CUE - Cooperation in Urban Environments: Theoretical Notation

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1 Model general notation

- A – Agent;
- P – Place;
- σ – trait value;
- D – critical Hamming distance of trait;
- R – critical radius;
- C – openness to contamination, or degree of interaction influence;
- M – memory size;

2 Simulation notation

- t – Timestep;
- i – Agent ID: unique integer number;
- j – Place ID: unique integer number;
- T – Total number of timesteps;
- x – position coordinate;
- y – position coordinate;

3 Agents

Agents are denoted by A. Any specific Agent is denoted by A^i, where i is the ID number of the Agent in a world of N_A Agents.

3.1 Parameters of Agents

Parameters are those static, constant attributes. They can be accessed by their names in the Agent subscript:

- A^i_D – D of A^i;
- A^i_R – R of A^i;
- A^i_C – C of A^i;
- A^i_M – M of A^i;

3.2 Variables of Agents

Variables are those mutable, changing attributes. They can be accessed by their names in the Agent subscript and by time step t in the Agent superscript:

- $A^i_\sigma(t)$ – σ of A^i in time step t;
- $A^i_x(t)$ – x position coordinate of A^i in time step t;
- $A^i_y(t)$ – y position coordinate of A^i in time step t;

4 Places

Places are denoted by P. Any specific Agent is denoted by P^j, where j is the ID number of the Place in a world of N_P Places. Any specific Place j is also denoted by it's own unique position: $P_{x,y}$ in 2D or P_x in 1D.

4.1 Parameters of Places

Parameters are those static, constant attributes. They can be accessed by their names in the Agent subscript:

- P^j_C – C of P^j;
- P^j_x – x position coordinate of P^j;
- P^j_y – y position coordinate of P^j;

4.2 Variables of Places

Variables are those mutable, changing attributes. They can be accessed by their names in the Place subscript and by time step t in the Place superscript:

- $P^j_\sigma(t)$ – σ of P^j in time step t;

5 Model equations

5.1 Interaction equations

When Agents interact with Places, both get some contamination from each other. Their next $t+1$ value of σ changes by the following equations. For Agents:

$$A^i_\sigma(t+1) = \frac{A^i_{\bar{\sigma}}(t) + P^j_\sigma(t) \cdot A^i_C}{1 + A^i_C} \quad \forall i, t \tag{1}$$

And for Places:

$$P^j_\sigma(t+1) = \frac{P^j_\sigma(t) + A^i_{\bar{\sigma}}(t) \cdot P^j_C}{1 + P^j_C} \quad \forall i, j, t \tag{2}$$

Where $P^j_x = A^i_x(t)$ and $P^j_x = A^i_y(t)$. The value of $A^{i,t}_{\bar{\sigma}}$ is a function of the Agent's previous traits (A_σ) and memory size (A_M):

$$A^i_{\bar{\sigma}}(t) = \Psi(A^i_\sigma, A^i_M) \quad \forall i, t \tag{3}$$

Currently, the function Ψ the average of σ values allocated in the Agent's memory:

$$A^i_{\bar{\sigma}}(t) = \frac{1}{A^i_M} \sum_{n=0}^{A^i_M} A^i_\sigma(t-n) \quad \forall i, t \tag{4}$$

5.2 Random walk

5.2.1 Set of candidate Places

At any time step, each Agent has a set $\mathbb{P}$ of candidate Places to move in so they can interact. This set is made of Places P within the Agent's window of sight of size A_R and below the Agent's interaction threshold A_D.

In the case of considering **euclidean** distances between positions:

$$P \in \mathbb{P} \;\Big|\; |P_x - A_x| \leq A_R \cap |P_y - A_y| \leq A_R \cap |P_\sigma - A_{\bar{\sigma}}| \leq A_D \tag{5}$$

In the case of considering **non-euclidean** distances between positions:

$$P \in \mathbb{P} \;\Big|\; \Phi(A_{x,y}, P_{x,y}) \leq A_R \cap |P_\sigma - A_{\bar{\sigma}}| \leq A_D \tag{6}$$

Where $\Phi(A_{x,y}, P_{x,y})$ is a function that returns the **path distance** from position $A_{x,y}$ to $P_{x,y}$.

5.2.2 Likelihood weight

If the number of Places in the set $\mathbb{P}$ of candidate is highet than 1, then Agents have to choose which Place to go in the next time step. This choice is basically random. However, the *likelihood weight* of a any candidate Place to be chosen is the sum of two components:

1. The trait of the Place: Agents have a tendency to go to Places like them;
2. The distance of the Place: Agents have a tendency to go to Places closer.

Hence:

$$P_L^j = f(P_\sigma, A_\sigma) + h(P_{x,y}, A_{x,y}) \tag{7}$$

Where P_L^j is the likelihood of the candidate Place P^j; $f(P_\sigma, A_\sigma)$ is the trait component, and; $h(P_{x,y}, A_{x,y})$ is the distance component. This value is turned into a sampling probability by normalization (divison by the sum of all places).

5.2.3 Uniform weighting function

The `uniform` weighting function is defined to set all available places the same likelihood to be chosen by a given Agent. Therefore:

$$P_L^j = \frac{1}{\mathbb{P}_N} \tag{8}$$

Where P_L^j is the likelihood of the candidate Place P^j and $\mathbb{P}_N$ is the number of candidate Places in $\mathbb{P}$.

5.2.4 Linear weighting function

The `linear` weighting function is defined to set the likelihoods of candidate Places proportinal to the σ discrepancy or the $\Phi(A_{x,y}, P_{x,y})$ distance. In the case of trait component:

$$f(P_\sigma, A_\sigma) = 1 - \frac{|A_\sigma - P_\sigma^j|}{\sum_{j=0}^{\mathbb{P}_N} |A_\sigma - P_\sigma^j|} \tag{9}$$

Where P_L^j is the likelihood of the candidate Place P^j and $\mathbb{P}_N$ is the number of candidate Places in $\mathbb{P}$. In the case of distance component:

$$h(P_{x,y}, A_{x,y}) = 1 - \frac{\Phi(A_{x,y}, P_{x,y}^j)}{\sum_{j=0}^{\mathbb{P}_N} \Phi(A_{x,y}, P_{x,y}^j)} \tag{10}$$

Where P_L^j is the likelihood of the candidate Place P^j and $\mathbb{P}_N$ is the number of candidate Places in $\mathbb{P}$.

5.2.5 Exponential weighting function

The `exponential` weighting function is defined to set the likelihoods of candidate Places to decay with the σ discrepancy or the $\Phi(A_{x,y}, P_{x,y})$ distance. In the case of trait component:

$$f(P_\sigma, A_\sigma) = e^{-\alpha * |A_\sigma - P_\sigma^j|} \tag{11}$$

In the case of distance component:

$$h(P_{x,y}, A_{x,y}) = e^{-\alpha * \Phi(A_{x,y}, P_{x,y}^j)} \tag{12}$$

By default, the α parameter is set to 1.

5.2.6 Gravity weighting function

The `gravity` weighting function is defined to set the likelihoods of candidate Places to decay with the σ discrepancy or the $\Phi(A_{x,y}, P_{x,y})$ distance. In the case of trait component:

$$f(P_\sigma, A_\sigma) = \frac{1}{|A_\sigma - P^j_\sigma|^\gamma} \tag{13}$$

In the case of distance component:

$$h(P_{x,y}, A_{x,y}) = \frac{1}{\Phi(A_{x,y}, P^j_{x,y})^\gamma} \tag{14}$$

By default, the γ parameter is set to 1.

5.2.7 Exception cases

When the set $\mathbb{P}$ of candidate Places is empty so $\mathbb{P}_N = 0$ then there is no interaction. This Agent will remain quiet until the suroundings Places yield an non-empty set $\mathbb{P}$.

When the set $\mathbb{P}$ of candidate Places holds only one candidade place so $\mathbb{P}_N = 1$ then the likelihood of interaction is unity (100%).