

CUE - Cooperation in Urban Environments: Theoretical Notation

January 5, 2023

1 Model general notation

- A – Agent;
- P – Place;
- σ – trait value;
- D – critical Hamming distance of trait;
- R – critical radius;
- C – openness to contamination, or degree of interaction influence;
- M – memory size;

2 Simulation notation

- t – Timestep;
- T – Total number of timesteps;
- x – position coordinate;
- y – position coordinate;

3 Agents

Agents are denoted by A . Any specific Agent is denoted by A^i , where i is the ID number of the Agent in a world of N_A Agents.

3.1 Parameters of Agents

Parameters are those static, constant attributes. They can be accessed by their names in the Agent subscript:

- A_D^i – D of A^i ;
- A_R^i – R of A^i ;
- A_C^i – C of A^i ;
- A_M^i – M of A^i ;

3.2 Variables of Agents

Variables are those mutable, changing attributes. They can be accessed by their names in the Agent subscript and by time step t in the Agent superscript:

- $A_\sigma^{i,t}$ – σ of A^i in time step t ;
- $A_x^{i,t}$ – x position of A^i in time step t ;
- $A_y^{i,t}$ – y position of A^i in time step t ;

4 Places

Places are denoted by P . Any specific Place is denoted by its own position: $P^{x,y}$ in 2D or P^x in 1D.

4.1 Parameters of Places

Parameters are those static, constant attributes. They can be accessed by their names in the Agent subscript:

- $P_C^{x,y}$ – C of $P^{x,y}$;

4.2 Variables of Places

Variables are those mutable, changing attributes. They can be accessed by their names in the Place subscript and by time step t in the Place superscript:

- $P_\sigma^{x,y,t}$ – σ of $P^{x,y,t}$ in time step t ;

5 Model equations

5.1 Interaction equations

When Agents interact with Places, both get some contamination from each other. Their next σ value changes by the following equations. For Agents:

$$A_{\sigma}^{i,t+1} = \frac{A_{\sigma}^{i,t} + P_{\sigma}^{X,Y,t} A_C^i}{1 + A_C^i} \quad \forall i, t \quad (1)$$

And for Places:

$$P_{\sigma}^{X,Y,t+1} = \frac{P_{\sigma}^{X,Y,t} + A_{\sigma}^{i,t} P_C^{X,Y}}{1 + P_C^{X,Y}} \quad \forall i, t \quad (2)$$

Where $X = A_x^{i,t}$ and $Y = A_y^{i,t}$. The value of $A_{\sigma}^{i,t}$ is the average of σ values allocated in the Agent's memory:

$$A_{\sigma}^{i,t} = \frac{1}{A_M^i} \sum_{n=0}^{A_M^i} A_{\sigma}^{i,t-n} \quad \forall i, t \quad (3)$$

5.2 Random walk

5.2.1 Set of candidate Places

At any time step, each Agent has a set \mathbb{P} of candidate Places to move in so they can interact. This set is made of Places P within the Agent's window of sight of size A_R and below the Agent's interaction threshold A_D . Hence:

$$P \in \mathbb{P} \mid |X - A_x| \leq A_R \cap |Y - A_y| \leq A_R \cap |P_{\sigma} - A_{\sigma}| \leq A_D \quad (4)$$

5.2.2 Uniform weighting function

The **uniform** weighting function is defined to set all available places the same likelihood to be chosen by a given Agent. Therefore:

$$P_L = \frac{1}{\mathbb{P}_N} \quad (5)$$

Where P_L is the likelihood of the candidate Place P and \mathbb{P}_N is the number of candidate Places in \mathbb{P} .

5.2.3 Linear weighting function

The **linear** weighting function is defined to set the likelihoods of candidate Places proportional to the σ discrepancy. Therefore, Agents are biased to go to Places like themselves. The function:

$$P_L = -\frac{2}{\delta_{max}^2} \delta + \frac{2}{\delta_{max}} \quad (6)$$

Where P_L is the likelihood of the candidate Place P , $\delta = |A_{\sigma} - P_{\sigma}|$ and δ_{max} is the highest δ value found in \mathbb{P} .

5.2.4 Exception case

When the set \mathbb{P} of candidate Places is empty so $\mathbb{P}_N = 0$ then the set is recomputed without the interaction threshold constrain and **uniform** function is used. In this case:

$$P \in \mathbb{P} \mid |X - A_x| \leq A_R \cap |Y - A_y| \leq A_R \quad (7)$$