# CUE - Cooperation in Urban Environments: Theoretical Notation

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## 1 Model general notation

- $A$ – Agent;
- $P$ – Place;
- $\sigma$ – trait value;
- $D$ – critical Hamming distance of trait;
- $R$ – critical radius;
- $C$ – openness to contamination, or degree of interaction influence;
- $M$ – memory size;

## 2 Simulation notation

- $t$ – Timestep;
- $T$ – Total number of timesteps;
- $x$ – position coordinate;
- $y$ – position coordinate;

## 3 Agents

Agents are denoted by $A$. Any specific Agent is denoted by $A^i$, where $i$ is the ID number of the Agent in a world of $N_A$ Agents.

## 3.1 Parameters of Agents

Parameters are those static, constant attributes. They can be accessed by their names in the Agent subscript:

- $A^i_D$ – $D$ of $A^i$;
- $A^i_R$ – $R$ of $A^i$;
- $A^i_C$ – $C$ of $A^i$;
- $A^i_M$ – $M$ of $A^i$;

## 3.2 Variables of Agents

Variables are those mutable, changing attributes. They can be accessed by their names in the Agent subscript and by time step $t$ in the Agent superscript:

- $A^{i,t}_\sigma$ – $\sigma$ of $A^i$ in time step $t$;
- $A^{i,t}_x$ – $x$ position of $A^i$ in time step $t$;
- $A^{i,t}_y$ – $y$ position of $A^i$ in time step $t$;

# 4 Places

Places are denoted by $P$. Any specific Place is denoted by it's own position: $P^{x,y}$ in 2D or $P^x$ in 1D.

## 4.1 Parameters of Places

Parameters are those static, constant attributes. They can be accessed by their names in the Agent subscript:

- $P^{x,y}_C$ – $C$ of $P^{x,y}$;

## 4.2 Variables of Places

Variables are those mutable, changing attributes. They can be accessed by their names in the Place subscript and by time step $t$ in the Place superscript:

- $P^{x,y,t}_\sigma$ – $\sigma$ of $P^{x,y,t}$ in time step $t$;

# 5 Model equations

## 5.1 Interaction equations

When Agents interact with Places, both get some contamination from each other. Their next $\sigma$ value changes by the following equations. For Agents:

$$A_{\sigma}^{i,t+1} = \frac{A_{\bar{\sigma}}^{i,t} + P_{\sigma}^{X,Y,t} A_{C}^{i}}{1 + A_{C}^{i}} \quad \forall i, t \tag{1}$$

And for Places:

$$P_{\sigma}^{X,Y,t+1} = \frac{P_{\sigma}^{X,Y,t} + A_{\bar{\sigma}}^{i,t} P_{C}^{x,y}}{1 + P_{C}^{X,Y}} \quad \forall i, t \tag{2}$$

Where $X = A_{x}^{i,t}$ and $Y = A_{y}^{i,t}$. The value of $A_{\bar{\sigma}}^{i,t}$ is the average of $\sigma$ values allocated in the Agent's memory:

$$A_{\bar{\sigma}}^{i,t} = \frac{1}{A_{M}^{i}} \sum_{n=0}^{A_{M}^{i}} A_{\sigma}^{i,t-n} \quad \forall i, t \tag{3}$$

## 5.2 Random walk

### 5.2.1 Set of candidate Places

At any time step, each Agent has a set $\mathbb{P}$ of candidate Places to move in so they can interact. This set is made of Places $P$ within the Agent's window of sight of size $A_R$ and below the Agent's interaction threshold $A_D$. Hence:

$$P \in \mathbb{P} \;\big|\; |X - A_x| \leq A_R \cap |Y - A_y| \leq A_R \cap |P_{\sigma} - A_{\bar{\sigma}}| \leq A_D \tag{4}$$

### 5.2.2 Uniform weighting function

The `uniform` weighting function is defined to set all available places the same likelihood to be chosen by a given Agent. Therefore:

$$P_L = \frac{1}{\mathbb{P}_N} \tag{5}$$

Where $P_L$ is the likelihood of the candidate Place $P$ and $\mathbb{P}_N$ is the number of candidate Places in $\mathbb{P}$.

### 5.2.3 Linear weighting function

The `linear` weighting function is defined to set the likelihoods of candidate Places proportinal to the $\sigma$ discrepancy. Therefore, Agents are biased to go to Places like themselves. The function:

$$P_L = -\frac{2}{\delta_{max}^{2}} \delta + \frac{2}{\delta_{max}} \tag{6}$$

Where $P_L$ is the likelihood of the candidate Place $P$, $\delta = |A_{\sigma} - P_{\sigma}|$ and $\delta_{max}$ is the highest $\delta$ value found in $\mathbb{P}$.

### 5.2.4 Exception case

When the set $\mathbb{P}$ of candidate Places is empty so $\mathbb{P}_N = 0$ then the set is recomputed without the interaction threshold constrain and `uniform` function is used. In this case:

$$P \in \mathbb{P} \,\Big|\, |X - A_x| \leq A_R \cap |Y - A_y| \leq A_R \tag{7}$$