller than overall population variance:

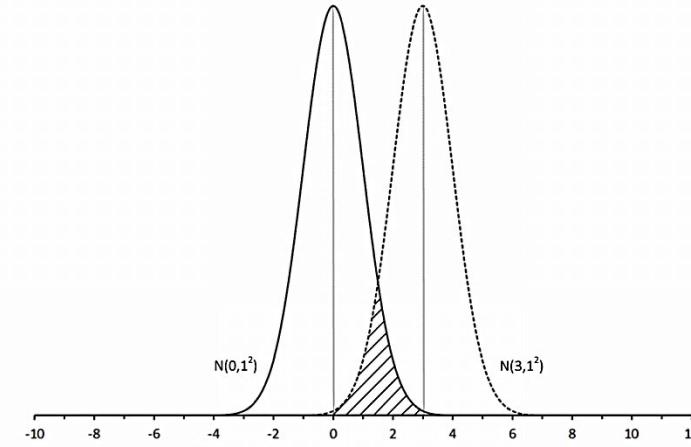
$\frac{\hat{\theta}_{mk}}{\hat{\theta}_m} > .90$ corresponds to a low degree of homogeneity

$\frac{\hat{\theta}_{mk}}{\hat{\theta}_m} < .60$ corresponds to a high degree of homogeneity

Class Homogeneity



$$\frac{\hat{\theta}_{mk}}{\hat{\theta}_m} = .88$$



$$\frac{\hat{\theta}_{mk}}{\hat{\theta}_m} = .55$$

Class Separation

- Well-separated classes have a small degree of overlap of the class-specific indicator distributions; that is,

- Standardized mean difference is large:
$$\hat{d}_{mjk} = \frac{\hat{\alpha}_{mj} - \hat{\alpha}_{mk}}{\hat{\sigma}_{mjk}}$$

corresponds to low separation—more than 50% overlap

$$|\hat{d}_{mjk}| < .85$$

corresponds to high separation—less than 20% overlap

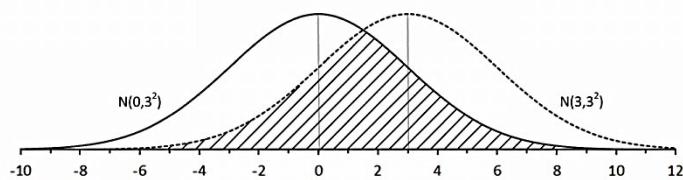
$$|\hat{d}_{mjk}| > 2.0$$

$$\hat{d}_{mjk} = \frac{\hat{\alpha}_{mj} - \hat{\alpha}_{mk}}{\hat{\sigma}_{mjk}}$$

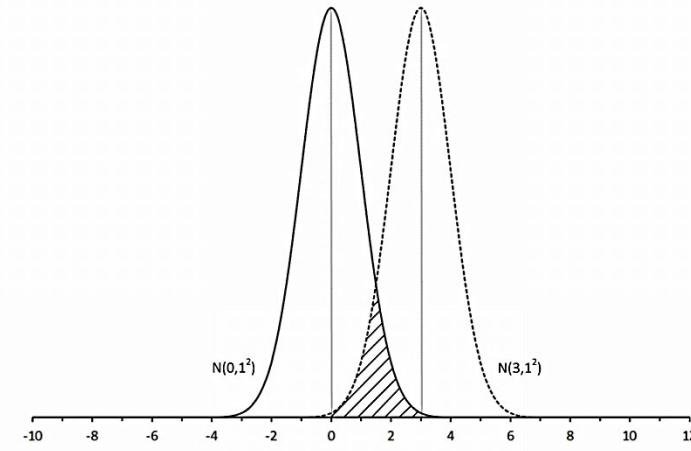
$$\hat{\sigma}_{mjk} = \sqrt{\frac{(\hat{\pi}_j)(n)(\hat{\theta}_{mmj}) + (\hat{\pi}_k)(n)(\hat{\theta}_{mmk})}{(\hat{\pi}_j + \hat{\pi}_k)n}}$$

Note: Keep an eye out for newer measures of class homogeneity and separation.

Class Separation



$$|\hat{d}_{mjk}| = 1.00$$



$$|\hat{d}_{mjk}| = 3.00$$

Some recommendations on writing up mixture modeling results

Rational for the use of Mixture Modeling

- We need to build an argument as to why we use mixture modeling:
 - Study the pattern of responses and how they relate to each other?
 - Hypothesize that there are different groups with respect to a set of outcomes?
 - Want to understand how a set of variables interact? And then perhaps relate these groups of interactions to other variable (covariates/distals)?
- Build the literature review of previous studies that relate to your topic but then try to highlight limitations and how your study/approach will address those limitations.
 - For example, I made this up: The Author (xxx) paper studies victimization using cut scores which highlighted differences in feelings of anxiety. In the current study we use a model-based approach to create groups using multiple indicators...

“The current study” section (can go by other names)

- Provides a specific rational as to why and how of your study.
- General statement that boils down the literature review into one or two paragraphs.
- High-level overview of the main goals of the study.
- Important summary paragraph for the reader. Helps to remind them what you are doing and what is to come.

This Study

A primary aim of this research was to explore whether there exist distinct groups of adolescents who differ based on the amount of community violence experienced and their emotional and behavioral responses to community violence. Among community-violence-exposed youth, we hypothesized that there would be distinct groups characterized by predominantly internalizing, predominantly externalizing, or both types of symptoms. A person-centered approach is optimally suited to explore this possibility. Whereas prior research has applied person-centered methods (e.g., latent class analysis [LCA]) to study community violence exposure among adolescents, these studies have focused only on identifying patterns of community violence exposure (e.g., Gaylord-Harden et al., 2016; Lambert, Nylund-Gibson, Copeland-Linder, & Ialongo, 2010). Our research differs by including adolescents' past year exposure to community violence and their proximal reports of depressive, anxious, and aggressive symptoms to explore patterns of recent emotional and behavioral adjustment among community-violence-exposed youth.

The current study with Rqs?

In addition, as there is much overlap in victimization experiences (Finkelhor, Ormrod, Turner, & Hamby, 2005), studies examining only one type of victimization may overestimate its association with subsequent revictimization. Also, aggression and victimization experiences overlap (Swearer & Hymel, 2015), and research needs to consider both when examining risk for later victimization/aggression. Using retrospective reports of childhood peer victimization and aggression assessed upon entry to college (Fall), we address how these impact reported peer victimization, peer aggression, hazing victimization, dating violence victimization, and sexual victimization experiences at the end of the first year of college (Spring) using latent class analysis (LCA). LCA is an example of a person-centered research approach that focuses on the processes assumed to be specific to people within a latent class, as opposed to a variable-centered approach that assumes that the process is the same across everyone (Hiatt, Laursen, Mooney, & Rubin, 2015). The use of LCA in this study provides us an opportunity to understand how the combination of different types of victimization and aggression experiences co-occur among youth, overcoming the limitations of previous research that may have studied them in isolation. LCA is becoming more widely used to explore multiple constructs at the same time. Specifically, we address the following research questions (RQ):

RQ1: What are the different latent classes of individuals involved in childhood peer victimization and aggression?

RQ2: How do these childhood latent classes relate to involvement in victimization and aggression over the first year of college in terms of individual types of peer victimization/aggression, hazing victimization, dating violence victimization, and sexual victimization?

RQ3: What are the victimization and aggression latent classes identified at the end of the first year of college?

RQ4: How do the childhood latent classes relate to the college latent classes?

Purpose of the Current Study

Despite the fact that Latino children represent the fastest growing subpopulation of students in the United States, relatively few studies have examined the literacy achievement and school readiness of these students, specifically. Previous research examining the literacy achievement of Latino students has predominantly focused on ELs, relied on cross-sectional data (NAEP), or has examined longitudinal data sets (ECLS) that were not fully inclusive of all Latino students (e.g., non-English proficient ELs were not included in analyses). Common findings among studies that have examined Latino children's literacy achievement across the elementary grades are that (a) Latino children enter kindergarten at a significant disadvantage in terms of early literacy skills, and (b) that additional research is needed to better understand how underlying differences among Latino children at kindergarten entry might be associated with differences in literacy achievement patterns during the early elementary grades (Reardon & Galindo, 2009; Roberts et al., 2010). In addition, previous research has identified discernible school readiness profiles among Latino students that are predictive of literacy achievement levels at the end of Grade 2 (Quirk et al., 2013); however, additional research is needed to better understand how Latino children's competencies at kindergarten entry are associated with longitudinal literacy achievement trajectories across the elementary school grades.

The current study addresses these gaps in the literature in several ways. First, using an independent sample of Latino chil-

dren this study utilized latent class analysis (LCA) to identify underlying patterns or profiles of children's school readiness and examined how readiness classes were associated with children's Grade 2 English-Language Arts (E-LA) achievement, which replicated the analyses from Quirk et al. (2013). Next, this study examined latent patterns in students' longitudinal E-LA achievement in Grades 2 through 5, providing a unique examination of literacy achievement trends among a sample of Latino students across the elementary grades. Finally, this study examined how patterns in Latino students' readiness during the first month of kindergarten were associated literacy achievement trends across the elementary school grades.

Method

Participants

The participants in this study ($N = 1,253$) included Latino students who entered kindergarten in a medium-sized school district in central California during the 2007–2008 academic year. Per

Method Section

- Method section strategy:
 - Provide a rational as to why mixture modeling is the chosen approach.
 - Describe details on how it was completed (e.g., software, details of analysis)
 - Describe how we evaluate model fit.
 - Scaffolding as to how results are presented

Analytic approach and model fit

Data analytic approach. LCA was employed to group individuals into “classes” according to patterns of indicator variables (i.e., community violence witnessing and victimization, and internalizing and externalizing behaviors) that existed in the data. The LCA used in this study was a mixed-mode LCA (Morgan, 2015), that is, an LCA with both binary and continuous indicators. Specifically, we used three continuous indicators (Grade 9 anxious, depressive, and aggressive symptoms) and two dichotomous indicators (Grade 9 past year witnessed community violence and past year victimization by community violence). Distinct response patterns across participants’ Grade 9 community violence exposure and their Grade 9 anxious, depressive, and aggressive symptoms were empirically identified. Analyses were conducted with *Mplus* version 8 (Muthén & Muthén, 1998–2017).

Model fit was assessed based on several fit indices: the Bayesian information criterion (BIC), consistent Akaike information criterion (CAIC), sample-sized adjusted Bayesian information criterion (saBIC), approximate weight of evidence criterion (AWE), Lo–Mendell–Rubin test (LMRT), bootstrap likelihood ratio test (BLRT), approximate correct model probability (cmP), and Bayes factor (BF). Lower values of the BIC, CAIC, saBIC, and AWE indicate better fit (Masyn, 2013). Low p values ($p < .05$) for the LMRT and BLRT indicate that the current model has significantly improved fit compared with a model with one less class (Nylund, Asparouhov, & Muthén, 2007). The cmP estimates the probability of each model within a set being correct, under the assumption that the correct model is present within that set (Masyn, 2013). Finally, the BF compares the fit between the present model (model K) and a model with one additional class (model $K + 1$). A BF of less than 3 is considered weak evidence for model K over model $K + 1$, whereas a BF between 3 and 10 is considered moderate evidence, and a BF greater than 10 is considered strong evidence for Model K (Masyn, 2013). Parsimony, class homogeneity, class separation, and the substantive meaning of classes also were considered in evaluating the model fit (Masyn, 2013).

Lambert, S.F., Tache, R.M., Liu, S.R., Nylund-Gibson, K., Ialongo, N.S. (2019). Individual Differences in patterns of community violence exposure and internalizing and externalizing behaviors. *Journal of Interpersonal Violence*.

Model Fit

Moore, S. A., Dowdy, E., Nylund-Gibson, K., & Furlong, M. J. (2019). An Empirical Approach to Complete Mental Health Classification in Adolescents. *School Mental Health*, 1-16.

Evaluations of relative fit assess model fit by comparing a target model to an alternative model with a different number of latent classes and include the information criteria statistics, such as the Bayesian information criteria (BIC; Schwartz, 1978), Bayes factor (BF), correct model probability (cmP), bootstrap likelihood ratio test (BLRT; McLachlan & Peel, 2000), and Vuong–Lo–Mendell–Rubin LRT (VLMR-LRT; Vuong, 1989). When interpreting the BF, values between 1 and 3 offer weak evidence, between 3 and 10 offer moderate evidence, and greater than 10 offer strong evidence for the current model (Wasserman, 1997). Larger cmP values indicate a greater likelihood of the model being the correct model out of all models tested (Masyn, 2013). The BLRT and the VLMR-LRT tests examine the fit of a k -class model with a $k - 1$ class solution, with nonsignificant p values indicating support for the $k - 1$ class solution. With regard to information criteria statistics, superior model fit is indicated by lower values. Accuracy of classification of individuals to latent classes within a given model was examined based upon estimates of posterior class probability (i.e., the likelihood of each individual's membership in a given class, based upon his or her pattern of responses) and relative entropy (Ramaswamy, DeSarbo, Reibstein, & Robinson, 1993). High entropy has been associated with values close to .80 (Clark & Muthén, 2009), with values closer to 1 indicating superior classification precision (Masyn, 2013). Each of the above criteria was evaluated in selecting and interpreting all models.

Please do not distribute

Data Analysis Plan

Latent class analysis (LCA; Lazarsfeld & Henry, 1968; Masyn, 2013) and growth mixture modeling (GMM; Muthén & Shedden, 1999) are exploratory analytical approaches that belong to a larger class of statistical models known as mixture models. In both models, patterns of responses are used to identify homogeneous subpopulations of individuals. In the current study, we first utilized LCA to replicate findings from a previous study (Quirk et al., 2013) by identifying classes of kindergarten readiness among an independent (new) sample of Latino children. Next, we estimated and compared the means of the distal outcome of Grade 2 E-LA NCE scores across the different latent classes. E-LA NCE mean differences across classes were estimated. Kindergarten readiness classes were derived from three components of readiness: social-emotional, physical, and cognitive.

Next, we used GMM to identify discernible trajectory classes of longitudinal E-LA achievement from Grades 2–5 using CST scores converted into NCE units. Once the GMM classes were established, we used latent transition analysis (LTA) to link the LCA and GMM models, which allowed us to examine students' transition patterns from kindergarten readiness classes to longitudinal E-LA achievement classes. Finally, we included covariates in each of the linked model components (i.e., LCA and GMM) to identify demographic characteristics that predicted latent class membership. This hybrid LTA model is a complex model, which uses two different mixture models as measurement models at each time point. For more on this type of model see Nylund-Gibson, Grimm, Quirk, and Furlong (2014).

All models were estimated using Mplus 7.11 (Muthén & Muthén, 1998–2012). Full information maximum likelihood (FIML) estimation was used, which allowed for item-level missing data under the missing at random (MAR) assumption. Students who had data on at least one of the school readiness items or from at least one of the CST years were included in the analyses. Data

Assessing model fit. For both the LCA and GMM, we considered several fit indices because no single statistical fit index has been shown to be a solely accurate indicator of model fit (Nylund, Asparouhov, & Muthén, 2007). The Bayesian Information Criterion (BIC; Schwarz, 1978) was used, as it is often trusted over other fit indices (Nylund et al., 2007). Lower BIC values suggest a preferred model. We also examined the Adjusted Bayesian Information Criterion (ABIC), which is interpreted similarly to the BIC. We also used the Bayes Factor (BF) that provides an information-heuristic comparison of two competing models and has shown promise for use in selecting latent class models (Masyn, 2013; Morovati, 2014). The BF calculates an approximate ratio of the probability of a model with k number of classes being "correct" compared to a model with $k + 1$ number of classes, assuming one of the models is indeed the "correct" model. Values between 1 and 3 are considered weak evidence for the k model, values 3 through 10 are considered moderate evidence, and values greater than 10 are considered strong evidence (Masyn, 2013). We also used the Lo-Mendell-Rubin (LMR) and the bootstrap likelihood ratio test (BLRT) to assess whether the addition of another latent class significantly improved model fit (Nylund et al., 2007). Significant p values indicate that the additional class significantly improved the model. We report the entropy, which ranges from 0 to 1, where larger values indicate better classification (Collins & Lanza, 2010); however, entropy is not used to assess the overall classification of individuals into latent classes because it is not a fit statistic. Models that have entropy values larger than .80 are considered to have high entropy (Clark & Muthén, 2009), which implies that there is a good classification of individuals into the latent classes.

The final step in this analysis was to link the LCA and GMM to model how students transitioned from their kindergarten readiness classes to the longitudinal E-LA classes. Figure 2 presents a diagrammatic representation of this final model. This was done using LTA, which allows for the inclusion of multiple latent class

Results

- Useful to provide a road map of results, especially when complicated.
- Helpful to have clear labels of sections:
 - “Latent Class Analysis” or “Deciding on the number of classes”
 - Covariate results
 - Distal Outcome results

Results

The results are divided into several subsections. First, we present the results of the unconditional LCA as well as how the LCA classes identified are associated with children’s Grade 2 E-LA achievement, which replicated the analyses from a previous study (Quirk et al., 2013) using an independent sample of Latino students. This is followed by presentation of the results of the unconditional GMM, which examined patterns in students’ longitudinal E-LA achievement levels across Grades 2–5. Finally, we present the results of the LTA, in which both the LCA and GMM models were linked, and discuss the transitions between readiness classes and longitudinal E-LA achievement classes. Descriptive statistics for all of the variables used in the analysis are included in Table 1.

RESULTS

We first present results to support the plausibility of the attitudinal trajectories (latent profiles at each grade level, stability of attitudes from seventh through 12th grade, relationship of the attitudinal trajectories with science and mathematics achievement, and STEM career attainment) and then describe gender differences in terms of the attitudinal trajectories.

ATTITUDINAL PROFILES AT EACH GRADE LEVEL

Based on empirical model results that were conducted on each grade level independently (Table 3), four attitudinal profiles were identified at each

Provide detail on the enumeration and class labeling

- Walk the reader through the table. Make an argument for how you decided on the number of classes.
 - There isn't a "right answer" here— so you're crafting a rational as to why you feel your solution is right.
- Describe how you labeled the classes and refer to the item probability plot.

3. Results

LCA was conducted by first examining a model with one class, then exploring models with more classes. Table 1 includes fit information for the LCA models with one through five latent classes. Examining the results in Table 1, as the number of classes increases the BIC increases; however, after the 3-class model the reduction in the BIC is small suggesting that increasing classes above 3 may not be meaningful. The non-significant VLMR *p*-value for the 4-class model (*p*=.06) also points toward a 3-class model. The BF value is greater than 10 for the 3-class model and smaller for all others, which is also support for the 3-class model. Taken together, these fit statistics aided in the decision that a 3-class model adequately described the subgroups in this population.

The item probability plots for the three class model are presented in Fig. 1. The item probability values differentiate the latent classes, and are interpreted as the probability that members of a particular class would endorse an item. Variables where class

members' probability proportions fall below .3 and above .7, indicating low and high probability of members endorsing the item, are important for classification (Masyn, 2013).

Class 1 represents 34% of the sample and is distinguished by having a low probability of concerns at intake. Members of this class have a low probability of reporting psychological, alcohol, drug, medical, peer, or family problems. Additionally, they are less likely to report a history of psychopathology, including suicidality and/or hallucinations than are the other groups. They have a high probability of reporting motivation for treatment. In regards to criminal offending, members of this class have a low probability of having a first arrest before the age of 16. Given few indicators of treatment needs and high motivation for success, they were deemed *Subthreshold Need* participants.

Class 2 accounts for 43% of the sample and represents a higher need level than that of the *Subthreshold Need* group. Participants in this class have a high probability of reporting mental health concerns. They are also more likely to endorse drug problems. As illustrated in Fig. 1, this group had a high probability of early substance use, but was unlikely to have been involved with the criminal justice system at an early age. This group is more likely to report a history of family substance abuse and psychological problems. While they are high in psychological concerns, they also present with strengths. In particular, this group is more likely to hold a high school diploma and be motivated for treatment. Based on their profile of presenting concerns, this group was labeled *Psychological Problems*.

The final group is distinguished by their increased likelihood of early involvement (before the age of 16) in substance use and other criminal behavior. Representing 24% of the sample, they were labeled the *Early Delinquent* class. This group was likely to be unemployed and not have a high school degree, and have a moderate probability of endorsing drug and psychological problems. Overall, this group was distinguished by their members having early involvement in the criminal justice system, a low probability of holding a high school education, and high probability of being unemployed.

Table 1
Fit indices for LCA models with 1–5 classes.

No. of classes	1	2	3	4	5
No. of free parameters	22	42	62	82	102
Log likelihood	−12522.17	−12256.94	−12090.26	−11999.06	−11931.14
BIC	25197.23	24805.78	24611.41	24568.01	24571.17
ABIC	25127.35	24672.38	24414.49	24307.57	24247.21
BLRT (<i>p</i> -value)	— ^b	0.00	0.00	0.00	0.00
VLMR (<i>p</i> -value)	— ^c	0.18	0.02 ^d	0.06	0.20
BF	— ^e	1.42E−109	1.64E+57 ^a	4.85	0.00
Cmp	— ^e	1.42E−109	1.00 ^f	6.08E−58	1.25E−58

Note. BIC = Bayesian Information Criterion; ABIC = adjusted BIC; BLRT = bootstrap likelihood ratio test; VLMR = Young–Lo–Mendell–Rubin.

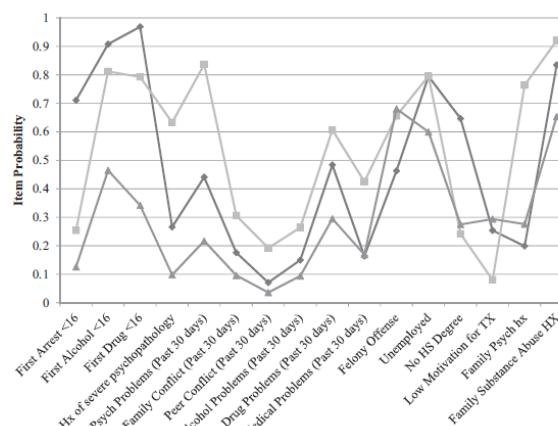
^a Best-fitting model according to that index.

^b BLRT not available for the one-class model.

^c VLMR not available for the one-class model.

^d BF not available for the one-class model.

^e Cmp not available for the one-class model.



Selection of final model

About the plot

Description of classes

Inclusion of covariates: gender and ethnic differences in constellations of school belonging

The manual three-step approach examined gender and ethnicity as covariates using the optimal three-profile class-varying, diagonal model. The latent profile variable was regressed onto the dichotomous covariates of gender and ethnicity using the *High School Belonging* profile as the normative comparison group. Specifically, two covariate comparisons were analysed: (a) the likelihood of being in the *Moderate School Belonging* profile versus the *High School Belonging* profile and (b) the likelihood of being in the *Low School Belonging* profile versus the *High School Belonging* profile for each covariate. Table 3 includes the logits, standard errors (SEs), *p*-values, and odds ratios for each gender and ethnicity covariate included in the model.

Compared to the *High School Belonging* profile, female students were significantly less likely to be in the *Low School Belonging* profile than male students ($\text{logit} = -.63; p = .02$). Similarly, compared to the *High School Belonging* profile, female students were significantly less likely to be in the *Moderate School Belonging* profile than male students ($\text{logit} = -.52; p = .06$); this difference was nonsignificant. No significant differences were seen for White students and non-White students when comparing all profiles. However, for

Covariates

[Wagle, R., Dowdy, E., Nylund-Gibson, K., Sharkey, J. D., Carter, D., & Furlong, M. J. \(2021\). School belonging constellations considering complete mental health in primary schools. *Educational and Developmental Psychologist*, 38\(2\), 173-185.](#)

the Latinx vs. non-Latinx variable, Latinx students were significantly less likely to belong to the *Moderate School Belonging* profile than non-Latinx students, compared to the *High School Belonging* profile ($\text{logit} = -.88, p = .01$). No other significant differences were found for gender or ethnicity.

Table 3. Log odds coefficients and odds ratios for the three-profile model with gender and ethnicity as covariates using the high school belonging profile as a reference group.

School Belonging Profile	Effect	Logit	SE	t	Odds Ratio	p-value
<i>Low school belonging</i>	Female	-.63	.28	-2.25	.53	.02
	Latinx	-.46	.34	-1.35	.63	.17
	White	-.10	.32	-0.29	.91	.77
<i>Moderate school belonging</i>	Female	-.52	.28	-1.88	.60	.06
	Latinx	-.88	.35	-2.57	.41	.01
	White	.28	.31	0.89	1.32	.37

Bolded values denote statistical significance, $p < .05$.

Constellations of School Belonging And Complete Mental Health differences

The final step of the analysis included examining the associations between latent profiles and mental health outcomes. Specifically, class-specific means of psychological strengths and psychological distress were estimated for each of the latent profiles, at the average of the gender and ethnicity covariates.

First, an omnibus test of association was conducted between the latent profile variable and the three proximal outcomes and found to be significant indicating significant relations between the profiles and psychological strengths, $\chi^2 = 314.21$, $df = 2$, $p < .01$, and both aspects of psychological distress: emotional, $\chi^2 = 132.33$, $df = 2$, $p < .01$, and behavioural difficulties, $\chi^2 = 72.39$, $df = 2$, $p < .01$.

To understand where class differences occurred, pairwise tests were examined. Results indicated that all pairwise comparisons were significantly different for all three distal outcomes. Precisely, students in the *High School Belonging* profile had significantly higher psychological strengths than students in the *Moderate School Belonging* and *Low School Belonging* profiles. Students in the *Moderate School Belonging* profile reported significantly higher psychological strengths than students in the *Low School Belonging* profile. Concerning psychological distress, students in the *High School Belonging* profile reported significantly lower emotional and behavioural difficulties than students in the *Moderate* and *Low School Belonging* profiles. Students in the *Moderate School Belonging* profile reported significantly lower emotional and behavioural difficulties than students in the *Low School Belonging*

Distals

Table 4. Model results for mean proximal outcome values within each latent school belonging profile.

Outcome	Latent Profile	Estimate	S.E.
Psychological Strengths	<i>Low School Belonging Class</i>	2.66	.04
	<i>Moderate School Belonging Class</i>	3.11	.03
	<i>High School Belonging Class</i>	3.51	.05
Emotional Difficulties	<i>Low School Belonging Class</i>	1.85	.03
	<i>Moderate School Belonging Class</i>	1.61	.03
	<i>High School Belonging Class</i>	1.38	.04
Behavioural Difficulties	<i>Low School Belonging Class</i>	1.56	.03
	<i>Moderate School Belonging Class</i>	1.35	.03
	<i>High School Belonging Class</i>	1.24	.04

All pairwise comparisons of distal outcomes are significantly different when comparing with class, $p < .001$.

profile. For students in all profiles, emotional difficulties were slightly higher than behavioural difficulties.

Differences in mental health were also based on the covariates of gender and ethnic identification. Female students reported higher psychological strengths ($p = .01$) and emotional difficulties ($p < .001$) than males. Gender differences for behavioural difficulties were non-significant ($p = .165$). White students reported lower emotional difficulties than non-White students, though this difference was nonsignificant ($p = .069$). Latinx students did not significantly differ on self-reported mental health indicators from non-Latinx students. Table 4 presents the class-specific means, standard errors, and p -values for each latent profile with demographic covariates held constant.

Heterogeneity Among Moderate Mental Health Students on the Mental Health Continuum-Short Form (MHC-SF)

Mei-ki Chan^{1,3}  · Michael J. Furlong² · Karen Nylund-Gibson³ · Erin Dowdy¹

After the intro

Empirical Approach to Classify Students' Mental Health

Latent profile analysis (LPA) uses empirical algorithms to categorize individuals based on their response patterns to relevant items. The current study used the MHC-SF domain (emotional, psychosocial, and social) means as indicators to

examine adolescents' mental health profiles. Provided that the three MHC-SF subjective well-being subscales are interrelated yet distinct (Keyes, 2005), some students may experience varying levels of well-being in each dimension. LPA can potentially provide a nuanced perspective to advance an understanding of students' well-being by identifying more than the three diagnostic categories proposed by Keyes (2005). Comparing MHC-SF categories using different approaches, such as through LPA and categorical diagnostic approaches, could help educators and researchers understand emergent mental health groups using other techniques and inform applications of the two classification approaches.

Current Study

This study aims to critically evaluate the utility of the MHC-SF for mental health screening and monitoring through exploring more detailed differentiation of the MHC-SF response profiles among US adolescents. LPA was employed to explore youth responses to the MHC-SF items across emotional, psychological, and social well-being. Considering previous MHC-SF person-centered studies (Reinhardt et al., 2020) and the theoretical assumptions of its three interrelated and distinctive mental health components (Keyes, 2005), we hypothesized that the LPA would identify Keyes' consistently low (i.e., similar to Languishing) and high (i.e., similar to Flourishing) profiles. Of interest and pertinent to the current study's contribution aims, we further hypothesized that more than three LPA classes would emerge due to the undifferentiated definition for the Moderate Mental Health classification. Subsequently, we examined the LPA profile associations with student psychological strengths and distress to assess the profiles' meaning and validity. In addition, we included psychological strengths and distress as proxies of quality of life given their robust and extensive associations with youth functioning in various aspects (e.g., substance use and academic achievement; Furlong et al., 2021; Dowdy et al., 2018). Finally, to inform educators' application of the current study's results, we evaluated the students' MHC-SF diagnostic categories' congruence with their LPA profiles. Comparing the two classification methods could help researchers and educators better understand the meaning of the empirically derived mental health profiles using the traditional MHC-SF diagnostic categories as a reference point.

Focus on one paper

Data Analysis Plan

Analyses were conducted on Mplus 8.4 (Muthén & Muthén, 2017) using maximum likelihood estimation with robust standard errors (MLR). The distributions of the three profile indicators were negatively skewed. Given the nested nature of the sample, the variables interclass correlations (ICC) were examined. The ICCs of the three mental health dimensions and two distal outcomes ranged from 0.014 and 0.009, suggesting that variables at the student level mostly accounted for the variances of these variables. The analysis consisted of three steps: (a) class enumeration, (b) estimating profiles' relations with distal outcomes, and (c) comparing mental health classification congruence between categorical diagnostic approach and latent profile analysis. In step 1, using the three composite scores from each dimension of the MHC-SF, 1-to 8-class LPA models were estimated. Provided that latent profiles can vary by their indicator means,

variances, and covariances, we analyzed four model structures for each number of latent profiles (Masyn, 2013):

1. Model 1: indicator variances were freely estimated but constrained to be equal across classes, with no within-class indicator covariances.
2. Model 2: indicator variances were estimated freely, and no within-class indicator covariance was specified.
3. Model 3: indicator variances were constrained to be equal across classes, and within-class indicator covariances were specified.
4. Model 4: indicator variances were constrained to be equal across classes, and indicator covariances for the overall model were specified.

The final model was selected based on the relative fit indices of the plausible competing models along with conceptual merits and profiles' meaning (Masyn, 2013).

Given no consensus on latent profile model fit indices (Masyn, 2013), several indices compared the model fit across models. The fit statistics, suggested by current best practices in mixture modeling, were: Bayesian information criterion (BIC), sample size adjusted BIC (saBIC), consistent Akaike information criterion (CAIC), approximate weight of evidence criterion (AWE), Bayes factor (BF), correct model probability (cmP), bootstrap likelihood ratio test (BLRT; McLachlan & Peel, 2000), and Vuong–Lo–Mendell–Rubin LRT (VLMR-LRT; Vuong, 1989). Lower information criterion values suggest a better model fit among the models compared (Nylund et al., 2007). Higher BF values and cmP values provide more robust evidence to the specific model as the best fitting relative to other models considered (Masyn, 2013). The BLRT and the VLMR-LRT tests compare the fit of a k -class model with a $k - 1$ class solution. Significant p values ($p < 0.05$) suggest there is evidence supporting the k class solution compared to the $k - 1$ class model (Nylund et al., 2007). Classification diagnosis of profiles' separation was conducted with high average posterior class probability (AvePP; i.e., > 0.70) and odds of correction classification ratio for Class k (OCC_k ; i.e., > 5), evaluating classification precision and separation (Masyn, 2013; Nagin, 2005).

In step 2, after confirming the final model for this study, the manual BCH method (Nylund-Gibson et al., 2019) examined profiles' association with students' social emotional strengths and psychological distress. Several demographic variables (i.e., students' socioeconomic circumstances, ethnicity, gender identity, and sexual orientation) were included as control variables. The manual BCH method was favored because it minimizes class shifting with auxiliary variables and can simultaneously assess the demographic covariates and distal outcomes of profiles (Asparouhov & Muthén, 2013). Wald tests assessed the significance of distal outcomes' estimated means differences between profiles, and

the demographic covariates were regressed on the latent profiles and each outcome.

In step 3, we calculated the proportion of classification agreement between the two classification methods to assess classification congruence. Each student's profile membership was coded according to their most likely assigned latent profile and also classified into *Flourishing*, *Languishing*, and *Moderate* groups following the MHC-SF categorical diagnostic scheme. The two sets of groupings were compared by cross-tabulation to assess classification congruence between the two methods.

Reviewing this now, I wish we didn't refer to the modeling phases as "steps" since that could easily be confused with 3-step procedures

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Results

Results

Tables 1 and 2 show descriptive information of the variables in the analysis. The overall Covitality score of psychological strengths showed large and positive correlations with all three dimensions of well-being (i.e., emotional, psychological, and social). Psychological distress had moderate and negative correlations with the three types of well-being.

Model Selection

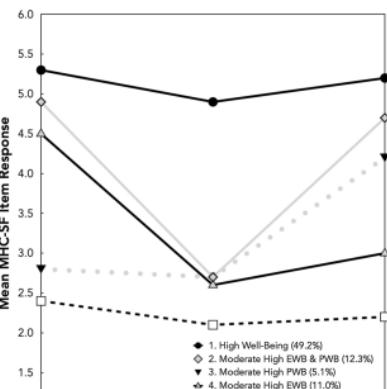
Table 3 displays the fit statistics of each Model estimated. The 1–8 class models converged for both Models 1 and 4. However, Model 2 did not converge after a 3-profile solution, and Model 3 did not converge after a 2-profile solution. Comparing across all converged models, we observed that Model 4 generally exhibited a better fit than Model 1 across the 1–8 profile solutions, as shown by the lower information criteria statistics, suggesting Model 4 provided a better fit to the data. In Model 4, the information criteria decreased for each additional class, but the decreasing magnitude became smaller after the fifth profile solution. However, the LMR-LRT indicated a six-profile solution in Model 4. Since the information given by fit statistics seemed to suggest a 4–6 profile solution, we examined these profiles closely.

The four-profile solution showed two ordered groups (consistently high and consistently low well-being across each of the three aspects of well-being)—the two profiles between the two ordered groups varied by responses to

Table 3 Fit statistics for LPA class enumeration (n = 10,880)

	<i>k</i>	LL	BIC	saBIC	CAIC	AWE	BLRT <i>p</i>	VLMR-LRT <i>p</i>	BF	cm <i>P</i>
Model 1	1	−58,891.76	117,844.71	117,825.64	117,850.71	117,923.89	—	—	<.001	<.001
	2	−49,425.82	98,953.62	98,826.79	98,963.62	99,085.60	<.001	<.001	<.001	<.001
	3	−45,535.66	91,214.09	91,036.53	91,228.09	91,398.86	<.001	<.001	<.001	<.001
	4	−44,034.34	88,252.24	88,023.95	88,270.24	88,489.80	<.001	<.001	<.001	<.001
	5	−43,450.04	87,124.43	86,845.41	87,146.43	87,414.79	<.001	<.001	<.001	<.001
	6	−43,017.39	86,299.92	85,970.17	86,325.92	86,643.07	<.001	<.001	<.001	<.001
	7	−42,598.02	85,501.98	85,121.49	85,531.98	85,897.91	<.001	<.001	<.001	<.001
	8	−42,244.11	84,834.95	84,403.73	84,868.95	85,283.68	<.001	<.001	1	1
Model 2	1	−58,891.76	117,844.70	117,825.64	117,850.70	117,923.89	—	—	<.001	<.001
	2	−47,842.72	95,818.01	95,653.13	95,831.01	95,989.58	<.001	<.001	<.001	<.001
Model 4	1	−45,301.13	90,694.04	90,665.44	90,703.04	90,812.82	—	—	<.001	<.001
	2	−44,004.33	88,141.23	87,976.36	88,154.23	88,312.80	<.001	<.001	<.001	<.001
	3	−43,100.80	86,374.96	86,159.36	86,391.96	86,599.33	<.001	<.001	<.001	<.001
	4	−42,628.32	85,470.80	85,204.46	85,491.80	85,747.95	<.001	<.001	<.001	<.001
	5	−41,991.62	84,238.19	83,921.12	84,263.19	84,568.13	<.001	<.001	<.001	<.001
	6	−41,712.25	83,720.24	83,352.44	83,749.24	84,102.98	<.001	<.001	<.001	<.001
	7	−41,518.19	83,372.91	82,954.38	83,405.91	83,808.44	<.001	.017	<.001	<.001
	8	−41,352.88	83,083.08	82,613.82	83,120.08	83,571.40	<.001	.022	1	1

K number of classes, *LL* model log likelihood, *BIC* Bayesian information criterion, *saBIC* sample size adjusted BIC, *CAIC* consistent Akaike information criterion, *AWE* approximate weight of evidence criterion, *BLRT* bootstrapped likelihood ratio test, *VLMR-LRT* Vuong–Lo–Mendell–Rubin adjusted likelihood ratio test, *p* *p* value, *BF* Bayes factor, *cmP* correct model probability; **Bold**=best fit statistic for each individual statistic. Model 1 indicates fixed variance across classes and no covariances specified. Model 2 indicates within-class variance are specified; Model 3 (within-profile covariance specified) was not listed because the models did not converge after 1 class. Model 4 indicates covariances specified for the overall model and fixed variance across classes



Please do not distort

IMMERSE Training



What comes next?

