Graph Theory

Graph

has no direction

↳ Vertices set (V)

↳ Edges set (E)

$x, y \in V(G)$

vertices

edge

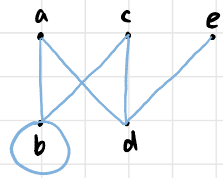
$xy \in E(G)$ → unordered pair

Size of graph

$|V(G)| = |G|$ → no of vertices

$||E(G)|| = ||G||$ → no of edges

no. of



Adjacent

↳ an edge b/w 2 vertices

Connected

↳ when every pair of distinct vertice is connected

Independent Vertex

↳ N(...) is empty set

↳ non adjacent vertices

↳ a = c, e

End Point

↳ vertices of edges

↳ a b

Incident

↳ adjacent edges

↳ a: {ab, ad}

Neighbours (N)

↳ N(a) = {b, d}

K-regular

↳ every vertex has same degree

DOV

Degree of vertex

↳ no of adjacent edges

↳ b = 3 , a = 2

odd degree

even degree

Isolated Vertex

↳ no adjacent vertices

↳ DOV = 0

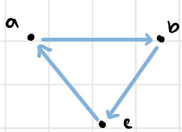
Multi edge

↳ multiple edges b/w same vertices

Simple graph

↳ no multiple edge

↳ no loop

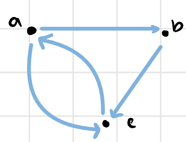


Multiple graph

↳ loops

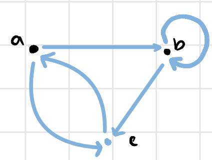
either

↳ multiple edge

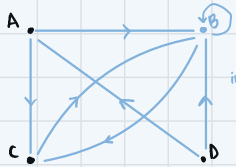


Pseudo graph

- loops and multiple same node edges



Directed graph

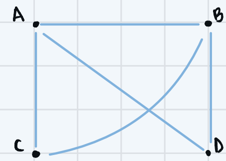


initial terminal

ABCD : vertices/nodes

⇄ : edges

Undirected graph



initial terminal

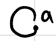
ABCD : vertices/nodes

⇄ : edges

indegree = $deg^-(v)$ $^-a: 1$ $^-b: 4$

outdegree = $deg^+(v)$ $^+a = 2$ $^+b: 2$

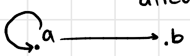
undirected



deg (a) = 2

self loop degree = 2

directed

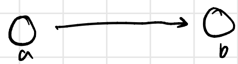


$deg^+(a) = 2$

$deg^-(a) = 1$

$deg^-(b) = 1$

initial



out
goes out

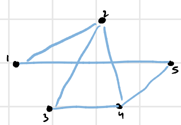
in
goes in

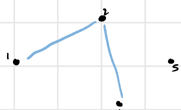
$deg^+(a) = 1$

$deg^-(b) = 1$

TABLE 1 Graph Terminology.			
Type	*Edges*	*Multiple Edges Allowed?*	*Loops Allowed?*
Simple graph	Undirected	No	No
Multigraph	Undirected	Yes	No
Pseudograph	Undirected	Yes	Yes
Simple directed graph	Directed	No	No
Directed multigraph	Directed	Yes	Yes
Mixed graph	Directed and undirected	Yes	Yes

G





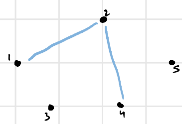
H_1

subgraph

G H_1

V > V

E > E



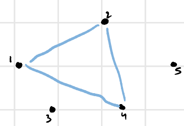
H_2

spanning subgraph

G H2

V = V

E > E

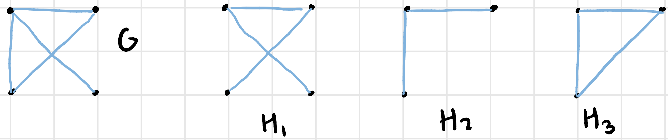


H_3

NOT a subgraph

as 1-4 edges does not exist

$V(H) \subseteq V(G)$
$E(H) \subseteq E(G)$



Subgraph
- ↳ subset vertices
- ↳ subset edges

Spanning subgraph
- ↳ same vertices
- ↳ subset edges
- ↳ H_1

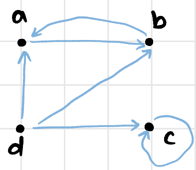
Induced subgraph
- ↳ subset vertices
- ↳ same edges
- ↳ H_3

Digraph (directed graph) → remove direction → Underlying Graph

- ↳ Head, Tail
- ↳ indegree, outdegree
- ↳ Vertices set (V)
- ↳ Arc set (A)
- ↳ sum of indegree = |edges|
- ↳ sum of outdegree = |edges|

Ex 1.6 $V(G_1) = \{a, b, c, d\}$ $A(G_1) = \{ab, ba, cc, dc, db, da\}$ → has directions

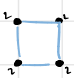
(start vertex, end vertex)



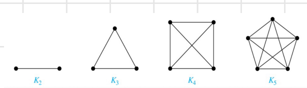
weighted graph
- ↳ edges with real numbers
  - ↳ probability tree

regular graph
- ↳ all vertices have same degree

$E = \frac{n * d}{2}$ (E: no of edges; n: no of vertex; d: degree of each vertex)



complete graph
- ↳ one edge b/w every vertice
- ↳ all connected
- ↳ simple graph
- ↳ regular graph



Properties of K_n
- ↳ degree of every vertex = $n-1$
- ↳ no of edges = $\frac{n(n-1)}{2} = {}^nC_2$

k-reg	complete
$\deg(v_i) = k$	$v_i \sim v_j$

Clique size of graph

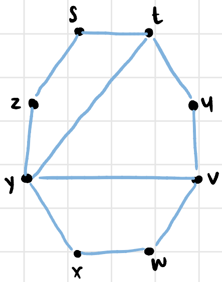
↳ $W(G)$ is max Z^+ (positive integer) such that

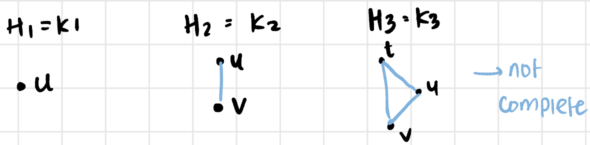
K_n is a subgraph of G

but K_{n+1} is not complete

Ex 1.a //

G_1

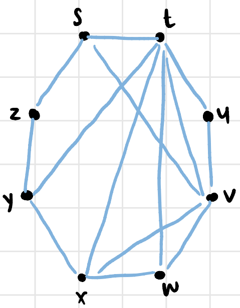


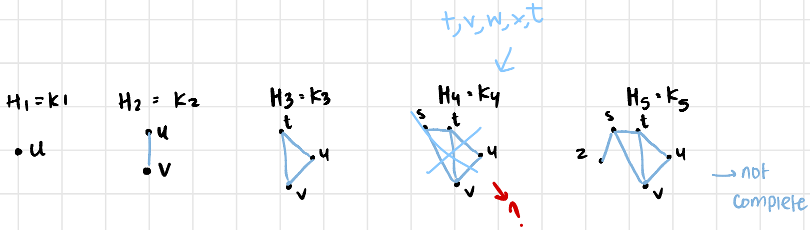


Since K_3 is not a subgraph of G_1

but K_2 is hence $W(G_1) = 2$

G_2





Since K_4 is not a subgraph of G_1

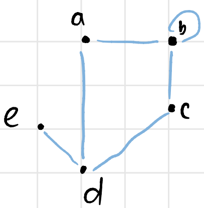
but K_5 is hence $W(G_2) = 4$

Plannar graph

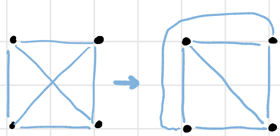
↳ dosent cross edges

↳ $n \geq 4$ then planar possible

if edge crossing exists → check if it can be drawn another way



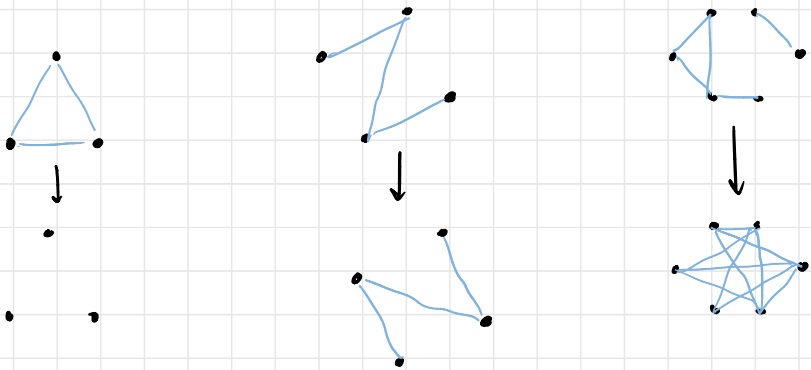
Plannar



Complement Graph

↳ G → $\overline{G}$

1.10



G

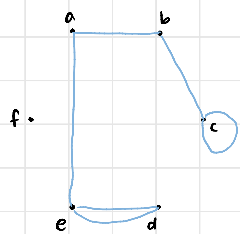
$\overline{G}$

Ex 1.1

V= {a,b,c,d,e,f}   E={ab, ae, bc, cc, de, ed}

if graph = simple
then compliment possible

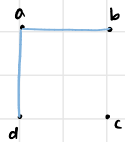
1) Draw graph G
2) Is G simple = No, cc is a loop
3) DOV = a=2, b=2, c=2, d=2, e=3, f=0
4) N(a) = {b,e}
5) edges incident to b = {ba, bc}



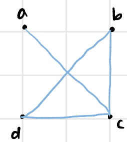
Ex 1.2

V= {a,b,c,d}   E={ab, ad}

1) Draw graph G
2) Is G simple = yes
3) DOV = a=2, b=1, c=0, d=1,
compliment
4) $\overline{G_2}$ = {ac, bc, bd, dc}



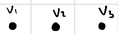
$\overline{G_2}$



TYPES OF SIMPLE GRAPH

Null graph (N_n)

↳ graph with 'n' vertices and 0 edges

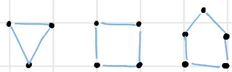


Cylic Graph (C_n, $n \geq 3$)

↳ simple graph

↳ make a cycle

↳ $V = E = n$, degree $= 2$

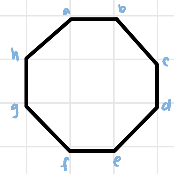


Biphatite graph

↳ adjacent vertices in other partition

↳ max edges $= \left\lfloor \frac{n^2}{4} \right\rfloor$ → floor

P_1	P_2
a	b
g	h
e	f
c	d



complete Biphatite graph ($K_{m,n}$)

↳ every vertex in P_1 is adjacent to every vertex in P_2

↳ max vertices : $m+n$

↳ max edges : $m * n$

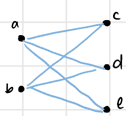
↳ max degree : m/n which ever is bigger

↳ $d(P_1) = V$ in P_2

↳ $d(P_2) = V$ in P_1

$K_{2,3}$

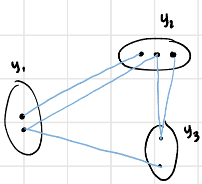
P_1	P_2
a	c
b	d
	e

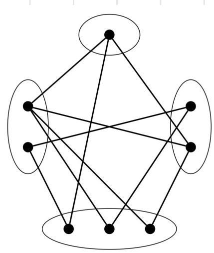


K-Biphatite graph

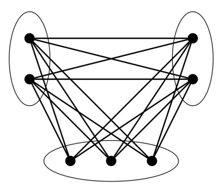
↳ can be partitioned into k sets

↳ so every edge has one endpoint x_i and the other in x_j





4-partite

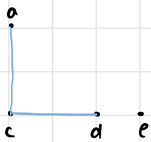


$K_{2,2,3}$

GRAPH COMBINATIONS

union

↳ combine both graphs

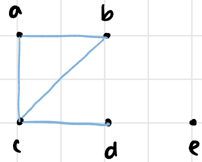


G

union

H

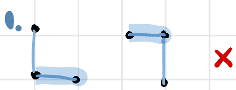
=



G ∪ H

Sum

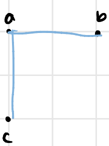
↳ no common vertices



Join

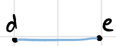
↳ sum then

↳ connect vertices of both graphs



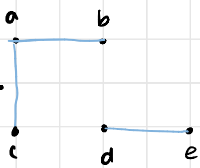
G

sum



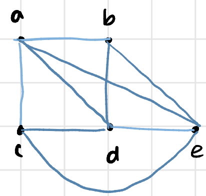
H

=



G + H

join



G join H

Isomorphisim

↳ same vertices, degree of vertices

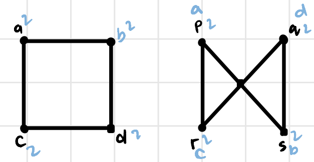
↳ same edges

↳ same max min length

↳ is connected

↳ has Euler and Hamilton circuit

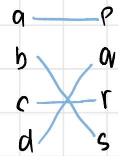
↳ bijective

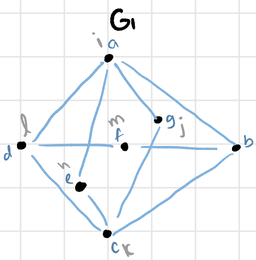


4 vertices

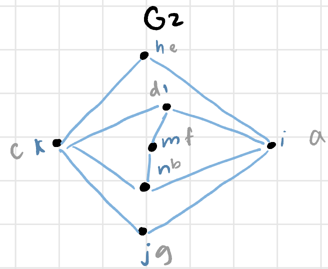
max lenght = 4

min length: 4





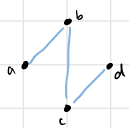
a — i
d — l
c — k
b — n
f — m
j — g
h — e



P.120

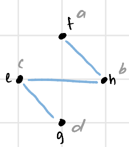
Self complementary ≅

↳ G complement is isomorphic to its graph



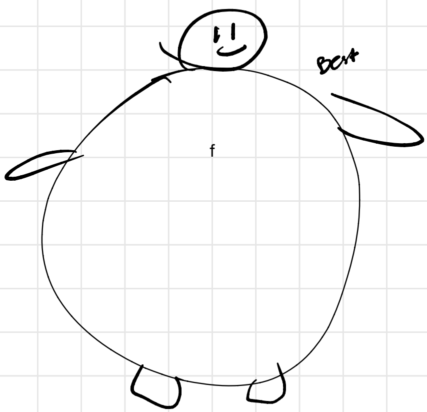
G

a — f
b — h
c — e
d — g



$\bar{G}$

↳ G complement



Adjancey matrix of undirected graph

↳ which vertices are neighbours

$A(G) \cdot [\underbrace{a_{ij}}_{\text{no of edges b/w } V_1 \text{ and } V_2}]_{n\times n}$

adajency list

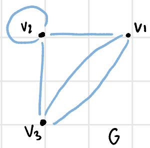
$$A(G) = \begin{array}{c|ccc} & V_1 & V_2 & V_3 \\ \hline V_1 & 0 & 1 & 2 \\ V_2 & 1 & 1 & 1 \\ V_3 & 2 & 1 & 0 \end{array}_{3\times 3}$$

edges

→ matrix is transpose

edges

no of loops



Adjancey matrix of directed graph

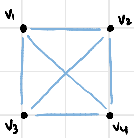
↳ count going out only

$A(G) \cdot [\underbrace{a_{ij}}_{\text{no of arcs when head is } V_i \text{ and tail is } V_j}]_{n\times n}$

$$A(G) = \begin{array}{c|ccc} & V_1 & V_2 & V_3 \\ \hline V_1 & 0 & 1 & 0 \\ V_2 & 1 & 0 & 0 \\ V_3 & 0 & 1 & 1 \end{array}_{3\times 3}$$

→ matrix is **NOT** transpose

IGd Find A(G)



$$A(G) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Handshaking Theorem

if g is a graph

always even
else graph dosent exist

sum of degree = $2 * edges$

$V \times$ degree of each $= 2e$

total no of odd vertices degree in any graph is always even

Beta index $= \frac{\text{Edges}}{\text{vertices}}$ $B = \frac{E}{V}$

TYPES OF PROOFS

DIRECT PROOFS

↳ $P \to q$ implication

↳ always assume P is true

↳ show q is also true

MATHAMETICAL INDUCTION

↳ check base case

↳ n: first tern, n: range → if not given assume 1
(LHS over "first tern", RHS over "range")

↳ induction step

↳ $n = k$

↳ $n = k+1$

↳ replace and prove

INDIRECT PROOFS

contraposition

↳ $\neg q \to \neg P$ contrapositive

contradiction

↳ assume statement = false

↳ $\neg q$

↳ if True then statement false — as our wrong assumption is true so it means the statement is false

↳ if False then statement true

DIRECTED PROOFS

Direct Proofs

- ↳ $P \to q$ implication
- ↳ always assume p is true
- ↳ show q is also true

even	$2k$
odd	$2k+1$
Prime	$n>1$, if $n=r.s$ then $r=1 \mid s=1$
composite	$n=r.s$, if $r\neq 1$ & $s\neq 1$
rational	$r=a/b$, $b\neq 0$
Perfect square	$n=k^2$
divides	n/d, $d\neq 0$

Q) Prove sum of 2 odd num is even

$$\underbrace{(\underbrace{2k+1}_{\text{odd number}}) + (\underbrace{2k+1}_{\text{odd number}})}_{\text{P assuming is true}} = \underbrace{2k}_{q}$$

$2k+1 + 2k+1$

$4k+2$

$2(2k+1)$

$m=2k+1$

$2m$ is even

Q) sum of 3 consecutive num is divisible by 3

$n+(n+1)+(n+2)$

$3n+3$

$3(1+n)$

$k=1+n \to$ not 0

$3k$

Q) odd integer n^2 is odd integer

$(2k+1)(2k+1)$

$4k^2+2k+2k+1$

$4k^2+4k+1$

$2(k^2+2k)+1$

$m=(k^2+2k)$

$2m+1$ is odd

Q) m and n are both Perfect squares then $m.n$ = perfect square

s^2 t^2

$(s.s)$ $(t.t)$

$st \cdot st$

$(st)^2$

Q) n = odd n^3+n = even

$(2k+1)^3 + (2k+1)$

$(4k^2+4k+1)2k+1 + (2k+1)$

$8k^3+8k^2+2k+4k^2+4k+1+2k+1$

$8k^3+12k^2+8k+2$

$2(4k^3+6k^2+4k+1)$

$m=4k^3+6k^2+4k+1$

$2m$ is even

Q) sum of 2 rational num = rational

$r=\frac{a}{b}$ $s=\frac{c}{d}$

$b\neq 0$ $d\neq 0$

$r+s$

$\frac{a}{b}+\frac{c}{d}$

$\frac{ad+cb}{bd}$ $ad+cb \to p$, $bd \to q$

$\frac{p}{q}$ is rational

Indirect Proofs

contraposition

↳ $\neg q \to \neg p$ contrapositive

Q) n is an integer $\overset{p}{3n+2 = \text{odd}}$ then $\overset{q}{n = \text{odd}}$

$\overset{\neg q}{n = \text{even}}$ → becuz contrapositive so take opposite

$n = 2k$

$3(2k)+2$

$6k+2$

$2(\underbrace{3k+1}_{m})$

$2m$

even

so True

Q) all intergs n, if $\overset{p}{n^2 = \text{even}}$ then $\overset{q}{n = \text{even}}$

$\overset{\neg q}{n = \text{odd}}$ → $\overset{\neg p}{n^2 = \text{odd}}$

$(2k+1)^2$

$4k^2+2k+1$

$2(\underbrace{2k^2+k}_{m})+1$

$2m+1$

True

contradiction

↳ assume statement = false

↳ $\neg q$

↳ if True then statement false — as our wrong assumption is true so it means the statement is false

↳ if False then statement true

Q) $n^2 = \text{even}$ then $n = \text{even}$

$n = \text{odd}$ $\neg q$

$(2k+1)^2$

$4k^2+4k+1$

$2(\underbrace{2k^2+2k}_{m})+1$

$2m+1 \neq \text{even}$ — so what we assumed is wrong so statement is true

hence statement is true

Q) $n^3+5 = \text{odd}$ then $n = \text{even}$

$n = \text{odd}$ $\neg q$

$(2k+1)^3+5$

$(4k^2+4k+1)\,2k+1$

$8k^3+8k^2+2k+4k^2+4k+1+5$

$8k^3+12k^2+6k+6$

$2(4k^3+6k^2+3k+3)$

$2k \neq \text{odd}$

so statement true

Q) $\sqrt{2}$ is irrational

$\sqrt{2} = \text{rational}$

$\sqrt{2} = \dfrac{a}{b}$ } as rational

$(\sqrt{2})^2 = a^2/b^2$

$2 = a^2/b^2$

→ $2b^2 = a^2$ — b even so take a ev

$b \neq 0$

$2b^2 = (2k)^2$

$2b^2 = 4k^2$

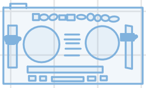
$b^2 = 2k^2$ so true

hence statement is false

Mathamatical induction

- check base case
  - n: first term (LHS), n: range (RHS) → if not given assume 1
- induction step
  - $n=k$
  - $n=k+1$
  - replace and prove

a) Prove $1+2+3+4+\dots n \;\; \frac{n(n+1)}{2}$, $n \geq 1$



Basis case

Leftside → n = first term, $n=1$ | rightside → n = value given as range, $n=1$ | compare

$1 = \frac{1(1+1)}{2}$

$1 = 1$ since equal move to next step

Induction step:

- Assume true $n=k$

$1+2+3+4+\dots k = \frac{k(k+1)}{2}$

- Show true $n=k+1$ based on above assumption

$\underbrace{1+2+3+4+\dots k}_{\text{same as above assumption so replace}} + k+1 = \frac{(k+1)(k+1+1)}{2}$

- $\frac{k(k+1)}{2} + k+1 = \frac{(k+1)(k+1+1)}{2}$

$\frac{k(k+1)}{2} + \frac{2}{2}(k+1) = \frac{(k+1)(k+1+1)}{2}$

so we can get same denominator

$k(k+1) + 2(k+1) = (k+1)(k+2)$

$k^2+k+2k+2 = k^2+2k+k+2$

hence equal so proven

b) $1+2+2^2 \dots 2^n = 2^{n+1}-1$ $\quad n \geq 0$

Basis step

$n=1$ $n=0$

$1 = 2^1 - 1$

$1 = 1$

Induction step

- $n=k$

$1+2+2^2 \dots 2^k = 2^{k+1}-1$

- $n=k+1$

$\underbrace{1+2+2^2 \dots 2^k}+2^{k+1} = 2^{k+1+1}-1$

$2^{k+1}-1+2^{k+1} = 2^{k+2}-1$

$2^{k+1}(1+1)-1$

$2^{k+1}\cdot 2^1 - 1$

$2^{k+2}-1 = 2^{k+2}-1$

hence equal so proven

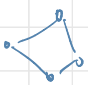
Havel Hakimi Theorem

→ total degree = 2 x edges (always even)
total d = 2e

↳ wether a simple graph exists or not

1. Arrange in descending order
2. Remove first element k
3. Subtract 1 from each by moving first element times k
4. Repeat from step 1
5. Till 0's are left

$(\cancel{2},2,2,2)$
reorder
$(1,1,2)$
dont forget
$(\cancel{2},1,1)$
$(0,0)$
YES POSSIBILITY OF GRAPH



$(\cancel{3},2,1,1,0)$
$(\cancel{1},0,0,0)$
$(0,-1,0,0)$
NOT POSSIBLE

Handshake theorem
no. of odd V should always be even
$3+1+1 = 5$ ✗

$(\cancel{8},5,4,3,3,1)$ element does not exist
NOT POSSIBLE

$(\cancel{7},6,5,4,4,3,2,1)$
$(\cancel{5},4,3,3,2,1,0)$
$(\cancel{3},2,2,1,0,0)$
$(\cancel{1},1,0,0,0)$
$(0,0,0,0,0)$
YES POSSIBILITY OF GRAPH

1.6

Score sequence

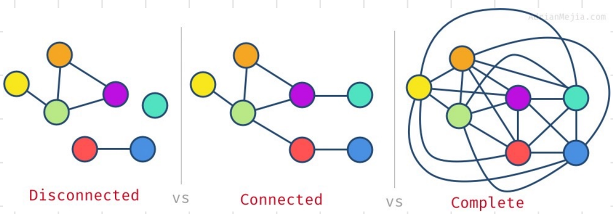
↳ listing out degrees
↳ increasing order
↳ at most (max) one $s_k = 0$
↳ at most one $s_k = n-1$
↳ $s_1 + s_2 + \dots + s_n = \frac{n(n-1)}{2} = {}^nC_2$
↳ $0 \le s_k \le n-1$

Degree sequence

↳ listing degrees of vertices
↳ decreasing order

Graphical

↳ degree sequence of a simple graph



2.1.2 TOURING A GRAPH

Walk none
- edges can repeat
- vertices can repeat
- closed walk (start, end same)
  - aka circuit

Trail edges → Eulerian
- cover edges once
- vertices can repeat

Path vertices → Hamilton
- cover vertices once
- edges can repeat
- open (start, end not same)

Circuit
- start, end same

Cycle
- starts and ends at same vertice
- edges cant repeat
- vertices cant repeat
- closed path

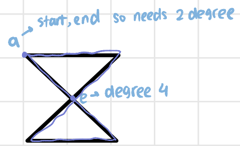
$\boxed{\text{length} = n-1}$

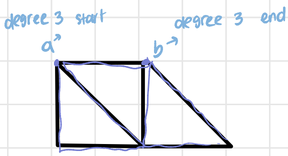
2.1.3

Euler Circuit
- cover all edges once
- end where start
- all vertices degree even
- connected

Euler Path / Semi Eulerian → edges
- cover all edges once
- start, end not same
- 2 vertices have odd degree
- connected

check with degree for euler

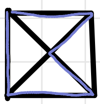




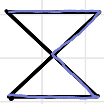
always make for hamilton

2.2

Hamilton Circuit
- cover all vertex once
- end where start



Hamilton Path → vertices
- cover all vertex once
- start, end not same



Hamilton Check
- connected
- no degree < 2
- not cut vertex
  - vertex whose removal splits graph

(4) If G contains a vertex x of degree 2 then both edges incident to x must be included in the cycle.

(5) If two edges incident to a vertex x must be included in the cycle, then all other edges incident to x cannot be used in the cycle.

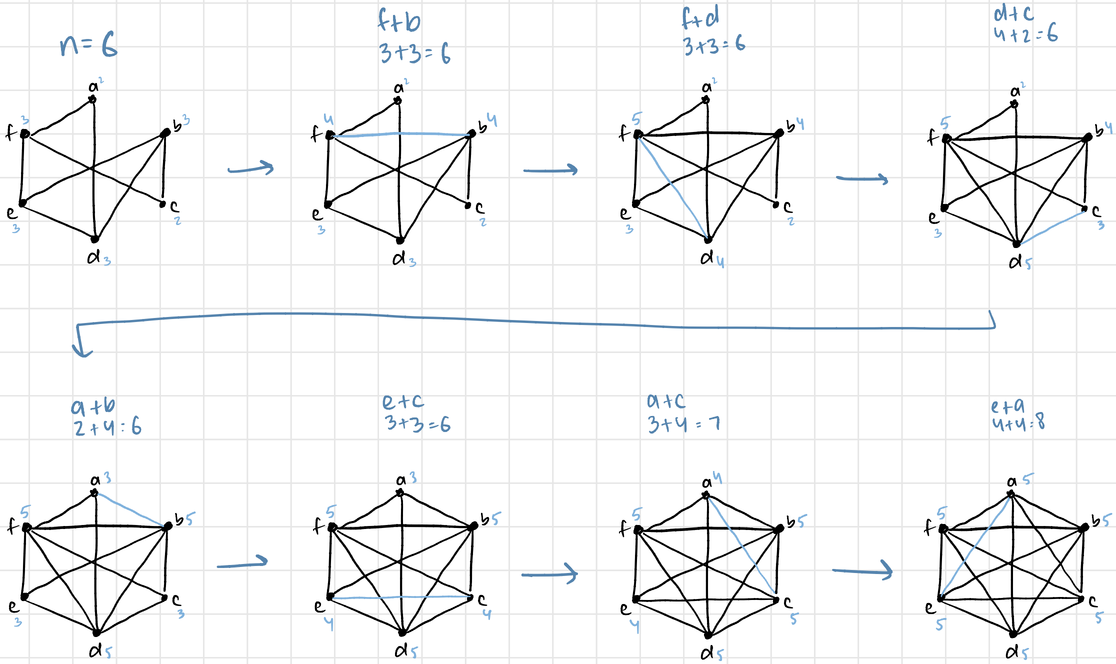
(6) If in the process of attempting to build a hamiltonian cycle, a cycle is formed that does not span G, then G cannot be hamiltonian.

Hamilton closure

↳ denoted by cl(G)

↳ add edges whose degree sum $\geq n$ no of vertices

↳ $deg(x) + deg(y) \geq n$



- if cl(G) is hamilton then (G) is hamilton
- if $V(G) \geq 3$, cl(G) is complete then (G) is hamilton

2.2.1 TRAVELLING SALESMAN PROBLEM

1. Brute Force → min hamilton cycle
2. Nearest Neighbour
3. Nearest Insersion
4. Cheapest link
5. Repitive Nearest Neighbour

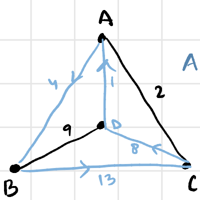
approx hamilton cycle

1. BRUTE Force Algo

↳ Choose any starting vertex → move connectedly

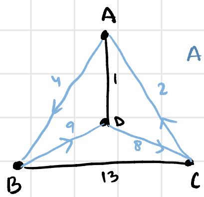
↳ check every possibility from that vertex

↳ find the shortest



A B C D A

$4+13+8+1 = 26$

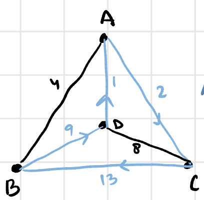


A B D C A

$4+9+8+2 = 23$

↓

Optimal



A C B D A

$2+13+9+1 = 25$

?

5. Nearest Insertion Algo

↳ choose smallest edge → move connectedly

↳ if same pick any

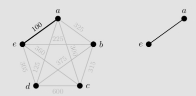
↳ pick smallest adjacent vertice to both prev vertices

↳ pick smallest adj vertice to all prev 3 vertice

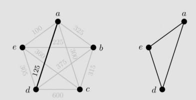
↳ delete a prev edge → biggest

↳ Repeat

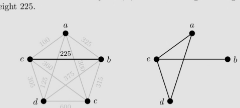
Step 1: The smallest weight edge is ae at 100.



Step 2: The closest vertex to either a or e is d through the edge ad of weight 125. Form a cycle by adding ad and de.



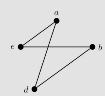
Step 3: The closest vertex to any of a, d, or e is b through the edge be with weight 225.



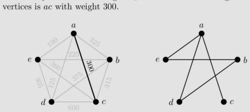
In adding edge be, either ae or de must be removed so that only two edges are incident to e. To determine which is the better choice, compute the following expressions:

$$be + ba - ea = 225 + 325 - 100 = 450$$
$$be + bd - ed = 225 + 375 - 305 = 295$$

Since the second total is smaller, we create a larger cycle by adding edge bd and removing ed.



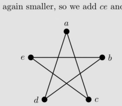
Step 4: The only vertex remaining is c, and the minimum edge to the other vertices is ac with weight 300.



Either ae or ad must be removed. As in the previous step, we compute the following expressions:

$$ca + cd - ad = 300 + 600 - 125 = 775$$
$$ca + ce - ae = 300 + 360 - 100 = 560$$

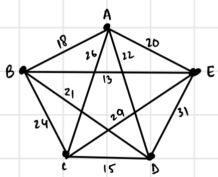
The second total is again smaller, so we add ce and remove ae.

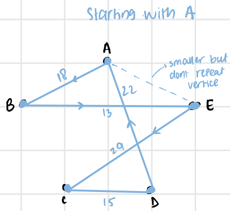


Output: The cycle is $a\,c\,e\,b\,d\,a$ with total weight 1385.

2. Nearest Neighbour Algo

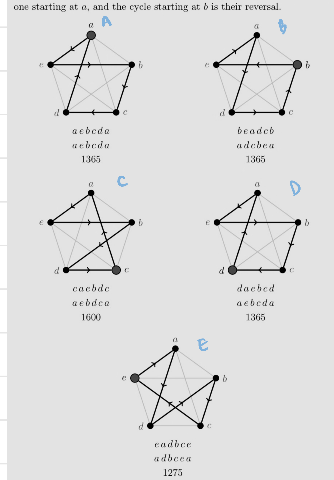
- ↳ choose any starting vertex
- ↳ Pick smallest adjacent → move connectedly
  - ↳ if same, pick any
- ↳ Repeat but dont use same vertex





3. Repititive Nearest Neighbour Algo

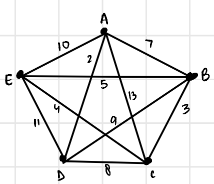
- ↳ choose any starting vertice
- ↳ apply nearest neighbour algo
- ↳ Repeat for each vertice
- ↳ find shortest

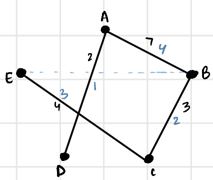


Solution: The five cycles are shown below, with the original name, the rewritten form with *a* as the reference point, and the total weight of the cycle. You should notice that the cycle starting at *d* is the same as the one starting at *a*, and the cycle starting at *b* is their reversal.

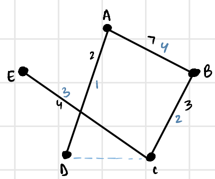
4. CHEAPEST LINK ALGO

- ↳ Pick smallest edge → move disconnectedly
  - ↳ if 2 same size, pick any
- ↳ Repeat BUT
  1. no vertex has 3 edges
  2. no cycle
  3. no circuit unless all vertices are used

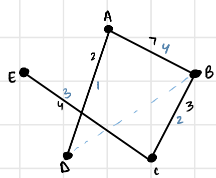




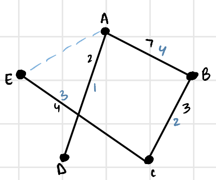
not 5 as BEC cycle



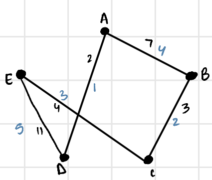
not 8 as a circuit w/o all vertices



not 9 as DBA cycle



not 10 as a circuit w/o all vertices



as 11 involves circuit with all vertices

→ completed so ignore 13

2	✓
3	✓
4	✓
5	✗
7	✓
8	✗
9	✗
10	✗
11	✓
13	

EDABCE

$11+2+7+3+4=27$

Relative error

$$\epsilon_r = \frac{\text{Solution} - \text{Optimal}}{\text{Optimal}}$$

Example 2.16 Find the relative error for each of the algorithms performed on the graph from Example 2.11.

Solution:

- Repetitive Nearest Neighbor: $\epsilon_r = \dfrac{1275 - 1270}{1270} = 0.003937 \approx 0.39\%$
- Cheapest Link: $\epsilon_r = \dfrac{1365 - 1270}{1270} = 0.074803 \approx 7.48\%$
- Nearest Insertion: $\epsilon_r = \dfrac{1385 - 1270}{1270} = 0.090551 \approx 9.05\%$

2.1.4

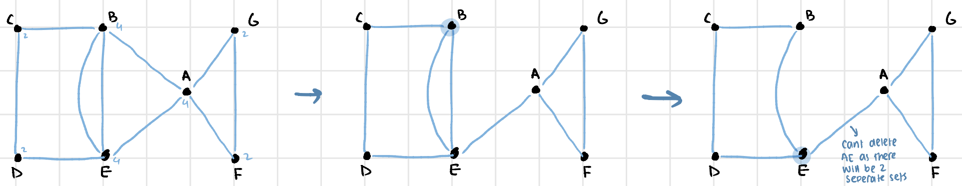
Fleurys Algorithm

↳ find Euler circuit

↳ has 0 or 2 odd vertices

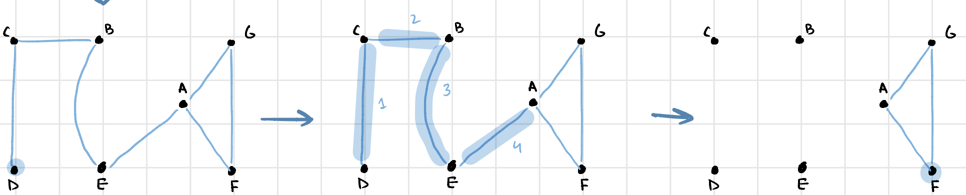
1. Start at any vertex → start at any odd degree vertice if Euler Path
2. choose any adjacent edge → if deleting it wont split graph in two
3. Write down the edge ↘ moves connectedly
4. Delete it from graph
5. Repeat unil circuit/trail is complete

Since all even degree, it is an euler circuit



AB

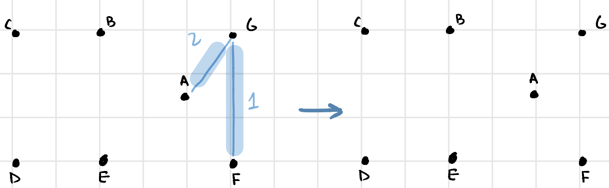
ABE



ABED

ABEDCBEA

ABEDCBEAF



ABEDCBEAFGA

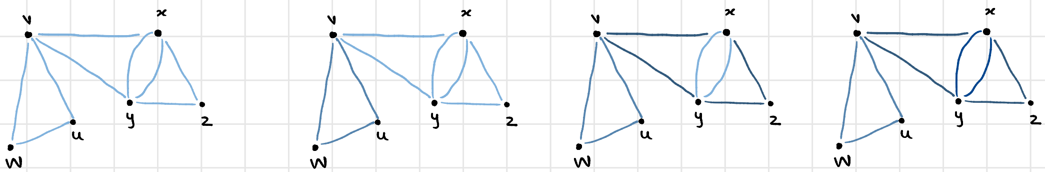
ABEDCBEAFGA

Hiernolzer's Algorithm

↳ find Euler circuit

↳ all vertices are even

1. Choose any starting vertice
2. Make boxes ending at the same vertice
3. Write in first instance of vertice
4. Repeat until all edges are used

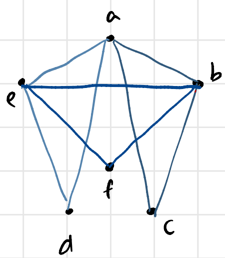


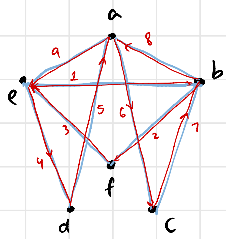
vuwv

vyzxvuwv

vyzxyxvuwv

Euler circuits





edae

edacbae

ebfedacbae

2.3.1 Dijkstras Algo

Dijkstra's Algorithm

Input: Weighted connected simple graph $G = (V, E, w)$ and designated *Start* vertex.

Steps:

1. For each vertex x of G, assign a label $L(x)$ so that $L(x) = (-, 0)$ if $x = Start$ and $L(x) = (-, \infty)$ otherwise. Highlight *Start*.
2. Let $u = Start$ and define F to be the neighbors of u. Update the labels for each vertex v in F as follows:

   if $w(u) + w(uv) < w(v)$, then redefine $L(v) = (u, w(u) + w(uv))$

   otherwise do not change $L(v)$
3. Highlight the vertex with lowest weight as well as the edge uv used to update the label. Redefine $u = v$.
4. Repeat Steps (2) and (3) until each vertex has been reached. In all future iterations, F consists of the un-highlighted neighbors of all previously highlighted vertices and the labels are updated only for those vertices that are adjacent to the last vertex that was highlighted.
5. The shortest path from *Start* to any other vertex is found by tracing back using the first component of the labels. The total weight of the path is the weight given in the second component of the ending vertex.

Output: Highlighted path from *Start* to any vertex x of weight $w(x)$.

distance → D(v) · vertex name → D(w)

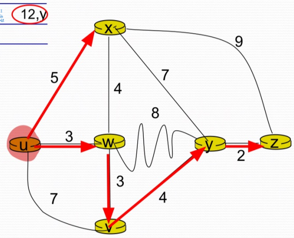
neighbouring nodes

Step	N'	D(v)	D(w)	D(x)	D(y)	D(z)
0	u	7,u	3,u (shortest)	5,u	∞	∞
1	+3 uw	3+3 6,w → small so replaced		5,u	11,w	∞
2	+5 uwx	6,w		4+3=7 big so didnt replace	11,w	14,x
3	+6 uwxv			6+4=10 small so replace	10,v	14,x
4	+10 uwxvy				10+2=12 small so replace	12,y
5	uwxvyz					

5 vertices so 5 columns

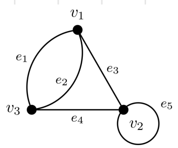
notes:

- construct shortest path tree by tracing predecessor nodes
- ties can exist (can be broken arbitrarily)



2.3.2 Walk using Matrices

↳ for any integr $n > 0$, the entry a_{ij} in A^n counts no of walks from v_i to v_j



$$\begin{array}{c c} & \begin{matrix} v_1 & v_2 & v_3 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \end{matrix} & \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix} \end{array}$$

walk from $v_1 \to 2$

length 1 : $v_1 \to v_2$

length 2: $v_1 \to v_2 \to v_2$

$v_1 \to v_3 \to v_2$

$v_1 \to v_3 \to v_2$

$1 \times 1 + 2 \times 1 = 3$

$$\begin{bmatrix} 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0 \times 1 + 1 \times 1 + 2 \times 1 = 3$$

$$\underset{A}{\begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix}} \times \underset{A}{\begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix}} = \begin{array}{c c} & \begin{matrix} v_1 & v_2 & v_3 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \end{matrix} & \underset{A^2}{\begin{bmatrix} 5 & 3 & 1 \\ 3 & 3 & 3 \\ 1 & 3 & 5 \end{bmatrix}} \end{array}$$

a_{ij} represents no of walks b/w v_i to v_j of using 2 edges

$$\underset{A^2}{\begin{bmatrix} 5 & 3 & 1 \\ 3 & 3 & 3 \\ 1 & 3 & 5 \end{bmatrix}} \underset{A^2}{\begin{bmatrix} 5 & 3 & 1 \\ 3 & 3 & 3 \\ 1 & 3 & 5 \end{bmatrix}} = \underset{A^3}{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}}$$

$v_1 \xrightarrow{e_3} v_2 \xrightarrow{e_4} v_3$

$v_3 \xrightarrow{e_1} v_1 \xrightarrow{e_3} v_2$

$v_3 \xrightarrow{e_1} v_1 \xrightarrow{e_3} v_2$

$v_3 \xrightarrow{e_4} v_2 \xrightarrow{e_5} v_2$

$v_3 \xrightarrow{e_4} v_2 \xrightarrow{e_4} v_3$

$v_3 \xrightarrow{e_2} v_1 \xrightarrow{e_2} v_3$

$v_3 \xrightarrow{e_1} v_1 \xrightarrow{e_1} v_3$

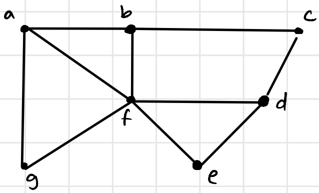
$v_3 \xrightarrow{e_2} v_1 \xrightarrow{e_1} v_3$

$v_3 \xrightarrow{e_1} v_1 \xrightarrow{e_2} v_3$

a_{ij} represents no of walks b/w v_i to v_j of using 3 edges

2.3.3

Distance, Diameter, Radius



distance $d(u,v)$

↳ length of shortest path b/w u, v

$d(a,e) = 2$

eccentricity of vertex u

↳ max distance from u to any other vertex

↳ $E(u) = \max d(u,v)$

$e(a) = 2$ $a,b = 1$ $a,c = 2$ $a,d = 2$

$a,e = 2$ $a,f = 1$ $a,g = 1$

diameter

↳ max eccentricity

$e(a) = 2$ $e(b) = 2$ $e(c) = 3$

$e(d) = 2$ $e(e) = 2$ $e(f) = 2$

max is $e(c) = 3$

Radius

↳ min eccentricity

$e(a) = 2$ $e(b) = 2$ $e(c) = 3$

$e(d) = 2$ $e(e) = 2$ $e(f) = 2$

min is $e(f) = 2$

circumfrence

↳ biggest cycle length

$g \to a \to b \to c \to d \to e \to f \to g$

$= 7$

central vertex

↳ which vertex has encentricity = radius

as $r = 2$

$v = a, d, e, b, f$

girth

↳ smallest cycle length

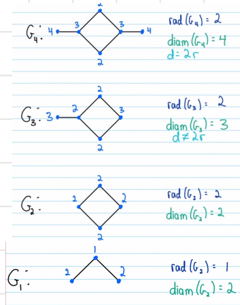
$g \to a \to f \to g$

$= 3$

center of graph

set of central point

$C(G) = (a, d, e, b, f)$



* if G is disconnected then $\overline{G}$ is connected and $\operatorname{diam}(\overline{G}) \leq 2$

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notes