CUE - Cooperation in Urban Environments: Theoretical Notation

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1 Model general notation

- A – Agent;
- P – Place;
- σ – trait value;
- D – critical Hamming distance of trait;
- R – critical radius;
- C – openness to contamination, or degree of interaction influence;
- M – memory size;

2 Simulation notation

- t – Timestep;
- T – Total number of timesteps;
- x – position coordinate;
- y – position coordinate;

3 Agents

Agents are denoted by A. Any specific Agent is denoted by A^i, where i is the ID number of the Agent in a world of N_A Agents.

3.1 Parameters of Agents

Parameters are those static, constant attributes. They can be accessed by their names in the Agent subscript:

- A^i_D – D of A^i;
- A^i_R – R of A^i;
- A^i_C – C of A^i;
- A^i_M – M of A^i;

3.2 Variables of Agents

Variables are those mutable, changing attributes. They can be accessed by their names in the Agent subscript and by time step t in the Agent superscript:

- $A^i_\sigma(t)$ – σ of A^i in time step t;
- $A^i_x(t)$ – x position of A^i in time step t;
- $A^i_y(t)$ – y position of A^i in time step t;

4 Places

Places are denoted by P. Any specific Place is denoted by it's own position: $P^{x,y}$ in 2D or P^x in 1D.

4.1 Parameters of Places

Parameters are those static, constant attributes. They can be accessed by their names in the Agent subscript:

- $P^{x,y}_C$ – C of $P^{x,y}$;

4.2 Variables of Places

Variables are those mutable, changing attributes. They can be accessed by their names in the Place subscript and by time step t in the Place superscript:

- $P^{x,y}_\sigma(t)$ – σ of $P^{x,y}$ in time step t;

5 Model equations

5.1 Interaction equations

When Agents interact with Places, both get some contamination from each other. Their next $t+1$ value of σ changes by the following equations. For Agents:

$$A^i_\sigma(t+1) = \frac{A^i_{\bar{\sigma}}(t) + P^{X,Y}_\sigma(t) \cdot A^i_C}{1 + A^i_C} \quad \forall i, t \tag{1}$$

And for Places:

$$P^{X,Y}_\sigma(t+1) = \frac{P^{X,Y}_\sigma(t) + A^i_{\bar{\sigma}}(t) \cdot P^{X,Y}_C}{1 + P^{X,Y}_C} \quad \forall i, t \tag{2}$$

Where $X = A^i_x(t)$ and $Y = A^i_y(t)$. The value of $A^{i,t}_{\bar{\sigma}}$ is a function of the Agent's previous traits (A_σ) and memory size (A_M):

$$A^i_{\bar{\sigma}}(t) = \Psi(A^i_\sigma, A^i_M) \quad \forall i, t \tag{3}$$

Currently, the function Ψ the average of σ values allocated in the Agent's memory:

$$A^i_{\bar{\sigma}}(t) = \frac{1}{A^i_M} \sum_{n=0}^{A^i_M} A^i_\sigma(t-n) \quad \forall i, t \tag{4}$$

5.2 Random walk

5.2.1 Set of candidate Places

At any time step, each Agent has a set $\mathbb{P}$ of candidate Places to move in so they can interact. This set is made of Places P within the Agent's window of sight of size A_R and below the Agent's interaction threshold A_D. Hence:

$$P \in \mathbb{P} \;\Big|\; |X - A_x| \leq A_R \cap |Y - A_y| \leq A_R \cap |P_\sigma - A_{\bar{\sigma}}| \leq A_D \tag{5}$$

5.2.2 Uniform weighting function

The `uniform` weighting function is defined to set all available places the same likelihood to be chosen by a given Agent. Therefore:

$$P_L = \frac{1}{\mathbb{P}_N} \tag{6}$$

Where P_L is the likelihood of the candidate Place P and $\mathbb{P}_N$ is the number of candidate Places in $\mathbb{P}$.

5.2.3 Linear weighting function

The `linear` weighting function is defined to set the likelihoods of candidate Places proportinal to the σ discrepancy. Therefore, Agents are biased to go to Places like themselves. The function:

$$P_L = -\frac{2}{\delta_{max}^2}\delta + \frac{2}{\delta_{max}} \tag{7}$$

Where P_L is the likelihood of the candidate Place P, $\delta = |A_\sigma - P_\sigma|$ and δ_{max} is the highest δ value found in $\mathbb{P}$.

5.2.4 Exception case

When the set $\mathbb{P}$ of candidate Places is empty so $\mathbb{P}_N = 0$ then the set is recomputed without the interaction threshold constrain and `uniform` function is used. In this case:

$$P \in \mathbb{P} \;\Big|\; |X - A_x| \leq A_R \cap |Y - A_y| \leq A_R \tag{8}$$